

AD-A214 781

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503</small>				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE October 1980	3. REPORT TYPE AND DATES COVERED Final		
4. TITLE AND SUBTITLE PROGRESS IN PREDICTING THE DEVELOPMENT AND TRANSITION ONSET IN FREE SHEAR LAYERS		5. FUNDING NUMBERS 61102F 2307/A2		
6. AUTHOR(S) Anthony Demetriades		8. PERFORMING ORGANIZATION REPORT NUMBER 89-1451		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Montana State University Bozeman, MT 59717		10. SPONSORING/MONITORING AGENCY REPORT NUMBER F49620-79-C-0210		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR BLDG 410 BAFB DC 20332-6448		11. SUPPLEMENTARY NOTES		
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release distribution unlimited.		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) In the present period the problem of transition in the arbitrary two-dimensional free shear layer was resolved within the context of the predictive approach based on the dissipation criterion. A formula for the transition Reynolds number was derived showing explicitly its increase with speed ratio, fast-side Mach number and total temperature ratio for the two streams. Comparison with data shows that qualitative and approximate quantitative predictions with this formula can be made. The theory for the arbitrarily asymmetric wake was also worked out, both for application to the non-equilibrium shear layer transition problem and for practical use in fluid-laser geometries. In the limit of the symmetric wake close to the trailing edge, the results are in excellent agreement with Goldstein's earlier theory. Application of the transition theory to boundary layers with surface roughness and pressure gradients was deferred since data used as starting points for the theory are yet unavailable.				
14. SUBJECT TERMS		15. NUMBER OF PAGES 23		
		16. PRICE CODE		
17. SECURITY CLASSIFICATION OF REPORT unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT	

FINAL REPORT
Progress In Predicting The Development
And Transition Onset In Free Shear
Layers

Prepared By
Anthony Demetriades

for

U.S. Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, DC 20332
under Contract F49620-79-C-0210

Supersonic Wind Tunnel Laboratory
Mechanical Engineering Department
Montana State University
Bozeman, MT 59717

October 1980

TABLE OF CONTENTS

1. Technical Summary	1
2. Transition Predictions for the Shear Layer	3
3. The Development of the Non-Equilibrium Shear Layer	5
4. Influence of Pressure Gradient and Wall Roughness on Transition	8
5. Publication of the Staff	11
6. Professional Personnel Active on the Research Efforts	11
7. Interactions and Coupling Activities	11
8. References	14
9. Figures	16

APPROVED	
DATE	
BY	
FOR	
REMARKS	

A-1



1. . Technical Summary

The general objective of this program in the 1979-80 period was to extend the application of the transition theory advanced by the Principal Investigator primarily to the case of the free shear layer, and secondarily to the case of boundary layers with surface roughness and pressure gradients.

Section 2 of this report outlines the successful application of the theory to the case of the free shear layer. The weight placed on this facet of the theory derives from the explicit interest within certain directorates, such as the Air Force Weapons Laboratory, in improving the design of high-energy fluid lasers. One of the many basic fluid-mechanical problems inherent in these devices is the prediction of laminar-turbulent transition along the shear layer separating two co-flowing streams; initially laminar flow in this layer is expected due to the small size of the working-fluid and additive-injectant nozzles. In this period the theory addressed homogeneous (same chemistry) mixing across a two-dimensional laminar shear layer of arbitrary "jump" conditions. A predictive formula was derived and was satisfactorily tested against the available data.

The development in Section 2 refers to a "momentumless" shear layer, i.e. one which has no wake component created by the shed boundary layers. This is somewhat idealistic, because the wake component is computed to be significant in laser cavities of finite length. In anticipation of fitting the transition theory to the true "non-equilibrium" shear layer, the problem of the arbitrary asymmetric two-dimensional laminar wake was solved and is discussed briefly in Section 3.

Together with its application to the wake, the transition approach adopted here brings to two the general classes of flows in which it has produced useable results. Its application to boundary layers has met with varying reactions from the community in view of the controversy surrounding the comparison data, in which the wind-tunnel stream turbulence apparently plays a yet unresolved role. Especially when pressure gradients and surface roughness are present, progress is made difficult. As Section 4 points out, the necessary data used as inputs to the data do not exist. At this point no more progress has been or can be made with the theory until such data begin appearing in the open literature.

Since the transition theory for boundary layer applications has therefore been carried as far as it can safely go, it appears that the most valuable contribution which can be made by this laboratory is to return to the experimental stability studies. The statement of work for the 1980-81 period therefore visualizes the performance of experiments in the Supersonic (Mach 3) Wind Tunnel of this laboratory, dealing with supersonic boundary-layer stability.

2. Transition Predictions for the Free Shear Layer

The work statement for the present contract required concentration on the transition characteristics of the free shear layer in two-dimensional flows. By way of background, the Principal Investigator had conducted experiments at AEDC/3 in the 1978-79 period, under AFOSR cognizance, designed to measure transition in such a layer. A strong motivation for performing the present work was to extend the theory of Reference 1 and 2 in order to interpret these transition measurements.

If one begins with the customary definition of the turbulence Reynolds number

$$Re_{\Lambda} \equiv \frac{u' \Lambda}{\nu} \quad (1)$$

it is possible to derive the variation of this quantity in the free shear layer (FSL) by using appropriate, although approximate, expressions for the velocity fluctuations u' , scale length Λ and kinematic viscosity variation. In the present instance, the scale variation was the least known and was therefore assumed to be a constant fraction of the FSL width, as was done in Reference 1. The latter width was in turn obtained from the work of Ortwerth and Shine (Reference 3) as

$$\delta_{n_t} = \frac{\pi \lambda^2}{\epsilon_0} g(M_1) \quad (2)$$

where ξ is the distance from the FSL origin ϵ_0 is the incompressible spreading parameter, M_1 the Mach number in the "fast" side of the layer and λ the customary "speed ratio"

$$\lambda \equiv \frac{u_1 - u_2}{u_1 + u_2} \quad (3)$$

The fluctuation intensity u' was obtained from essentially empirical correlations exemplified by that of Reference 3:

$$u' = 0.16 \Gamma(M_1) \frac{2\lambda}{\lambda+1} u_1 \quad (4)$$

where $\Gamma(M_1)$ is another function of the fast-side Mach number. Finally the kinematic viscosity was found by combining the so-called Crocco relation for the FSL, with the velocity converted to λ :

$$\frac{T_{DSL}}{T_1} = \frac{1}{2} \left(1 + \frac{\gamma-1}{2} M_1^2 \right) \left(1 + \frac{T_{02}}{T_{01}} \right) - \frac{(\gamma-1)M_1^2}{2(1+\lambda)^2} \quad (5)$$

By utilizing the relations mentioned above, an expression was derived for the transition Reynolds number based on "fast-side" properties and the distance from the virtual origin of turbulence. To convert this to a Reynolds number based on the actual origin of the pre-transitional (laminar) flow the recent work of Moeny (Reference 4) was utilized, which provides expressions for arbitrary laminar FSL growth. In the end, the transition Reynolds number based on the fast-side properties of the arbitrary FSL is

$$Re_{x_T} = \frac{C''}{\rho^2 G^2} \left(\frac{T_{DSL}}{T_1} \right)^{2(k+1)} \left(\frac{\lambda+1}{\lambda} \right)^2 \quad (6)$$

where G is a function of λ , M_1 and T_{02}/T_{01} and k the exponent of the temperature-viscosity relation. This is a perfectly general formula on predicting the transition distance from the origin of an initially laminar shear layer. Its implications are that the transition zone moves downstream as λ decreases, i.e. as the velocities of the two streams approach each other; it also moves downstream as the "fast" side Mach number increases. A similar downstream movement occurs when the total temperature of the fast stream exceeds that of the slow stream. Of these results the former two were intuitively expected but had been never before expressed quantitatively.

