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Final Technical Report

AFOSR 77-3173

General Similarity Solution of the Tricomi Equations and Its Applications

List of Research Objectives. **AFOSR-TR- 89-1412**

(i) develop the formalism for the general similarity solution of the Tricomic equation which is the hodograph equation of transonic small-disturbance theory.

(ii) study higher order approximations in perturbation theory, for transonic flow with the aim of clarifying the ambiguity of the transonic similarity parameter K.

(iii) study the nose singularity for a rounded nose airfoil in transonic flow and develop a local approximation based on the exact equations.

Summary of Research

Almost all the research objectives above have been achieved.

Using Lie group methods and Backelund transformation, a complete catalog of similarity solutions has been found. It seems likely that some of these solutions would find applications in special transonic flow problems formulated in the hodograph. In one example, Dr. E. Tse has used these solutions as part of a flow calculation of flow past a lifting airfoil at $M_\infty = 1$ (Ref 1). This work is reported in publications 1. and 2. and we also expect a future report.

Higher order approximations to transonic small-disturbance theory were studied with the aim of clarifying the role of transonic similarity parameter k_1 and stretched coordinate $\tilde{y}$. Two prototypical examples, supersonic expansion flow past a corner and the attached shock wave on a wedge were worked out to second-order. The potential is expanded as

$$\Phi(x,y;M_\infty,\delta) = U\{x + \delta^{4/3}\phi_1(x,\tilde{y};k_1) + \delta^{4/3}\phi_2(x,\tilde{y};k_1) + \cdots\}$$

where

$$M_\infty^2 = 1 + \delta^{2/3}k_1 + \delta^{4/3}k_2(k_1) + \cdots$$

$$\tilde{y} = \delta^{1/3}y\,\{1 + \delta^{4/3}\Lambda(k_1) + \cdots\}$$

The limit process has $\delta \to 0$, k_1 fixed. We want to ascertain if there is a special choice of k_1, Λ so that the correction to the surface pressure due to the second approximation is as small as possible. For the expansion fan we must have

$$\Lambda = \frac{1}{3(\gamma+1)^2}\left\{\left((\gamma+1)\left(\frac{6}{5}\gamma+2\right) + \frac{4}{5}\gamma k_1^{3/2}\right)\xi_b^2 - \frac{4}{5}\gamma k_1^{5/2}\right\}$$

and

$$k_2 = 2k_1\Lambda, \quad \xi = \frac{x}{\tilde{y}}$$

in order for any second order solution to exist which can satisfy the boundary conditions. For the shock wave on the other hand there is no definite result. If we make the Mach angle structure correct to $O(\delta^{4/3})$ we get again

$$\Lambda = \frac{k_2}{2k_1}$$

We can make $\phi_{2x} = 0$ on the surface so that $C_p = -2\delta^{2/3}\phi_{1x} + O(\delta^2)$ by choosing

$$k_2(k_1) = \frac{\gamma+1}{2(k_1\phi_{1x}^2-1)}\left\{\frac{\gamma-3}{k_1} + 4\frac{k_1^2}{\gamma+1} + k_1\phi_{1x} - \left(\frac{\gamma-3}{2k_1} + 2\frac{k_1^2}{\gamma+1}\right)\left(3-k_1\phi_{1x}^2\right)\right\}$$

The main conclusion of this research is thus negative. There is no simple choice of k_2 (k_1) that is valid for both these cases. Thus there is no way to extrapolate to more general cases. Details of these results appear in Publ. 3.

Publ. 3 also details the solution of the problem stated in the research objective (iii). A local exact equation was derived for the neighborhood of the nose region. This equation contains basically all the terms in the full potential equation. A numerical solution was carried out which matches to the small-disturbance singular solution. This singular solution is written up in Publ. 4.

Julian D. Cole

Julian D. Cole
Principal Investigator

Publications and Reports

1. Feng, C.K.. The General Similarity Solution of the Tricomic Equation with Application to Transonic Flow. Ph.D. Thesis. UCLA. 1977. 126 pp.

2. Feng, C.K.. The General Similarity Solution of the Tricomic Equation. Paper presented at Israel Congress of Aeronautics. 1978. To appear in Israel Journal.

3. Kusunose, Kazuhiro. Two-Dimensional Flow Past Convex and Concave Corners at Transonic Speed and Two-Dimensional Flow around a Parabolic Nose at Subsonic and Transonic Speeds. Ph.D. Thesis. UCLA. 1979. 137 pp.

4. Cole, J.D. The Nose Singularity of Transonic Small Disturbance Theory for a Lifting Airfoil in Preparation.

References

1. J.D. Cole and E. Tse. Airfoils at Sonic Velocity. Paper presented at Meeting of Fluid Dynamics Division. APS. Nov. 1978.