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The errors in linear reduced models of structures, their effects, and means of compensation are discussed. An analysis of natural frequency errors resulting from Guyan reduction shows that errors are reduced as the complementary structure of a reduced model is stiffened. A minimum-ratio criterion is proposed for selecting a best set of retained degrees of freedom. Further improvement of reduced models can be achieved by using analytical model improvement (AMI), which accounts for the specific modal parameters in the frequency range of interest. The effectiveness of AMI under various conditions is evaluated. Also investigated is the capability of various reduced models to accurately represent the effects of structural changes. Numerical results of eigensolution and forced response computations confirm that reduced models formulated in accordance with the minimum ratio criterion and improved by AMI are excellent bases for efficient structural design studies. The approach is especially applicable to the design of large space structures because it (1) takes full advantage of Guyan reduction; (continued on reverse)

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A-1

SUMMARY

In this report the errors in linear reduced models of structures, their effects, and means of compensation are discussed. An analysis of natural frequency errors resulting from Guyan reduction shows that errors are reduced as the complementary structure of a reduced model is stiffened. A semi-analytic minimum-ratio criterion is proposed for selecting a best set of retained degrees of freedom. Numerical examples indicate that applying this criterion results in good reduced models. Further improvement of reduced models can be achieved by using Analytical Model Improvement (AMI) program, which accounts for the specific modal parameters in the frequency range of interest. The effectiveness of AMI under various conditions is evaluated. Also investigated is the capability of various reduced models to accurately represent the effects of structural changes. Numerical results of eigensolution and forced response computations confirm that reduced models formulated in accordance with the minimum ratio criterion and improved by AMI are excellent bases for efficient structural design studies. The approach is especially applicable to the design of large space structures because it (1) takes full advantage of Guyan reduction; (2) simplifies the task of choosing retained DOF; (3) incorporates a powerful error compensation algorithm; and (4) provides an accurate analytical base for structural modifications.

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NOMENCLATURE

A	matrix defined in Eq. (10b)
B	matrix defined in Eq. (10c)
H	matrix defining an eigenproblem
I	identity matrix
K	system stiffness matrix
M	system mass matrix
m	number of modes
N	number of DOF of a full model
n	number of DOF of a reduced model
p	N-n
x	displacement vector
α	vector defined in Eq. (10a)
ϵ	the square of natural frequency error ratio
Λ	diagonal natural frequency matrix of the complementary structure
Φ	mode shape matrix of the complementary structure

ψ mode shape of a (Guyan) reduced model

ω the square of a natural frequency

Subscripts

$(\bar{\ })$ modified model due to structural changes

r reduced model or retained DOF of reduced model

s DOF to be eliminated

0 refers to Guyan reduction

1 submatrix associated with retained DOF

2 submatrix coupling 1 and 4

4 submatrix associated with DOF to be eliminated

SECTION I

INTRODUCTION

The use of large space structures for surveillance, reconnaissance, detection, and tracking will be increased in the near future. Minimizing the structure's mass is crucial for launching cost reduction. This among other considerations leads to the design of highly flexible structures. The overall size of an antenna boom-solar panel assembly is on the order of 100m and its first several elastic modes are below 0.1 Hz. These characteristics cause problems in the dynamic analysis and design of control systems. For instance, disturbances can create significant structural deformations far beyond stringent mission tolerances, and control must be applied to suppress the vibration level.

It is the size and flexibility of structures and the requirement for active control which make it necessary for the dynamic characteristics of such systems to be determined accurately. Furthermore, since the dynamic characteristics of large space structures in the actual space environment cannot be rigorously determined from ground vibration testing, their design and the design of control systems for them will require precise analytical models.

Detailed finite element analysis, however, will result in a large order dynamic model. Prior to further studies, it will be necessary to reduce the order of the analytical model to allow economical computations. For this purpose, essentially two types of techniques are developed: component mode synthesis and reduction of physical coordinates. The former employs selected component modes to represent system structural response. Component modes used can be eigenvectors[1], Ritz vectors solved from the corresponding static problem[2], or their combinations[3]. Similarly, reduction of physical coordinates may apply to both component and system levels. This latter class includes Guyan reduction[4] and generalized dynamic reduction[5]. Certainly, it is possible to combine two types of reduction techniques if the design is based on substructure syntheses. For example, Guyan reduction can be applied to various component levels to bring down the number of degrees of freedom, an eigensolution can then be obtained from the assembled structure and, finally, a modal transformation can be performed to formulate the system equation. In the present research, we focus on reduction of physical coordinates without using substructure syntheses techniques.

As indicated in References 6 and 7, a reduced model is inherently non-linear, even when the full model is linear. Thus, all linear dynamic models having a reduced number of degrees of freedom (DOF) are imprecise[7]. It is important then to evaluate the effects of these errors and to develop means to correct them.

In practice, attempts to minimize these imprecisions are made by using a set of modes whose frequencies cover the range of interest (notably, the truncated normal mode approach), or by eliminating unimportant DOF and retaining a well distributed set of coordinates. These intuitive procedures may be adequate when the reduction order is small and when test data is available to qualitatively validate the models. This is not the case for large space structures. More sophisticated algorithms have also been developed to improve the accuracy through analytical selection of retained DOF[8,9] or using substructure techniques[10,11]. The former approach eliminates DOF by performing reduction one DOF at a time while the latter involves matrix inversions of high rank. Neither can be pursued for the design of large systems where major reduction is a main concern.

Moreover, even if these methods can be used with confidence, there is no commonly accepted method for compensating for errors.

The generalized dynamic reduction[5] applies subspace iteration techniques to compensate errors in a Guyan reduction. Low modes of the structure are used recursively to make up the lost dynamic portion. The iterative nature makes the algorithm computationally inefficient for large structure systems and the procedure may be sensitive to noise resulting from the orthogonalization procedure. It is also expected that this compensation method has little effect of those modes beyond, say, the first dozen. For large space structures, it is reported that as high as 100 elastic modes may be considered in structure/control system interaction analysis[12].

Despite its poor accuracy for higher elastic modes, Guyan reduction[4], mainly due to its efficiency, is still popular in practical applications. The present approach will take advantage of this. But to ease the burden of cautiously choosing retained DOF, as encountered in any reduction procedure, a semi-analytical, one time only guideline would definitely be helpful to analysts dealing with large systems. Also, when a reduced model is obtained, the question arises as to the possibility of improving the model through compensation for errors for specified modes. A qualified improvement procedure should be computationally efficient, applicable to realistically large models, and satisfy dynamic constraints. Another important problem is the accuracy with which a reduced model can reflect structural changes. Evaluation of this capability would decide the usefulness of a reduction and compensation procedure for system design, especially in the preliminary design stage where intensive modifications are inevitable.

This report addresses these issues.

SECTION II

ANALYSIS OF INACCURACIES

Exact Reduced Eigenproblem

A linear structure subject to applied forces can be described by

$$M\ddot{y} + Ky = f(t) \tag{1}$$

where the system mass and stiffness matrices are of order N. The associated eigenequation

$$H(\omega) = (K - \omega M)\, x = 0 \tag{2}$$

uniquely defines the dynamic characteristics of the system. In Equation 2, ω is the square of a natural frequency and x the corresponding mode shape vector. When the order of M and K is large (as in a detailed finite element modeling of structures), it is desirable to use x_r, a subset of x containing all retained DOF, to describe the system. That is, a reduced model (M_r and K_r) of order $n < N$ is sought which satisfies the equation

$$H_r(\omega)x_r = (K_r - \omega M_r)x_r = 0 \tag{3}$$

The exact reduced model is the one preserving all information of the full model. To formulate reduced models, M and K are reordered in such a way that the upper left submatrices correspond to x_r, that is,

$$\left(\begin{bmatrix} K_1 & K_2 \\ K_2^T & K_4 \end{bmatrix} - \omega \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_4 \end{bmatrix} \right) \begin{Bmatrix} x_r \\ x_s \end{Bmatrix} = 0 \tag{4}$$

where the subset x_s contains all DOF to be condensed out and is related to x_r as:

$$x_s = -(K_4 - \omega M_4)^{-1} (K_2^T - \omega M_2^T)\, x_r \tag{5}$$

Substituting Eq. (5) back to the first part of Eq. (4) leads to

$$K_r = K_1 - K_2 K_4^{-1} K_2^T \tag{6a}$$

$$M_r = M_1 - \left[K_2 (K_4 - \omega M_4)^{-1} M_2^T + M_2 (K_4 - \omega M_4)^{-1} K_2^T\right]$$
$$+ K_2 K_4^{-1} M_4 (K_4 - \omega M_4)^{-1} K_2^T + \omega M_2 (K_4 - \omega M_4)^{-1} M_2^T \tag{6b}$$

It is observed that no approximation has been introduced in condensation procedures up to this point. The reduced model in Eq. (6) contains complete information of the full model. However, since M_r is frequency dependent the reduced eigenproblem, Eq. (3), is nonlinear. Solution of this equation is difficult because it involves extensive iterations. A useful approximation is simply to impose $\omega = 0$ on Eq. (6b) which leads to the well-known Guyan reduction

$$K_r = D^T K D \tag{7a}$$

$$M_{r0} = M_r\ (\omega = 0) = D^T M D \tag{7b}$$

$$H_{r0}(\omega) = K_r - \omega M_{r0} \tag{7c}$$

where $D = \begin{bmatrix} I \\ -K_4^{-1} K_2^T \end{bmatrix}$. Eq. (7) is exact only when $\omega = 0$, hence is a quasi-static condensation.

Application of Guyan reduction to dynamic analysis, as is often done, can be poor. The errors depend on the subset x_r chosen and the frequency range of interest. Questions then arise as to the choice of x_r which best represents the dynamic characteristics of the full model and to the accuracy of the quasi-static approximation.

Reformulation of Reduced Eigenproblem

In Eqs. (4) - (6), submatrices of a full model have been used to define a reduced model. A structure having M_4 and K_4 as its mass and stiffness matrices is defined to be the "complementary structure" of the reduced model. Conceptually, it is achieved by constraining all x_r in the full model to be zero and using x_s as its DOF (see Figure 1).

Let Λ (pxp), p = N - n, be a diagonal matrix whose nonzero elements (λ_i, i=1, ..., p) are the square of natural frequencies of the complementary structure and Φ(pxp) the corresponding mode shape matrix. The eigenequation and the orthogonality property of this structure are given by

$$(K_4 - \lambda_i M_4)\ \phi_i = 0 \tag{8a}$$

$$\Phi^T M_4 \Phi = I \tag{8b}$$

$$\Phi^T K_4 \Phi = \Lambda \tag{8c}$$

where ϕ_i is the ith mode shape vector.