Figures 1 and 2 show the theoretical results compared with the rather sparse FSL transition data available at present, which are taken from References 5, 6 and 7. Considering the relative unsophisticated approach adopted in the present theory, the comparison is rather flattering to the theory. More important, the theory specifies the parameters which are significant to the FSL transition process, in itself an element of progress over the previous state of the art.

3: The Development of The Non-Equilibrium Shear Layer

A large body of theoretical literature exists on the shear layer problem, concentrating mostly on the "far" or "equilibrium" shear layer developing far from the flow origin. Typical of the classical solutions is the work of Mills (Reference 8) which considers arbitrary jumps across the layer and gives solutions for both the laminar and the turbulent cases. By contrast there has been little work done on the "non-equilibrium" problem, that is the shear layer region just downstream of the trailing edge (T. E.) of the splitter plate (or "partition") separating, initially, the two dissimilar streams. A solution to this problem is offered here.

Since the partition sheds its two boundary layers into the stream, the non-equilibrium flow problem in its general form requires prediction of the development of a composite profile downstream of the T. E. There is a "shear layer component" generated by the uniform but arbitrary streams on either side of the partition; and a "wake component" consisting of the merging boundary layers. These two layers are generally taken to be different in momentum thickness and thus the wake component, which will eventually decay to zero, is asymmetric in the initial conditions, while the shear-layer component is asymmetric in the boundary conditions. The problem solved here is that of the wake component, and corresponds to the arbitrary two-dimensional compressible laminar wake generated by unequal (asymmetric) boundary layer thickness $\theta_1 \neq \theta_2$.

A useful landmark for comparison was the classical solution for the incompressible, symmetric flow ($\theta_1 = \theta_2$) past the trailing edge produced many years ago by Goldstein (Reference 9). Using the Oseen approximation Kubota (Reference 10) extended Goldstein's results and derived the asymptotic ("linear") solution of large distances from the T. E. for the arbitrary (compressible) wake; Gold (Reference 11) then showed, using the same method, that the same linear solution obtains for any arbitrary initial profile.

The present solution was obtained by prescribing the velocity profiles at the T. E. and then using the Gold solution to see how the wake develops, for arbitrary initial $\theta_1 \neq \theta_2$. Simple exponential functions were chosen to represent the initial Blasius profiles, so that the complete profile is an asymmetric cusp. A simple analytic solution resulted for the velocity field downstream of the T. E. as follows:

$$1 - \frac{u}{u_e} = \frac{1}{2} \left[e^{-xP^2 + yP} (1 - \operatorname{Erf}[\sqrt{x}(P + \frac{y}{2x})]) + e^{-x-y} (1 + \operatorname{Erf}[\sqrt{x}(\frac{y}{2x} - 1)]) \right] \quad (7)$$

where u is the velocity, u_e the edge (infinity) velocity, P the asymmetry ratio θ_1/θ_2 , and

$$x \equiv \frac{x^*}{\theta_1 Re_{\theta_1}}, \quad Re_{\theta_1} \equiv \frac{u_e \theta_1}{\nu_e}$$

$$y \equiv \frac{y^*}{\theta_1} = \frac{1}{\theta_1} \int_0^y \frac{y}{\theta_2} dy \quad ()^* = \text{physical quantity. (8)}$$

This solution meets all the standard validity tests such as the boundary conditions for large y , recovery of the initial profile, momentum balance, etc. The solution indicates that a non-dimensional longitudinal variable more appropriate than x is

$$x' \equiv \frac{x^*}{\Theta Re_{\Theta}}, \quad \Theta \equiv \theta_1 + \theta_2 \quad (9)$$

and at large x' the asymptotic solution of Kubota and others is recovered.

The results provided by eq. (7) are plotted on Figures 3 through 7. The first three of these show the lateral velocity profiles for the symmetric case (Fig. 3) and for an asymmetric-case example ($P = 10$). In both instances it is seen that the initial cusp-like profile becomes smooth immediately after the T. E. The gradual symmetrization of the initially asymmetric profile for $P = 10$ is evident as one proceeds downstream. At large distances the universal asymptotic profile predicted by Kubota is achieved.

It is seen that the profile peak shifts away from the axis after the T. E. and does not return to it until very far downstream. This excursion is not practically significant, but it invites caution when plotting the "defect" or center-line velocity; this defect does not, therefore, represent the minimum velocity point (unless $P = 1$) and is thus called the "pseudodeflect". Plots of this quantity appear on Figure 6, which carries two points representing the minimum-velocity points for the $P = 10$ case. The conclusion drawn from this Figure is that asymmetries, even large ones, affect the profile but not the magnitude of the minimum velocity in the wake. Except for positions extremely close to the T. E., in fact, the solution is only of order 20% - 30% lower than the asymptotic solution which can be considered to be the crudest possible estimate.

The present solution is linearized in the sense that its momentum deficit exactly at the T. E. does not precisely equal . It is all that more surprising, therefore, to note its comparison with Goldstein's theory (which is in some ways also approximate) and with experimental data on Figure 7, which comparison is done for $P = 1$, no data or other theories being available for $P \neq 1$. In fact, the equation (7) gives results which are always within 4% of Goldstein's predictions.

The present results, besides providing the wake-component solution for use in solving the non-equilibrium FSL problem, are directly applicable, say, to fluid laser problems involving laminar mixing in multiple nozzles.

