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RESEARCH PROJECT

FINAL REPORT

ONR Contract No. N00014-88-K-0047/P00001

Phase Space and Path Integral Methods  
Applied to Direct and Inverse  
Ocean Seismo-Acoustic Modeling

CSM Proposal No.: 2684

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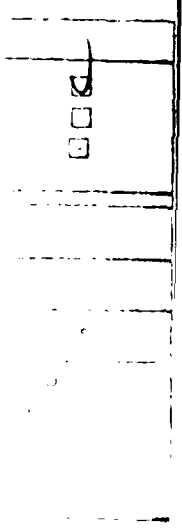
**TITLE:** Phase Space and Path Integral Methods Applied to Direct and Inverse Ocean Seismo-Acoustic Modeling

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**OBJECTIVE:** The long-range objective is to develop and apply "microscopic" phase space methods and global path integral constructions to gain a deeper theoretical and computational understanding of electromagnetic, seismic, and acoustic direct and inverse wave propagation problems. Specifically, this project focuses on the development of new, multidimensional algorithms for direct (forward) acoustic propagation and generalized acoustic tomography at the level of the reduced scalar Helmholtz equation.

**BACKGROUND:** The analysis, understanding, and fast, accurate numerical computation of the wave equation are quite difficult for rapidly changing, multidimensional environments extending over many wavelengths. This is particularly so for environments characterized by a refractive index field with a compact region of arbitrary variability superimposed upon a transversely inhomogeneous background profile. For both the forward and inverse problems, the entire domain is in the scattering regime. There is, generally, no accessible asymptotically homogeneous regime where simplified wave fields can be constructed or appropriate data collected in these model ocean seismo-acoustic environments.

**APPROACH:** The introduction and widespread application of the parabolic (paraxial) approximation marked a significant advance in wave propagation modeling for ocean seismo-acoustic and other strong channeling environments. In many respects, this is a most natural motivation for the application of phase space factorization and path integral methods to the Helmholtz equation. The parabolic is, after all, an approximation to the full square root (pseudo-differential) operator, while the associated split-step FFT computational algorithm follows from a direct integration of the phase space Feynman path integral. It is then quite natural, in attempting to extend the ordinary parabolic approximation beyond its limited region of validity, to examine more closely the full square root operator and the more general path integral representation associated with its corresponding propagator (fundamental solution). For the one-way theory, both the square root Helmholtz operator construction and the phase space path integral representation and marching algorithm depend crucially upon a detailed analysis of the operator symbol. The construction of exact, numerical, and uniform perturbation solutions to the Weyl composition equation for the operator symbol and their application to the development of the marching algorithm are considered. For the two-way theory, the evaluation of the Feynman/Garrod approximate path integral and the construction of a rigorous path integral for the homogeneous limit are considered. It is in addressing the full two-way problem and attempting to construct true path functionals (sum over paths) that a more physically



transparent underlying structure is introduced (even at the level of the one-way problem). For the inverse analysis, the generalized Fourier integral operator structure of the propagators and the semigroup property of the one-way equation are considered to exactly reconstruct the refractive index profile for transversely inhomogeneous environments. This is intended to extend current ocean tomographic analysis in both frequency domain and inhomogeneity strength.

**RESULTS:** During this project period, a number of exact solutions to the Weyl composition equation were constructed and examined. These include multidimensional coupled and uncoupled quadratic cases as well as several non-quadratic cases which provide appropriate models for ocean seismo-acoustic environments. Additionally, for every exact solution case, the symbols exactly corresponding to operator rational approximations were also constructed and studied. This connects with the work of Lee, St. Mary, Greene, etc. The leading-order uniform high-frequency approximate symbol was also further developed. Two numerical algorithms for the construction of the Weyl symbol are under development and testing. One, based on the spectral representation, is currently operational. The other, based on directly solving the Weyl composition equation in a basis set, soon will be operational. Finally, the steps for the exact refractive index profile reconstruction for transversely inhomogeneous environments were completely outlined based on phase space and path integral concepts. A particular high-frequency approximation, which can be applied in the exact procedure, and is appropriate for the environments under consideration, was examined for the exact symbol construction cases.

The following papers acknowledging ONR support appeared (or will soon appear) during this contract period.

