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Ground Target Location Errors Derived from Measurements Collected from a Variety of Hypothetical Satellite Sentinel Systems

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A large number of symmetric circular orbit satellite constellations were tested for their worth in providing continuous surveillance of five selected corps-sized regions over various parts of the world. The results of this work when combined with results from a previous report can be used to evaluate the target location mensuration capability of a variety of target mensuration systems located on satellite platforms defined by the constellations.

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PREFACE

This study was conducted under DA Project 4A762707A855, "Topographic Mapping Technology."

The study was conducted during the spring and summer of 1988 under the supervision of Mr. Donald R. Barnes, Chief, Space Concepts Division; and Dr. Joseph J. Del Vecchio, Director, Space Programs Laboratory.

Col. David F. Maune, EN, was Commander and Director, and Mr. Walter E. Boge was Technical Director during the report preparation.

GROUND TARGET LOCATION ERRORS DERIVED FROM MEASUREMENTS COLLECTED FROM A VARIETY OF HYPOTHETICAL SATELLITE SENTINEL SYSTEMS

INTRODUCTION

A hypothetical sentinel system defined by a variety of possible satellite constellations is evaluated for its ground point target mensuration capability. The satellite constellations were designed to provide single, double, triple and quadruple ground coverage.[1] Five corps-sized regions were selected for the target location evaluation. Each of the trial constellations was allowed to run for 24 hours and each target point within each target region was tested every 5 minutes to see if the programmed number of lines of sight occurred. In every case, the average distance to target and its standard deviation were calculated for each corps-sized region as well as a summary average and summary standard deviation over all corps-sized regions. In the case of double coverage and greater, the average angular distance and its standard deviation were calculated between pairs of target trackers as viewed simultaneously from the target. In the cases of triple coverage and greater, the average value and its standard deviation of minimum PDOP (Position Dilution of Precision) were calculated from the simultaneous slant range observations on the target. The average distance to target, average angle, and average minimum PDOP can be used in conjunction with another ETL report to derive expected 99% target spherical errors as a function of expected target tracking system error.[2]

EXPERIMENT

Real time exterior orientation is a requirement of the hypothetical sentinel target tracking system. The position component will be derived from knowledge of the target tracker's orbit. The orbit will be determined through ground tracking aided by GPS (Global Positioning System) calculations where applicable. The attitude component will be derived from a star camera-gyro system, which is discussed in more detail below.

Attitude. The following comments pertain to the general problem of target camera determination when attitude information is transferred to the target camera from one or more stellar cameras rigidly locked to the target camera. More than one star camera is sometimes needed to cancel the adverse effect in the target camera caused by a poor estimate of the star camera yaw angle (rotation about the star camera Z-axis). It should be noted here that an inaccurate estimate of the yaw angle will cause little or no problem in the target camera if targets are measured at the principal point and if the star camera and the target camera are parallel. Estimates of the adverse effect of a variety of one-stellar and two-stellar camera configurations on target image positions are given in tables 2 and 3.

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[1] T.J. Lang. "Symmetric Circular Orbit Satellite Constellations For Continuous Global Coverage," AAS/AIAA Astrodynamics Specialist Conference. Kalispell, Montana, August 1987.

[2] M.A. Crombie. Target Location Errors Derived From A Hypothetical Target Tracking System. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, 22060-5546. Report ETL-0531, February 1989.

Star Camera - Gyro System. A hypothetical real time attitude system is conceived along the lines described by Strikwerda.[3] In that work, star cameras were conceived around solid state digital imaging array technology for real time computation of highly accurate attitude on the order of one arc second or less. That system includes one or more star cameras rigidly oriented with respect to a 3-axis gyro system, wherein vehicle rotation rates are measured about three mutually orthogonal gyro axes. The angular rate data are integrated over time to produce instantaneous attitude data and also to provide estimates of star camera attitudes at future exposure times. The estimated star camera attitudes are used as first estimates in a least-squares adjustment process and also to determine a set of stars that will most likely be imaged by the pertinent star camera. It is expected that approximately five star images per exposure will be available for the least-squares attitude determination. The two attitude estimates, namely the one derived by integration and the other derived by the star camera adjustment, are combined by a discrete Kalman filter process to produce an optimal estimate of the attitude at specific times.

Star Camera. At present, plans are underway to evaluate star camera performance in the NASA space shuttle. The star camera, being fabricated under contract, is a narrow angle, solid-state focal plane device composed of an array of 512 by 403 detectors. One-half of the detectors in the 512 dimension are exposed to the star field and the other half are used for storage. Detector dimensions are 20 micrometers (along the 512 dimension) by 16 micrometers (along the 403 dimension). The focal length is 41.0 millimeters which provides a 7.2 degree field-of-view by a 9.0 degree field-of-view. Starlight is slightly defocused in the camera in order to spread a star image over 9 to 16 neighboring detectors. Star image coordinates are determined by a centroid process. It is expected that coordinate accuracy can be determined to 2 percent of a detector dimension. The scan times of the field-of-view are performed rapidly enough so that the exposure times for the several star images can be considered to be at the same instant.

An estimate of the star camera attitude, derived from the gyro subsystem, is used as a first estimate in a least-squares process and used also to select a subset of star directions from the star catalog data base. A sophisticated star identification procedure is used to coordinate star images with star directions. A refined attitude estimate is produced by a least-squares adjustment of the star image observations.

The shuttle experiment will also incorporate an on-board processor element designed to demonstrate the practical feasibility of real time autonomous attitude determination. A successful completion of this experiment is anticipated to further substantiate the hypothetical real time attitude system previously described.

Star Camera - Target Camera Relative Orientation. The interlock angles for two star cameras with respect to the target camera are defined in the following way. Consider figure 1, where the X_{S1} -axis is in the X_TY_T-plane. The same is true for the X_{S2}-axis (see figure 2). From figure 1, β_1 is the elevation angle of the Z_{S1}-axis above the X_TY_T-plane. Since the X_{S1}-axis is in the X_TY_T-plane, a rotation of $\omega_{S1} = 90° - \beta_1$ about the X_{S1}-axis will rotate the Z_{S1}-axis into the Z_T-axis. From figure 1, α_1 is the angular distance of the $-X_T$-axis to the Y'_{S1}-axis measured clockwise in the X_TY_T-plane. A rotation of $K_{S1} = -(90° - \alpha_1)$ about the Z'_{S1}-axis $= Z_T$-axis will complete the rotation of the first star camera reference frame into the target camera reference frame. The roll, pitch, and yaw angles that affect this transformation are

[3] Thomas E. Strikwerda and John L. Junkins. Star Pattern Recognition and Spacecraft Attitude Determination, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546, ETL-0260, May 1981, AD-A103 806.

$$\omega_{S1} = 90° - \beta_1$$

$$\rho_{S1} = 0°$$

$$K_{S1} = -(90° - \alpha_1)$$

The parameters (α_1, β_1) are chosen by the user to reflect a desired orientation.

From figure 2, β_2 is the elevation angle of the Z_{S2}-axis above the $X_T Y_T$-plane. Since the X_{S2}-axis is in the $X_T Y_T$-plane, a rotation of $\omega_{S2} = -(90° - \beta_2)$ about the X_{S2}-axis will rotate the Z_{S2}-axis into the Z_T-axis. From figure 2, α_2 is the angular distance of the $-X_T$-axis to the $-Y'_{S2}$-axis measured counter clockwise in the $X_T Y_T$-plane. A rotation of $K_{S2} = (90° - \alpha_2)$ about the $Z'_{S2} = Z_T$-axis will complete the rotation of the second star camera reference frame into the target camera reference frame. The roll, pitch, and yaw angles that affect this transformation are

$$\omega_{S2} = -(90° - \beta_2)$$

$$\rho_{S2} = 0°$$

$$K_{S2} = (90° - \alpha_2)$$

Note that if $\alpha = \beta = 90°$ for either star camera, the star camera and the target camera are parallel.

