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Fluctuations in Geophysical and Boundary Layer Flows

Final Technical Report

Grant # AFOSR - 78-3655

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REVIEW OF INVESTIGATION PROGRESS

A. Initial-Value Problems in Stratified Shear Flow

The question of the stability of a stratified shear flow is addressed through the investigation of the initial-value problem defined for a two-layer fluid of infinite extent with uniform velocity and density in each layer. Solutions of this problem have application to the generations of unstable motions in the atmosphere.

The eigenvalues of this problem can be expressed as the solutions of a quadratic polynomial in the frequency with coefficients which are functions of the wavenumbers, and relative densities and velocities. The value of the discriminant of this equation determines if the system is stable or unstable. Figure 1 is the locus of the neutral values of the discriminant as a function of velocity and wavenumber. The graph indicates that there is a threshold velocity below which the system is stable. Figure 2 is an example of the values of the imaginary part of the complex frequency for waves traveling at angles of obliquity of 0°, 30° and 60° to the mean velocity.

Solutions to the initial-value problem driven by a Gaussian pulse showing the initial distortion and the asymptotic development are shown in Figures 3 and 4, respectively.

The initial distortion is evaluated using a power series expansion and approximation of the complex dispersion relation as a polynomial in the horizontal wavenumbers. The expansion for non-dimensional time parameter 0, 0.05 and 0.75 are shown in Figure 3.

The two-dimensional asymptotic expansion using saddle-point techniques for this non-conservative system is evaluated at values of the time parameter of 20 and 60 and is shown in Figure 4. It is apparent that the system tends to become two-dimensional (long standing wave patterns) in the large.

A complete report of this work is contained in the Ph.D. dissertation of J.E. Bradt with a version for publication to be submitted.

A second piece of research as been completed (submitted for publication) and concerns the question of other singularities in the eigenvalue spectrum. A recent paper by Chimonas (G. Chimonas, 1979, Journal of Fluid Mechanics, 90, 1-69) drew attention to the fact that essential singularities are present and thereby result in (a) algebraic growth in time for the perturbations and (b) are not form preserving. Unfortunately, Chimonas did a purely inviscid calculation and our analysis shows that even a small bit of viscosity (large Reynolds number) negates this result. Nevertheless, the general Laplace transform technique does reveal that considerable care must be given to perturbation problems in stratified shear flow.

The general study of the problem of shear flow-internal wave interaction continues as well. A modelled situation is proving quite productive, particularly when the major goal is a representative of a spectrum of the fluctuating energy. Specifically, the work includes (a) viscous effects, (b) a constant mean shear, (c) a constant Brunt-Väsäiliä frequency (exponential density stratification), but omits any effects due to boundaries. This last point seems reasonable enough in view of the fact that we are concerned with motions in the interior of the atmosphere. Moreover, the technique used for solution is novel and is capable of producing results for arbitrary three-dimensional disturbances with the aid of an analogue computer. Both a Master's thesis and a publishable version of this work will soon be available.

B. Fluctuations in Geophysical Boundary Layers

Concentration on the analysis of perturbations in a turbulent Ekman layer has been the center of this research since the last report. Besides desiring the characteristics of the structure in the physical situation, an answer to the important question of body force effects on turbulence was sought. In order to reduce the system to a workable (and understandable) means it was decided to allow the important parameter to be the relative scale depth of the logarithmic portion of the boundary layer (near wall) to that of the total shear zone (including spiral). When this ratio is too small, the layer is unstable, a result quite different from conventional flat plate turbulent boundary layers where it is well known that a fully turbulent layer is stable. The ramifications of this are being explored further.

A report on both the laminar (wave packets) and turbulent Ekman layers will be available with the Ph.D dissertation of G.F. Spooner.

C. Internally or Externally Perturbed Boundary Layers over a Flat Plate

We consider steady viscous incompressible flow over a semi-infinite flat plate under the influence of either internal or external time-dependent perturbations. Experimentally, an internal perturbation may be realized by a vibrating ribbon and an external perturbation corresponds to small fluctuations superimposed on a uniform freestream.

We show that in either case, when the amplitude of perturbations σ is less than the boundary layer thickness ε but larger than ε^2 the unsteady correction terms to the conventional boundary layer equations are governed by certain linear variational equations with variable coefficients; these coefficients involve the Blasius velocity functions. For the case of internal perturbations we obtain certain unsteady boundary conditions to be imposed on the flat plate while for external perturbations the unsteady boundary conditions are specified at infinity in the vertical direction.