- L. Fishman, "Exact and Operator Rational Approximate Solutions of the Helmholtz, Weyl Composition Equation in Underwater Acoustics — the Quadratic Profile," *Wave Motion*, submitted for publication (1989).
- L. Fishman, "Phase Space and Functional Integral Methods in Direct and Inverse Scattering," on **Problèmes Inverses, Proceedings of R.C.P. 264**, edited by P. Sabatier (to appear in 1990).
- L. Fishman, "Exact Solutions of the Helmholtz, Weyl Composition Equation in Underwater Acoustics — the Quadratic Profile," in **Mathematical and Computer Modeling in Science and Technology (The Seventh International Conference)** edited by X.J.R. Avula (Pergamon Press, Oxford, 1990).
- L. Fishman, "Exact and Uniform Perturbation Solutions of the Helmholtz, Weyl Composition Equation in Underwater Acoustics — the Quadratic Profile," in **Computational Acoustics II**, edited by D. Lee, A. Cakmak, and R. Vichnevetsky (Elsevier, Amsterdam, 1990).

- L. Fishman, "Symbol Analysis and the Construction of One-Way Direct Wave Propagation Theories in Ocean Seismo-Acoustics," in **Proceedings of the Fifth International Symposium on Numerical Methods in Engineering, Vol. 2**, edited by R. Gruber, J. Periaux, and R.P. Shaw (Computational Mechanics Publications/Springer-Verlag, Southampton/New York, 1989), p. 49.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Direct and Inverse Wave Propagation Theories in Ocean Seismo-Acoustics," in **13th International Congress on Acoustics, Vol. 4**, edited by P. Pravica (Sava Centar, Belgrade, 1989), p. 399.
- L. Fishman, "Numerical Solutions of the Helmholtz, Weyl Composition Equation in Ocean Seismo-Acoustics," in **13th International Congress on Acoustics, Vol. 5**, edited by P. Pravica (Sava Centar, Belgrade, 1989), p. 149.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories," in **Proceedings of the 1989 URSI International Symposium on Electromagnetic Theory**, edited by S. Ström (Royal Institute of Technology, Stockholm, 1989), p. 157.
- L. Fishman, "Symbol Analysis and the One-Way Helmholtz Equation," in **IMACS Transactions on Scientific Computing, Paris 1988. Volume 1: Numerical and Applied Mathematics**, edited by W.F. Ames and C. Brezinski (J.C. Baltzer AG, Basel, 1989).
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories in Ocean Seismo-Acoustics," in **Numerical Simulation and Analysis in Guided-Wave Optics and Optoelectronics, 1989 Technical Digest Series Volume 3**, edited by W.J. Tomlinson, R.J. Hawkins, and D. Yevick (OSA, Washington, D.C., 1989), p. 93.
- L. Fishman, "Phase Space Factorization and Functional Integral Methods in Direct and Inverse Scattering — Symbol Analysis," in **Wave Phenomena: Theoretical, Computational, and Practical Aspects**, edited by H. Morris, J. Day, and L. Lam (Springer-Verlag, New York, 1989), p. 76.
- L. Fishman, "Phase Space and Path Integral Methods In Computational Acoustics," in **Proceedings Volume 2: 12th IMACS World Congress on System Simulation and Scientific Computation**, edited by R. Vichnevetsky, P. Borne, and J. Vignes (IMACS, Villeneuve D'Ascq, 1988), p. 244.

Abstracts were accepted or already appeared during this contract period for the following conferences:

URSI Winter Meeting, Boulder, CO (1990).  
Joint AMS-SIAM Symposium, Albuquerque, NM (1990).  
SPIE Conference on Inverse Problems, San Diego, CA (1990).  
13th International Congress on Acoustics, Belgrade Yugoslavia (1989).  
ICA Sea Acoustics Meeting, Dubrovnik, Yugoslavia (1989).  
URSI International Symposium, Stockholm, Sweden (1989).  
International Symposium on Numerical Methods in Engineering, Lausanne, Switzerland (1989).  
Progress in Electromagnetic Research Symposium, Cambridge, MA (1989).  
1989 Annual SIAM Meeting, San Diego, CA (1989).  
ASA Spring meeting, Syracuse, NY (1989).  
International Conference on Mathematical and Computer Modeling, Chicago, IL (1989).  
Problèmes Inverses, R.C.P. 264, Montpellier, France (1989).  
URSI Winter Meeting, Boulder, CO (1989).  
OSA Topical Meeting, Houston, TX (1989).  
IMACS Symposium on Computational Acoustics, Princeton, NJ (1989).  
International Congress on Computational and Applied Mathematics, Leuven, Belgium (1988).  
12th IMACS World Congress, Paris, France (1988).  
1988 Annual SIAM Meeting, Minneapolis, MN (1988).  
The First Woodward Conference on Wave Phenomena, San Jose, CA (1988).  
ASA Spring Meeting, Seattle, WA (1988).

A book on field-splitting methods in direct and inverse scattering, based on a SIAM minisymposium (Minneapolis, 1988), is in the final stages. This is a collaborative effort with James Coronas (Ames Laboratory, ISU), the late Robert Krueger (Ames Laboratory), Vaughn Weston (Purdue Univ.), and John McCoy (Catholic Univ.).

