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Final Report for AFOSR Grant 85–0330
Joseph Glover, Principal Investigator
Kai Lai Chung, Co–Principal Investigator
May 28, 1989

1. Semilinear Equations.

A major thrust of the original proposal was to find new probabilistic methods for dealing with semilinear partial differential equations. Mathematicians are currently devoting more of their attention to studying nonlinear partial differential equations since they recognize that descriptions of physical phenomena must incorporate nonlinear behavior. Glover succeeded in finding a method for solving systems of semilinear elliptic equations by a new procedure which does not need the old hypotheses of quasi–monotone systems [1]. It combines probability, analysis and a transfinite induction scheme to solve equations of the form

$$-\Delta u_1 + f_1(u_1, \cdots, u_n) = \mu_1$$
$$\vdots$$
$$-\Delta u_n + f_n(u_1, \cdots, u_n) = \mu_n$$

on a domain E in $\mathbf{R}^d$ subject to boundary conditions $u_1 = u_2 = \cdots = u_n = 0$ on the boundary of E. This result was published in May, 1987, and Glover subsequently gave a one–hour invited lecture on the subject of solution of semilinear equations at a week–long AMS Summer Research Conference in July, 1987 at Cornell University. He wrote an article [2] for the proceedings of the conference which synthesizes most of the known results on these equations.

In an article accepted for publication [3], Glover, Lazer and McKenna investigated particular semilinear equations which exhibit catastrophic behavior. They concentrated on one equation which models a bridge suspended by one cable, taking into account the fact that the restoring forces are different if the bridge bed is displaced upwards or downwards. The equation displays the following behavior, which is best explained in terms of the bridge it models. Subject the bridge to an oscillation of magnitude b (such as might be produced by a brisk wind). The resulting steady state oscillation of the bridge is of similar magnitude. But if one increases the magnitude of the driving oscillation by a very small number (in simulations, on the order of one ten thousandth), the magnitude of the bridge oscillation increases by a factor of five to ten. This is a direct consequence of the fact that the equation modelling the phenomenon is nonlinear. These results have caught researchers in the field by surprise.

2. Gauge and Conditional Gauge Theorem.

The gauge theorem was originally established for the Feynman–Kac functional

$$exp[\int_0^T q(B_t)\, dt]$$

where B_t is the Brownian motion in $\mathbf{R}^d$, q a suitable function and T the exit time from a domain. The theorem has now been extended twofold in an article of Chung and Rao [4]: the process may now be one in a general class of Markov processes (containing discontinuous Markov processes), and the functional may be a general multiplicative functional of the form $exp[A(T)]$, where A_t is an additive functional for the process. The finiteness of the generalized gauge function at any point in the domain implies its boundedness in the closure of the domain. In this case, a boundary value problem for the associated equation is solved by a simple representation. The special case of the Feynman–Kac functional leads to the reduced Schrodinger equation with Dirichlet data:

$$(\Delta/2 + q)\varphi = 0 \;\; in \; D; \quad \varphi = 0 \;\; on \;\; \partial D$$

The new extension comprises a variety of similar partial differential equations and integral equations. For instance, when A_t is the local time on a smooth boundary, the gauge relates to the solution of a mixed boundary value problem. Chung gave a one hour invited lecture on the topic of the gauge theorem and probabilistic approaches to solving partial differential equations at the Prague Conference on Potential Theory in July, 1987. He has written a synthesis of the subject for the proceedings of this conference [5].

Chung has obtained a new sharp estimate for the Green function in a ball [6]. If $d \geq 3$ and B is an open ball, let $p(x)$ be the distance from x to ∂B, and let

$$F(x,y) = min\{|x-y|^{2-d}, p(x)p(y)|x-y|^{-d}\}$$

If G is the Green function for B, then there are constants c_1 and c_2 depending on d so that $c_1 F \leq G \leq c_2 F$.

3. Energy and Dirichlet Spaces.

In collaboration with M. Rao and W. Hansen, Glover showed that the electrostatic capacity of a symmetric potential theory determines the potential theory uniquely [7]. One consequence of this result is that two symmetric Markov processes with the same capacities must be time changes of one another. In terms of analysis, this result states that from the knowledge of its capacity, one can recover a self–adjoint elliptic differential operator up to a multiplicative factor.

An outgrowth of this work was Glover's remark that for a nonsymmetric potential kernel, the sum of the potential kernel and its dual $u(x,y)+u(y,x)$ is often another potential kernel $v(x,y)$. Whenever this holds, the maximum principle of potential theory holds. This is the most difficult principle of potential theory to verify in the nonsymmetric case, and general conditions guaranteeing it holds are much sought after. In [8], Glover and Rao examined this method of symmetrizing a Markov process and another method based on Dirichlet space considerations. They give a set of sufficient conditions to guarantee that $u(x,y)+u(y,x)$ is a potential kernel, and they posed several questions which remain unanswered. In a subsequent paper [9], they gave a sufficient condition for the original process to satisfy the maximum principle.

4. Markov Processes with Random Times of Birth and Death.

In collaboration with R. K. Getoor, Glover wrote an article extending and simplifying a theorem of Kuznetsov allowing construction of Markov processes with random times of birth and death. Such processes arise in linear Markov process theory as well as in nonlinear situations [10].

5. Symmetry Groups and Markov Processes.

In two recent articles accepted for publication [11],[12], Glover has introduced a group of symmetries of the excessive functions of a Markov process X_t and a group of symmetries of the excessive measures of a Markov process. In the first article, written in collaboration with J. Mitro, he gives an algebraic method to construct Markov functions Φ. These functions have the property that $\Phi(X_{\tau(t)})$ is a Markov process for some suitably chosen time change $\tau(t)$ of the process X_t. The existence and construction of such Markov functions has been a classical problem in Markov process theory. The second article looks at the same problem from the dual point of view. In addition to the immediate use of the techniques in constructing Markov functions, it has become clear that the study of the symmetry groups is an important fundamental technique in the study of Markov processes and potential theory which has been overlooked.

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