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There are few methods available for analyzing structures or structures with controls that have high-modal densities and/or unmodelable excitations (these are essential characteristics of large space structures). Admittance models do not require that a structure be described in modal coordinates or that excitations be characterized as determinate functions of space.

Methods have been developed for the analytical and experimental modeling of actuators and combining them with structural models. A scheme for optimal design of controller impedance using only measured admittance functions was derived and tested. The necessary elements for a hardware demonstration of the theory have been identified. A scheme for design and testing of a structure for verification of the equivalent excitation and structural modification equations has been developed.

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This report documents CSA Engineering's development of admittance modeling techniques for active control of structures. This work was directed by Dr. Anthony Amos and performed under contract F49620-88-C-0024.

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## 1. Introduction

The ability to analyze structures with active control systems has progressed rapidly in the last few years. The SDI and Space Station projects have been a driving force in this work. Even with these advances, the available techniques fall far short of providing the tools necessary to analyze realistic systems.

Most structural (single- and multi-component) and control system analyses require accurate knowledge of physical coordinate modal data. Accumulating experience indicates<sup>1</sup> that the high modal-density associated with space structures makes design and accurate performance predictions of active controls on complex structures intractable at this time. Acquiring a high quality modal model of a complex structure is usually impossible.

Even if a good modal model can be constructed, it is of limited use without an accurate load model. The number and complexity of the various forces acting on a space structure render their characterization an equally impractical task. Many of the excitation forces are simply not known in terms of deterministic functions of space and time.

A method is needed that retains the desirable features of modal models (computational efficiency, combining multiple structures through substructuring), but avoids the problems associated with high modal-density, high bandwidth, lack of load characterization methods, and residual effects. There are a few techniques available which appear promising.

CSA Engineering has recently developed a software package called the Generalized Admittance Matrix Analyzer (GAMMA) [2]. It implements the method of admittance modeling for prediction of the dynamic response of connected structures. The method has certain theoretical and practical attributes that appear well-suited to the above problems. In particular, it suggests a method by which space structures and their various in-service excitation sources could be modeled from test data in a way that would be both efficient and general for predicting dynamic response. GAMMA has the facilities required for accessing and manipulating both test and finite element data to perform such response calculations.

### 1.1 Description of Admittance Modeling

Admittance techniques may be applied to structural dynamic analysis, control systems, and combined systems to solve many types of problems. It is particularly suited to situations where a normal-modes model of a structure cannot be accurately obtained by analysis or test and/or where excitations to a structure cannot be modeled

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<sup>1</sup>See, for instance, Pappa, R., "Comparison of modal identification techniques using a hybrid-data approach." *Proc. First NASA/DOD Controls/Structures Interaction Technology Conference*, 1986.

instance, the frequency-response function (FRF) element,  $H_{ij}$ , of matrix function  $\mathbf{H}$ , refers to a response quantity (e.g., acceleration) at DOF  $i$  due to an input (e.g., force) at DOF  $j$ .

Since FRF's are complex quantities in frequency, a matrix function is clearly a three-dimensional complex entity. Though matrix functions are three dimensional, none of the operations involving matrix functions are three dimensional. Operations proceed on a discrete frequency-by-discrete-frequency basis. Because of this feature, admittance models are theoretically insensitive to resolution. An admittance equation may be solved at any frequency or set of frequencies without affecting (or even requiring) the solution at any other frequency.

Inability in finding common terminology, mathematics, and design/analysis tools has slowed the development of combined structure/controller systems. The admittance modeling technique has a terminology and mathematics that is familiar to both engineering communities. An engineer familiar with analyses based on FRF's (transfer functions to controls/networks engineers) is ready to utilize admittance techniques.

Not only do admittance modeling techniques have a well-known terminology, but the technique allows each separate field to retain all of their day-to-day design and analysis routines. All that is required is for system elements (structures, sensors, controllers, excitations, etc.) to be finally described with FRF or waveforms. Thus, structural engineers do not need to be fluent in state-space representations, and controls engineers need not be fluent in finite element methods.

## 2. Background

Classical structural modification problems are represented very elegantly using admittance equations [1]. The modified structure can be represented using an admittance representation of the original structure and an admittance representation of the attached structure at the connection DOF's. The equations suggest that *any* system which can be represented as admittances at physical DOF's can be attached to the original structure, i.e., the attached system need not be a physical structure.