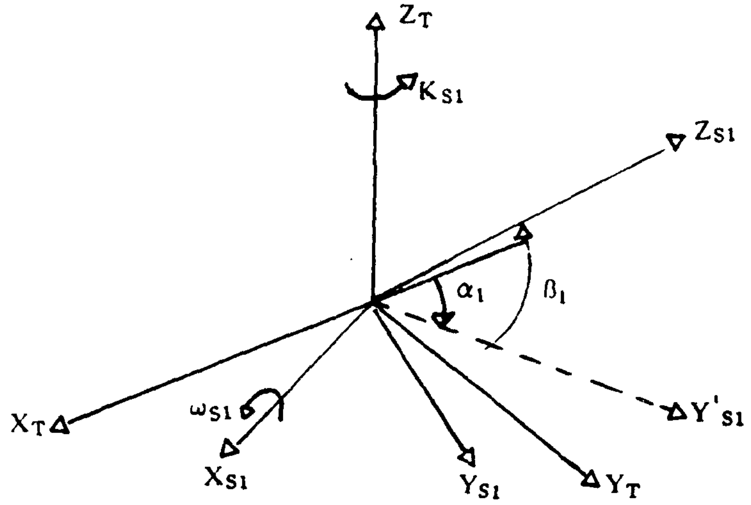

Figure 1. Relative Orientation of First Star Camera

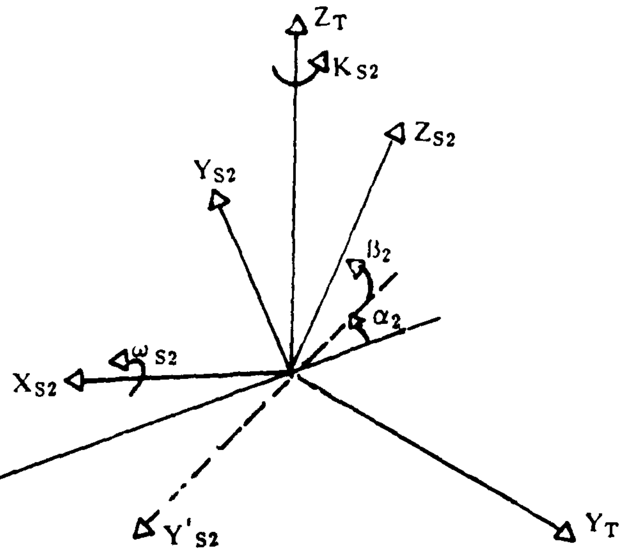

Figure 2. Relative Orientation of Second Star Camera

Sentinel Constellations. A variety of sentinel constellations are evaluated to determine how they perform as platforms for ground target location determination.[4] The constellations are characterized by symmetric circular orbits, wherein every satellite within a specific constellation has a common inclination and a common elevation above the earth. The constellations fall into four groups depending on whether they are designed to provide single, double, triple, or quadruple ground coverage.

A specific constellation is characterized by six parameters. A set of three integers (T/P/F) is used to define the total number of satellites in the constellation (T), the number of orbital planes (P), and the relative phasing parameter (F). Since the satellites in a particular constellation are symmetrically arranged, there must be T/P satellites in a given plane, all equally spaced in central angle. The P orbital planes must be equally spaced in right ascension of the ascending node. The relative phasing parameter F is used to relate satellites in one orbital plane to those in another plane. For example, if there is a satellite at its ascending node in one orbital plane, then the argument of latitude of a satellite in an adjacent plane will be $F \cdot \frac{360}{T}$ degrees.

Three more parameters are used to completely define the constellation. The first is i, the inclination of the orbital planes. The second is ϵ, the elevation angle of the satellite viewing cone. The angle ϵ is the smallest viewing angle that a ray from the satellite makes with a plane tangent to the earth. The viewing angle was set to 5 degrees so as to avoid ground clutter. The third parameter is Θ, the angle at the center of the earth subtended by the satellite viewing cone. The elevation h of the satellite above the earth is determined from the following equation:

$$\text{Cos}\,(\Theta + \epsilon) = \frac{\text{Cos}\,\epsilon}{1 + h/R_E}$$

where R_E is the earth radius. The orbits were determined by using the following values for the earth radius R_E and GM, the product of the constant of gravitation and the mass of the earth.

$$R_E = 6378144.0 \text{ meters}$$

$$GM = 3.986018 \times 10^{14} \text{ meter}^3/\text{second}^2$$

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[4] T.J. Lang. "Symmetric Circular Orbit Satellite Constellations For Continuous Global Coverage," AAS/AIAA Astrodynamics Specialist Conference. Kalispell, Montana, August 1987.

Target Areas. Five corps-sized target areas were designated for the error analysis. The locations of the five target areas are specified in table 1.

Table 1. Target Areas

Target Area	Location	ϕ (degrees)	λ(degrees)	H (meters)	Az (degrees)
1	South Nicaragua Central America	12.0	-85.0	500.0	0.0
2	Al Basrah Persian Gulf	30.5	48.0	50.0	60.0
3	Fulda South Germany	50.5	9.66	600.0	80.0
4	Damascus Syria	33.5	36.5	600.0	90.0
5	Seward Peninsula Alaska	65.0	-163.0	1500.0	90.0

Nine target points were regularly spaced over each approximately 335 by 335 kilometer square target areas. The azimuth angle pertains to the orientation of the central line of points with respect to the central meridian defined by λ. The central target point location is defined by ϕ and λ. Each point within a target area was assigned an elevation above the sphere defined by H.

Target Mensuration. One of the objectives of this effort was to determine how well ground targets could be determined from images of targets sensed by target trackers based on platforms borne by a constellation of satellites. Associated with the effort was a requirement to determine if the constellations described above could be used, and specifically to determine if they did in fact meet their programmed coverage specifications.

The coverage requirement was tested simply by operating each constellation for 24 hours and at 5-minute intervals, determining whether every point in each of the five test areas was observed simultaneously by at least the specified number of satellites. Target measuring accuracy for all four coverage modes was examined by measuring average distances to target and their standard deviation, and then relating those values to expected errors of a hypothetical target tracking system.[5] In the case of double coverage and greater, target measuring accuracy was examined by measuring average angular distance, and its standard deviation, between pairs of target trackers as viewed simultaneoulsy from the target. In the case of triple coverage and greater, target measuring accuracy was examined by measuring minimum PDOP, and its standard deviation, calculated from three simultaneous slant range observations on the target. Position dilution of precision (PDOP) is the square root of the trace of the covariance matrix associated with the least squares adjustment of the three slant range observations divided by the expected standard error in slant range. Minimum PDOP and average angular values were also related to the referenced hypothetical target tracking system.

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[5] M.A. Crombie. Target Location Errors Derived From A Hypothetical Target Tracking System. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546. Report ETL-0531, February 1989.

NUMERICAL RESULTS

Attitude. The covariance matrix of the three fundamental angles of a star camera rotation matrix was estimated by Monte Carlo methods. The rotation matrix, A_{PS}, relates the star coordinate reference system to the star camera coordinate frame. The fundamental angles are defined to be roll (ω_S), pitch (P_S), and yaw (K_S). One of the objectives of the work is to estimate the covariance matrix of the three fundamental angles of the target camera rotation matrix A_{PT}. It was assumed in the simulation that $A_{PS} = I$. The assumption is not realistic, but the following artifice can be used to obtain the covariance matrix of the three angles associated with A_{PT}.[6] Let

$$A_{PT} = A_{ST} A_{S^1 S} A_{PS^1}$$

where $$A_{PS^1} = A^{T}_{S^1 T} A_{PT}$$

and $$A_{S^1 S} = I$$

A_{ST} is the constant rotation matrix which relates the star camera to the target camera. A_{PT} is the rotation matrix which relates object space to the target camera. The (3x3) matrix of partial derivatives of (ω_{PT}, P_{PT}, K_{PT}) with respect to (ω_S, P_S, K_S) are given in a previous report.[7] If E is defined to be that matrix and if σ_{PS} is the star camera covariance matrix, then the target camera covariance matrix is

$$\sigma_{PT} = E \sigma_{PS} E^T$$

σ_{PS} was estimated by averaging 25,000 two star least squares solutions. The numerical results turned out to be

$$\sigma_{PS} = \begin{pmatrix} 2.9667 \times 10^{-10} & -9.8103 \times 10^{-16} & 1.3637 \times 10^{-11} \\ -9.8103 \times 10^{-16} & 2.9622 \times 10^{-10} & 2.7628 \times 10^{-12} \\ 1.3637 \times 10^{-11} & 2.7628 \times 10^{-12} & 8.8682 \times 10^{-8} \end{pmatrix}$$

The measuring error associated with each star image was assumed to be one micrometer. σ_{PS} can be made more general by multiplying the numerical result by $2M^2/N$ where $N \geq 2$ is the number of star measurements and where M is the standard deviation of the star image measuring error.