4. Influence of Pressure Gradient and Wall Roughness on Transition

The theory formulated by this author to describe transition in the boundary layer is based on a minimum possible turbulence Reynolds number

$$Re_{\lambda} \equiv \frac{u' \lambda}{\nu} \quad (10)$$

which is required for the maintenance of turbulence. The calculation procedure requires that the layer is first assumed turbulent throughout and Re_{λ} is calculated both along and across this turbulent flow. In Reference 2 this is done by finding u' , λ and ν as functions of the geometric coordinates and flow parameters, as follows. The velocity fluctuation u' is taken from experimental data (e.g. References 16 and 17) to be of the form

$$u' \left(\frac{y}{\delta}, M_e, \frac{T_w}{T_0}, \dots \right) = C_1 \left(\frac{y}{\delta} \right) \left[\frac{T_w}{T_0} \right]^{\frac{1}{2}} \quad (11)$$

while the integral scale λ is obtained from the turbulent thickness δ_t :

$$\lambda = C_2 \delta_t \quad (12)$$

and the kinematic viscosity ν from

$$\nu = \nu_0 (p_e / p)^{n+1/2} \quad (13)$$

where the exponent n arises in the usual viscosity-temperature relation. Insertion of the latter three relations into (10) then results in

$$\frac{Re_{\lambda}}{Re_{\lambda}^{1/2}} = 0.0767 C_1 \left(\frac{y}{\delta} \right) H(M_e, Re_{\theta}, \dots) \left[F(M_e) \right]^{7/12} \left[\frac{p}{p_0} \left(\frac{y}{\delta} \right) \right]^{n+1/2} \quad (14)$$

This formula allows one to compute Re_{λ} as a function of the distance y/δ ; its maximum value is picked and the transition momentum Reynolds number $Re_{\lambda T}$ is computed for that particular set of flow parameters M_e , T_w/T_0 , etc. In this manner results were obtained for zero-pressure-gradient boundary layers at Mach numbers from 0 to 10 and temperature ratios T_w/T_0 from 0.1 to 1. The results showed many of the familiar trends of experimental observations and even provided an explanation of the well-known "reversal" phenomenon of transition.

1.1 Study of the Transition Criterion with Roughness and Pressure Gradient

A substantial amount of time was devoted in the present contract period in attempting to extend the above theory to the case of a boundary-layer with pressure gradient and roughened surfaces. This effort was largely unsuccessful for the following reasons.

As can be seen from equation (11) through (13), the present theory requires certain inputs for computing the turbulence Reynolds number Re_Λ . At first glance from eq. (10) these inputs are the integral scale Λ , the wideband fluctuation intensity u' and the kinematic viscosity profile $\nu(y/\delta)$. By the time the expression for the transition (momentum) Reynolds number (14) is written down additional requirements have developed e.g. for the form factor $H = \delta^*/\theta$, the wall friction or friction coefficient and the correct form of the turbulent boundary layer velocity profile. Such information is available, often with accuracy, only for the "simpler" boundary layers; indeed, the earlier computations were carried out and numerical results obtained for the flat, smooth wall; when such simplicity is removed, data on the quantities mentioned above become very scarce.

As an example, a thorough search was made for the necessary information in boundary layers with favorable pressure gradients (FPG). It is known that the wall friction decreases for such gradients, so that the right-hand-side of eq. (14) decreases because it is proportioned to some positive power of c_f . Such an increase predicts, on the basis of eq. (14), an increase of the transition Reynolds number, which is also experimentally observed. Superficially, therefore, the present theory produces the right trend, which originally steered the work toward verifying this trend quantitatively. The right-hand-side of (14), as it was discovered however, is affected by the FPG indirectly via the effect of the latter on Λ and u' . If, for example, the FPG "stretched" the longitudinal scale Λ then its benign effect on c_f could be cancelled out. No data on the FPG effect on scale lengths could be found, or data of the FPG effect on the fluctuations. No fluctuation data were reported during the 1978 Stanford Conference (Reference 18) or during the 1980 Stanford Conference on Complex Turbulent Flows (Reference 19).

Nor were the Δ and u' the only ones absent. There is very little available on the friction variation, too. The 1977 AGARD Compendium (Reference 21) lists 16 potential contributions to the knowledge of the FPG effect on c_f . Of these, eleven do not report the value of the pressure gradient parameter, one has large normal pressure gradients and the remainder are suspected to represent transitional flows!

A similar situation was found to prevail for the effect of roughness on transition. Again, since c_f (or τ_w) increases with roughness, one finds from eq. (14) that transition moves forward as the roughness height increases, for example; a remarkably simple means of finding the roughness effect on transition quantitatively thus appears at hand. However, there is nothing known on the effect of roughness on Δ , so that final conclusions are impossible.

To summarize, no progress toward applying eq. (14) to a roughened wall or one with pressure gradients could be made because the inputs necessary to that equation are unavailable. To the extent which a list of required experiments to fill the gaps was produced, the study had positive aspects. This list, intended to fulfill needs for a more general understanding of turbulent boundary layers, is as follows:

1) The general requirement is for the production of experiments with compressible boundary layers under conditions of (a) surface curvatures, (b) favorable and adverse pressure gradient, (c) surface roughness. Mean (average) flow measurements must include surface friction coefficients, the Crocco relationship (i.e. the temperature velocity relation) and the velocity profile, specifically the exponent value in the relation

$$\frac{u}{u_e} = \left(\frac{y}{\delta}\right)^{1/5} \quad (15)$$

or alternately, the value of the free parameters in the Clauser-Coles description of the boundary layer. The form factor H/δ is also required.

2) Local wideband r.m.s. fluctuation intensities of the velocity fluctuations and the longitudinal scales must be also measured. It is important to obtain these properties as profiles across the layer, rather than as single values at a point.

3) The standards set in the 1968 and 1980 Stanford Conferences are important to maintain in performing and documenting these experiments.

Publications of the Staff

The following documents describe work performed, in their entirety or in part, under this contract:

- 1) Demetriades, A.: "Consequences of a Necessary Threshold for Boundary-Layer Transition", submitted to the AIAA Journal, October 1979.
- 2) Demetriades, A.: "Necessary Conditions for Transition in a Free Shear Layer", AFOSR TR 80-0442 (MSU/SWT Report No. 80-1), Montana State University, Bozeman, MT, February 1980.
- 3) Demetriades, A., Ortwerth, P. J. and Moeny, W. M.: "Growth and Transition Behavior of Arbitrary Laminar Free Shear Layers", AIAA Paper 80-1402, Snowmass, CO, July 1980; submitted for publication to the AIAA Journal, 1981.
- 4) Chevallier, T.: "Development of an Impact Pressure Probe for Flow Vector Measurements", M.S. Thesis. Department of Mechanical Engineering, Montana State University, Bozeman, MT 59717, December 1980.

Professional Personnel Active on the Research Effort

The majority of the work reported here was performed by Dr. Anthony Demetriades, Professor of Mechanical Engineering at Montana State University and Director, Supersonic Wind-Tunnel Laboratory.

The Principal Investigator was assisted by Travis Chevallier, graduate student. Mr. Chevallier is scheduled to receive his M. S. degree from the University in December 1980.

Interactions and Coupling Activities

a) Meetings and Exchanges

The Principal Investigator delivered a paper entitled "Growth and Transition Behavior of Arbitrary Laminar Free Shear Layers", in the 13th Fluid and Plasma Dynamics Conference of the AIAA in Snowmass, CO, July 1980.

At the invitation of its organizers, he also attended the 1980-81 AFOSR-HTIM-Stanford Conference on Complex Turbulent Flows held at Stanford University in September 1980.

The Principal Investigator has also been invited by the Director of Technology, USAF/AEDC, to attend an organizational meeting at AEDC for a working group to recommend hypersonic turbulence research and related topics to that organization. The meeting is scheduled for November 1980.

