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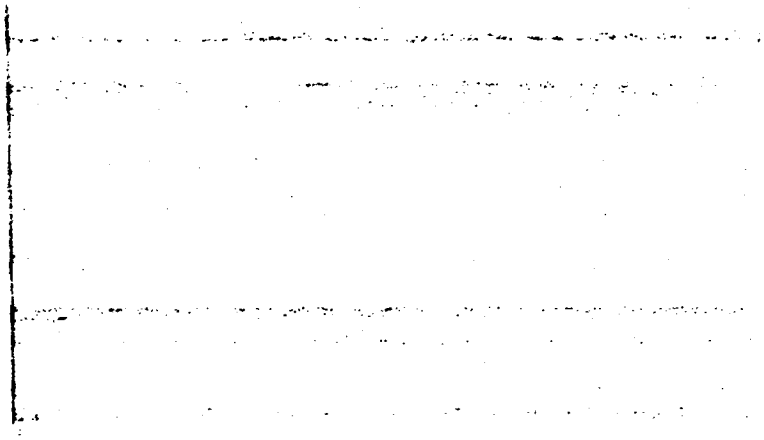

DEPARTMENT OF MATHEMATICS, STATISTICS, AND COMPUTER SCIENCE

Final Scientific Report to Air Force Office of Scientific Research Contract AFOSR 85-0320

Design of Experiments and Reliability Models

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SUMMARY

In this report we list the scientific achievements and activities sponsored by the U.S. Air Force Office of Scientific Research under contract AFOSR 85-0320.

I. INTRODUCTION

We report here the achievements and activities sponsored by the U.S. Air Force Office of Scientific Research under contract AFOSR 85-0320. Our research activities have been concentrated on two main areas: Design of Experiments and Reliability Models.

Our research efforts in design of experiments have included the following important areas:

1. Efficient designs for experiments involving several factors;
2. Efficient designs for repeated measurements models;
3. Trade off in designs;
4. Flexible orthogonal arrays;
5. Efficient designs for comparing test treatments with controls;
6. Designs for collecting data through sampling.

Our research efforts in reliability have been mainly directed to the following important areas:

1. Multistate reliability models;
2. Optimal assembly of coherent systems (both in the binary and multistate cases);
3. Redundancy importance and allocation of spares in coherent systems;
4. Closure properties of classes of life distributions;
5. Optimal inspection policies.

Our research efforts in both areas have led to the production of a total of 39 papers, of which 23 are in print, 11 are in press, 3 have been submitted for publication and 2 are under preparation. In addition, two research monographs on orthogonal arrays and survey sampling are in the final stages of preparation.

Training graduate students for Ph.D. degrees in statistics have been an integral part of our research activities. One student has already completed his Ph.D. training and he is currently on the faculty of Iowa State University. Three other students have completed major portions of their research towards their Ph.D dissertations.

II. THEMES OF RESEARCH

Our main objective in design of experiments was the development of economical and efficient techniques for the enhancement of data collection and analysis in scientific experiments. Our contributions have been reported in scientific journals, research monographs and catalogs of efficient and economical designs catered to the experimenter's needs. Our discoveries have wide applications to scientific problems encountered by the Air Force such as communication engineering, equipment testing and aerospace medicine.

Our most significant achievements can be broadly classified in six categories.

1. Efficient designs for experiments involving several factors.

   Engineers and scientists working for the U.S. Air Force often conduct experiments which involve several factors. Designing such experiments which are economical, efficient and are capable of providing valid data for decision making is a very difficult and expensive task. Experiments of these types involve factor screening and identifying levels at which the output products are highly reliable and their performance level are at the highest possible. Our discoveries on complete and incomplete orthogonal arrays address these important problems and provide solutions for many situations.

2. Efficient designs for repeated measurements models.

The materials and conditions under which scientific experiments are performed are often expensive. One way to reduce cost is to recycle and reuse these materials for additional experiments. In doing so, the past history of the material should be taken into account. Our discoveries in repeated measurements design can handle problems of the type explained above. In addition our results can be used whenever the U.S. Air Force is interested to discover the degree of fatigue due to the repeated use of its equipments.

3. Trade-off technique.

Collecting scientific data under undesirable situations is expensive and very time consuming. Many of the scientific experiments performed by and for the U.S. Air Force are of these types since the circumstances are often unfriendly and hostile. The trade-off technique, introduced into the literature by us, enables the experimenter to substitute a hostile experimental location by a more congenial one without losing any of the statistical content of the experiment.

4. Flexible Orthogonal arrays.

Scientists rarely know the exact model which is responsible for generating the data. At best, they can approximate the model of their scientific data by a member of a class of models. Unfortunately, many available efficient designs for collecting data are heavily model dependent. A slight change in the model makes these efficient designs inefficient and in many cases statistically useless. We have been able to identify and characterize a class of orthogonal arrays which are efficient for fitting two or more models to the data.