The derivation of these results proceeds systematically from the Navier-Stokes equations using conventional asymptotic arguments and does not involve the ad hoc assumption of parallel flow in the boundary layer. In fact, both the horizontal and vertical velocity components of the Blasius solution occur in our variational equations.

A preliminary program for numerically solving the variational equations for the case of a sinusoidal internal perturbation is complete, the results are qualitatively in accordance with experimental observations in that the disturbance region diffuses in the vertical direction and is convected downstream. This numerical procedure is only valid for moderate frequencies and does not apply for the more interesting case of high frequency disturbances. To analyze this limit we have rescaled the variables in our variational equations and we are currently working on a second numerical code.

Paralleling these numerical studies we have derived an asymptotic approximation of the perturbation equations in the limit of large excitation frequency. In this limit the basic problem reduces to a weakly nonlinear diffusion equation with an oscillating boundary condition. An approximation valid for short times has been derived and indicates that the disturbance in the boundary layer grows with time. In order to derive a uniformly valid result for long times we plan to develop a multiple variable solution. Multiple variable expansions have been used with much success in wave propagation problems as well as in some examples of weakly nonlinear elliptic equations. However there are no significant results available for weakly nonlinear parabolic problems with time dependent boundary conditions. We propose to concentrate on this point during the next grant period.

An initial report on this work will be given at the forthcoming meeting of the Division of Fluid Dynamics, American Physical Society, to be held at the University of Notre Dame in November 1979.

The alternative approach to analysis of the continuous spectrum-early time problem for the boundary layer is still is progress. Basic equations are available with a means for solution still not complete. This work is of high priority.

D. Report on a Turbulence Initiation Calculational Procedure

Analytical and experimental results obtained by a number of investigators (see, for example, Landahl, M.T., JFM 56, 1972, p. 775; Landahl, M.T. and Criminale, W.O., JFM 79, 1977; Wygnanski, I., Haritonidis, J.H., and Kaplan, R.E., JFM 92, 1979, p. 505; Gaster, M. and Grant, I., Proc. Roy. Soc. Lond. 347, 1975, p. 253; Kim, H.T., Kline, S.J., and Reynolds, W.C., JFM 50, 1971, p. 133, and the references contained therein) help focus attention on both the importance and the complexity of the nonlinear processes which take the place after the first appearance of a turbulent spot in a boundary layer. It appears useful to attempt to supplement those investigations with a reasonably accurate computational approach to the problem, and for this purpose we have considered several possible techniques, and have experimented briefly with a program based on one of them. The program appears promising, but it is clear that much remains to be done.

Using Cartesian index notation (and the usual summation convention), the Navier-Stokes equations for incompressible flow may be written

$$\frac{\partial u_k}{\partial t} + u_{k,s} u_s = -\frac{1}{\rho} P_{,k} + F_k + \nu u_{k,jj} \qquad (1)$$

where u_k is the k'th component of velocity, ρ is the density, F_k the component of body force, and ν the kinematic viscosity. Taking the curl of Eq. (1) we obtain after simple manipulation an equation for the vorticity vector ε_i:

$$\frac{\partial \xi_i}{\partial t} + \xi_{i,s} u_s - \xi_r u_{i,r} = \varepsilon_{ijk} F_{k,j} + \nu \xi_{i,jj} \qquad (2)$$

and we note also that

$$u_{i,jj} = - \varepsilon_{ijk} \zeta_{k,j} \tag{3}$$

The advantage of using a vorticity equation is that the pressure derivative is eliminated. Our reason for including a body force term is twofold. First, its use provides a convenient and flexible triggering mechanism, which avoids the necessity of inserting a perturbation in either the entering or existing laminar flow. Second, there is experimental evidence to the effect that a longitudinal flow vortex can play an important role in the growth of turbulence near an initial turbulent spot; one way in which such a vortex can be generated is if there is an unequal division of flow around a turbulent spot (the instability of parallel channel flows is well known), and this kind of inequality can be easily triggered by the use of an appropriate small body force distribution.

Our approach is to take time steps in vorticity, using Eq. (2), and to then use Eq. (3) to determine the new velocity field. The region of interest is any simple geometrical region containing the turbulent spot, with the boundaries sufficiently far removed that in the initial portion of the turbulence build-up there will only be a negligible perturbation felt at these boundaries. This will of course not be precisely true, and in fact it will not be at all correct on, say, a flat plate boundary near which the turbulence is being generated. Consequently, a correction procedure, near any such boundary, will be necessary.

