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Large-scale, low-resolution simulation models frequently group nuclear weapons effects against area targets into a few aggregated state vectors. This can create serious inaccuracies in the treatment of multiple burst and other time-dependent effects. This paper presents improved techniques for updating nuclear weapon effects against area targets, allowing the analyst to model the processes of delayed injury (damage), recovery (repair) over time, and the effects of subsequent nuclear bursts. Our approach is to store the effects in a state vector of coefficients for a piecewise Pareto distribution. Examples illustrating the benefits of this approach are provided.

This procedure will be implemented in the US Army Concepts Analysis Agency IWFORCEM theater model, but is applicable to any low-resolution simulation that includes nuclear effects representation.

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NOVEMBER 1988

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REQUIREMENTS DIRECTORATE

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UPDATING NUCLEAR EFFECTS IN THEATER MODELS

ABSTRACT

Large-scale, low-resolution simulation models frequently group nuclear weapons effects against area targets into a few aggregated state vectors. This can create serious inaccuracies in the treatment of multiple burst and other time-dependent effects. This paper presents improved techniques for updating nuclear weapon effects against area targets, allowing the analyst to model the processes of delayed injury (damage), recovery (repair) over time, and the effects of subsequent nuclear bursts. Our approach is to store the effects in a state vector of coefficients for a piecewise Pareto distribution. Examples illustrating the benefits of this approach are provided.

This procedure will be implemented in the US Army Concepts Analysis Agency IWFORCEM theater model, but is applicable to any low-resolution simulation that includes nuclear effects representation.

THE METHODOLOGY MAY BE APPLIED TO any simulation model (a "low resolution" model) that combines or aggregates the exposure history and location, for each target element (personnel and equipment represented in the simulation model) receiving nuclear effects in a unit, across that unit, rather than tracking each element separately. The simulation should have a nuclear effects module that calculates the amount of target area covered by any given level of effect, given target area size and shape, actual ground zero, weapon characteristics, etc.

MAJOR ASSUMPTIONS include the following:

1. Target elements are uniformly dispersed over a circular or rectangular target area.
2. The effects received by target elements are aggregated into a single state vector for each unit; the unit (e.g., division) may be composed of subordinate units (e.g., battalions).
3. The probability that any given target element has received a given level of effect is identical for all target elements; this is equivalent to assuming that the unit population is randomly "mixed" after the effects of each detonation have been calculated. This assumption may not hold if a target receives effects from multiple bursts within a short interval of time; however, it is consistent with the manner in which multiple effects are normally computed in simulation models.
4. For each type of target element separately represented in the simulation, there is either a single dominant effect which is consistent throughout the simulation, or the previous exposure history for each effect is recorded in separate state vectors.

THE MAJOR LIMITATIONS of this methodology are:

1. The assumption of uniform dispersion of target elements with respect to the target area and with respect to the previous exposure history. This assumption should be consistent with the simulation model's treatment of conventional casualties and other combat effects.
2. The methodology computes each nuclear effect separately, but does not provide a mechanism for adding exposures to different effects (e.g., exposure to both blast and nuclear radiation).

THE RESEARCH PAPER IS DIRECTED TOWARD analysts working in simulation modeling that are familiar with basic probability theory. It is not intended for a lay audience.

THE RESEARCH PAPER WAS COMPLETED IN November 1988.

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COMMENTS AND QUESTIONS may be directed to MAJ Mark A. Youngren, US Army Concepts Analysis Agency, CSCA-RQN, 8120 Woodmont Ave., Bethesda, MD 20814-2797.

CONTENTS

CHAPTER 1

SUMMARY

1-1. INTRODUCTION

Large-scale, low-resolution simulation models cannot model the history of nuclear effects exposure for every soldier and piece of equipment on the battlefield. As a result, such models group nuclear weapons effects against area targets into a few aggregated state vectors. This can create serious inaccuracies in the treatment of multiple burst and other time-dependent effects when updating the current state of each unit in the simulation.