Let U be the (2x3) matrix of partial derivatives of the target coordinates with respect to (ω_{PT}, P_{PT}, K_{PT}), then the (2x2) covariance matrix of the target coordinates due to star camera errors given in a previous report[8] is

$$\sigma_T = U E \sigma_{PS} E^T U^T$$

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[6] M. A. Crombie. Mapping Camera Image Errors Due to Star Camera Identification and Measuring Errors. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546. Report ETL-RN-73-1, January 1973, AD-759 491.

[7] Ibid.

[8] Ibid.

where

$$\sigma_{PS} = \frac{2M^2}{N} \begin{pmatrix} 2.9667 \times 10^{-10} & -9.8103 \times 10^{-16} & 1.3637 \times 10^{-11} \\ -9.8103 \times 10^{-16} & 2.9622 \times 10^{-10} & 2.7628 \times 10^{-12} \\ 1.3637 \times 10^{-11} & 2.7628 \times 10^{-12} & 8.8682 \times 10^{-8} \end{pmatrix}$$

where

M = Star measurement error in micrometers

and

N = Expected number of star images

If K star cameras are used for orientation, then

$$\sigma_T = U \left(\sum_{k=1}^{K} \left(E_k \, \sigma_{PS_k} \, E_k^T \right)^{-1} \right)^{-1} U^T$$

If, on average, five stars are measured with a measuring error of 1/50 of a pixel, then N = 5 and M = 0.4/μm. The standard deviations associated with A_{PS} are

$$\sigma_{\omega S} = 0.90 \text{ seconds}$$
$$\sigma_{PS} = 0.90 \text{ seconds}$$
$$\sigma_{KS} = 15.54 \text{ seconds}$$

Note that the standard error in the yaw angle is over 17 times larger than the other standard errors. It was stated in the section entitled "Experiment" that a large error in yaw would not cause a large error in the target camera coordinates if the target image was at the principal point. Suppose the target camera focal length is 75.000 millimeters, the target image coordinates are zero and $A_{ST} = I$; then

$$\sigma_T = \begin{pmatrix} 0.11 & 0 \\ 0 & 0.11 \end{pmatrix} \text{micrometers}^2$$

and

$$\gamma(99\%) = 0.99 \text{ micrometers}$$

$\gamma(99\%)$ is the 99 percent circular error for the target error due to star camera error. If the target camera focal length is Q * 75.000 millimeters then $\sigma_T(Q) = Q^2 * \sigma_T(75.000)$ and $\gamma(99\%)_Q = Q * \gamma(99\%) * 75.000$. These relations do not hold if the target image is not at the principal point.

Suppose now that the target camera has a field of view of 10 degrees in the X and Y directions. Suppose also that the star camera's orientation with respect to the target camera is defined by β, the elevation angle of the star camera optical axis above the focal plane of the target camera (see figure 1 or figure 2). (99%) circular errors for target coordinate errors due to star camera errors are presented in table 2. The parameter δ pertains to the angular distance of a target image from the optical axis. δ = 14 degrees pertains to the corners of the square format. The target camera focal length is 75.000 millimeters and the star camera errors are those described previously.

Table 2. γ(99%) Circular Errors For One Star Camera

				ß			
δ	0	15	30	45	60	75	90
0	14.6	14.1	12.6	10.3	7.3	3.9	1.0
5	14.6	14.4	13.3	11.2	8.4	5.1	1.6
10	14.6	14.7	13.9	12.1	9.6	6.3	2.7
14	14.6	15.0	14.4	12.9	10.4	7.3	3.7

Note that the smallest errors occur when the two optical axes are parallel. Note, too, that an error of 3.7 micrometers translates into a 10.2 arc second target directional error when the focal length is 75 millimeters.[9]

If a one star camera attitude capability is not adequate, then the two star camera attitude system described previously can be employed. Table 3 lists the γ99% circular errors in micrometers when two star cameras defined by (α,ß) are used. Results for 49 two star camera configurations are given in table 3. It appears that the best two star camera configuration is defined to be in the range $45° \leq \alpha \leq 90°$ and $30° \leq ß \leq 75°$.

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[9] M.A. Crombie. Mapping Camera Image Errors Due to Star Camera Identification and Measuring Errors. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, 22060-5546. ETL-RN-73-1, January 1973, AD-759 491.

Table 3. γ(99%) Circular Errors For Two Star Cameras

		α						
ß	δ	0°	15°	30°	45°	60°	75°	90°
0°	0°	10.3	2.3	1.2	1.0	1.2	2.3	10.3
	5°	10.3	2.3	1.2	1.0	1.2	2.3	10.4
	10°	10.3	2.3	1.2	1.0	1.3	2.3	10.6
	14°	10.3	2.3	1.2	1.0	1.3	2.4	10.9
15°	0°	9.9	2.3	1.2	1.0	1.1	1.7	2.3
	5°	10.2	2.3	1.3	1.0	1.1	1.7	2.3
	10°	10.4	2.4	1.3	1.0	1.2	1.7	2.3
	14°	10.6	2.4	1.3	1.0	1.2	1.8	2.4
30°	0°	8.9	2.2	1.2	0.9	1.0	1.1	1.2
	5°	9.4	2.4	1.3	1.0	1.0	1.1	1.2
	10°	9.8	2.5	1.3	1.0	1.0	1.2	1.3
	14°	10.2	2.6	1.4	1.0	1.0	1.2	1.3
45°	0°	7.3	2.2	1.2	0.9	0.9	0.9	0.9
	5°	7.9	2.4	1.3	1.0	0.9	0.9	0.9
	10°	8.6	2.6	1.4	1.0	0.9	0.9	0.9
	14°	9.1	2.7	1.5	1.1	0.9	0.9	0.9
60°	0°	5.2	2.1	1.2	0.9	0.8	0.8	0.8
	5°	6.0	2.4	1.4	1.0	0.8	0.8	0.8
	10°	6.7	2.7	1.5	1.1	0.9	0.8	0.8
	14°	7.4	3.0	1.7	1.2	1.0	0.9	0.8
75°	0°	2.7	1.8	1.1	0.9	0.8	0.7	0.7
	5°	3.6	2.3	1.5	1.1	0.9	0.8	0.7
	10°	4.5	2.9	1.8	1.3	1.0	0.9	0.8
	14°	5.2	3.4	2.1	1.5	1.2	1.0	0.9
90°	0°	0.7	0.7	0.7	0.7	0.7	0.7	.07
	5°	1.1	1.1	1.1	1.1	1.1	1.1	1.1
	10°	1.9	1.9	1.9	1.9	1.9	1.9	1.9
	14°	2.6	2.6	2.6	2.6	2.6	2.6	2.6

Sentinel Coverage. Summary results for coverage tests and accuracy parameters are presented in tables 4, 5, 6, and 7, where table 4 pertains to single coverage, table 5 pertains to double coverage, and so on. These data are averages taken from all nine points from the five test regions. Three lines of data are presented for each constellation. The first line of data is

$$T/P/F, \Theta, i, PER, h$$

where

- T: Total number of satellites
- P: Number of orbital planes
- F: Relative phasing parameter
- Θ: Central coverage angle in degrees
- i: Inclination in degrees
- PER: Orbital period in hours
- h: Satellite height in nautical miles

The minimum viewing angle ϵ was set to 5 degrees for all constellations. The eccentricity e was set to zero for all constellations.

The second line of data is

$$DIST, \sigma_{DIST}, \alpha, \sigma_{\alpha}, PDOP, \sigma_{PDOP}$$

where

- DIST: Average distance to target in meters divided by 10^6.
- σ_{DIST}: Standard deviation of distance to target in meters divided by 10^6.
- α: Average angle from target to two target trackers in degrees.
- σ_{α}: Standard deviation of α in degrees.
- PDOP: Average value of minimum PDOP calculated from three target trackers.
- σ_{PDOP}: Standard deviation of minimum PDOP.

Note that table 4 pertains to single coverage and therefore contains only distance to target data. Table 5 pertains to double coverage and therefore contains distance and angular data. Tables 6 and 7 contain complete sets of accuracy parameter data.

The third line of data consists of relative frequencies beginning at P_0 and extending to P_N, where N is the largest number of target trackers that viewed the target. P_0 is the relative frequency of the target not being viewed by any of the target trackers, P_1 is the relative frequency of the target being viewed by exactly one target tracker, and so on.