To take a time step in Eq. (2), we have so far used a simple explicit procedure. Although this can be chosen to be both stable and accurate, and has in fact been quite satisfactory for the initial investigation, it is recognized that larger time steps are feasible for implicit procedures of the kinds developed for diffusion type equations. Possible improvement in this particular procedure will be examined. Using the new vorticity values, Eq. (3) has the form of a Poisson equation for each component of velocity. Assuming first that the boundary values of velocity are those of the laminar flow, new interior values of velocity corresponding to the new vorticities may be obtained by a successive optimal over-relaxation procedure which (for a prismatical region, say) is very efficient. The number of iteration sweeps required to reduce the error by a factor of 100 is of the order of the number of mesh points along a side; the initial error is kept small by using, for a first guess, either the velocity values appropriate to the previous time step or extrapolations from those values.

The coupling between Eqs. (2) and (3) required for boundary corrections arises at the next step in the program. In order to compute interior vorticity values for the next time step, we need new vorticity values on the boundary. However, we do not have a priori values for velocity derivatives (and hence vorticities) on the boundary; thus we must extrapolate to obtain these values from interior values. This kind of calculation is known to lead to possible inaccuracies and even instabilities, and it may be necessary to devise safeguards against such inaccuracies. There are some situations in which analytical information involving derivative values are available from the above equations, and if so the numerical calculations can be simplified.

Another problem which must ultimately be dealt with is that of mesh refinement near critical regions, for the purpose of enhanced accuracy. Our experience with other problems of diffusion or potential flow character leads us to expect this possibility to be useful (in that it reduces the amount of computer time required for a desired accuracy of result), we have not experimented with this feature in the present problem.

A preliminary program has been written, and applied to the initial phases of turbulence growth near a flat plate boundary. The mean stream flow here was

Poiseuille flow character, with an initially parabolic velocity distribution. Non-dimensionalization was used, and, in terms of the distance to the inflection point in the parabolic profile, the Reynolds numbers used were primarily in the range of a few hundreds to a few thousands. Disturbances were generated by impulsive localized body force contributions, and with about 1500 mesh points, a few tens of time steps were taken.

With the limited time and computational facilities available, only very preliminary results could be obtained. However, the program did appear to be both accurate and stable, at least for the initial part of the motion. To date, both the space and time meshes are crude, the control of perturbed boundary values is primitive, and -- as previously mentioned -- refinements aimed at increasing accuracy or stability have not been made. Our general feeling is that the method appears to be well worth further investigation as a tool in better understanding the growth of turbulent regions, and we propose to undertake such an investigation.