A book on computational methods in underwater acoustics is underway for Springer-Verlag. This is a joint effort with John DeSanto (CSM, CWP).

The following talks were given during the contract period.

- L. Fishman, "Phase Space and Functional Integral Methods in Direct and Inverse Scattering, Problèmes Inverses, Université des Sciences et Techniques du Languedoc, Montpellier, France, November 27-December 1, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse

Wave Propagation Theories in Ocean Seismo-Acoustics." Fifth International Symposium on Numerical Methods in Engineering, E.P.F.L., Lausanne, Switzerland, September 11-15, 1989.

- L. Fishman, "Phase Space and Functional Integral Methods in Underwater Acoustics," SACLANT Undersea Research Centre, La Spezia, Italy, September 8, 1989.
- L. Fishman, "Numerical Solutions of the Helmholtz, Weyl Composition Equation in Ocean Seismo Acoustics," 13th ICA Meeting on Sea Acoustics, Dubrovnik, Yugoslavia, September 4-6, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Direct and Inverse Wave Propagation Theories in Ocean Seismo-Acoustics," 13th International Congress on Acoustics, Belgrade, Yugoslavia, August 24-31, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories," URSI International Symposium on Electromagnetic Theory, Royal Institute of Technology, Stockholm, Sweden, August 14-17, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories in Ocean Seismo-Acoustics," Seventh International Conference on Mathematical and Computer Modelling, Chicago, Illinois, August 2-5, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories," Progress in Electromagnetic Research Symposium, Boston, Massachusetts, July 25-27, 1989.
- L. Fishman, "Exact and Uniform Perturbation Solutions of the Weyl Composition Equation," SIAM 1989 Annual Meeting, San Diego, California, July 16-21, 1989.
- L. Fishman, "Phase Space and Path Integral Methods in Direct and Inverse Scattering," Acoustical Society of America Meeting, Syracuse, New York, May 22-26, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories in Ocean Seismo-Acoustics," Second IMACS International Symposium on Computational Acoustics, Princeton University, Princeton, New Jersey, March 15-17, 1989.
- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories," OSA Topical Meeting on Numerical Simulation and Analysis in Guided-Wave Optics and Optoelectronics, Houston, Texas, February 5, 1989.

- L. Fishman, "Symbol Analysis and the Construction of One-Way Forward and Inverse Wave Propagation Theories in Ocean Seismo-Acoustics." URSI National Meeting, University of Colorado, Boulder, Colorado, January 4-7, 1989.
- L. Fishman, "Numerical Construction of the Square Root Helmholtz Operator Weyl Symbol," SACLANT Undersea Research Centre, La Spezia, Italy, August 17, 1988.
- L. Fishman, "Phase Space and Functional Integral Methods in the Mathematical and Computational Construction of Direct and Inverse Wave Propagation Theories." International Congress on Computational and Applied Mathematics, Catholic University of Leuven, Leuven, Belgium, July 25-30, 1988.
- L. Fishman, "Phase Space and Path Integral Methods in Computational Acoustics," 12th IMACS World Congress, Paris, France, July 18-22, 1988.
- L. Fishman, "Phase Space Factorization and Functional Integral Methods in Direct and Inverse Scattering," SIAM 1988 Annual Meeting, Minneapolis, Minnesota, July 10-15, 1988.
- L. Fishman, "Phase Space Factorization and Functional Integral Methods in Direct and Inverse Scattering," University of Lowell, Lowell, Massachusetts, June 22-29, 1988.
- L. Fishman, "Phase Space Factorization and Functional Integral Methods in Direct and Inverse Scattering," The First Woodward Conference on Wave Phenomena, San Jose State University, San Jose, California, June 2-3, 1988.
- L. Fishman, "Phase Space and Path Integral Methods in Computational Acoustics," Acoustical Society of America Meeting, Seattle, Washington, May 16-20, 1988.
- L. Fishman, "Modern Mathematical Methods in Underwater Acoustics," Defence Research Establishment Pacific, Victoria, British Columbia, Canada, May 12, 1988.

The following scientific collaborations were established and maintained during the contract period.

**Stephen C. Wales** (NRL, Washington, D.C.) — continued work on numerical aspects of project. Visits to NRL.

**Mike Porter** (SACLANT Centre, La Spezia, Italy) — continued work on numerical symbol construction; visits to SACLANT.

**Mike Fiddy** (University of Lowell, Lowell, MA) — continued work on inverse problems associated with method; visits to Lowell and CWP; one of Mike's students has arrived at CWP for a post-doc.

