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EVALUATION OF A DISCRETE MODIFICATION OF THE CONTINUOUS AMSAA RELIABILITY GROWTH MODEL

by

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Block 19. Abstract (continued)

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Evaluation of a Discrete Modification of the Continuous AMSAA Reliability Growth Model

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A new discrete reliability growth model is created by modifying the often used Army Material Systems Analysis Activity (AMSAA) continuous reliability growth model. The new model is labeled the AMSAA-D model. Its accuracy is evaluated and compared with three other existing discrete reliability growth models. The results show the AMSAA-D model is at least as accurate as the other models. In particular it is more accurate than an AMSAA discrete model which requires computer supported numerical methods to calculate the reliability estimates from test data. The AMSAA-D reliability estimates can be made with a hand-held calculator. Computer simulations were used to generate test data needed for the evaluation. The simulated test plan assumes that repeated tests on a system are performed until a predetermined number of failures occur, at which time a design change is made to the system to improve its reliability. Ten reliability values are specified to define a reliability growth pattern. Five hundred replications of each growth pattern are simulated. For each replication, reliability estimates are calculated for each of the ten sets of generated test data using equations from each of the four growth models. Averages and sample mean square error values

across the 500 replications are used to determine accuracy. Sensitivity of the AMSAA-D model to the number of failures before system modification and to the number of possible failure causes is also evaluated. Results of all evaluations are presented graphically.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

Early in the acquisition process of a system, a variety of test programs are conducted to detect weaknesses in its design features and manufacturing processes. These tests are performed under tight budget constraints; consequently the number of component, assembly and system tests is severely limited. These development testing programs are designed to induce early failures and determine associated failure causes. This information is needed to make appropriate changes in the design and production processes. Hopefully these changes will yield an increase in the reliability of the system under acquisition. Current DoD policy requires reliability growth management and assessment programs for major systems acquisitions.

Reliability growth methodology used to estimate the actual reliability growth patterns should be able to track a variety of growth patterns reasonably well. Many reliability growth models have been developed and reported in the literature. New ones are continually being developed and proposed for use by contractors.

A discrete version of the well known Army Material System Analysis Activity (AMSAA) or DUANE model is analyzed in this thesis. This discrete model uses the number of observed tests to achieve a prescribed number of failures as the primary data input. This model is evaluated and compared with other established models which have been extensively tested and described in former theses [Ref. 1,2,3]. The same Monte Carlo simulation is used as in the former theses to make the comparisons. The simulation was originally developed by James E. Drake [Ref. 1], then twice updated and enlarged in its performance and ability to track different reliability growth patterns by James D. Chandler, Jr. [Ref. 2] and Pam Markiewicz [Ref. 3]. The program and its features are well described in these theses and will not be included here. Only the program introduced to estimate the new model is presented.

In Chapter II the analysis of the proposed model is developed, the simulation data presented and the results evaluated. A summary and recommendations for further studies is presented in Chapter III.

II. RELIABILITY GROWTH MODELING

A proposed reliability growth model should be able to track accurately all likely reliability growth patterns. During the development phase of a system, the actual pattern of reliability growth is dependent on many factors. Reliability progress paths may be steadily increasing or they may indicate early stagnation and degradation prior to continued growth. They may also demonstrate no reliability progress at all [Ref. 4]. Figure 1 shows the true reliability growth patterns which will be used for evaluation purposes in this thesis. The different patterns are numbered from 1 to 6. Pattern numbers cited later in this thesis refer to Figure 1.

Reliability growth usually occurs in discrete jumps; therefore it is meaningful to break the system tests into subphases. Tests within the same phase have the same probability of success, i.e., the same reliability. Reliability may change from phase to phase. Hopefully it is increasing as the items under test undergo change from phase to phase.

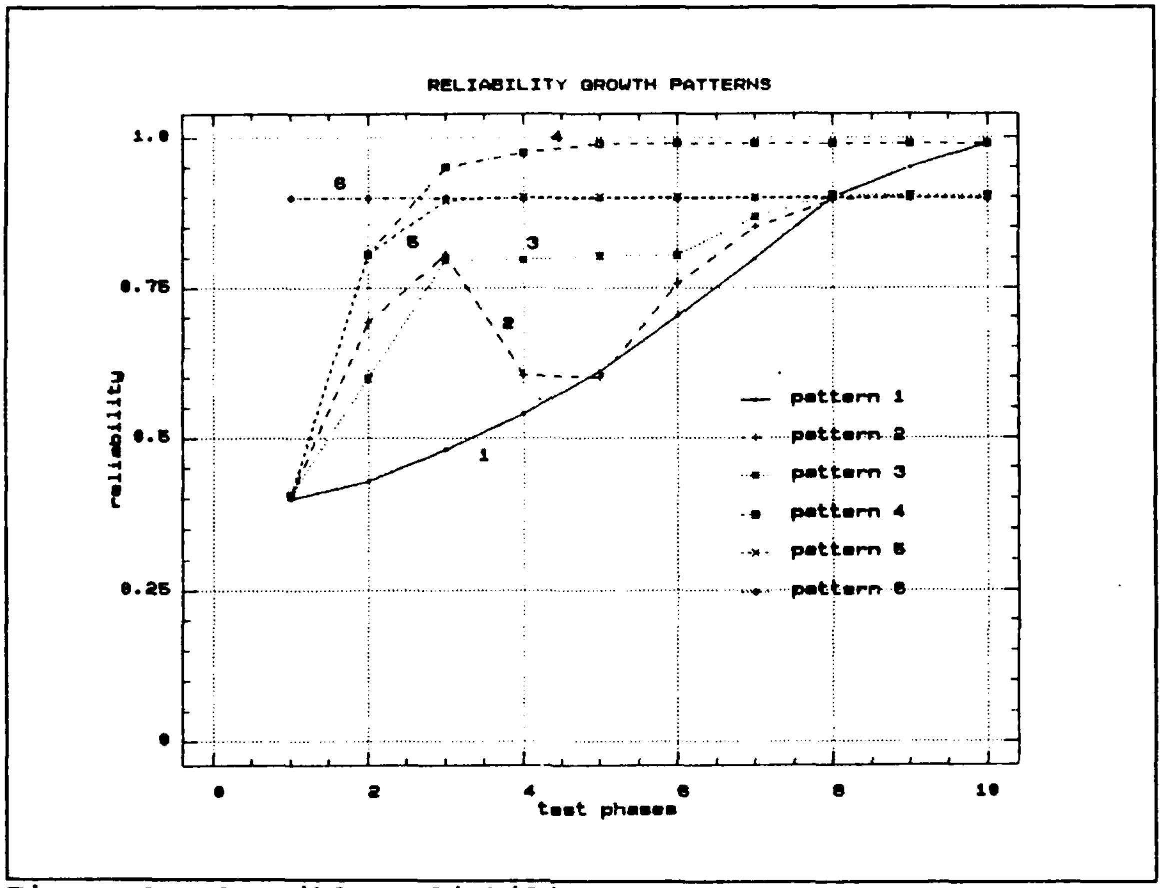


Figure 1 : Possible reliability growth patterns

A. THE DISCRETE AMSAA MODEL

1. Theory

The Discrete AMSAA model was developed in 1983 by L. Crow [Ref. 5] It stems from a cumulative continuous model of the form

$$K(t) = \lambda t^{\beta}$$

where K(t) denotes total number of failures after t hours of testing.

In the discrete model

$$K_i = \sum_{j=1}^{i} M_j$$

where M_j denotes the number of failures in test phase j.

$$T_i = \sum_{j=1}^{i} N_j$$

where N_j denotes the number of tests in test phase j.

The model then states

$$E(K_1) = E(M_1) = (1 - R_1)N_1 = \lambda T_1^{\beta}$$

$$E(K_2) = E(M_1 + M_2) = \lambda T_1^{\beta} + (1 - R_2)N_2 = \lambda T_2^{\beta}$$

Thus $(1 - R_2)N_2 = \lambda T_2^{\beta} - \lambda T_1^{\beta}$

Continuing in this manner the AMSAA discrete model states

$$(1 - R_i)N_i = \lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}$$

Maximum likelihood estimates for λ and β are used to obtain the MLE for R_i. The estimates are updated sequentially as the test results of the next test phase become available.

2. Some Results

The results shown in the following figures are taken directly from a thesis by Major Rio M.Thalieb [Ref. 6].

Figure 2 shows the estimation of the reliability growth when the actual reliability is following pattern 1.

Figures 16 and 17 in Appendix D show the estimation of the discrete AMSAA model for patterns 2 and 4.

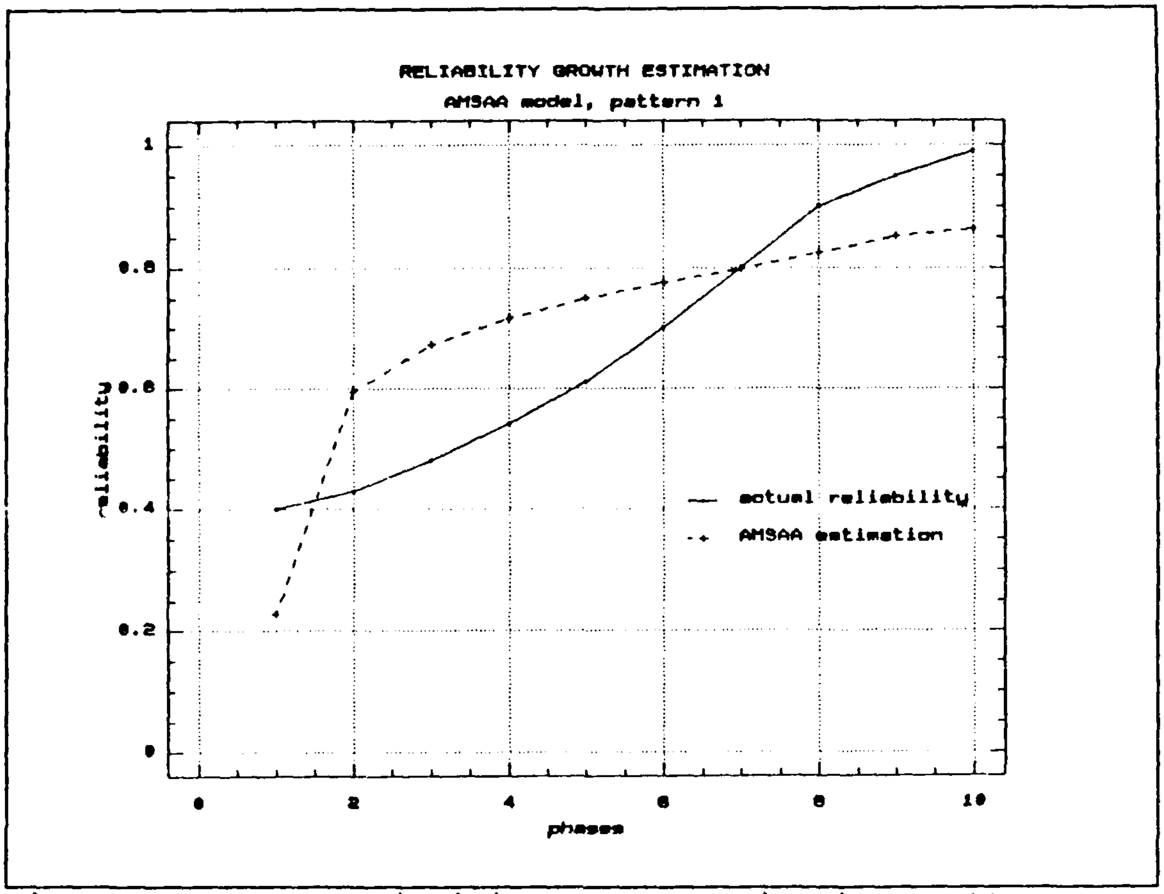


Figure 2 : AMSAA reliability growth estimation, pattern 1

It is clear that the discrete AMSAA model is not able to track all possible reliability growth patterns, especially when the growth is not steadily concave increasing. This is an expected result due to the fact that the results of all phases are combined to estimate the reliability.

B. THE AMSAA-D MODEL

The model analyzed in this thesis is a discrete variation of the continuous AMSAA model for estimating the failure rate of a system. The input random variables to the continuous model are the total number of failures and the total accumulated test time up to the point of estimation [Ref. 4]. It is different from the discrete AMSAA model developed by Duane and evaluated in the thesis of R.M. Thalieb.[Ref. 6]. The equation used by AMSAA to model the instantaneous failure rate f_{TT} is

$$f_{TT} = b(1 - a)TT^{-a} \qquad \text{(II. 1)}$$

where TT denotes the total test time and a and b are parameters. The concept for this model was first presented by Duane [Ref. 7]. A large amount of development test data he examined indicated that the logarithm of the failure rate was a linear function of the logarithm of the total accumulated test time. Comments on the development of this model appear in Appendix E. Woods [Ref. 4] showed that a modification to this model, using regression estimation methods rather than maximum likelihood estimation methods, provided a more accurate growth model. The new discrete model developed here uses similar regression estimation methods.

1. Theoretical analysis

The discrete modification of the continuous AMSAA model developed here should be distinguished from the Discrete AMSAA model and will hereafter be referred to as the AMSAA-D model. We shall adopt the following notation :

j or k : test phase index, corresponds to design change index

f_j : actual failure rate during phase j. Equal to probability of failure for a test.

f_{tk} : failure rate at the end of phase k as determined by the model.

F_j : total number of failures observed in phase j

N_j : number of trials in phase j

N_{tk} : total accumulated number of trials up to and including phase k

$$(N_{tk} = N_1 + N_2 + \ldots + N_k)$$

C_j : number of possible failure causes in phase j

We shall model the failure rate f_{tk} after a total number of tests N_{tk} have been accumulated over k phases by

$$f_{tk} = b(1 - a)N_{tk}^{-a} \qquad (II.2)$$

This provides a discrete version of the model given by equation (II.1). Regression methods similar to those used by Woods [Ref. 4], will be used to estimate the parameters a and b chronologically as test phases are completed. That is, all observed numbers of failures F_1, F_2, ... F_k and the observed total numbers of trials N_1, N_2, ... N_k are used to obtain the current estimate of f_{tk} , k=1,2 ....

Each test phase is terminated after a predetermined number of failures has occurred. Consequently the number of failures per phase will always be greater than 0. The maximum likelihood estimate for f_j is F_j/N_j. Its expected value is larger than f_j, the probability of failure on each trial. There are numerous ways to decrease the bias. The reduction factor chosen here is $2C_j/(1+2C_j)$ primarily because it resembles the bias reduction factor used by Woods [Ref. 4]. Therefore a more nearly unbiased estimate $\hat{f}_j$ for f_j is used where

$$\hat{f}_j = \begin{cases} \dfrac{2C_j}{1+2C_j}\dfrac{F_j}{N_j} & \text{if } C_j \geq 3 \\ \\ 0.85\,\dfrac{F_j}{N_j} & \text{if } C_j = 1,2 \end{cases} \qquad \text{(II.3)}$$

A rationale for equation (II.3) can be developed from a continuous analog. If $X_i = \min(T_i, t_0)$, $i = 1,2, \ldots, N$ where $T_i, \ldots, T_N$ are iid exponential variables, then a nearly unbiased estimate $\hat{\lambda}$ for the common failure rate λ is given by

$$\frac{2N}{1+2N}\,\frac{F}{\Sigma X_i}$$

where F is the number of failures ($T_i < t_0$). This bias correction factor, $2N/(1+2N)$ is developed in Reference 8. Woods [Ref. 4] uses this correction factor for the continuous model. It is modified here by replacing N_j with C_j. Table I shows a comparison of both correction factors $2C_j/(1+2C_j)$ and $2N_j/(1+2N_j)$. The chosen correction factor in equation (II.3) is a constant when the number of failure causes or the number of subsystems in each phase are fixed. Table I shows both the smallest and the largest values of the correction factor $2N_j/(1+2N_j)$ across all 500 replications and six growth patterns for j = 1 and j = 10, i.e. for phases 1 and 10. These four values are grouped by number of failure causes. The number of failure causes is an input parameter for the simulation program. The number of failure causes is important when a fraction of a failure for a specific cause can be removed after a fixed number of successfull system tests occur without failure for the same cause. This failure reduction process is called failure discounting and is permitted by specifying input parameters to the computer program used to run these simulations. Failure discounting was not allowed in any simulation in this thesis.

TABLE I : COMPARISON OF CORRECTION FACTORS

			# of failure causes C_j			
			1	3	5	7
AMSAA-D $2C_j/(1+2C_j)$			0.85	0.8571	0.9091	0.9333
continuous AMSAA model $2N_j/(1+2N_j)$	phase 1	min	0.8571	0.8571	0.8571	0.8571
		max	0.9524	0.9565	0.9565	0.9333
	phase 10	min	0.9524	0.9091	0.96	0.9231
		max	0.9915	0.996	0.9948	0.9922

Least square estimation procedures are used to obtain estimates for a and b as the testing in each new phase is completed. This provides update estimates for a and b and thus for the failure rate f_{tj}, $j = 1,2 \ldots$. Equations for the least square estimates were first developed by Neal [Ref. 9]. He uses the transformations

$$X_j = \ln N_{tj} \qquad \text{and}$$

$$Y_j = \ln f_{tj}$$

to obtain a linear relationship between Y_j and X_j. His equations for the least squares estimates $\hat{a}_k$ and $\hat{b}_k$ are as follows :

$$\hat{a}_k = \frac{\sum_{j=1}^{k} X_j Y_j - \bar{Y}_k \sum_{j=1}^{k} X_j}{\bar{X}_k \sum_{j=1}^{k} X_j - \sum_{j=1}^{k} X_j^2}$$

$$\hat{b}_k = \frac{1}{1 - \hat{a}_k} e^{\bar{Y}_k + \hat{a}_k \bar{X}_k}$$

where $\bar{X}_k = \frac{1}{k} \Sigma X_j$

$\bar{Y}_k = \frac{1}{k} \Sigma Y_j$

$k \geq 2$.

The regression estimation can be made only after two phases of testing have been completed. Therefore the failure rate is estimated for $k \geq 2$ as

$$\hat{f}_{tk} = (1 - \hat{a}_k) \hat{b}_k N_{tk}^{-\hat{a}_k} \qquad (\text{II.4})$$

The term $f_{t1} = f_1$ is estimated with equation (II.3).

2. **Reliability estimate as a linear function of the failure rate estimate.**

To compare the results of this model with other models, it is necessary to convert the failure rate estimate into a reliability value.

A discrete failure rate is defined as

$$f_n = \frac{P[N=n]}{P[N \geq n]} \quad ,n = 1,2,3... \qquad \text{(II.5)}$$

where the random variable N is the number of cycles (trials) up to and including the cycle on which the r^{th} failure occurs [Ref. 10]. For the geometric distribution r = 1 and $f_n = 1-p$. When estimating system reliability in each phase, one basic assumption is that the unknown reliability, p, stays constant within that phase. Even though we are observing the number of trials to the r^{th} failure with common probability of success p on each trial, we shall take the failure rate to be the unreliability 1-p. This means that the failure rate is just the probability of failure on a single trial inside one single phase. Consequently one method for converting the least squares failure rate estimates into reliability estimates is from the expression :

$$\hat{R}_{tk} = 1 - \hat{f}_{tk} \qquad \text{(II.6)}$$

where f_{tk} is given in equation (II.4). In order to avoid subscripts in graphs, equation (II.6) will be written as

$R(N) = 1 - F(N)$ in the graphs. In the text that follows it is referred to as "the linear model". A more detailed discussion of the relationship between failure rate and reliability for discrete distributions is provided in Appendix C.

3. Reliability estimate as an exponential function of the failure rate estimate.

Analysis of the first simulations indicated that the linear model consistently underestimated the actual system reliability on the average. To increase the value of the average point estimate, another equation was introduced for estimating reliability using the failure rate. This equation is

$$\hat{R}_{tk} = e^{-\hat{f}_{tk}} \qquad \text{(II.7)}$$

and is referred to as " the exponential power model " in the text. In the figures it is noted as

$$R(N) = \exp(f(N)).$$

4. Simulation results

The simulations were replicated 500 times for each combination of input parameters. These parameters are

- the phase reliabilities $p_1 \ldots p_{10}$. Six different sets of these 10 values establish six different reliability growth patterns,
- the number of failure causes per phase

- the number of failures before corrective action in each phase.

Table II gives an overview of all simulated parameter combinations.

Table II : ALL SIMULATED PARAMETER COMBINATIONS

# of causes	# of fail/phase	pattern					
		1	2	3	4	5	6
3	3	x	x	x	x	x	x
5	1	x	x	x	x	x	x
	2	x	x	x	x	x	x
	3	x	x	x	x	x	x
	5	x	x	x	x	x	x
	7	x	x	x	x	x	x
7	3	x	x	x	x	x	x

During each replication the estimated system reliability as well as the estimation error, Err_j, was computed. Err_j is defined as

$$Err_j = R_j - \hat{R}_{tj} \qquad \text{(II.8)}$$

where

R_j : actual system reliability in phase j

$\hat{R}_{tj}$: estimated system reliability at the end of phase j

These computations are performed for both AMSAA-D models (the linear and the exponential power model). They are also computed for the Maximum Likelihood Estimation model and the exponential Regression Estimate model; hereafter referred to as the MLEwd and Exp.Reg.Estimate models. These models have been evaluated by Chandler [Ref. 2]. Additional comments on these models are given in Appendix E. After all replications are completed the average system reliability estimate and the Mean Square Error are computed for each estimation method as

$$\bar{\hat{R}}_{tj} = \frac{1}{nrep} \sum_{i=1}^{nrep} (\hat{R}_i)_{tj} \qquad \text{for all } j \qquad (\text{II.9a})$$

$$MSE_j = \frac{1}{nrep} \sum_{i=1}^{nrep} [Err_i]^2 \qquad \text{for all } j \qquad (\text{II.9b})$$

where j : phase index

nrep : total number of replications (= 500)

$\bar{\hat{R}}_{tj}$: average reliability estimate over nrep replications for phase j

MSE : Mean Square Error

This average reliability estimate and the Mean Square Error are plotted and graphed to show the performance of the growth models.

The MLEwd and the exp.Reg.Estimate models are used to compare the performance of both AMSAA-D models relative to these former established growth models. The AMSAA-D model does not allow the application of any discounting method. Therefore the comparisons are all made without any discounting applied to the MLEwd and the exp.reg.Estimate.

a. Sensitivity of the reliability growth estimation with respect to different growth patterns

The evaluation of the sensitivity of both AMSAA-D growth models with respect to different growth patterns is discussed with the following parameter setting :

- number of failure causes : 5
- number of failures/phase : 3

The results and conclusions, however, are in general the same for all other parameter combinations as is shown in Figures 18 - 35 in Appendix D.

Figure 3 illustrates the performance of both AMSAA-D models (linear and exponential) when the true reliability growth follows the path stipulated in pattern 1.

The exponential power model in the first 6 phases far overestimates the actual reliability. The linear model tracks the convex growth pattern very well over all phases. Also it underestimates the actual reliability, which is usually a desired behavior. In comparison with the MLEwd and the exp.Reg.Estimate, the linear model performs better over all phases with respect to the average of the point estimates $\hat{R}_{t1}$, $\hat{R}_{t2}$, ...

The Mean Square Error for all four methods is shown in Figure 3. The exponential power model is the worst estimation model with respect to the average reliability estimate, but it has the smallest Mean Square Error and is less dependent on the phase. But from phase 8 on, all models have nearly the same Mean Square Error.

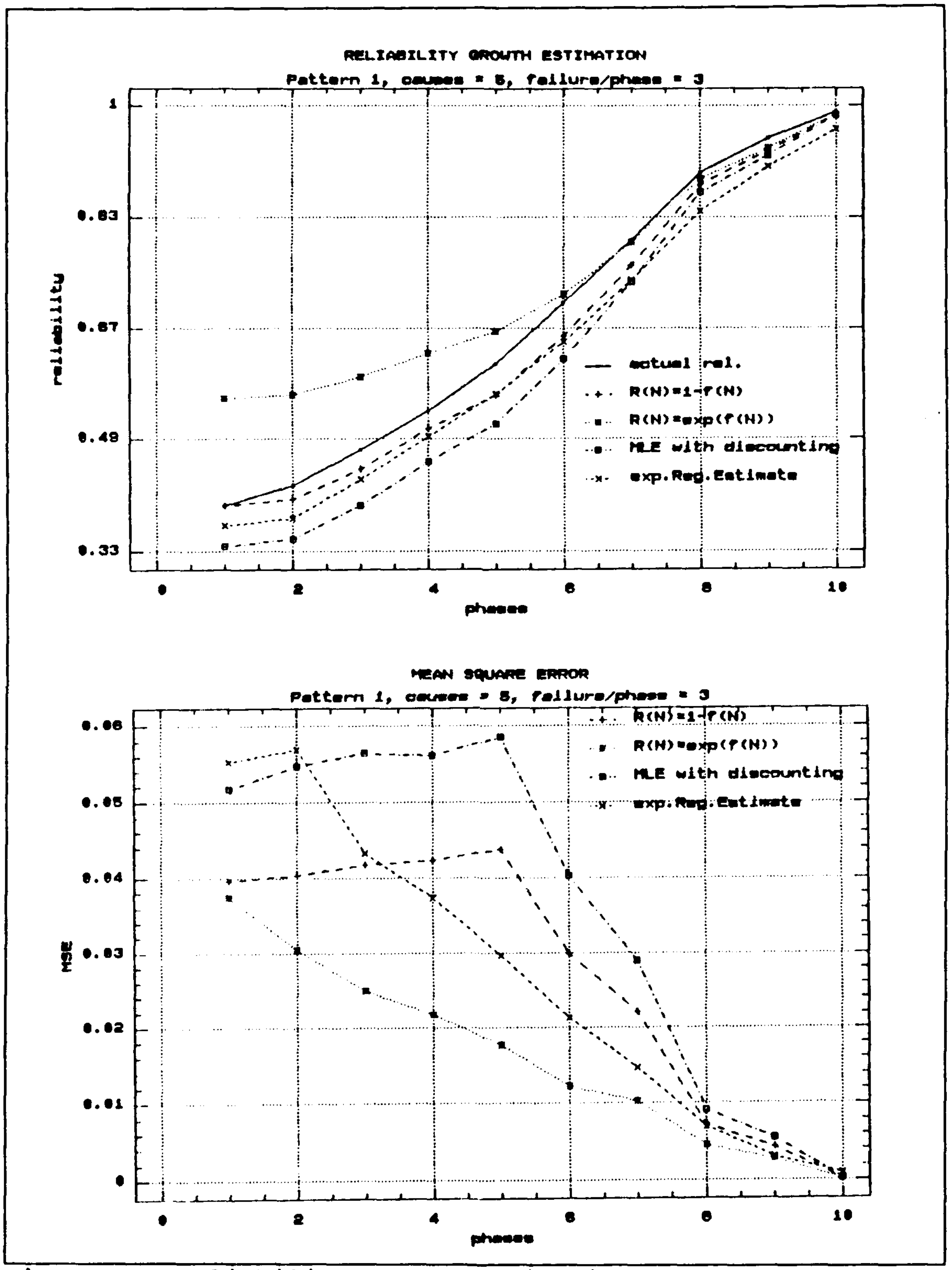


Figure 3 : Reliability growth estimation, pattern 1

Figure 4 illustrates the performance of all models under pattern 2.

Both AMSAA-D models track the decline of the actual reliability very well. The linear model again always underestimates the actual system reliability, and the exponential power model overestimates the actual system reliability up to phase 6. It also smoothes the decline and the following incline of the actual reliability.

The linear model performs better with respect to the average reliability estimate, when compared with the MLEwd and the exp.Reg.Estimate. Also, from phase 3 on, both AMSAA-D models have the smaller Mean Square Error and this Mean Square Error is not very strongly influenced by the phase as is the case for the MLEwd and the exp.Reg.Estimate.

For pattern 3 the general behavior of all models, as shown in Figure 5, is similar to the behavior for patterns 1 and 2.