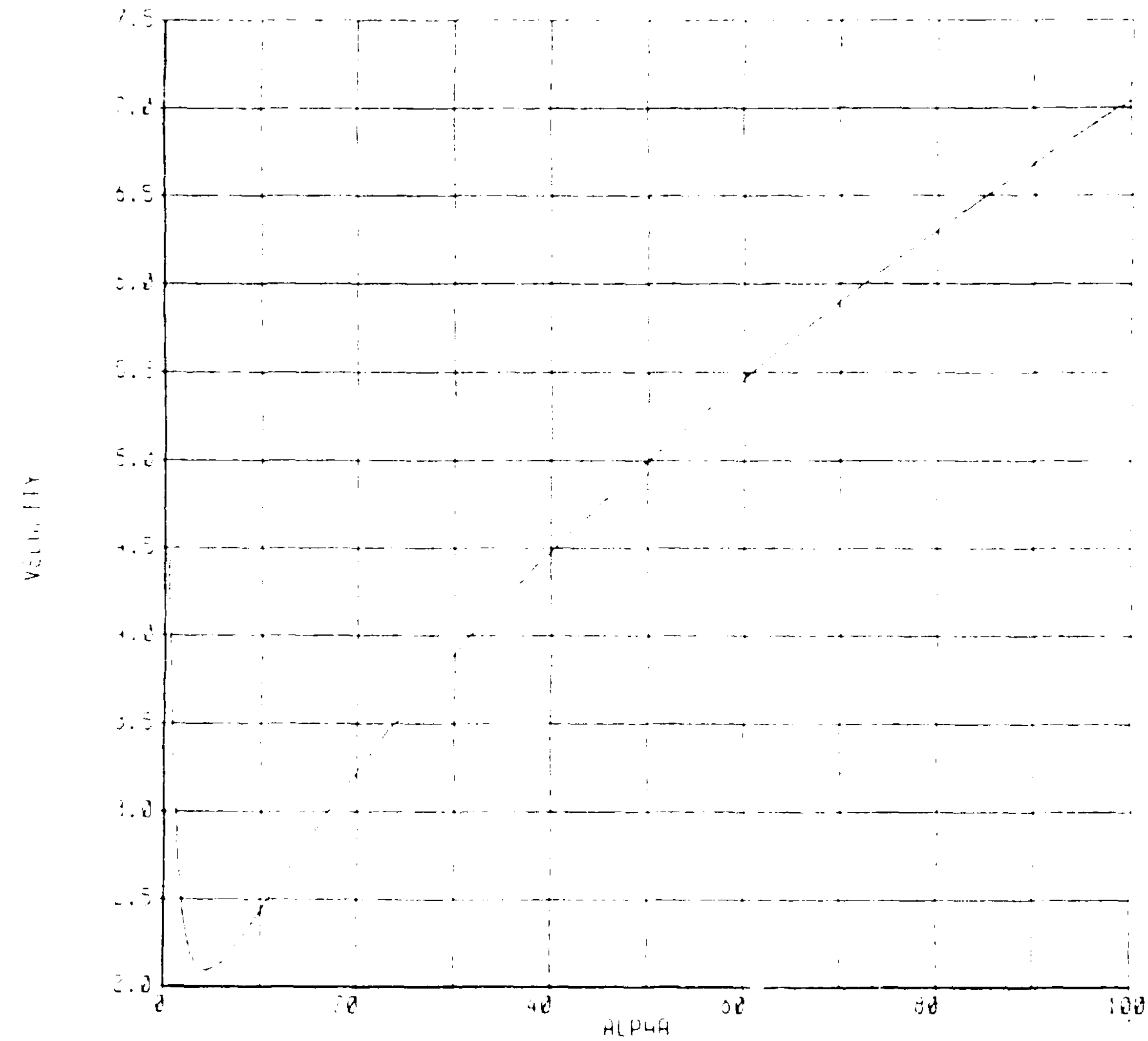


Fig. 1. Locus of neutral curve as function of velocity and wavenumber.