This paper develops a methodology, based on a probability model, which permits us to determine the current levels of exposure to nuclear effects within a unit at any point in time during a simulation with reasonable accuracy. Furthermore, the model will:

- Represent the effects of delayed injury or damage that degrade unit capabilities over time.
- Represent the effects of recovery or repair that improve unit capabilities over time. These two processes will oppose each other.
- Determine the effect of subsequent nuclear bursts on a previously exposed unit.
- Represent the impact of residual nuclear radiation.
- Account for protection provided by vehicles, emplacements, etc.
- Account for the impact of conventional casualties and replacements.
- Aggregate small unit effects into larger unit effects.

This model does not attempt to solve the real problem of translating the level of effect into an operational impact on each target element or on the unit. Assumptions on how personnel and equipment exposures to given levels of effect will affect the unit are, of necessity, already present in combat simulation models. Given whatever assumptions already found in our large, low-resolution simulation model, the techniques discussed in this paper will more accurately record and update the nuclear effects on the unit over time.

1-2. RESULTS

We can approximate the distribution of nuclear effects such as nuclear radiation, thermal radiation, or blast received by a target element using a piecewise Pareto distribution with a discrete component. The levels of each nuclear effect are divided into categories i, $i = 0,1,\ldots,N$, according to the operational effect that each category has upon the unit within the combat simulation (see paragraph 2-2). The parameters of the piecewise Pareto distribution are formed from the percentages of elements P_i that are initially placed within each category i based on the nuclear effects algorithms invoked at the time of burst, and the threshold levels used to define the upper level of each category i, U_i, for $i = 0,1, \ldots, N$, where a total of N categories are defined. The probability that any element receives a level of effect X (for example, a dose of radiation) at the time of the detonation is:

$$P[X = x] = \begin{cases} P_0 & x = 0, \\ P_i\, c_i\, x^{-2} & U_{i-1} \le x < U_i,\ i = 1, \ldots, N;\quad U_N \equiv \infty \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{where } c_i = P_i \Big/ \left[\frac{1}{U_{i-1}} - \frac{1}{U_i} \right]. \qquad [1\text{-}1]$$

This distribution requires only the storage of the percent P_i of elements that fall within each category i of the nuclear effect for each unit.

The processes of delayed injury and recovery will cause the percentages of elements falling within each category to shift. If the *rate* of delayed injury or recovery is time-dependent, we store an additional value δ_t for each unit, where δ_t is a multiplying factor that indicates the amount of increase or decrease in the level of effect experienced after a time t has elapsed, due to delayed injury or recovery. An example of the updating formulas that result from this approach is given in Appendix A. We may also store an additional value γ_t for each unit if we track exposure to effects such as residual nuclear radiation. Given the P_i's, δ_t, and γ_t, we can compute at any time the current number of elements falling within each category i, as well as the probability that any element receives a particular level of effect.

At any time t, the current level of effect X_t is

$$X_t = \delta_t X + \gamma_t \qquad [1\text{-}2]$$

where X is distributed as [1-1], a piecewise Pareto distribution.

We also use the piecewise Pareto distribution to account for the additive effect of multiple bursts. Although the sum of two random variables having Pareto distributions is not Pareto, we show that we may closely approximate this sum with a Pareto random variable. Provided we assume that the target unit population is "mixed" between bursts, we can use our distribution [1-1] for any unit in our simulation, regardless of how many weapons have detonated near the unit.An example of the updating formulas used for multiple bursts is given in Appendix A.

Details on how to separately account for blast, thermal radiation, and residual nuclear radiation, as well as protection from nuclear effects, is provided in the main body, using the piecewise Pareto approach. We also discuss the procedure used to aggregate and disaggregate the effects distributions between smaller and larger units.

The piecewise Pareto distribution allows us to easily compute equations that can be used to update the nuclear effects to units over time. The details are provided in the main body of this paper; an example is summarized below.

1-3. AN EXAMPLE

To illustrate the applications of the techniques given in this paper, we develop an example showing two nuclear bursts and the effect of radiation on personnel, to include biological recovery from radiation exposure. The results are summarized in Tables 1-1 through 1-4. Comparable results have been included using a simple unclassified nuclear effects algorithm (the "actual" results) (USANCA [1981]).and using a uniform distributional assumption.