All models perform well under pattern 4 and 5 as it can be seen in Figures 6 and 7. For pattern 5 only the exp.reg.Estimate shows an overestimation of the actual reliability over all phases. Again, the decline of the Mean Square Error from phase 1 to phase 3 for all models is very significant.

Figure 8 shows the behavior under pattern 6. All models underestimate the actual reliability quite well in the average. The Mean Square Error is nearly the same for all models and is also nearly constant over all phases. Only the

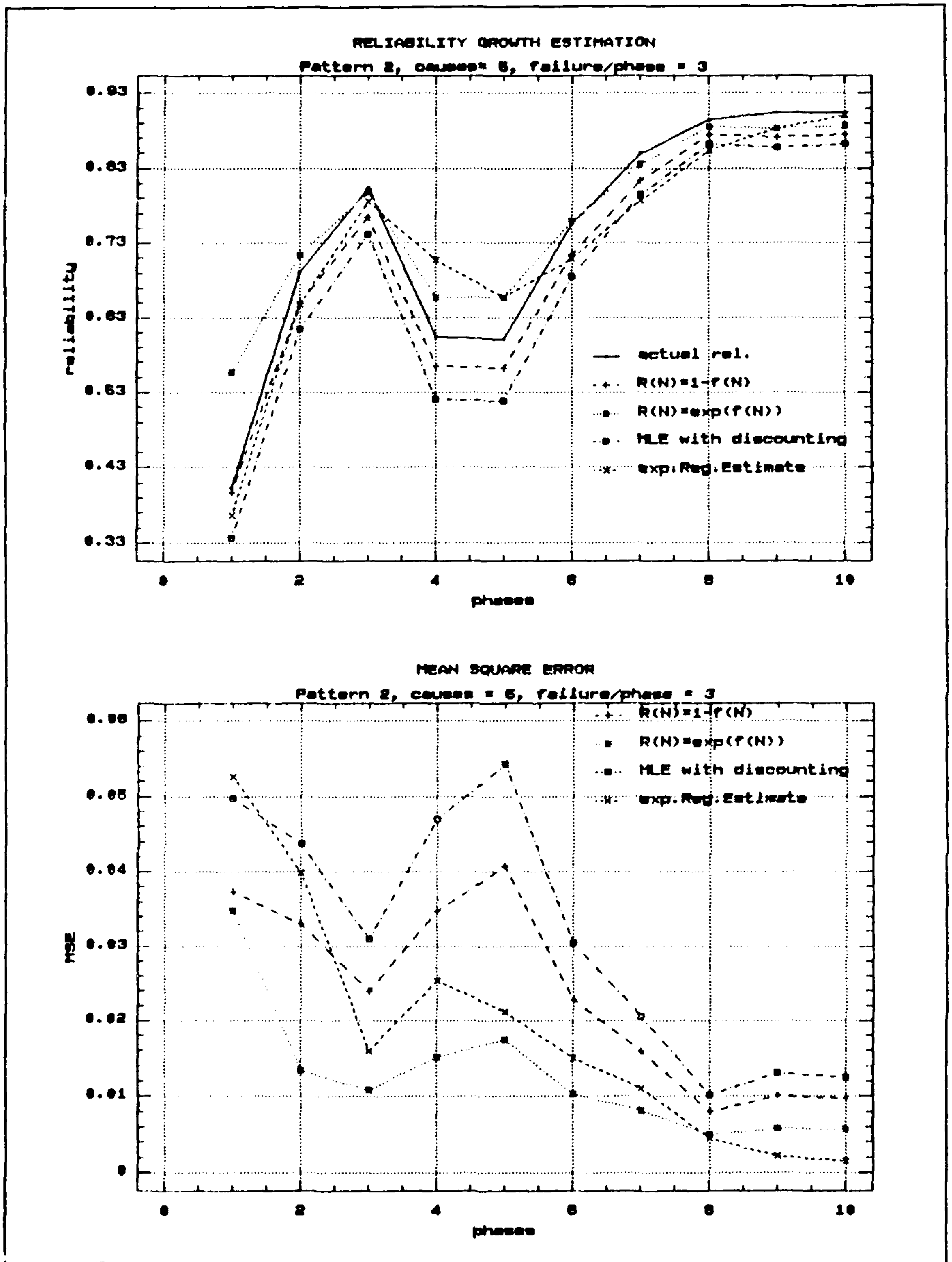


Figure 4 : Reliability growth estimation, pattern 2

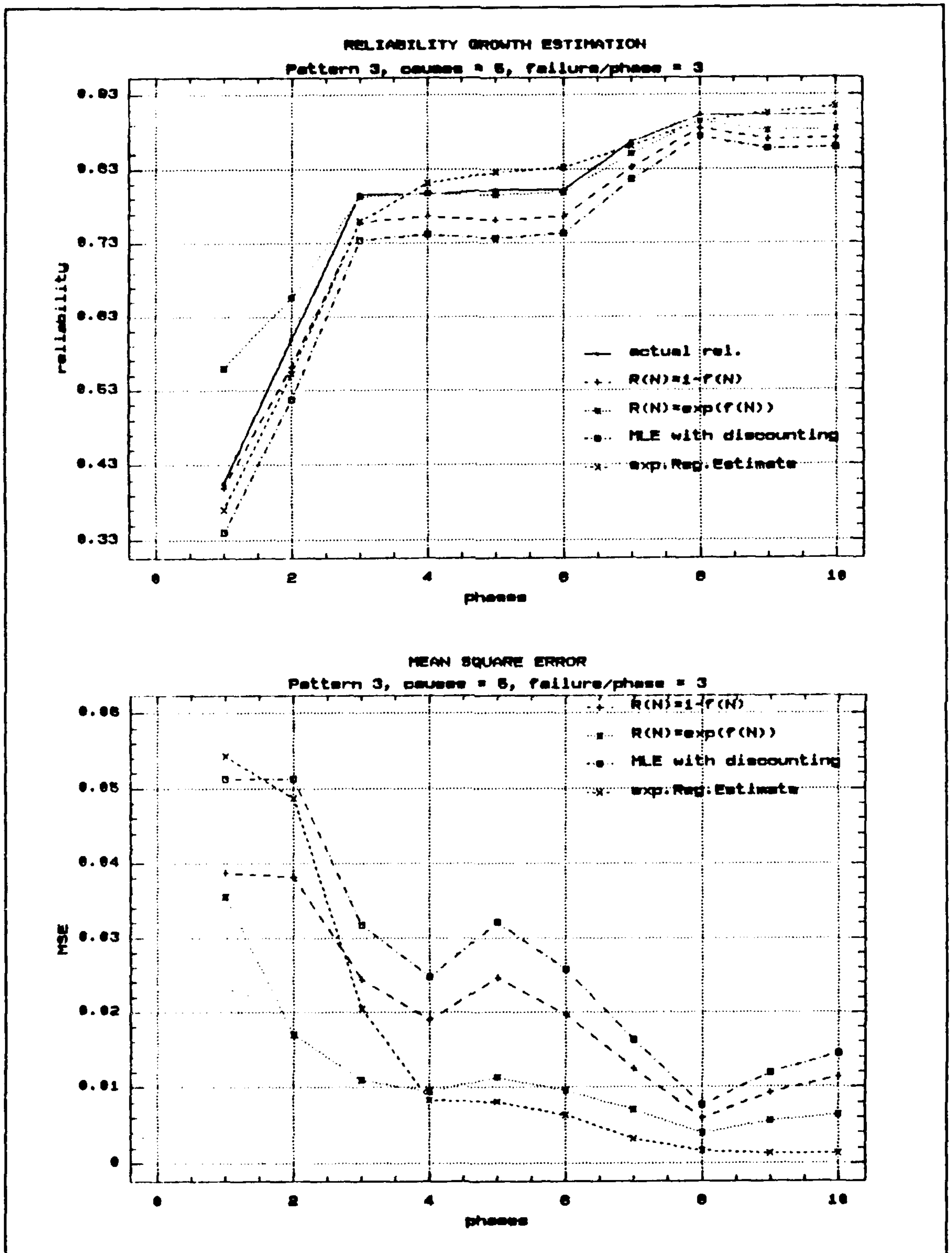


Figure 5 : Reliability growth estimation, pattern 3

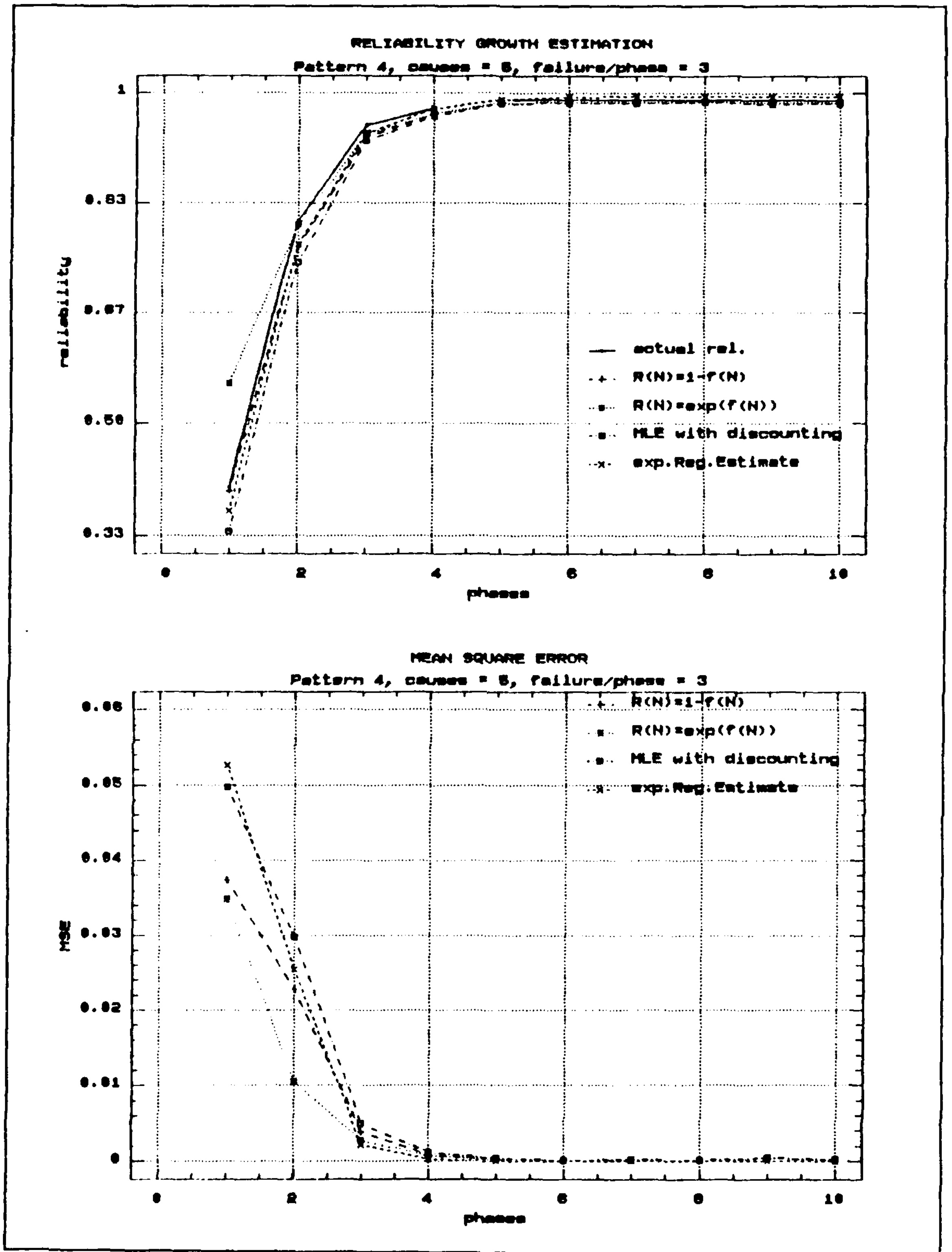


Figure 6 : Reliability growth estimation, pattern 4

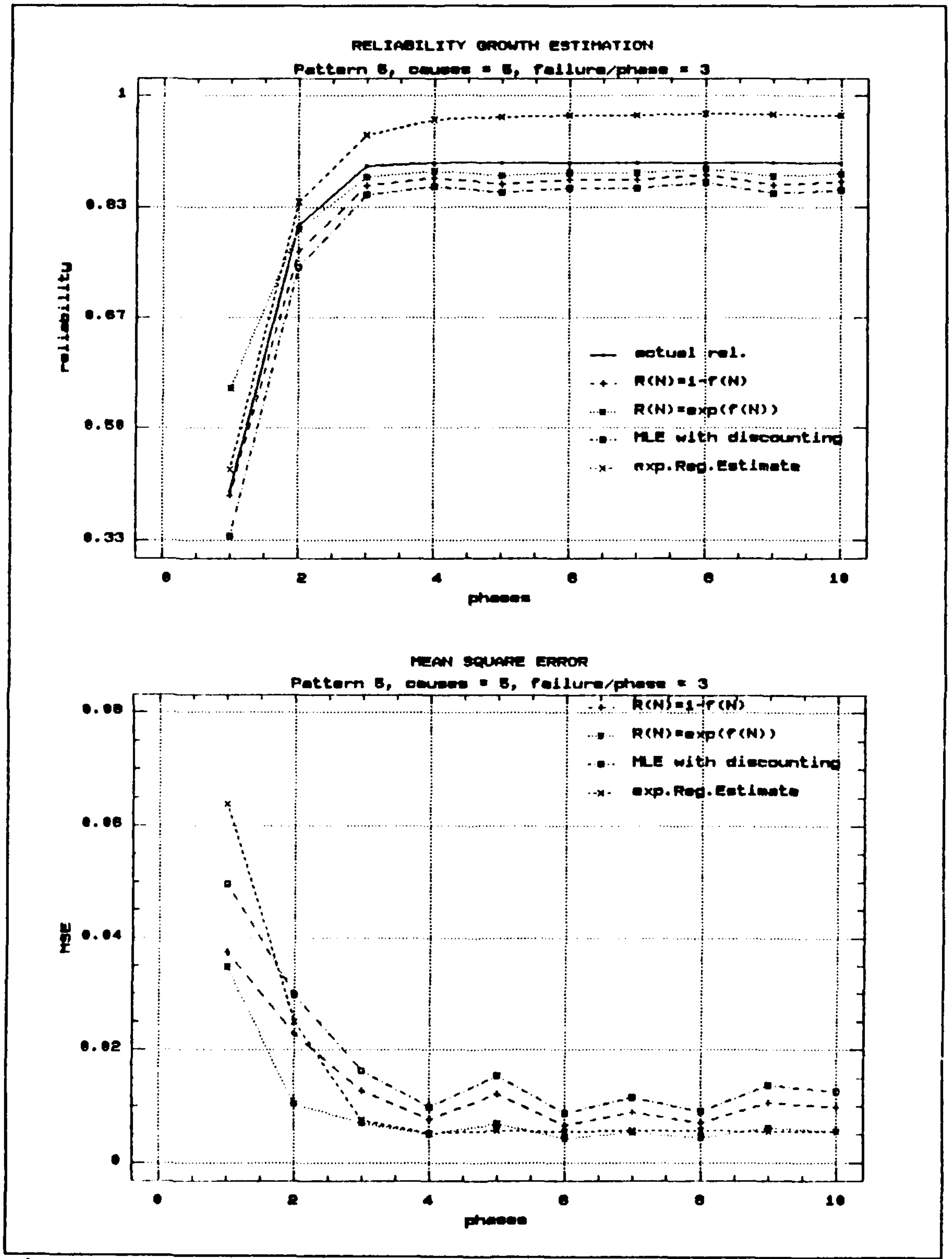


Figure 7 : Reliability growth estimation, pattern 5

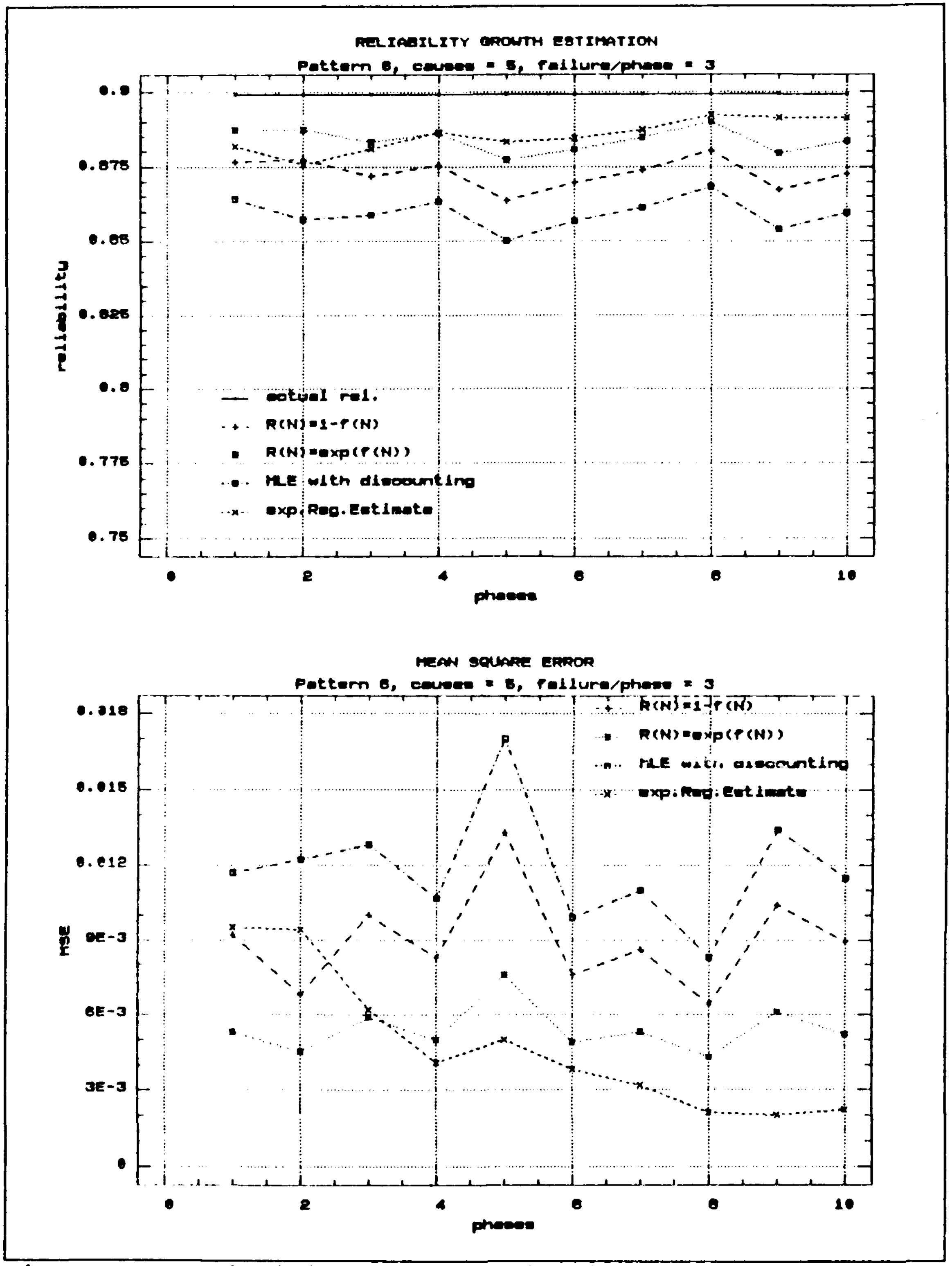


Figure 8 : Reliability growth estimation, pattern 6

exp.Reg.Estimate shows a slight decline in the Mean Square Error from phase 1 to 10.

Figures 9 and 10 illustrate the behavior of both AMSAA-D models with respect to the Mean Square Error and with phases and patterns as variables.

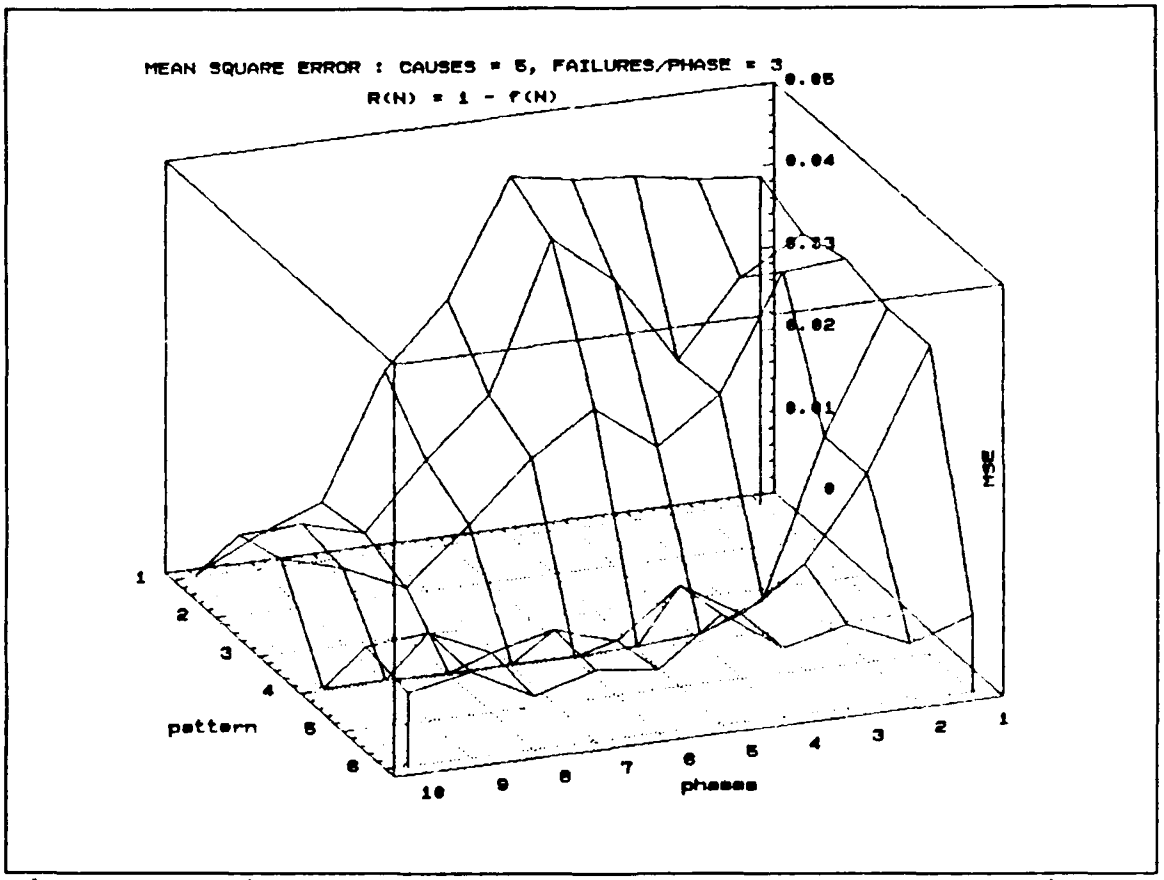


Figure 9 : Linear model, Mean Square Error as a function of phases and patterns

Both models show a high Mean Square Error in the early phases. This is reasonable because in the early phases less information is available to get accurate reliability estimates. The exponential power model (Figure 10) shows a constant decline of the Mean Square Error for pattern 1,

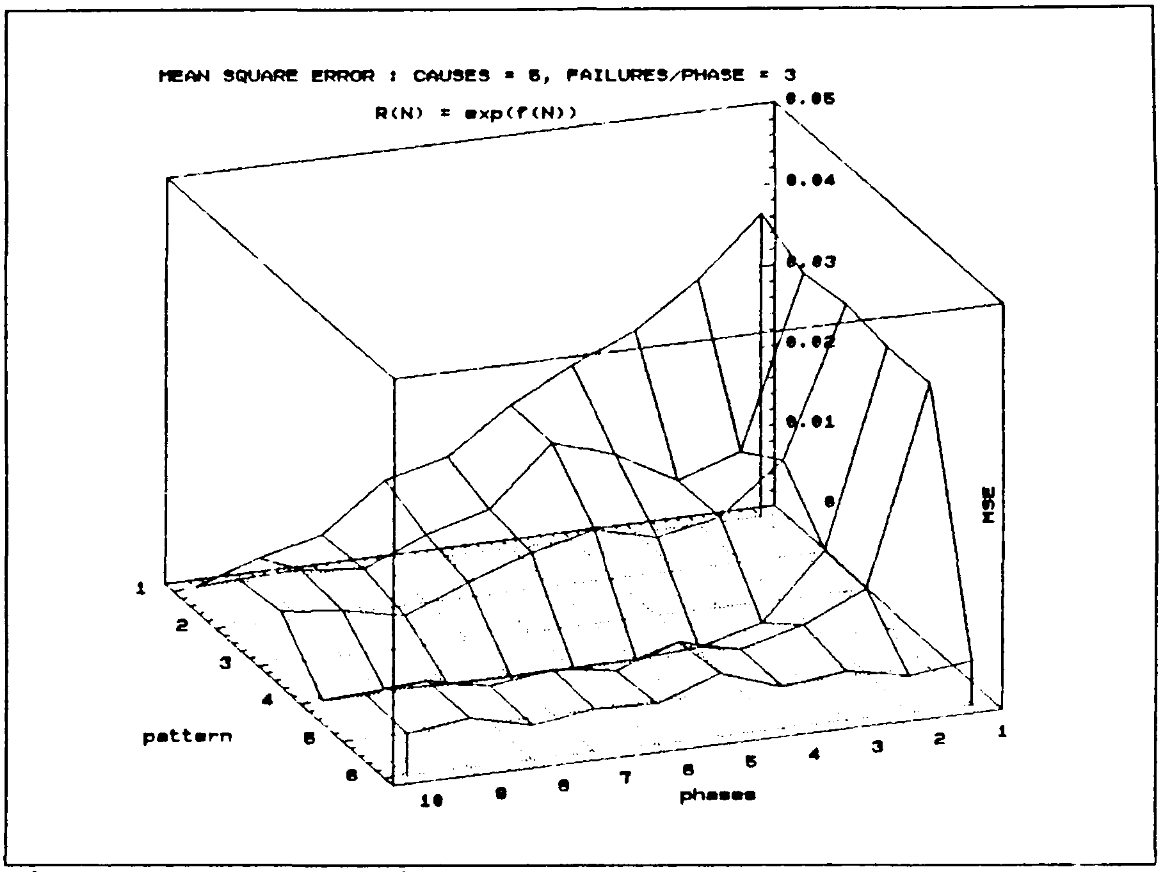


Figure 10 : Exponential power model, Mean Square Error as a function of phases and patterns

whereas for the pattern 2 - 5 the Mean Square error drops rapidly after phase 1 and stays nearly constant thereafter.

For both models pattern 6 is less dramatic because both models show a nearly constant low Mean Square Error over all phases.

b. Sensitivity of the reliability growth estimation to the number of failure causes.

To evaluate the sensitivity of the reliability growth estimation to the number of failure causes the simulation was run for each pattern with the number of failure causes set at 3, 5 and 7. Figure 11 and the Figures 36 to 40 in Appendix D show three dimensional plots of the Mean Square Error as a function of the phase and the number of failure causes for the linear and the exponential power model. The Mean Square Error was chosen as a measure of the sensitivity, because the estimation of the actual reliability doesn't show great differences due to the change in the number of failure causes. The graphs of the reliability estimation and their related Mean Square Error for all different parameter settings is enclosed in Appendix D (Figures 18 - 35).

As can be seen, the general behavior of the growth estimation due to the number of failure causes is relatively independent of the simulated growth pattern. Also the difference between the two models is not very large. Both models show, in general, an increasing Mean Square Error when the number of failure causes is increased. This increase is overall a little steeper for the linear model, but it becomes less dependent in the number of causes in later phases.

The exponential power model shows a more differentiated behavior. It has a steep increase of the Mean Square Error for pattern 1 in the first 5 phases as the number of causes is increased. This is true in all other patterns only for phase 1. For all other patterns and phases greater than 1 the Mean Square Error of the exponential power model is only slightly increasing with increasing number of causes. The slope is reduced even more within the last phases.