5. Efficient designs for comparing test treatments with controls.

The U.S. Air Force is often involved with experiments to compare existing equipments and techniques with their modern, freshly developed counterparts. It is highly desirable that such experiments be economical and easy to conduct and yet result in conclusions which are of the hightest statistical quality as regards their reliability and validity. Our research has produced highly efficient and economical designs which are applicable to a wide variety of experiments which compare new equipments with existing ones.

6. Designs for collecting data through sampling.

Valid and efficient sampling designs for collecting data in situations where sampled data must reflect the temporal and special variabilities of the material are rarely found in the literature on experimental design. This is because the mathematical and statistical problems associated with such studies are very difficult and complicated. We have been able to identify a series of designs which can be used in these situations where contiguous units provide similar information, such as sampling atmosphere of a specific zone, or sampling a waste site. In the area of collecting data via sampling we have also developed a general technique which is useful when the auxiliary information is helpful in efficiently determining the units which are to be sampled.

We now give a brief outline of the accomplishments of our research efforts in each of the reliability topics mentioned in the introduction.

1. Multistate reliability models.

(A) In this area we have been able to define a new and rich class of multistate systems. The monotone structure function of a system in this

class is required to be a Schur-concave function. This expresses in a mathematical form the common feature that systems of this class share, namely that they perform at a higher level when the states of their components are more homogenous. Examples of such systems abound and the potent property of Schur-concavity allows us to derive several structural properties shared by members of this class. Having derived most of the important structural properties we turned our attention to the probabilistic consequences of such properties. Among other results, we have derived important bounds on the probability that a system in this class perform at a level j or higher, where j is a specified level of performance. We have also shown that the expected state of performance of a system is also a Schur-concave function in the parameters defining the joint distribution of its components.

Our results in this direction are contained in [32].

(B) We have also obtained some results in the optimal allocation of multistate components to k series systems so that some performance characteristic like expected number of systems functioning at level j or higher, the probability that at least one of the system functions at level j or higher, etc. is maximized. Applications of such results to binary models in which components are backed up by spares have been obtained. Those results are contained in [23].

(C) Recently Block, Griffith and Savits (1987), studied the class of L-superadditive structures by imposing the following condition on the structure function ϕ:

$$\phi(\underline{x} \vee \underline{y}) + \phi(\underline{x} \wedge \underline{y}) \geq \phi(\underline{x}) + \phi(\underline{y}) \text{ for all } \underline{x} \text{ and } \underline{y},$$

where $\underline{x} \vee \underline{y}$ ($\underline{x} \wedge \underline{y}$) is the vector of componentwise maximums (minimums).

Such functions have the interesting interpretation of describing whether a multistate system is more series-like than parallel-like.

A vector $\underline{x}$ is said to be a critical connection vector to level $k > 0$ for a multistate structure Φ if $\Phi(\underline{x}) = k$ and $\underline{y} < \underline{x}$ implies $\Phi(\underline{y}) < k$, where $\underline{y} < \underline{x}$ means $y_i \leq x_i$ for each i and strict inequality holds for at least one i. Critical connection vectors to the various performance levels of a multistate structure ϕ play a central role similar to the one played by minimal path vectors in the binary case. So naturally, Block, Griffith and Savits (1987) examined critical connection vectors to levels of performance of an LSP structure function. They conjectured an upper bound for the number of such vectors and proved it for some special cases. In [34], we proved this conjecture and we also showed that imposing some coherence conditions on an LSP structure can severely limit the number of its critical connection vectors to its various levels of performance.

2. Optimal assembly of coherent systems (both in the binary and multistate cases)

(A) In our first paper an optimal assembly of coherent systems, which was sponsored by grant AFOSR 80-0170, we tacitly assumed that all the components are of the same type and thus can be interchanged freely. Our research efforts under the present grant have concentrated on a more general formulation of the problem in which components are allowed to be of different types. We have continued to exploit the elegance and power of majorization and Schur functions to obtain useful results. Hers is an example of such results. Suppose we have k components of type j with reliabilities $p_1^j \leq p_2^j \leq \ldots \leq p_k^j$, $j=1,\ldots,m$. We wish to build k series systems each containing one component of each type. Let N be the number of working components. To maximize N (stochastically): the

best components are assembled together, the next best components are assembled together,..., and finally the worst components are assembled together.

Our results in this direction are contained in [21].

(B) See topic 1 (B) for a brief outline of our results on optimal assembly of multistate systems.

3. Redundancy importance and allocation of spares in coherent systems.

Our research efforts under the present grant have led us to the investigation of the following interesting and practically motivated problem: Consider a coherent system with n components whose reliabilities are given by $p_1 \leq p_2 \leq \ldots \leq p_n$. When we introduce an active redundancy to component i (i.e. we connect component i in parallel to a component of the same type whose reliability is p_i) the reliability of that component (position) is increased from p_i to $1 - (1-p_i)(1-p_i)$, $1 \leq i \leq n$. Suppose that due to cost and/or time considerations we are allowed to introduce active redundancies to only $r \leq n$ components, and assume we have a choice in selecting those r components. Which components should we choose to introduce the maximum improvement of the reliability of the system?