Note that under linearity assumption, x_s is the sum of modal superposition when x_r is constrained to zero and the displacement due to relaxation of the constraint, i.e.,

$$x_s = \Phi\alpha - (K_4^{-1}\ K_2^T\ )\ x_r \tag{9}$$

The last term in Eq. (9) actually is the static expression (ω = 0) of the right hand side of Eq. (5). The modal coefficient vector α is solved from Eq. (5), (8), and (9)[11]

$$\alpha = A(\omega)\ B\ x_r \tag{10a}$$

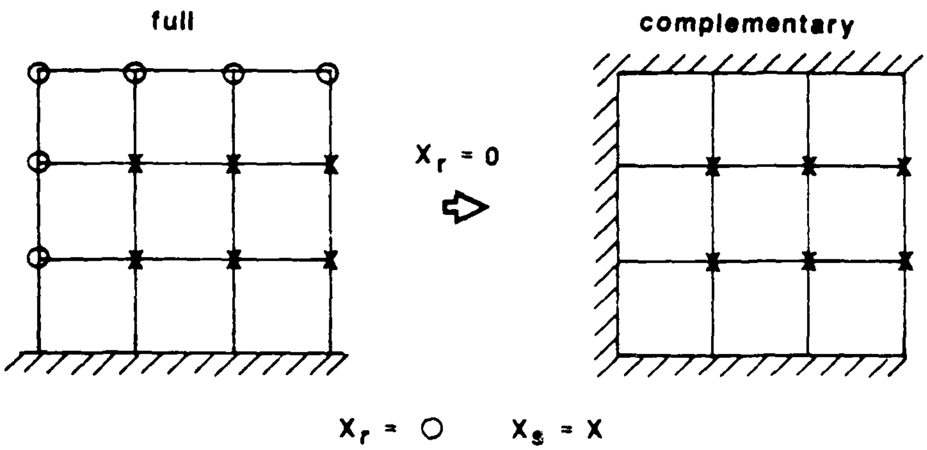

Figure 1. Definition of Complementary Structure.

in which

$$A(\omega) = \omega\ (\Lambda - \omega I)^{-1} \tag{10b}$$

$$B = \Phi^T M_2^T - \Lambda^{-1} \Phi^T K_2^T \tag{10c}$$

Thus,

$$x_s = [\Phi A(\omega)\ B - K_4^{-1}\ K_2^T]\ x_r \tag{11}$$

Substituting Eq. (11) into Eq. (4) results in the exact reduced eigen-equation

$$H_r(\omega)\ x_r = 0 \tag{12a}$$

$$H_r(\omega) = K_r - \omega\ [M_{r0} + B^T A(\omega) B] \tag{12b}$$

where K_r and M_{r0} were defined in Eq. (7). Formally, Eq. (12b) is different from Eq. (7c) only in the additional term $B^T A(\omega) B$ in the bracket. Although Eqs. (3) and (6), and (12) are equivalent and exact, the new formulation (Eq. (12)) is considered more efficient than its old version (Eqs. (3) and (6)) because the only frequency dependent matrix $A(\omega)$ is diagonal. However, no attempt is made here to solve Eq. (12) since it still needs intensive iterations. This formulation is used as a base for the error analysis in the next subsection.

Error of Natural Frequency

A perturbation analysis is performed below to investigate the natural frequency errors introduced by Guyan reduction. It is assumed tacitly that $\omega \neq 0$ which precludes a trivial case.

Let (ω_i, ψ_i) be an eigenpair defined by a (Guyan) reduced model, i.e.,

$$H_{r0}(\omega_i)\,\psi_i = 0 \tag{13a}$$

where ψ_i has been normalized so that

$$\psi_i^T M_r \psi_i = 1 \tag{13b}$$

Expand $H_r(\omega)$ and x_r with respect to their corresponding approximate functions at ω_i, the exact eigenproblem (12) becomes

$$[K_r - \omega_i M_{r0} - (\omega_i \Delta M_r + \Delta\omega_i M_{r0}) - \Delta\omega_i\ \Delta M_r]\,(\psi_i + \Delta\psi_i) = 0 \tag{14a}$$

where

$$\Delta M_r = B^T A(\omega) B \tag{14b}$$

Premultiplying Eq. (14a) by $(\psi_i + \Delta\psi_i)^T$ and applying Eq. (13) to simplify the result, then

$$\Delta\omega_i = -\omega_i\ \psi_i^T\ \Delta M_r\ (\omega_i)\ \psi_i \tag{15}$$

Therefore, the error ratio of a natural frequency is

$$\begin{aligned}\epsilon_i &= -\Delta\omega_i/\omega_i = \psi_i^T\ B^T A\ (\omega_i)\ B\psi_i \\ &= [(\Phi^T M_2^T - \Lambda^{-1}\Phi^T K_2^T)\ \psi_i]^T\ (\Lambda - \omega_i I)^{-1}\ (\Phi^T M_2^T - \Lambda^{-1}\Phi^T K_2^T)\ \psi_i\end{aligned} \tag{16}$$

Note that if A is positive definite then $\Delta\omega_i < 0$, i.e., the natural frequency of a (Guyan) reduced model is bigger than its corresponding exact value of the full model.

In Eq. (16), the error of natural frequency of a reduced model through quasi-static condensation is expressed in terms of the eigenpair (ω_i, ψ_i) of the reduced model and the eigensolutions (Λ, Φ) of the complementary structure. In the case of a major reduction in degrees of freedom from the full model, Eq. (16) is not very promising because complete eigensolutions of the complementary structure are time-consuming when the order of eigenequation (p) is large. Further development is possible if the scope of the analysis is confined to error-bound calculation.

Let λ_1 be the smallest eigenvalue in Λ. When $\omega_i << \lambda_1$, an inequality can be obtained from Eq. (16)

$$\epsilon_i < \omega_i/\lambda_1 \left[\psi_i^T(\Phi^T M_2^T - \Lambda^{-1} \Phi^T K_2^T)^T (\Phi^T M_2^T - \Lambda^{-1}\Phi^T K_2^T) \psi_i\right] \tag{17}$$

Using Eqs. (8b) and (7b), the expression in the bracket in Eq. (17) is rewritten as

$$I - \psi_i^T (M_1 - M_2 M_4^{-1} M_2^T) \psi_i \tag{18}$$

It is noted that the expression in the parentheses in Eq. (18) is the same as that of in Eq. (6a) if M is replaced by K. In other words, the parenthesized matrix is an exact condensation of a system with M as its stiffness matrix and thus, is positive semidefinite. Since Eq. (16) guarantees ϵ_i being positive for $\omega_i < \lambda_1$, (i.e., an eigenvalue of a reduced model is greater than the corresponding eigenvalue of the exact model), combination of Eqs. (17) and (18) leads to

$$\epsilon_i < \omega_i/\lambda_1 \tag{19}$$

Eq. (19) gives an upper bound of errors due to Guyan reduction. The error bound is a function of natural frequency sought (ω_i) and, as expected, becomes larger when the natural frequency is higher. The only information required from complementary structure is the smallest eigenvalue λ_1. Many efficient algorithms are available to compute the leading eigenvalue of a large order eigenproblem. A similar derivation was shown in Reference 13 as an intermediate step to find absolute error bound. Here Equation 19 is used as a base from which to draw useful information for the retained DOF selection.

Choice of x_r

Eq. (19) also reveals that an optimal subset x_r chosen from the full model should maximize λ_I to make the error bound low. In other words, the retained degrees of freedom x_r must be selected in such a way that the corresponding complementary structure is as "stiff" as possible.

For a given set of full matrices M and K, no once-for-all criterion on the choice of x_r can be drawn analytically because there is no functional relationship between eigenvalues of a system and the individual elements of the system. It is acknowledged that an "analytical" method on selection of x_r based on simple criterion and incorporated with frontal solver techniques was proposed. This approach requires reducing degrees of freedom one by one and in each loop a Guyan reduction be performed. It may be useful for a relatively small system. In the case of major reduction from a large structure, as is usually done, this procedure would be prohibitively expensive and thus is not recommended here.

From Eq. (8a), it is justifiable qualitatively to expect that a complementary structure is stiffer if all diagonal element ratios $(K_4)_{ii}/(M_4)_{ii}$ become larger. This means that as a rule of thumb a best set of degrees of freedom is the one having minimum K_{ii}/M_{ii} $(i = 1, \ldots, n)$ ratios. The greatest advantage of this criterion is its simplicity although it is not an analytical result. There is no guarantee that the reduced model so obtained will preserve all n lowest natural frequencies of the full model. This guideline will be referred to as the minimum ratio criterion in the future discussion. The "analytical selection" method adopted in References 8 and 9 uses the same criterion, but is on an iterative basis.

Frequency Response

The mobility matrix of a linear dynamic system, Eq. (1), is defined by

$$Z(\omega) = (\omega^{-1}K - M)^{-1} \tag{20}$$

The (i, j)th element of $Z(\omega)$ characterizes the steady-state response of jth DOF when a unit force of frequency ω is applied at ith DOF of the system. To evaluate how good a reduced model can predict the forced response of the full model, it is useful to compare certain mobility elements of interest, for instance, the driving point mobility (a diagonal element of $Z(\omega)$) of both models.

Validation of the Criterion

A delta wing is modeled as a coupled system of three rigid ribs (modal representation) and five skin finite element substructures using Kaman's DYSCO (Dynamic System Coupler) program. In Figure 2, each skin lumped-mass is denoted by a solid circle and each rib center of gravity is denoted by an X. Vertical displacements of the skin and ribs and pitch displacements of the ribs are considered. The total number of DOF represented in Figure 2 is 61, including 21 implicit (dependent) DOF. All skin DOF coincident with ribs are considered to be implicit, that is they can be expressed as linear combinations of the rib vertical and pitch DOF. Thus, the model has 40 independent DOF.

In DYSCO the required input data are the finite element mass and stiffness matrices for each skin substructure and the modal definition, center of gravity location, and implicit degree of freedom definitions for each rib. The coupled 40 degree of freedom model is then assembled automatically by the program. Figure 3 identifies the final 40 degrees of freedom which define a full model. An eigenanalysis of the full model was performed using DYSCO and the first 10 modes are shown in Figure 4.