We assume that we have an circular area target with a radius of 1,300 meters, with previously unexposed, unprotected personnel distributed uniformly within the target area. The unit personnel are exposed to the effects of a 1-kiloton weapon aimed at the unit detonating 100 meters from the center of the target area. After the burst, we get the results shown in Table 1-1.

Table 1-1. Parameters P_i for the First Nuclear Burst, Day 0.

P_0	P_1	P_2	P_3	P_4	P_5
0	.009	.146	.295	.393	.157

When we account for biological recovery from radiation over 3 days, we get Table 1-2:

Table 1-2. Parameters P_i for the First Nuclear Burst, Day 3.

$P_0^{(3)}$	$P_1^{(3)}$	$P_2^{(3)}$	$P_3^{(3)}$	$P_4^{(3)}$	$P_5^{(3)}$
0	.027	.155	.293	.367	.157

On day 3, a weapon detonates at a distance of 2.3 kilometers away from the center of the target area. The results for this burst only are displayed in Table 1-3.

Table 1-3. Parameters P_i for the Second Nuclear Burst.

P_0	P_1	P_2	P_3	P_4	P_5
.017	.920	.039	.024	0	0

Using the formulas provided in Appendix A to combine the effects of the two bursts yields the combined effects of the two doses:

Table 1-4. Parameters P_i' for the Combined Bursts, Day 3.

P_0'	P_1'	P_2'	P_3'	P_4'	P_5'
0	.026	.146	.295	.376	.157

For comparison, we update the effect after 30 days and compare the results of this model with those calculated using a simple uniform distribution within each category and using a detailed simulation. The results are displayed in Figure 1-1. We can see that the model proposed within this paper provides a 30-day distribution fairly close to that obtained from a detailed high-resolution simulation, while the uniform distribution did not. Using our model, we can realize an improvement in the representation of the status, over time, of elements in a theater simulation model; this improved representation should improve the fidelity of simulation results which depend upon the status of elements within each modeled unit.

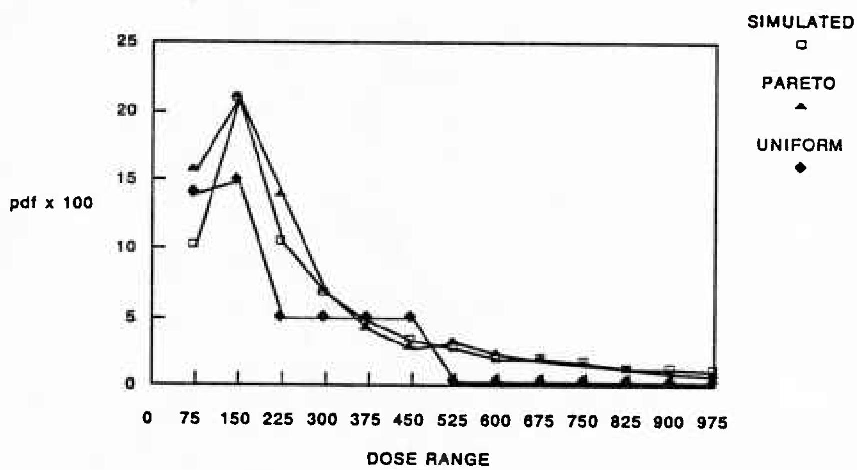

Figure 1-1. Simulated and Calculated Segments of the Radiation Exposure Probability Density Function (pdf) 27 Days After the Second Nuclear Burst

CHAPTER 2

BACKGROUND

2-1. MOTIVATION

Combat models incorporating tactical nuclear warfare generally simulate the effect of the detonation of nuclear weapons using standard algorithms to compute the level of various weapon effects (e.g., blast, thermal, and nuclear radiation) that will be experienced. Regardless of the level of resolution of the model, the levels of effects caused by the weapon can be categorized according to the operational impact of the damage or injury caused by each level of effects upon the unit. For example, radiation exposures against personnel falling within a given range are classed as latent lethal; exposures falling within a higher range are immediate transient incapacitating, etc. Once the categories have been defined, the percentage of personnel and equipment that receive specified levels of effect within each category is treated as proportional to the area of the target covered by the corresponding levels of effect.