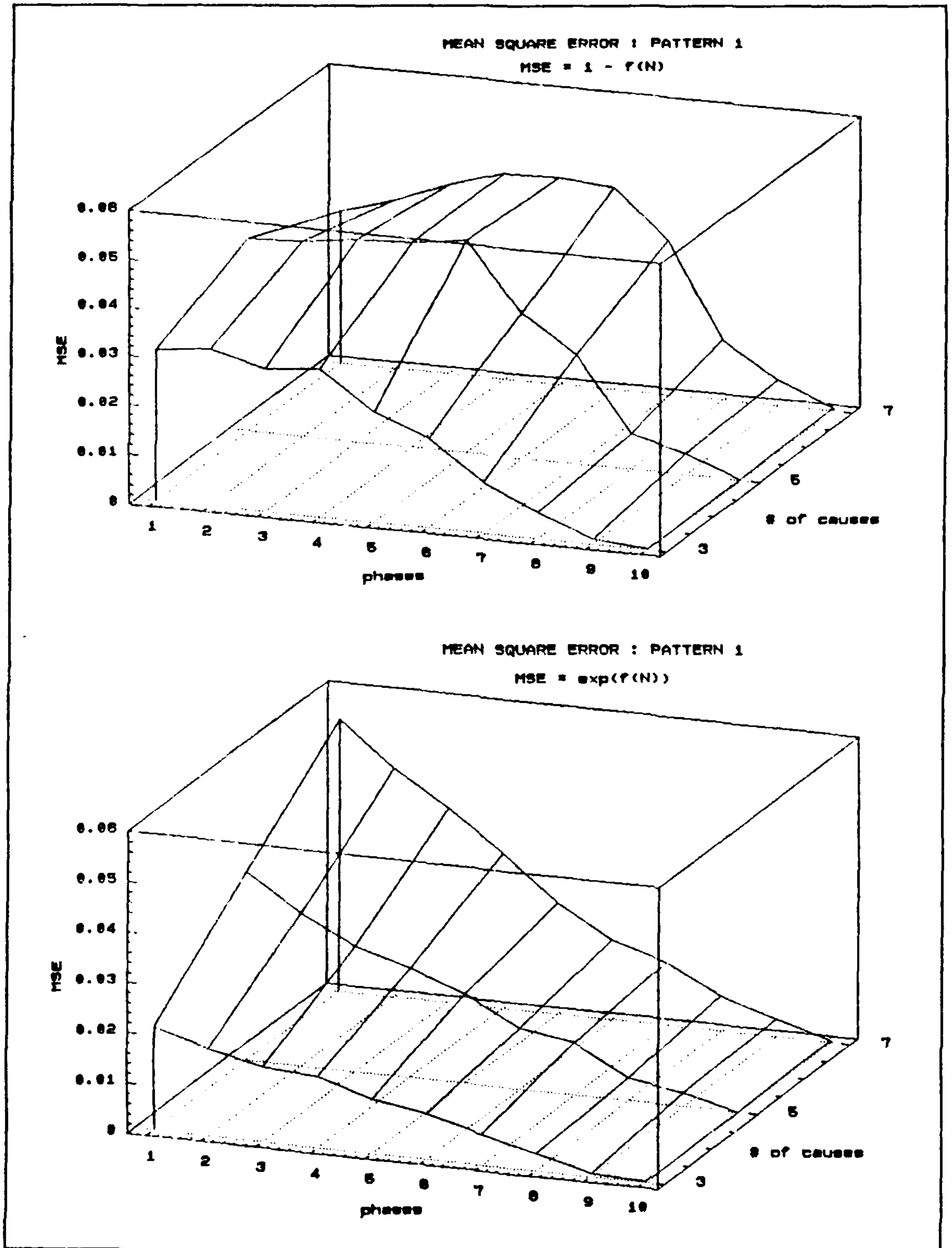


Figure 11: Mean Square Error as a function of phases and failure causes, both AMSAA-D models, pattern 1

To catch the general trend in a different way a total average Mean Square Error was computed as

$$\text{tot.avg.MSE} = \frac{1}{\text{npat}} \sum_{p=1}^{\text{npat}} \left(\frac{1}{\text{nphase}} \sum_{j=1}^{\text{nphase}} \text{MSE}_{jp} \right) \qquad \text{(II.10)}$$

where

MSE_{jp} : Mean Square Error as in equation (II.9),

j : phase index (here 1 to 10),

p : pattern index (here 1 to 6),

nphase : number of phases (here 10) and

npat : number of patterns (here 6).

The result is shown in Figure 12. It also affirms the trend estimation from the three dimensional graphs. The total average Mean Square Error increases for both models. This is also true for the other two models, MLEwd and exp.Regr.Estimate, shown for comparison. But the slope of the linear model is steeper than the slope of the exponential power model when increasing the number of causes from 3 to 5.

c. Sensitivity of the reliability growth estimation to the number of failures per phase

The evaluation of the sensitivity of the reliability growth estimation to the number of failures per phase was done with the following parameter settings :

- all patterns (1 to 6)
- number of failure causes = 5

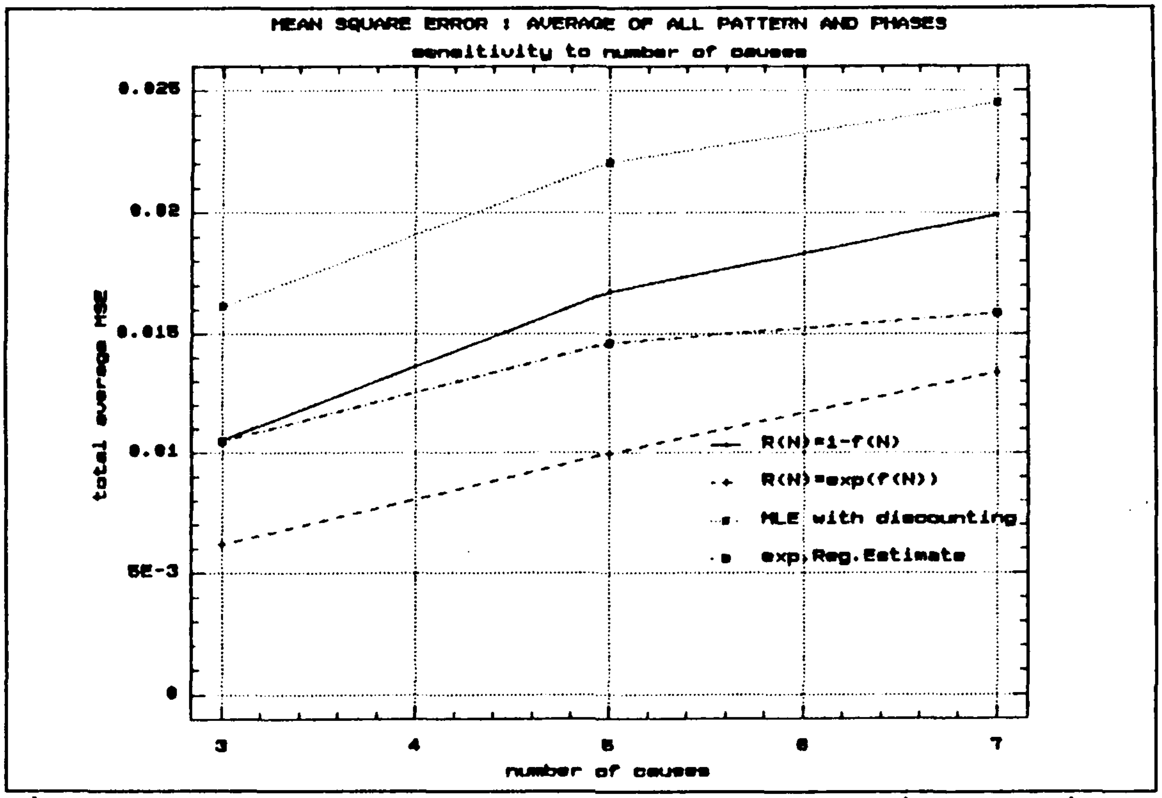


Figure 12 : Average Mean Square Error as a function of failure causes

Figure 13 and Figures 41 to 45 in Appendix D show the three dimensional plots of the variation of the Mean Square Error over all phases with varied failures/phase for all patterns.

From all Figures, it can clearly be seen that the greatest impact on the reduction of the Mean Square Error occurs where the number of failures/phase changes from 1 to 2. In general an increase in the number of failures/phase from 1 to 2 will reduce the mean Square Error by at least 50% in most of the cases. This is relatively independent of the phase. But the reduction in phase 1 is always the smallest for

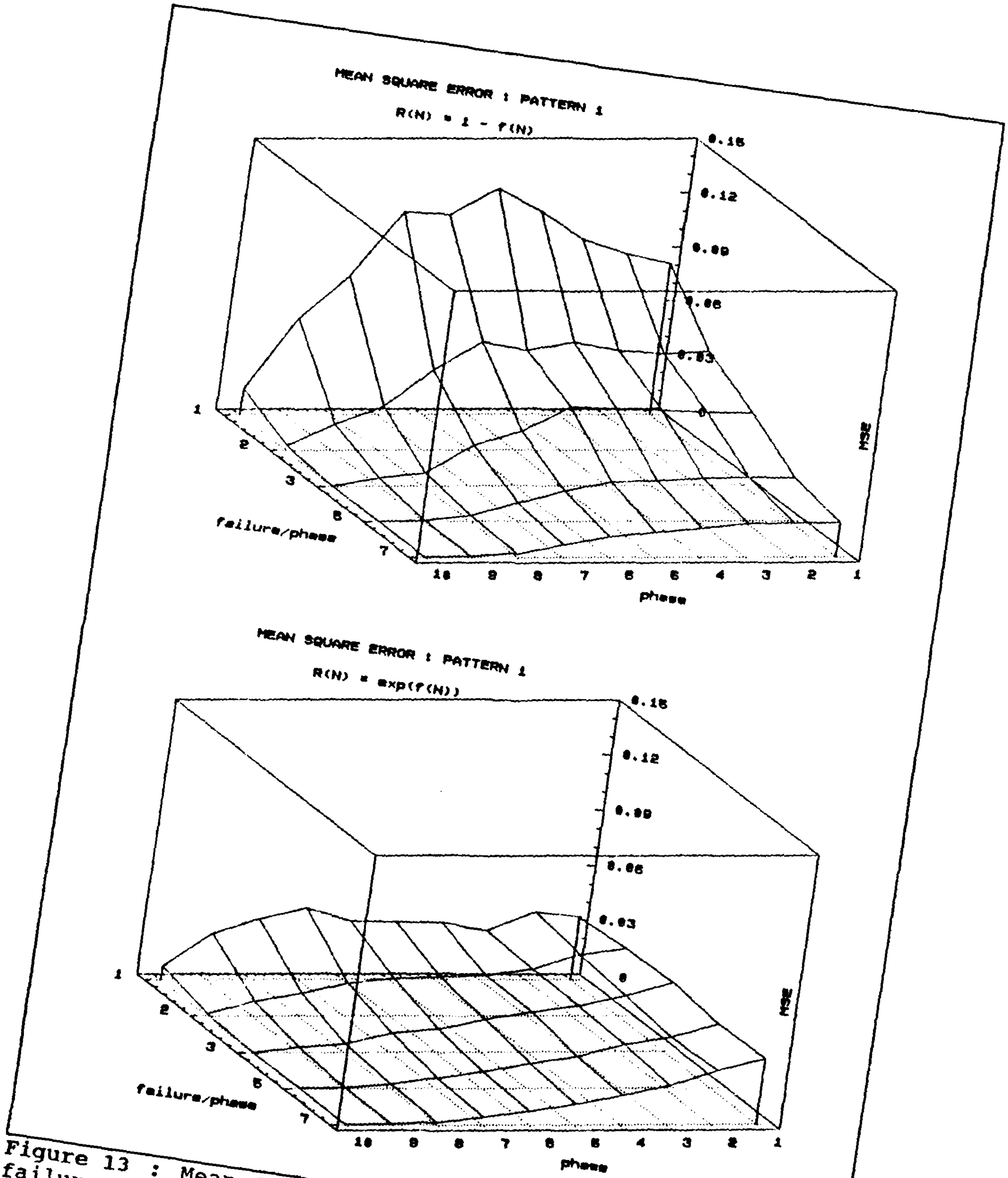


Figure 13 : Mean Square Error as a function of phases and failures per phase, both AMSAA-D models, pattern 1

each pattern. Note that there is no decrease of the Mean Square Error for the exponential power model in the first phase for the patterns 1 to 5 when the number of failures/phase is increased. Only pattern 6 shows a decrease in phase 1 also.

The variation of the total average Mean Square Error over all phases and patterns, computed as in equation II.10, with increasing number of failures/phase is shown in Figure 14. It again affirms in general the trend evaluated from the three dimensional graphs. The greatest reduction of the Mean Square error is achieved by increasing the number of failures/phase from 1 to 2. This results in an average decrease of the Mean Square Error from 0.08 to 0.03 for the linear model and from 0.03 to about 0.015 for the exponential power model. This result is true in general for the Maximum Likelihood Estimation and the exponential Regression Estimate, which are also shown in Figures 13 and 14 for comparison.

d. Comparison with the discrete AMSAA model

The results show the AMSAA-D model is more accurate than the discrete AMSAA model. The AMSAA-D model appears to account more accurately for different reliabilities in different phases, but still uses information from previous phases to yield more accurate estimates of reliability at later stages in the growth process. It makes good use of the fact that the actual reliability is not constant over all phases, but stays constant within a phase. The discrete AMSAA model is not able to track all different possible patterns of

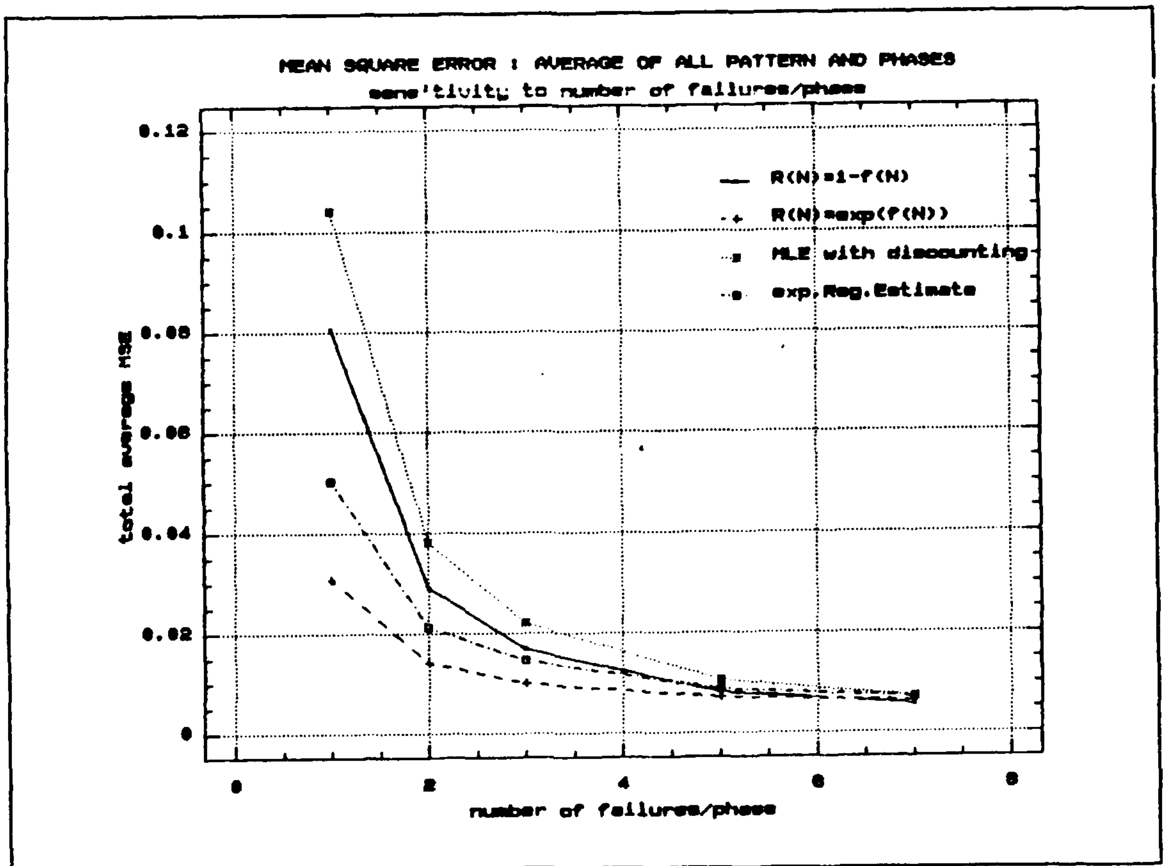


Figure 14 : Average Mean Square Error as a function of failures per phase

reliability growth as can be seen in Figure 15, which is taken from R.Thalieb [Ref. 6] for pattern 2 with 3 failures/phase and 1 failure cause.

e. Conclusion

The linear AMSAA-D model performs reasonably well and is relatively independent of the pattern of actual reliability growth. It always underestimates the actual reliability and is therefore a conservative approach. It also performs at least as good as the formerly established MLEwd and exp.Regr.Estimate models when using Mean Square Error as

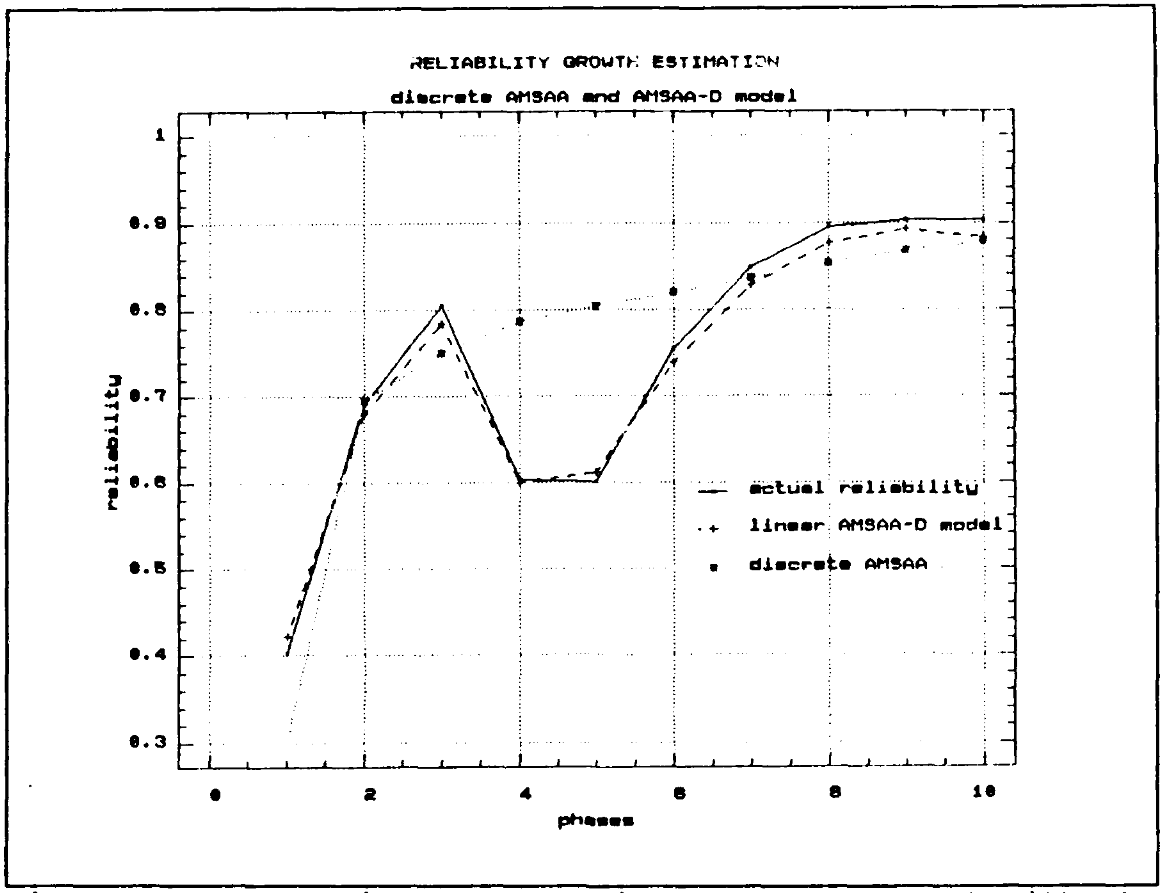


Figure 15 : Comparison of the discrete AMSAA model with the AMSAA-D model, pattern 2

a measure of performance. It is not very sensitive to the number of failure causes and this low sensitivity is even decreasing within later phases of testing. When the number of failures/phase are increased from 1 to 2, the Mean Square Error can be reduced by about 50%.

The exponential power model has a very low Mean Square Error. In general, it is about 50% less than the Mean Square Error of the other models. It overestimates the actual reliability in the early phases for nearly all of the six growth patterns. The impact on accuracy due to an increase in the number of failure causes is nearly the same as for the linear model. The same can be said for the impact of an increase in the number of failures/phase.

The linear AMSAA-D model is a conservative, but reasonable accurate tool, for estimating actual system reliability growth. It is not very sensitive to the number of failure causes and an increase in the number of failures per phase can reduce the Mean Square Error significantly.

III. SUMMARY, RECOMMENDATIONS

A. SUMMARY

The objective of this thesis was to evaluate the accuracy of a discrete modification of the continuous AMSAA model (AMSAA-D model) for the reliability growth estimation. The sensitivity of the model to different growth patterns and system parameters as the number of subsystems, the number of phases and the number of failures till phase termination was evaluated. The accuracy of this model was compared with that for the MLEwd and the exponential regression estimate models [Ref. 1,2,3]. Although possible improvements due to different discounting and weighing methods for the MLEWD and the exponential regression estimate models have been demonstrated in these former theses, comparisons in this thesis were made without using discounting or weighting of data.

Throughout the simulations the AMSAA-D model illustrated a reasonable accuracy and was at least as good as the MLEWD or the exponential regression estimate. It is able to track all possible growth patterns reasonably well. Its sensitivity to the number of failure causes and the number of failures within a phase compares favorably with that of other established models.

B. RECOMMENDATIONS

The following are recommendations for further study :

1. The AMSAA-D model should be modified by using the minimum variance unbiased estimate of the failure rate instead of the failure rate estimate using a correction factor.
2. Since the exponential regression estimate can be improved by weighting the data, similar methods should be analyzed for the AMSAA-D model.

APPENDIX A

USER'S GUIDE TO : RELIABILITY GROWTH WITH THE AMSAA-D MODEL

(DISCONT)

APPENDIX A : USER'S GUIDE TO RELIABILITY GROWTH WITH THE AMSAA-D MODEL (DISCONT)

1. Introduction

In order to use the Fortran program DISCONT the user must possess three files :

- Input file "like" PT1C5F3 DATA A1
- DISCONT FORTRAN A1
- DISCONT EXEC A1

A sample of each of these files along with a sample run using DISCONT EXEC A1 and a sample output is contained in Appendix B. The input file and the exec file can be changed to meet the needs of the user. In the current form, the exec file produces a large number of intermediate calculations for both the DISCONT FORTRAN program and Lt P. Markiewicz's program DRG FORTRAN [Ref. 3]. These calculations may not be of interest and therefore may be eliminated with no effect to the simulation. A detailed explanation of each file, which is created in addition to Lt P.Markiewicz's version is contained in the following sections.

2. The DISCONT EXEC A1 file

The DISCONT EXEC A1 contains all necessary file definitions and commands needed to run DISCONT FORTRAN A1. Appendix B 2. shows a sample run using DISCONT EXEC A1. DISCONT FORTRAN A1 is an extension of the DRG FORTRAN A1 program of Lt P.Markiewicz [Ref. 3]. Therefore in addition to the output created for the discretized continuous models there are all outputs and models available, which have been used by Lt P.Markiewicz [Ref. 3].

For the use of DISCONT FORTRAN A1 it is necessary to specify an Input file in line 24 of DISCONT EXEC A1. This input file must have all necessary inputs in the same format as mentioned in Appendix A of Lt P.Markiewicz thesis [Ref. 3]. The filename and type can be chosen free as long as it matches the format of the CP/CMS requirements of a maximum of eight characters.

For the purpose of this thesis, 42 different input files have been created to allow the evaluation of the sensitivity of the model to the following different input parameters

- type of growth pattern
- number of failure causes and
- number of failures per phase.

The filenames, used in the 42 input files, express the setting of these different input parameters. For example, the filename PT1C5F3 means

- pattern : 1
- number of failure causes : 5
- number of failures/phase : 3

The full name of the input file is then PT1C5F3 DATA A1.

The output for the AMSAA-D models as well as the results of the models chosen for comparison is written to GROWTH EST A1. A sample output is given in Appendix B.

Also a control output is available. It allows us to track the intermediate results of single simulation runs for the first and last 10 replications, independent of the number of replications chosen. This control output is written to GROWTH CONTROL A1.

The next output file is FAILURE RATE A1. It contains the actual reliability as well as the average of the estimated failure rates of each replication, which is the basis for the computations of the estimated reliability in the linear and the exponential power model.

The last output file is MSE COMPARE A1. This file contains results used to evaluate the influence of changes in the number of failure causes and the number of failures/phase. It contains the average Mean Square Error, averaged over all phases from the distinct Mean Square Error for each phase, computed for the four growth models

- 1 .... linear model
- 2 .... exponential power model
- 3 .... MLE with discounting
- 4 .... exponential Regression Estimate

In addition to these files, related to the AMSAA-D models, all results of Lt P.Markiewicz's simulation program DRG FORTRAN A1 are computed and written to the same output files as in their original program. The only difference is that the filenames have in addition an "X" in front of their original name. In this way it is possible to run both simulations, DISCONT and DRG, one after the other without overwriting the first created output files. Therefore the file descriptions in Appendix A of the thesis of Lt P.Markiewicz are totally valid.

3. The input data file

These instructions should be used in connection with a sample input (see Appendix B 2.). The sequence of all data inputs is mandatory as it is shown in the sample input file. The input is read from file device number 10.

For the discretized continuous models the input can be the same as for the DRG FORTRAN program of Lt. P.Markiewicz. All steps noted in her thesis in Appendix A 3. are still valid. The only difference is that <u>the first line of the input file should contain the filename of the input file</u> to get this remark stated on top of the output file GROWTH EST A1. Due to this additional first line the only change in the steps of Appendix A 3 of Ref. 3 is that the mentioned line numbers must be increased by one. But all other input variations/possibilities stay the same.

