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The PI has continued his seminal work in both theory and computation of chaotic dynamics. There were 10 papers produced during the funding period. These papers concentrated on characterizing new features of formation/destruction of factal basin boundaries together with an important numerical procedure, "shadowing", which allows extremely robust trajectory determination.

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FINAL SCIENTIFIC REPORT
GRANT AFOSR-~~81-0217~~ 87-0110

James A. Yorke, Principal Investigator
October 1988

The following papers have been published:

1. E. Coven, I. Kan, and J. A. Yorke, 'Pseudo-orbit shadowing in the family of tent maps,' Transactions Amer. Math. Soc. 308 (1988) 227-241.

2. H. E. Nusse and J. A. Yorke, 'Is every approximate trajectory of some process near an exact trajectory of a nearby process?, Commun. Math. Phys. 114 (1988) 363- 379.

3. E. J. Kostelich and J. A. Yorke, 'Noise reduction in dynamical systems,' Phys. Rev. A 38 (1988) 1649-1652.

4. E. J. Kostelich and J. A. Yorke, 'Using dynamic embedding methods to analyze experimental data,' Proceedings AMS-SIAM Summer Research Conference, July 1987.

The work on shadowing has a rather practical spin-off: noise reduction. Establishing that a trajectory can be shadowed by a true trajectory is carried out in practice by finding out where the true trajectory is with high precision. That means we can replace a noisy trajectory with a much more precise one and eliminate a large amount of the noise. This approach requires that we know the underlying dynamical process. When dealing with experimental data, we do not know the underlying process. Nonetheless, the shadowing approach is useful. Eckmann and Ruelle argued that one could approximate the unknown dynamical process by looking at a segment of an experimental time series and finding other segments with almost the same pattern. By finding perhaps 50 segments with very similar patterns, one can try to determine what the underlying dynamics is for similar patterns. We then use shadowing on those segments to eliminate much of the noise. This method is only useful if the physical system has a low dimensional chaotic attractor.

5. H. E. Nusse and J. A. Yorke, 'Period halving for $x_{n+1} = MF(x_n)$ where F has negative Schwarzian derivative,' Phys. Lett. A 127 (1988) 328-334.

This paper gives an example showing that the monotonic development of chaos does not hold in general for a class of one dimensional maps that are a natural generalization of the logistic map. The map in this example has a negative Schwarzian derivative. For this example, chaos apparently increases as the parameter is increased until some critical parameter value is reached, and then it decreases as the parameter is increased further.

6. S. M. Hammel, J. A. Yorke, and C. Grebogi, Numerical orbits of chaotic processes represent true orbits, Bull. Amer. Math. Soc. <u>19</u> (1988) 465-469.

The following papers have been submitted for publication:

7. H. E. Nusse and J. A. Yorke, A procedure for finding numerical trajectories on chaotic saddles, Physica D, Nonlinear Phenomena. This paper develops numerical algorithms for observing chaotic trajectories that do not sit on chaotic attractors. An example discussed is the horseshoe map on a rectangle. It has chaotic trajectories that remain inside the rectangle. The algorithms are proved to be valid in this case and a large group of examples are studied. Another paper is in preparation (see #10) which will discuss the validity more generally for hyperbolic systems. The examples illustrate that the procedure works well on computers for long periods of time even when the system lacks hyperbolicity.

8. I. Kan and J. A. Yorke, Antimonotonicity: Concurrent creation and annihilation of periodic orbits, Bulletin Am. Math. Soc. This is the first of a pair of papers. These papers illustrate the idea that chaos does not develop monotonically as a parameter is varied. We say it is "antimonotonic". "Antimonotonicity" is the following property: if the system becomes more chaotic as a parameter is varied over an interval, then over some subinterval it becomes less chaotic. The technical definition is in terms of whether homoclinic points are added to the system. If so, we say chaos increases. The results of the first paper establish these results for a standard prototype situation. The second paper will show that the results of the prototype are valid for general two dimensional systems that depend on a parameter assuming only mild non-degeneracy conditions. The work on this second paper has begun.

These papers are soon to be submitted:

9. K. T. Alligood and J. A. Yorke, Accessible saddles on fractal basin boundaries. This paper had its basic origins in questions that arose when Yorke gave a lecture on fractal basin boundaries at AFOSR. Fractal basin boundaries have incredibly complicated structures. This paper finds a simple way of viewing these structures. When the attractors (in R^2) are fixed points, as illustrated by the Poincare return map of the forced damped pendulum, the basin of attraction is topologically an open disk. The boundary of this disk is not a circle but rather is the entire fractal basin

boundary. Nonetheless, we show that the map acts to twist the basin giving it a rotation number. We analyze what happens when this rotation number is rational. In that case, there is an envelope enclosing the entire boundary. This envelope consists of the stable manifolds of periodic orbits with periods corresponding to the rational rotation number of the basin.

10. H. E. Nusse and J. A. Yorke, Analysis of a procedure for finding numerical trajectories close to chaotic saddle hyperbolic sets. (In draft form)

James A. Yorke

Invited Lectures 1988

JANUARY

Houston, Texas - Invited Speaker at Dynamics Days, Texas

FEBRUARY

Boston, Massachusetts - Invited Speaker at AAAS Annual Meeting

MARCH

Columbus, Ohio - Invited Speaker at International Conference on Differential Equations

APRIL

Syracuse, New York - Invited Speaker at Joint Physics-Mathematics Colloquium

National Bureau of Standards, Gaithersburg, MD - Lecture in Minicourse on Chaos

Northwestern University, Evanston, Illinois - Invited Speaker at Midwest Dynamical Systems Conference

MAY

Princeton University, Princeton, NJ - Symposium on Visualizations in Scientific Computing

Metre Corp, McLean, VA - Seminar on Chaotic Dynamics

JUNE

Utrecht, The Netherlands - Invited lecturer at Symposium on Chaotic Dynamical Systems (5 lectures)

Düsseldorf, W. Germany - Invited speaker at Dynamics Days

JULY

Minneapolis, Minnesota - Invited speaker at SIAM Minisymposia (2 lectures)

AUGUST

Brown University, Providence, RI - Invited lecturer at AMS Short Course on Chaos and Fractals

Brown University, Providence, RI - Invited speaker at Conference on Differential Equations

James Yorke

Invited Lectures 1988

OCTOBER

Johns Hopkins Univeristy, Applied Physics Laboratory, Columbia, MD - Keynote Speaker

Princeton Plasma Physics Laboratory, Princeton, NJ - Invited Lecture

Catholic University, Washington, DC - Invited Lecture at Saenz Symposium

University of Indiana, Bloomington - Colloquium Lecture

University of Maryland, College Park, MD - Numerical Analysis Seminar talk

NOVEMBER

Vanderbilt University, Nashville, TN - Shanks Lecture Series (2 lectures)