High-resolution models generally maintain information on the effects experienced by and the location of each combat element modeled (a combat element is something we represent explicitly in a simulation -- e.g., a soldier, a tank, etc.). The impact on the unit at any time is determined by computing the proportion of elements within each category. When the model keeps track of the previous exposure suffered by each target element, the effects of subsequent nuclear bursts (which will create damage or injury additive to the previous injury) and other time-dependent effects, such as delayed injury and recovery, can easily be determined.

Low-resolution models involving large forces, such as corps and armies, cannot afford to simulate every element. They compute the percentage of assets exposed to effects within each category, storing only the percentages pertaining to each unit. In this case, the computation of time-dependent effects becomes difficult. For example, when we compute the probability that any person within that category will become sick so many days after exposure (and thereby become combat ineffective), the probability will depend upon the actual dose received within the category. If that person is exposed to radiation at a later date, the combined effect will clearly depend upon the previous exposure.

In the absence of the actual distribution of effect received by elements within each category, a uniform distribution is frequently assumed. For example, if an element has received a previous exposure to radiation within a category from 150 to 450 rad, all values between 150 and 450 are considered equally likely. Unfortunately, this uniform assumption yields significantly different results from those obtained from high-resolution simulations. Our intermediate goal is to establish a reasonable, analytically tractable approximating distribution for the effects received by any target element. Meeting our intermediate goal allows us to meet our objective: to create a set of algorithms that will allow us to update nuclear effects over time, accounting for delayed injury, recovery, and multiple burst exposures with a reasonable degree of accuracy, without having to perform a high-resolution simulation of each target element.

Because low resolution models do not track the location and exposure history for each individual combat element, it is necessary to make some assumptions. First, elements are assumed to be evenly (randomly) distributed across the target unit area, which has a single recorded location (normally center-of-mass) and defined shape. Because we do not know the previous exposure history for any given element in the unit, we must assume an equal probability for each element with respect to the distribution of previous exposures. This methodology is applied to compute conventional effects within models (for example, attrition is applied evenly within each category of element within the smallest entity defined as a unit), and these assumptions carry over to the nuclear effects representation. The author is not aware of any method for avoiding these assumptions short of keeping track of the individual locations and previous exposures, which makes the model "high resolution". The assumptions employed to compute the nuclear effects must be consistent with those employed to compute other combat effects.

2-2. THE SCENARIO

We begin by considering a target unit that is affected by the detonation of a nuclear weapon at or near the target location. The detonation of a nuclear weapon will cause levels of several effects (blast, thermal radiation, and immediate nuclear radiation) which we model as a circular pattern around the actual ground zero (AGZ) of the weapon.

In a theater-level model, we generally adopt a simple target representation. Each target area is represented by a circle of known radius R_t, with personnel and equipment of various types distributed uniformly within it. A rectangular target area with elements uniformly distributed may also be used (see page 3-3).

As mentioned above, low-resolution models do not attempt to track the actual level of effect received by each target element. Rather, the set of all possible values of each effect is partitioned into categories and the percent of the unit area covered by effects within each category is determined. Treating all target elements as uniformly distributed within the target area allows us to translate the area calculation into a percentage of the unit onhand assets that receive an effect within each category. The unit status is stored in a state vector $\mathbf{P} \equiv \{ P_0, P_1, \ldots, P_k \}$, where each P_i equals the proportion of unit assets receiving effects in category i.

The number of categories k to use depends on how these categories are used within the simulation. For example, if the simulation only represented the operational impact of 3 categories within the model - unhurt, walking wounded, and dead, then a logical choice for k might be 2 (category 0 = level of effect less than that required to create "walking wounded" however that is defined within the model, category 2 = level of effect required to kill, and category 1 = everything in between). More categories would give a better fit for the piecewise Pareto distribution, but would not improve the representation of the operational impact within the model.

This paper uses the radiation effect against personnel to demonstrate the techniques involved. However, the techniques described may be applied separately to blast, thermal radiation, or immediate nuclear radiation against any defined target element type, using any consistent set of category definitions. The detailed issues of combining various effects (e.g., the cumulative effect of blast damage, thermal burns, and radiation exposure) are outside the scope of this paper. Standard techniques may be used, such as assuming a dominant effect (assuming that one effect dominates the others, so only it needs to be represented) for various weapons against type target elements.