5) Coupling Activities

The work performed in this period relates closely to and benefits other USAF directorates and government agencies. The work of course actively supports the high-energy-laser program at the Air Force Weapons Laboratory (Dr. P. J. Ortwerth). The Principal Investigator remains in close contact with AFWL, exchanges technical data with them and has steered an appropriate portion of this year's work toward basic aerodynamics problems affecting fluid laser design.

The Principal Investigator remains in touch with the Air Force's BMD group at Norton Air Force Base, California, through a subcontract received by the MSU/SWT Lab from the AVCO Systems Division, BMD's prime contractor on the BMD HTSCT program. The SWT Laboratory task is to measure experimentally boundary layer transition over a surface with large favorable pressure gradient, various degrees of surface cooling and a two-dimensional type of surface roughness. These experiments will be done in the Laboratory's Mach 3 Wind-Tunnel and are naturally closely related to this year's effort to predict theoretically the location of transition under these conditions. Thus, despite the lack of success evident so far in applying the transition theory to such conditions (due to lack of available data, see Section 4) empirical transition data will be at least obtained and the interest of the MSU researchers will remain focused on the problem for some time.

An interesting example of the possible benefits to USAF/BMD accruing from the AFOSR-funded work on boundary layer stability and transition is the following case history. In the recent past this investigator has been supported by AFOSR to study the pre-transitional instability of high-speed boundary layers. These studies led to the discovery of very intense oscillations of the hypersonic boundary layer flow near the location of transition to turbulence (References 12, 13 and 14). Recently this

investigator was called in as a consultant to help evaluate a sub-surface (stethoscopic) acoustic transducer, developed by BMO to resolve transition-related problems in hypervelocity vehicle design (Reference 15). While studying taped signals of the transducer output, he found the same oscillations to correlate well with the occurrence of transition near the sensor, and enabled him to qualify the sensor as a good transition detector. The transducer should be very useful to BMO during ground or flight tests, and its utility was greatly abetted by the earlier, basic research done under AFOSR auspices.

In July 1980, representatives from BMO and their MTSCCT contractor visited the Supersonic Wind-Tunnel Laboratory and were briefed on the on-going AFOSR program and related programs, as well as the forthcoming rough-wall boundary-layer stability studies funded by AFOSR at MSU during the 1980-81 period.

The Supersonic Wind-Tunnel Laboratory at MSU has been also working on a program for the Army's Ballistic Missile Defense directorate at Huntsville, AL (Mr. J. Papadopoulos) to measure and correlate hypersonic and supersonic boundary layer spectra for applications to ICBM interceptor design. One important aim of this work is the measurement and correlation of the turbulence intensity u' and integral scale Λ in turbulent boundary layers over a wide range of M_e and T_w/T_o . This affords a good opportunity to cross-check the inputs of u' and Λ into the theory of the OSR-funded effort (see Section 4, eq's. 11 and 12). In fact it was found that the values already chosen for the transition theory ($\Lambda \approx 2.0\delta$ and u' as given by eq. (11)) were quite accurate.

References

1. Demetriades A.: "Transition to Turbulence in Two-Dimensional Wakes", AIAA Journal, Vol. 16, No. 6, pp. 587-591, June 1978.
2. Demetriades, A.: "Stability, Transition and Turbulence in Mixing Layers and Adjacent Streams", Aeronutronic Report No. U-6488, Newport Beach, CA, November 1978.
3. Ortwerth, P. J. and Shine, A. J.: "On the Scaling of Plane Turbulent Shear Layers", Laser Digest AFWL-TR-77-118, Spring, 1977, p. 115.
4. Ortwerth, P. J., Moeny, W. M. and Happ, H. J.: "A Momentum Integral Model of the Laminar Non-Reactive Shear Layer", TETRA ITR 80-008 Tetra Corp., Albuquerque, NM, April 1980.
5. Demetriades, A., Ortwerth, P. J. and Moeny, W. M.: "Growth and Transition Behavior of Arbitrary Laminar Free Shear Layers", AIAA Paper 80-1402, Snowmass, CO, July 1980.
6. Birch, S. F. and Keyes, J. W.: "Transition in Compressible Free Shear Layers", J.S.R. Vol. 19, No. 8, p. 623, August 1972.
7. Crawford, D. H.: "Investigation of the Flow over a Spiked-Nose Hemisphere-Cylinder at a Mach Number of 6.8", NASA TN D-118, Washington, D.C., 1959.
8. Mills, R. D.: "Numerical and Experimental Investigations of the Shear Layer Between Two Parallel Streams", JFM Vol. 33, Part 3, 1968, p. 591.
9. Goldstein, S.: "On the Two-Dimensional Steady Flow of a Viscous Fluid Behind a Solid Body," Part I: Proc. of the Royal Soc. of London, series A, Vol. 142, 1933, p. 545; Part II, ibid. p. 563
10. Kubota, T.: "Laminar Wakes With Stream-wise Pressure Gradient", GALCIT Hypersonic Project IM-9, CIT, Pasadena, CA, May 1962.
11. Gold, H.: "Laminar Wake With Arbitrary Initial Profiles", AIAA J., Vol 2, No. 5, pp. 948, May 1964.
12. Demetriades, A.: "Laminar Boundary Layer Stability Measurements at Mach 7 Including Wall Temperature Effects", Aeronutronic Report No. U-6381, Newport Beach, CA, November 1977.
13. Demetriades, A.: "Boundary Layer Stability Observations at Mach Number 7", J. of App. Mech., ASME, Vol. 99, No. 1, pp. 7-10, March 1977.

14. Demetriades, A.: "New Experiments on Hypersonic Boundary Layer Stability Including Wall Temperature Effects", 1978 Heat Transfer and Fluid Mechanics Institute Proceedings, Stanford University Press, 1978, pp. 39-55.
15. Peckham, V. and Winnicki, R.: "Acoustic Transition Monitor Program Report, Volume I: BLAM Characterization and Development", SAMSO TR 79-8, Los Angeles, CA, 1979.
16. Laderman, A. J. and Demetriades, A.: "Turbulent Shear Stresses in Compressible Boundary Layers", AIAA J., Vol. 17, No. 7, pp. 736-744, July 1979.
17. Demetriades, A. and Laderman, A. J.: "Wake Fluid Measurements Program", Aeronutronics Report U-6207, Newport Beach, CA, October 1975.
18. Coles, D. et al: Proceedings of the AFOSR-IFP-Stanford Conference on Computation of Turbulent Boundary Layers, Vol. I and II, Stanford University, 1969.
19. Kline, S. et al: The 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows, Stanford, University, 1980.
20. Fernholz, H. H. and Finley, P. J.: "A Critical Compilation of Compressible Turbulent Boundary Layer Data", AGARDograph No. 223, Neuilly Sur Seine, France, June 1977.