Several interesting results have been obtained in this direction and they are all contained in [22] and [33].

4. Closure properties of classes of life distributions.

The class of new better than used in expectation (NBUE) life distributions and the class of life distributions with decreasing mean residual life (DMRL) are among the important classes of the life distributions in reliability. However the basic question as to whether

or not these two classes are closed under the formation of parallel systems with i.i.d. units has remained open.

Our research efforts under the present grant have shown that the NBUE class is closed under the above mentioned operation and that the class of absolutely continuous DMRL distributions enjoys the same closure property. These results are contained in [20].

5. Optimal inspection policies.

We have recently started investigating some special situations in which we tackle another important optimization question, namely: How to "optimally" inspect a system that has failed in order to identify the components responsible for the failure. Among the possible optimality criteria to be used are: (i) mimnimzing the expected value (or stochastically) the number of components that need to be checked, (ii) minimizing the expected value (or stochastically) the number of subsystems that need inspection. Our preliminary results are contained in [40].

III. Training Ph.D Students

Training Ph.D students have been an integral part of our research activities. One graduate student has already obtained his Ph.D degree and he wrote his dissertation on efficient designs for comparing test treatments with controls. Two other students in the area of designs have completed major portions of their research and are expected to graduate by summer, 1989. One of them has worked on repeated measurements designs, while the other has worked on factorial designs and designs for sample survey. A fourth student in the area of reliability has completed a major portion of his research and is expected to graduate by the summer of 1989. The topic of his dissertation

is multistate reliability models.

IV. RESEARCH VISITS AND SCIENTIFIC CONFERENCES

Members of our research team have been repeatedly invited to visit various campuses to collaborate on research projects with other well known researchers in the areas of design of experiments and reliability. Such campuses include: University of California at Berkely, Cornell University, the Institute for Mathematics and its Applications at the University of Minnesota, the University of Pittsburgh and the Reliability Center at Florida State University.

We have also presented invited talks at many major national and international statistical conferences. These include: the 1986 Joint Annual Statistical Meetings at Chicago; the First International Conference on Statistical Computing in Izmir, Turkey, 1987; the First International Conference-Workshop on Optimal Design and Analysis, Neuchatel, Switzerland 1988, and the 1988 Annual Meeting of American Statistical Association at New Orleans.

Several prominent researchers have visited us for various lengths of time to collaborate on research projects.

V. LIST OF PUBLICATIONS

In his section we list 39 papers and 2 research monographs; twenty-three of these are already in print, eleven in press, three under consideration for publication and the remaining four are currently in the final stage of preparation. Copies of the papers have already been submitted to the Air Force Office of Scientific Research. Additional copies can be obtained through the Statistical Laboratory of the University of Illinois at Chicago.

[1]

CHARACTERIZATION OF TRIPLY BALANCED MATRICES WITH APPLICATIONS TO SURVEY SAMPLING

By: A.S. Hedayat and H. Pesotan

Abstract: R x L triply balanced matrices arise in estimating the mean square errors of nonlinear statistics in survey samplings. It is shown that: (1) any R x L exact triply balanced matrix and an orthogonal array $OA(R,L,2,3;\lambda)$ are one and the same object up to a possible notational change of the two symbols of the array. (2) R is a multiple of 8 and $L \leq \frac{1}{2}R$. (3) The problem of the construction of R x L exact triply balanced matrices. $3 \leq L \leq \frac{1}{2}R$, is completely resolved modulo the existence of Hadamard matrices of order $\frac{1}{2}R$. (4) There is no sequence of R X L matrices which are nearly triply balanced in the sense of Rao and Wu(1985) if $R < 2L$.

Publication Status: In print: J. Statistical Planning and Inference 15(1986), 11-17

[2]

FRACTIONAL FACTORAL DESIGN IN THE FORM OF INCOMPLETE ORTHOGONAL ARRAYS

By: A.S. Hedayat and J. Stufken

Abstract: In this paper we study certain fractional factorial designs, which are known in the literature as incomplete orthogonal arrays. We indicate situations in which these designs can be of practical interest and study their statistical properties.

Publication status: In print. Statistical Design: Theory and Practice (Edited by C.E. McCulloch, S.J. Schwager, G. Casella and S.R. Searle), Cornell University, Ithaca, New York, (1986), 101-115.

[3]

OPTIMAL DESIGNS FOR COMPARISONS BETWEEN TWO SETS OF TREATMENTS*

By: Dibyen Majumdar

Abstract: Suppose v treatments are to be compared in b blocks of size k each. Also suppose that the treatments are divided into 2 sets of u and w=v-u treatments. A-optimal designs are obtained for estimating all the differences of two treatments, one from each set. Optimal row-column designs are also obtained. Some new optimal designs for comparing several treatments with a single control are obtained as special cases.

