STRUCTURAL UNDERSTANDING IN PROBLEM SOLVING

FINAL REPORT

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Abstract

A common instructional objective in domains of math and science is the capability to use formulas and arithmetic procedures to solve problems. Although students are explicitly taught the relevant formulas and principles, are shown worked-out examples, and are given practice, they frequently experience considerable difficulty when asked to solve similar problems. Previous research suggests that difficulties often result from mechanical application of rules and formulas with little understanding of important structural relations between elements in the problem domain. The objective of this research was to analyze in greater detail what is meant by structural understanding within the domain of basic electricity, the role this understanding plays in performance and learning, and the extent to which important structural relations can be taught more directly. The research involved (a) theoretical analyses that represent detailed hypotheses about the knowledge underlying performance in this domain and (b) empirical studies involving the collection of verbal protocols from subjects as they learn to solve basic electricity problems under different instructional treatments.

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Objectives

The main objective of this research contract was to provide a basis for improving instruction in the Navy Basic Electricity and Electronics (BE&E) course, specifically, to improve students' ability to solve paper-and-pencil problems involving direct current (D-C) and alternating current (A-C) circuits. The academic attrition rates for the BE&E course is as high as 30% (Atwater & Abrahams, 1980; Swanson & Boothe-Kewley, 1985) and setback rates are also high. BE&E is a preparatory course providing personnel with the knowledge and applied skills of basic electricity and electronics that have been designated as entry-level prerequisites for the subsequent A-1 schools. Unless students demonstrate competency in these prerequisites, they cannot continue with more advanced training.

The project objectives were (a) to provide theoretical analyses of the knowledge required to solve electricity problems and (b) to conduct empirical studies to identify which components of knowledge students acquire or fail to acquire under different forms of instruction.

Technical Approach

The research was guided by information obtained from two main kinds of empirical studies: protocol studies of individual subjects and group studies. The goal of the protocol studies was to provide detailed information about the knowledge underlying problem-solving performance and learning: for example, information about the sequence and types of solution steps, whether subjects reason using equations or by analogy, and whether subjects reason using quantitative or qualitative relations.

Group studies were performed in collaboration with Dr. William E. Montague, Navy Personnel Research and Development Center, to obtain performance data from actual BE&E students and graduates solving paper-and-pencil problems like the ones used in the protocol studies. These data, together with information obtained from the theoretical analyses, were used to develop diagonostic tests for monitoring knowledge and skill acquisition in BE&E. The group data also provided a valuable test of the generality of the findings from the protocol studies.

Scientific Importance

The research contributed to theories of problem-solving and human cognition. The major issue the research addressed was how general concepts could lead to important gains in understanding at the outset of learning a new domain. Recent comparisons of the knowledge in physics and other domains (e.g., geometry, medical diagnosis) that underlies expert and novice performance suggest that problem-solving skill depends in large part upon having an understanding of important structural relations between the concepts in a domain. Sophisticated understanding, built up over thousands of hours of experience, enables experts to construct elaborate problem representations that guide efficient solutions. Novices lack this understanding, of course. They rely on local problem features and weak general strategies that often lead to inefficient or unsuccessful solutions, especially as problem complexity increases. Novices nevertheless understand general concepts that might be used to structure problem situations in ways that could result in improved performance and provide a basis for learning more domain-specific concepts.

A related issue concerns the distinction between rote performance and performance with understanding. Understanding in problem solving has been discussed before by Gestalt psychologists (e.g., Duncker, 1935/1945; Katona, 1940; Wertheimer, 1945/1959) and by educational psychologists concerned with meaningful learning (e.g., Brownell, 1928; Bruner, 1960; Dienes, 1960, 1963, 1966). Many compelling arguments and demonstrations were offered to emphasize the importance of understanding for problem-solving performance and efficiency, for retention, and for transfer of knowledge to related problems. Only recently have the theoretical and methodological tools been available for providing explicit analyses of what constitutes understanding in a domain and how these structures relate to facts, rules, and strategies used during problem solving. The research contributes to these analyses.

Accomplishments

Protocol Studies

Reasoning about DC circuits using Ohm's and Kirchhoff's laws. A protocol study was conducted to identify the components of knowledge that subjects acquired and failed to acquire during the course of instruction. The existing set of BE&E problems were modified and extended to provide more specific information about the components of knowledge identified in the theoretical analysis that are available to a student at the outset of instruction, and the components of knowledge learned during the course of instruction. Five subjects participated in the study. The results indicate that

- Subjects could solve isolated equations when numbers were provided (quantitative problems) but had difficulty reasoning about qualitative relations between variables in these equations (qualitative problems).

- Similarly, subjects could reason correctly about circuit problems when numbers were provided but made frequent errors when reasoning about qualitative relations between current, voltage, and resistance.

- Pretest performance appears to be correlated with performance on circuit problems. However, successful performance on the pretest does not guarantee successful performance on circuit problems.

These data support and extend earlier findings (Riley, 1984).

Learning by analogy. This research indicated that many subjects (and BE&E students) attempt to memorize algorithms for manipulating equations to solve particular problems, without understanding the relationships between the elements in the equations, let alone how these equations relate to the relations between current, voltage, and resistance in electrical circuits. This project explored teaching subjects important structural relations between circuit variables using a concrete analogy. We analyzed in greater detail protocol tapes from a previous instructional study (Montague, Riley, & Konoske, 1985) that supplemented the standard BE&E course with instructions to think about current, voltage, and resistance in terms of a concrete analogy. The analogy was hypothesized to facilitate the acquisition of the structural knowledge by making circuit constraints more salient and by providing subjects with simple procedures for taking those constraints explicitly into account. It was also hypothesized that subjects would be able to map their understanding of circuit constraints from the analogy to understand the same constraints expressed as equations.

The results indicate:

- Subjects performed quite well on both quantitative and qualitative problems when they used the analogy.

- Subjects had difficulty envisioning solutions to some problems, forcing them to work out the analogy on paper and pencil.

- Subjects frequently preferred to use equations to solve problems, unless prompted to use the analogy.

- When subjects relied on equations, they frequently made errors of the kind observed in previous studies.

Thus, it seems that the analogy does facilitate performance but mapping knowledge of constraints from the analogy to circuit equations is a subtle and complex issue that requires considerable additional empirical and theoretical work.

Group Data

Dr. Montague administered diagnostic tests to three groups of BE&E students who had completed the instruction on DC circuits and passed the standard tests. Some of the students had also successfully completed advanced instruction on AC circuits. The diagnostic tests included questions about qualitative changes similar to those used in the protocol studies. We focused initally on qualitative problems since these problems are quite difficult for students and analyzed patterns of correct responses and errors for individual subjects. In all three groups, proportions of errors were quite high, sometimes approaching 1.00. These findings indicate that even when students perform successfully on the standard tests administered in BE&E, they may lack an understanding of important circuit concepts. The types of errors made by subjects in the three groups were similar to those made by subjects in the protocol studies, which gives us additional insight into the nature of students' difficulties.

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