The approach taken here is that the attached structure is a control system. The attachment DOF's represent sensors and actuators. Clearly, the sensors and actuators do not need to be colocated, and the sensors and actuators need not be represented in pairs.<sup>2</sup>

The equations developed for the structural modification problem may be generalized to include attached structures that do not have an admittance description. This

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<sup>2</sup>Actual sensor and actuator frequency response functions may be used in representing the control system; they need not be analytical. In contrast to modern control theory, including observer/controller dynamics in the model does not increase the order of the system.

Rewriting Equation 1 in partitioned form

$$\begin{bmatrix} X_{Ai} \\ \mathbf{X}_{A2} \end{bmatrix} = \begin{bmatrix} H_{Aii} & \mathbf{H}_{Ai2} \\ \mathbf{H}_{A2i} & \mathbf{H}_{A22} \end{bmatrix} \begin{bmatrix} L_{Ai} \\ \mathbf{L}_{A2} \end{bmatrix} \quad (2)$$

Equation 2 is general; it applies to Body A whether or not it is coupled to Body B. However, the objective is to find the drive-point admittance at DOF  $i$  after coupling. We define this quantity as

$$H_{(A+B)ii} = \left. \frac{X_{Ai}}{L_{Ai}} \right|_{\text{after coupling}} \quad (3)$$

Coupling amounts to imposing the following constraints on the forces and motions at DOF Set 2.

$$\mathbf{Z}_{B22}\mathbf{X}_{B2} = \mathbf{L}_{B2} \quad (4)$$

$$\mathbf{X}_{B2} = \mathbf{X}_{A2} \quad (5)$$

$$\mathbf{L}_{B2} = -\mathbf{L}_{A2} \quad (6)$$

Equation 4 is simply the definition of the drive-point impedance matrix for Body B, that is excited only by its connections to Body A. The latter two equations are, respectively, the conditions of compatibility and equilibrium at the connection points.

Applying the constraints of Eqs. 4 through 6 to Equation 2 and solving for the quantity in Equation 3 gives the desired result

$$H_{(A+B)ii} = H_{Aii} - \mathbf{H}_{A2i}^T \mathbf{Z}_{B22} [\mathbf{H}_{A22} \mathbf{Z}_{B22} + \mathbf{I}]^{-1} \mathbf{H}_{A2i} \quad (7)$$

where

$H_{(A+B)ii}$  = drive-point FRF at DOF  $i$  on Body A after modification

$H_{Aii}$  = drive-point FRF at DOF  $i$  on Body A before modification

$\mathbf{H}_{A22}$  =  $n \times n$  admittance matrix for Body A at DOF Set 2

$\mathbf{Z}_{B22}$  =  $n \times n$  impedance matrix for Body B at DOF Set 2

$\mathbf{H}_{A2i}$  =  $n \times 1$  cross-admittance matrix of Body A between DOF Set 2 and DOF  $i$

Equation 7 may be expanded to include the case of a single cross-admittance function or an entire admittance matrix as the desired result.

software tools for dealing with statistical quantities can be used. Moments (mean, standard deviation, etc.), RMS-type quantities, crest factors, power, etc., can all be used and also provide ways of doing comparative analyses and trending.

For structural modification problems different performance measures are used. Moments, particularly RMS, are natural choices and of course time-domain measures can be readily applied after transformation.

Figure 2 shows the transformations used in structural analysis. For CSI problems additional transformations need to be included. Figure 3 shows some of the transformations that will be necessary for a hardware verification effort.

Figure 4 shows a schematic of the structure used for all of the test problems that are documented in this report. The truss elements are round tubular aluminum and all joints are modeled as welded. Tube diameter and overall length were chosen to give a bare structure with eight or more modes below 200 Hz. NASTRAN was used for all analytical analyses of the bare truss. A damping level of 0.005 critical was imposed for all modes. (This damping level is typical for a carefully made truss of this type.) A linear velocity feedback controller was implemented in three different ways

1. using NASTRAN's transfer function facility
2. using NASTRAN's viscous element, and
3. by directly synthesizing the control impedance in *GAMMA*.

Figure 5 shows the results from these three analyses overplotted on the original undamped structure. Figure 6 shows the impulse response function of the tip X-direction for the original undamped structure (top) and closed-loop system(bottom). Note that the original system appears unstable; it is not but the inverse transform makes it appear to be.