Publication status: In print: J. Statistical Planning and Inference 14(1986), 359-372.

[4]

ON A STATISTICAL OPTIMALITY OF MAGIC SQUARES

By: A. S. Hedayat

Abstract: It is shown that an optimal way of running a simple linear regression with n^2 equally spaced levels in an n x n square is to distribute the levels in the square in the form of a magic square.

Publication status: In print. Statistics and Probability Letters 5, (1987), 191-192.

[5]

A LINEAR ALGEBRAIC ALGORITHM FOR REDUCING THE SUPPORT SIZE OF T-DESIGNS AND TO GENERATE A BASIS FOR TRADES.

By: G.B. Khosrovshahi and E.S. Mahmoodian

Abstract: A simple linear algebraic algorithm to generate a basis of the null space of a given integral matrix is utilized to present a computer algorithm, which in general, is used to reduce the support size of a given design as in a theorem of Foody-Hedayat (Theorem 4.1, 1977), and in particular, it is used to produce a basis for trades. The computations based on this algorithm is of order of a polynomial function.

Publication status: In print. Communications in Statistics: Computations and Simulations 16(1987), 1015-1038.

[6]

A-OPTIMAL BLOCKS DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL

by: John Stufken

Abstract: We consider the problem of comparing test treatments with a control in a proper block design. We derive sufficient conditions for the A-optimality of both R-type and S-type designs, and demonstrate how these conditions can be used to obtain families of optimal designs. We give an example for the construction of the desired S-type designs. A table with optimal R-type designs ($3 \leq k \leq 10$, $k \leq V \leq 30$) is also given.

Publication status: In print. Annals of Statistics 15, (1987), 1629-1638.

[7]

OPTIMAL STEP TYPE DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL

By: C.S. Cheng, D. Majumdar, J. Stufken and T.E. Ture

Abstract: The problem of obtaining A-optimal designs for comparing v test treatments with a control in b blocks of size k each is considered. A condition on the parameters (v,b,k) is identified for which optimal step type designs can be obtained. Families of such designs are given. Methods of searching for highly efficient designs are proposed for situations where it is difficult to determine an A-optimal design.

Under the usual additive homoscedastic model, an A-optimal design minimizes the average variance of the least squares estimaters of the control - test treatment comparisons. Majumdar and Notz (1983) gave a method for finding A-optimal designs. Their optimal designs can basically be of two types. Using the terminology of Hedayat and Majumdar (1984), they are: rectangular (or R) type, in which every block has the same number of replications of the control, and step (or S-)type, in which some blocks contain the control t times and the other $t+1$ times. Optimal R-type designs were studied by Hedayat and Majumdar (1985). Families of such designs, particularly when each block has one replication of the control, were given in that paper. In this article, we intend to study optimal S-type designs. Step type designs are more complicated than rectangular type designs; the latter being a balanced incomplete block (BIB) design in the test treatments augmented by an equal number of controls in each block, but the former does not have such a simple characterization. Consequently, both the optimality and the construction of such designs are more involved.

Publication status: In print. J. American Statistical Association 83(1988), 477-482.

[8]

A GRAPHICAL PROOF OF THE NONEXISTENCE OF BIB(7,b,r,3,λ|16) DESIGNS

by: W. Foody and A.S. Hedayat

Abstract: The notation BIB(v,b,r,k,λ|b*) is used to denote a BIB(v,b,r,k,λ) design having exactly b* distinct blocks. These b* distinct blocks are said to form the support of the design. A new graphical concept called a figure around a variety within the support is introduced and studied when k = 3. Via a series of graphical results, it is shown that it is not possible to construct a BIB(7,b,4,3,λ) design with support size 16. Therefore this result with those in Hedayat and Li (Ann. Statistics (1979)) form a complete description of the pairs (b,b*) for which a BIB(7,b,r,3,λ|b*) design exists.

Publication status: In print. J. Statistical Planning and Inference 20(1988), 77-90.

[9]

OPTIMAL DESIGNS FOR COMPARING TEST TREATMENTS WITH CONTROLS

by: A.S. Hedayat, M. Jacroux and D. Majumdar

Abstract: This paper outlines existing knowledge on optimal designs for comparing test treatments with controls under 0-, 1-, and 2-way elimination of heterogeneity models. The results are motivated through numerical examples.

Publication status: In print. Statistical Science 3(1988), 462-491 (with discussions).

[10]

MODEL ROBUST OPIMAL DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL

by: A.S. Hedayat and Dibyen Majumdar

Abstract: Families of designs are obtained for comparing test treatments with a control. They are simultaneously A- and MV-optimal for either one-way or two-way elimination of heterogeneity when the model of response is homoscedastic and linear additive. These designs can be easily cataloged.

Publication status: In print. J. Statistical Planning and Inference 18(1988), 25-33.