Table 4. Single Coverage Summary Results

5/5/1	69.150	43.660	9.809	9117.810			
19.363	1.423						
0.517	0.068	0.042	0.069	0.113	0.191		
5/5/3	75.500	51.800	20.881	17343.210			
34.778	1.513						
0.006	0.137	0.789	0.068				
6/6/4	66.420	53.130	7.785	7323.660			
16.027	1.357						
0.126	0.184	0.428	0.261				
6/2/0	66.720	52.240	7.970	7494.180			
16.384	1.367						
0.000	0.237	0.681	0.080	0.001			
7/7/5	60.260	55.690	5.172	4754.030			
11.140	1.292						
0.001	0.316	0.651	0.032				
7/7/1	60.500	48.000	5.243	4829.300			
11.250	1.265						
0.574	0.048	0.045	0.034	0.054	0.110	0.090	0.045
8/8/6	56.520	61.870	4.252	3750.860			
9.267	1.159						
0.023	0.268	0.675	0.034				
8/2/1	56.900	48.200	4.331	3840.070			
9.358	1.198						
0.000	0.292	0.518	0.178	0.011			

Table 4. Single Coverage Summary Results (continued)

9/9/7	54.810	70.540	3.926	3378.620				
8.586	1.165							
0.004	0.331	0.615	0.048	0.002				
9/9/1	57.280	49.880	4.413	3931.850				
9.522	1.194							
0.578	0.047	0.044	0.028	0.027	0.042	0.085	0.094	0.055
9/9/2	57.900	61.300	4.554	4087.390				
9.940	1.206							
0.000	0.151	0.658	0.187	0.003				
9/3/0	61.900	70.500	5.697	5300.730				
12.329	1.280							
0.000	0.036	0.728	0.234	0.002				
10/10/7	51.535	47.930	3.419	2777.83				
7.227	1.092							
0.004	0.174	0.676	0.137	0.008				
10/5/2	52.231	57.110	3.516	2894.780				
7.556	1.100							
0.241	0.180	0.151	0.253	0.174				
10/5/1	52.300	47.400	3.526	2906.670				
7.503	1.107							
0.112	0.143	0.394	0.260	0.091				
10/10/2	52.500	48.800	3.555	2941.420				
7.588	1.112							
0.000	0.135	0.688	0.176	0.001				

Table 4. Single Coverage Summary Results (continued)

10/2/0	53.200	47.700	3.660	3066.770
7.820	1.125			
0.000	0.264	0.424	0.244	0.068
11/11/4	47.610	53.790	2.959	2206.000
6.141	1.023			
0.016	0.267	0.597	0.120	
12/3/1	47.900	50.730	2.989	2243.740
6.199	1.022			
0.000	0.072	0.791	0.137	
12/3/2	48.300	58.800	3.031	2296.870
6.361	1.037			
0.001	0.140	0.769	0.089	
12/6/3	49.500	66.700	3.164	2464.160
6.751	1.075			
0.000	0.255	0.573	0.172	
12/12/2	49.600	48.500	3.176	2478.660
6.668	1.058			
0.001	0.112	0.559	0.315	0.014
12/12/10	50.200	57.500	3.247	2567.570
6.901	1.073			
0.000	0.107	0.662	0.219	0.012
12/2/1	50.400	46.500	3.272	2597.960
6.879	1.074			
0.000	0.211	0.384	0.264	0.141

Table 4. Single Coverage Summary Results (continued)

13/13/5	43.760	58.440	2.616	1760.510
5.241	0.957			
0.076	0.388	0.263	0.256	0.017
14/7/4	41.960	53.980	2.483	1582.880
4.833	0.922			
0.027	0.207	0.683	0.082	0.001
14/2/0	49.300	46.400	3.141	2435.430
6.551	1.052			
0.000	0.130	0.367	0.253	0.250
15/3/1	42.130	53.510	2.495	1598.920
4.869	0.922			
0.011	0.161	0.681	0.137	0.010
15/5/1	42.700	53.500	2.536	1653.810
4.987	0.933			
0.043	0.179	0.493	0.281	0.005
15/15/6	42.700	65.300	2.536	1653.810
5.000	0.945			
0.090	0.271	0.447	0.174	0.018
16/8/5	40.100	56.500	2.361	1416.500
4.477	0.890			
0.021	0.265	0.567	0.146	
16/16/2	43.700	51.500	2.611	1754.300
5.186	0.949			
0.031	0.104	0.415	0.414	0.035

Table 4. Single Coverage Summary Results (continued)

16/16/9	45.100	49.200	2.725	1904.650			
5.488	0.975						
0.351	0.095	0.077	0.071	0.095	0.187	0.112	0.013
17/17/7	38.900	55.500	2.289	1317.480			
4.257	0.870						
0.000	0.274	0.589	0.137				
17/17/13	41.000	53.500	2.418	1494.960			
4.643	0.900						
0.003	0.120	0.668	0.172	0.037			
18/6/2	38.400	56.600	2.261	1278.010			
4.163	0.857						
0.123	0.316	0.259	0.154	0.145	0.003		
18/9/6	39.700	62.700	2.336	1382.800			
4.401	0.883						
0.041	0.268	0.494	0.184	0.013			
19/19/5	37.100	57.400	2.191	1179.990			
3.930	0.831						
0.051	0.241	0.539	0.169				
19/19/8	37.500	57.300	2.212	1209.460			
4.005	0.834						
0.030	0.161	0.720	0.089				
20/10/7	36.600	56.800	2.165	1143.990			
3.846	0.823						
0.052	0.225	0.516	0.195	0.011			

Table 4. Single Coverage Summary Results (continued)

20/20/12	40.800	69.400	2.405	1477.200
4.612	0.911			
0.070	0.099	0.586	0.204	0.042
21/7/3	36.700	61.100	2.170	1151.120
3.868	0.828			
0.155	0.253	0.260	0.215	0.116
21/21/9	37.300	62.000	2.201	1194.650
3.974	0.837			
0.010	0.191	0.654	0.143	0.002
22/22/6	35.240	58.400	2.099	1050.550
3.620	0.797			
0.050	0.281	0.446	0.196	0.027
22/11/8	35.740	58.000	2.123	1084.160
3.706	0.806			
0.001	0.183	0.695	0.119	0.002
22/22/18	37.190	60.100	2.195	1186.570
3.947	0.835			
0.012	0.124	0.682	0.168	0.014
23/23/14	34.700	58.700	2.075	1015.18
3.541	0.786			
0.000	0.225	0.667	0.103	0.005
23/23/10	35.420	57.300	2.108	1062.550
3.653	0.801			
0.021	0.185	0.573	0.213	0.009

Table 4. Single Coverage Summary Results (continued)

23/23/16	36.100	56.600	2.140	1108.890		
3.766	0.811					
0.143	0.194	0.350	0.111	0.113	0.086	0.002
24/6/1	35.640	58.400	2.118	1077.37		
3.690	0.804					
0.060	0.185	0.371	0.366	0.017		
24/24/20	35.970	74.400	2.134	1099.910		
3.732	0.818					
0.011	0.281	0.609	0.088	0.010	0.001	
24/12/7	36.740	55.800	2.172	1153.980		
3.871	0.823					
0.029	0.149	0.416	0.293	0.113		

Table 5. Double Coverage Summary Results

7/7/2	75.970	61.810	22.517	18415.470	
36.884	1.500	76.651	13.537		
0.000	0.013	0.456	0.463	0.068	
8/8/2	71.000	57.100	11.767	10737.770	
22.163	1.389	76.466	14.163		
0.000	0.000	0.462	0.524	0.015	
8/8/6	74.000	58.000	16.798	14536.700	
29.603	1.466	52.893	21.023		
0.000	0.000	0.359	0.430	0.209	0.002
8/8/5	74.200	56.500	17.261	14865.610	
30.162	1.501	58.521	23.385		
0.028	0.090	0.148	0.370	0.365	
9/3/2	66.200	62.100	7.653	7202.130	
15.910	1.335	55.017	25.422		
0.000	0.017	0.400	0.542	0.040	0.001
9/3/0	66.800	65.500	8.021	7540.620	
16.521	1.316	59.223	17.788		
0.000	0.000	0.508	0.438	0.054	
10/10/2	64.100	61.600	6.571	6173.370	
13.954	1.313	69.052	16.311		
0.000	0.000	0.420	0.466	0.114	
10/5/2	65.151	52.560	7.077	6660.430	
14.794	1.342	54.368	20.424		
0.087	0.143	0.102	0.193	0.326	0.148

Table 5. Double Coverage Summary Results (continued)