SUBSTRUCTURE DEFINITION

SKIN	# DOF	RIB	# DOF	# IDOF*
1	19	1	2	10
2	17	2	2	7
3	15	3	2	4
4	18			
5	15			

(*IDOF = IMPLICIT DOF)

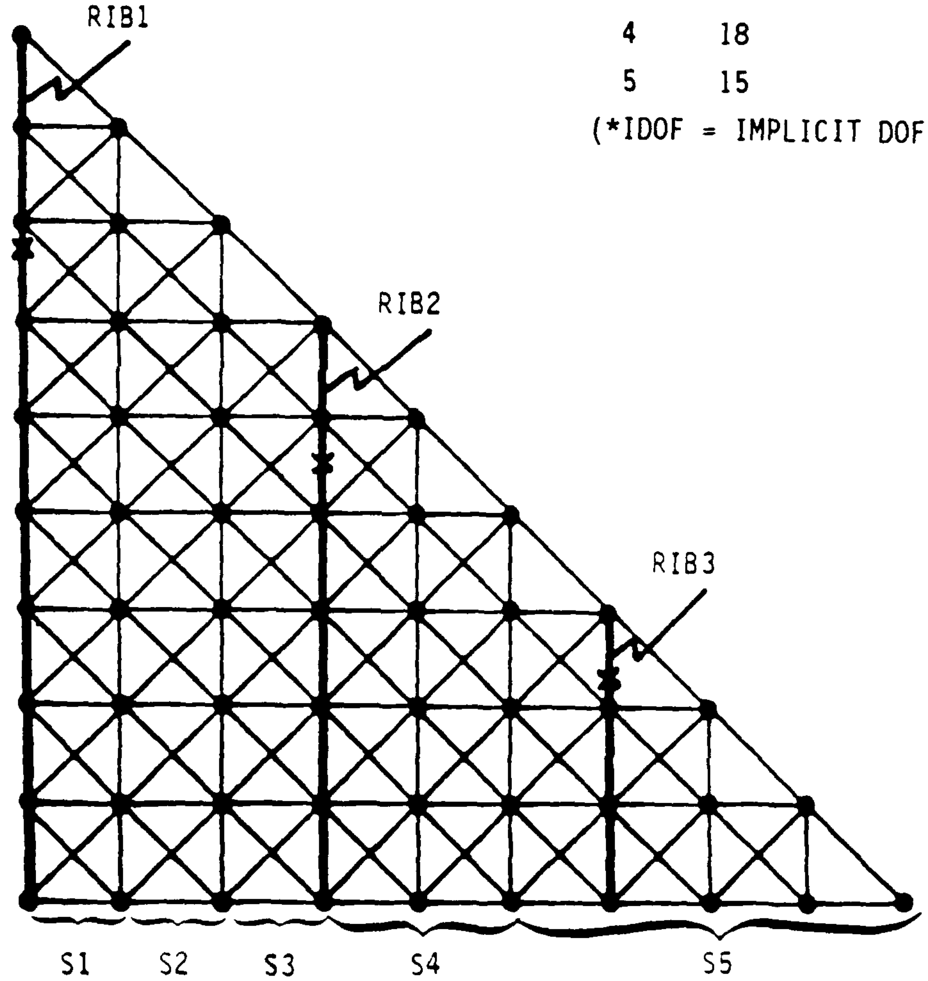

Figure 2. A Delta Wing Model.

SYSTEM DOF

1	ZCG	1000
2	PTCH	1000
3	Z	11
4	Z	12
5	Z	13
6	Z	14
7	Z	15
8	Z	16
9	Z	17
10	Z	18
11	Z	19
12	Z	20
13	Z	21
14	Z	22
15	Z	23
16	Z	24
17	Z	25
18	Z	26
19	Z	27
20	ZCG	2000
21	PTCH	2000
22	Z	35
23	Z	36
24	Z	37
25	Z	38
26	Z	39
27	Z	40
28	Z	41
29	Z	42
30	Z	43
31	Z	44
32	Z	45
33	ZCG	3000
34	PTCH	3000
35	Z	50
36	Z	51
37	Z	52
38	Z	53
39	Z	54
40	Z	55

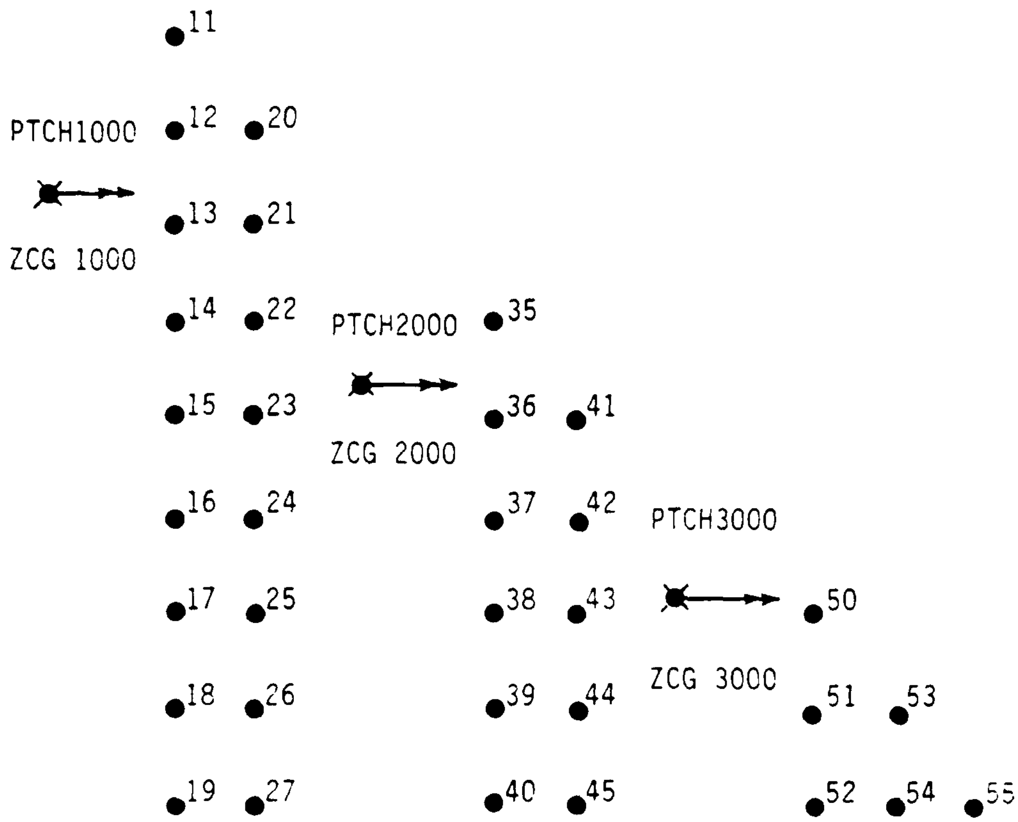

Figure 3. Coupled Model of 40 DOF.

MODE	1	2	3	4	5
FREQ HZ	1.5629E-02	1.7387E+01	2.7273E+01	3.9556E+01	4.5129E+01
RAD/S	9.8202E-02	1.0925E+02	1.7136E+02	2.4854E+02	2.8355E+02
GEN MASS	7.6001E-01	3.2744E-01	1.1609E-01	2.3540E-01	5.0097E-02
SYS DOF					
ZCG 1000	1.0000	0.8555	0.0113	-0.4355	0.0065
PTCH1000	-0.0000	0.0119	0.0061	-0.0001	-0.0020
Z 11	1.0000	1.0000	0.0832	-0.3550	0.0115
Z 12	1.0000	0.7251	-0.0198	-0.1354	0.0578
Z 13	1.0000	0.5732	-0.1149	-0.0845	0.0910
Z 14	1.0000	0.3565	-0.2076	-0.0853	0.1042
Z 15	1.0000	0.1403	-0.3963	-0.1001	[illegible]
Z 16	1.0000	-0.0737	-0.3818	-0.3341	[illegible]
Z 17	1.0000	-0.2793	-0.4679	-0.3576	[illegible]
Z 18	1.0000	-0.4587	-0.5450	-0.4733	[illegible]
Z 19	1.0000	-0.5491	-0.5938	-0.5474	[illegible]
Z 20	1.0000	0.6375	-0.0793	0.3311	0.1375
Z 21	1.0000	0.4463	-0.1436	0.4659	0.1743
Z 22	1.0000	0.2596	-0.1974	0.4854	0.1191
Z 23	1.0000	0.0717	-0.2439	[illegible]	[illegible]
Z 24	1.0000	-0.1140	-0.2880	[illegible]	0.0047
Z 25	1.0000	-0.2914	-0.3393	-0.3274	-0.0593
Z 26	1.0000	-0.4425	-0.3637	-0.4687	-0.1168
Z 27	1.0000	-0.5196	-0.3966	-0.6096	-0.1531
ZCG 2000	1.0000	0.0757	-0.1700	0.3198	0.0691
PTCH2000	-0.0000	0.0077	-0.0001	0.0176	0.0051
Z 35	1.0000	0.0157	-0.1045	1.0000	-0.0117
Z 36	1.0000	-0.1276	-0.0556	0.6987	-0.1350
Z 37	1.0000	-0.2646	-0.0043	0.3965	-0.2142
Z 38	1.0000	-0.3924	0.0355	0.0584	-0.2636
Z 39	1.0000	-0.5025	0.0644	-0.2511	-0.2956
Z 40	1.0000	-0.5589	0.0781	-0.4208	-0.3222
Z 41	1.0000	-0.2539	0.0779	0.7002	-0.2814
Z 42	1.0000	-0.3712	0.1643	0.4369	-0.3189
Z 43	1.0000	-0.4739	0.2429	0.1645	-0.2986
Z 44	1.0000	-0.5520	0.2957	-0.1271	-0.2096
Z 45	1.0000	-0.5927	0.3255	-0.2227	-0.1682
ZCG 3000	1.0000	-0.5358	0.4206	0.3093	-0.3394
PTCH3000	0.0000	0.0030	-0.0036	0.0151	-0.0114
Z 50	1.0000	-0.6220	0.6009	0.0785	-0.0218
Z 51	1.0000	-0.6684	0.6829	-0.1381	0.2229
Z 52	1.0000	-0.6963	0.7401	-0.2474	0.3672
Z 53	1.0000	-0.7156	0.8289	-0.2191	0.5320
Z 54	1.0000	-0.7348	0.8774	-0.2870	0.6640
Z 55	1.0000	-0.7712	1.0000	-0.3661	1.0000

Figure 4. The First 10 Modes of the Full Model.