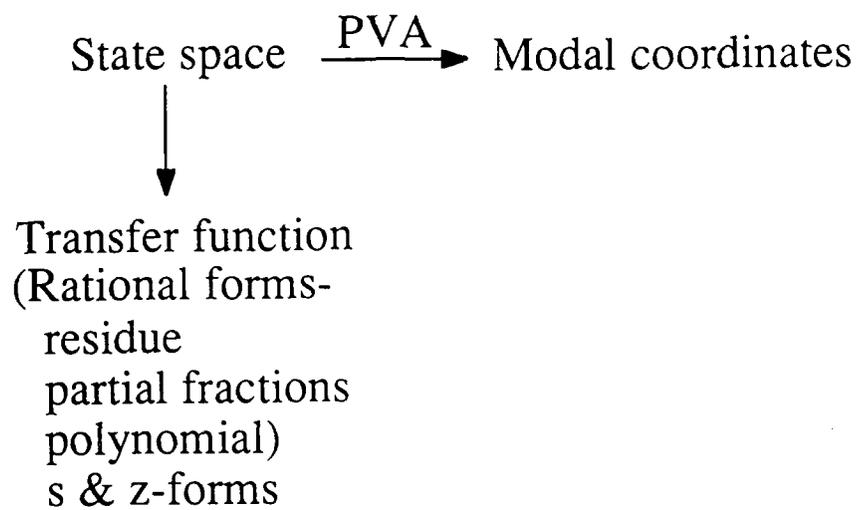


Figure 3. Transformations for control system design.

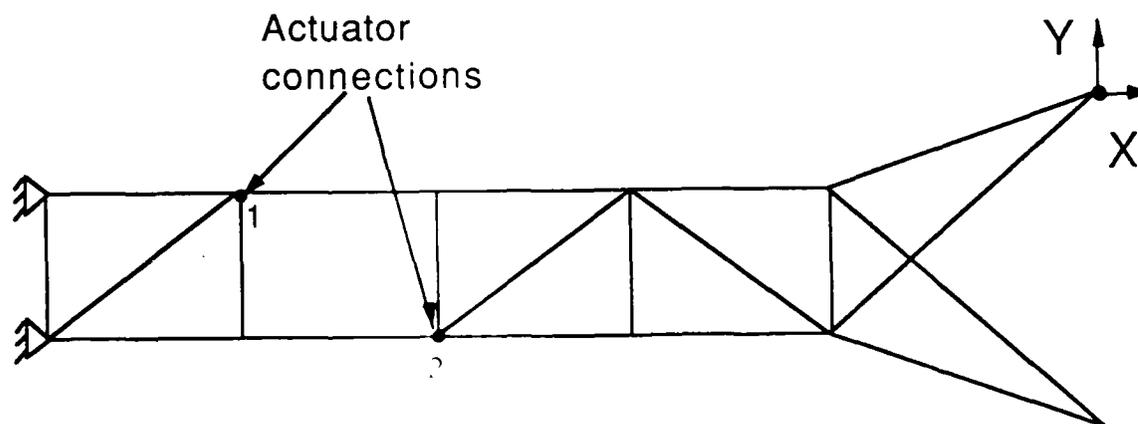


Figure 4. Structure schematic.

## 5. Control System Synthesis

This section presents a scheme for turning the models developed above into a realistic experimental setup. Some of the issues involved in a hardware demonstration include

- A testable, linear structure.
- Linear actuators.
- Controller (analog or digital).
- Motion sensors.
- Signal conditioning and power electronics.
- A rationale for arriving at a suitable control.
- Method for translating the control rationale into a form usable by the controller.
- Facilities and tools (admittance testing hardware and software, control system design software, controller hardware and software, monitoring hardware and software and a means for making them all work together).

Building a testable, linear structure is well within the reason of a small scale experiment.<sup>3</sup> However, there are few linear actuators with good combinations of generated force and bandwidth. One of the few candidates are the piezoelectric based struts. JPL has demonstrated a simple and effective active strut based on commercial materials and sensors [5].<sup>4</sup> This strut has good force delivery (though low-stroke) and more than adequate bandwidth for a laboratory experiment.

Most of the other issues involve evaluating and integrating various commercially available electronics and software. The design and testing routines are fairly standard with one exception: the sixth item above—the control rationale.