10/10/7	65.173	62.830	7.088	6671.190		
14.902	1.319	53.393	21.779			
0.000	0.001	0.341	0.535	0.123	0.001	
10/10/8	65.500	49.400	7.260	6834.050		
15.091	1.356	65.640	12.956			
0.000	0.000	0.218	0.557	0.222	0.002	
10/2/0	73.100	44.400	14.953	13194.260		
26.990	1.480	70.329	19.154			
0.000	0.000	0.035	0.261	0.693	0.012	
11/11/9	62.000	52.700	5.733	5336.680		
12.264	1.277	56.126	21.126			
0.000	0.001	0.342	0.366	0.285	0.006	
12/3/1	56.600	57.000	4.268	3769.440		
9.283	1.180	55.547	16.701			
0.000	0.000	0.452	0.426	0.121	0.001	
12/6/2	56.600	54.000	4.268	3769.440		
9.249	1.179	55.325	20.540			
0.182	0.142	0.091	0.124	0.270	0.179	0.012
12/3/2	56.700	58.500	4.289	3792.810		
9.349	1.180	57.609	19.105			
0.000	0.000	0.421	0.500	0.076	0.002	
12/12/10	59.300	56.800	4.903	4467.480		
10.633	1.222	59.399	16.041			
0.000	0.002	0.310	0.479	0.210		

Table 5. Double Coverage Summary Results (continued)

12/2/0	63.700	45.700	6.395	6000.930		
13.504	1.329	66.334	21.092			
0.000	0.000	0.143	0.239	0.549	0.068	
12/2/1	64.300	45.000	6.662	6262.160		
14.003	1.344	66.951	19.646			
0.000	0.000	0.108	0.239	0.576	0.072	0.004
13/13/3	54.700	52.800	3.907	3356.190		
8.439	1.143	60.110	20.463			
0.000	0.034	0.335	0.433	0.198		
14/14/10	52.400	53.800	3.540	2923.990		
7.597	1.099	54.659	23.954			
0.009	0.103	0.255	0.372	0.261		
14/14/3	52.800	53.500	3.599	2994.430		
7.700	1.113	60.949	21.660			
0.000	0.033	0.275	0.515	0.177		
14/2/0	59.000	44.600	4.824	4382.440		
10.387	1.246	63.962	19.133			
0.000	0.000	0.160	0.217	0.459	0.141	0.022
15/3/1	51.300	55.300	3.388	2739.510		
7.242	1.084	56.593	21.799			
0.000	0.005	0.340	0.489	0.157	0.009	
15/15/11	51.500	58.600	3.415	2772.080		
7.332	1.091	60.016	18.745			
0.008	0.085	0.257	0.426	0.223		

Table 5. Double Coverage Summary Results (continued)

16/4/2	49.860	52.100	3.206	2516.790	
6.763	1.057	54.037	22.031		
0.000	0.000	0.244	0.585	0.170	0.001
16/4/3	50.350	58.800	3.266	2590.320	
6.966	1.072	52.376	20.208		
0.000	0.003	0.271	0.629	0.094	0.002
16/8/3	50.360	51.800	3.267	2591.850	
6.917	1.067	69.572	15.331		
0.000	0.000	0.229	0.548	0.222	0.001
16/16/6	50.630	52.200	3.301	2633.370	
6.995	1.073	54.724	25.476		
0.014	0.069	0.285	0.181	0.426	0.025
17/17/3	47.780	55.000	2.976	2228.040	
6.204	1.023	62.569	22.240		
0.013	0.068	0.281	0.376	0.262	
17/17/10	50.250	49.800	3.254	2575.130	
6.860	1.084	67.272	14.473		
0.000	0.000	0.149	0.508	0.330	0.013
17/17/14	51.280	56.500	3.385	2736.280	
7.245	1.084	60.880	16.622		
0.000	0.000	0.203	0.473	0.280	0.044
18/18/14	46.710	53.600	2.870	2092.890	
5.915	1.005	58.939	19.892		
0.000	0.002	0.210	0.688	0.100	

Table 5. Double Coverage Summary Results (continued)

18/18/3	47.830	55.700	2.981	2234.570	
6.221	1.024	63.029	23.143		
0.016	0.051	0.223	0.380	0.330	
18/18/4	48.300	51.500	3.031	2296.870	
6.321	1.029	58.708	16.559		
0.000	0.000	0.233	0.429	0.308	0.030
18/9/6	48.600	55.000	3.063	2337.560	
6.430	1.034	53.285	21.759		
0.000	0.008	0.151	0.603	0.235	0.003
19/19/11	45.870	55.300	2.793	1992.520	
5.716	0.989	66.733	16.383		
0.000	0.000	0.327	0.460	0.203	0.009
19/19/15	45.920	55.200	2.797	1998.360	
5.728	0.990	53.683	22.464		
0.000	0.021	0.262	0.512	0.205	
19/19/7	46.300	59.000	2.832	2043.290	
5.841	1.006	54.710	24.202		
0.032	0.135	0.304	0.168	0.217	0.145
20/4/2	44.180	54.500	2.649	1804.540	
5.317	0.960	58.127	18.718		
0.000	0.000	0.303	0.551	0.132	0.014
20/20/3	45.910	53.800	2.796	1997.190	
5.699	0.992	62.982	22.999		
0.016	0.038	0.215	0.279	0.428	0.025

Table 5. Double Coverage Summary Results (continued)

21/21/4	44.060	56.800	2.640	1791.850		
5.297	0.962	58.789	17.685			
0.000	0.005	0.302	0.480	0.206	0.006	
21/21/12	44.210	55.700	2.652	1807.720		
5.328	0.961	64.754	17.163			
0.000	0.000	0.300	0.460	0.219	0.022	
21/21/9	44.400	56.700	2.667	1828.010		
5.375	0.980	54.220	21.352			
0.000	0.000	0.264	0.546	0.186	0.003	0.001
22/22/4	42.630	54.700	2.531	1646.980		
4.979	0.934	58.759	19.640			
0.000	0.003	0.304	0.471	0.208	0.013	
22/11/3	43.400	54.700	2.588	1723.580		
5.144	0.946	63.110	16.828			
0.000	0.000	0.266	0.477	0.221	0.036	
22/11/8	43.430	56.300	2.590	1726.630		
5.159	0.949	62.286	17.952			
0.000	0.000	0.280	0.477	0.230	0.013	
23/23/18	42.390	54.600	2.513	1623.750		
4.926	0.927	54.797	24.043			
0.007	0.076	0.202	0.339	0.359	0.017	
23/23/10	43.600	60.400	2.603	1744.000		
5.200	0.955	52.564	20.589			
0.000	0.015	0.241	0.512	0.216	0.016	

Table 5. Double Coverage Summary Results (continued)

23/23/9	43.700	56.500	2.611	1754.300	
5.207	0.965	69.669	14.767		
0.000	0.014	0.139	0.596	0.240	0.012
24/12/3	41.340	57.400	2.441	1525.580	
4.724	0.912	62.832	17.803		
0.000	0.000	0.336	0.457	0.178	0.029
24/12/9	41.870	57.000	2.477	1574.440	
4.826	0.920	49.337	21.345		
0.000	0.000	0.360	0.344	0.246	0.049
24/24/14	42.110	54.200	2.494	1597.030	
4.867	0.924	58.409	24.219		
0.011	0.023	0.149	0.456	0.361	

Table 6. Triple Coverage Summary Results

9/9/3	83.036	59.320	220.681	96664.565			
181.729	1.8005	62.119	23.470	25.289	91.647		
0.000	0.000	0.146	0.263	0.316	0.273	0.001	
9/9/7	84.877	58.576	14076.426	1594725.547			
2956.335	1.667	62.951	18.862	11.454	42.505		
0.000	0.000	0.004	0.268	0.414	0.262	0.051	
10/10/8	80.300	60.000	59.694	38427.360			
74.035	1.572	66.371	17.743	4.374	33.129		
0.000	0.000	0.001	0.174	0.470	0.351	0.003	
11/11/3	74.600	59.800	18.255	15561.620			
31.609	1.485	62.122	23.353	5.120	12.389		
0.000	0.000	0.016	0.213	0.557	0.163	0.047	0.004
11/11/9	75.80	52.00	21.901	18014.930			
36.126	1.521	66.189	23.180	3.663	8.875		
0.000	0.000	0.024	0.127	0.421	0.336	0.092	
12/4/2	70.900	60.000	11.644	10639.210			
22.362	1.410	54.027	22.481	10.157	44.707		
0.000	0.000	0.000	0.270	0.536	0.160	0.034	
12/4/3	71.100	49.500	11.891	10837.760			
22.661	1.453	55.127	21.924	13.738	71.384		
0.000	0.000	0.000	0.140	0.610	0.173	0.075	0.001
12/12/10	71.900	53.800	12.976	11693.260			
24.283	1.470	65.100	19.414	4.462	22.338		
0.000	0.000	0.003	0.203	0.485	0.261	0.048	