MODE	6	7	8	9	10
FREQ HZ	5.3893E+01	6.3110E+01	8.5738E+01	8.6371E+01	9.0966E+01
RAD/S	3.3862E+02	3.9653E+02	5.3871E+02	5.4269E+02	5.7156E+02
GEN MASS	2.3811E-01	3.5737E-02	5.4391E-02	5.4300E-02	5.2959E-02
SYS DOF					
ZCG 1000	-0.2028	-0.0364	-0.0314	-0.1611	-0.0701
PTCH1000	0.0040	0.0013	-0.0016	0.0002	-0.0010
Z 11	-0.1032	-0.0378	0.3045	0.5059	0.6392
Z 12	-0.1092	-0.0538	0.3156	0.5093	0.5155
Z 13	-0.1062	-0.0596	0.2712	0.4263	0.3136
Z 14	-0.1160	-0.0527	0.1516	0.2130	0.0112
Z 15	-0.1337	-0.0376	0.0460	0.0572	-0.1659
Z 16	-0.1579	-0.0177	-0.0418	-0.0115	-0.1993
Z 17	-0.1842	0.0039	-0.1375	-0.0144	-0.1450
Z 18	-0.2106	0.0250	-0.2669	0.0291	-0.0707
Z 19	-0.2413	0.0397	-0.4424	0.0773	-0.0421
Z 20	0.0232	-0.0642	0.5347	1.0000	0.8804
Z 21	0.0960	-0.0409	0.3008	0.6136	0.3171
Z 22	0.1648	-0.0036	0.1037	0.3105	-0.0764
Z 23	0.2085	0.0491	-0.0190	0.1713	-0.2239
Z 24	0.2459	0.1067	-0.1240	0.1191	-0.2477
Z 25	0.2810	0.1660	-0.2268	0.1327	-0.1728
Z 26	0.3162	0.2248	-0.3834	0.1928	-0.0800
Z 27	0.3600	0.2843	-0.6065	0.3045	-0.0414
ZCG 2000	0.4344	0.0721	-0.0757	0.0443	-0.1721
PTCH2000	-0.0041	-0.0034	0.0002	-0.0015	-0.0029
Z 35	0.3079	0.1188	-0.1800	-0.6348	0.5364
Z 36	0.2576	0.1437	-0.0915	-0.5273	0.5333
Z 37	0.2760	0.1400	0.0188	-0.3783	0.4099
Z 38	0.4777	0.1052	0.1778	-0.1472	0.1238
Z 39	0.7585	0.0637	0.3850	-0.0136	-0.1162
Z 40	1.0000	0.0449	0.6522	0.0365	-0.3025
Z 41	-0.2520	0.1449	-0.0642	-0.8654	1.0000
Z 42	-0.2500	0.0558	0.0828	-0.4431	0.4729
Z 43	-0.0134	-0.0731	0.2338	-0.1656	0.0637
Z 44	0.5860	-0.2259	0.4397	-0.1051	-0.1654
Z 45	0.8852	-0.3587	0.7333	-0.1327	-0.3639
ZCG 3000	-0.8755	-0.1384	0.0308	0.1109	-0.1611
PTCH3000	-0.0458	0.0118	0.0003	0.0059	-0.0039
Z 50	-0.5732	-0.1975	-0.3803	0.2374	-0.0959
Z 51	0.0055	-0.2465	-0.4483	0.1594	0.1143
Z 52	0.2588	-0.2687	-0.6244	0.1429	0.2848
Z 53	-0.2704	0.1615	-0.4374	0.1869	0.1429
Z 54	-0.1921	0.2662	-0.4646	0.1379	0.2820
Z 55	-0.5420	1.0000	1.0000	-0.3436	-0.3361

Figure 4. The First 10 Modes of the Full Model (Cont'd).

Various reduced models can be obtained by applying Guyan reduction to the full model. Five of them having 10 or 15 DOF and their eigenvalue errors of the first several modes are shown in Figure 5. Selection of DOF for RM4 and RM30 are based on intuitive judgement while in the "worst" model (RM20), two dominant rib DOF (PTCH 1000 and PTCH 3000) are intentionally reduced out. The rest two, RM11 and RM40 are formulated based on the minimum ratio criterion.

It is observed that both RM4 and RM11 preserve (in the sense that $-\Delta\omega_i$ and ϵ_i are small) the first eight modes of the full model and are considered as good reduced models. In this case, use of the criterion does not make RM11 much different from a reasonably formulated RM4 since choosing 15 out of 40 may not be viewed as a truly major reduction in system DOF. A more crucial comparison is made for the three 10 DOF models. Note that in Figure 5, RM40 preserves the first seven modes, RM30 missed the fifth and modes beyond the sixth while RM20 preserves only modes 1, 2, and 4. Obviously, the reduced model with the minimum ratio criterion is superior to the other two.

Other evidence of the minimum ratio criterion being a good guideline for the selection of retained DOF is seen from frequency responses. In Figure 6 the driving point mobilities at the wing tip (Z55) for RM4 and RM11 are plotted against that for the full model in the frequency range of 15 to 95 Hz. Similarly, the driving point mobilities at Z19 for RM30, RM40 and the full model are given in Figure 7. Obviously, RM11 and RM40 are better than their intuitively chosen counterparts RM4 and RM30.

REDUCED MODEL (†)	n	DOF	$-\Delta\omega_i$ (HZ) / $\epsilon_i^{1/2}$ (%)								
			2††	3	4	5	6	7	8	9	10
RM11 (Y)	15	ZCG 1000, 2000, 3000 PTCH 1000, 2000, 3000 Z 35, 40, 41, 45 50, 52-55	.055	.236	.361	.496	.371	.774	3.91	7.72**	12.2
			.3	.9	.9	1.1	.7	1.2	4.4	8.2	11.9
RM4 (N)	15	ZCG 1000, 2000, 3000 PTCH 1000, 2000, 3000 Z 11, 12, 18-20 27, 53-55	.041	.219	.491	.903	1.31	1.27	4.85	8.24**	6.74
			.2	.8	1.2	2.0	2.4	2.0	5.4	8.7	6.9
RM40 "Best" (Y)	10	ZCG 1000, 2000, 3000 PTCH 1000, 2000, 3000 Z 11, 19, 54, 55	.065	.404	.608	1.25	1.42	1.54	12.4**		
			.3	1.5	1.5	2.7	2.6	2.4	12.7		
RM30 "Medium" (N)	10	ZCG 1000, 2000, 3000 PTCH 1000, 2000, 3000 Z 11, 12, 18, 19	.135	1.56	.637	7.35*	3.03	31.0**			
			.7	5.4	1.6	14.0	5.3	37.0			
RM20 "Worst" (N)	10	ZCG 1000, 2000, 3000 PTCH 2000 Z 11, 12, 18-20, 27	.452	4.65*	.900	19.8**					
			2.5	14.5	2.2	30.6					

† Based on minimum ratio criterion? †† $\omega_1 = 0$

* Large discrepancy; the corresponding mode in the full model is considered missed.

** The indicated and higher modes are considered missed.

Figure 5. Comparison of Various Reduced Models.

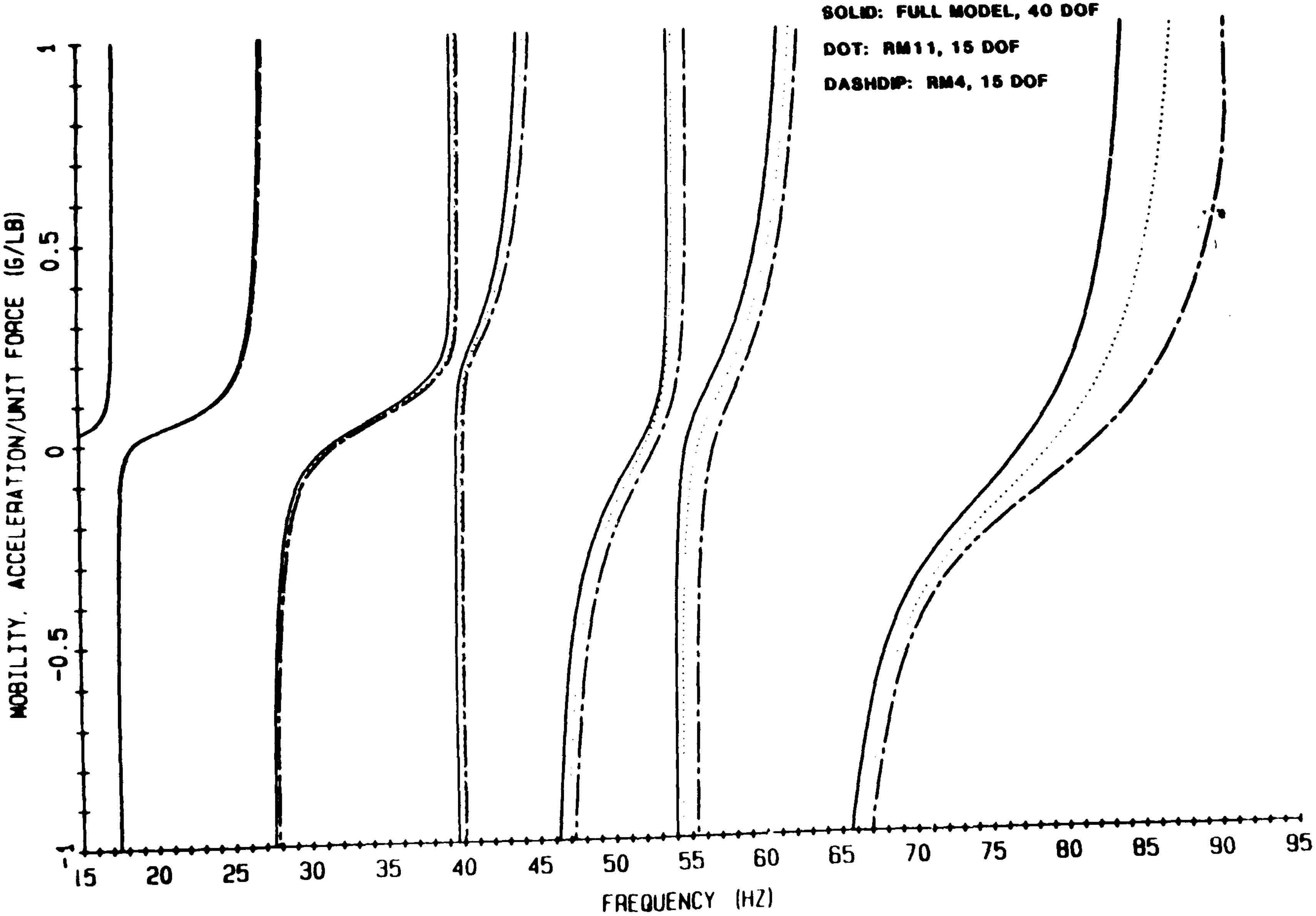

Figure 6. Driving Point Mobilities, Wing Tip (Z55).

