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Final Scientific Report

### Grant AFOSR 74-2649

# DEVELOPMENT OF ANALYTICAL AND SEMI-NUMERICAL METHODS OF FLOW CALCULATION

1 December 1973 - 30 November 1978

The original 1973 proposal listed five promising methods of combining the computer with analysis for flow calculations. By the following year we had decided to concentrate on two of those used in tandem: first extending a regular perturbation series to high order by delegating the arithmetic labor to the computer, and second analyzing and improving that series.

We have tried to develop and popularize this two-part semi-numerical scheme by solving a variety of realistic problems, and by discussing the results at meetings and seminars as well as in print.

### Specific problems solved

The following problems in viscous and compressible flow were solved during the course of the Grant, and have been or will be published, as indicated:

- 1. Impulsive heating of the boundary layer on a plate (Van Dyke 1975)
- 2. Viscous decay of a cubical array of vortices -- the Taylor-Green model of turbulence (Van Dyke 1975)
- 3. Transonic flow through a nozzle (Van Dyke 1976)
- 4. Laminar flow up a uniformly heated vertical pipe (Van Dyke 1977)
- 5. Laminar flow through a loosely coiled pipe (Van Dyke 1978)
- 6. Viscous flow produced by a spinning sphere (Van Dyke 1979)
- 7. Viscous flow inside a moving circular boundary (Conway 1978)
- 8. Laminar flow through a uniformly heated horizontal pipe (Van Dyke, to appear)
- 9. Transonic flow past a circle by the M² expansion (Van Dyke & Guttmann, to appear)

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- 10. Laminar flow through a rotating pipe (Mansour, to appear)
- 12. Free convection in a circular cavity (Mansour, to appear)
- 13. The strong shock wave from a plane piston started impulsively and then stopped (Reddall, to appear)

#### Unexpected results

In each of the problems listed above, applying the technique of series extension and improvement has served to provide a more complete or accurate solution than would be possible with other numerical or analytical techniques. In addition, we have time and again discovered that conventionally accepted results are incorrect. We cite five examples (of which the first two were started under the previous Contract F 44620-69-C-0036):

<u>Highest water wave</u>. Stokes conjectured that his 1849 series solution would converge up to the highest wave, with a 120degree sharp peak. However Schwartz (J. Fluid Mech. 1974) showed that the Stokes series does not converge there until it has been suitably modified.

<u>Self-similar blast wave</u>. It is generally accepted that the strong shock wave produced by a blunt body in steady flight, or by the analogous unsteady movement of a piston, decays like the "blast wave" produced by an intense concentrated explosion. However Reddall (to appear) finds that the decay is somewhat faster, because this turns out to be one of the curious class of problems that are in Russian called "self-similar solutions of the second kind."

Friction in coiled pipe. All existing boundary-layer models suggest that the friction factor for fully developed laminar flow through a loosely coiled pipe grows eventually as the square-root of the appropriate similarity parameter, the so-called Dean number. However, our calculations (Van Dyke 1978) show that it grows instead as the one-quarter power, in accord with experiments for loose coiling.

<u>Transonic controversy</u>. Since the 1940's most experts in transonic flow have supported the argument of Busemann and Guderley that transonic flows past airfoils with shock-free supersonic zones are exceptional, unstable, and isolated. However, our extension of the M² expansion for the circle (Van Dyke & Guttmann, to appear) shows that there is instead a continuous range of shock-free flows up to  $4\frac{1}{2}$  per cent above the critical Mach number.

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Enstrophy catastrophe. There is a belief among many workers in turbulence theory that the continual increase in the vorticity of a fluid that results from the stretching of vortex lines is held in check only by viscous diffusion, so that in the absence of viscosity (at infinite Reynolds number) the mean-square vorticity (the "enstrophy") would become infinite within a finite time -- the so-called enstrophy catastrophe. However, our computer extension of the Taylor-Gree problem of vortex decay (Van Dyke 1975) suggests that this catastrophe does not occur. We hope to confirm this by more detailed calculations.

From a more analytical viewpoint, an unexpected discovery of the last year has been that the singularities that limit the convergence of a power series need not lie on the real axis. In the problems of a spinning sphere and the blast wave from a piston we find that convergence is limited by a conjugate pair of singularities located in the complex plane of the expansion quantity. This information proves vital in extending the range of utility of the series, for which we have found some useful techniques.

#### Talks and Publications

During the five years of the Grant, the principal investigator and his Ph.D. students have presented their work at a dozen national and international meetings and a score of university and industrial seminars in seven different countries. In addition, the following journal publications carry acknowledgement of support by the Grant:

- VAN DYKE, M. 197, Computer extension of perturbation series in fluid mechanics. SIAM J. Appl. Math. <u>28</u>, 720-734
- VAN DYKE, M. 1976 Extension, analysis, and improvement of perturbation series. 10th Symposium on Naval Hydrodynamics, U. S. Govt. Printing Crice, 449-457
- VAN DYKE, M. 1977 From zero to infinite Reynolds number by computer extension of Stokes series. <u>Singular Perturbations</u> <u>and Boundary Layer Theory</u> (A. Dold & B. Eckmann, eds.), Lecture Notes in Mathematics, No. 594, Springer, Berlin, 506-517.
- CONWAY, B. A. 1978 Extension of Stokes series for flow in a circular boundary. Phys. Fluids <u>21</u>, 289-290

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- VAN DYKE, M. 1978 Extended Stokes series: laminar flow through a loosely coiled pipe. J. Fluid Mech. <u>86</u>, 129-145
- VAN DYKE, M. 1979 Semi-analytical applications of the computer. Fluid Dynamics Transactions, Warsaw (to appear).

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