4. Output file GROWTH EST A1

The output file shows in its first line the filename of the input file used for the actual simulation run. The names of the variables are :

- PHASE ... self explanatory
- ACTREL ... actual reliability
- LINREL ... reliability estimate, using the linear model
- MSEL ... Mean Square Error of the linear model
- RELEXP ... reliability estimate using the exponential power model
- MSEE ... Mean Square Error of the exponential power model
- MLEWD ... Maximum Likelihood estimate with discounting
- MSEWD ... Mean Square Error of the MLEWD
- REGEST ... exponential regression estimate
- MSEREG ... Mean Square Error of the exp.Reg.Estimate

APPENDIX B

FILES AND VARIABLES

1. Sample DISCONT EXEC A1 file
2. Sample input file : PT1C5F3 DATA A1
3. Sample simulation run, using DISCONT EXEC A1, recorded session
4. Sample output GROWTH EST A1
5. Main variables used for the discrete modification of the continuous AMSAA model (AMSAA-D model)

1. **Sample DISCONT EXEC A1 file**

```
&TRACE OFF
*******************************************************
&FN = DISCONT
&FN1 = GROWTH
&TYPE THIS PROGRAM USES A DISCRETIZED CONTINUOUS MODEL
&TYPE DO YOU NEED TO COMPILE YOUR PROGRAM ? (Y/N)
&READ VAR &R_COMPILE
&IF &R_COMPILE NE Y &GOTO -RUN
-H FORTVS &FN
&IF &RC EQ 0 &GOTO -RUN
&TYPE Your program did not compile; check for errors.
&TYPE DO YOU WISH TO VIEW THE PROGRAM LISTING FILE? (Y/N)
&READ VAR &RSP1
&IF &RSP1 EQ Y BROWSE &FN LISTING A
&TYPE DO YOU WISH TO XEDIT THE PROGRAM FILE? (Y/N)
&READ VAR &RESP1
&IF &RESP1 NE Y &EXIT 1
&COMMAND XEDIT &FN FORTRAN A
&TYPE DO YOU WISH TO RUN THE PROGRAM AGAIN? (Y/N)
&READ VAR &RESP2
&IF &RESP2 EQ Y &GOTO -H
&EXIT 1
-RUN
```

```
FILEDEF 10 DISK PT2C7F3 DATA A1
FILEDEF 82 DISK XA1 NUM A1
FILEDEF 84 DISK XA9 NUM A1
FILEDEF 30 DISK XJRELIAB LISTING A1 (LRECL 133
FILEDEF 35 DISK XPRELIAB LISTING A1 (LRECL 133
FILEDEF 20 DISK XJTHESIS OUT A1
FILEDEF 25 DISK XPTHESIS OUT A1
FILEDEF 81 DISK XPMATRIXA LISTING (LRECL 133
FILEDEF 83 DISK XJMATRIXA LISTING (LRECL 133
FILEDEF 87 DISK XPREGMAT DATA A1
FILEDEF 88 DISK XJREGMAT DATA A1
FILEDEF 90 DISK XYSTAR LISTING (LRECL 133
FILEDEF 89 DISK XTRIALS DATA A1
FILEDEF 40 DISK XEST OUT A1
FILEDEF 50 DISK XJMLEWD OUT A1
FILEDEF 55 DISK XPMLEWD OUT A1
FILEDEF 60 DISK XJMLESP OUT A1
FILEDEF 65 DISK XPMLESP OUT A1
FILEDEF 70 DISK XJREGEST OUT A1
FILEDEF 75 DISK XPREGEST OUT A1
FILEDEF 15 DISK XPWRES1 OUT A1
FILEDEF 39 DISK XPWRES2 OUT A1
FILEDEF 49 DISK XPWRES3 OUT A1
FILEDEF 16 DISK XJWRES1 OUT A1
FILEDEF 38 DISK XJWRES2 OUT A1
FILEDEF 48 DISK XJWRES3 OUT A1
```

```
FILEDEF 52 DISK XMLEWD1 NUM A1
FILEDEF 51 DISK XMLEWD1 SDV A1
FILEDEF 54 DISK XMLEWD9 NUM A1
FILEDEF 53 DISK XMLEWD9 SDV A1
FILEDEF 72 DISK XREG8 NUM A1
FILEDEF 71 DISK XREG8 SDV A1
FILEDEF 74 DISK XREG16 NUM A1
FILEDEF 73 DISK XREG16 SDV A1
FILEDEF 77 DISK XM1P1 NUM A1
FILEDEF 76 DISK XM1P1 SDV A1
FILEDEF 79 DISK XM2P1 NUM A1
FILEDEF 78 DISK XM2P1 SDV A1
FILEDEF 92 DISK XM3P1 NUM A1
FILEDEF 91 DISK XM3P1 SDV A1
FILEDEF 18 DISK XM1P9 NUM A1
FILEDEF 17 DISK XM1P9 SDV A1
FILEDEF 94 DISK XM2P9 NUM A1
FILEDEF 93 DISK XM2P9 SDV A1
FILEDEF 96 DISK XM3P9 NUM A1
FILEDEF 95 DISK XM3P9 SDV A1
FILEDEF 62 DISK XMLESP1 NUM A1
FILEDEF 61 DISK XMLESP1 SDV A1
FILEDEF 64 DISK XMLESP9 NUM A1
FILEDEF 63 DISK XMLESP9 SDV A1
FILEDEF 11 DISK GROWTH CONTROL A1
FILEDEF 12 DISK FAILURE RATE A1
```

```
FILEDEF 13 DISK GROWTH EST A1

FILEDEF 14 DISK ERROR EST A1

FILEDEF 19 DISK MSE COMPARE A1

FILEDEF 21 DISK LLOYD EST A1

FILEDEF 22 DISK FRATE COMPARE A1

FILEDEF 06 TERMINAL

LOAD &FN (START

&IF &RC EQ 0 &SKIP 9

&TYPE Your program did not run correctly; check for
errors.

&TYPE Do you wish to XEDIT the program file? (Y)

&READ VAR &RESP3

&IF &RESP3 NE Y &EXIT 2

&COMMAND XEDIT &FN FORTRAN A

&TYPE Do you wish to run the program again? (Y)

&READ VAR &RESP4

&IF &RESP4 EQ Y &GOTO -H

&EXIT 2

&TYPE LLOYD ESTIMATION/EXPECTATION OUTPUT IS IN "LLOYD EST
A1"

&TYPE

&TYPE CONTROL OUTPUT IS IN THE FILE "GROWTH CONTROL A1"

&TYPE

&TYPE YOUR ESTIMATION OUTPUT IS IN THE FILE &FN1 EST A1

&TYPE DO YOU WISH  TO BROWSE YOUR ESTIMATION OUTPUT? (Y/N)

&READ VAR &RESP
```

```
&IF &RESP EQ Y &COMMAND BROWSE &FN1 EST A1

&TYPE PRINT YOUR OUTPUT FILE?  (Y/N)

&READ VAR &RESP7

&IF &RESP7 EQ Y &COMMAND PRINT &FN1 EST A1

-REDO

&TYPE DO YOU WISH TO CHANGE THE INPUT DATA FILE (Y/N)

&READ VAR &RESP5

&IF &RESP5 EQ Y XEDIT &FN EXEC A1

&TYPE DO YOU WISH TO RUN THE PROGRAM AGAIN? (Y/N)

&READ VAR &RESP6

&RESP56 = &CONCAT OF &RESP5 &RESP6

&IF &RESP56 EQ YY &GOTO -H

&IF &RESP6 EQ Y &GOTO -RUN

&EXIT
```

2. **Sample input file : PT1C5F3 DATA A1**

```
PT1C5F3        PATTERN 1, 5 FAILURE CAUSES,

             3   FAILURES/PHASE

5              NUMBER OF FAILURE CAUSES

10             NUMBER OF PHASES ( NPHASE )

1              FIXED RELIABILITY OPTION

             ( 1: YES ; 0: NO )

3              NUMBER OF FAILURES IN PHASE 1

3              NUMBER OF FAILURES IN PHASE 2

3              NUMBER OF FAILURES IN PHASE 3

3              NUMBER OF FAILURES IN PHASE 4

3              NUMBER OF FAILURES IN PHASE 5
```

3	NUMBER OF FAILURES IN PHASE 6
3	NUMBER OF FAILURES IN PHASE 7
3	NUMBER OF FAILURES IN PHASE 8
3	NUMBER OF FAILURES IN PHASE 9
3	NUMBER OF FAILURES IN PHASE 10
.85	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 1
.86	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 2
.90	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 3
.91	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 4
.93	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 5
.95	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 6
.97	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 7
.99	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 8
.99	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 9
.998	PROB. OF SUCCESS FROM CAUSE 1 IN PHASE 10
.84	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 1
.85	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 2
.87	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 3
.90	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 4
.92	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 5
.95	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 6
.97	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 7
.99	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 8
.99	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 9
.998	PROB. OF SUCCESS FROM CAUSE 2 IN PHASE 10
.83	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 1

.84	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 2
.86	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 3
.88	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 4
.90	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 5
.93	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 6
.96	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 7
.98	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 8
.99	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 9
.998	PROB. OF SUCCESS FROM CAUSE 3 IN PHASE 10
.83	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 1
.84	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 2
.85	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 3
.87	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 4
.89	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 5
.92	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 6
.94	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 7
.975	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 8
.99	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 9
.998	PROB. OF SUCCESS FROM CAUSE 4 IN PHASE 10
.81	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 1
.83	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 2
.84	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 3
.86	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 4
.89	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 5
.91	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 6
.94	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 7

.961	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 8
.99	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 9
.998	PROB. OF SUCCESS FROM CAUSE 5 IN PHASE 10
1	NUMBER OF TRIALS AFTER FAILURE BEFORE A DISCOUNT IS APPLIED
0.0	FRACTION EACH FAILURE IS DISCOUNTED
624712.0	RANDOM NUMBER SEED FOR GGUBFS UNIFORM (0,1) GENERATOR
.75	FRACTION RELIABILITY IMPROVES AFTER FAILING IN A PHASE
500	NUMBER OF DESIRED REPETITIONS FOR THE SIMULATION
0	INTERMEDIATE INPUT OPTION(1:INT. OUTPUT; 0: NO INT. OUTPUT)
0	SAVE ALL MLE W/ DISCOUNTING ESTIMATES (1: YES; 0: NO)
0	SAVE ALL MLE SINGLE PHASE ESTIMATES (1: YES; 0: NO)
0	SAVE ALL UNWT'D REGRESSION ESTIMATES (1: YES; 0: NO)
0	SAVE ALL METHOD 1 WT'D REG. ESTIMATES (1: YES; 0: NO)
0	SAVE ALL METHOD 2 WT'D REG. ESTIMATES (1: YES; 0: NO)
0	SAVE ALL METHOD 3 WT'D REG. ESTIMATES (1: YES; 0: NO)

1	DISCOUNTING OPTION (1: STRAIGHT % ; 2: LLOYD METHOD)
.9	PERCENT C.I. FOR LLOYD DISCOUNTING METHOD (MUST HAVE A VALUE)
1	LLOYD DISCOUNT INTERVAL
.03	WEIGHT FOR PHASE 1
.03	WEIGHT FOR PHASE 2
.03	WEIGHT FOR PHASE 1
.03	WEIGHT FOR PHASE 2
.03	WEIGHT FOR PHASE 1
.15	WEIGHT FOR PHASE 2
.15	WEIGHT FOR PHASE 1
.15	WEIGHT FOR PHASE 8
.2	WEIGHT FOR PHASE 9
.2	WEIGHT FOR PHASE 10

3. **Sample simulation run, using DISCONT EXEC A1, recorded session**

```
BEGIN RECORDING OF TERMINAL SESSION
Ready; T=0.01/0.04 08:49:01
DISCONT
THIS PROGRAM USES A DISCRETIZED CONTINUOUS MODEL
DO YOU NEED TO COMPILE YOUR PROGRAM ? (Y/N)
Y
VS FORTRAN COMPILER ENTERED.  08:49:17
+2S=$.33
+2S=$.33
```

```
**MAIN** END OF COMPILATION 1 ******

VS FORTRAN COMPILER EXITED.   08:49:26


Execution begins...

+2S=$.33

+2S=$.33

+2S=$.33

+2S=$.33

+2S=$.33

LLOYD ESTIMATION/EXPECTATION OUTPUT IS IN "LLOYD EST A1"


CONTROL OUTPUT IS IN THE FILE "GROWTH CONTROL A1"


YOUR ESTIMATION OUTPUT IS IN THE FILE GROWTH EST A1

DO YOU WISH TO BROWSE YOUR ESTIMATION OUTPUT? (Y/N)

Y

PRINT YOUR OUTPUT FILE? (Y/N)

Y

DO YOU WISH TO CHANGE THE INPUT DATA FILE (Y/N)

Y

DO YOU WISH TO RUN THE PROGRAM AGAIN? (Y/N)

Y

VS FORTRAN COMPILER ENTERED.  08:50:56

+2S=$.33

+2S=$.33
```

```
**MAIN** END OF COMPILATION 1 ******

VS FORTRAN COMPILER EXITED.   08:51:05



Execution begins...

+2S=$.33

+2S=$.33

+2S=$.33

+2S=$.33

+2S=$.33

LLOYD ESTIMATION/EXPECTATION OUTPUT IS IN "LLOYD EST A1"


CONTROL OUTPUT IS IN THE FILE "GROWTH CONTROL A1"


YOUR ESTIMATION OUTPUT IS IN THE FILE GROWTH EST A1

DO YOU WISH TO BROWSE YOUR ESTIMATION OUTPUT? (Y/N)

Y

PRINT YOUR OUTPUT FILE? (Y/N)

N

DO YOU WISH TO CHANGE THE INPUT DATA FILE (Y/N)

N

DO YOU WISH TO RUN THE PROGRAM AGAIN? (Y/N)

N

Ready; T=27.06/30.92 08:52:03

EXEC REC OFF
```

4. Sample output GROWTH EST A1

SIMULATION INPUT FILE : PT1C5F3

PHASE	ACTREL	LINREL	MSEL	RELEXP	MSEE	MLEWD	MSEWD	REGEST	MSEREG
1	.39842	.39805	.0397	.55853	.0375	.33786	.0517	.36841	.0553
2	.42811	.40747	.0404	.56384	.0304	.34822	.0547	.37926	.0569
3	.48079	.45315	.0418	.59035	.0250	.39846	.0564	.43754	.0432
4	.53924	.51145	.0423	.62577	.0217	.46259	.0561	.50047	.0374
5	.60995	.56290	.0437	.65869	.0177	.51918	.0585	.56433	.0296
6	.70268	.65305	.0298	.71604	.0122	.61836	.0402	.64388	.0213
7	.79812	.75894	.0221	.79329	.0103	.73483	.0289	.73642	.0147
8	.89996	.88128	.0071	.89087	.0045	.86940	.0091	.84127	.0069
9	.95099	.93208	.0043	.93596	.0027	.92529	.0054	.90832	.0030
10	.99004	.98662	.0002	.98678	.0001	.98528	.0002	.96385	.0009

5. **Main variables used for the discrete modification of the continuous AMSAA model (AMSAA-D model)**

TRTOT	number of trials in a phase
NFAPH(K)	number of failures in phase k
NFAPHK	number of failures in a phase
FRATE	failure rate estimate of the AMSAA-D model
FRHAT(K)	failure rate estimate for phase k using the AMSAA-D model
FSHAT(K)	minimum variance unbiased failure rate estimate for phase k
XAHAT,XBHAT	regression estimates $\hat{a}$, $\hat{b}$ in one phase and repetition
Z**	auxiliary variables
AREL(K)	actual reliability in phase k
RHAT(K)	failure rate estimate at phase k
XI	ln of the total number of trials up to phase k
YI	ln of the failure rate estimate at phase k

LSSUM(4,K)	summands of the least square estimate $\hat{a}$ in phase k
ANUMER	numerator to compute $\hat{a}$
ADENOM	denominator to compute $\hat{a}$
ESTRN(K)	failure rate estimate in phase k using the linear model
ESTRNE(K)	failure rate estimate in phase k using the exponential power model
RNHAT(K)	summation of the reliability estimate using the linear model
RNHATE(K)	summation of the reliability estimate using the exponential power model.
AVFR(K)	average failure rate estimate for phase k
AVRN(K)	average reliability estimate for phase k, using the linear model
MSE(K)	Mean Square Error of the linear model reliability estimation
AVRNE(K)	average reliability estimate for phase k, using the exponential power model
MSEE(K)	Mean Square Error of the exponential power model reliability estimation
MLEWD(3,K)	average reliability using the maximum likelihood estimation with discounting
MSEWD(K)	Mean Square Error of the MLEwd

REGEST(3,K) average reliability using the exponential regression estimate

MSEREG(K) Mean Square Error of the exp.Regr.Estimate

MSEERR(I,K) average Mean Square Error over all phases k for the i methods.

i = 1 : AMSAA-D, linear model

i = 2 : AMSAA-D, exponential power model

i = 3 : MLEwd

i = 4 : exp.Regr.Estimate

AVGMSE(I) average Mean Square Error over all phases for method i

APPENDIX C

FAILURE RATE FOR THE GEOMETRIC AND PASCAL DISTRIBUTION

The failure rate for a discrete probability distribution is defined as

$$f_n = \frac{P[N=n]}{P[N\geq n]} \quad ,n = 1,2,3... \qquad (\text{C.1})$$

In this treatment the random variable N is the number of trials up to and including the r^{th} failure; that is, N has a Pascal distribution.

1. **Case 1 : The number of failures equals 1.**

In this case the number of Bernoulli trials, N, required to obtain the first failure, has a geometric distribution with parameter p. Thus :

$$P[N = n] = p^{n-1}(1-p) \quad , \quad n = 1,2,... \qquad (\text{C.2})$$

It follows that

$$P[N > n] = p^n$$

Therefore from equation (C.1),the failure rate function f_n is

$$f = \frac{p^{n-1}(1-p)}{p^{n-1}}$$

or

$$f = 1 - p . \qquad (C.3)$$

The parameter p of the geometric distribution represents the reliability of the item under test and therefore it follows that

$$R_{tk} = 1 - f_{tk} \qquad (C.4)$$

This relation suggests the use of equation (II.6) to convert the estimated failure rate into a reliability value; i.e.

$$\hat{R}_{tk} = 1 - \hat{f}_{tk} \qquad (C.5)$$

Even when the number of failures in a phase is $r > 1$, we are still interested in modeling the growth of the probability of success on a single trial. Consequently equation C.5 provides the correct expression for an estimate of reliability in terms of the failure rate estimate.

2. **Case 2 : The number of failures equals r.**

In this case N has a Pascal distribution with parameters p and r.

$$P[N = n] = \binom{n-1}{r-1} p^{n-r} (1 - p)^r$$

and

$$P[N \geq n] = P[N > n-1] = P[Y \leq r-1]$$

where Y is binomial with parameters 1-p and n-1. Therefore

$$P[N \geq n] = \sum_{j=0}^{r-1} \binom{n-1}{j} p^{n-1-j} (1 - p)^j$$

and

$$f_n = \frac{\binom{n-1}{r-1} p^{n-r} (1-p)^r}{\sum_{j=0}^{r-1} \binom{n-1}{j} p^{n-1-j} (1-p)^j} \qquad (C.6)$$

It is important to realize that even though C.6 is the failure rate function when testing until r failures occur, it is not this failure rate that we are modeling in the AMSAA-D model. We are modeling the growth of p only; i.e. the growth of the probability of success (reliability) on a single trial.

APPENDIX D

FIGURES 16 TO 45

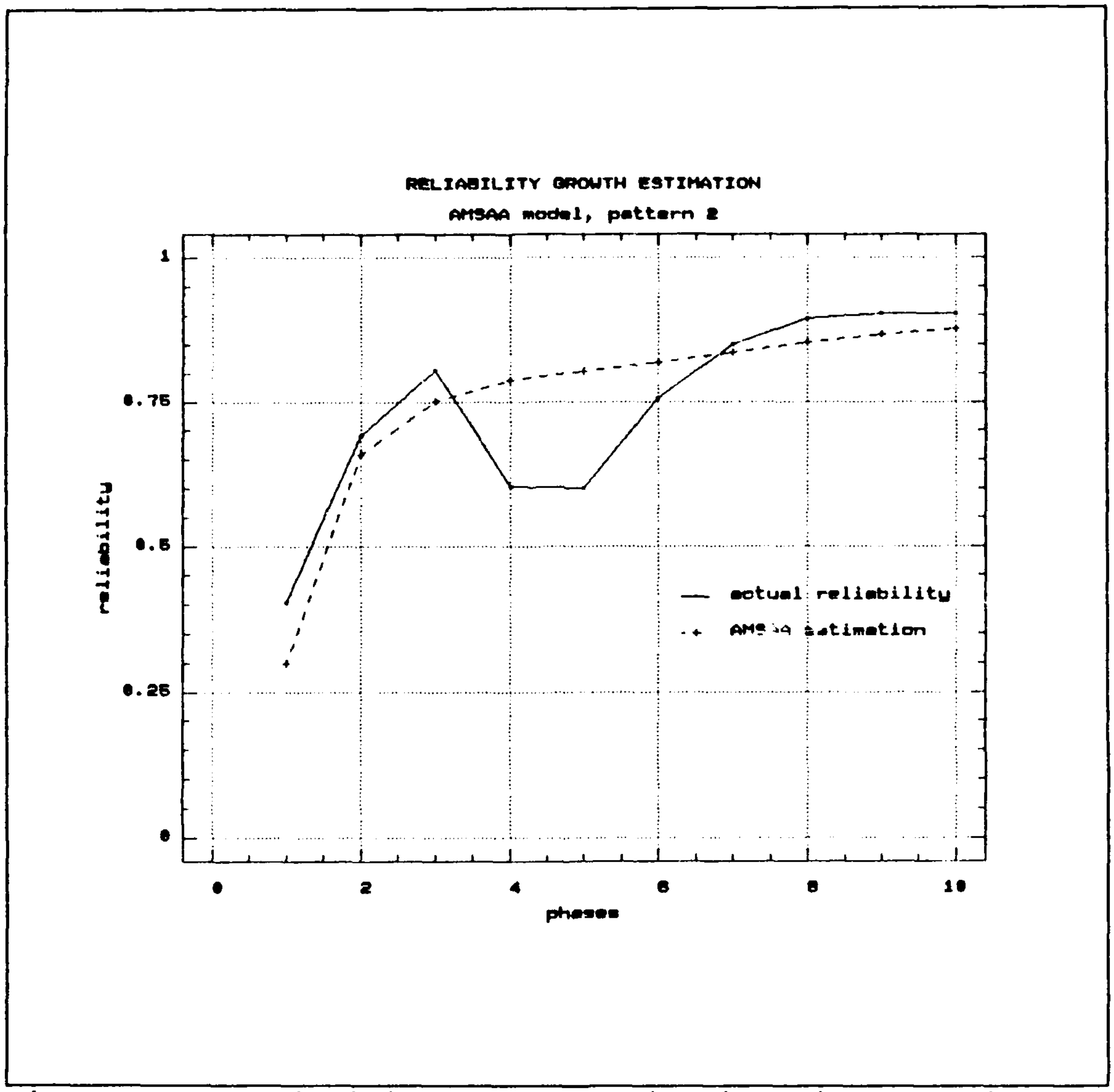


Figure 16 : Reliability growth estimation with the discrete AMSAA model, pattern 2

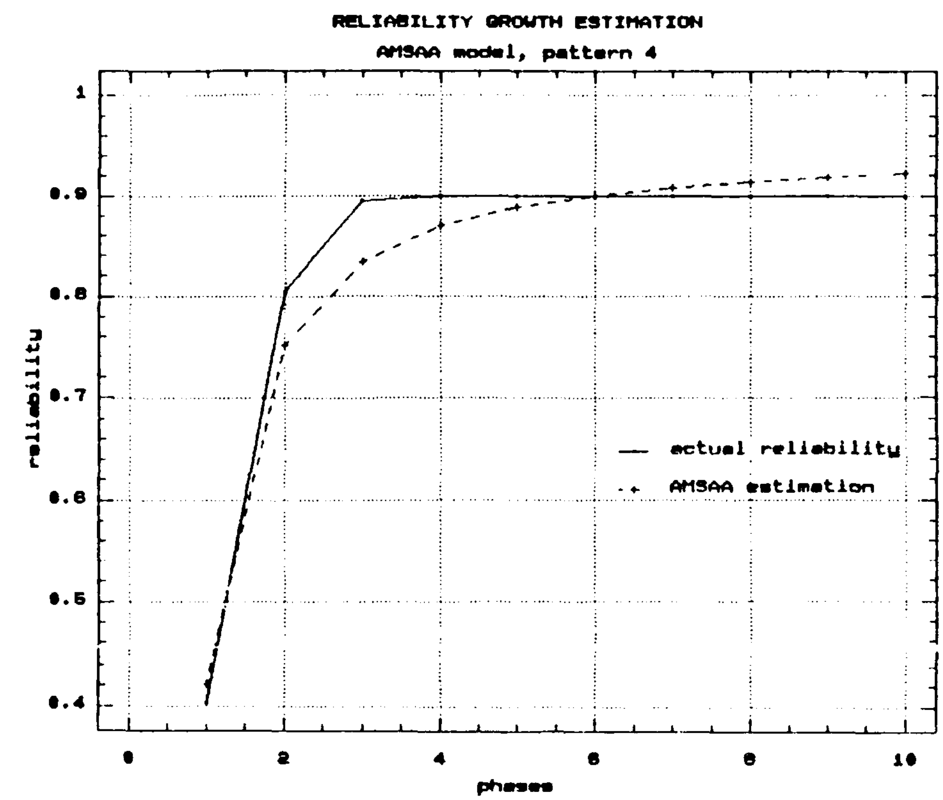


Figure 17 : Reliability growth estimation with the discrete AMSAA model, pattern 4

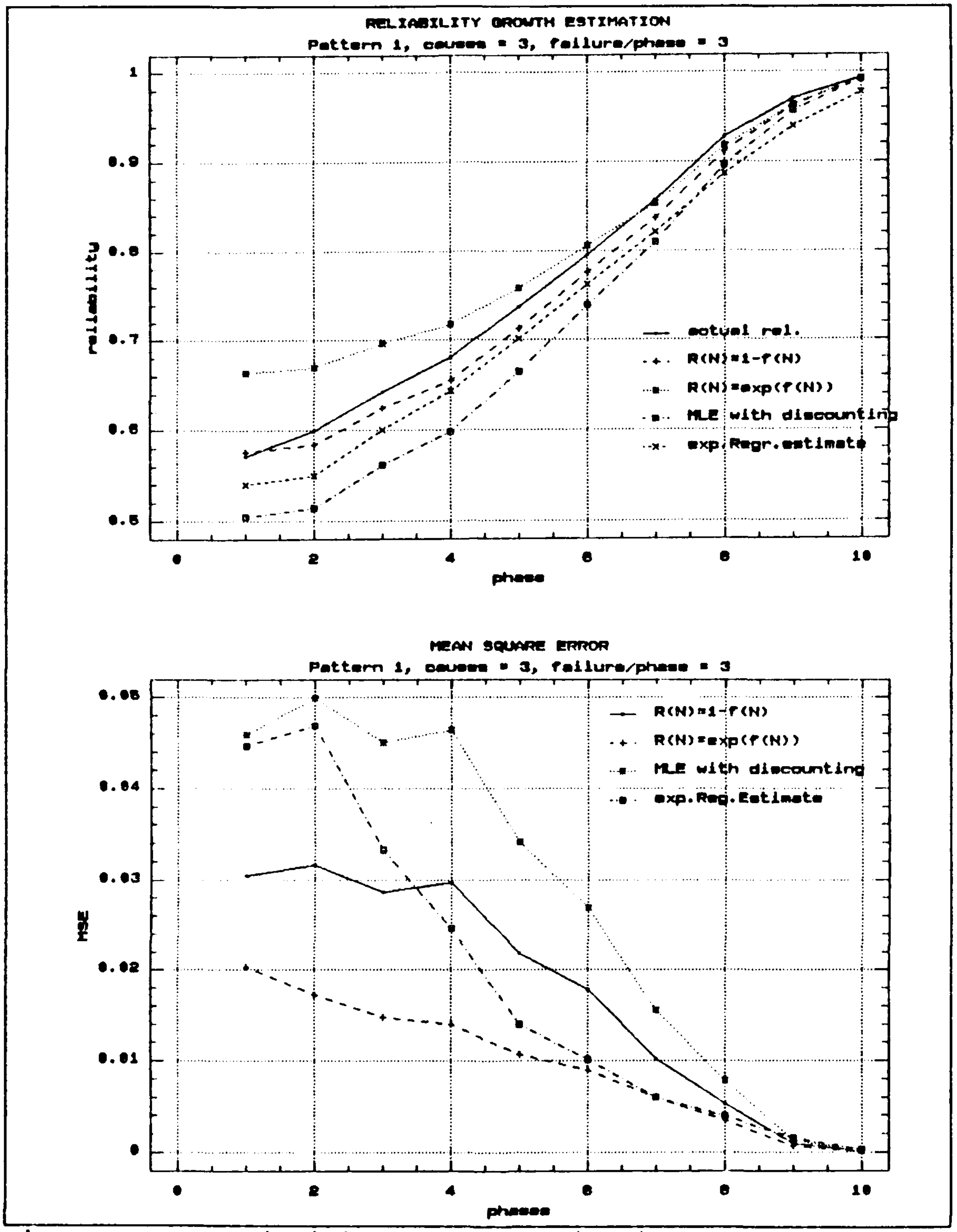


Figure 18 : Reliability growth estimation and Mean Square Error, pattern 1, 3 failure causes, 3 failures/phase