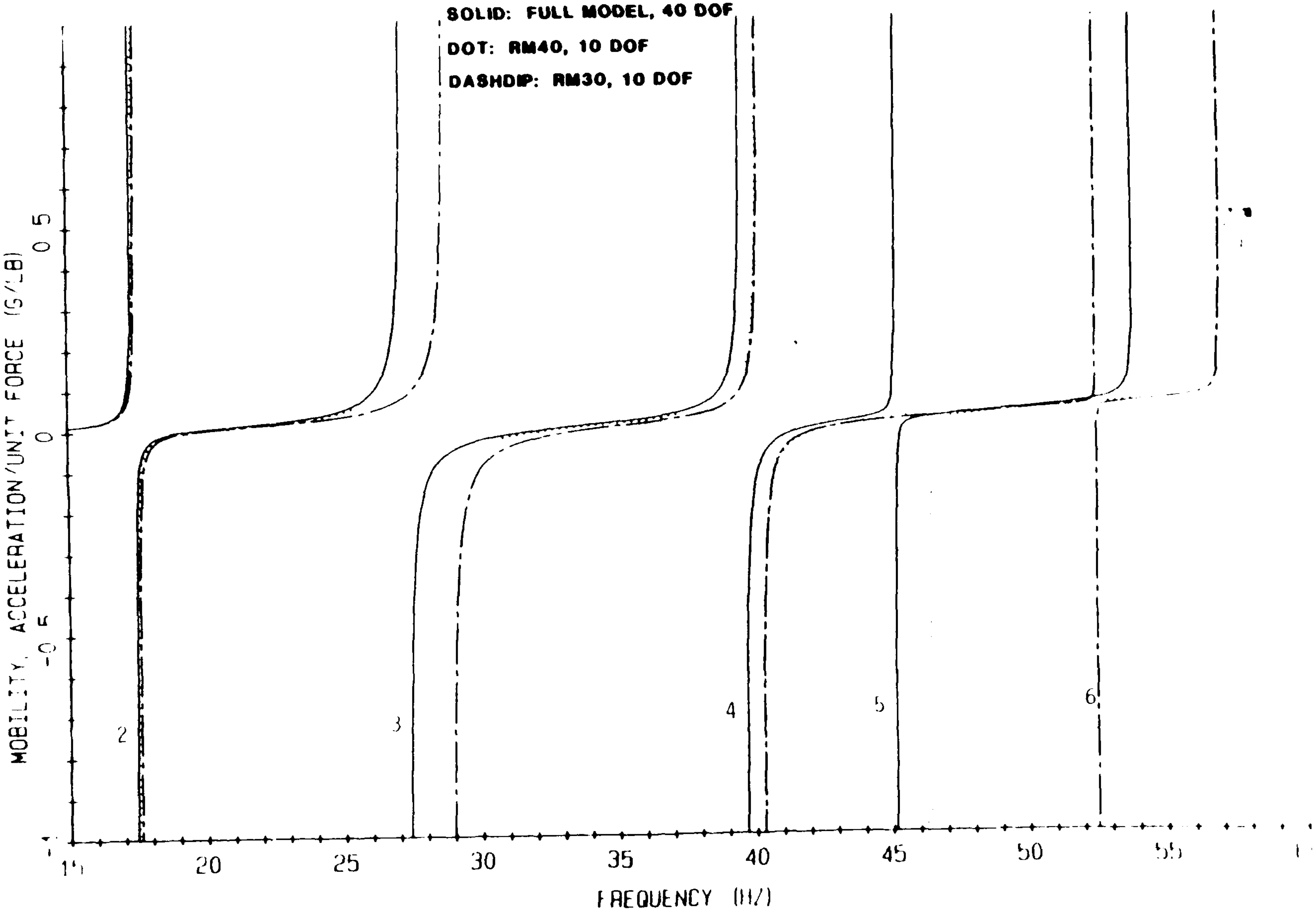

Figure 7. Driving Point Mobilities, Wing Root Trailing Edge (Z19).

SECTION III

ERROR COMPENSATION FOR REDUCED MODELS

AMI

Although it can result in better models, some deficiencies still exist in using the minimum ratio criterion. First of all, there is no guarantee that the resultant reduced model can preserve all natural frequencies of the full model in the frequency range of interest. The situation is even worse if the range of interest is not in the lowest frequency range. Furthermore, accuracies of the corresponding eigenvectors may be very poor and cannot be estimated analytically. Thus, for the compensation of these errors, a procedure called AMI (Analytical Model Improvement) is applied to modify reduced models.

AMI is a procedure which finds the smallest changes in the given model so that the improved one satisfies orthogonality relationships and the dynamic equation and exactly predicts specific dynamic characteristics (eigensolutions in the assigned frequency range) of the full model. This algorithm was originally designed for structural system identification[14,15], whose physical[7,16] and mathematical[16,17] foundations have been extensively discussed. The bases of this identification method are a well-formulated analytical model and a modal matrix to be matched. In the present model reduction problem, they are the reduced model and the specific modes of the full model, respectively. The program is applicable to realistically large models and is computationally efficient.

Improvement of Reduced Models

The following procedures are used to modify a reduced model:

1. Extract eigensolutions of the given full model over a frequency range of interest. In practice, the number of eigensolutions extracted is very small compared to the order of the full

model. Many algorithms are available, notably those of NASTRAN, to effectively perform this procedure.

2. Formulate a reduced model by using Guyan reduction.

3. Provide the modes to be matched (i.e., the eigensolutions extracted in step 1) to AMI for the improvement of the reduced model.

4. Compute and plot mobilities of interest of the full and reduced models for comparison.

Figure 8 shows some results of AMI improvement when the procedures are applied to the models described in Figure 5. In column 3, m ($m \leq n$) denotes the number of modes to be matched and column 4 specifies these modes. The percentage changes of mass and stiffness matrices are shown in columns (5) to (10). A measure given in (5) and (8) is the ratio of the root mean square (rms) of the changes to the rms of the original matrix of a reduced model. The other two measures named the absolute mean ratio of the diagonal changes and rms of the changes divided by the square root of product of the two corresponding diagonal elements are listed in columns (6) and (9) and columns (7) and (10), respectively. These last two are important measures since mass matrices are strongly diagonal. Apparently, large changes in matrices of a reduced model imply that the modes to be matched deviate far from any subject of eigensolutions of the model. In RM4, the number of modes to be matched (m) are chosen to be 5, 7, 10, and 15, and large m value means imposing more constraints on the AMI procedures. It is noted that m/n may not exceed one half or the improvement procedures would be too constrained to allow small modifications on the reduced model to match the assigned m modes.

To investigate the sensitivity of AMI to a missed mode, two cases are performed on RM20, RM30 and RM40 for comparison. In the first case, AMI is carried out to match the first five modes (the first one being rigid body mode) of the full model, while in the second case, the third mode is intentionally dropped. It is observed that the changes are about the same order for both

REDUCED MODEL (1)	n (2)	m (3)	MODES TO BE MATCHED (4)	ΔM(%)			ΔK(%)		
				(5)†	(6)*	(7)**	(8)†	(9)*	(10)**
RM4	15	5	1-5	1	.6	.5	1	.3	.3
		7	1-7	3	2	1.6	1	1	.6
RM11	15	7	1-7	3	1	.9	1	.4	.3
			1-3; 5-8	5	3	3	4	3	3
RM40 "Best"	10	5	1-5	2	.8	.6	2.5	.8	.7
			1-2; 4-6	3	1.6	1.8	.5	.8	.7
			3-7	1.4	1.4	1.7	3.2	2.8	2.2
RM30 "Medium"	10	5	1-5	22	21	23	7	7	8
			1-2; 4-6	42	8	7	35	13	13
			3-7	196	73	79	163	161	166
RM30 "Worst"	10	5	1-5	1459	525	533	730	257	261
			1-2; 4-6	23	65	61	26	22	20
			3-7	3699	5594	5566	4046	1952	1924

† = $\mathrm{rms}(\Delta A)/\mathrm{rms}(A)$ * = $1/n \; \Sigma|\Delta A_{ii}/A_{ii}|$ ** = $\mathrm{rms}(\Delta A)/(A_{ii}A_{jj})^{1/2}$

Figure 8. AMI Improvement.

cases if the unimproved reduced models (RM40 and RM30) are reasonably formulated. Evidently, RM20 is a poor reduced model as is seen from very large changes in the first case. Since changes are relatively small in the second case, RM20 seems not that bad if the third mode is not included in AMI. This can be attributed to the fact that the unimproved RM20 missed the third mode of the full model (see Figure 5). Therefore, AMI is numerically stable whenever the unimproved reduced model is well formulated even if certain low modes are not included in the improvement procedures.

Also shown in Figure 8 are the percentage changes of the three 10 DOF models when modes 3 to 7 of the full model are assigned to AMI. It is anticipated that the current changes are higher than the corresponding ones which match the first five modes.

Forced responses are given in Figures 9 to 12. For a good reduced model (RM11) based on the minimum ratio criterion, Figure 9 shows that AMI still can improve it to give better mobility prediction. As expected, improvement becomes more significant when frequencies are larger. For various 10 DOF reduced models (RM40, RM30 and RM20), Figures 10 to 12 show driving point mobilities at wing root trailing edge (Z19) plotted against the full model mobility. It is evident that the AMI improved models (based on either the first 5 modes or the modes 3 to 7) excellently match the specified modes of interest even the corresponding unimproved model (for instance, RM20) is highly inaccurate.