### 5.1 Control Rationale

In the JPL work [5] they used a common approach to CSI problems: decompose the structural model into modal components using an experimental modal analysis. In the particular configuration they were using, the structure had so little damping that they had to resort to some custom signal processing software to minimize leakage errors. In addition, the structure had few modes in the bandwidth of interest. Here the experimental modal analysis was used primarily to provide damping estimates for a companion finite element model and to verify the finite element model. The

---

<sup>3</sup>CSA has tested, modified, and verified the linearity of joint balls and quick-connects [3,4] that are to be used in scale-model testing of the Space Station. These components are not expensive and would be an ideal candidate for a demonstration experiment.

<sup>4</sup>A very good paper dealing with a number of CSI issues in a practical manner.

tip admittance (see Figure 4). This admittance is calculated from a NASTRAN finite element model and passed to MATRIX<sub>X</sub>. The admittance was then corrupted by adding white noise (15% by RMS). The *SYSTEM/ID* portion of MATRIX<sub>X</sub> was then used to calculate an approximate maximum-likelihood estimate of the dynamic system. The approximate model parameters are then used in a recursive maximum-likelihood model to arrive at the final estimate. Both techniques produce a  $z$ -domain transfer function of the form:

$$y(z) = \frac{1}{\text{Den}(z)} \text{Num}(z)U(z) + \text{Numc}(z)V(z) \quad (9)$$

where Num, Den, and Numc are polynomials of specified order. (Numc represents the noise process.) The output parameters can be put directly into state-space form where further design could be done.

## 6. Actuator Analysis & Testing

In the Background section above nothing was said about how to acquire the describing admittances (or impedances) of a sensor or actuator. It was simply assumed that they had been acquired by some means. Also, nothing was explicitly stated concerning the control system state when the describing admittances were acquired. That is, if the modeling equation represents closed-loop behavior, how can closed-loop actuator impedances ( $Z_{B22}$ ) be acquired except in the actual closed-loop situation?

### 6.1 Modeling a Piezoelectric Actuator

The procedure for determining how a particular type of actuator should be modeled and tested is presented using a particular case. This particular actuator has most of the features of real actuators but is not totally general. Using the concepts developed in this section the testing and modeling procedure for a different class of actuator (or sensor or controller) can also be derived.

The actuator type that will be described is currently being developed by a number of sources for use in large space structures and mirror control applications. The active piezoelectric (PZT) strut has a number of attractive features with a few drawbacks. A typical strut is shown in Figure 8. This actuator is configured as a strut that can directly replace any non-active strut in a truss structure. This actuator is composed of several distinct components:

**Piezoceramic** The piezoelectric elements are cylinders of man-made piezoceramic. The outer and inner surfaces of the cylinder are metalized (silver) and an electric field is imposed radially (this is done by silver-soldering wires on the inner and

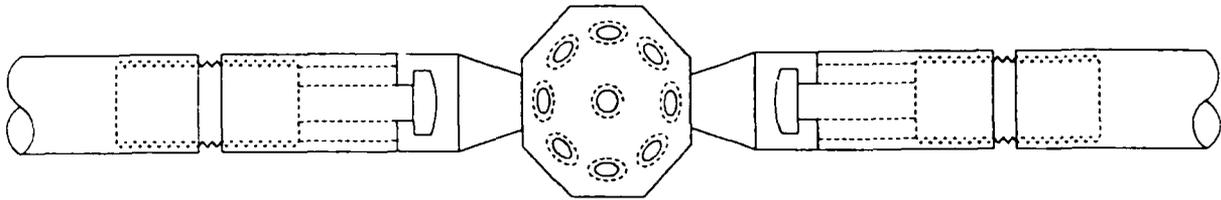


Figure 9. Quick-connect joints and a node ball

outer surfaces). This type of piezoelectric material has a high  $d_{31}$  piezoelectric constant (strain developed in the axial direction for a field applied radially), so that the cylinder lengthens when a voltage is applied across the inner and outer surfaces. Multiple cylinders (2 in the figure) are used to develop more displacement.

**Witness Tube** The witness tube is firmly attached to one end of the strut and passes through the center of the entire strut and terminates in a *target*. Attached to the other end of the strut is a non-contacting sensor. Thus the witness tube and sensor measure relative axial displacement of the ends of the strut.

