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A Method for Calculating Asymmetric Flow Through Nozzles

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FOR THE COMMANDER

Lester L Lyles

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The asymmetric steady-state, inviscous, adiabatic flow of a perfect gas through a subsonic/supersonic nozzle is considered. The nozzle itself is either of rectangular cross section or is axisymmetric. A first-order small asymmetry induced by an entrance flow which is oblique to the nozzle axis and has a transverse pressure gradient is allowed. The cross section of the nozzle is assumed to vary only slowly as a function of the axial distance; the case considered here is therefore an extension of the well-known theory of the quasi-one-dimensional flow of a perfect gas treated in standard textbooks. An integral method is used to obtain approximate results. The method is simple, yet in a test case (supersonic flow at an angle of attack through a rectangular channel) where the exact first-order result is known, a comparison shows surprisingly good agreement. Numerical results are tabulated for axisymmetric nozzles with polynomial Mach number dependence.

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1. INTRODUCTION

High-speed flows that have a small lateral asymmetry occur in rocket motors that have canted nozzles (to avoid impingement of the plume on adjacent structures) or nozzles that can be vectored. Interest in this type of flow has been renewed recently, since it is thought to play a role in an instability observed in certain spin-stabilized, solid-propellant, upper-stage rockets. In this case, the flow at the nozzle entrance is oblique to the nozzle axis as a consequence of the Coriolis force acting on the combustion gas in a precessing and nutating rocket (Ref. 1,2).

Nonaxisymmetric flows through nozzles have also been considered by Walters, (Ref. 3) both experimentally and theoretically. However, in that case, the nozzles themselves were nonaxisymmetric, having an obliquely machined throat section. Other authors have studied nonaxisymmetric flows in nozzles, but generally they have confined themselves to the supersonic part of the flow, computed by the method of characteristics, starting from an assumed inclined sonic surface at the nozzle throat (Refs. 4,5). None of these studies is directly applicable to the case encountered in canted nozzles or in unstable spinning and precessing rocket vehicles.

In these applications, the flow--as a correction to the axisymmetric flow--is sufficiently closely approximated by assuming a steady-state, inviscous and adiabatic flow of a perfect gas. Based on the assumption of constant reservoir conditions upstream of the nozzle, the flow is isentropic and isoenergetic (constant total enthalpy) everywhere. Analogously to the well-known, elementary theory of quasi-one-dimensional flow, the nozzle cross section is assumed to vary only slowly with axial distance. Since the lateral asymmetry caused by the angle of attack or transverse pressure gradient of the entrance flow is also assumed small, the effects due to the changing cross section and those due to the lateral asymmetry can both be treated as small perturbations superposed on the one-dimensional zero-order flow.

and similarly for the density ρ, enthalpy h, and the Cartesian velocity components u,v,w; hence, for instance, for u we write

$$u = u_o(z) + \varepsilon u_1 + \text{s.f.p.} \tag{1b}$$

The symmetric flow perturbation (s.f.p.) terms are written informally merely as a reminder; it will be clear from the subsequent development that, by reason of symmetry, they do not contribute to any of the integrals defined below.

The first-order asymmetric terms ρ_1, h_1, w_1 can all be expressed in terms of p_1. Thus, from the assumption of a perfect gas and constant entropy, $\rho/\rho_o(z) = [p/p_o(z)]^{1/\gamma}$ where γ is the ratio of the specific heats. Since this relation applies separately to the symmetric and the asymmetric case and superposition holds, it follows, after neglecting second- and higher-order terms, that

$$\rho_1/\rho_o(z) = \frac{1}{\gamma}\, p_1/p_o(z) \tag{2a}$$

and similarly

$$h_1/h_o(z) = \frac{\gamma - 1}{\gamma}\, p_1/p_o(z) \tag{2b}$$

From conservation of energy, $h + 1/2(u^2 + v^2 + w^2) = \text{const.}$, and neglecting again second-order terms, it follows that $w_1/w_o = -h_1/w_o^2$. Introducing the Mach number $M_o(z)$ of the quasi-one-dimensional flow, $M_o = w_o/\ (\gamma - 1)h_o$, we can express the asymmetric perturbation of the axial velocity component by

$$w_1/w_o(z) = -\,[\gamma M_o^2(z)]^{-1}\, p_1/p_o(z) \tag{2c}$$

2. CONSERVATION EQUATIONS

We consider a control volume bounded by transverse planes at z and z + dz and by the nozzle walls. We designate by $\varepsilon P_{1x}(z)$ the transverse momentum (which is in the x-direction) carried by the flow through the transverse plane, per unit time. In the case of the rectangular nozzle, P_{1x} is taken per unit width in the y-direction. From conservation of momentum to the lowest significant order