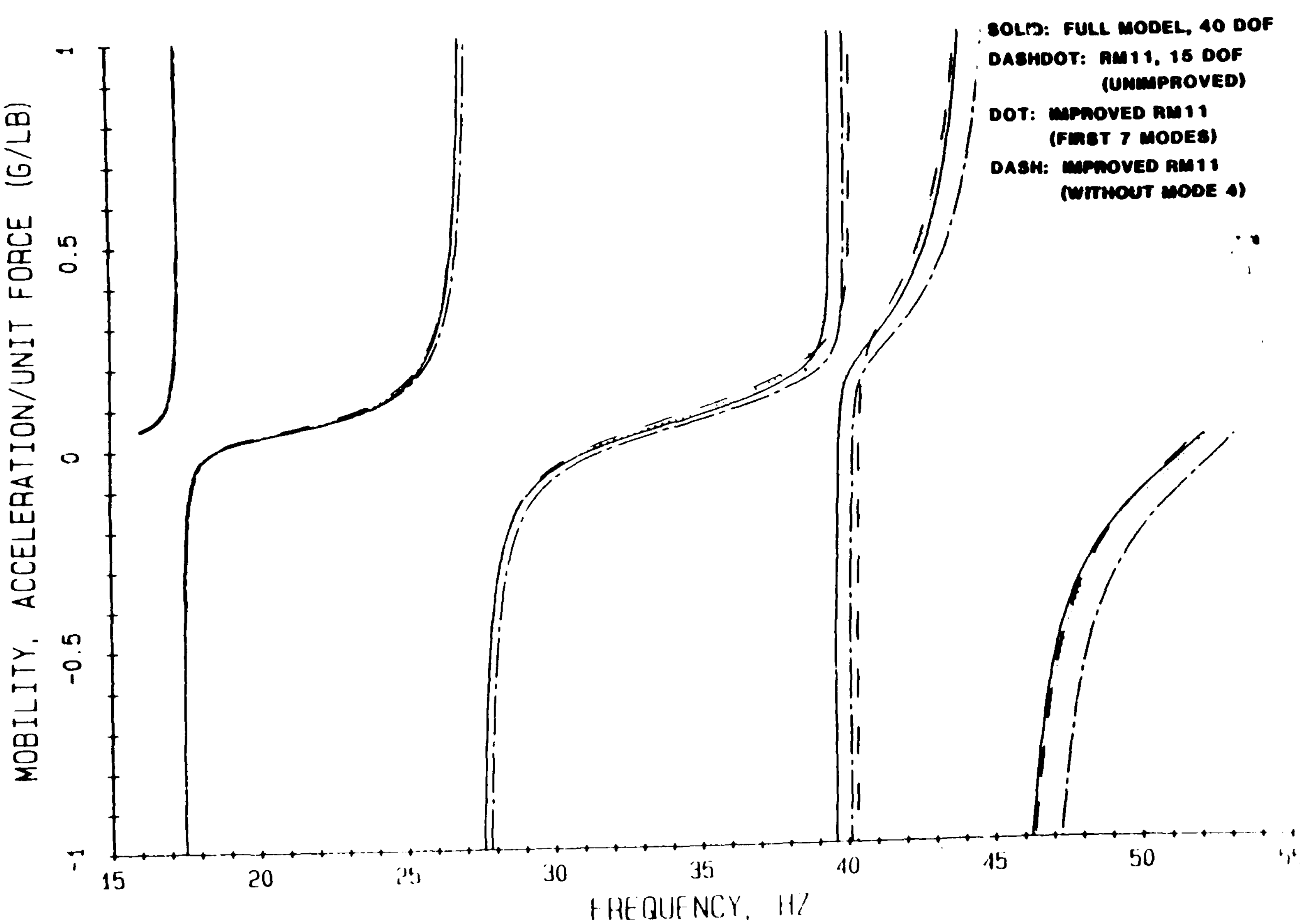

Figure 9. Frequency Domain Solution, Driving Pt. Mobilities: Wingtip (Z55), RM11.

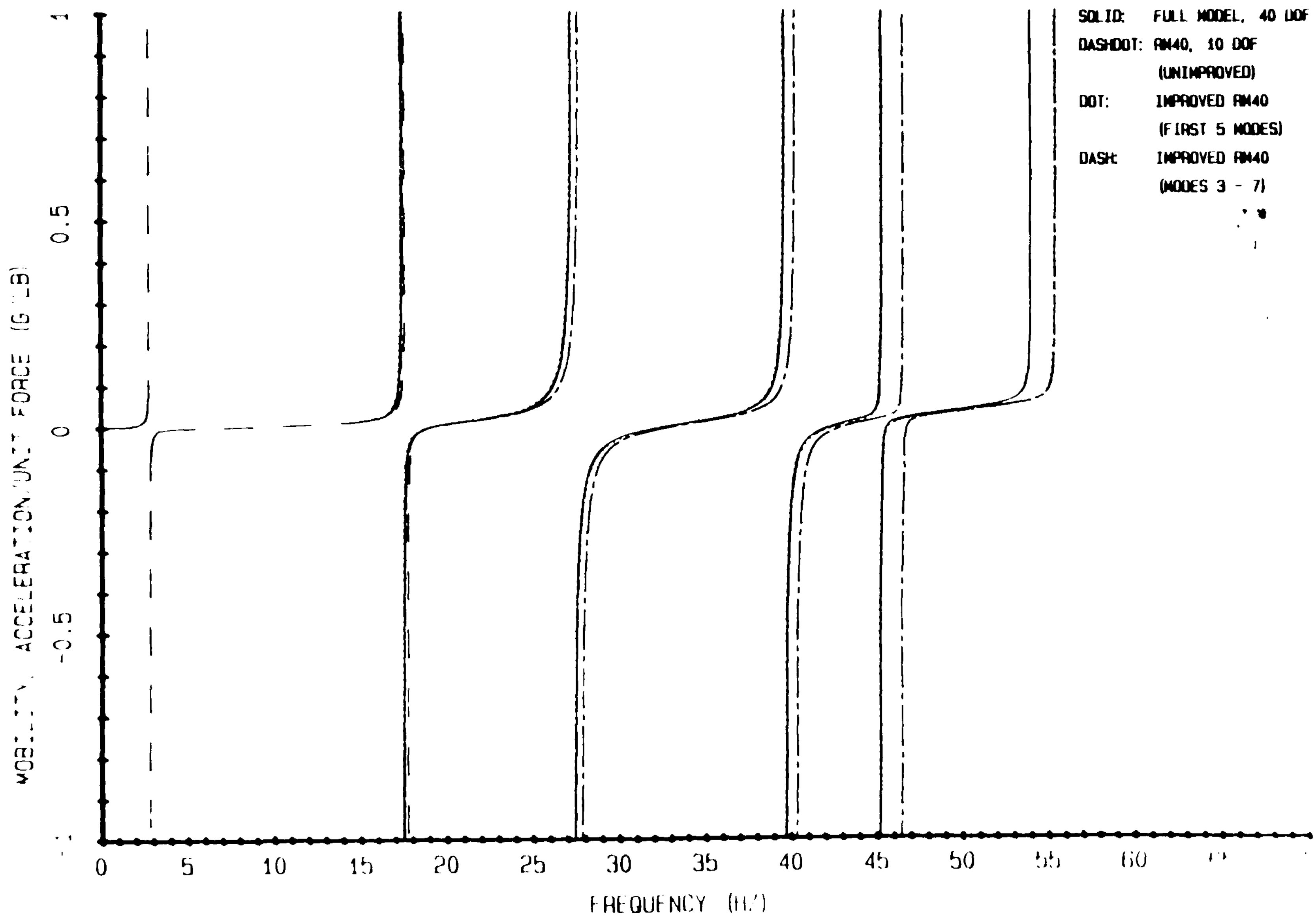

Figure 10. Frequency Domain Solution, Driving Pt. Mobilities: Wing Root Trailing Edge (Z19), RM40.

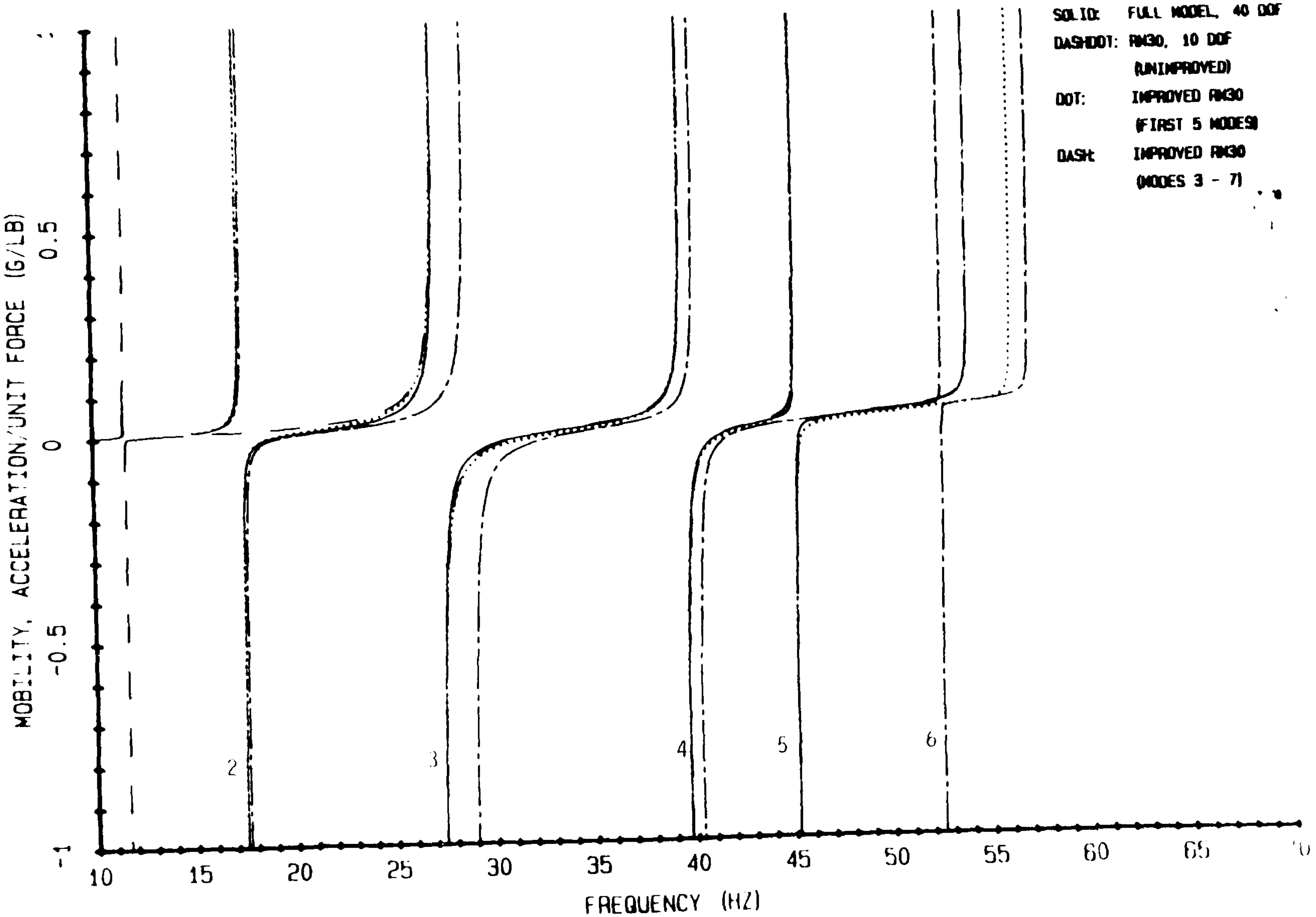

Figure 11. Frequency Domain Solution, Driving Pt. Mobilities: Wing Root Trailing Edge (Z19), RM30.

