

AD-A209 822

TECHNICAL REPORT ON NONLINEAR WAVES
OFFICE OF NAVAL RESEARCH GRANT #N00014-88-K-0447

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JUNE 15, 1989

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Overview

The current research funding has been used to support the research activities of Professors Mark J. Ablowitz and Athanassios S. Fokas and associates. The investigators have been working in the general area of nonlinear wave propagation for over twenty years. The main focus of this work is the understanding of the nonlinear phenomena involved with the wave propagation arising in physical problems. The work has application to numerous areas of physics, engineering and mathematics. Applications include fluid dynamics, waves in stratified fluids, surface gravity waves and wave excitation phenomena related to moving pressure distributions; numerical approximation and computation; nonlinear optics; and plasma physics. Moreover the study of solutions to some of the underlying nonlinear evolution equations has led naturally to the investigation and new results in the separate but closely related field of inverse scattering. Developments to both one and multidimensional inverse problems have been made. (K8) ←

During the period of support of this grant there has been substantial activity. The investigators have over 30 research papers published and more than 10 papers accepted for publication; with still a number of papers ready to be submitted. There are a number of research directions and problems we are pursuing. These include the following.

(i) Development of solutions to multidimensional nonlinear evolution equations of physical significance. Prototypes are the so-called Kadomtsev-Petviashvili and Davey-Stewartson equations. These equations arise in a variety of physical instances such as water waves, stratified fluid flow, plasma waves etc. We have found some new and important results. The nature of the boundary value problems and solutions of the equations in the so-called strong coupling limit have recently been uncovered. The role of the boundary conditions and a number of the essential differences between one and two (spatial) dimensional problems have been clarified. The role

of the boundary conditions is important in the understanding of why highly localized multidimensional soliton solutions exist for one of the Davey-Stewartson (D-S) equations (i.e. DS-I but not DS-II). These multidimensional soliton solutions have been found by a novel application of the Inverse Scattering Transform. Specifically the soliton solutions correspond to a nontrivial mean field contribution at infinity. Considering the asymptotic description by which the D-S equations are derived, this means that energy must be inserted to the physical field near the boundary. Moreover rigorous theorems regarding the existence of solutions have been obtained under certain assumptions.

(ii) Solutions of discrete nonlinear evolution equations and numerical computation. In our studies we have found the following surprising situation. Namely associated with the integrable nonlinear Schrodinger equations are standard numerical schemes which exhibit at intermediate levels of mesh refinement a weak form of temporal chaos. On the other hand not all approximations exhibit this instability. In fact, difference schemes developed by Inverse Scattering Transform (IST) methods do not exhibit this spurious chaos. All schemes agree when the mesh is sufficiently refined.

(iii) Inverse problems associated with multidimensional problems. A key element in this work is the DBAR method developed by the principal investigators and their colleagues a few years ago. The method has been extended from the study of two dimensional inverse problems to n dimensions ($n > 2$). In principal the method can be applied to inverse problems arising in geophysics and acoustics. Rigorous theorems have been obtained for a number of interesting multidimensional problems.

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(iv) Cellular Automata and solitons. The principal investigators and their associates have been studying a class of cellular automata which admit solitons as special solutions. Interesting new classes of solutions have been found. Recently these automata have been generalized to two and three dimensions and we have found instances of soliton interaction. These systems are not reversible, which is quite a novel and interesting aspect.

(v) Painleve equations. In our study of Painleve equations we have developed a method to linearize these classical nonlinear ordinary differential equations. The linearization is provided by a system of Riemann-Hilbert boundary value problems which can be related to a system of linear integral equations. A rigorous theory is under development.

(vi) Moving pressure distributions in surface gravity waves and semi-infinite boundary value problems. A general theory to study semi-infinite boundary value problems has been formulated. Solutions to these systems are under study and a connection between certain boundary value problems on the semi-infinite line and forced nonlinear wave equations has been obtained.

(vii) Solutions to a class of nonlinear singular integro-differential equations have been developed. These include the well known Benjamin-Ono, Intermediate Long Wave and Sine-Hilbert equations which arise in stratified fluid dynamics. We have recently been studying multidimensional nonlinear singular integro-differential equations and have a number of interesting results.

(viii) Analysis and development of the symmetries, Hamiltonian structures and recursion operators for two spatial dimensional problems has been constructed. A general theoretical framework has been developed.

The broad attack is to understand the behavior of coherent structures and solutions to nonlinear equations arising in physical problems. The results obtained and wide ranging interest by scientists and engineers

in our work have motivated many of the studies. In what follows is a list of recent publications.

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