$$\frac{dP_{1x}}{dz} = -\left[(p_1)_{x=R(z)} - (p_1)_{x=-R(z)}\right] \qquad \text{if } \sigma = 1 \tag{3a}$$

$$\frac{dP_{1x}}{dz} = -R(z)\int_{\phi=0}^{2\pi} (p_1)_{r=R(z)} \cos\phi \, d\phi \qquad \text{if } \sigma = 2 \tag{3b}$$

for the rectangular ($\sigma = 1$) and axisymmetric ($\sigma = 2$) nozzle, respectively.

The transverse angular momentum about the origin of the coordinate system is in the y-direction. The amount carried through a plane normal to the nozzle axis, per unit time (and per unit width in the case of the rectangular nozzle), is designated by $\varepsilon L_{1y}(z)$. When second- and higher-order terms are dropped, the momentum flux ρw^2 in the z-direction becomes

$$\rho w^2 = \rho_o w_o^2 + 2\varepsilon\rho_o w_o w_1 + \varepsilon w_o^2 \rho_1 + \text{s.f.p.}$$

By symmetry, the zero and symmetric first-order terms in ρw^2 make no contribution to the integration over x, so that for the rectangular nozzle

$$L_{1y} = zP_{1x} - \int_{x=-R(z)}^{R(z)} (2\rho_o w_o w_1 + w_o^2 \rho_1) x dx \qquad \text{if } \sigma = 1 \tag{4a}$$

The first term on the right comes from the u component of the velocity, integrated over the cross section; the second term is from w.

By essentially the same argument, for the axisymmetric nozzle

$$L_{1y} = zP_{1x} - \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} (2\rho_o w_o w_1 + w_o^2 \rho_1) r^2 \cos\phi \, dr \, d\phi \qquad \text{if } \sigma = 2 \qquad (4b)$$

Conservation of angular momentum then requires that

$$\frac{dL_{1y}}{dz} = \frac{d}{dz} \int_{x=-R(z)}^{R(z)} p_1 x dx + \left(z + R\frac{dR}{dz}\right) \frac{dP_{1x}}{dz} \qquad \text{if } \sigma = 1 \qquad (5a)$$

$$\frac{dL_{1y}}{dz} = \frac{d}{dz} \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} p_1 r^2 \cos\phi \, dr \, d\phi + \left(z + R\,\frac{dR}{dz}\right) \frac{dP_{1x}}{dz} \qquad \text{if } \sigma = 2 \qquad (5b)$$

where Eqs. (3a) and (3b), respectively, have been used and where again the zero-order and first-order symmetric terms make no contribution to the integrals. The first term on the right represents the moment from forces acting on the transverse planes bounding the control volume; the last term results from the wall pressure.

The term L_{1y} is eliminated from these equations by differentiating Eqs. (4) and substituting the result into Eqs. (5). When Eqs. (2) also are used to eliminate ρ_1 and w_1, it follows that

$$\frac{d}{dz} \int_{x=-R(z)}^{R(z)} (M_o^2 - 1) p_1 x dx + R\,\frac{dR}{dz}\,\frac{dP_{1x}}{dz} - P_{1x} = 0 \qquad \text{if } \sigma = 1$$

$$\frac{d}{dz} \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} (M_o^2 - 1) p_1 r^2 \cos\phi \, dr \, d\phi + R\frac{dR}{dz}\,\frac{dP_{1x}}{dz} - P_{1x} = 0 \qquad \text{if } \sigma = 2$$

where use has been made of the expression for the dynamic pressure of the quasi-one-dimensional flow (Ref. 6) $(\rho_o/2)w_o^2 = (\gamma/2)p_o M_o^2$. Differentiating again with respect to z and using Eqs. (3) gives the following equations for p_1, where now all other first-order perturbation terms have been eliminated:

$$\frac{d^2}{dz^2}\left\{\left(M_o^2 - 1\right)\int_{x=-R(z)}^{R(z)} p_1 x dx\right\} - R\frac{dR}{dz}\frac{d}{dz}\left[(p_1)_{x=R(z)} - (p_1)_{x=-R(z)}\right]$$