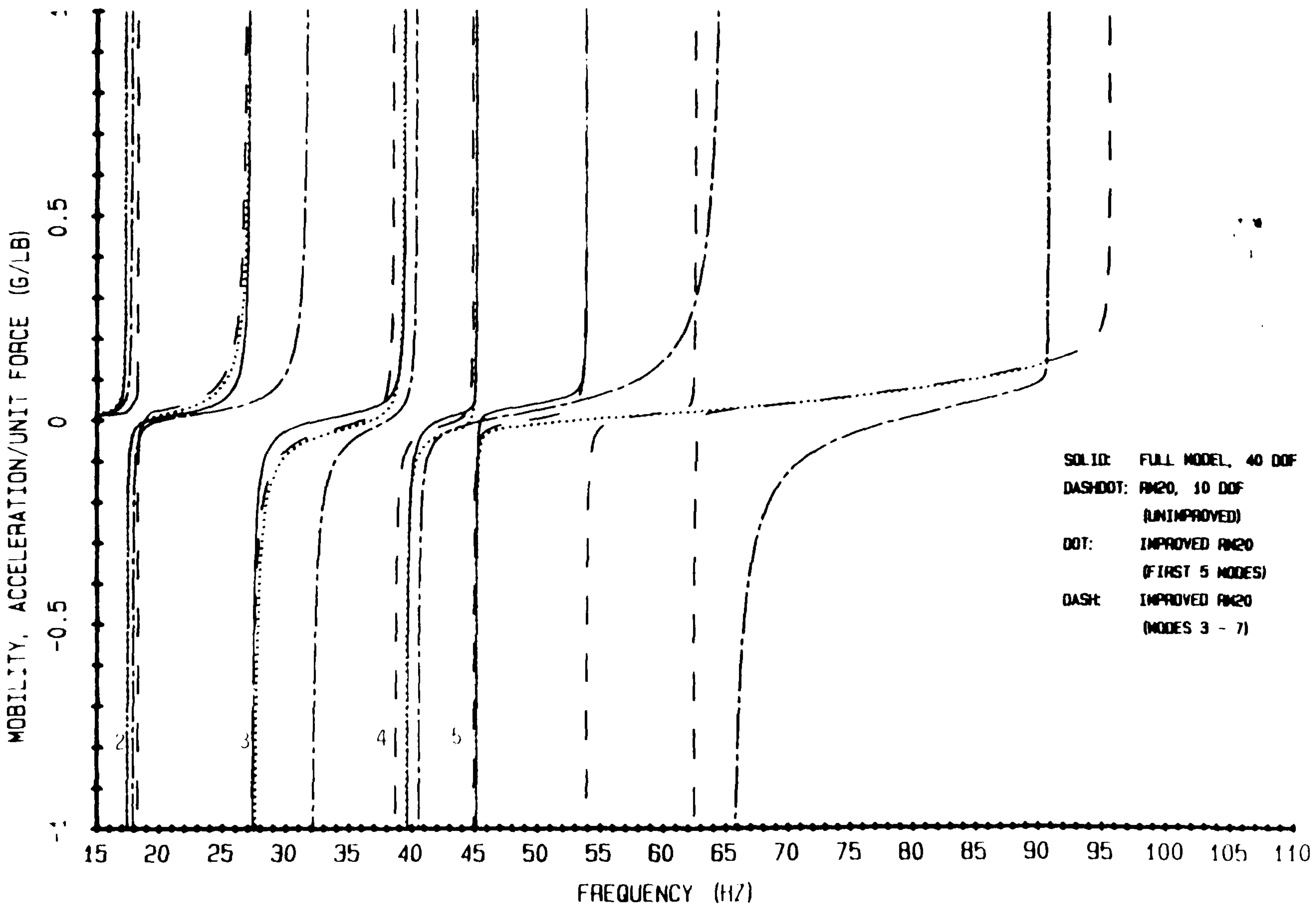

Figure 12. Frequency Domain Solution, Driving Pt. Mobilities: Wing Root Trailing Edge (Z19), RM20.

SECTION IV

EFFECTS OF STRUCTURAL CHANGES

Structural Modification

Design of a large space structure requires intensive interactions among project departments. When a design change is made, the whole analysis loop must be repeated, which includes reformulation of full and reduced models and thus is quite inefficient. It is desirable to be able to modify reduced models directly without a new analysis loop.

To clarify this idea, four sets of models are defined as follows. Before structural modifications, a full model is identified by M and K, and a reduced model by M_r and K_r. The corresponding full and reduced models are denoted by $\bar{M}$, $\bar{K}$ and $\bar{M}_r$, $\bar{K}_r$ respectively after modification of the structure. There are two approaches to achieve a modified reduced model $\bar{M}_r$, $\bar{K}_r$. The first obtains $\bar{M}$, $\bar{K}$ by directly modifying the original full model and then reduces the modified full model to get $\bar{M}_r$, $\bar{K}_r$. This approach is exact but tedious. The second approach avoids changing a full model. Rather, it reduces M, K to M_r, K_r and modifies this small system to achieve the modified reduced model ($\bar{M}_r$, $\bar{K}_r$). The latter approach certainly is much more efficient. The question remaining is the error it may introduce. That is, we have to investigate the capability of $\bar{M}_r$ and $\bar{K}_r$ to represent the effects of changes if the shortcut is taken.

Evaluation of Various Reduced Models

Figure 13 shows how we evaluate the capacity of various reduced models to accurately represent the effects of changes. In the figure, eigensolutions and/or frequency response from modified reduced model $\bar{M}_r$ and $\bar{K}_r$ (which are obtained through the shortcut as shown in the last section) are compared against those from modified full model $\bar{M}$ and $\bar{K}$. For numerical comparison we use the 40 DOF delta wing model (see Figure 3) as the original full model and the results are given in Figures 14 to 16. The modified full model is

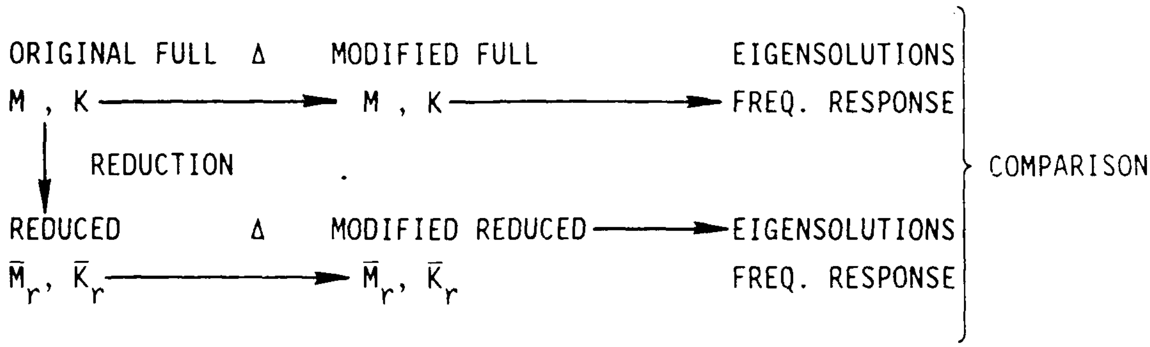

Figure 13. Evaluation of Reduced Model Capability To Represent the Effects of Changes.

MODEL	FREQUENCIES OF ELASTIC MODES (Hz)				
	2††	3	4	5	6
FO	17.39	27.27	39.56	45.13	53.89
FM	21.96 [26]	31.64 [16]	39.53 [-.06]	45.15 [.05]	53.83 [-.1]
RM30M	22.66 (3.2)	32.78 (3.6)	40.15 (1.6)	52.48 (16)	56.85 (5.6)
IM30M	21.74 (-1)	32.53 (2.8)	39.55 (.06)	45.17 (.05)	55.97 (4)
RM40M	22.18 (1.0)	31.99 (1.1)	40.15 (1.6)	46.38 (2.7)	55.24 (2.6)
IM40M	21.94 (-.08)	31.86 (.7)	39.53 (0)	45.16 (.01)	55.22 (2.6)

$\dagger\dagger\omega_1 = 0$

Figure 14. Frequencies of Various Modified Models.

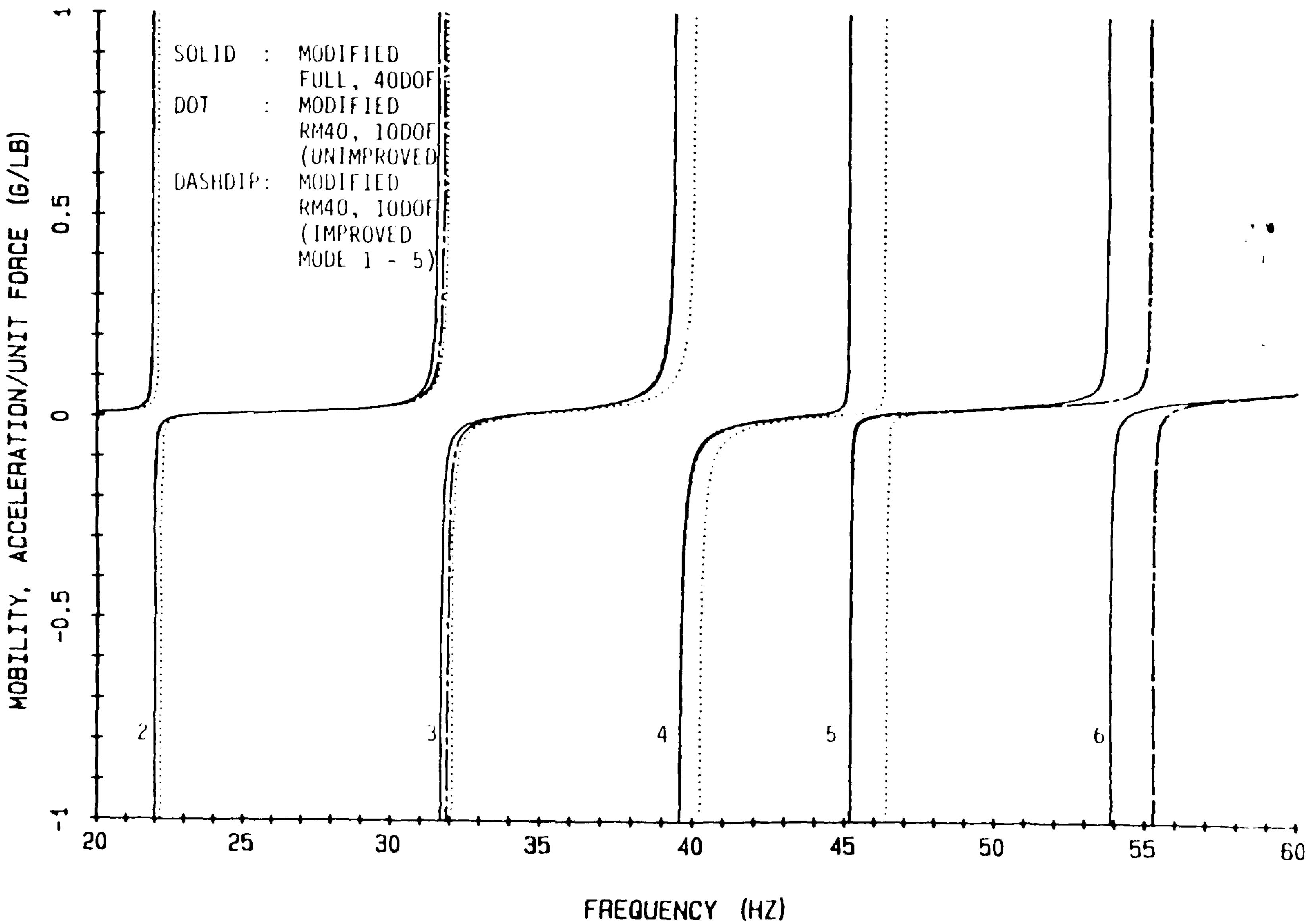

Figure 15. Comparison of Modified Models.