For the purpose of demonstration, we categorize radiation effects against personnel as follows:

Category 0 : 0 - 2 rad (no significant exposure)
Category 1 : 2 - 75 rad (exposure; no operational effect)
Category 2 : 75 - 150 rad (operational radiation casualty)
Category 3 : 150 - 450 rad (radiation burdened)
Category 4 : 450 - 8000 rad (radiation disabled)
Category 5 : Over 8000 rad (radiation fatality).

These categories are currently used in CAA's Integrated Warfare Force Evaluation Model (IWFORCEM) (Booz·Allen & Hamilton [1988]) [except for category 0, which we added to refine our representation of the additive effects of subsequent bursts]. These definitions are useful for IWFORCEM; other categories may prove useful in other applications. The breadth of some of the categories (e.g., category 4) makes the need to distinguish between exposures at the upper and lower end of the category evident.

We assume that standard nuclear effects algorithms are available to compute the following: given the detonation of a weapon of yield W at distance D from the center of a circular target of radius R_t, find the area of the target covered by radiation (or other nuclear effects) at or above a specified level U. For example, a weapon that creates radiation levels at or above 150 rad out to a distance of 1200 m, detonating 1500 m from a target with radius 700 m, will cover approximately 20 percent of the target area with radiation at or above 150 rad [NOTE: All nuclear effects computed in this paper are based on UNCLASSIFIED algorithms provided by the US Army Nuclear and Chemical Agency.]

2-3. THE APPROACH

In order to provide a reasonable basis for defining the distribution of nuclear effects, we regard the components of our unit state vector $\mathbf{P}$ not as the relative proportions of assets in each category, but instead as parameters which characterize the probability distribution function for the level of effect realized by any randomly selected element within the unit.

If we define the random variable X_t as the level of the (dominant) effect experienced by a target element in a unit t units of time after the most recent nuclear detonation affecting that unit, then we model X_t as:

$$X_t = \delta_t X_0 + \gamma_t \qquad [2\text{-}1]$$

where X_0 is the level of effect at the time $t = 0$, δ_t is a multiplying factor that indicates the amount of increase or decrease in the level of effect experienced after a time t has elapsed, due to delayed injury or recovery, and γ_t is an offset parameter. We will show that the distribution of X_t can be closely approximated by an offset piecewise Pareto distribution for both single and multiple bursts. To implement this concept, we replace the k-dimensional vector $\mathbf{P}$ with a (k+2)-dimensional vector $\mathbf{P}^{(0)} \equiv \{ P_0, P_1, \ldots, P_k, t_0, \gamma_0 \}$, which characterizes the distribution of X_t.

We justify our distribution for X_t by establishing the distribution of individual target elements as a function of range from the weapon AGZ. From this, we derive a piecewise Pareto distribution for the level of effect realized by any target element. We show that subsequent burst effects can also be approximated by a piecewise Pareto distribution, which allows us to establish our procedures for updating nuclear effects on the target unit over time. We discuss various modifying factors such as protection, conventional attrition, and exposure to residual radiation, demonstrate how to aggregate small unit distributions into larger unit distributions, and conclude with a brief discussion of implementation.

The appendices provide examples illustrating how these techniques may be used in a nuclear simulation. Appendix A gives sample equations for updating radiation effects over time and combining multiple bursts. Appendix B develops a specific example using two weapons directed at a unit at different times, updating the effects over a 30-day period. The results are compared to a high-resolution simulation and to those obtained using a uniform distribution for the levels of effect.

2-4. A PERSPECTIVE ON $\mathbb{P}$

The vector $\mathbb{P}^{(t)} \equiv \{ P_0^{(t)}, P_1^{(t)}, \ldots, P_5^{(t)}, \delta_t, \gamma_t \}$ describes at any time t the current state of a unit in our simulation. The transitions from one state to another over time, representing the processes of delayed injury (damage) or recovery (repair), depend upon the distribution of personnel and equipment within each category. $\mathbb{P}^{(t)}$ defines the parameters of the distribution of the levels of effect relative to the distribution computed at the time of the most recent detonation; thus, our transition to a new state, say $\mathbb{P}^{(t+1)}$, may be calculated from the vector $\mathbb{P}^{(0)}$. If subsequent bursts are not anticipated, it is simple to develop analytic results in lieu of simulation.