[11]

DESIGNS IN SURVEY SAMPLING AVOIDING CONTIGUOUS UNITS

by: A.S. Hedayat, C.R. Rao and J. Stufken

Abstract: Various ideas to obtain improved estimates of population characteristics in the presence of some additional knowledge on the sampling units are available in the literature. We discuss one of such ideas, recently introduced in a paper by Hedayat, Rao and Stufken. It applies if there exists some ordering of the units under which contiguous units are anticipated to provide similar data. In such a situation, it is intuitively appealing that more information on the population can be obtained if the sample avoids pairs of contiguous units. The required ordering of the units will in most such situations be induced by a natural entity, such as time, location, and so on. It may be an ordering in one or more dimensions. As an example, small neighboring plots at a dump site for chemical waste tend to give similar measurements on the present amount of some chemical compound. With the plots as the sampling units, location induces a two-dimensional ordering of the units. A class of sampling designs for this setup is introduced, and a discussion on alternative sampling design is included. Results on the existence, construction and the implementation of such designs are given.

Publication status: In print. Handbook of statistics (P.R. Krishnaiah and C.R. Rao, Editors), Vol. 6(1988), pp. 575-583, Elsevier Science Publisher.

[12]

SAMPLING PLANS EXCLUDING CONTIGUOUS UNITS

by: A.S. Hedayat, C.R. Rao and J. Stufken

Abstract: We consider fixed size sampling plans for which has the second order inclusion probabilities are zero for pairs of contiguous units and constant for pairs of non-contiguous units. A practical motivation for the use of such plans is pointed out and a statistical condition is identified under which these plans are more efficient than the corresponding simple random sampling plans. Results on the existence and construction of these plans are obtained.

Publication Status: In print. J. Statistical Planning and Inference 19(1988), 159-170.

[13]

TWO-SYMBOL ORTHOGONAL ARRAYS

by: A.S. Hedayat and J. Stufken

Abstract: Results on the construction and maximum number of factors in orthogonal arrays are discussed, with emphasis on arrays in which the factors are at two levels.

Publication status: In print. In: Optimal Design and Analysis of Experiments (Y. Dodge, V.V. Fedorov and H.P. Wynn, Editors) (1988), 47-58, Elsevier Science Publisher.

[14]

BIB(9,18T,8T,4,3T) DESIGNS WITH REPEATED BLOCKS

by: G.G. Khosrovshahi and E.S. Mahmoodian

Abstract: The set of all distinct blocks of a BIB(v,b,r,k,λ) design is referred to as the support of a design. Via a computer program based on the methods of trade-off and composition of designs, a table of 105 BIB designs based on v = 9, k = 4 with support sizes $18 \leq b^* \leq \binom{9}{4} = 126$ except for $b^* = 19,20,21,23$ have been constructed. BIB designs with v = 9, k = 4 and $b^* < 18$ are shown to be non-existent.

Publication status: In print. J Statistical Planning and Inference, 18(1988), 125-131.

[15]

ON BIB DESIGNS WITH VARIOUS SUPPORT SIZES FOR v=9 and k=3

by: G.B. Khosrovshahi and E.S. Mahmoodian

Abstract: The set of all distinct blocks of a BIBD(v,b,r,k,λ) is referred to as the support of the design. In this paper, the family of BIB designs with v=9 and k=3 is studied from the view of possible support sizes, b*'s. A table is constructed of designs with support sizes belonging to (12,18,20,21,...,84), for minimum possible b in each case and for any larger admissible b. In constructing this table the methods of trade-off and composition of designs are utilized.

Publication status: In print. Commun. Statist. - Simmula., 17(1988), 765-770.

[16]

OPTIMAL REPEATED MEASUREMENTS DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL.

by: D. Majumdar

Abstract: A-optimal and MV-optimal repeated measurements designs are given both for direct and residual treatment effects, for comparing several test treatments with a control. The models considered are basically of two types: without preperiods and the circular model. It is shown that some known balanced and strongly balanced uniform repeated measurements designs can be modified to obtain optimal designs for this problem. Some other methods of finding optimal designs are also given.

Publication status: In print. Commun. in Statistics A-Theory and Methods 17(1988), 3687-3703.

[17]

OPTIMAL BLOCK DESIGNS FOR COMPARING NEW TREATMENT WITH A STANDARD TREATMENT

by: Dibyen Majumdar

Abstract: Prior information on the standard treatment is utilized to obtain an optimal design for comparing v new treatments with a standard treatment in b blocks each of size $k \leq v$. The optimal design is a BTIB design. The nature of the optimal designs is explored; examples are studied. The theorems are proved in the set up of exact design theory.

Publication status: In print. Optimal Design and Analysis of Experiments (Y. Dodge, V.V. Federov and H.P. Wynn, eds.), Elsevier Science Publishers B.V., North Holland, Amsterdam, (1988), 15-27.