**David J. Thomson and Gary Brooke** (DREP Victoria, B.C.) — visits to CWP and DREP.

**James Corones, the late Robert Krueger** (Ames Laboratory, Iowa State University, Ames Iowa) and **Vaughan Weston** (Purdue University, W. Lafayette, IN) — final stages of a book on field-splitting methods in direct and inverse scattering.

**John A. DeSanto** (CWP, CSM) — daily conversations and joint work with Thomson and Brooke. Work with post-doc Wombell has commenced. Book in progress on computational methods in ocean acoustics

**Steve Pruess and Frank Hagin** (CWP, CSM) — development of a second (distinct) method for numerical symbol construction.

During the contract period, this work was spread throughout the scientific community in the following additional ways.

**(1) Interest in the Seismic Community**

Over the past five years, there has been a growing interest in the phase space-based code from the seismology community. This has been spearheaded primarily by **Neil Frazer** (HIG, University of Hawaii, Honolulu, HA), and to a lesser extent, and in a more theoretical direction, by **Colin J. Thomson** (Department of Geological Sciences, Queens University, Kingston, Ontario, Kingston, Ontario, Canada). Neil has used the code quite extensively.

**(2) Incorporation into the Educational Curriculum**

Over the past five years, the phase space/path integral development has been introduced in graduate level wave propagation courses at the Catholic University of America, the Colorado School of Mines, the University of Hawaii, and the University of Lowell.

**(3) Reprint Requests**

During the contract period, over fifty reprint requests were answered relating to this work. Material was sent to the United States, Canada, Western Europe, Eastern Europe, India, and China. Requests came from government laboratories, private industry, university departments (chemistry, physics, mathematics, engineering), and even one medical research center.



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17 COSATI CODES		18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	Helmholtz equation, factorization, pseudo-differential operator, path integral, marching algorithm, high-angle parabolic equations, acoustic tomography	
19 ABSTRACT (Continue on reverse if necessary and identify by block number) This work addresses wave propagation in extended, inhomogeneous, multidimensional environments capable of channeling energy over many wavelengths. Sound propagation in the ocean and electromagnetic guided-wave propagation are two examples. Even for the zeroth-order picture of a transversely inhomogeneous environment (which is often an appropriate starting point), the domain of interest is entirely within the scattering regime, with the subsequent absence of an "asymptotically free", or homogeneous, region in which to construct wave fields and collect data. For both forward modeling and tomographic experiments, then, this situation is quite different from the usual formulations of obstacle and quantum scattering theory. The extended and near-one-way nature of the environments Please see reverse.			
20 DISTRIBUTION AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED//UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21 ABSTRACT SECURITY CLASSIFICATION	
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motivated the initial development and application of phase space and functional integral methods for both forward and inverse wave propagation calculations. In this approach, the  $n$ -dimensional one-way problem provides the starting point and is central to the analysis.

The objective of this research is to further develop and apply these "microscopic" phase space (pseudo-differential and generalized Fourier integral operator) methods and global path integral constructions to derive new multidimensional algorithms for forward acoustic propagation and generalized acoustic tomography at the level of the  $n$ -dimensional scalar Helmholtz equation. The frequency domain problem is chosen because there are corresponding experiments being conducted and in the  $\Psi$ DO context, it presents novel problems. As the Weyl operator symbol is pivotal to the one-way analysis, the studies center around the construction of exact, numerical, and uniform perturbation solutions of the (Helmholtz) Weyl composition equation for the square root operator symbol. This has direct application to the path integral representation of the propagator and the subsequent numerical marching algorithm. Beyond that, however, the explicit construction of exact and approximate symbols corresponding to the square root of the indefinite Helmholtz operator should be of interest to both the PDE and  $\Psi$ DO communities. The inverse studies concentrate on making explicit the previously-outlined inversion approach. This analysis seeks to exploit (1) the generalized Fourier integral operator structure of both the finite (macroscopic) and infinitesimal (microscopic) propagators, (2) the semigroup property of the one-way Helmholtz equation, and (3) the Weyl composition equation to solve the refractive index profile reconstruction problem for transversely inhomogeneous environments. This should be appropriate for certain ocean and seismic problems, for example. The transition from the one-way analysis to the full two-way problem is accomplished through the construction of a path integral for the two-way Helmholtz propagator. This would provide a global solution representation, a hoped-for transparency of the underlying physics, and the possibility of direct sum-over-paths computational algorithms exploiting recent advances in massively parallel computing. The studies in this area are preliminary. The Monte Carlo evaluation of the Feynman/Garrod approximate two-way propagator and the construction of an exact, explicit path integral representation for the homogeneous Helmholtz propagator are the initial goals. The numerical computation of path integrals should be of interest to a wide range of mathematical physicists and applied scientists.