$$+\left[1 - \frac{d}{dz}\left(R\frac{dR}{dz}\right)\right]\left[(p_1)_{x=R(z)} - (p_1)_{x=-R(z)}\right] = 0 \qquad \text{if } \sigma = 1 \qquad (6a)$$

$$\frac{d^2}{dz^2}\left\{\left(M_o^2 - 1\right)\int_{r=0}^{R(z)}\int_{\phi=0}^{2\pi} p_1 r^2 \cos\phi \, dr \, d\phi\right\} - R\frac{dR}{dz}\frac{d}{dz}\left[R\int_{\phi=0}^{2\pi}(p_1)_{r=R(z)}\cos\phi \, d\phi\right]$$

$$+\left[1 - \frac{d}{dz}\left(R\frac{dR}{dz}\right)\right] R\int_{\phi=0}^{2\pi}(p_1)_{r=R(z)}\cos\phi \, d\phi = 0 \qquad \text{if } \sigma = 2 \qquad (6b)$$

3. INTEGRAL METHOD

Integral methods are typically based on prescribing for the dependent variable a simple functional form that may depend on one or several parameters and satisfies the boundary conditions. The parameters then are determined such that the integral relations--in this case, the conservation equations for the transverse momentum and angular momentum integrated over the nozzle cross-section--are satisfied. With a judicious choice of the functional dependence, useful results, although of limited accuracy, can often be obtained.

In the present case, we prescribe for the asymmetric perturbation term p_1 a linear dependence on the transverse coordinate, in the form therefore

$$\frac{p_1}{p_o(z)} = \frac{g_1(z)\, x}{R(z)} \tag{7}$$

where the nondimensional coefficient $g_1(z)$ is to be determined.

We designate by R^* the nozzle half width ($\sigma = 1$) or nozzle radius ($\sigma = 2$) at the throat (a fictitious throat if the quasi-one-dimensional flow is subsonic throughout) where $M_o = 1$, and $p_o = p_o^*$ and define the nondimensional quantities

$$\xi = R/R^* \quad , \quad \zeta = z/R^* \quad , \quad k_1(\zeta) = \frac{R^*}{R(z)} \frac{p_o(z)}{p_o^*} g_1(z) \tag{8}$$

Carrying out the integrations in Eqs. (6) yields a combined relation for the rectangular and axisymmetric nozzles, in the form of a second-order differential equation for $k_1(\zeta)$

$$\frac{1}{\sigma + 2} \frac{d^2}{d\zeta^2}\left[\xi^{\sigma+2}\left(M_o^2 - 1\right) k_1\right] - \frac{d}{d\zeta}\left[\frac{d\xi}{d\zeta}\, \xi^{\sigma+1}\, k_1\right] + \xi^{\sigma}\, k_1 = 0 \tag{9}$$

Equation (9) represents the principal result of Sections 1 through 3 and is suitable for computer programming of the direct problem (prescribed zero order Mach number as a function of axial distance) or of the inverse problem (prescribed nozzle contour). In either case, it is advantageous to express the function $\xi(\zeta)$ in Eq. (9) by the Mach number $M_o(\zeta)$. From a well-known formula for quasi-one-dimensional flows (Ref. 6)

$$\xi = \left\{ \frac{1}{M_o^2} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M_o^2\right)\right]^{(\gamma+1)/(\gamma-1)} \right\}^{1/(2\sigma)} \tag{10}$$

As one would expect, Eq. (9) exhibits the transition at $M_o = 1$ between an essentially exponential behavior of $k_1(\zeta)$ and, on the other hand, a wavelike character as the sign of the coefficient of the highest (second) derivative changes.

Boundary Conditions

As is well known, classical one-dimensional nozzle theory requires the Mach number to be unity at a throat where a transition from subsonic to supersonic flow occurs. An analogous condition occurs in the present case and is caused by the vanishing of the coefficient in Eq. (9) of the second derivative of $k_1(\zeta)$ at $M_o = 1$. Carrying out the differentiations indicated in Eq. (9) and letting at the throat $M_o = 1$, $\xi = 1$, $d\xi/d\zeta = 0$ results in the condition for the solution to be regular at $\zeta = 0$

$$\frac{2}{\sigma + 2} \frac{dM_o^2}{d\zeta} \frac{dk_1}{d\zeta} + \left[1 - \frac{d^2}{d\zeta^2}\left(\xi - \frac{M_o^2}{\sigma + 2}\right)\right] k_1 = 0 \qquad \text{at } \zeta = 0 \tag{11a}$$

This condition, therefore, relates the first derivative of k_1 to k_1 at a sonic throat.