[18]

ON BOUNDS FOR THE EFFICIENCY OF BLOCK DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL

by: John Stufken

Abstract: In this paper we study the class of augmented balanced incomplete block designs, which are used for comparing a control treatment with a set of test treatments. Under the A-criterion we establish a condition that enables us to determine the most efficient augmented design and we suggest some methods to compute a lower bound for the efficiency of these designs. For $3 \leq k \leq 10$, $v \geq k$ we list the parameters of the most efficient designs with a lower bound for their efficiency or, if known, mention their optimality.

Publication status: In print. J. Statistical Planning and Inference 19(1988), 361-372.

[19]

ON THE MAXIMUN NUMBER OF CONSTRAINTS IN ORTHOGONAL ARRAYS

by: A.S. Hedayat and J. Stufken

Abstract: In this paper we show that Bush's bound for the maximum number of constraints in an orthogonal array of index unity is uniformly better than Rao's bound. In addition it is shown, using an argument similar to that needed in the proof of the above result, that Noda's characterization of parameters in orthogonal arrays of strength 4 achieveing equality in Rao's bound, leads easily to a similar characterization in arrays of strength 5.

Publication status: In print. Ann. Statist. 17(1989), 448-451.

[20]

CLOSURE OF NBUE AND DMRL CLASSES UNDER FORMATION OF PARALLEL SYSTEMS

by: A. Abouammoh and E. El-Neweihi

Abstract: The class of new better than used in expectation life distributions is shown to be closed under the formation of parallel systems with independent and identically disturbed components. The class of differentiable life distributions with decreasing mean residual life is also proved to have the same closure property.

Publication status: In print. Statistics and Probability Letters, 4, (1986), 223-225.

(The research for this paper was sponsored by Grant AFOSR 85-0320. However, an oversight resulted in giving the credit, in print, to Grant AFOSR 80-0170).

[21]

OPTIMAL ASSEMBLY OF SYSTEMS USING SCHUR FUNCTIONS AND MAJORIZATION

by: E. El-Neweihi, F. Proschan, and J. Sethuraman

Abstract: A general assembly of n systems from k types of components is considered. The techniques of majorization and Schur function are utilized to pinpoint the optimal assembly under several critiera. Earlier results of Derman, Leiberman, and Ross [2] and El-Neweihi, Proschan, and Sethuraman [3] are generalized.

Publication status: In print. Naval Research Logistic Quarterly, 34, (1987), 705-712.

[22]

ACTIVE REDUNDANCY ALLOCATION IN COHERENT SYSTEMS

by: P. Boland, E.El-Neweihi, and F. Proschan

Abstract: We introduce in this paper a new measure of component importance, called redundancy importance, in coherent systems. It is a measure of importance for the situation in which an active redundancy is to be made in a coherent system. This measure of component importance is compared with both the (Birnbaum) reliability importance and the structural importance of a component in a coherent system. Various models of component redundancy are studied, with particular reference to k-out-of-n systems, parallel-series systems, and series-parallel systems.

Publication status: In print. Probab. in the Eng. and Informational Sciences, 2, (1988), 343-353.

[23]

OPTIMAL ALLOCATION OF MULTISTATE COMPONENTS

by: Emad El-Neweihi, Frank Proschan and Jayaram Sethuraman

Abstract: In this paper we present some results in the optimal allocation of multistate components to k series systems so that some performance characteristic like expected number of systems functioning at level α or higher, the probability that at least one of the systems functions at level α or higher etc. is maximized. Our basic mathematical tools are majorization and Schur functions; the methods used and some of the theorems obtained are those of "Optimal Allocation of Components in Parallel-Series and Series-Parallel Systems" El-Neweihi, E., Proschan, F., and Sethuraman, J., Report (1984). In addition, we show how these results may be used to obtain fruitful applications in reliability theory.

Publication status: In print. Handbook of Statistics, 7, (1988), 417-432.

[24]

THE THEORY OF TRADE-OFF FOR T-DESIGNS

by: A.S. Hedayat

Abstract. A trade for a t-design consists of two disjoint collections of blocks, T_1, T_2 such that T_1 covers every t-subset of varieties as often as T_2 does. The theory of trade-off deals with building trades and using them for the purpose of constructing t-designs based on various numbers of distinct blocks. Special attention is given to 2-designs, that is BIB designs.

Publication status: In press. IMA Volume in Math. and its Applications #21 (D. Ray-Chaudhuri, ed.), Springer-Verlag, to appear.

[25]

NEW PROPERTIES OF ORTHOGONAL ARRAYS AND THEIR STATISTICAL APPLICATIONS

by: A.S. Hedayat

Abstract: It is shown that an orthogonal array of strength t is more than a fractional factorial design of resolution t + 1. The practical usefulness of this result is shown. The notion of flexible orthogonal arrays of strength t is introduced and its practical usefulness is demonstrated. An efficient way of generating the design and information matrices associated with orthogonal arrays in the context of orthogonal polynomial models is presented.