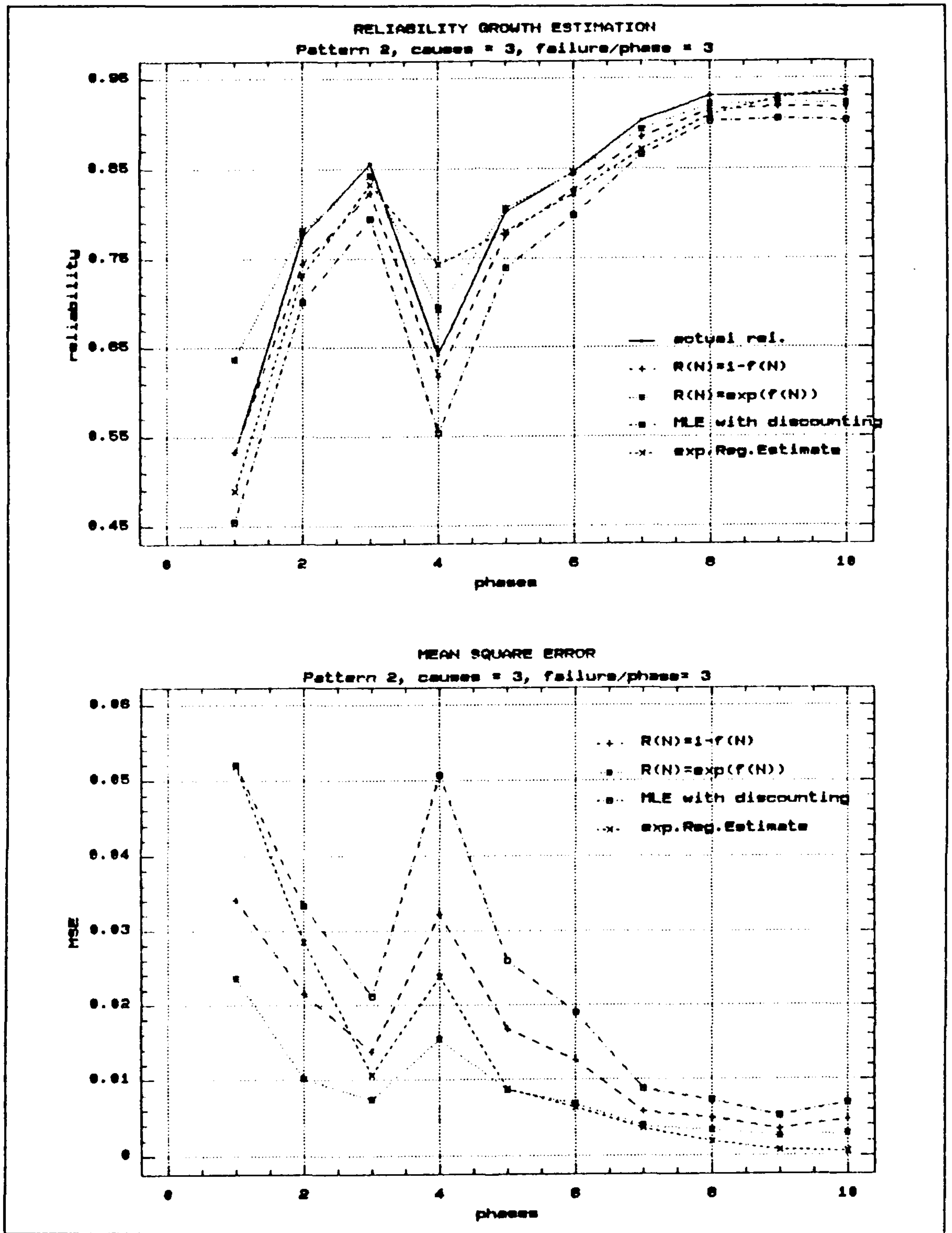


Figure 19 : Reliability growth estimation and Mean Square Error, pattern 2, 3 failure causes, 3 failures/phase

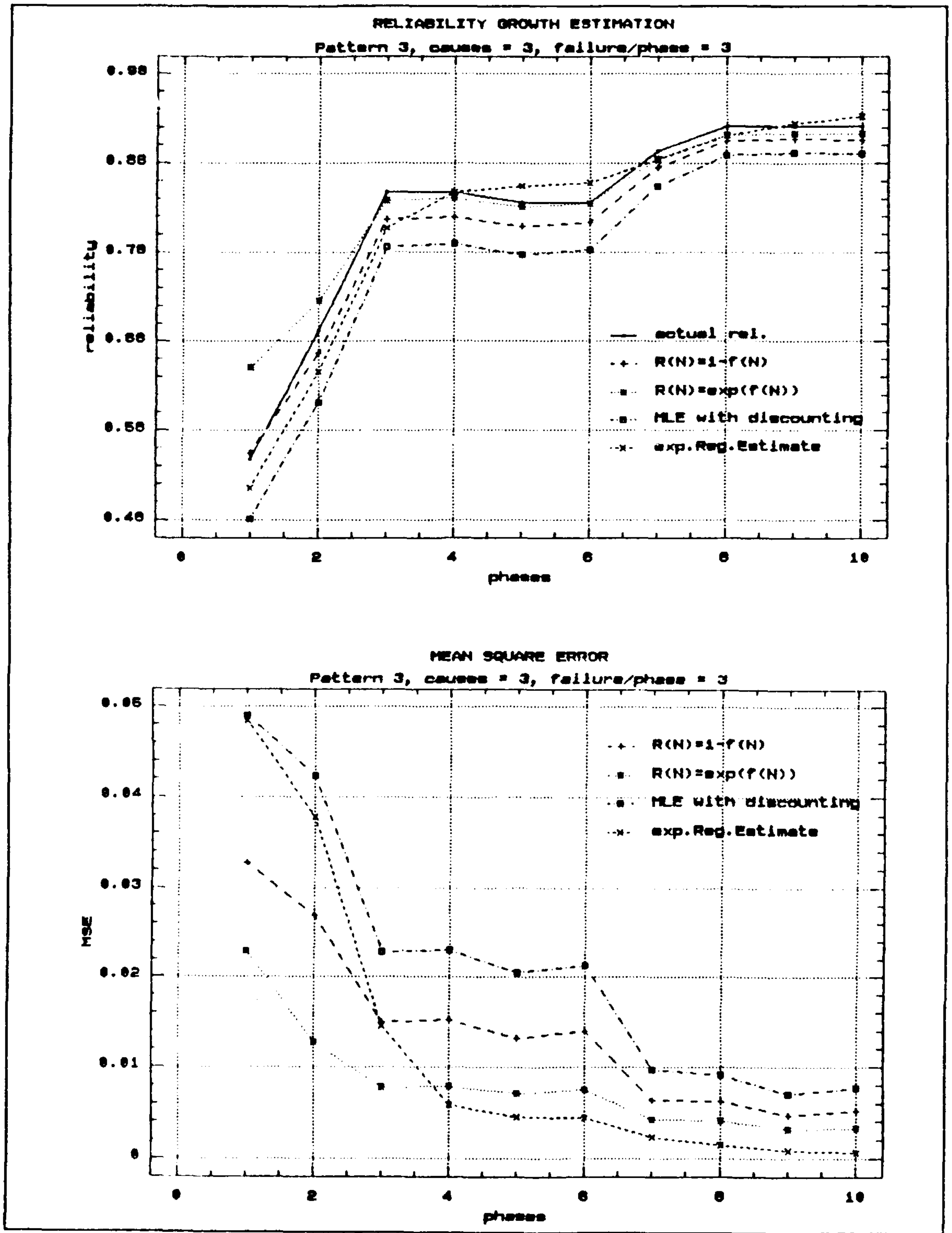


Figure 20 : Reliability growth estimation and Mean Square Error, pattern 3, 3 failure causes, 3 failures/phase

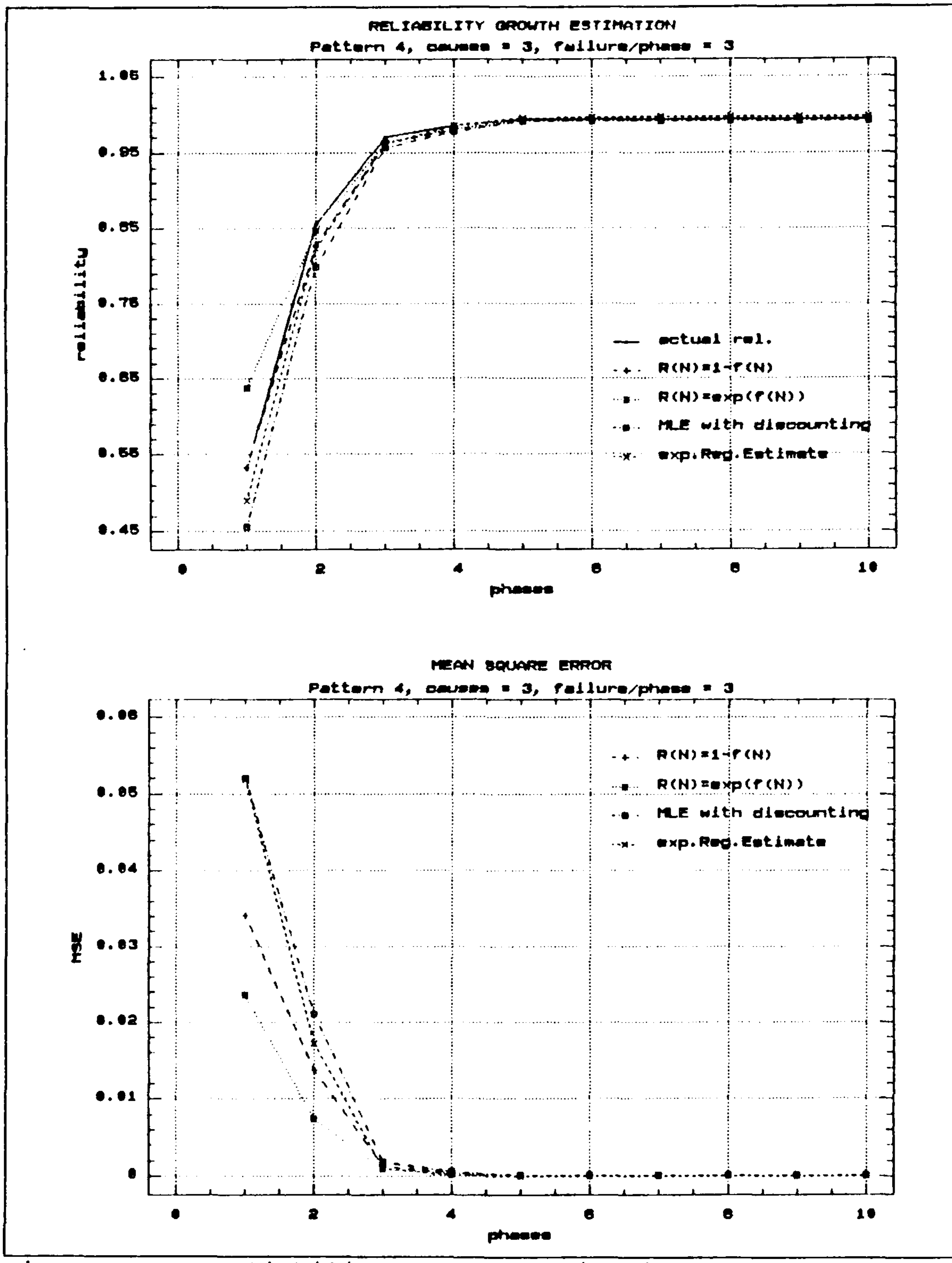


Figure 21 : Reliability growth estimation and Mean Square Error, pattern 4, 3 failure causes, 3 failures/phase

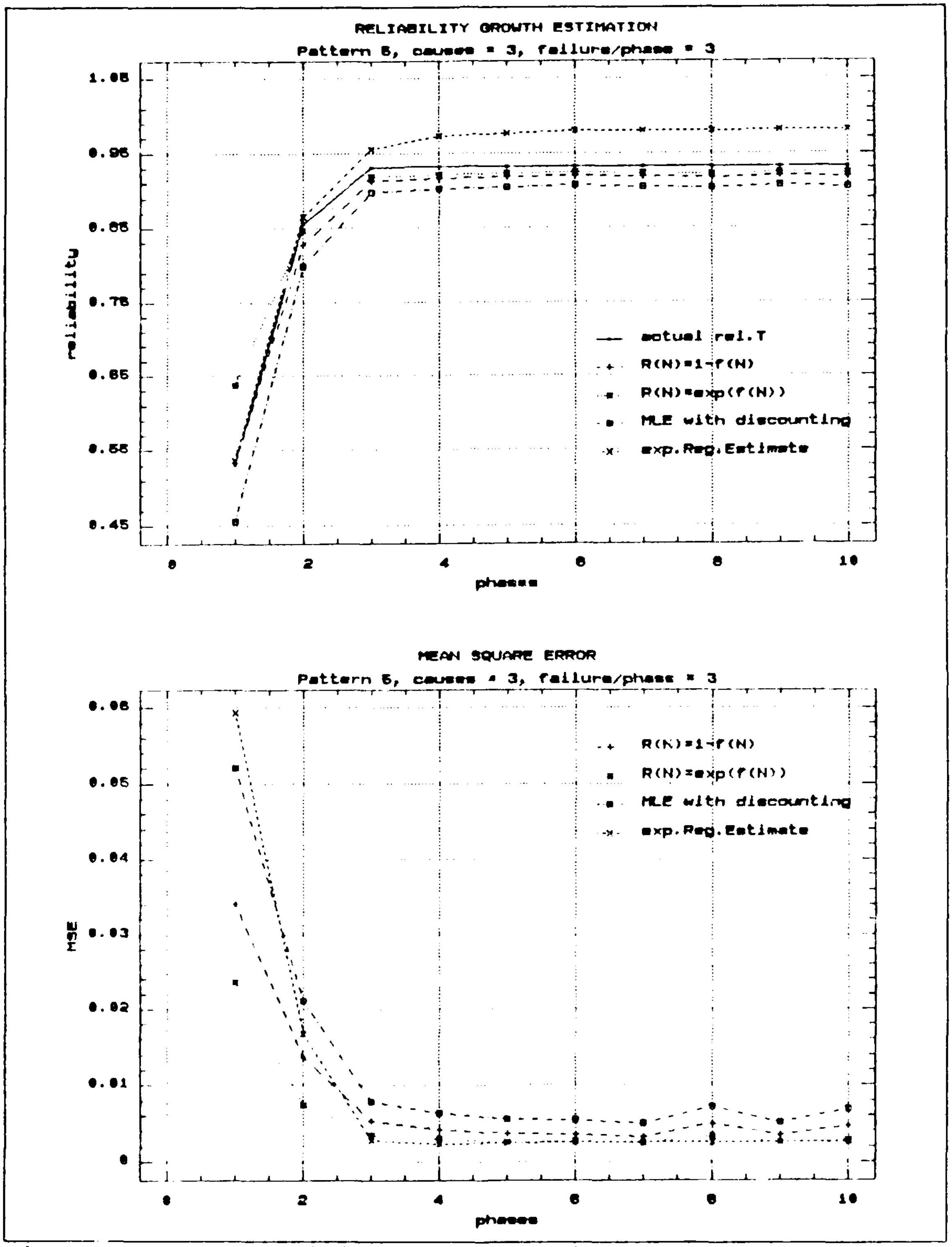


Figure 22 : Reliability growth estimation and Mean Square Error, pattern 5, 3 failure causes, 3 failures/phase

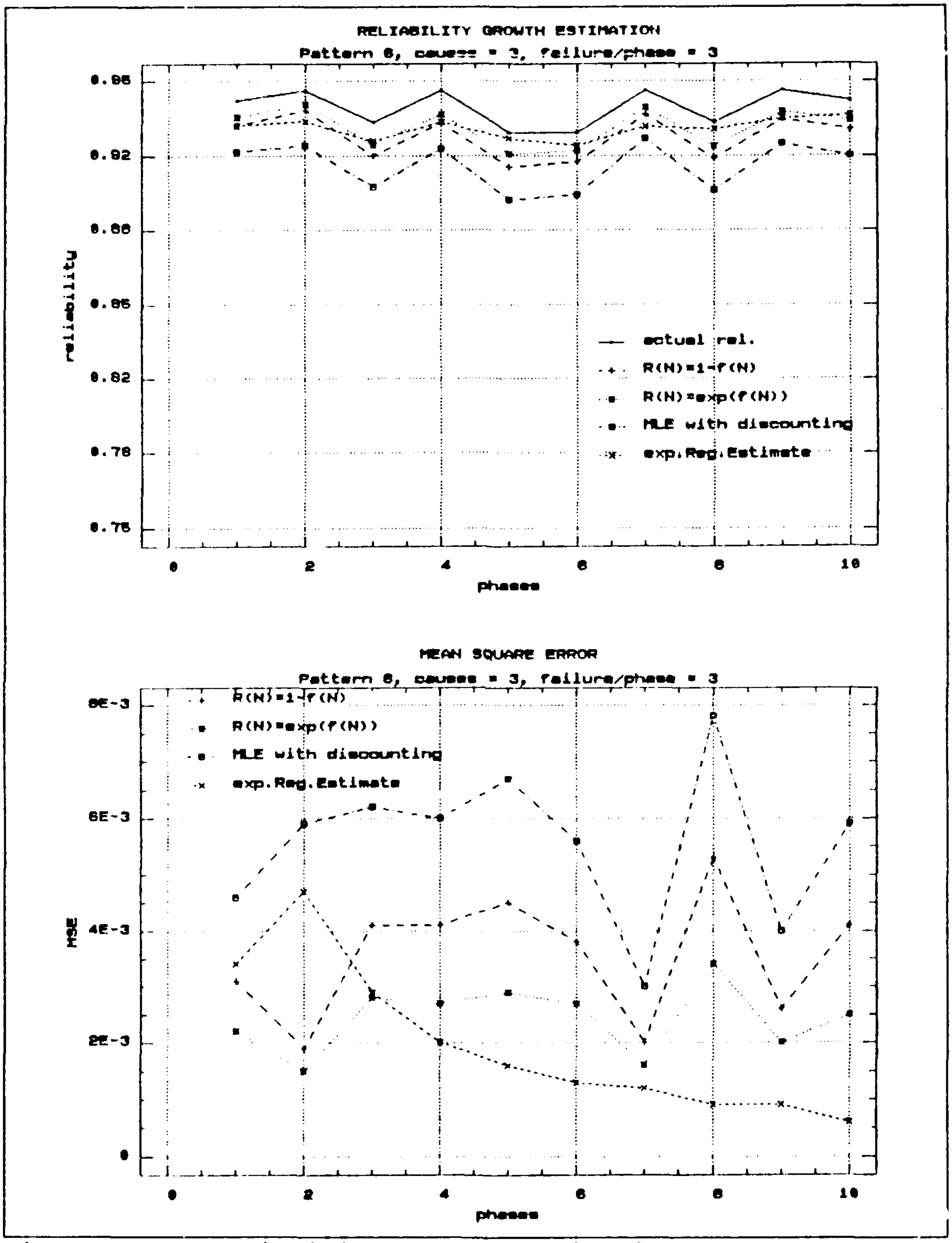


Figure 23 : Reliability growth estimation and Mean Square Error, pattern 6, 3 failure causes, 3 failures/phase

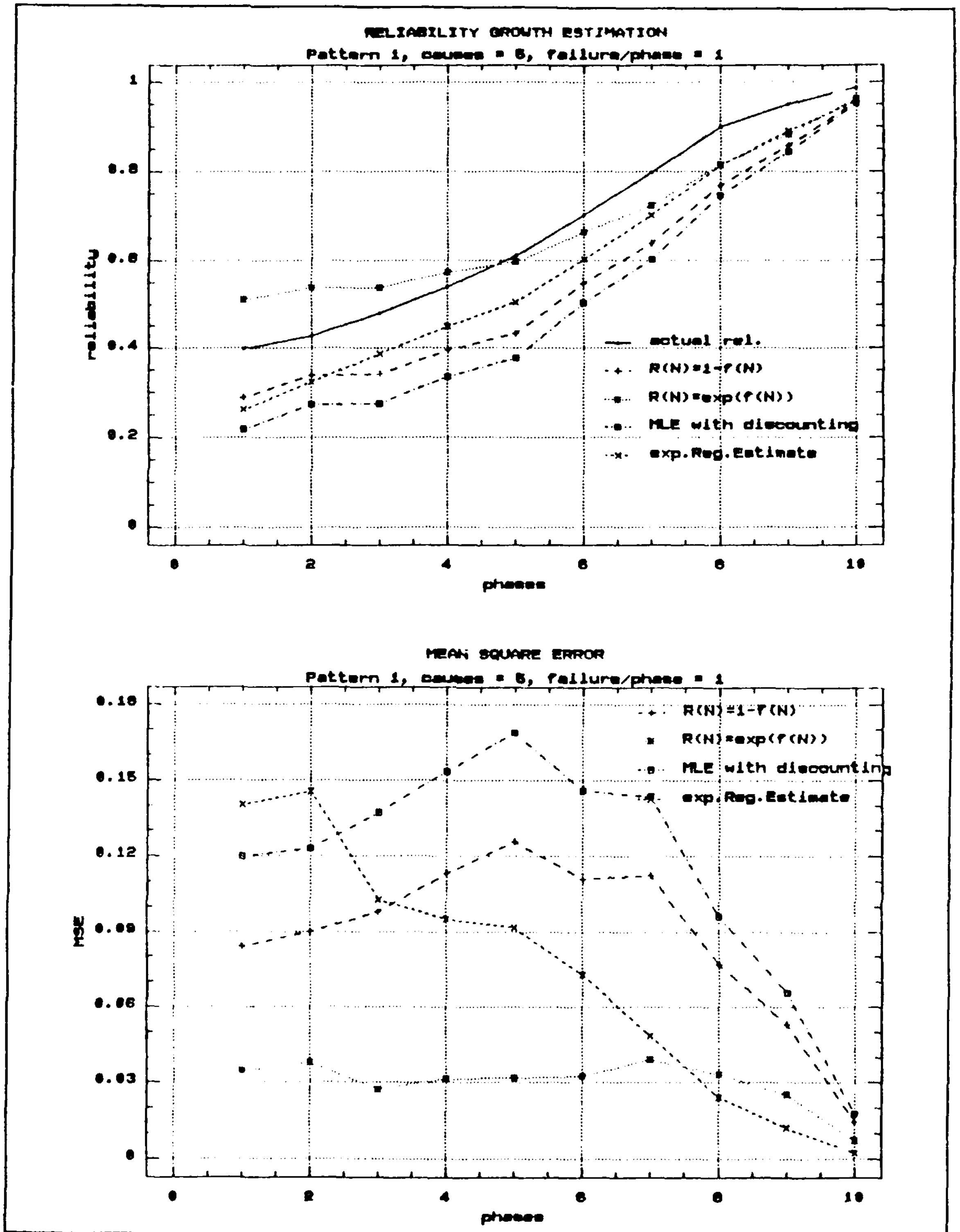


Figure 24 : Reliability growth estimation and Mean Square Error, pattern 1, 5 failure causes, 1 failure/phase

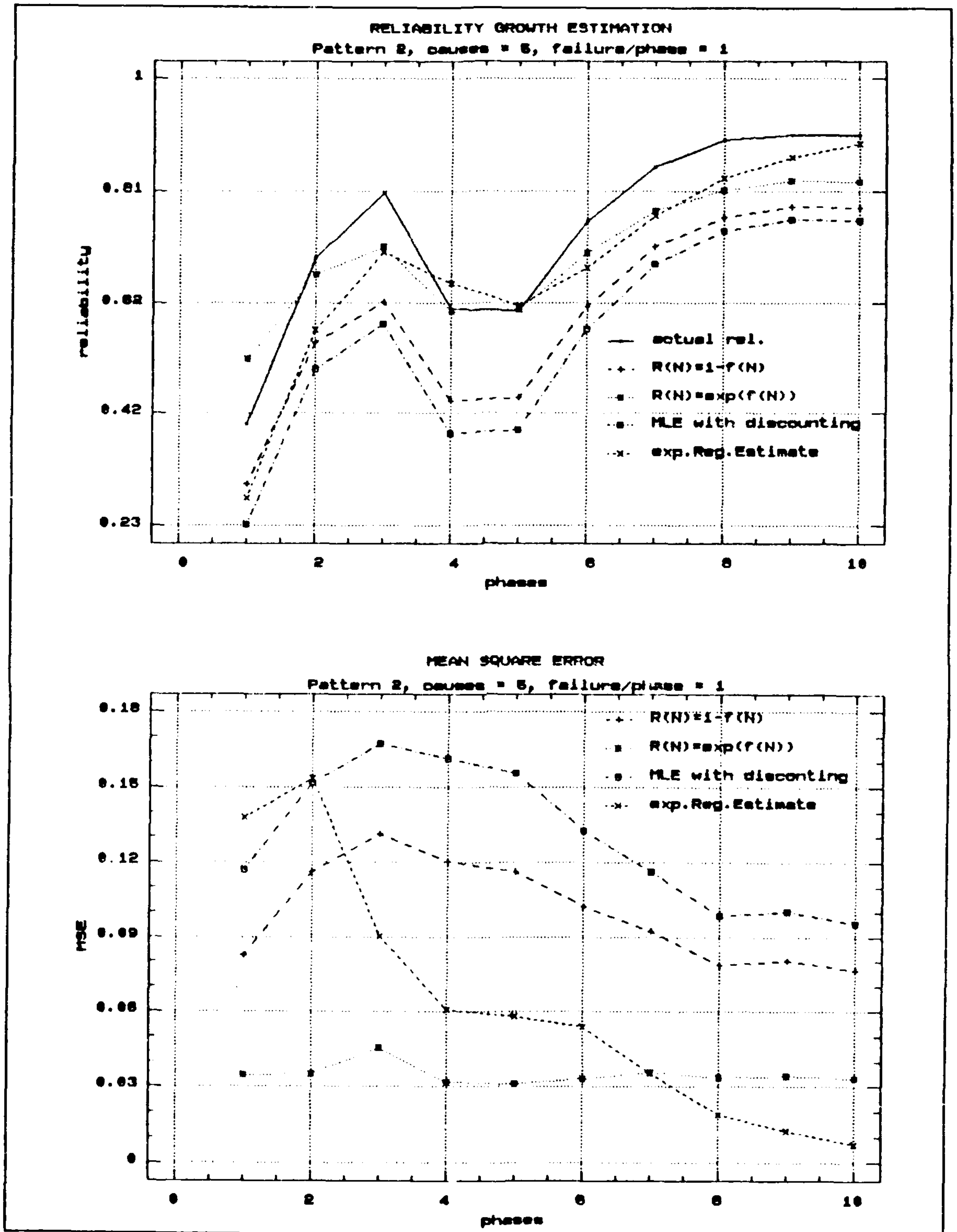


Figure 25 : Reliability growth estimation and Mean Square Error, pattern 2, 5 failure causes, 1 failure/phase