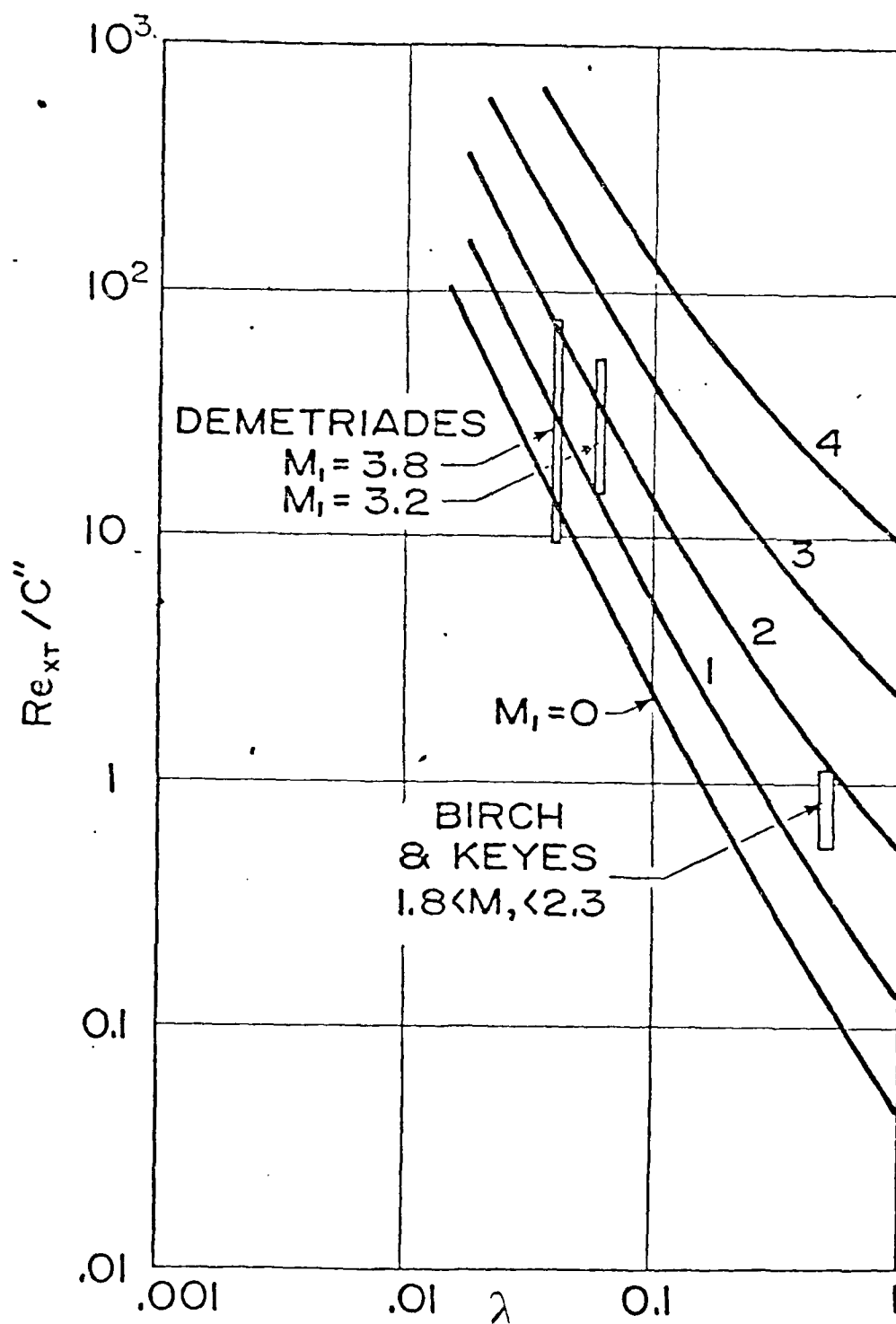


Figure 1. The transition predictions for the arbitrary two-dimensional adiabatic shear layer in air, compared with experimental data.