A second boundary condition results from prescribing the transverse pressure gradient $\varepsilon\partial p_1/\partial x$ at an initial (upstream) location $\zeta = \zeta_i$, hence the condition

$$k_1 = \frac{R^*}{p_o^*} \frac{\partial p_1}{\partial x} \qquad \text{at } \zeta = \zeta_i \qquad (11b)$$

In applications to rocket motors, the transverse pressure gradient at the entrance to the nozzle needs to be determined from pressure matching with the flow field in the motor chamber adjacent to the nozzle. Since the combustion gas velocity in the chamber is typically far below the speed of sound, the calculation is simplified by the assumption of incompressibility, although it is sometimes complicated by the geometry of the boundaries. This latter calculation is outside the scope of the present report.

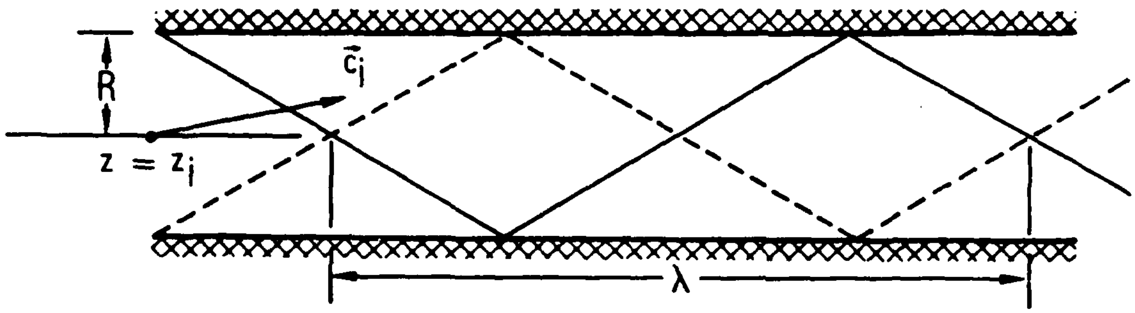
After $k_1(\zeta)$ has been obtained from a solution of Eq. (9), the transverse pressure gradient is obtained from Eqs. (7) and (8). In turn, one obtains the asymmetric force εF_1 (per unit axial length) exerted by the gas on the nozzle wall. This force is often the quantity of greatest interest in applications. For instance, for the axisymmetric nozzle, after integrating over the polar angle ϕ

$$F_1 = \pi(p_o^*/R^*)\, R^2(\zeta)k_1(\zeta) \qquad (12)$$

Numerical results for the homogeneous case, applied to a parameterized family of nozzle contours, are discussed in Section 5.

4. A TEST CASE

Supersonic flow, incident at a small angle on a rectangular duct of uniform width (Fig. 1), represents an example in which an exact--at least in the sense of a first-order perturbation result--solution is easily calculated. It is of interest, therefore, to compare, as a test case, results obtained from Eq. (9) with the exact, but more restricted, solution in this special case. The exact first-order solution, which is well known, is described by a periodic pattern of triangular and rhombic regions bounded by Mach lines, in which the flow properties are constant, but changing discontinuously across the Mach lines.



Note: R = half width; C_i = incident flow velocity; λ = period

Figure 1. Supersonic Flow with Small Angle of Incidence Through Rectangular Duct

With M_o again designating the unperturbed Mach number, and R the half width, the period λ of the flow perturbation is given by

$$\lambda = 4R\sqrt{M_o^2 - 1} \qquad \text{(exact first-order theory)} \qquad (13a)$$

On the other hand, from Eq. (9), since here $d\xi/d\zeta = dM_o^2/d\zeta = 0$ and $\sigma = 1$

$$\frac{d^2k_1}{d\zeta^2} + \frac{3}{\xi^2(M_o^2 - 1)} k_1 = 0$$

The transverse pressure gradient, and similarly the other perturbation quantities, therefore, have a sinusoidal dependence on the axial coordinate, with period

$$\lambda = \frac{2\pi}{\sqrt{3}} R\sqrt{M_o^2 - 1} = 3.628 R\sqrt{M_o^2 - 1} \qquad \text{(integral method)} \qquad (13b)$$

The integral method, therefore, gives the correct Mach number dependence although with a multiplier which differs from the correct one by approximately 10%.