Publication status: In press. In Design and Analysis of Experiments with Application to Physical and Engineering Sciences (S. Ghosh, Editor), Marcel Dekker Inc., to appear.

[26]

THE POSSIBLE SUPPORT SIZES FOR BIB DESIGNS WITH v=8 and k=4

by: A.S. Hedayat, I.N. Landgev and J. Stufken

Abstract: Hedayat and Hwang [2] studied the support sizes for BIB designs with $v = 8$ and $k = 4$. They established the existence or nonexistence for all possible support sizes, with the exceptions of 15, 16, 17 and 19. We will show that there are no such BIB designs with support sizes 15 and 16, while there are designs with support sizes 17 and 19. These latter designs require at least 42 blocks. In addition, we will provide an answer to their question on self-complementary designs.

Publication status: In press: J. Combinatorial theory, Ser. A, 50(1989), to appear.

[27]

RESULTS ON THE SUPPORT OF BIB DESIGNS

By: A.S. Hedayat, I.N. Landgev and V.D. Tonchev

Abstract: The support of a BIB design is the set of all distinct blocks in the design. The notation BIB(v,b,r,k,$\lambda|b^*$) is used to denote a BIB(v,b,r,k λ) design with precisely b^* distinct blocks in the design. New lower bounds on b^* are obtained by utilizing the information about those blocks in the support that are repeated λ times in the design. These results on the support together with other new and old results are used:

(1) To show the nonexistence of BIB(8,56t,21t,3,6t|b^*) designs with $b^* \leq 21$, the design with the support size of 22 being already available;

(2) To show the nonexistence of BIB(11,55t,15t,3,3t|b^*) designs with $b^* \leq 24$, and moreover, to show the existence of a BIB(11,55,15,3,3|25) design by actual construction,

(3) To construct a BIB(12,44,11,3,2|28) design. This design has the minimum support size within the family of BIB(12,44t,11t,3,2t) designs.

Publication status: In press: J. Statistical Planning and Inference 22(1989), to appear.

[28]

ON CONSTRUCTION OF ΠPS SAMPLING DESIGNS THROUGH A METHOD OF EMPTYING BOXES.

by: A.S. Hedayat, B.-Y. Lin and J. Stufken

Abstract: We present a simple but universal technique for the construction of ΠPS sampling designs. A tool that is used in the construction consists of playing a game in which objects are removed from N boxes, n at a time, and at most one from each box at a time. Necessary and sufficient conditions on N, n and the contents of the boxes are established such that all boxes can be emptied by this process.

It is shown that every ΠPS design can be derived from such a game. Sampling designs with additional properties are obtained through additional restrictions on emptying boxes. Various rigorous methods are presented, complemented by numerous suggestions. The emphasis is on controlling sample selection probabilities and inequalities for the first and second-order inclusion probabilities. The method is very adaptive to computer use.

Publication status: In press: Ann. Statist. 18(1989), to appear.

[29]

ON A RELATION BETWEEN PAIRWISE AND VARIANCE BALANCED BLOCK DESIGNS.

by: A.S. Hedayat and J. Stufken

Abstract: We show that the problems of constructing pairwise balanced and variance balanced block designs are equivalent. This provides incentive to study only the direct construction for the simplest of these designs, namely pairwise balanced designs.

Publication status: In press: Amer. Statist. Assoc. 84(1989), to appear.

[30]

SOME MATHEMATICAL RESULTS ON INCOMPLETE ORTHOGONAL ARRAYS

by: A.S. Hedayat and J. Stufken

Abstract: Let H be a proper subset of size h of S with s elements. A $k \times N$ array based on S is called an incomplete orthogonal array of strength t and index λ based on S and H if the columns of any $t \times N$ subarray contain each element from $S^t - H^t$ λ times, while those from H^t do no appear at all. Such an array is denoted by $IOA(N,k,(s,h),t)$. New results include: (1) In any such array $k \leq s/h + t-1$ regardless of the value of λ; (2) $k \leq f(\lambda s^{t-1},s,t-1)+1$ where $f(N,s,t)$ denotes the largest value of k for which $OA(N,k,s,t)$ exists; (3) If $s_1 > s_2$ where s_2 is a prime power and s_1 is a power of s_2, then there exist $IOA(s_1 - s_2, s_2+1, (s_1,s_2),t)$; (4) Some families of incomplete orthogonal Latin squares and orthogonal arrays are constructed.

Publication status: In Press: Sankhya, (1989), special issue in honor of R.C. Base, to appear.

[31]

OPTIMAL BLOCK DESIGNS FOR COMPARING TEST TREATMENTS WITH A CONTROL WHEN $k \geq v$

by: Mike Jacroux and Dibyen Majumdar

Abstract: The problem of comparing v test treatments with a control in b blocks of size k each is considered for the case k > v. Some sufficient conditions for designs to be A and MV-optimal in these experimental situations are derived and examples are given to demonstrate how the sufficient conditions obtained can be applied. Some infinite families of optimal designs that satisfy the sufficient conditions obtained are also given. For cases where the derived sufficient conditions are not applicable, some "approximately" optimal designs are suggested for usage.