In the case of subsequent bursts, we shift from a state vector $\mathbb{P}^{(t)}$ to a combined effects state vector $\mathbb{P}^{(0)*}$ and adjust our base time t_0 to t. The transition matrix from $\mathbb{P}^{(t)}$ to $\mathbb{P}^{(0)*}$ will depend upon the location and type (size, height of burst, etc.) of the burst causing the transition and the time t elapsed since the previous detonation. Direct analysis of subsequent bursts (which defines the weapon and location parameters) in the context of a model and scenario is straightforward; the techniques introduced in this paper may be used to determine $\mathbb{P}^{(0)*}$.

CHAPTER 3

METHODOLOGY DEVELOPMENT

3-1. THE DISTRIBUTION OF TARGET ELEMENTS AS A FUNCTION OF RANGE TO THE AGZ

In order to compute the level of nuclear effects received by target elements, it is necessary to determine the distribution of elements as a function of their range to the point of detonation (actual ground zero, or AGZ). We begin by considering the distribution of elements within the target area relative to the center of the target. An element could be a soldier, an item of equipment, etc.

Let S = radial coordinate of an individual element, $0 \leq S \leq R_t$

Let T = angular coordinate of an individual element, $0 \leq T < 2\pi$

Since we assume that the elements are uniformly dispersed over a circular target area,

$$\lim_{\substack{ds \to 0 \\ dt \to 0}} P\,[\, s \leq S < s + ds,\ t \leq T < t + dt\ ;\ 0 \leq s \leq R_t\ ;\ 0 \leq t < 2\pi \,]$$

is equal to a constant for all s, t. Equivalently, if we pick any radial distance s, each point on the circumference of a circle of radius s is equally likely to be the location of a target element. Thus

$$f_S(s)\,ds \;\simeq\; \frac{[\text{area of circle of radius } s+ds] - [\text{area of circle of radius } s]}{[\text{ area of the target }]}$$

$$f_S(s) \;=\; \frac{2\pi s}{\pi R_t^2} = \frac{2s}{R_t^2}\,, \qquad\qquad 0 \leq s \leq R_t.$$

Similarly, $f_T(t) = \frac{1}{2\pi}$, $0 \leq t < 2\pi$. A uniform dispersion implies that S and T are independent; thus

$$P[\,S = s, T = t\,] = \frac{2s}{R_t^2} \cdot \frac{1}{2\pi} = \frac{s}{\pi R_t^2}\ ,\ 0 \leq s \leq R_t\ ,\ 0 \leq t < 2\pi. \qquad [3\text{-}1]$$

The areas that receive the levels of effect in category 0, 1, 2, etc., form a series of concentric circles with an outside radius R_W , where R_W is defined to be the maximum distance at which the weapon is assumed to affect a particular type of target element. Since category 0 denotes levels of effect equivalent to no exposure, R_W will be the distance from the AGZ at which category 0 begins. Let us label the radial distances from outer to inner as follows (see Figure 3-1. Also note that $R_0 = R_W$):

R_0 = closest distance to AGZ at which Category 0 begins.
R_1 = closest distance to AGZ at which Category 1 begins.
R_2 = closest distance to AGZ at which Category 2 begins.
R_3 = closest distance to AGZ at which Category 3 begins.
R_4 = closest distance to AGZ at which Category 4 begins.

We define constants U_i as the level of effect received by a target element at each distance R_i. Note that U_i can also be viewed as the upper threshold or upper boundary of category i, $i = 0, \ldots, 5$. For convenience in notation, we define R_5 as 0 and U_5 as ∞. If we examine Figure 3-1, we see that the potential coverage area for the effect within each category (except Category 5) forms a ring or annulus shape. Therefore, the geometry of the area within a target that receives an effect within a particular category is defined by the intersection of a ring or disk (representing the weapon effects) with another disk (representing the target).