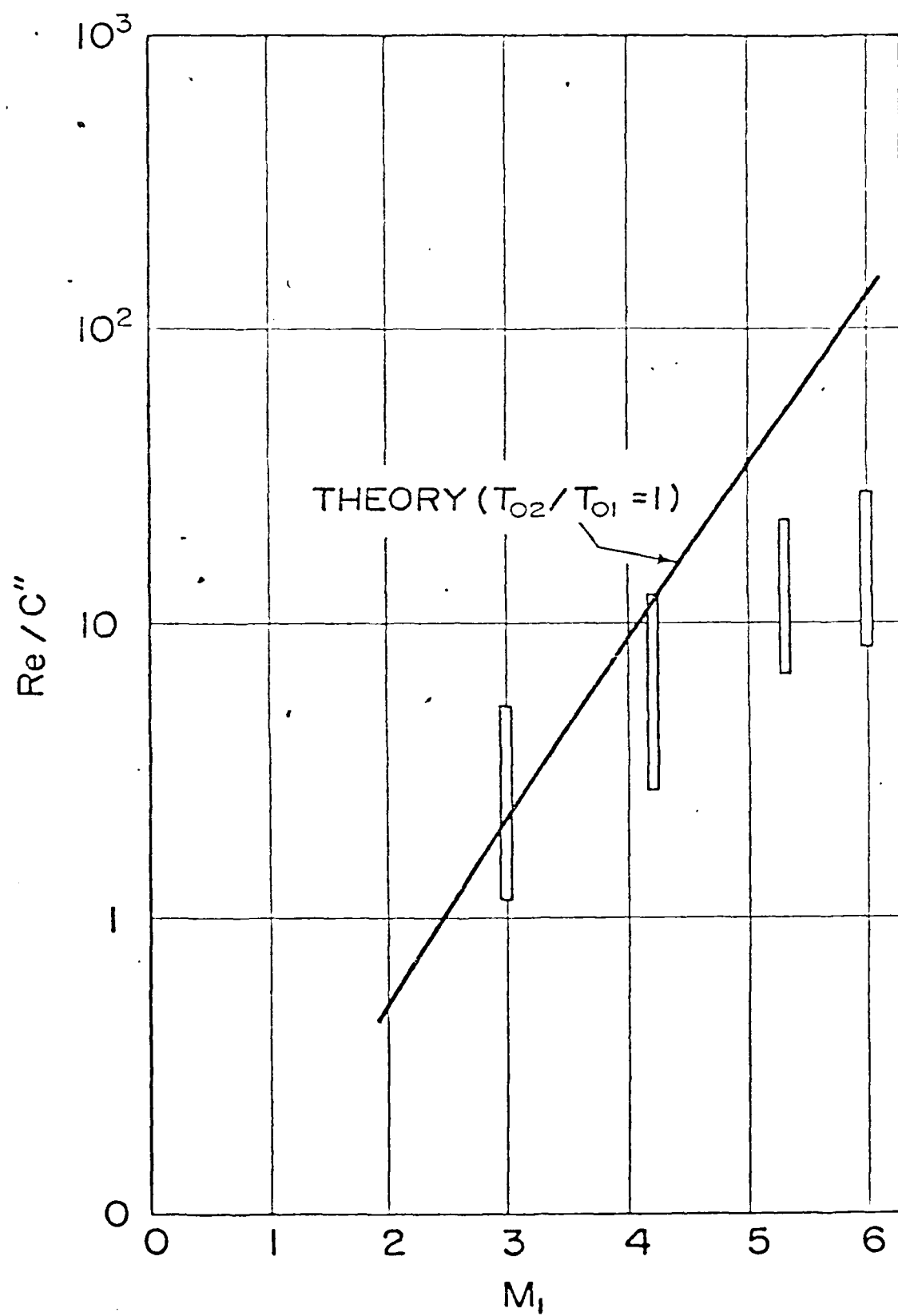
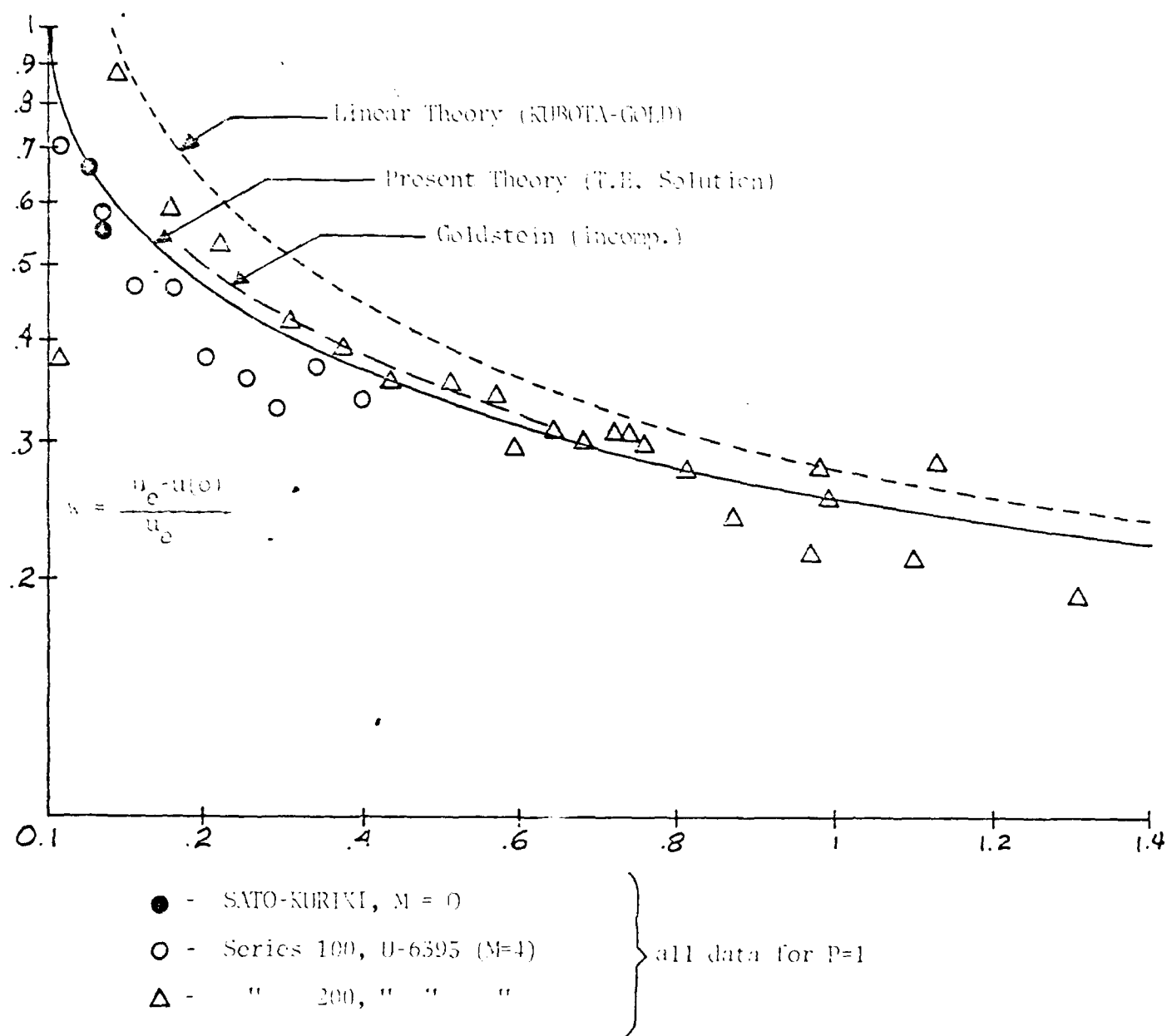


Figure 2. The transition prediction for the shear layer compared with the data of Crawford (Reference 7).

Figure 1. Comparison of the present solution with Goldstein's theory and with experimental data.



$$X^* = \frac{X^*}{Re}$$

Figure 6. The pseudodeflect w computed for the symmetric ($P=1$) and completely asymmetric ($P=0$) cases and for one intermediate case ($P=0.1, 10$). The typical true defect (minimum velocity) for the latter case is exemplified by two points. The "delayed" solution is an alternate calculation not discussed here.