5. NOZZLES WITH POLYNOMIAL MACH NUMBER DEPENDENCE

As an example of the application of Eq. (9), we consider an axisymmetric Laval nozzle for which the square of the unperturbed (quasi-one-dimensional) Mach number varies with axial distance as a second-degree polynomial, for which we choose

$$M_o^2 - 1 = a\zeta(1 - b\zeta), \qquad a > 0,\ b \geq 0 \tag{14}$$

where a and b are constants. The nozzle contour is easily calculated from Eq. (10). For suitably chosen constants, relation (14) results in quite realistic nozzle contours (Fig. 2).

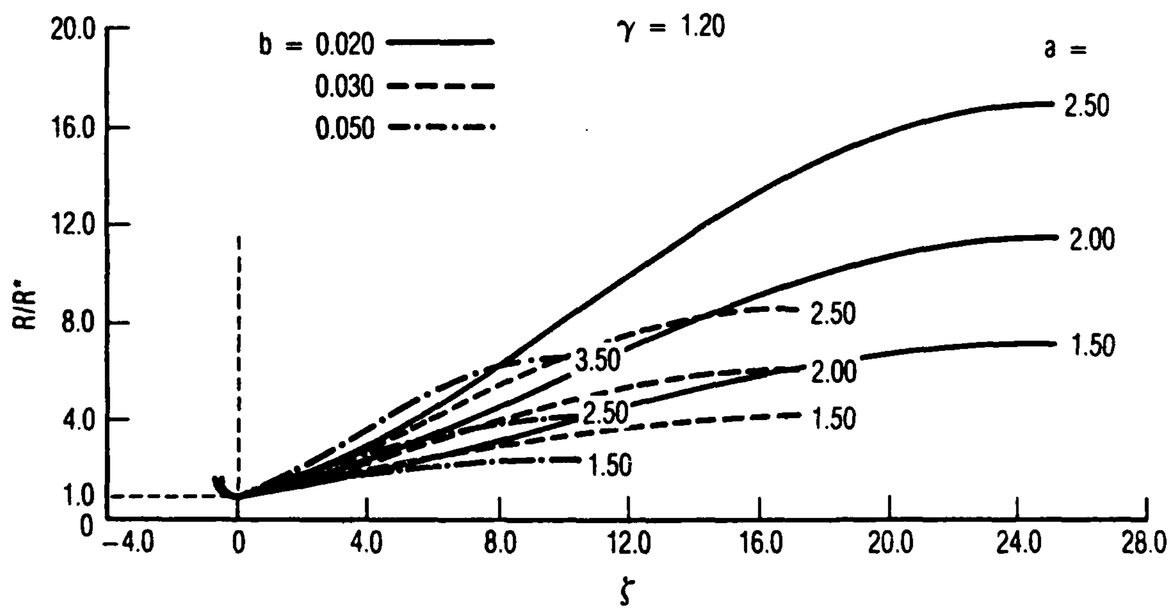


Figure 2. Nozzle Contours Satisfying Eq. (14), for $\gamma = 1.20$

At $M_o = 0$ the slope of the nozzle contour becomes infinite. This occurs at $\zeta = -(2b)^{-1}(\sqrt{1 + 4b/a} - 1)$. Clearly, the assumption of quasi-one-dimensional flow breaks down for ζ approaching this value. At $\zeta = (2b)^{-1}$, the Mach number M_o and, hence, the nozzle radius each reach a maximum, which occurs in the supersonic part of the flow.

The transverse pressure gradient in the form of the function $k_1(\zeta)$ and the lateral force per unit axial length exerted on the nozzle, in the form of the nondimensional ratio $F_1(\zeta)/(p^*R^*)$, are computed for several values of the nozzle parameters a and b. They are graphed in Figures 3 and 4. (For the range of b that is of practical interest for realistic nozzle contours, the curves in Figure 3 very nearly coincide and depend only on the parameter a.) The solutions are computed for a ratio of the specific heats $\gamma = 1.20$, a value that is representative of many rocket motor combustion gases, and are normalized so that $k_1 = 1$ at the nozzle throat. It is evident that the largest transverse pressure gradients and nozzle side forces occur in the subsonic and transonic sections of the nozzle where most of the readjustment of the flow direction takes place. It is also evident, particularly for the smaller values of the parameter a, that the transverse pressure gradient downstream of the throat at first reverses sign, an indication of the reflection on the nozzle walls of the Mach cones associated with the turning of the flow.

Table 1 lists numerical values for the side force on the nozzle. The effect of Mach cone reflections reversing the sign of the force in the diverging (supersonic) part of the nozzle is again evident.