**Preload Tube** Piezoceramics have low tensile strength and must therefore be either preloaded or used strictly in a compression mode. The preload tube must be gauged to provide the necessary preload without imposing any more stiffness than is absolutely necessary. (The piezoceramic cylinders must overcome the preload tube stiffness in order to provide any axial strain to the strut.)

**Quick-connect** Figure 9 is a more detailed (and somewhat different) view of the quick-connect joint. (This joint is the one currently being developed for scale-model work on the space-station.) These joints are very linear and impart a negligible amount of damping [3].

This type of actuator can be represented as a linear spring in parallel with a force generator (Figure 10). This is one of the essential features of piezoelectric materials. With this model and knowledge of the material properties of the piezoceramic the strut properties can be predicted using a simple finite element model (shown later).

The strut can be treated as an equivalent excitation (see [1] Equation 20 or Equation 21) in terms of free motions or blocked forces depending upon which condition more closely resembles the in-service condition.<sup>5</sup> Free motions are usually easier to

<sup>5</sup>Note that either condition has an exact solution and that the choice of test boundary conditions

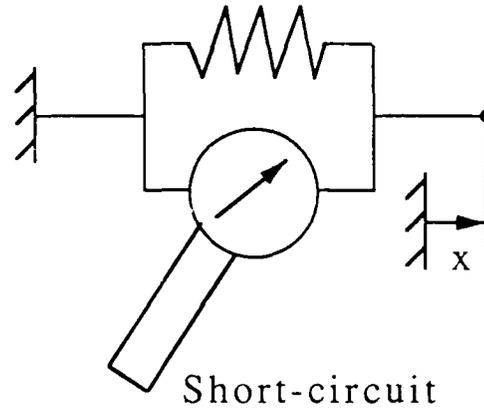


Figure 11. Passive stiffness measurement setup—PZT short-circuited.

surement is  $\mathbf{H}_{A21}$ . The final closed-loop prediction is just

$$\mathbf{H}_{B3A1} = \mathbf{H}_{B32}[\mathbf{H}_{B22} + \mathbf{H}_{A22}]^{-1}\mathbf{H}_{A21} \quad (10)$$

$\mathbf{H}_{B3A1}$  is  $X_i/V$ ; the closed-loop FRF between input DOF (voltage) and response—the desired result.

## 6.2 Predicting Strut Dynamics

In addition to making measurements on a PZT strut it is highly desirable to be able to construct an analytical model before prototyping. Figure 12 shows the load path (top) of the PZT strut (Figure 8). This load path can be represented by the simple model shown at the bottom. Constructing an analytical model of this equivalent model is very straightforward in a finite element program. The admittance model is very similar to the model developed in Section 6.1 but the PZT cylinders are now Body A and the remainder of the strut is Body B. The required properties of the PZT cylinders ( $\mathbf{H}_{A21}$  and  $\mathbf{H}_{A22}$ ) can be estimated directly from its dimensions and material properties: specifically the  $d_{ij}$  and  $g_{ij}$  constants.

The  $\mathbf{H}_{A21}$  function comes directly from the definition of the voltage constant  $g_{ij}$ . Assuming the PZT is annular in shape with electrodes on the inner and outer curved surfaces we have

$$\frac{v}{\Delta x} = g_{31}Y_{33}^D \quad (11)$$

or

$$\mathbf{H}_{A21} = \frac{X}{V} = \frac{1}{g_{31}Y_{33}^D} \quad (12)$$

where

$Y_{33}^E$  = the constant-field(short-circuited) Youngs modulus in the axial (3-3) direction.

### 6.3 Test Problem & Results

The model in Figure 4 was used to verify the process developed in Section 6.2. The structural modification equation (Equation 7) was essentially solved for the combined strut and truss and the strut was modeled using the equivalent excitation method in Section 6.2.

The passive compliance of the strut (Equation 16) was modeled directly using bar elements whose properties were derived to match the actuator depicted in Figure 8. The passive stiffness of the PZT cylinder ( $k_z^E$ ) was taken from manufacturers data sheets for a high-performance piezoceramic. Once again, note that the passive stiffness of a PZT is measured in the zero-field condition, i.e., short-circuited (not to be confused with zero applied-field, or open-circuit). With these components modeled the passive stiffness of the strut is assembled (entirely in NASTRAN) and this strut is placed into the model of Figure 4 and the free-free admittance of the strut alone is computed in a separate NASTRAN model. This second calculation is used in the *GAMMA* comparison model.