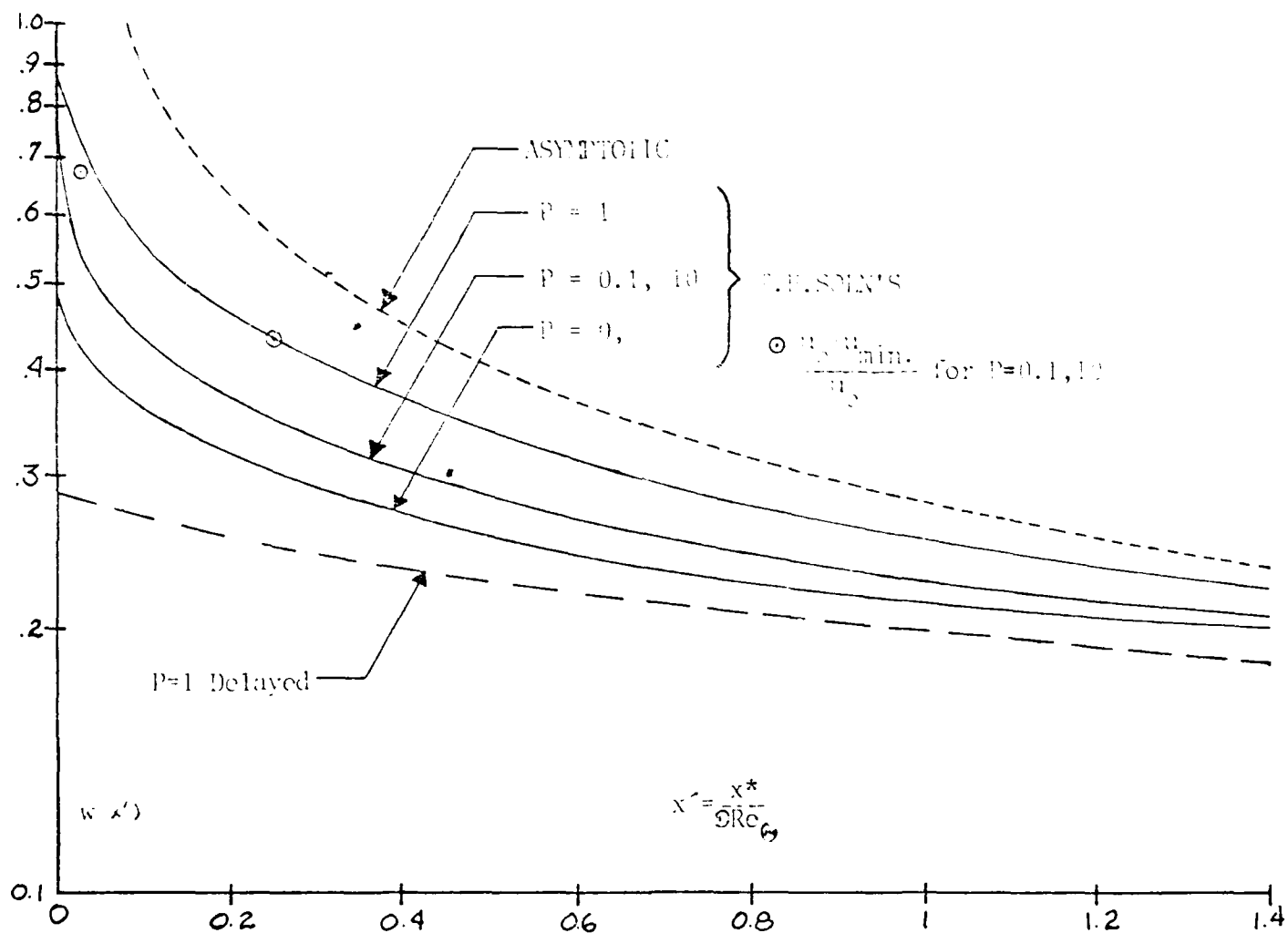


Figure 5: The asymmetric wake-component solution ($p=10$) far from the trailing edge.

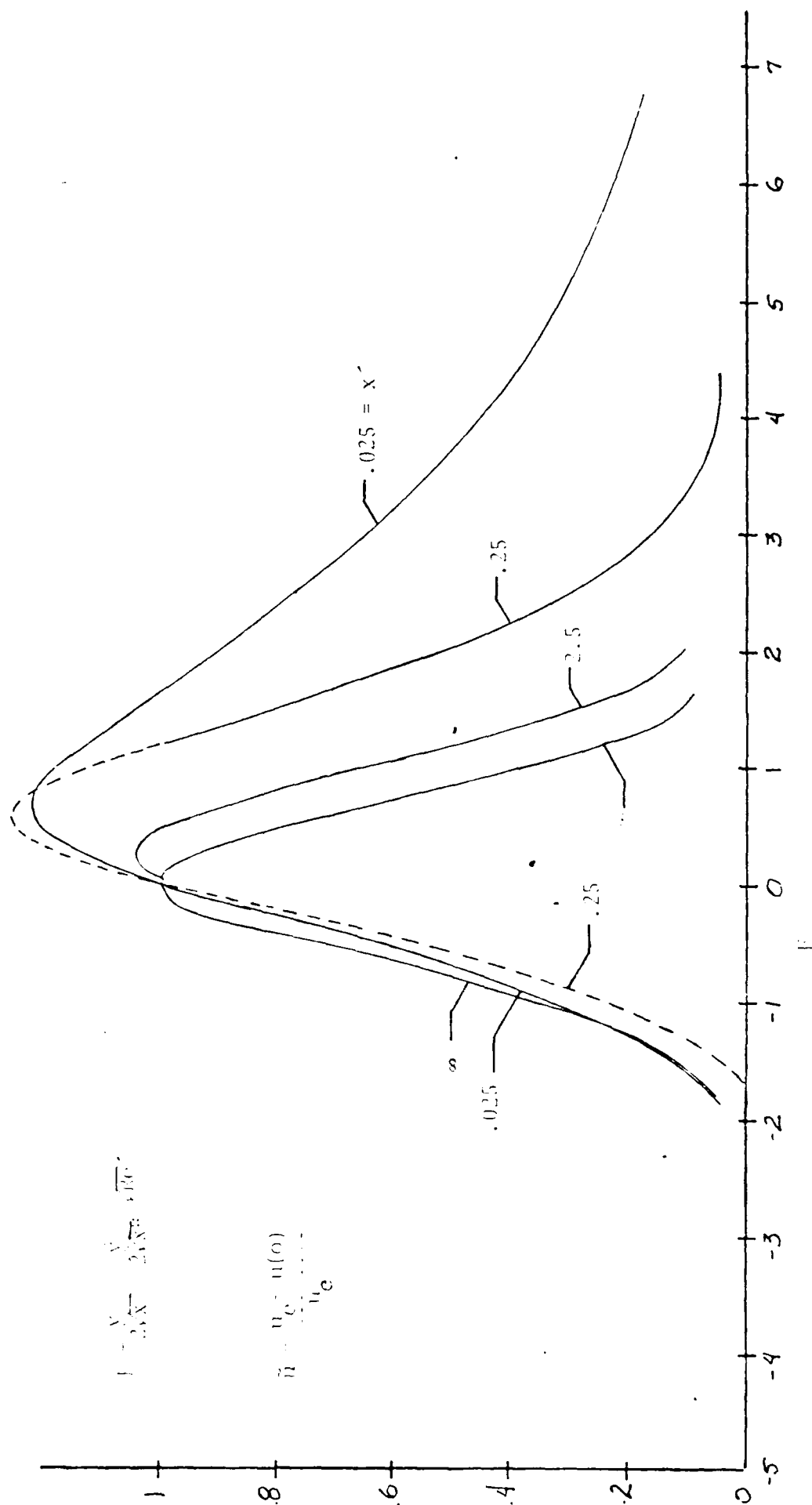


Figure 4: Evolution of the wake-component solution: the $P=10$ (asymmetric) case near the trailing edge.