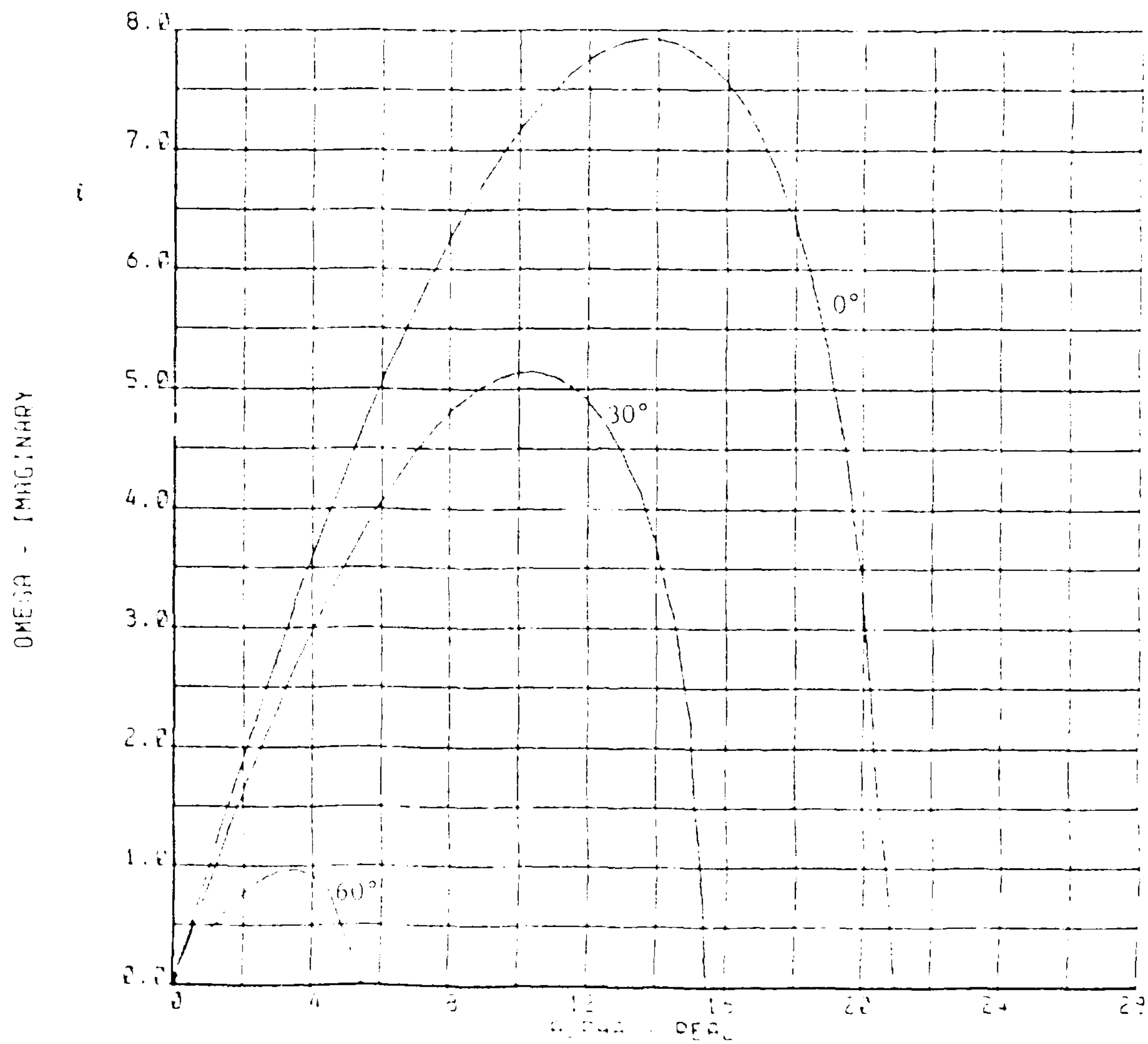


Fig. 2. Growth factor as a function of wavenumber.