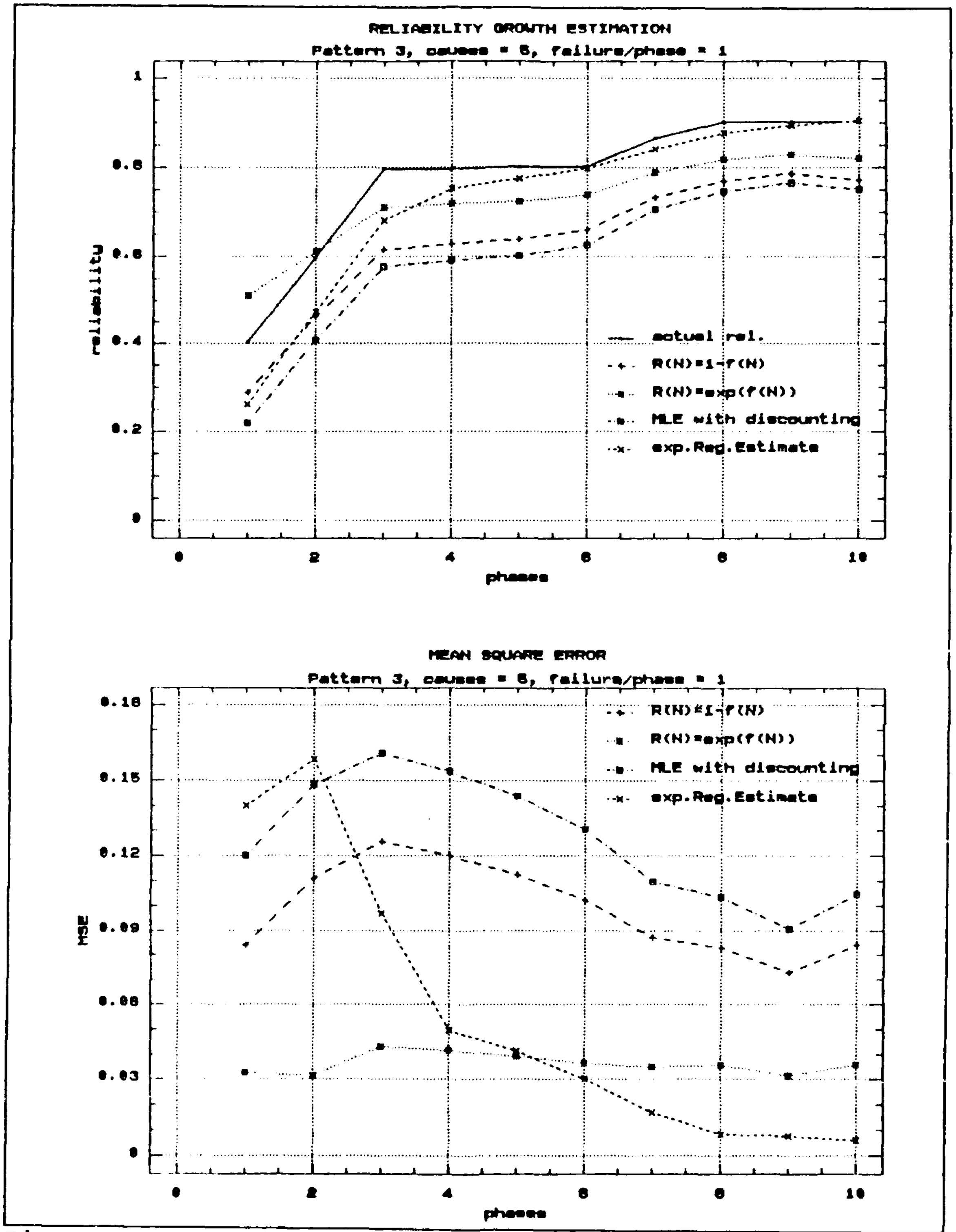


Figure 26 : Reliability growth estimation and Mean Square Error, pattern 3, 5 failure causes, 1 failure/phase

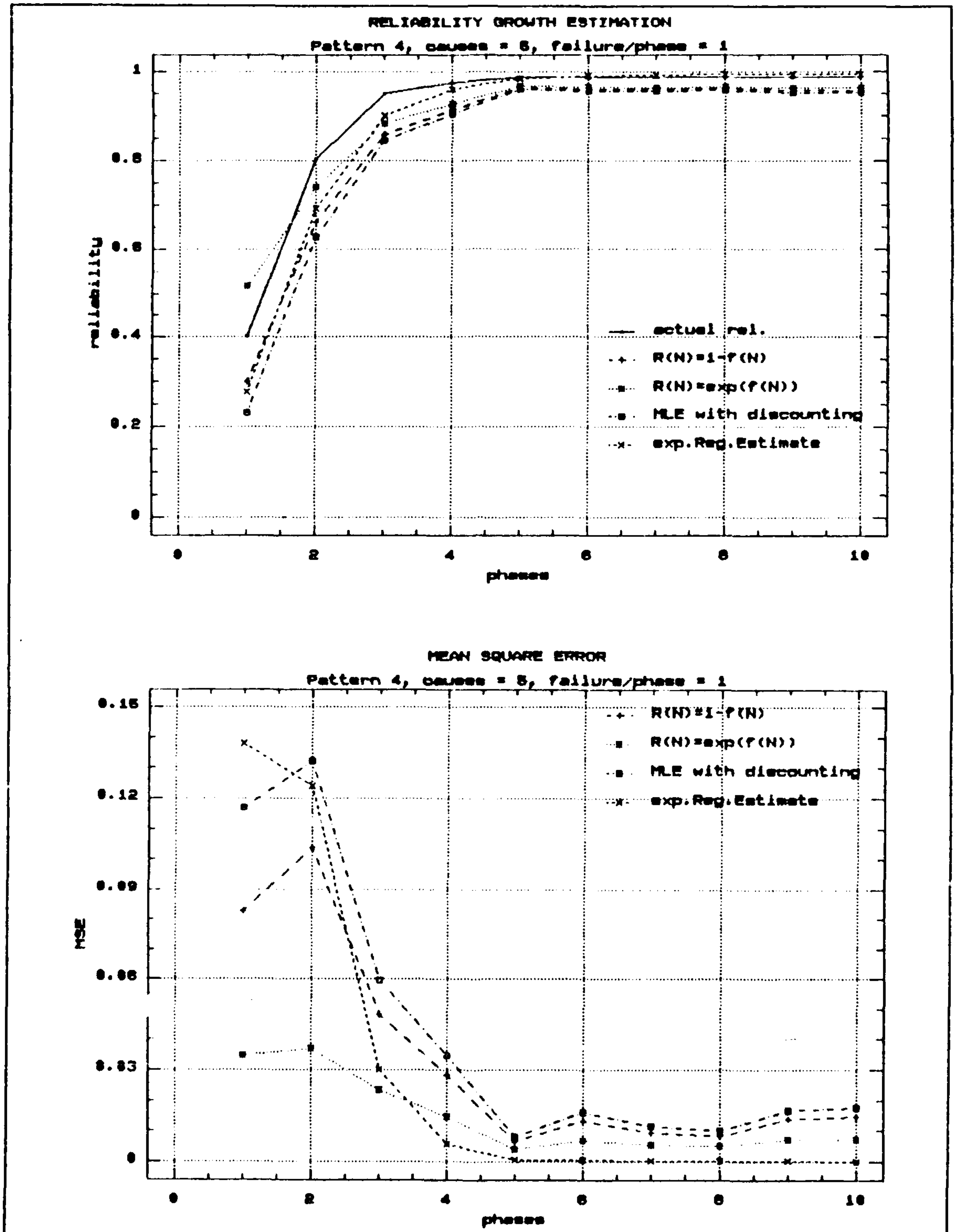


Figure 27 : Reliability growth estimation and Mean Square Error, pattern 4, 5 failure causes, 1 failure/phase

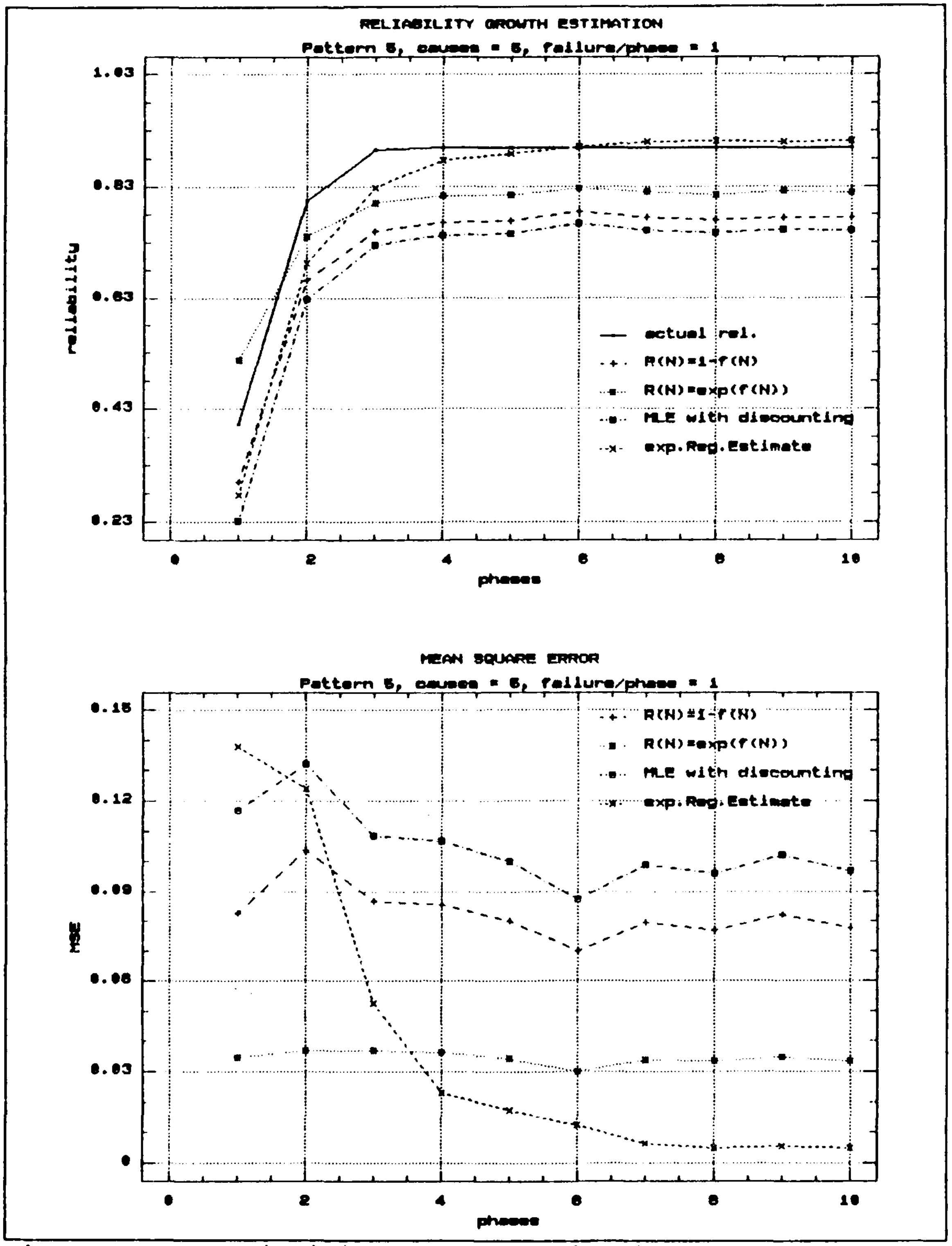


Figure 28 : Reliability growth estimation and Mean Square Error, pattern 5, 5 failure causes, 1 failure/phase

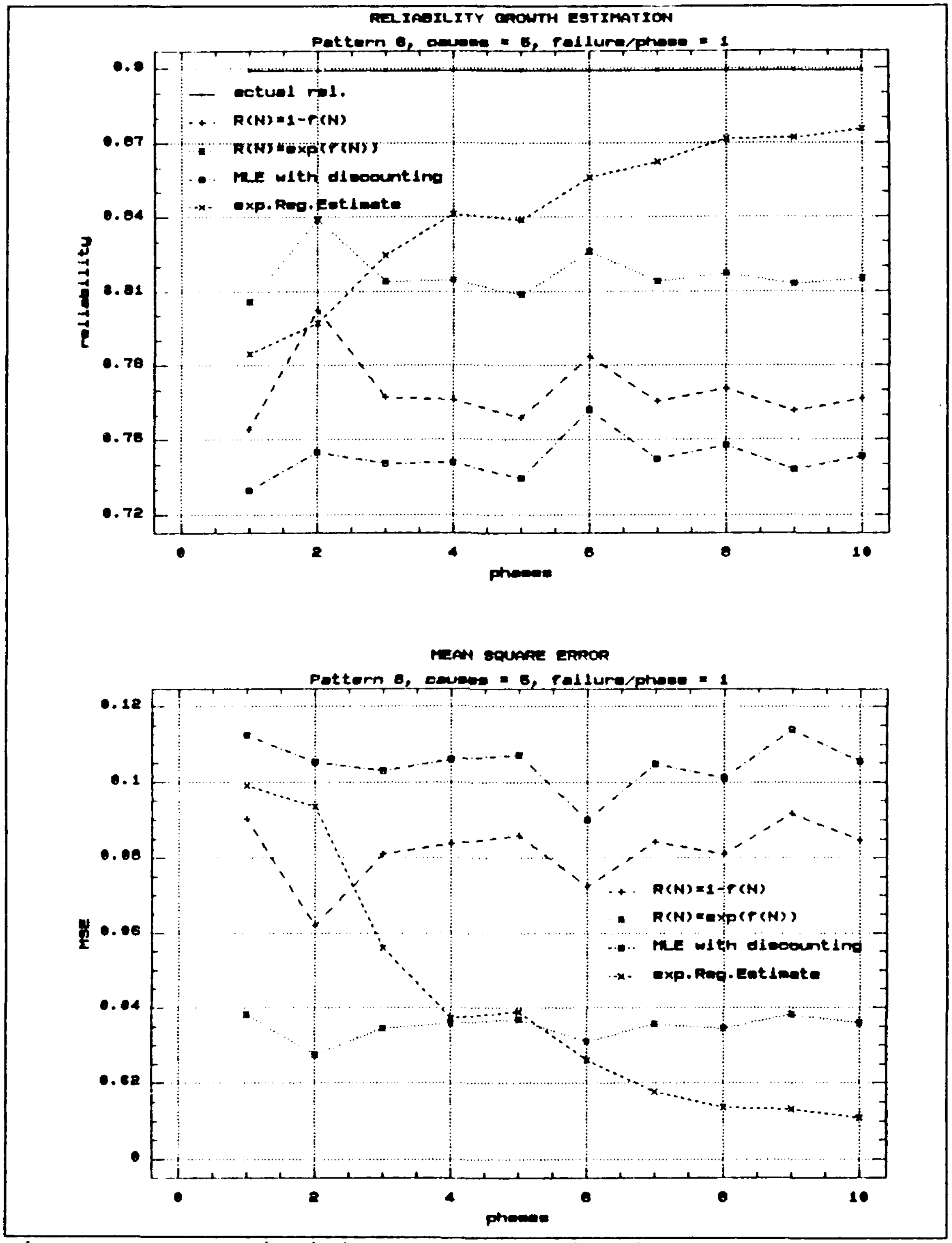


Figure 29 : Reliability growth estimation and Mean Square Error, pattern 6, 5 failure causes, 1 failure/phase

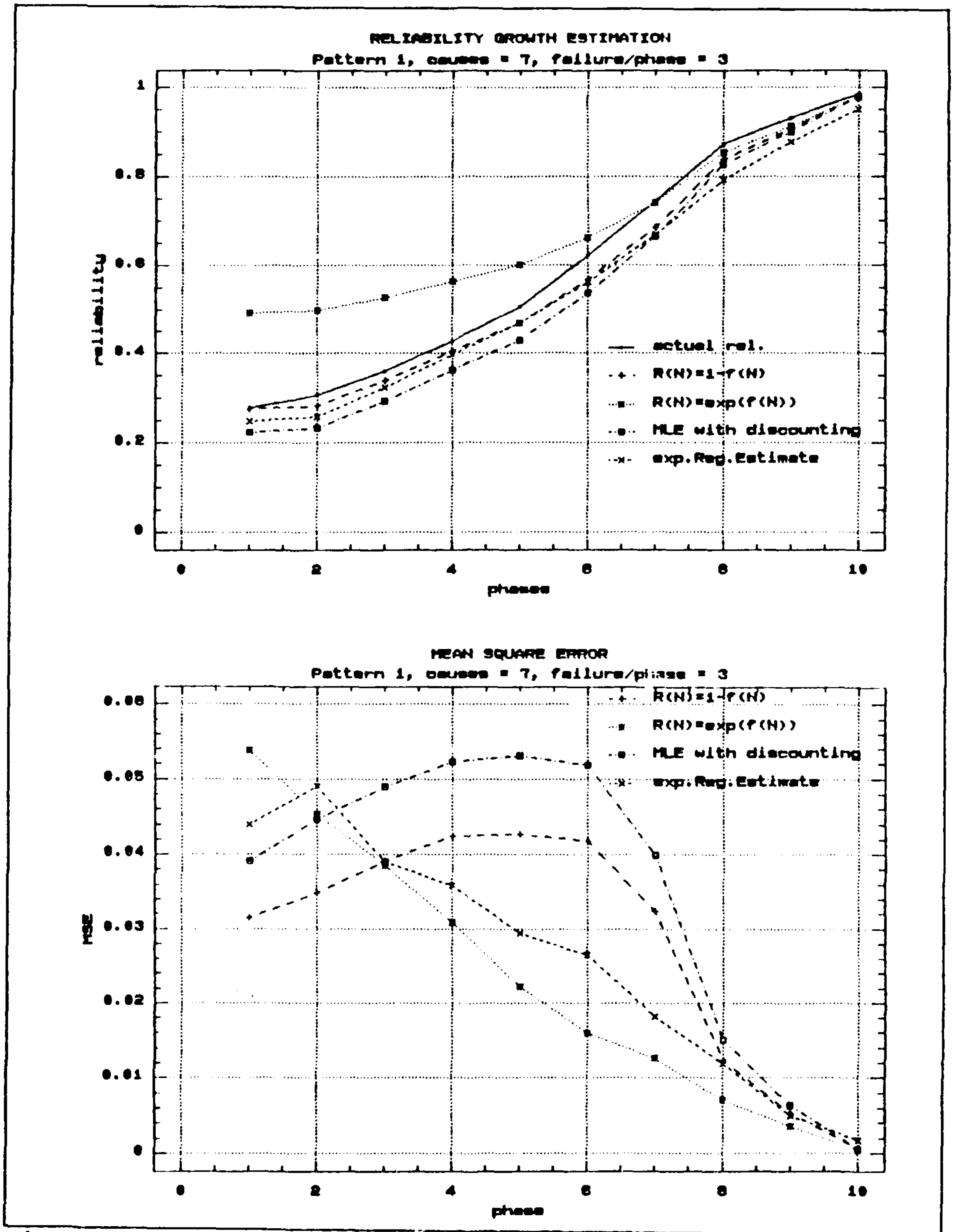


Figure 30 : Reliability growth estimation and Mean Square Error, pattern 1, 7 failure causes, 3 failures/phase

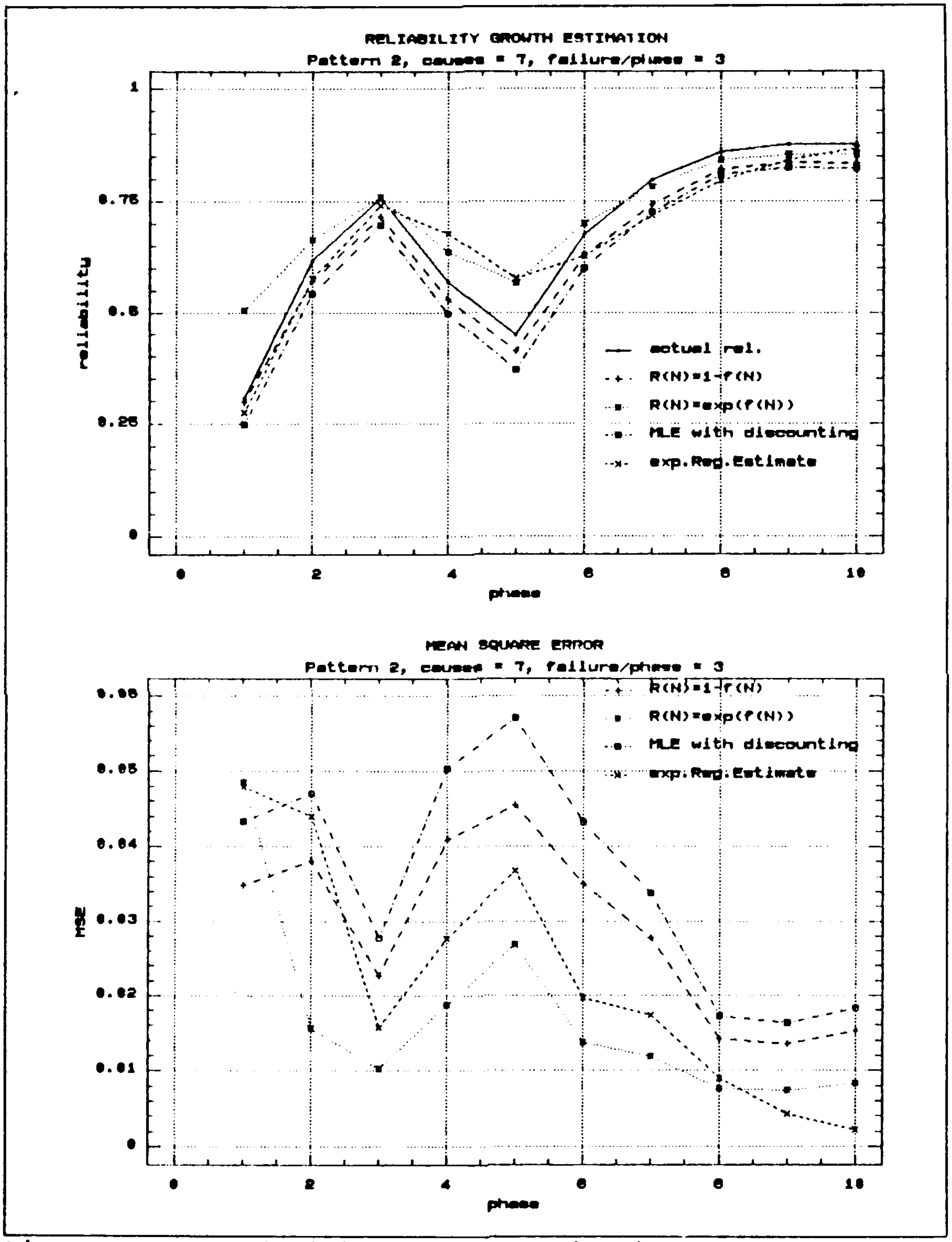


Figure 31 : Reliability growth estimation and Mean Square Error, pattern 2, 7 failure causes, 3 failures/phase

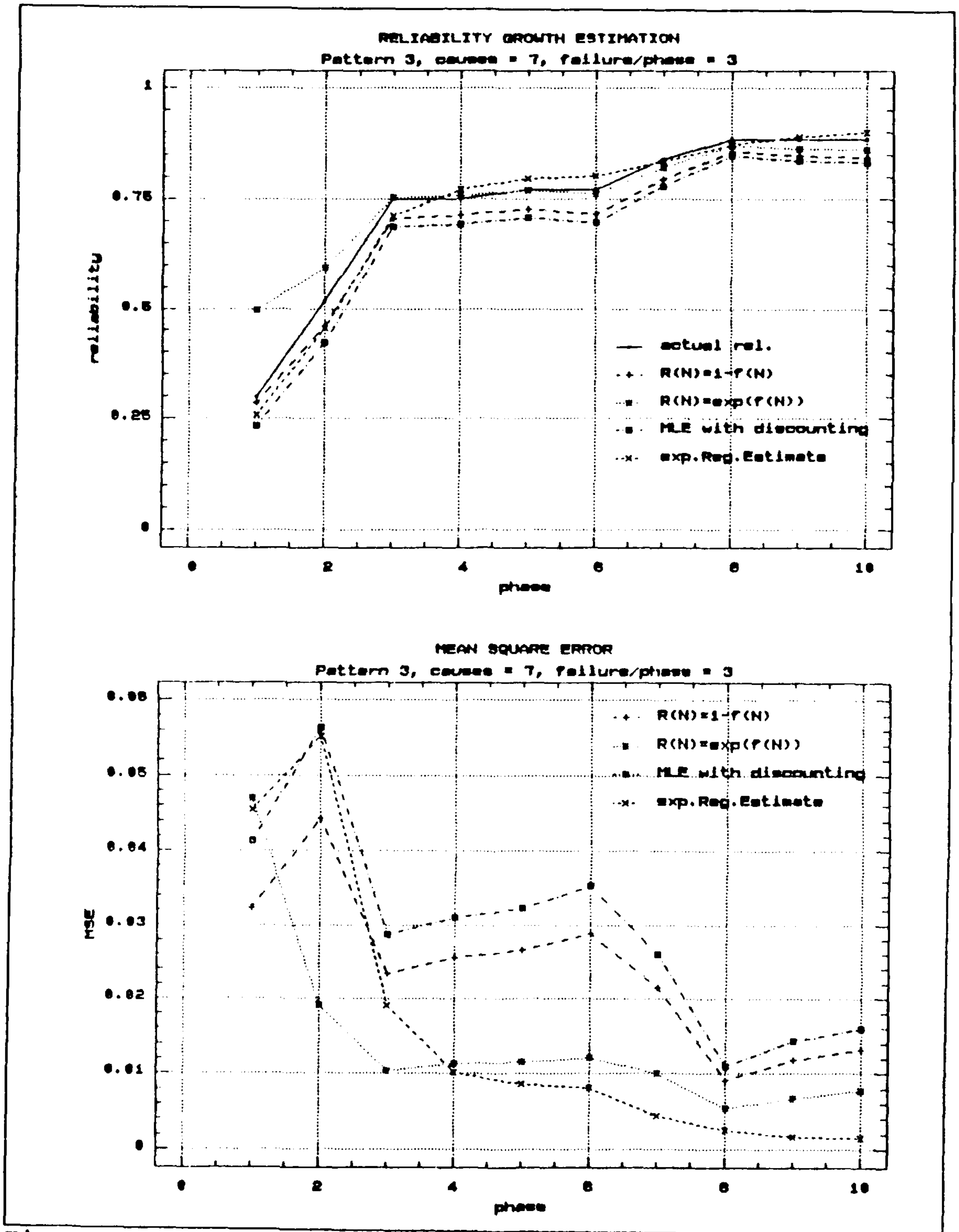


Figure 32 : Reliability growth estimation and Mean Square Error, pattern 3, 7 failure causes, 3 failures/phase