The active part of the model is simulated using dynamic loads (DLOAD in NASTRAN). Equal and opposite loads are applied to the strut grid points where the PZT elements attach to the metal lengths of the strut. The load levels are chosen from the blocked force levels achieved by applying 750 volts to the PZT cylinders. The free-strain values of the PZT are calculated from data sheets in the no-preload case and used in the *GAMMA* model.<sup>6</sup>

The closed-loop control is simply linear velocity feedback (a viscous damper) and the results showed identical agreement between the all-NASTRAN model and the *GAMMA* model.

Besides being a demonstration of the equivalence of the two approaches this modeling exercise would be genuinely useful in a demonstration project or application. The ability to model and size elements (and thus pick instrumentation, PZT's, and power amplifiers) without resort to experiment would save considerable time and money, and would definitely contribute to an overall better system. In addition, by finding an equivalent analytical representation, dynamic optimization techniques could further improve the system. (In fact, the sizing of the preload tube for the actuator is best done using a dynamic optimization program such as ADS/NASOPT.)

### 6.4 Summary

Two tests are required to characterize the PZT strut.

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<sup>6</sup>Neglecting the preload tube is neither correct nor is it wrong. The effect of leaving the tube out is that the strut will generate a somewhat higher strain for a given voltage. The same strain could be developed but would require a higher voltage. In effect, the results are qualitatively unchanged but are now uncalibrated, i.e., the system seems to have a higher performance than it really would.

impedance that will lead to the smallest closed-loop admittance? The second term ( $\mathbf{H}_{A2i}^T \mathbf{Z}_{B22} [\mathbf{H}_{A22} \mathbf{Z}_{B22} + \mathbf{I}]^{-1} \mathbf{H}_{A2i}$ ) can be rewritten so that the equation is

$$H_{(A+B)ii} = \mathbf{H}_{A2i}^T \mathbf{Z} \mathbf{H}_{A2i} \quad (17)$$

Note that  $\mathbf{H}_{A2i}$  is not square (as derived above). But, in order to solve for  $\mathbf{Z}_{B22}$ , the inverse of  $\mathbf{H}_{A2i}$  must be computed. A least squares solution for this problem exists<sup>9</sup> and can be written as

$$\mathbf{Z} = \mathbf{H}_{A2i}^{T+} \mathbf{H}_{A2i} \mathbf{H}_{A2i}^+ \quad (18)$$

The '+' operator is termed the pseudo-inverse. Without going into the derivation,  $\mathbf{H}_{A2i}^+$  can be computed very simply for the case where the matrix order is  $1 \times 1$ .

$$\mathbf{H}_{A2i}^+ = \mathbf{H}_{A2i}^T (\mathbf{H}_{A2i}^T \mathbf{H}_{A2i})^{-1} \quad (19)$$

or, more simply as

$$\mathbf{H}_{A2i}^+ = \frac{\mathbf{H}_{A2i}^T}{(\mathbf{H}_{A2i}^T \mathbf{H}_{A2i})} \quad (20)$$

Finally, substitute this back into Equation 7, use Equation 18 and, solving for the controller impedance gives

$$\mathbf{Z}_{B22} = [\mathbf{Z}^{-1} + \mathbf{H}_{A22}]^{-1} \quad (21)$$

In the case where  $\mathbf{Z}_{B22}$  is non-singular the equivalent admittance is

$$\mathbf{Z}_{B22}^{-1} = \mathbf{H}_{B22} \quad (22)$$

or

$$\mathbf{H}_{B22} = \mathbf{Z}^{-1} - \mathbf{H}_{A22} \quad (23)$$

## 7.2 Test Problem & Results

The PZT active strut and truss structure of Section 5 was chosen as the test problem configuration. Since there were no side constraints on the controller impedance the controller essentially was *assumed* to have

- as many DOF's as there were discrete spectral lines,
- mass and/or stiffness and/or damping.

In other words, having no side constraints on the controller impedance made the model as general as is possible with second-order admittance models.

Two problems were run

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<sup>9</sup>The underdetermined case is termed the *dual* to the overdetermined case. There are far fewer solution techniques available for the the underdetermined case than for the overdetermined case.