In some applications, such as in spin-stabilized solid-propellant rockets, the condition at the nozzle exit plane is of particular interest, since it determines the angular momentum flux of the combustion gas and hence the resulting moment about the vehicle's center of mass. Table 2, therefore, lists the transverse force F_{1e} at the exit plane, per unit axial length, relative to the force F_1^* at the nozzle throat, for nozzles with the polynomial Mach number dependence (14) and zero nozzle divergence at the exit plane.

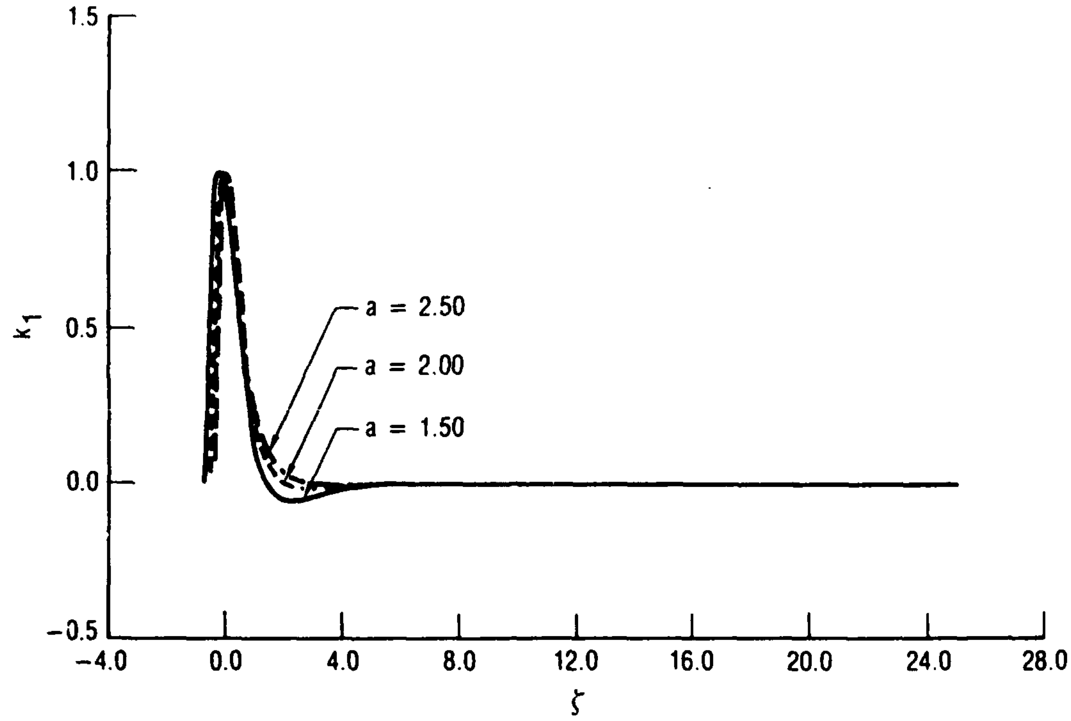


Figure 3. Function k_1 (ζ) Computed from Eq. (9) and Normalized to k_1 = 1 at Nozzle Throat