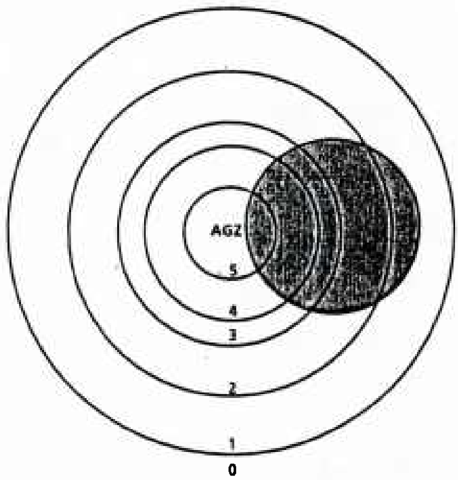

Figure 3-1. Weapon Effects Radii Overlapping a Target Unit

Consider the following possible geometries (see Figure 3-2): Case 1 represents a miss for a given level of effect against a type target element within the target area. The effect against elements within the target is obviously nil, and we need not consider this case further in this paper. Cases 2 through 4 are more interesting. The shaded area i indicates the area of the target overlapped by the effect within Category i. Case 3 includes the case where we examine the overlap of Category 5 on the target area.

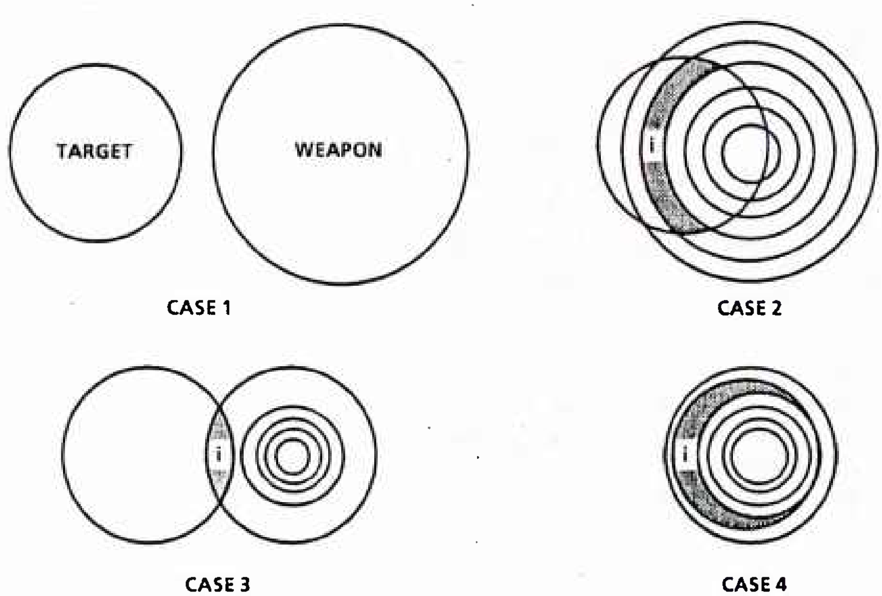

Figure 3-2. Some Possible Overlap Geometries

In each case, we can compute the area of the overlap, A_i, using standard disk-on-disk calculations (cf. DeRiggi and Helmbold [1987]). This value is an output of any standard effects algorithms. In order to obtain the distribution of the target elements as a simple function of the range to the AGZ, we will approximate the overlap i with a shape i', where i and i' have identical areas A_i. The overlap area i' is defined as the segment of a ring formed by R_i and R_{i-1} within an angular measure α_i (see Figure 3-3). The areas i' are shown in Figure 3-4 for Cases 2 through 4. It is clear that i' is very close to i in Case 2; the fit is not as good in Cases 3 and 4, but should provide a reasonable approximation that is analytically tractable. If we assume a rectangular target, we also approximate the area of the overlap of the circular weapon effects radii on the rectangular target area by segments i' of a ring. The fit is not as good as disk-on-disk, but it still forms a reasonable approximation.