Table 6. Triple Coverage Summary Results (Continued)

12/12/2	73.700	53.200	16.142	14065.300			
28.598	1.482	63.482	15.375	4.175	23.062		
0.000	0.000	0.000	0.147	0.501	0.258	0.095	
12/3/1	79.000	61.700	41.430	29378.420			
57.222	1.538	67.547	18.674	2.345	4.277		
0.000	0.000	0.018	0.057	0.283	0.479	0.163	
13/13/4	68.000	50.000	8.857	8290.660			
17.860	1.407	61.547	21.676	5.864	33.382		
0.000	0.000	0.031	0.278	0.305	0.275	0.111	
14/14/4	66.100	47.600	7.594	7147.840			
15.698	1.359	63.952	16.559	2.507	4.269		
0.000	0.000	0.006	0.107	0.466	0.356	0.065	
14/14/12	66.400	49.400	7.773	7312.490			
16.013	1.367	66.719	23.748	2.703	4.833		
0.000	0.000	0.009	0.113	0.485	0.314	0.078	
14/2/0	77.500	45.300	29.691	22840.980			
44.965	1.532	61.907	16.164	2.246	2.997		
0.000	0.000	0.000	0.015	0.021	0.216	0.745	0.003
15/15/6	63.200	57.000	6.187	5794.500			
13.177	1.296	58.156	20.987	4.076	19.503		
0.000	0.000	0.006	0.145	0.605	0.243		
15/15/2	65.500	71.800	7.260	6834.050			
15.277	1.348	61.045	13.127	6.334	20.389		
0.000	0.000	0.003	0.339	0.324	0.272	0.062	

Table 6. Triple Coverage Summary Results (Continued)

16/16/14	61.670	54.900	5.618	5219.270			
12.058	1.271	64.519	19.994	2.690	9.570		
0.000	0.000	0.002	0.115	0.560	0.315	0.008	
16/16/4	61.840	53.200	5.676	5279.320			
12.118	1.346	63.663	19.290	3.990	15.106		
0.000	0.000	0.053	0.180	0.403	0.294	0.070	
16/4/3	63.970	57.800	6.513	6116.590			
13.806	1.303	58.217	20.421	6.725	40.071		
0.000	0.000	0.000	0.213	0.329	0.292	0.160	0.006
17/17/15	60.080	58.300	5.119	4698.550			
11.098	1.233	62.953	22.013	2.819	7.153		
0.000	0.000	0.001	0.106	0.647	0.200	0.046	
17/17/7	60.910	53.500	5.369	4961.500			
11.564	1.257	67.329	14.835	2.039	3.918		
0.000	0.000	0.000	0.092	0.459	0.397	0.052	
17/17/4	61.870	46.700	5.687	5290.010			
12.119	1.375	65.601	18.226	2.959	11.206		
0.000	0.000	0.003	0.081	0.368	0.375	0.174	
18/18/4	57.700	51.600	4.508	4036.410			
9.742	1.212	64.719	17.796	3.143	13.334		
0.000	0.000	0.038	0.143	0.409	0.348	0.062	
18/9/4	57.870	60.100	4.547	4079.690			
9.909	1.204	57.863	24.023	8.355	35.265		
0.000	0.000	0.015	0.236	0.456	0.229	0.064	

Table 6. Triple Coverage Summary Results (Continued)

18/18/16	58.680	55.500	4.742	4293.950			
10.286	1.212	62.775	18.740	2.375	4.325		
0.000	0.000	0.002	0.055	0.607	0.305	0.031	
19/19/14	57.050	53.900	4.363	3875.990			
9.463	1.185	67.944	16.047	2.019	1.225		
0.000	0.000	0.000	0.047	0.588	0.320	0.045	0.001
19/19/17	57.270	53.500	4.411	3929.400			
9.555	1.190	62.030	19.550	2.703	12.736		
0.000	0.000	0.003	0.070	0.494	0.377	0.056	
19/19/5	57.830	54.000	4.537	4069.450			
9.906	1.139	61.277	17.727	2.663	13.929		
0.000	0.000	0.000	0.051	0.418	0.344	0.187	0.001
20/5/3	53.550	51.800	3.715	3131.670			
7.986	1.123	53.731	24.347	5.839	19.763		
0.000	0.000	0.004	0.127	0.617	0.206	0.044	0.002
20/20/12	55.160	52.500	3.989	3451.150			
8.622	1.152	59.198	23.457	3.639	8.541		
0.000	0.000	0.000	0.073	0.546	0.325	0.054	0.003
20/20/8	55.880	50.200	4.124	3606.140			
8.907	1.169	67.183	15.877	1.970	1.321		
0.000	0.000	0.000	0.046	0.463	0.396	0.091	0.003
21/21/5	51.970	55.100	3.479	2850.300			
7.464	1.096	56.611	19.641	7.268	35.971		
0.000	0.000	0.004	0.327	0.343	0.253	0.073	

Table 6. Triple Coverage Summary Results (Continued)

21/3/2	53.190	54.700	3.659	3064.930				
7.882	1.116	62.987	21.611	2.883	11.129			
0.000	0.000	0.001	0.074	0.678	0.179	0.068		
21/3/0	54.380	52.200	3.852	3291.920				
8.306	1.138	63.377	21.387	2.205	3.217			
0.000	0.000	0.000	0.025	0.581	0.282	0.111	0.001	
22/22/16	50.730	52.400	3.314	2648.930				
7.035	1.074	68.343	16.256	2.501	10.138			
0.000	0.000	0.001	0.149	0.561	0.278	0.001		
22/11/5	52.780	52.100	3.596	2990.860				
7.710	1.109	63.289	19.881	3.154	17.237			
0.000	0.000	0.015	0.120	0.346	0.435	0.084	0.001	
22/11/7	52.800	55.700	3.599	2994.430				
7.752	1.109	53.323	23.622	4.441	25.119			
0.000	0.000	0.002	0.055	0.595	0.304	0.035	0.007	0.003
23/23/14	51.320	55.200	3.390	2742.750				
7.248	1.084	63.961	18.216	2.324	2.689			
0.000	0.000	0.007	0.132	0.425	0.388	0.048		
23/23/10	51.890	54.500	3.468	2836.820				
7.431	1.093	58.095	22.065	6.078	47.613			
0.000	0.000	0.001	0.153	0.405	0.298	0.120	0.024	
23/23/17	52.240	52.300	3.517	2896.330				
7.531	1.099	59.715	22.485	4.386	25.427			
0.000	0.000	0.001	0.096	0.401	0.331	0.167	0.005	

Table 6. Triple Coverage Summary Results (Continued)

24/12/5	49.170	51.800	3.126	2416.930			
6.562	1.046	62.964	18.493	3.658	15.469		
0.000	0.000	0.025	0.152	0.420	0.303	0.099	
24/8/4	49.590	54.200	3.174	2477.210			
6.703	1.053	67.683	16.663	2.072	2.362		
0.000	0.000	0.000	0.098	0.559	0.309	0.034	
24/24/14	50.310	52.400	3.261	2584.240			
6.901	1.068	57.271	22.945	3.623	7.388		
0.000	0.000	0.001	0.033	0.523	0.394	0.045	0.004
24/3/2	50.300	54.000	3.260	2582.720			
6.913	1.065	62.123	20.597	2.506	7.655		
0.000	0.000	0.000	0.037	0.571	0.349	0.042	

Table 7. Quadruple Coverage Summary Results

12/12/2	82.395	57.602	144.498	72042.550			
136.321	1.612	66.060	17.231	1.945	0.426		
0.000	0.000	0.000	0.000	0.202	0.420	0.336	0.042
12/12/4	83.163	55.550	244.948	103582.890			
194.555	1.769	65.264	19.357	1.867	0.212		
0.000	0.000	0.000	0.000	0.189	0.537	0.265	0.009
12/12/10	83.722	55.132	420.346	150383.080			
281.402	1.654	65.662	18.667	1.873	0.282		
0.000	0.000	0.000	0.000	0.119	0.424	0.419	0.038
12/12/8	83.787	51.639	454.576	158624.750			
296.568	1.750	67.444	20.256	1.788	0.093		
0.000	0.000	0.000	0.000	0.065	0.590	0.296	0.049
13/13/2	77.146	45.743	27.718	21663.180			
42.722	1.541	61.404	18.011	2.653	1.486		
0.000	0.000	0.000	0.001	0.212	0.475	0.250	0.061
14/14/4	75.780	69.210	21.831	17968.780			
36.151	1.498	63.060	19.940	2.132	2.425		
0.000	0.000	0.000	0.022	0.178	0.506	0.247	0.046
14/14/2	75.820	62.380	21.972	18061.280			
36.267	1.503	64.517	17.001	2.126	1.346		
0.000	0.000	0.000	0.015	0.202	0.408	0.308	0.067
14/7/4	75.910	62.040	22.297	18272.380			
36.598	1.481	63.618	22.414	2.718	17.949		
0.000	0.000	0.000	0.022	0.337	0.234	0.293	0.115