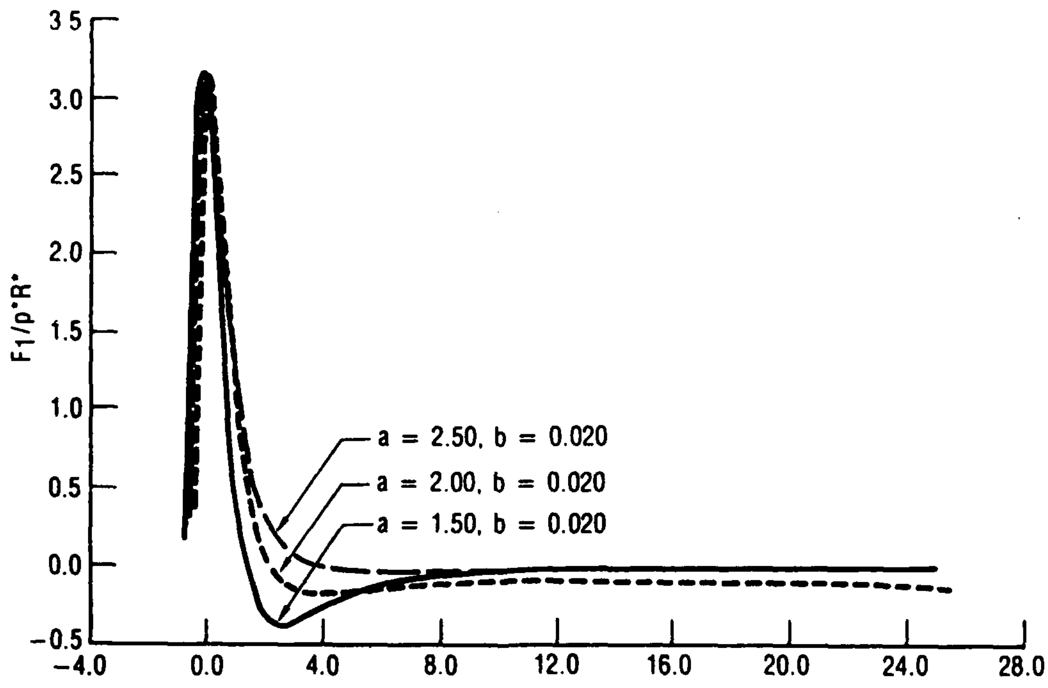


Figure 4. Nondimensional Ratio $F_1(\zeta)/(p^*R^*)$ of the Transverse Force F_1 on the Nozzle Wall, per Unit Axial Length

Table 1. Function $F_1(\zeta)/(p^*R^*)$ for the Nozzle Contours Shown in Figure 2 for γ = 1.20

ζ	a = 1.50 b = 0.020	a = 2.00 b = 0.020	a = 2.50 b = 0.020	a = 1.50 b = 0.030	a = 2.00 b = 0.030	a = 2.50 b = 0.030	a = 1.50 b = 0.050	a = 2.50 b = 0.050	a = 3.50 b = 0.050
-0.6	+ 0.918			+ 0.861			+ 0.750		
-0.4	+ 2.774	+ 1.327		+ 2.717	+ 1.300		+ 2.663		
-0.2	+ 3.315	+ 2.810	+ 2.368	+ 3.305	+ 2.801	+ 2.359	+ 3.287	+ 2.342	+ 1.466
0	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141
0.2	+ 2.627	+ 2.879	+ 3.047	+ 2.632	+ 2.884	+ 3.052	+ 2.640	+ 3.062	+ 3.263
0.4	+ 2.017	+ 2.391	+ 2.616	+ 2.022	+ 2.398	+ 2.625	+ 2.033	+ 2.642	+ 2.849
0.6	+ 1.436	+ 1.870	+ 2.118	+ 1.441	+ 1.877	+ 2.129	+ 1.449	+ 2.150	+ 2.345
0.8	+ 0.940	+ 1.399	+ 1.662	+ 0.942	+ 1.406	+ 1.673	+ 0.946	+ 1.695	+ 1.898
1.0	+ 0.542	+ 1.008	+ 1.282	+ 0.541	+ 1.013	+ 1.292	+ 0.538	+ 1.314	+ 1.538
1.2	+ 0.238	+ 0.697	+ 0.980	+ 0.234	+ 0.699	+ 0.989	+ 0.223	+ 1.007	+ 1.260
1.4	+ 0.015	+ 0.457	+ 0.746	+ 0.007	+ 0.456	+ 0.753	- 0.011	+ 0.766	+ 1.048
1.6	- 0.143	+ 0.276	+ 0.568	- 0.155	+ 0.271	+ 0.572	- 0.179	+ 0.579	+ 0.886
1.8	- 0.250	+ 0.141	+ 0.432	- 0.265	+ 0.133	+ 0.433	- 0.295	+ 0.435	+ 0.762
2.0	- 0.319	+ 0.041	+ 0.329	- 0.335	+ 0.031	+ 0.328	- 0.370	+ 0.325	+ 0.667
2.5	- 0.381	- 0.101	+ 0.168	- 0.401	- 0.118	+ 0.161	- 0.442	+ 0.145	+ 0.508
3.0	- 0.364	- 0.159	+ 0.085	- 0.382	- 0.180	+ 0.075	- 0.422	+ 0.050	+ 0.417
3.5	- 0.318	- 0.178	+ 0.042	- 0.334	- 0.200	+ 0.029	- 0.366	- 0.003	+ 0.362
4.0	- 0.267	- 0.178	+ 0.019	- 0.279	- 0.202	+ 0.005	- 0.301	- 0.032	+ 0.327
4.5	- 0.220	- 0.171	+ 0.006	- 0.228	- 0.195	- 0.009	- 0.238	- 0.050	+ 0.304
5.0	- 0.181	- 0.162	- 0.001	- 0.183	- 0.185	- 0.017	- 0.181	- 0.061	+ 0.289
6.0	- 0.122	- 0.143	- 0.007	- 0.116	- 0.165	- 0.024	- 0.088	- 0.072	+ 0.275
7.0	- 0.084	- 0.127	- 0.009	- 0.072	- 0.149	- 0.026	- 0.020	- 0.079	+ 0 275
8.0	- 0.060	- 0.116	- 0.010	- 0.043	- 0.137	- 0.027	+ 0.033	- 0.085	+ 0.287
9.0	- 0.044	- 0.108	- 0.010	- 0.023	- 0.128	- 0.027	+ 0.070	- 0.093	+ 0.311
10.0	- 0.034	- 0.101	- 0.010	- 0.009	- 0.123	- 0.027	+ 0.103	- 0.103	+ 0.351