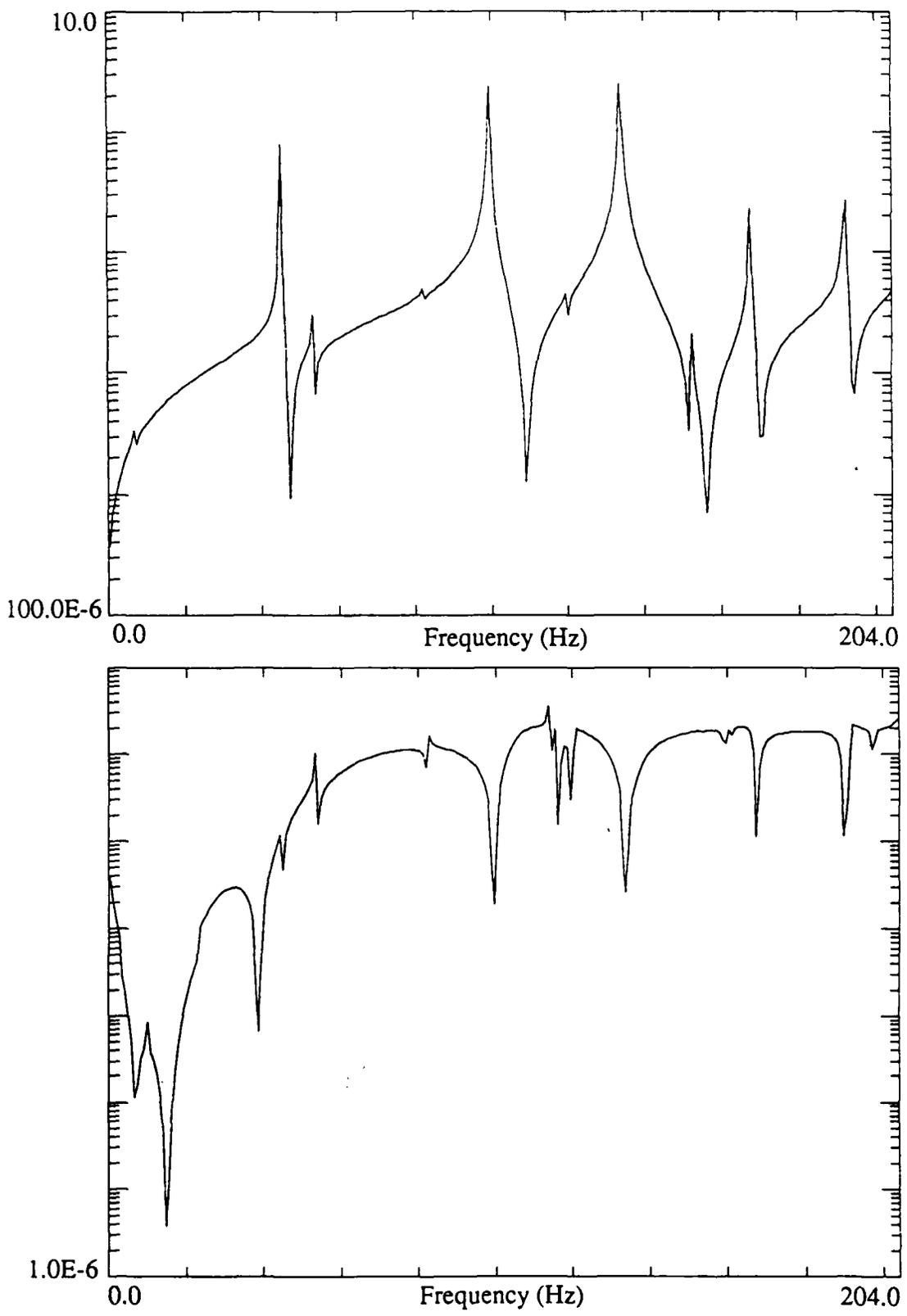


Figure 13. Open-loop driving point admittance (top), optimum controller impedance (bottom).