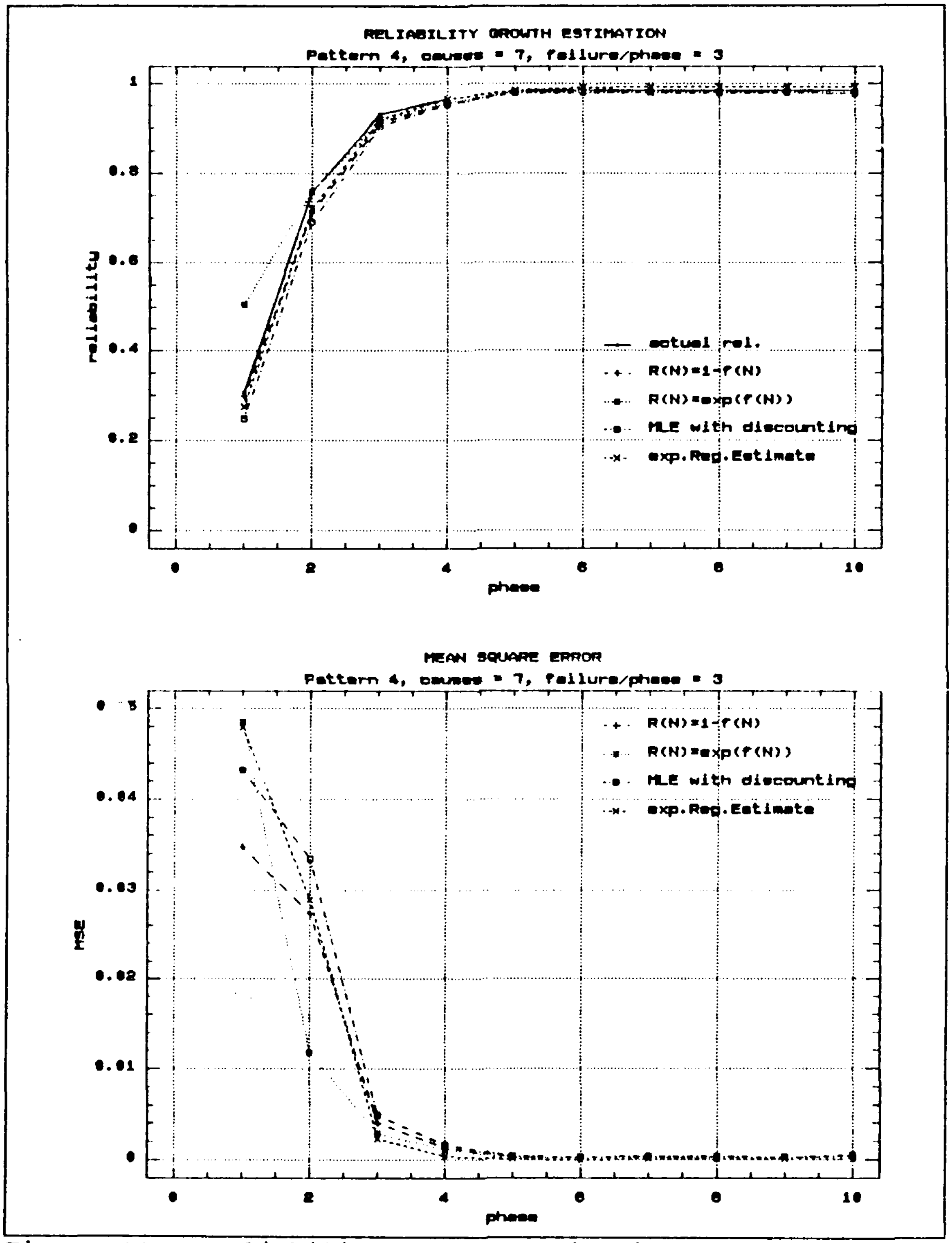


Figure 33 : Reliability growth estimation and Mean Square Error, pattern 4, 7 failure causes, 3 failures/phase

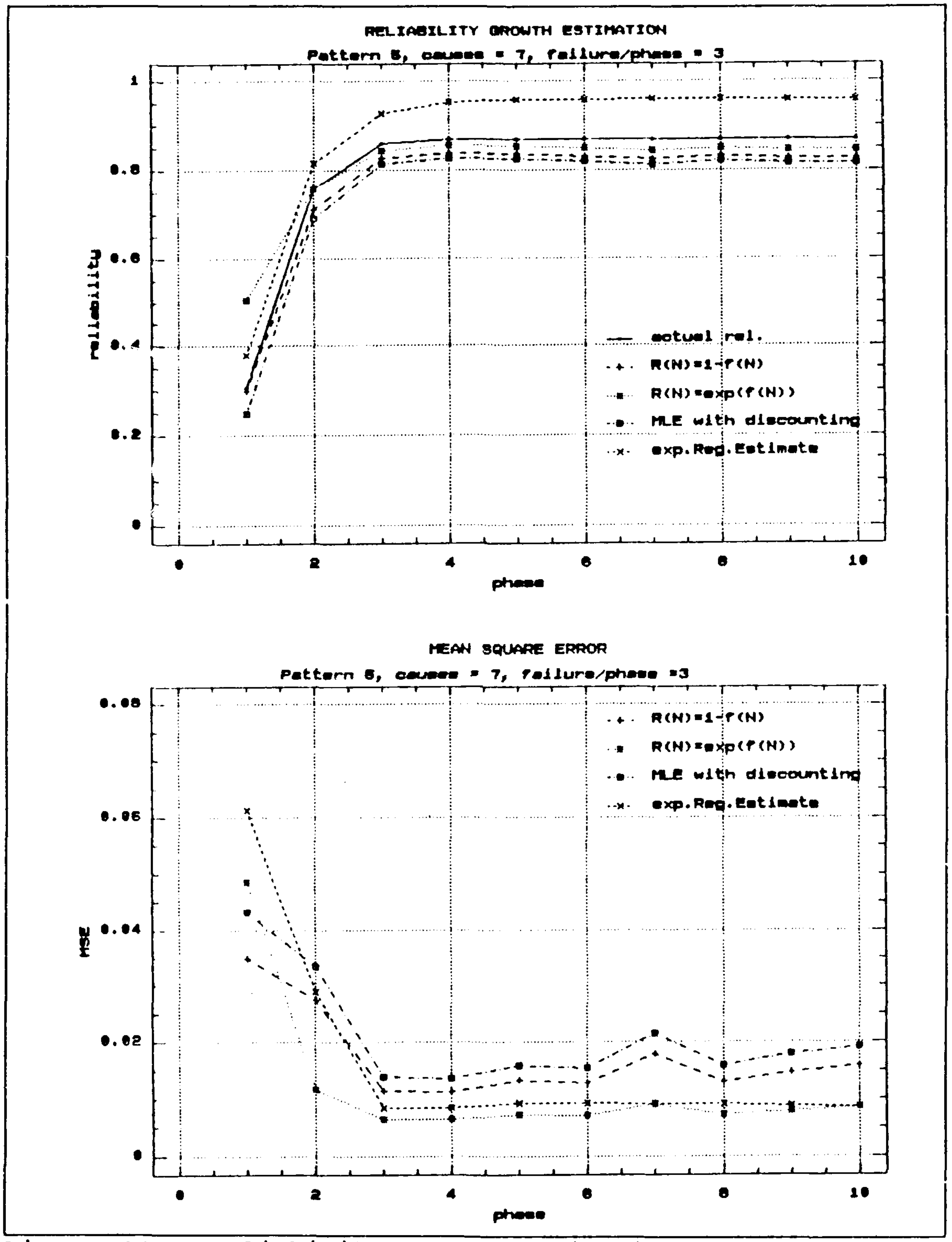


Figure 34 : Reliability growth estimation and Mean Square Error, pattern 5, 7 failure causes, 3 failures/phase

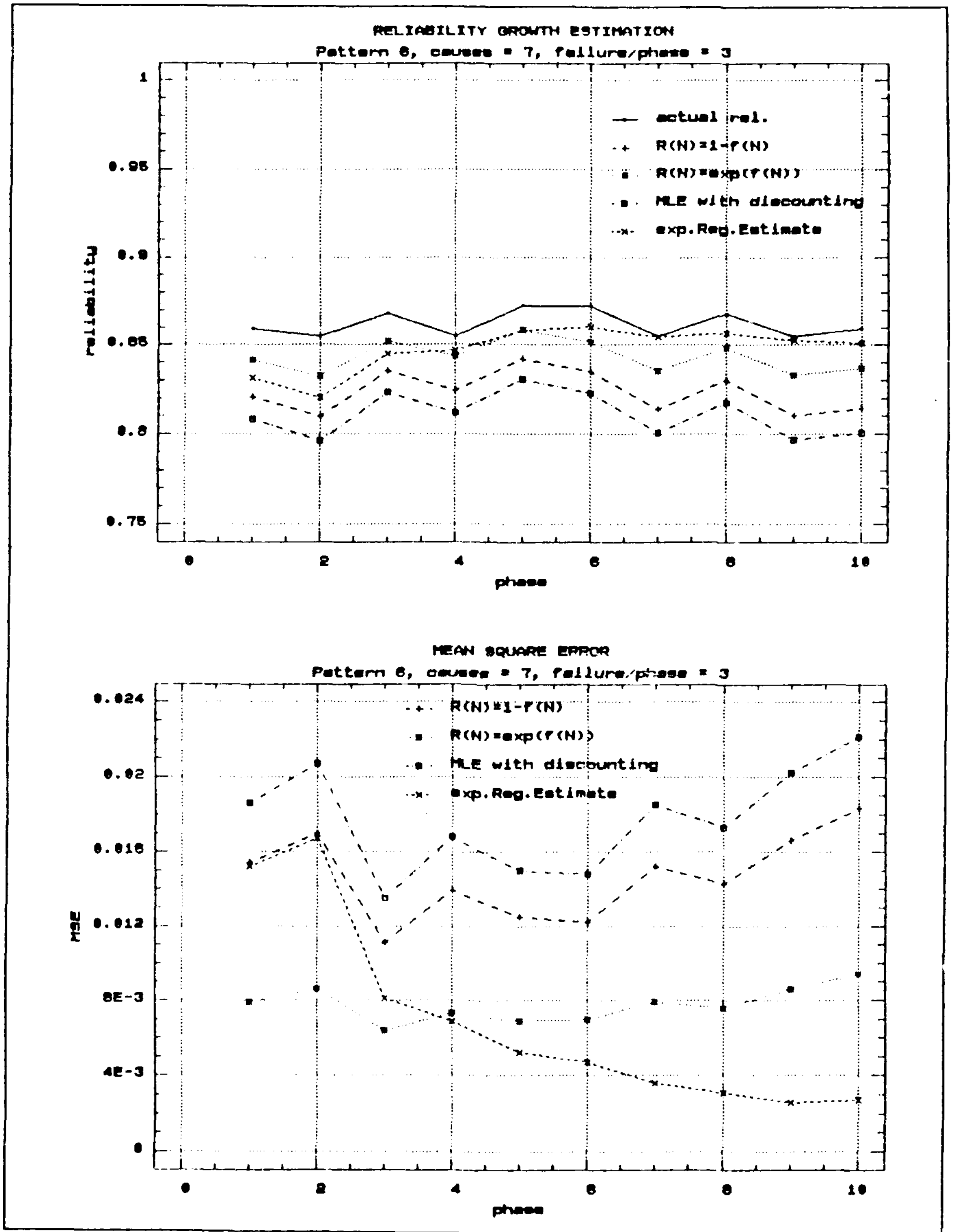


Figure 35 : Reliability growth estimation and Mean Square Error, pattern 6, 7 failure causes, 3 failures/phase

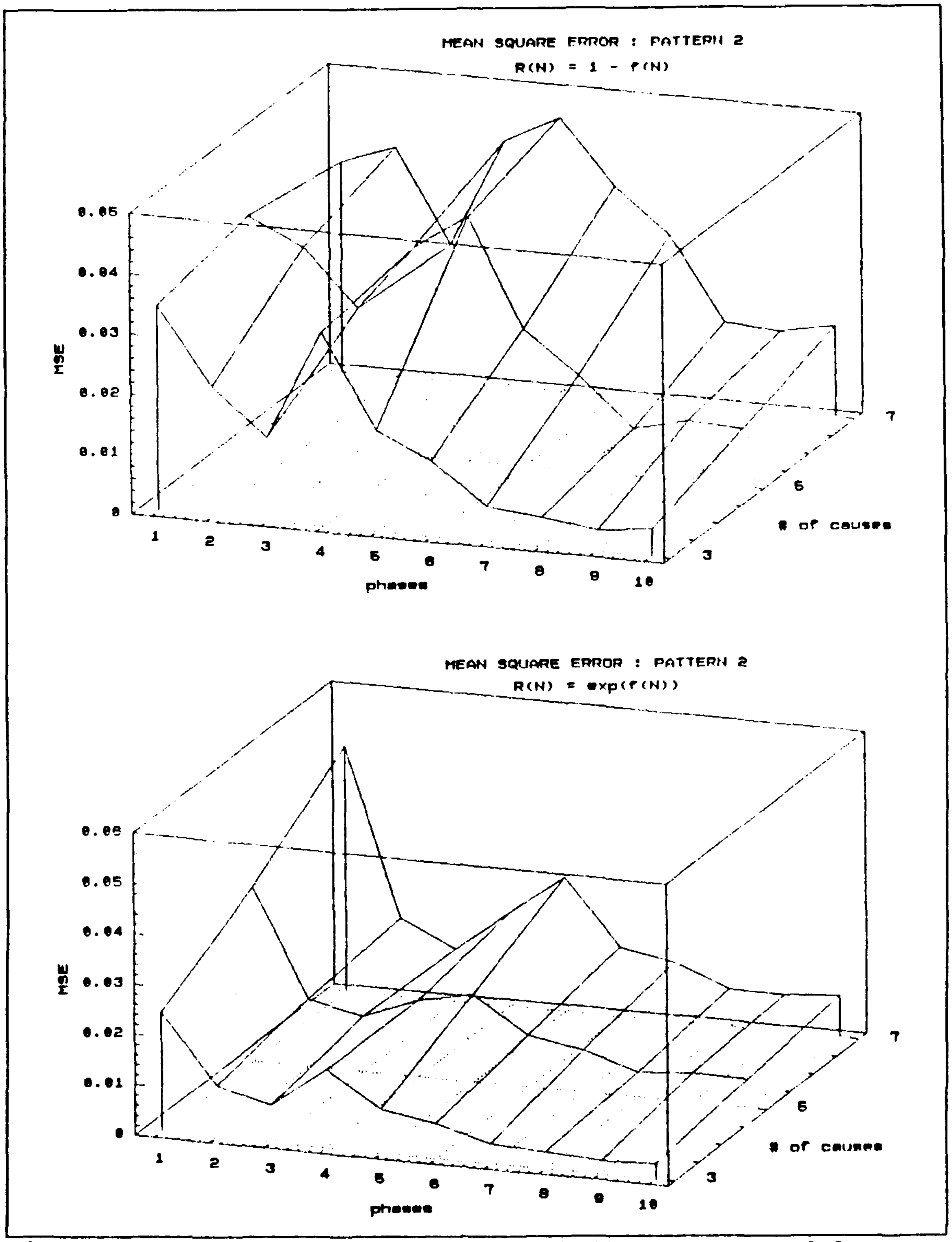


Figure 36 : Mean Square Error for both AMSAA-D models as a function of phases and number of failure causes, pattern 2, failures/phase = 3

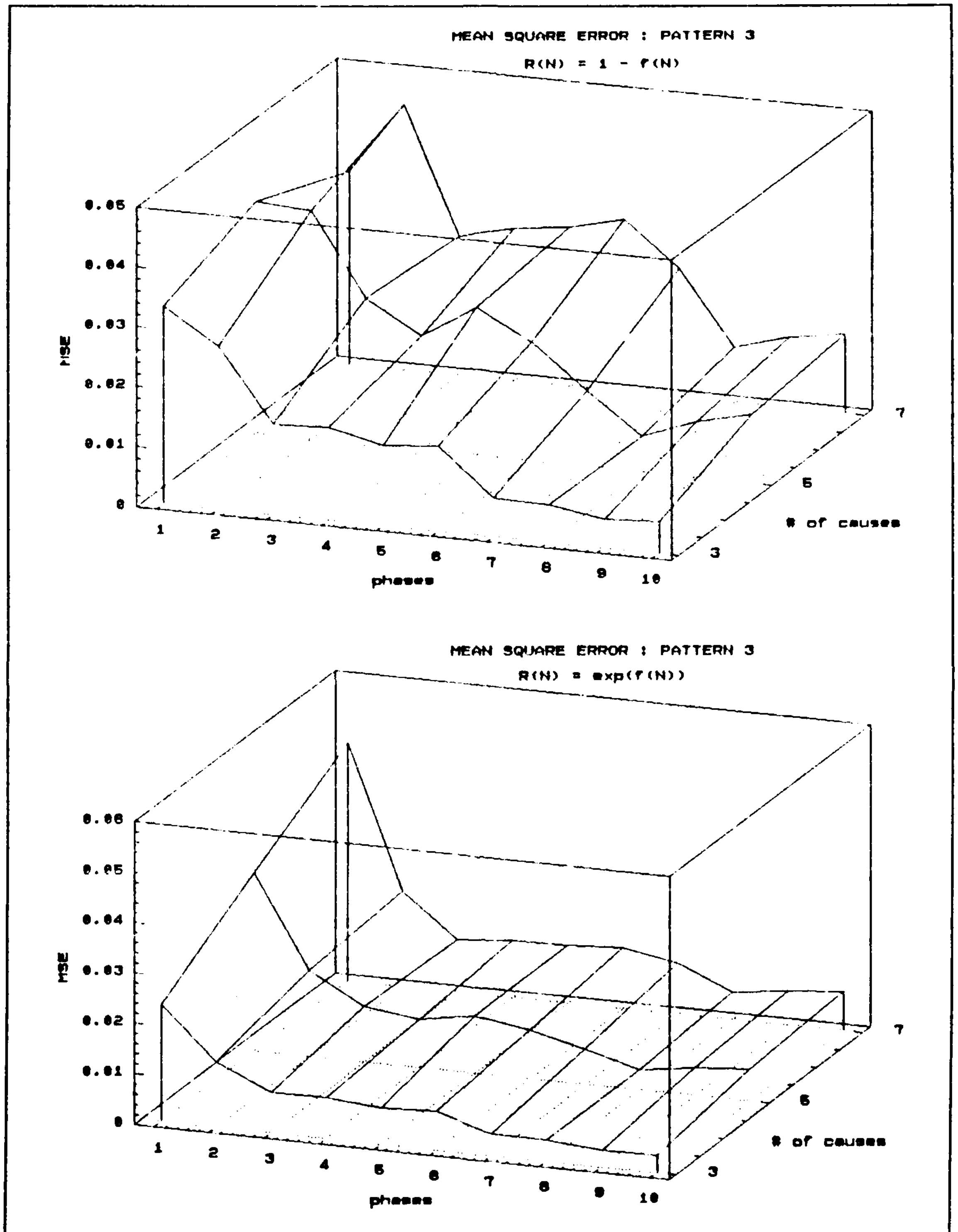


Figure 37 : Mean Square Error for both AMSAA-D models as a function of phases and number of failure causes, pattern 3, failures/phase = 3

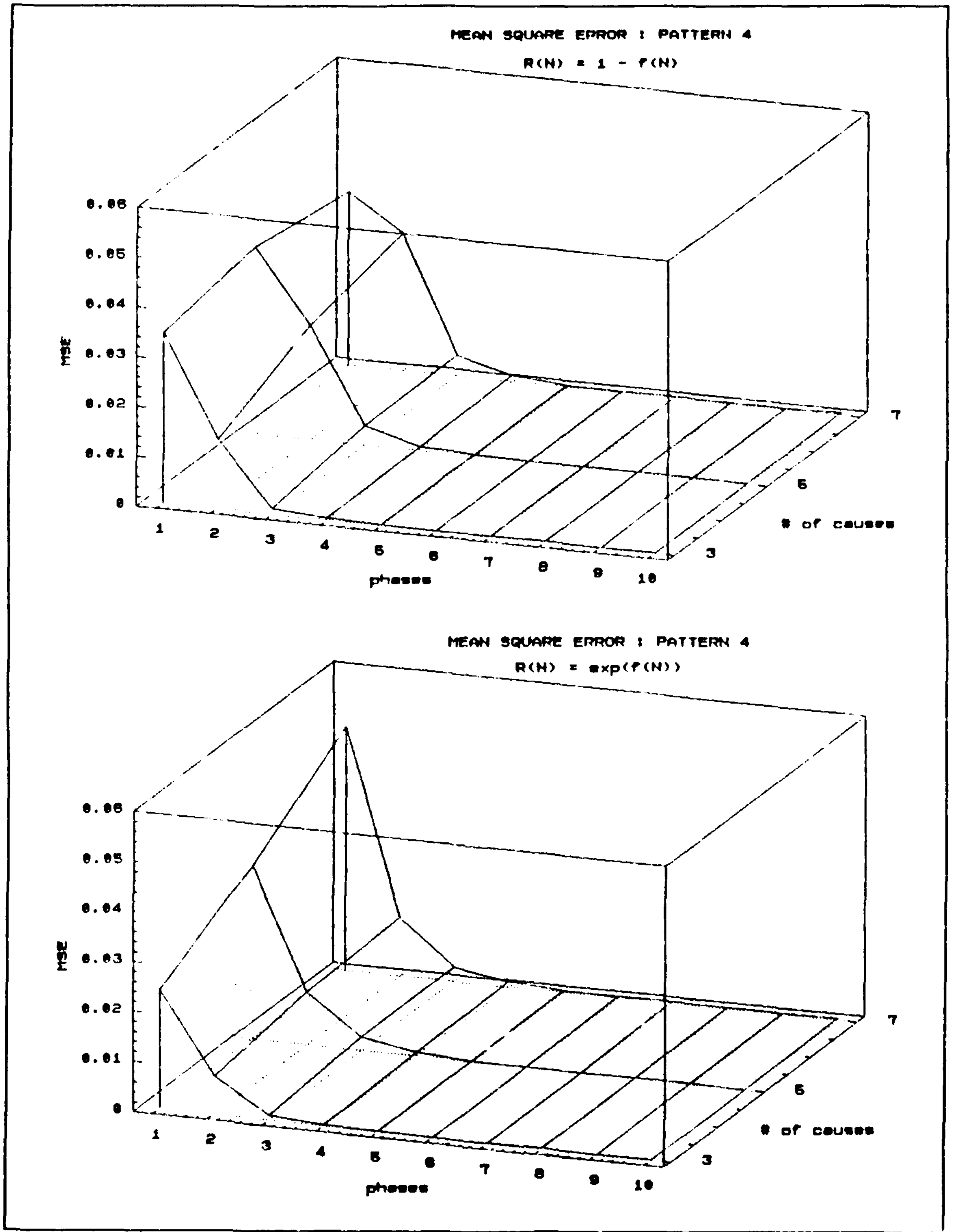


Figure 38 : Mean Square Error for both AMSAA-D models as a function of phases and number of failure causes, pattern 4, failures/phase = 3

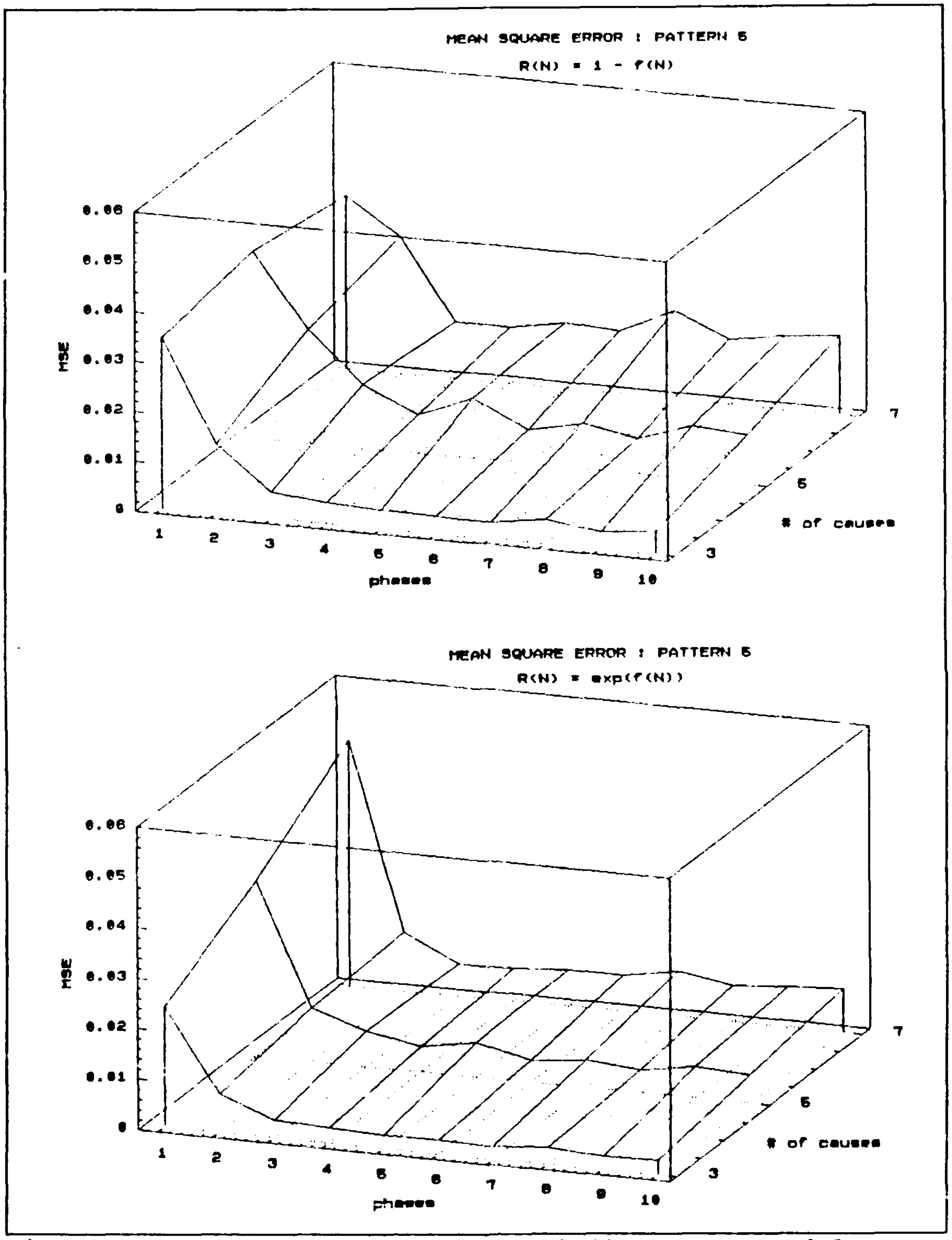


Figure 39 : Mean Square Error for both AMSAA-D models as a function of phases and number of failure causes, pattern 5, failures/phase = 3