Table 7. Quadruple Coverage Summary Results (continued)

14/14/11	76.000	70.100	22.629	18487.730				
37.109	1.498	62.277	23.606	2.420	3.766			
0.000	0.000	0.004	0.020	0.264	0.375	0.292	0.032	0.014
14/14/9	76.000	61.800	22.629	18487.730				
37.026	1.516	64.780	23.395	2.463	1.775			
0.000	0.000	0.000	0.025	0.184	0.438	0.294	0.058	0.001
15/15/2	70.900	55.700	11.644	10639.210				
21.959	1.412	61.509	17.486	2.835	1.104			
0.000	0.000	0.000	0.000	0.436	0.349	0.215		
15/5/4	71.700	67.300	12.689	11469.640				
23.982	1.431	64.032	19.065	1.980	0.500			
0.000	0.000	0.000	0.000	0.334	0.443	0.202	0.021	
15/3/1	73.100	54.400	14.953	13194.260				
27.106	1.479	66.476	21.182	1.878	0.677			
0.000	0.000	0.000	0.000	0.139	0.362	0.492	0.006	
16/16/5	69.030	57.600	9.702	9025.850				
19.310	1.406	62.668	19.530	4.155	31.785			
0.000	0.000	0.000	0.045	0.255	0.340	0.318	0.036	0.007
16/16/10	69.130	56.800	9.791	9102.390				
19.446	1.404	63.075	22.044	2.527	11.107			
0.000	0.000	0.000	0.014	0.357	0.225	0.306	0.097	
16/8/5	71.020	59.000	11.791	10757.650				
22.585	1.428	61.780	19.537	2.161	8.850			
0.000	0.000	0.000	0.012	0.139	0.480	0.243	0.124	0.002

Table 7. Quadruple Coverage Summary Results (continued)

17/17/11	67.020	50.500	8.164	7670.510					
16.715	1.389	65.737	21.701	2.082	2.718				
0.000	0.000	0.000	0.026	0.156	0.362	0.369	0.087		
17/17/5	67.420	52.100	8.435	7915.140					
17.190	1.383	59.117	21.204	2.968	10.532				
0.000	0.000	0.000	0.006	0.224	0.226	0.320	0.118	0.050	
17/17/7	68.180	59.700	8.995	8412.550					
18.176	1.375	66.228	15.847	1.883	0.205				
0.000	0.000	0.000	0.002	0.093	0.582	0.316	0.006		
18/3/2	64.180	54.700	6.607	6208.680					
13.958	1.313	63.952	20.466	1.948	0.692				
0.000	0.000	0.000	0.000	0.275	0.378	0.327	0.019		
18/3/1	64.210	56.700	6.621	6221.990					
14.000	1.310	63.628	20.317	1.914	0.271				
0.000	0.000	0.000	0.000	0.310	0.368	0.298	0.025		
18/6/5	66.240	57.200	7.677	7224.010					
15.909	1.337	64.817	23.363	1.919	0.757				
0.000	0.000	0.000	0.006	0.106	0.505	0.314	0.067	0.003	
19/19/5	63.040	58.100	6.123	5730.480					
13.073	1.291	61.038	18.468	2.099	1.775				
0.000	0.000	0.000	0.009	0.282	0.413	0.201	0.086	0.009	0.001
19/19/8	63.860	59.600	6.465	6069.100					
13.731	1.295	63.527	22.309	2.869	17.998				
0.000	0.000	0.002	0.049	0.215	0.288	0.389	0.057		

Table 7. Quadruple Coverage Summary Results (continued)

19/19/6	64.050	56.700	6.549	6151.450					
13.854	1.302	64.554	18.179	1.919	0.474				
0.000	0.000	0.000	0.000	0.188	0.417	0.336	0.060		
20/10/7	62.050	59.500	5.750	5354.770					
12.360	1.270	61.122	18.938	2.172	0.475				
0.000	0.000	0.000	0.005	0.266	0.402	0.226	0.090	0.011	
20/20/8	62.150	55.500	5.786	5391.200					
12.404	1.276	65.296	14.457	1.869	0.122				
0.000	0.000	0.000	0.000	0.124	0.467	0.380	0.029		
20/20/11	63.000	47.500	6.107	5714.630					
12.962	1.314	46.429	22.069	7.787	45.735				
0.113	0.052	0.079	0.050	0.037	0.060	0.078	0.148	0.151	0.232
21/3/2	58.800	54.400	4.773	4326.870					
10.345	1.217	61.459	19.911	1.975	0.550				
0.000	0.000	0.000	0.000	0.292	0.391	0.261	0.056		
21/3/1	58.850	55.500	4.785	4340.680					
10.380	1.215	62.170	19.380	1.977	0.860				
0.000	0.000	0.000	0.000	0.294	0.385	0.295	0.026		
21/3/0	58.920	55.900	4.803	4360.100					
10.421	1.216	61.647	19.435	1.976	0.492				
0.000	0.000	0.000	0.000	0.296	0.389	0.282	0.033		
22/22/16	58.070	57.400	4.594	4131.341					
10.002	1.197	65.590	16.000	1.871	0.255				
0.000	0.000	0.000	0.001	0.137	0.608	0.220	0.035		

Table 7. Quadruple Coverage Summary Results (continued)

22/22/6	59.030	55.700	4.832	4390.850					
10.477	1.219	55.856	20.268	2.445	1.095				
0.000	0.000	0.000	0.000	0.193	0.377	0.367	0.051	0.011	0.001
22/22/9	59.980	61.000	5.090	4668.090					
11.068	1.231	61.556	22.356	2.949	18.154				
0.000	0.000	0.000	0.029	0.238	0.252	0.352	0.118	0.012	
23/23/17	56.830	56.700	4.316	3823.450					
9.382	1.180	60.904	22.966	2.594	5.823				
0.000	0.000	0.000	0.018	0.274	0.370	0.247	0.075	0.015	
23/23/14	57.170	54.100	4.389	3905.010					
9.522	1.187	65.287	19.681	1.864	0.240				
0.000	0.000	0.000	0.001	0.126	0.508	0.299	0.065		
23/23/16	58.660	60.020	4.737	4288.490					
10.328	1.211	49.850	22.460	4.845	9.424				
0.000	0.000	0.004	0.054	0.185	0.192	0.459	0.106		
24/12/5	54.960	55.300	3.953	3409.490					
8.569	1.145	65.073	19.490	1.974	2.472				
0.000	0.000	0.001	0.013	0.203	0.472	0.263	0.048		
24/3/2	55.060	54.400	3.971	3430.240					
8.597	1.149	60.380	18.835	1.998	0.508				
0.000	0.000	0.000	0.000	0.239	0.405	0.313	0.042	0.001	
24/3/0	55.140	55.600	3.985	3446.950					
8.641	1.149	59.542	19.367	2.065	0.568				
0.000	0.000	0.000	0.000	0.264	0.403	0.267	0.064	0.002	

Table 7. Quadruple Coverage Summary Results (continued)

24/6/4	55.170	54.600	3.991	3453.250				
8.647	1.151	57.580	20.301	2.444	0.428			
0.000	0.000	0.000	0.001	0.206	0.478	0.246	0.066	0.004
24/24/20	55.450	56.600	4.043	3512.620				
8.782	1.156	59.464	22.076	2.602	9.216			
0.000	0.000	0.000	0.043	0.274	0.325	0.223	0.107	0.028

DISCUSSION

A sentinel system composed of a constellation of satellite platforms (Lightsats, perhaps) is postulated and evaluated for its target location capability. The arguments are purely geometric with no consideration given to the complex command, control, and communication problems such a system would create. Basically, two questions were addressed in this study. The first question pertains to the problem of continuous coverage and to the multiplicity of coverage. Satellite constellations that were designed to provide one-, two-, three-, and four-fold continuous ground coverage were analyzed in this study. The second question addresses the accuracy with which ground target locations can be estimated from measurements collected from sensors located on the satellite platforms. The target sensors are characterized by their measurement errors rather than by parametric specifications. The same is true of the location errors of the satellite platforms. Standard errors here were specified as 1, 5, 10, or 15 meters in each of the three geocentric coordinates. One component of the sensor attitude system, namely the star camera, was specified. One reason for this is that the overall work effort is in support of an ETL-initiated, USACE-approved NASA shuttle experiment intended to validate the concept of precision real time attitude keyed to a digital image stellar camera operating in space. Standard errors in this case were specified as ½, 1, 2, or 3 seconds about each of the three mutually orthogonal platform reference axes.