Table 2. Ratio F_{1e}/F_1^* of the Transverse Force F_{1e} on the Nozzle Wall, per Unit Axial Length, at the Nozzle Exit Plane, Relative to the Corresponding Force F_1^* at the Throat

	F_{1e}/F_1^*
a = 1.50, b = 0.020	-2.468×10^{-3}
a = 2.00, b = 0.020	-3.811×10^{-2}
a = 2.50, b = 0.020	-4.730×10^{-3}
a = 1.50, b = 0.030	$+9.298 \times 10^{-3}$
a = 2.00, b = 0.030	-4.230×10^{-2}
a = 2.50, b = 0.030	-1.205×10^{-2}
a = 1.50, b = 0.050	$+3.267 \times 10^{-2}$
a = 2.50, b = 0.050	-3.279×10^{-2}
a = 3.50, b = 0.050	$+1.116 \times 10^{-1}$

Note: For $\gamma = 1.20$, and zero nozzle divergence angle at the exit plane.

REFERENCES

1. Flandro, G. A., "Interaction of Inertial Waves in a Spinning Solid Propellant Rocket Motor with Spacecraft Motion," Air Force Rocket Propulsion Laboratory Report, October 1982. Also, "Fluid Mechanics of Spinning Rockets," Air Force Rocket Propulsion Report TR-86-072, January 1987.

2. Meyer, R. X., "Convective Instability in Solid Propellant Rocket Motors," AAS Paper 83-368, Advances in the Astronautical Sciences, Vol. 54, 1983.

3. Walters, A. G., "Nonsymmetric Flow in Laval Type Nozzles," Royal Soc. Phil. Transactions, A Vol. 273, 1972, pp. 185-235.

4. Darwell, H. M. and G. F. P. Trubridge, "Design of Rocket Nozzles to Reduce Gas Misalignment," J. of Spacecraft and Rockets, Vol. 5, No. 1, 1968, pp. 36-41.

5. Hoffman, J. D. and A. R. Maykut, "Gas Dynamic Gain of Supersonic Thrust Nozzles," J. of Spacecraft and Rockets, Vol. 11, No. 10, 1974, pp. 697-704.

6. Liepmann, H. W. and A. Roshko, Elements of Gas Dynamics, John Wiley and Sons, Inc., Chapters 2 and 5, 1957.

NOMENCLATURE

a, b	parameters [Eq. 14)]
$F_1(z)$	transverse force on nozzle, per unit axial length
g_1, k_1	functions characterizing transverse pressure gradient [Eqs. (7) and (8)]
h	enthalpy
$L_{1y}(z)$	transverse component of angular momentum
$M_o(z)$	Mach number of zero-order (unperturbed) flow
p	pressure
$P_{1x}(z)$	transverse component of momentum
R(z)	half-width of rectangular nozzle, and radius of axisymmetric nozzle
u,v,w	Cartesian velocity components
x,y,z; r,ϕ,z	Cartesian and cylindrical coordinates
γ	ratio of specific heats
ξ, ζ	nondimensional lengths [Eq. (8)]
ρ	density
σ	$\sigma = 1$ for rectangular nozzle; $\sigma = 2$ for axisymmetric nozzle
$(\ )_o$	zero-order term
$(\ )_1$	first-order asymmetric term
$(\ )^*$	condition at $M_o = 1$
$(\ )_i$	initial condition
$(\ )_e$	nozzle exit plane condition