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In joint work with M.J. Norris, a new uniqueness theorem for ordinary differen equations was proved. This theorem was then applied to solve an old problem of electrodynamics. The mathematical model involves in charged particles movi on the x-axis, each being influenced by the retarded fields of all the orthers Given appropriate past histories of the trajectories, it is now proved that unique solutions will exist as long as no two particles collide.

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THE TWO-BODY PROBLEM OF CLASSICAL ELECTRODYNAMICS

Final Technical Report for AFOSR Contract F49620-79-C-0129

Period covered: 1 July 1979 - 30 June 1980

Principal Investigator: Rodney D. Driver
Professor of Mathematics
University of Rhode Island
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I. Research Objectives

The overall goal of this long-term research project is to examine various competing models for classical electrodynamics to determine whether or not they make sense mathematically. One purely mathematical test which can be applied to a model is a study of existence, uniqueness, and properties of solutions of the resulting mathematical two-body or n-body problem.

II. Status of the Research Effort

Under the present contract, progress has been made for the first time on the simplest n-body problem of classical electrodynamics. The motion of n particles on the x-axis is assumed to be governed by a relativistic model in which each particle is influenced by (and only by) the retarded fields produced by the other particles.

The resulting system of "ordinary" differential equations involves $n(n-1)$ delays which depend on the unknown trajectories. In the usual spirit of delay differential equations, one specifies past histories for the trajectories and then seeks solutions which satisfy the equations of motion in the future.

But now the problem is further complicated by the fact that the past histories of velocities should only be assumed to be absolutely continuous--not necessarily continuously differentiable.

(This lack of smoothness was indicated by consideration of another problem--that of two charged particles moving in three dimensional space.)

Joint work with Dr. Michael J. Norris of Sandia National Laboratories, Albuquerque, has resulted in two papers on the problem. The first [1] proves a new uniqueness theorem for a system of ordinary differential equations (without the usual Lipschitz condition.) This result is needed because of the lack of smoothness of velocities mentioned above. In the second paper [2] this new uniqueness theorem is applied to the n-body problem. The resulting theorem asserts the existence of a unique solution--subject to appropriate given past histories--which can be continued as long as no two or more particles collide.

III. Publications

[1] M. J. Norris and R. D. Driver, A uniqueness theorem for ordinary differential equations, SIAM J. Math. Anal., to appear.

[2] A collinear n-body problem of classical electrodynamics, in Nonlinear Phenomena in Mathematical Sciences, to appear.

Abstracts of these two papers are attached.

IV. Interactions

The Principal Investigator participated in a five-day International Conference on Nonlinear Phenomena in Mathematical Sciences at the University of Texas, Arlington, 16-20 June 1980. The electrodynamics paper [2] was presented as an invited paper at this meeting.

This same conference led to discussions with Drs. V. M. Bogdan and G. G. Johnson of the Johnson Space Center (NASA) in Houston. One problem arising in automatic remote control of an orbiting satellite leads to differential equations with state dependent delays reminiscent of those in the electrodynamics two-body problem.

Another NASA problem was raised in a telephone inquiry from Dr. Richard C. Brown at Marshall Space Flight Center in Huntsville. Here the question had to do with a model for automatic docking in space. The control system contains an inherent delay due to the electronic and mechanical response times. And this delay--simply represented in the equation $x''(t) + kx(t-r) = 0$ with $k > 0$ and $r > 0$--leads to instability regardless of how small the delay may be. An elementary method for showing this and for predicting the asymptotic behavior of solutions was given by R. D. Driver, J. Differential Equations 21 (1976) 148-166.

A Uniqueness Theorem for Ordinary Differential Equations

M. J. Norris† and R. D. Driver‡

Abstract. The uniqueness theorem of this paper answers an open question for a system of differential equations arising in a certain n-body problem of classical electrodynamics. The essence of the result can be illustrated using the scalar prototype equation $x' = g_1(x) + g_2(t + x)$ with $x(0) = 0$. The solution of the latter will be unique provided g_1 and g_2 are continuous positive functions of bounded variation.

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†Applied Mathematics Department 5640, Sandia National Laboratories, Albuquerque, NM 87185. Work supported in part by the U. S. Department of Energy under Contract DE-AC04-76DP00789.

‡Department of Mathematics, University of Rhode Island, Kingston, RI 02881. Work supported in part by AFOSR Contract [redacted].
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A Collinear n-Body Problem of Classical Electrodynamics

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Abstract

One model for the motion of n charged particles on the x-axis leads to a system of delay differential equations with delays dependent on the unknown trajectories. If appropriate past histories of the trajectories are given, say on $[\alpha,0]$, then for sufficiently small $t \geq 0$ one has a system of n^2 ordinary differential equations of the form

$$y' = f(t,y) \quad \text{with} \quad y(0) = y_0 \quad \text{given.} \tag{*}$$

The function f, which involves the known past histories of the trajectories, is continuous; so existence of solutions is assured. However, f does not satisfy the Lipschitz condition usually used for proving uniqueness.

The key new result is that the solution of (*) is unique provided, for some integer $m \leq n^2$,

$$f_i(t,\xi) < 1 \quad \text{for} \quad i = 1,\dots, m, \quad \text{and}$$

$$||f(t,\xi) - f(t,\eta)|| \leq K \sum_{i=1}^{m} |g_i(t-\xi_i) - g_i(t-\eta_i)| + K \sum_{i=m+1}^{n^2} |\xi_i - \eta_i|,$$

where $K > 0$ is constant and each g_i is a continuous function of bounded variation.

This generalized Lipschitz-type condition is indeed satisfied in the electrodynamics case. The m components of y which play the special role in the above uniqueness criterion are the $n(n-1)$ delays of the original n-body problem.

Eventually one finds that solutions of the original equations of motion exist and are unique as long as no two particles collide.