A simple coverage evaluation was performed by allowing each of the constellations to operate for 24 hours, where each of nine target points within five target regions was tested every 5 minutes to see if the programmed number of lines of sight occurred. Results are presented in tables 4, 5, 6, and 7 and are summarized below in table 8. Programmed coverage was achieved if: for single coverage $P_0 = 0$; for double coverage $P_0 = P_1 = 0$; for triple coverage $P_0 = P_1 = P_2 = 0$; and for quadruple coverage $P_0 = P_1 = P_2 = P_3 = 0$. P_0 is the relative frequency of the target not being viewed by any of the satellites, P_1 is the relative frequency of the target being viewed by exactly one satellite, and so on.

Table 8. Fraction of Programmed Coverage

	Percent of Full Coverage				
	100%	99%	98%	95%	90%
Single	0.250	0.423	0.500	0.673	0.808
Double	0.462	0.635	0.712	0.789	0.904
Triple	0.295	0.773	0.864	0.955	0.977
Quadruple	0.405	0.619	0.738	0.929	0.976

The column headings pertain to the percentages of times the tested constellations met coverage specifications. For example, 40.5 percent of the 4-fold coverage constellations tested provided full coverage 100 percent of the time, whereas 92.9 percent of those tested provide full coverage 95 percent of the time.

Of the 52 one-fold coverage constellations tested, 10 were such that all of the targets were not in view over 10 percent of the time. Three of the 10 were such that the targets were not in view over 50 percent of the time. Of the 52 two-fold coverage constellations tested, 5 were such that the programmed coverage failed over 10 percent of the time. Of the 44 three-fold coverage constellations tested, only one was such that the programmed coverage failed over 10 percent of the time. The same was true for the 42 four-fold coverage constellations tested.

A review of the data from which tables 4, 5, 6, and 7 were derived indicates that outages are functions of latitude. That is, if more than one target region recorded outages for a particular constellation, then those regions were generally either at the lower or upper latitudes. Since the constellations are symmetrically organized over the earth, it is highly likely that the outage regions at a particular latitude would be regularly spaced in longitude. Test areas over the entire earth would have to be evaluated to show this. However, such an effort is beyond the scope of this particular experiment.

Four kinds of target location mensuration procedures were considered in this study. The first is a single station intersection of a target where the exterior orientation of the sensor is known, along with the slant range to a target identified at the principal point of a solid state imaging device. This is the only kind of target location mensuration that can be performed from a 1-fold coverage constellation in an exact manner. Single station intersection can be performed without knowing the slant range if the target height above the spheroid is known or estimated. The second method of target location mensuration is a two-station intersection scheme where the exterior orientation of both stations is known along with the slant ranges to a target identified at the principal points of both solid state sensors. This method of target determination requires simultaneity of target observation from 2-fold coverage and larger constellations. The third method of target location mensuration is similar to the second method, except that the slant range distances are not measured. The fourth method of target location mensuration is a three-station intersection scheme where the positions of the three stations are known along with the three individual slant ranges to the target. This method of target mensuration requires simultaneity of target observation from 3-fold coverage and larger constellations.

Some of the summary results given in tables 4, 5, 6, and 7 must be correlated with the tables referred to previously[10] in order to produce target location error estimates. Errors associated with the first mensuration procedure can be obtained from table H-1 of that report. Errors in that report are expressed as 99 percent spherical errors and are a function of reference system position errors, reference system attitude errors, target mensuration errors, and slant range errors. In addition, the target errors are characterized by the distance to the target from the satellites. Errors associated with the second mensuration procedure can be obtained from tables H-2 through H-5. The target location errors are characterized exactly as before, but with an additional geometrical parameter, namely the angle between the two rays as measured at the target. Errors associated with the third mensuration procedure can be obtained from tables H-14 and H-15. Note that the summary output in tables 4, 5, 6, and 7 include the average distance to the target and associated standard deviation. Note, too, that the summary output in tables 5, 6, and 7 include the average angular separation of the two rays at the target and the associated standard deviation. Target location errors for the fourth mensuration type can be estimated from the following formula:

$$\text{SP (99\%)} = \frac{\text{PDOP}}{3} * 3.367 * \text{SIGR}$$

where

PDOP = Position Dilution of Precision

SIGR = Slant Range Measuring Error

The summary output in tables 6 and 7 include the average minimum PDOP and the associated standard deviation. The formula was evaluated for a range of SIGR and PDOP values and the results are presented below in table 9.

[10] M.A. Crombie. Target Location Errors Derived From A Hypothetical Target Tracking System. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, 22060-5546. Report ETL-0531, February 1989.

Table 9. Y99% Spherical Errors of Position

Slant Range Errors (meters)

PDOP	5	10	15	20	25
1.0	10	19	29	39	49
1.5	15	29	43	58	73
2.0	19	39	58	78	97
2.5	24	48	73	97	121
3.0	29	58	88	117	146
3.5	34	68	102	136	170
4.0	39	78	117	156	194
4.5	44	87	131	175	219
5.0	49	97	146	194	243
5.5	53	107	160	214	267
6.0	58	117	175	233	292
6.5	63	126	190	253	316
7.0	68	136	204	272	340
7.5	73	146	219	292	364
8.0	78	156	233	311	389
8.5	83	165	248	330	413
9.0	87	175	262	350	437
9.5	92	185	277	369	462
10.0	97	194	292	389	486

The results of this study are intended to be used together with the results described in the earlier report[11] to evaluate the several constellations for their worth in surveillance and target location estimation. In all, 190 constellations were tested and 67 satisfied the programmed coverage specifications 100 percent of the time. The two sets of results can be combined in many ways to evaluate a variety of questions. For example, note from tables 4 and 5 that the constellations designated as 12/2/1 and 12/3/1 satisfy the programmed coverage requirements for 1-fold and 2-fold coverage. Suppose the vehicle position standard error is 10 meters for each coordinate, the attitude standard error is 2 arc seconds about each reference axis, and the target measuring error is 5 arc seconds. Suppose further that if a slant range measuring capability exists, then its standard error is 10 meters. The appropriate average distances and average observation angles are taken from tables 4 and 5 of this work and used as interpolation parameters in the previous work[12] to produce table 10 of 99 percent confidence spheres about estimated target locations.

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[11] M.A. Crombie. Target Location Errors Derived From A Hypothetical Target Tracking System. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, 22060-5546. Report ETL-0531, February 1989.

[12] Ibid.

Table 10. Numerical Example

1-Fold

	12/2/1		12/3/1	
h	2597.96	nautical miles	2243.74	nautical miles
i	46.50	degrees	50.73	degrees
Error	512	meters	462	meters

2-Fold

	12/2/1		12/3/1	
h	6262.16	nautical miles	3769.44	nautical miles
i	45.00	degrees	57.00	degrees
Error (with Slant Range)	503	meters	337	meters
Error (without Slant Range)	1349	meters	986	meters

The results in table 10 and others like them can be used as one argument in a comparative analysis of competing target tracking systems. Considerations such as C^3 sensor costs, vehicle costs, and launch costs are beyond the scope of this work.

CONCLUSIONS

1. Results presented in this work, when combined with results from a previous work[13], can be used in a sentinel system target location error analysis.

2. A more stringent outage test should be performed for the various constellations on a large computer to determine coverage deficiencies over the entire world.

3. A comparison should be made between errors derived from single station intersection using measured slant range and those derived from single station intersection without slant range, but with estimated target height.

4. An investigation should be made to determine the best constellations, not necessarily circular and symmetric, that meet coverage specifications over selected areas of the world rather than the entire world.

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[13] M.A. Crombie. Target Location Errors Derived From A Hypothetical Target Tracking System. U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, 22060-5546. Report ETL-0531, February 1989.