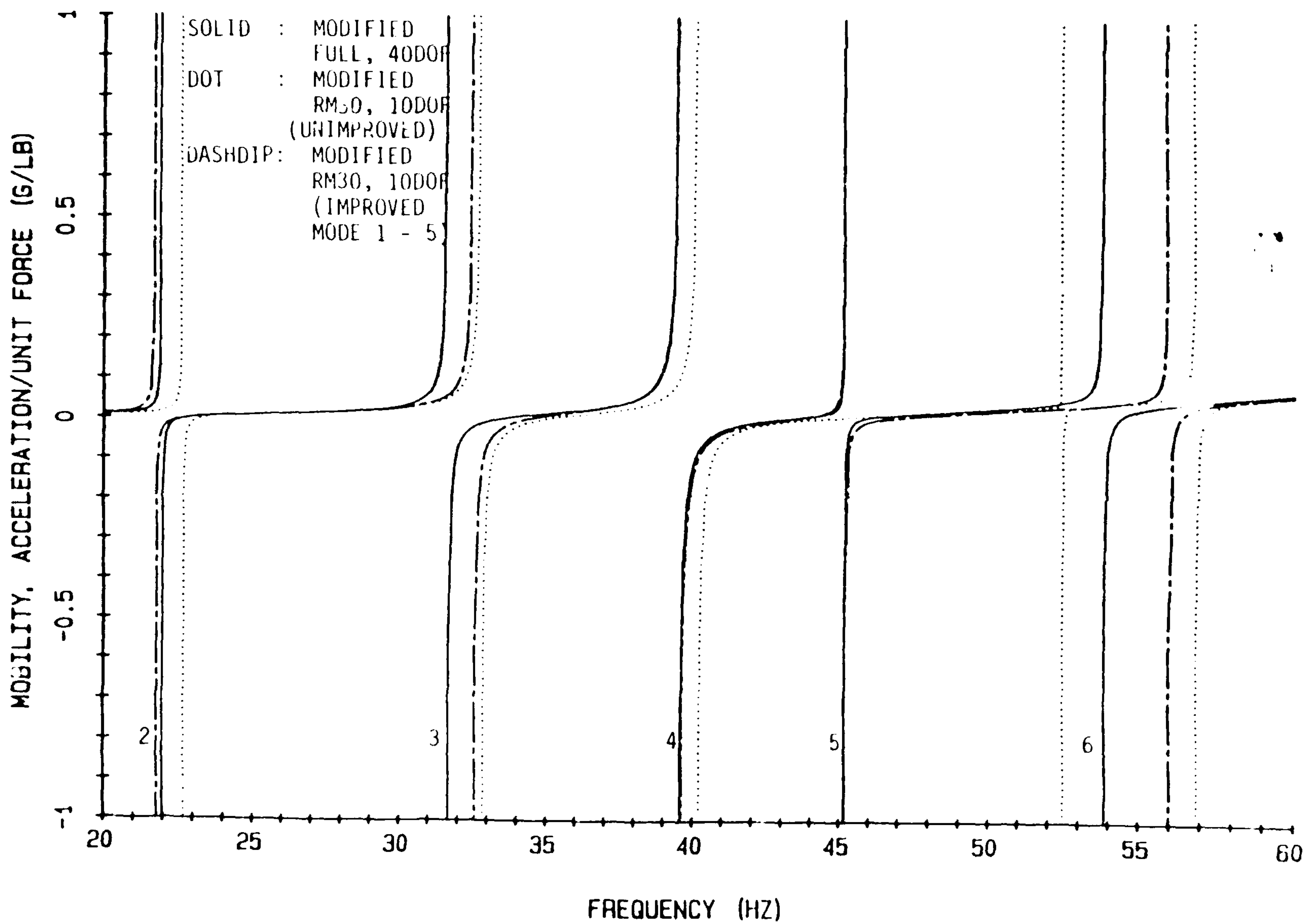

Figure 16. Comparison of Modified Models.

achieved by doubling mass and stiffness elements at Z11 and Z19 (see Figure 3). The changes are significant as can be seen from a 26% increase in natural frequency of the first elastic mode (compare the solid lines labeled 2 in Figures 7 and 15). The other modified reduced models in Figures 15 and 16 are obtained by changing the associated elements of the AMI improved and unimproved RM30 and RM40 (see Figures 5 and 8) accordingly.

Figure 14 lists frequencies of the first five elastic modes of two full and four reduced models. The two full models are the original (FO, as shown in Figure 3) and the modified (FM). The four reduced models are all modified ones which include RM30M, IM30M, RM40M and IM40M. Note that here "modified" (e.g., FM, RM30M and IM30M) is referred to 100% increase of mass and stiffness element values at Z11 and Z19 from the corresponding unmodified full (FO) or reduced (RM30, IM30) model. IM30 and IM40 are the AMI improvement (matching modes 1 to 5 of FO) of RM30 and RM40 before modification. In the figure, the percentage frequency change of a FM mode relative to the associated FO mode is given in brackets while the percentage change of a reduced model mode relative to the associated FM mode is given in parentheses. Obviously, from the eigenvalue comparison the improved IM30M and IM40M are better than their unimproved counterparts RM30M and RM40M in the frequency range shown. Also, the modified reduced models applying the minimum ratio criterion (RM40M and IM40M) are better than the corresponding models (RM30M and IM30M) without applying the criterion.

In Figure 15, the modified, improved reduced model (DASHDIP) shows much better agreement with the modified full model (SOLID) than the modified, unimproved one (DOT) in the specified frequency range. The same conclusion also can be drawn from Figure 16. This validates the effectiveness of using AMI improved reduced models as a baseline for design modification. Furthermore, comparison of the two DASHDIP cases in Figures 15 and 16 indicates that the modified improved RM40 better reflects the structural changes than the modified improved RM30. Therefore, it is effective to use the AMI improved reduced models whose DOF are chosen using the minimum ratio criterion as bases of modification when changes in structural design are made.

SECTION V

CONCLUSIONS

This report deals with several important aspects of dynamic model reduction and related issues. Guyan reduction is here applied to the design of large space systems because the procedure is highly efficient and the structure is quite flexible. The minimum-ratio criterion, which retains a subset of DOF whose corresponding diagonal element ratio of stiffness over mass matrices are the smallest, is drawn from an analysis of natural frequency errors. Comparison of eigensolutions and frequency responses of various reduced models reveals that the criterion is a good guideline for the selection of retained DOF. The reduced model is then improved by AMI program to exactly match desirable modes of the associated full model. Extensive studies show that this compensation algorithm is computationally efficient, numerically stable and formulated on sound physical grounds. Numerical solutions also indicate that the reduced model, which is based on the selection criterion and improved by AMI, excellently represents the structural modifications in the frequency band of interest. The proposed approach is, therefore, highly recommended for the design of large space structures.

SECTION VI

REFERENCES

1. Hurty, Walter C., "Dynamic Analysis of Structural Systems Using Component Modes." AIAA Journal, Vol. 3, April 1965.

2. Wilson, E.L., Yuan, M., and Dickens, J.M., "Dynamic Analysis by Direct Superposition of Ritz Vectors," Earthquake Engineering and Structural Dynamics, Vol. 10, pp. 813-821.

3. Kline, K.A., "Dynamic Analysis Using a Reduced Basis of Exact Modes and Ritz Vectors," AIAA Journal, Vol. 24, 1986, pp. 2022-2029.

4. Guyan, R.J., "Reduction of Stiffness and Mass Matrices," AIAA Journal, Vol. 3, 1965, pp. 380.

5. Joseph, J.A. (ed.), "MSC/NASTRAN Application Manual," Vol. 1, The MacNeal- Schwendler Corp., 1978.

6. Kidder, R.L., "Reduction of Structural Frequency Equations," AIAA Journal, Vol. 11, 1973, pp. 892.

7. Berman, A., "System Identification of Structural Dynamic Models - Theoretical and Practical Bounds," AIAA SDM Meeting, Palm Springs, CA, May 1984, AIAA-84-0929.

8. Hensell, R.D., and Ong, J.H., "Automatic Masters for Eigenvalue Economization," Earthquake Engineering and Structural Dynamics, Vol. 3, April 1975, pp. 375-383.

9. Shah, V.N., "Analytical Selection of Masters for the Reduced Eigenvalue Problem," International Journal for Numerical Methods in Engineering, Vol. 18, 1982, pp. 89-98.

10. Leung, Y.T., "An Accurate Method of Dynamic Substructuring With Simplified Computation," International Journal for Numerical Methods in Engineering, Vol. 14, 1979, pp. 1241-1256.

11. Petersmann, N., "Calculation of Eigenvalues Using Substructures and Dynamic Condensation," Second International Conference on Recent Advances in Structural Dynamics, Southampton, England, April 1984.

12. Garibotti, J.F., "Requirements and Issues for the Control of Flexible Space Structures," AIAA SDM Meeting, Palm Springs, CA, May 1984, AIAA-84-1025.

13. Thomas, D.L., "Errors in Natural Frequency Calculations Using Eigenvalue Economization," International Journal for Numerical Methods in Engineering, Vol. 18, 1982, pp. 1521-1527.

14. Berman, A., "Mass Matrix Correction Using an Incomplete Set of Measured Modes," AIAA Journal, Vol. 17, October 1979, pp. 1147-1148.

15. Berman, A., and Nagy, E.J., "Improvement of a Large Analytical Model Using Test Data," AIAA Journal, Vol. 21, 1983, pp. 1168-1173.

16. Chen, S.Y., and Fuh, J.S., "Reply by Authors to J.A. Brandon," AIAA Journal, Vol. 23, 1985, pp. 1840-1841.

17. Chen, S.Y., and Fuh, J.S., "Application of the Generalized Inverse in Structural System Identification," AIAA Journal, Vol. 22, December 1984, pp. 1827-1828.