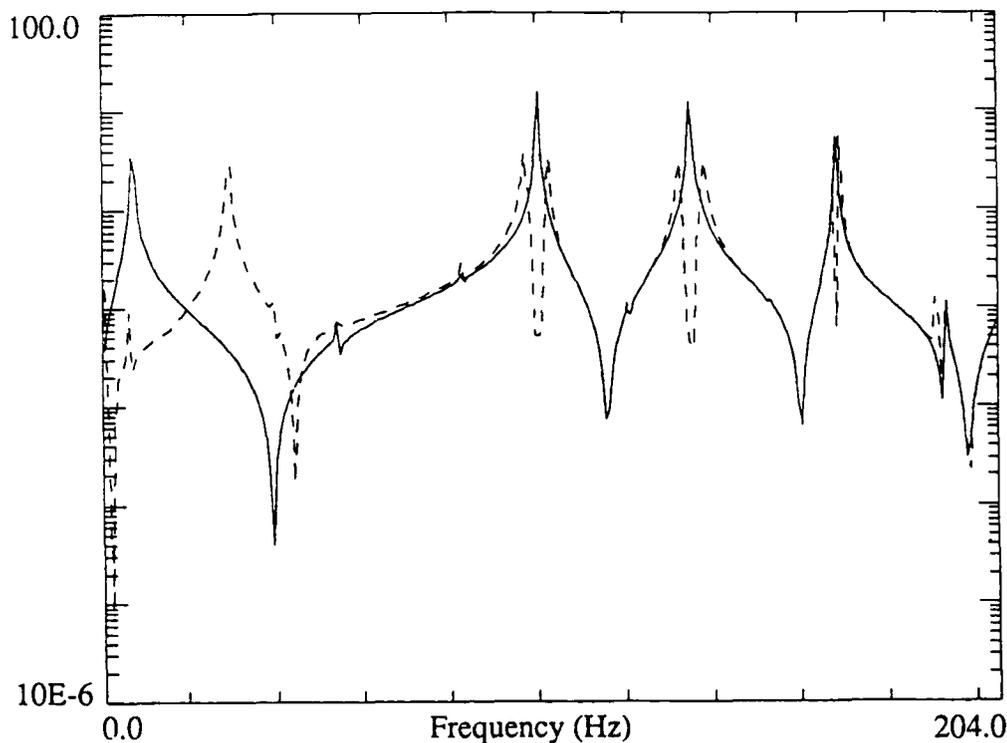


Figure 15. Closed-loop driving point and open-loop admittance (Y-direction).

By choosing DOF Set  $i$  to have only one entry the pseudo-inverse computation in Equation 20 could be performed using standard *GAMMA* commands without resorting to writing a general pseudo-inverse facility. However, by restricting the basis function ( $\mathbf{H}_{A2i}$ ) to order 1 we may be imposing an inadvertant constraint. Restricting underdetermined systems to the absolute minimum degrees-of-freedom can result in no real solution.

The results raise a question: can a concept similar (or identical) to *observability* in modern control theory be constructed for an admittance model? And if observability can be determined can controllability be determined and then worked into a design scheme?

## 8. Conclusions & Recommendations

### 8.1 Conclusions

This project has found a practical way to test and analyze active struts with embedded PZT's. An analytical technique for predicting closed-loop behavior for structures with embedded actuators was developed and tested. The procedure will be useful

in an actual hardware demonstration effort by permitting sizing calculations of strut components.

A procedure for using measured admittances in control system design was tested and verified. The procedure makes use of standard software and measurement techniques and is directly applicable to hardware experiments.

A simple optimization scheme was derived and implemented. The results were not verified by an independent analysis (none is known that could do so) but the results are realistic and promising.

The combination of the results of this project and previous admittance modeling work now permits realistic, small-scale hardware testing of the main features of admittance modeling of structures with active controls. These experiments should be able to provide high-quality answers and numerous insights into this modeling technique.

## 8.2 Recommendations

Although there are many control design techniques based on frequency response methods these methods assume that a parameterized model of the plant is available. Making full use of the features of admittance modeling will require that control system design can be done using measured admittances. Further work is required to develop and verify such techniques. These techniques should

- Identify constraints on types of controllers and forms of control representation.
- Obtain stability characteristics using admittance data.
- Attempt to construct analogs to modern control concepts such as observability, and controllability.
- Attempt to construct improved optimization criteria and techniques. These techniques should address not only optimal control but optimal combined structures with controls.

CSA would like to emphasize most the development of design tools based on measured admittances. In addition; as different classes of linear actuators are considered seriously for use in LSS applications derivation of the actual testing and modeling procedures will need to be repeated in accordance with the process derived in this project.

## 8.3 Phase II

The Phase II proposal presents a comprehensive plan for demonstrating the equivalent excitation and structural modification admittance models for a structure with an