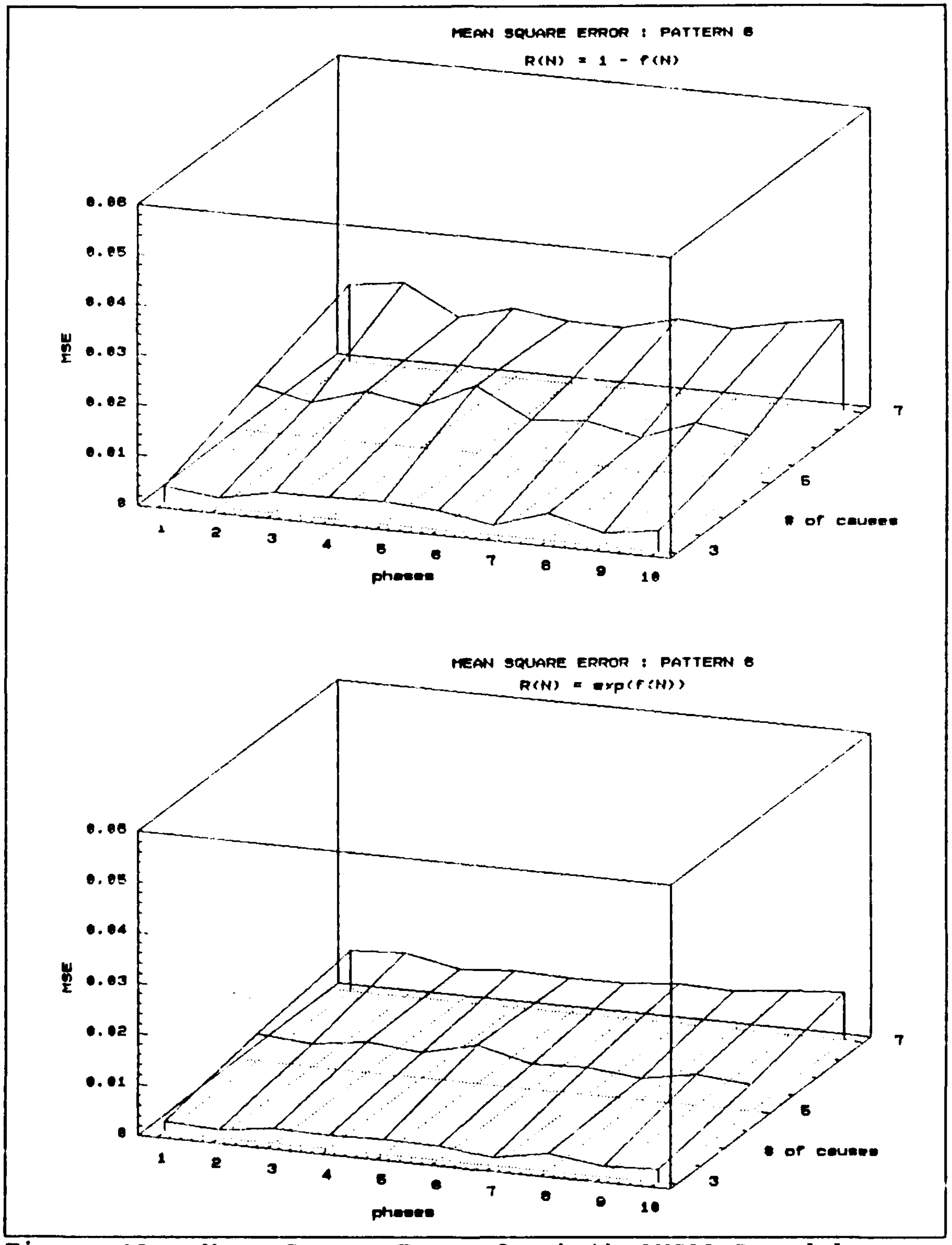
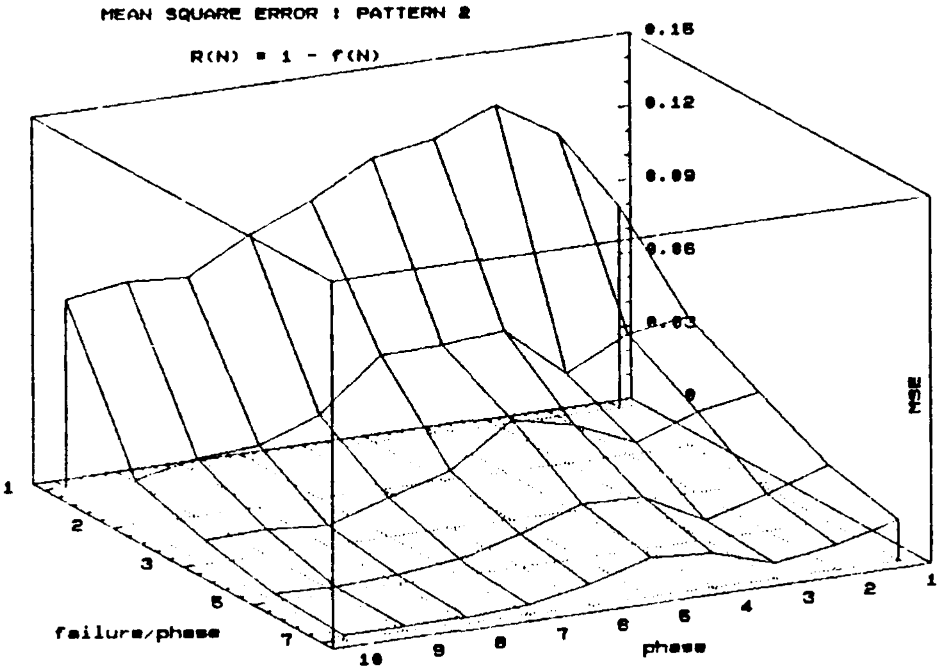


Figure 40 : Mean Square Error for both AMSAA-D models as a function of phases and number of failure causes, pattern 6, failures/phase = 3



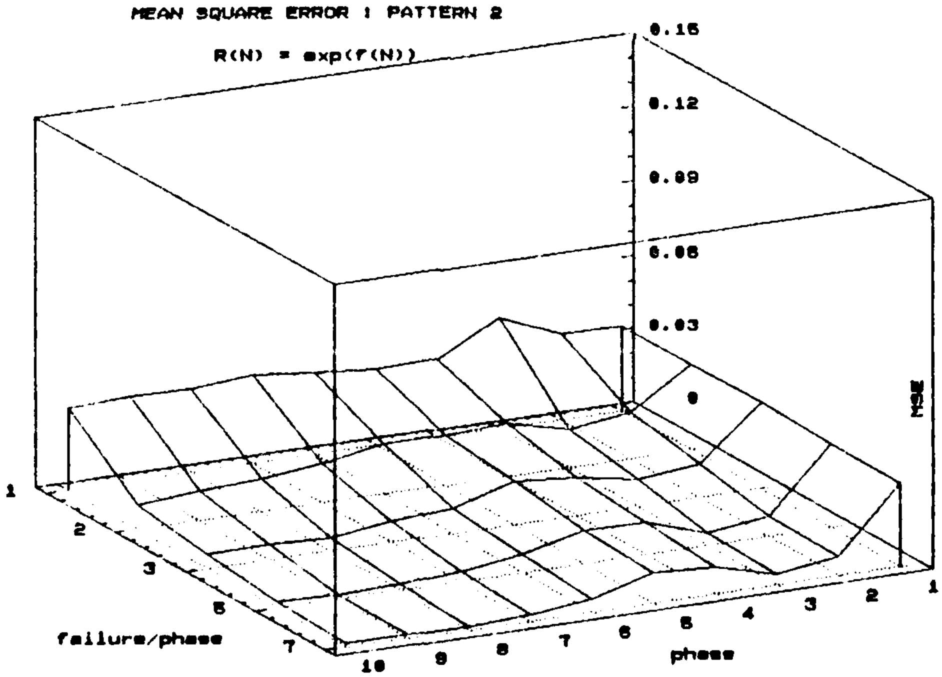


Figure 41 : Mean Square Error for both AMSAA-D models as a function of phases and failures/phase, pattern 2, failure causes = 5

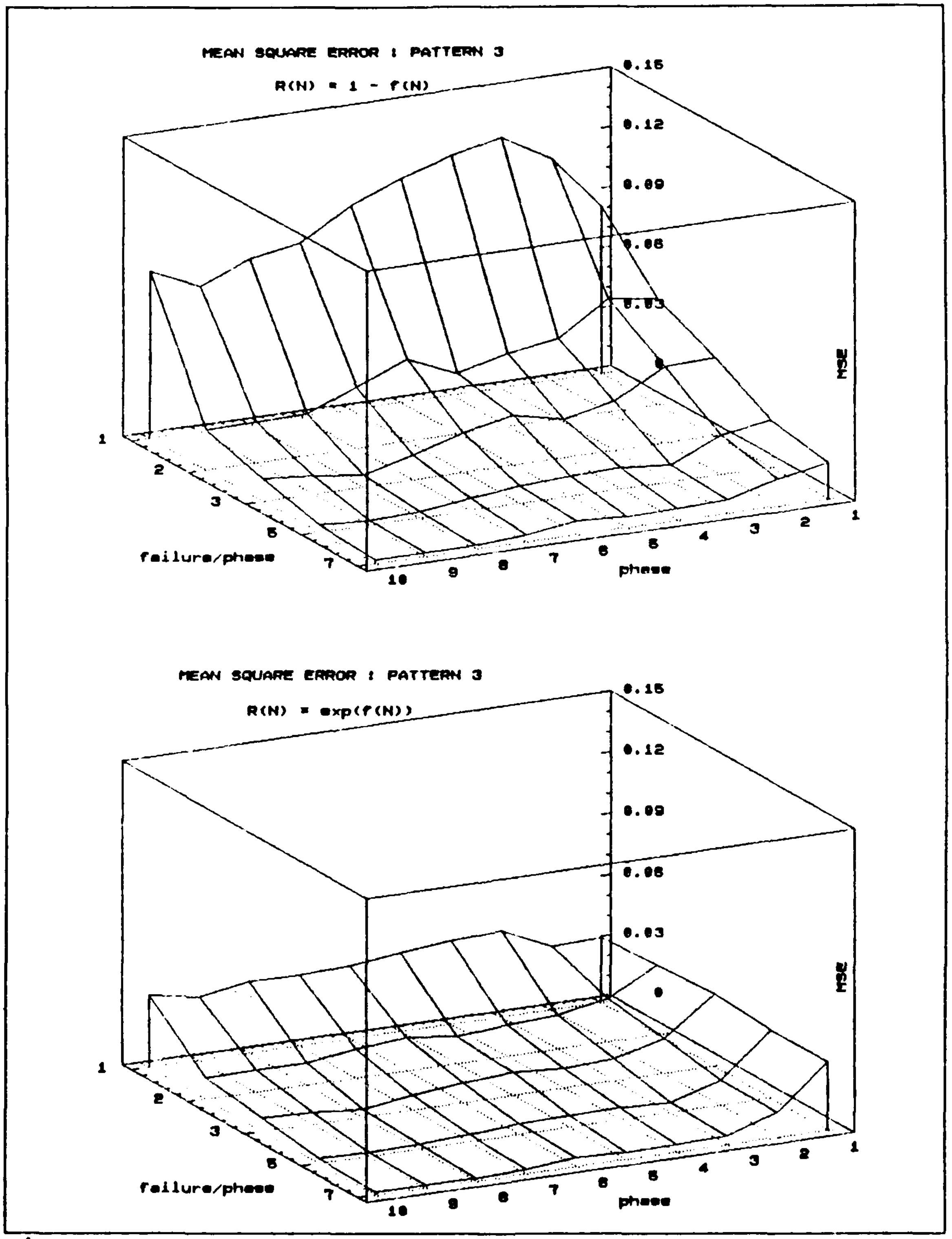


Figure 42 : Mean Square Error for both AMSAA-D models as a function of phases and failures/phase, pattern 3, failure causes = 5

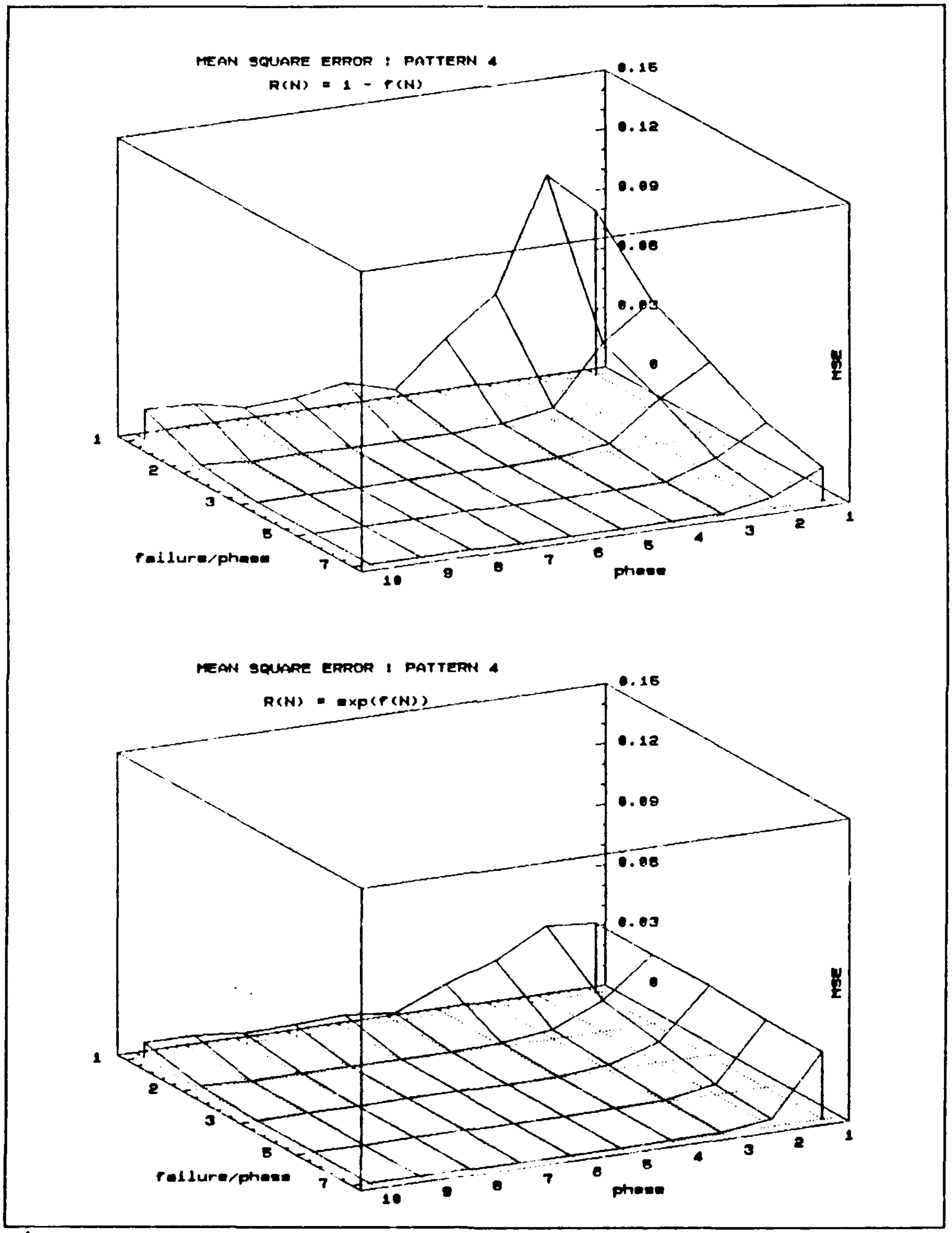


Figure 43 : Mean Square Error for both AMSAA-D models as a function of phases and failures/phase, pattern 4, failure causes = 5

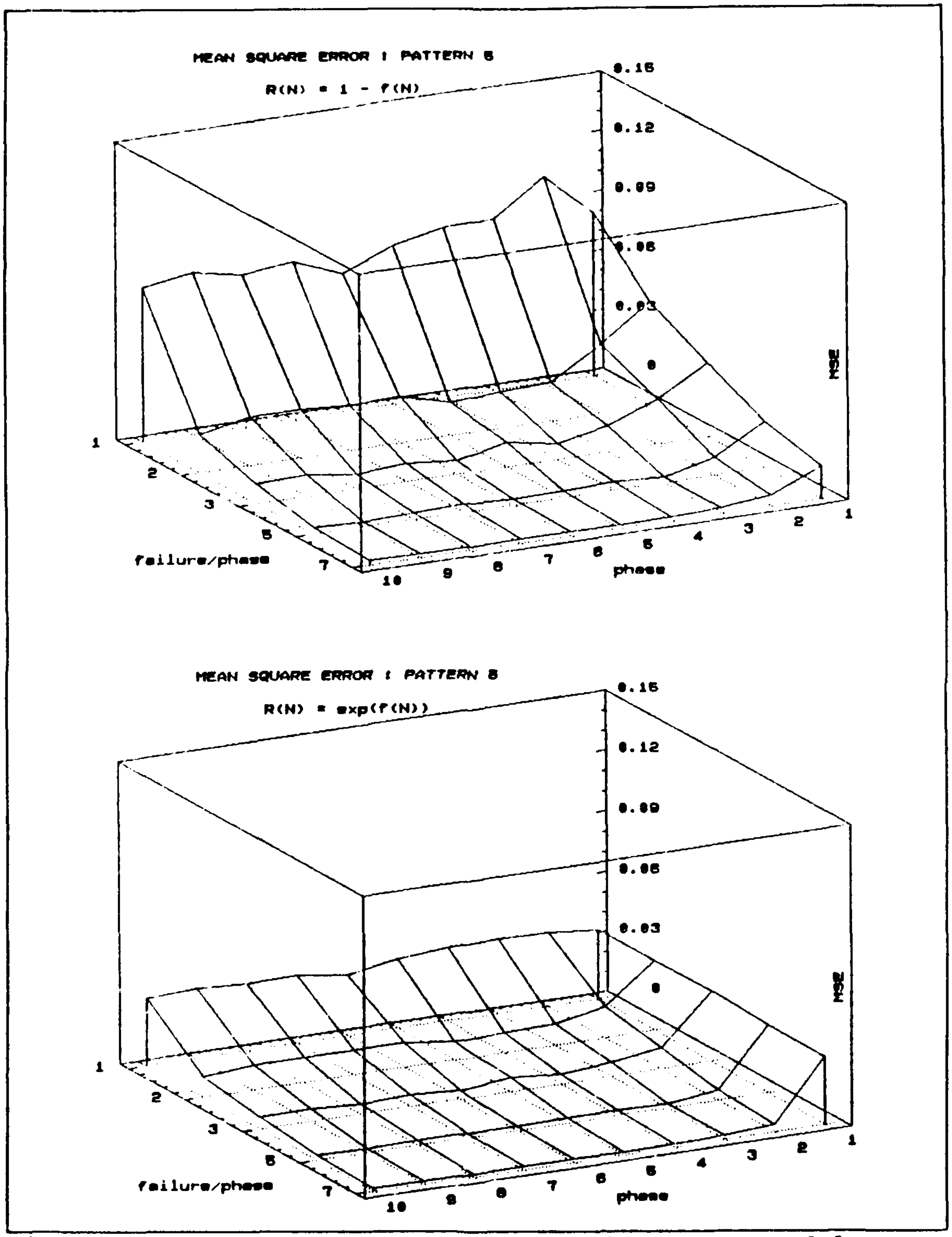


Figure 44 : Mean Square Error for both AMSAA-D models as a function of phases and failures/phase, pattern 5, failure causes = 5

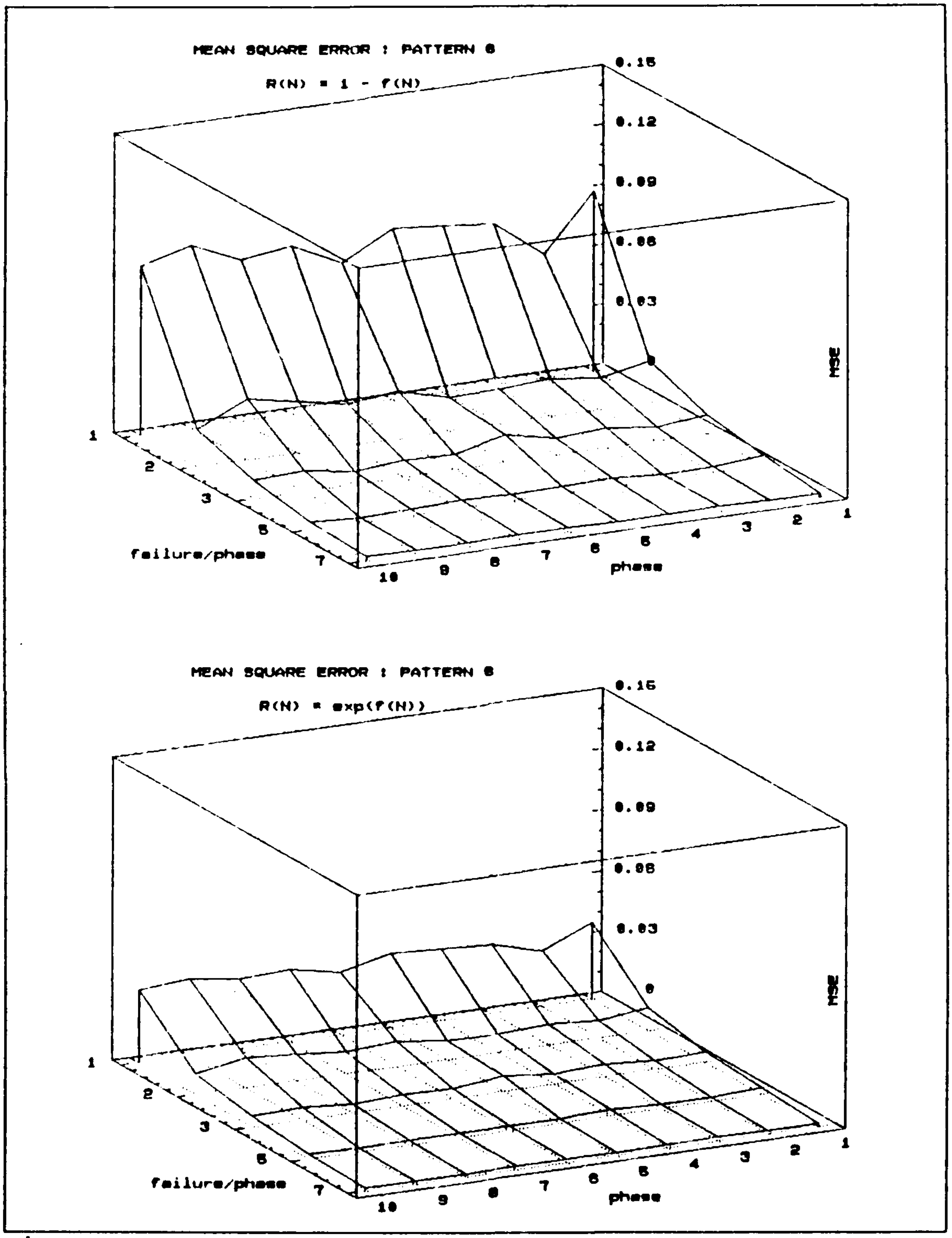


Figure 45 : Mean Square Error for both AMSAA-D models as a function of phases and failures/phase, pattern 6, failure causes = 5

APPENDIX E

GLOSSARY OF THE DIFFERENT RELIABILITY GROWTH MODELS

1. Maximum Likelihood Estimate Model
2. The Exponential Regression Model
3. The Discrete AMSAA Model
4. The AMSAA-D Model
    a. Linear AMSAA-D Model
    b. Exponential Power AMSAA-D Model

1. **Maximum Likelihood Estimate Model**

The reliability estimate $\hat{R}_{tk}$ of the true reliability R is computed as

$$\hat{R}_{tk} = \frac{\text{total trials - total failures}}{\text{total trials}}$$

If failure discounting is applied then the reliability estimate $\hat{R}_{tk}$ is evaluated as

$$\hat{R}_{tk} = \frac{\text{total trials - total adjusted failures}}{\text{total trials}}$$

The estimation is performed after each phase and, if failure discounting is applied, the actual test data are updated with the use of test data from previous phases. This model is denoted as MLEwd.

2. **The Exponential Regression Model**

The exponential regression model obtains sequentially updated estimates $\hat{R}_{tk}$ of the true reliability R_{tk} after the k^{th} phase. The basic model for R_{tk} is :

$$R_{tk} = 1 - e^{-A_k}$$

The parameter $A_k = \alpha + \beta k$ is estimated by $\hat{A}_k$ where

$$\hat{A}_k = \hat{\alpha}_k + \hat{\beta}_k k$$

The estimates $\hat{\alpha}_k$ and $\hat{\beta}_k$ are least square regression estimates for α and β. Equations for $\hat{\alpha}_k$ and $\hat{\beta}_k$ are developed in Reference 4.

The exponential regression estimate $\hat{R}_{tk}$ of the true reliability R_{tk} is then finally :

$$\hat{R}_{tk} = 1 - e^{-(\hat{\alpha}_k + \hat{\beta}_k k)} , \quad k = 1,2,...$$

This model is denoted as exp.Regr.Estimate.

3. The Discrete AMSAA Model

In the Discrete AMSAA model, the equation for the probability of success, R_k, on each trial in the i^{th} testing phase is given by

$$(1 - R_k)N_k = \lambda T_k^{\beta} - \lambda T_{k-1}^{\beta}$$

where

N_k : number of tests in test phase i

$$T_k = \sum_{j=1}^{k} N_j$$

After each new test phase is completed the maximum likelihood estimates for λ and β are recomputed to provide the MLE for R_k. The two equations which must be solved after testing is completed in phase k are as follows [Ref. 6]:

$$\sum_{i=1}^{k}\left\{\frac{M_i}{\left[\lambda T_i^{\beta}-\lambda T_{i-1}^{\beta}\right]}-\frac{N_i-M_i}{\left[N_i-\lambda T_i^{\beta}+\lambda T_{i-1}^{\beta}\right]}\right\}\left[\lambda T_i^{\beta}\ln T_i-\lambda T_{i-1}^{\beta}\ln T_{i-1}\right]=0$$

and,

$$\sum_{i=1}^{k}\left\{\frac{M_i}{\left[\lambda T_i^{\beta}-\lambda T_{i-1}^{\beta}\right]}-\frac{N_i-M_i}{\left[N_i-\lambda T_i^{\beta}+\lambda T_{i-1}^{\beta}\right]}\right\}\left[T_i^{\beta}-T_{i-1}^{\beta}\right]=0$$

Numerical methods and a computer are needed to solve these equations.

4. **The AMSAA-D Model**

The Continuous AMSAA Model expresses the instantaneous failure rate f_{TT_k} after TT_k time units of testing by

$$f_{TT_k} = b(1 - a)TT_k^{-a}$$

where TT_k is the total accumulated test time over k phases. This model is also known as the Duane Model. After examining a large variety of data, Duane noticed that the plot of the cumulative failure rate λ_T versus cumulative test time on log-log paper produced a straight line. From this he inferred a linear relationship between log λ_T and log T. Consequently, he wrote the following model for cumulative failure rate

$$\lambda_T = kT^{-a}$$

where $\lambda_T = \frac{N(T)}{T}$

$N(T)$: total failures up to time T

k and a are unknown parameters.

Thus

$$\frac{N(T)}{T} = kT^{-a}$$

$$N(T) = kT^{1-a}$$

The instantaneous failure rate f_T is the change in N(T) per unit time. Thus

$$f_T = \frac{dN(T)}{dT} = (1 - a)kT^{-a}$$

The AMSAA model departs at this point from Duane by using maximum likelihood methods to estimate the unknown parameters. Letting $a = 1-\beta$ we have

$$f_T = \beta kT^{\beta-1}$$

which is the Weibull failure rate function. Consequently AMSAA develops maximum likelihood estimates for β and k assuming time to failure within a phase has a Weibull distribution. This assumption makes the accuracy of their model succeptible to variations in the underlying failure distribution. Woods [Ref. 4] uses regression methods to estimate the parameters and does not assume an underlying distribution. This makes his procedure more robust. Similar procedures are followed in the AMSAA-D model.

The AMSAA-D model relates the failure rate f_{tk} and the end of k test phases to the total number of failures N_{tk} across the first k test phases by

$$f_{tk} = b(1 - a)N_{tk}^{-a}$$

a. **Linear AMSAA-D Model**

In this model the reliability estimate $\hat{R}_{tk}$ of the true reliability R_{tk} at the end of phase k is computed as

$$\hat{R}_{tk} = 1 - \hat{f}_{tk}.$$

b. **The exponential Power AMSAA-D Model**

In this model the reliability estimate $\hat{R}_{tk}$ of the true reliability R_{tk} at the end of phase k is computed as

$$\hat{R}_{tk} = e^{-\hat{f}_{tk}}$$

LIST OF REFERENCES

1. Drake, J.E., *Discrete Reliability Growth Models Using Failure Discounting*, M.S. Thesis, Naval Postgraduate School, Monterey, CA, September 1987.

2. Chandler, James D. Jr., *Estimating Reliability With Discrete Growth Models*, M.S. Thesis, Naval Postgraduate School, Monterey, CA, March 1988.

3. Markiewicz, Pamela A., *Discrete Reliability Growth*, M.S. Thesis, Naval Postgraduate School, Monterey, CA, September 1988.

4. Woods, W.M., *Reliability Growth Models*, Conference proceedings No. 261, AGARD paper presented at the Meeting of the Avionics Panel, Ankara, Turkey, 9-13 April 1979.

5. Crow, L.H., *AMSAA Discrete Reliability Growth Model*, US Army Material Systems Analysis Activity, Aberdeen Proving Ground, MD, 1983.

6. Thalieb, Rio M., *An Accuracy Analysis Of Army Material Systems Analysis Activity Discrete Reliability Growth Model*, M.S. Thesis, Naval Postgraduate School, Monterey, CA, March 1988.

7. Duane, J.T., *Learning Curve Approach to Reliability Monitoring*, IEEE Transaction On Aerospace, Volume 2 Number 2, April 1964.

8. Department of Navy, Special Projects Office, *Statistical Exposition Of The Guide Manual For Reliability Measurement Program*, NAVORD OD 29304/ADDENDUM, 15 November 1967.

9. Neil, R.O., *An Evaluation Of Three Reliability Growth Models*, M.S. Thesis, Naval Postgraduate School, Monterey, CA, June 1978.

10. Department of the Navy, Strategic Systems Project Office, *Reliability Evaluation Program Manual*, NAVORD OD 29304 Revision A, Proposed, 1 April 1973.

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