Publication status: In press. J. Statistical Planning and Inference.

[32]

SCHUR STRUCTURE FUNCTIONS

by: A.M. Abouammoh, E.El-Neweihi, and F. Proschan

Abstract: We define two new classes of multistate coherent systems by requiring, among other conditions, that their structure functions be Schur-concave (Schur-convex). The M+1 performance levels of both the systems and their components are represented by the set $\{0,1,\ldots,M\}$. We present basic structural properties of the new classes. In particular we study in some detail the number and form of the critical upper(lower) vectors to the various performance levels. We also present some probabilistic aspects of the new classes.

Publication status: In press. Probab. in the Eng. and Informational Sciences.

[33]

REDUNDANCY IMPORTANCE AND ALLOCATION OF SPARES IN COHERENT SYSTEMS

by: Philip J. Boland, Emad El-Neweihi and Frank Proschan

Abstract: We study the models in which a set of spares is available for redundancy in a coherent system. In some circumstances, parallel (or active) redundancy is used to improve the reliability of the system, while in others series redundancy is used to improve a different measure of utility. Hence we define the two concepts of parallel and series redundancy importance for components in a coherent system relative to an available set of redundant spares. These measures of importance are compared with the structural importance and reliability importance of a component. Various results for the optimal allocation of redundant spares are given, with particular reference to k out of n systems and modules of coherent systems.

Publication status: In press. Journal of Statistical Planning and Inference (a special issue on reliability).

[34]

THE NUMBER OF CRITICAL CONNECTION VECTORS OF L-SUPERADDITIVE STRUCTURE FUNCTIONS

by: Emad El-Neweihi and Fan C. Meng

Abstract A conjecture due to Block, Griffith and Savits (1987), concerning the number of critical connection vectors to the various performance levels of a discrete L-superadditive structure function, is proved. When the components of the discrete L-superadditive structure function are further assumed to satisfy a certain relevance condition due to Griffith (1980), it is shown that there is exactly one critical connection vector to each performance level.

Publication status: In press. Advances in Applied Probability.

[35]

SAMPLING DESIGNS USEFUL FOR SOLID WASTE SAMPLING

by: A. Hedayat and J. Stufken

Abstract: Information on sampling units can often be used in survey sampling to reduce the variance in estimating a population total corresponding to some variable of interest. Various procedures have been introduced to deal with various types of information. We suggest suitable plans for a situation that occurs, among others, in solid waste sampling. The sampling units are obtained by dividing the waste site into small, equally shaped plots. A small number of these plots will then be selected in order to study the contamination levels due to hazardous waste at this site. It is anticipated that contiguous plots will give similar observations. Intuitively, it is therefore reasonable to use sampling plans that avoid contiguous units completely, or at least, result in samples that are unlikely to contain contiguous units. Such samples are more likely to be representative for the entire population. We present various ideas and results that lead to or can lead to sampling plans which are appealing for these situations.

Publication status: Submitted for publication.

[36]

ADMISSIBLE EXTENSION OF THE SAMPLE MEAN AND THE HORVITZ-THOMPSON ESTIMATOR UTILIZING ADDITIONAL RESOURCES

by: A.S. Hedayat and K.W. Pu

Abstract: The problem of extending a given sampling strategy with the sample mean or the HTE as the underlying estimator, when additional resources are available, is considered. A method of obtaining such an admissible and improved extension of the initial strategy with a design of full support is given. Some applications are discussed.

Publication status: Submitted for publication.

[37]

OPTIMAL TWO-PERIOD REPEATED MEASUREMENTS DESIGNS

by: A.S. Hedayat and W. Zhao

Abstract: For the class of repeated measurements designs based on t treatments, n experimental units and two periods, the following results are obtained.

(1) For uncorrelated errors, a family of universally optimal designs is constructed if t divides n. In addition, the equivalence of the information matrices of such repeated measurements designs and of certain block designs is established. The implication of this equivalence on the optimality of both repeated measurements designs and block designs is explored.

(2) A family of A-optimal designs is constructed for arbitrary n whether or not the errors are correlated.

Publication status: Revised and resubmitted to Ann. Statistics.

[38]

DESIGN AND INFERENCE IN FINITE POPULATION SAMPLING

by: A.S. Hedayat and B.K. Sinha

Publication status: In the final stages of preparation.

[39]

ORTHOGONAL ARRAYS AND THEIR APPLICATIONS

by: A.S. Hedayat and J. Stufken

Publication status: In preparation.

[40]

OPTIMAL INSPECTION POLICIES

by: P. Boland, E. El-Neweihi, and F. Proschan

Publication status: In preparation.

[41]

NEW CLASSES OF MULTISTATE COHERENT SYSTEMS

by: E. El-Neweihi and F.C. Meng

Publication status: In preparation.