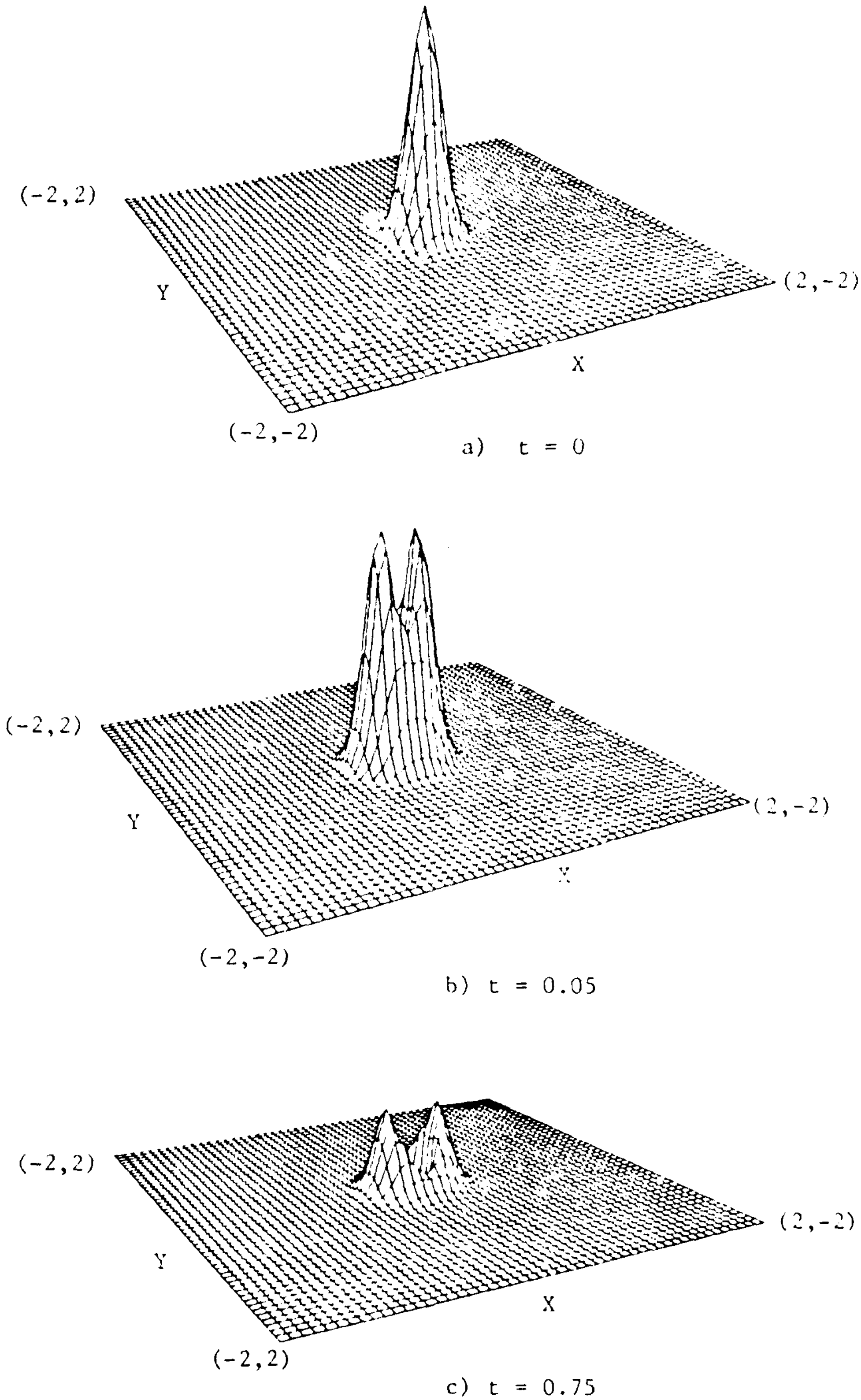
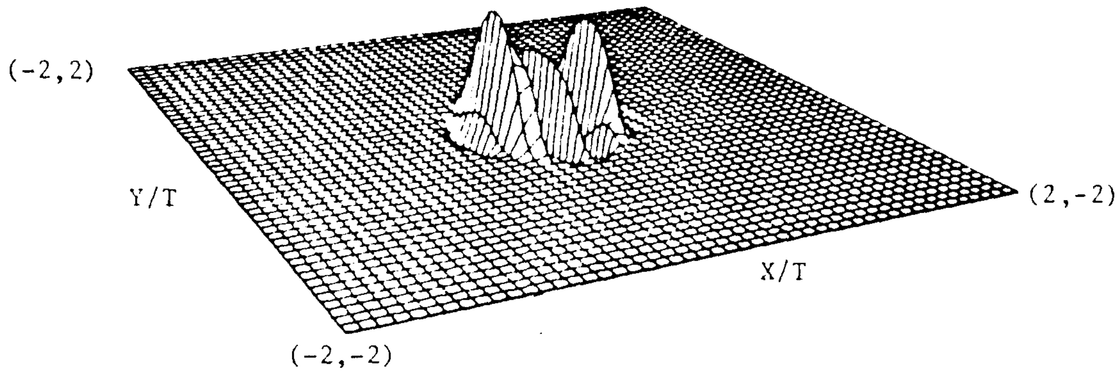
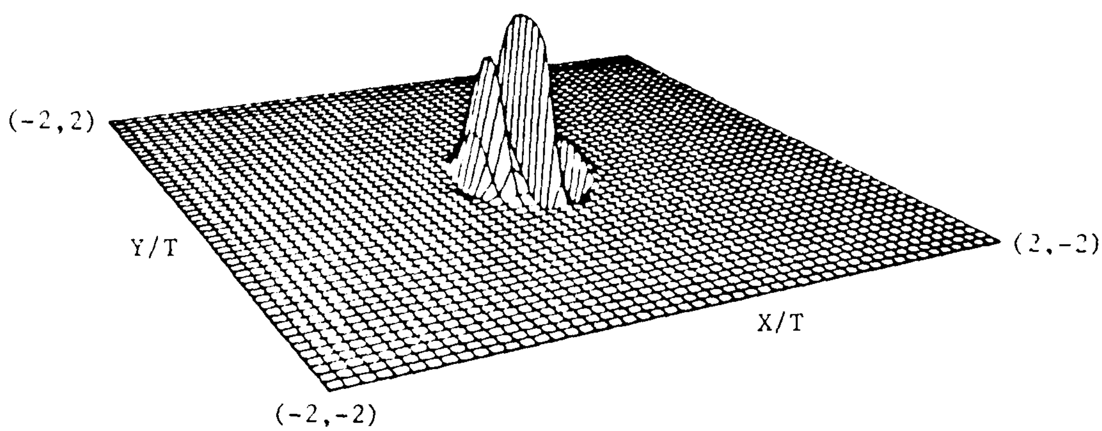


Fig. 3. Initial distortion of Gaussian pulse of standard deviation $\lambda = 0.1$.



a) t = 20



b) t = 60

Fig. 4. Asymptotic expansion of Gaussian pulse of standard deviation $\lambda = 0.1$.