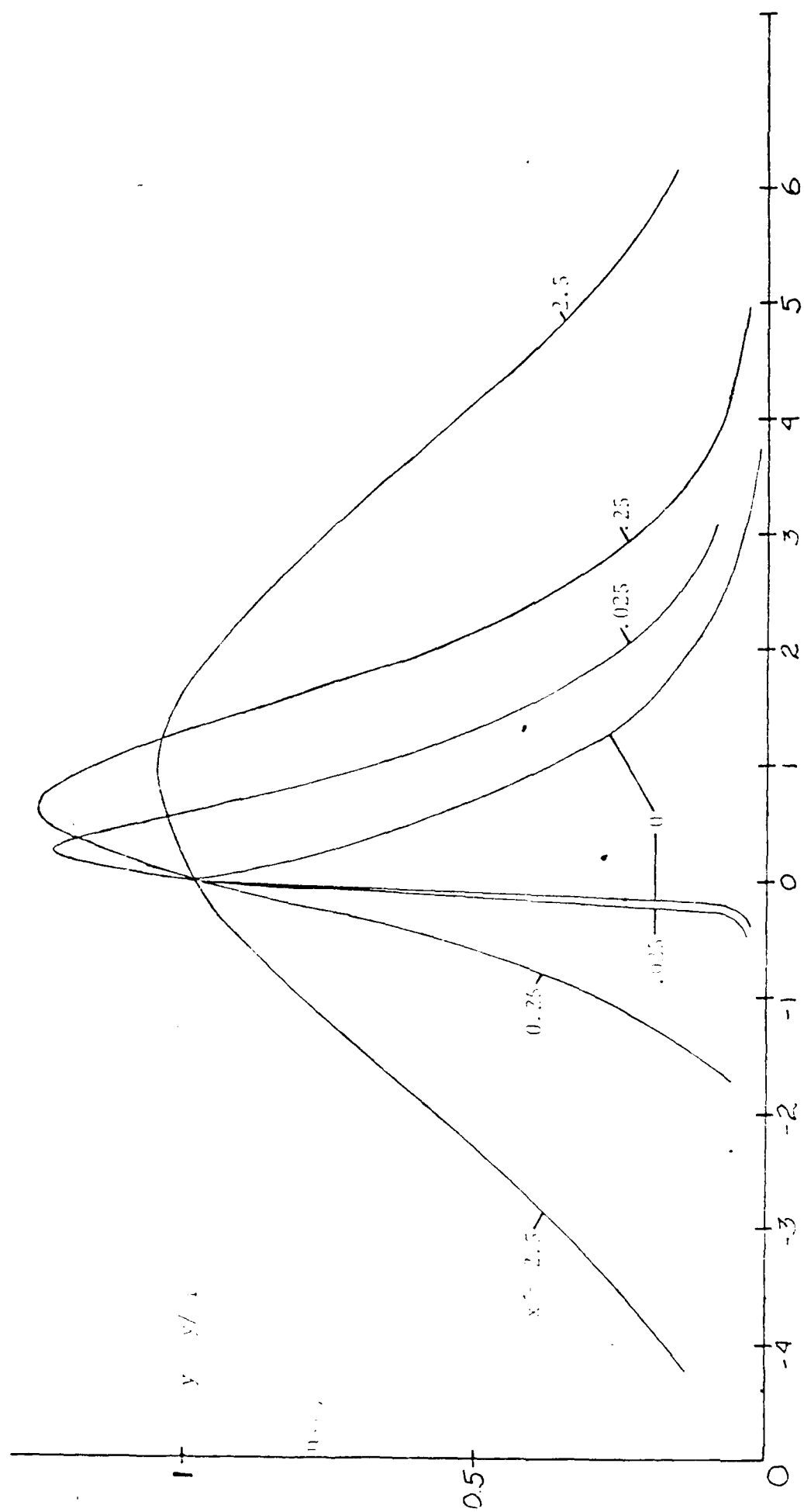
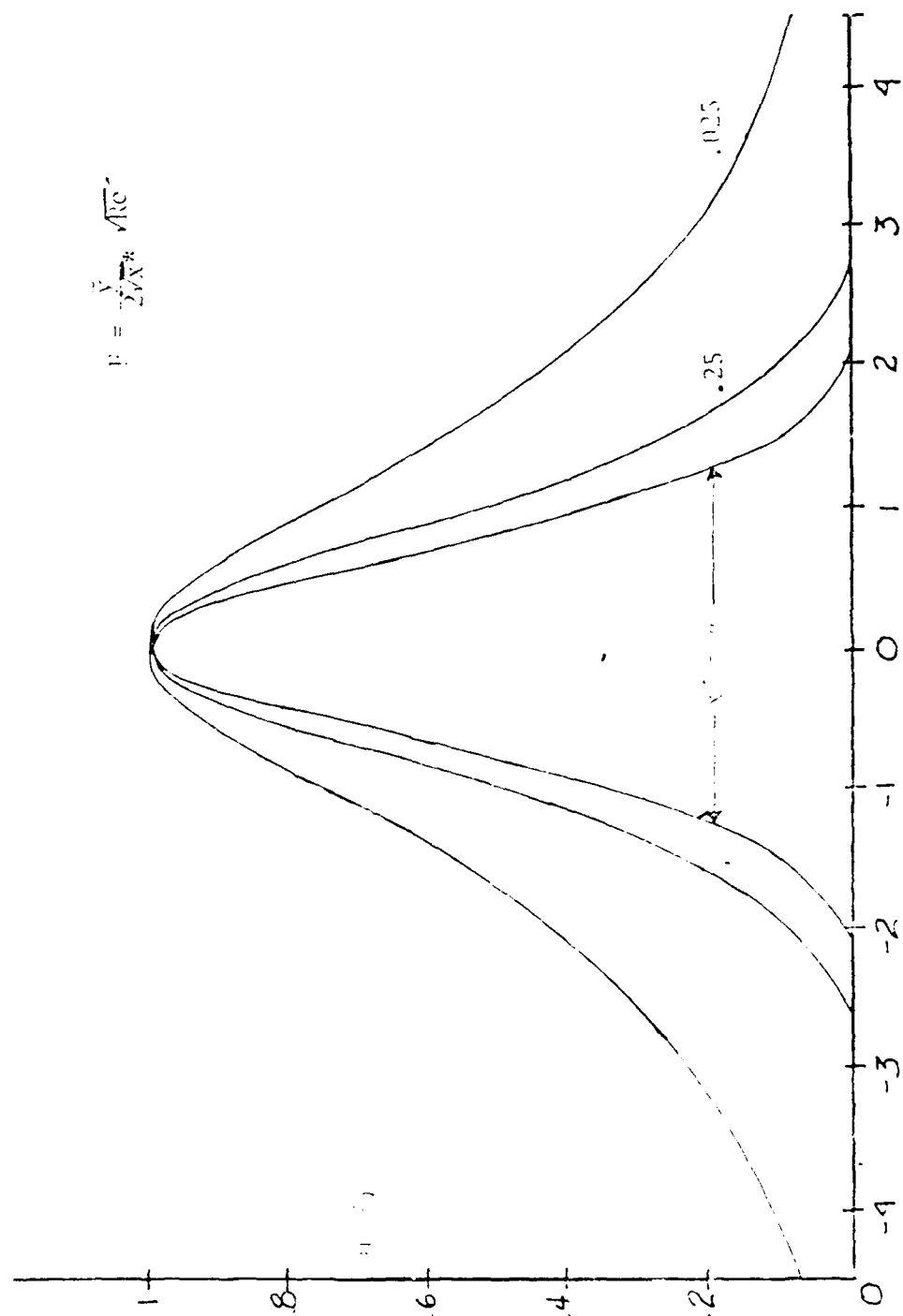


Figure 10. The symmetric case ($\beta = 1$) far from the trailing edge.



$$F = \frac{V}{2\sqrt{\lambda}} \sqrt{RC}$$

Figure 10. Evolution of the wake-component solution: the symmetric case ($P=1$) near the trailing edge.