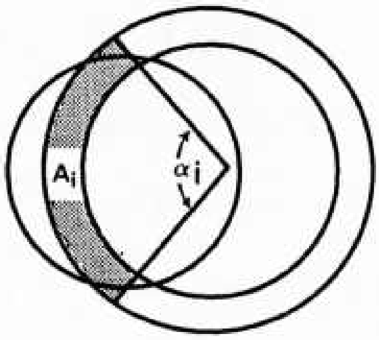

Figure 3-3. Example of Approximating Area i'

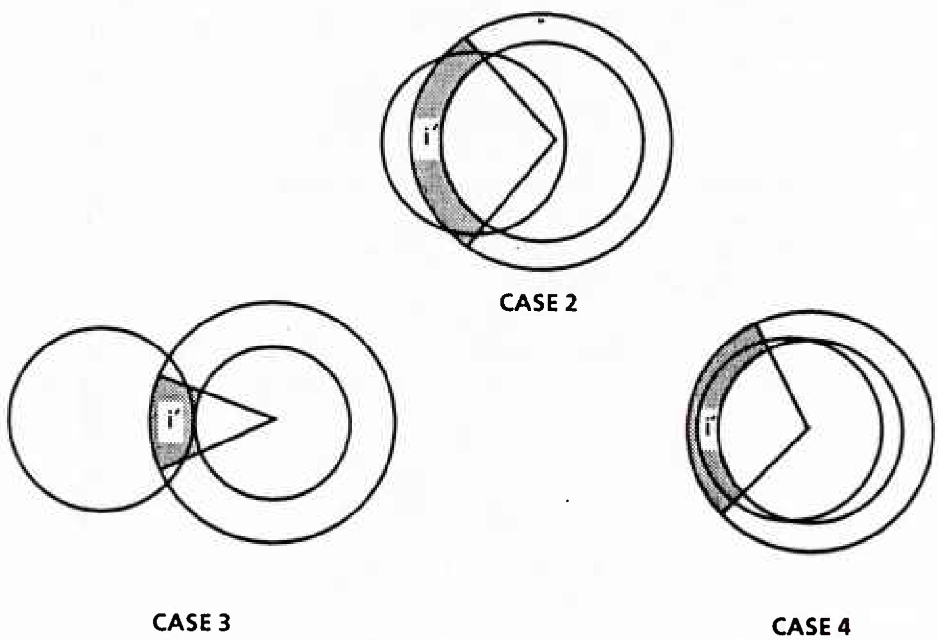

Figure 3-4. Examples of Area i' for Cases 2, 3, and 4.

In all cases, if P_i is the proportion of the target area covered by effects within Category i,

$$P_i = \frac{A_i}{\text{target area}} = \frac{A_i}{\pi R_t^2}\,. \tag{3-2}$$

Note that P_i can also be regarded as the probability that any randomly selected target element received a level of effect within Category *i*.

Let us consider the area A_i formed by *i'* (see Figure 3-4). Since the areas are the same,

$$P_i \equiv P[\text{ element in area } i\,] \simeq P[\text{ element in area } i'\,] = \frac{A_i}{\pi R_t^2}.$$

Since the target elements are uniformly dispersed within *i'*, they will be uniformly dispersed with respect to coordinates defined at any origin. If we define coordinates S_W and T_W *relative to the weapon AGZ*, then from [3-1], $P[\,S_W = s,\, T_W = t\,] \propto \frac{s}{\pi R_W^2}$. Therefore,

$$\frac{A_i}{\pi R_t^2} = \int_{s = R_i}^{R_{i-1}} \int_{t = \theta}^{\theta + \alpha_i} \frac{k\,s}{\pi R_W^2}\, dt\, ds = \frac{k\,\alpha_i}{2\pi R_W^2}\,(\,R_{i-1}^2 - R_i^2\,) \text{ for some k.}$$

Clearly, $k = \frac{R_W^2}{R_t^2}\,\frac{A_i}{\alpha_i/2}\,\frac{1}{(\,R_{i-1}^2 - R_i^2\,)}$. However, from Figure 3-3 it is clear that

$A_i = \frac{\alpha_i}{2}\,(\,R_{i-1}^2 - R_i^2\,)$, so $k = \frac{R_W^2}{R_t^2}$, and

$P[\,S_W = s,\, T_W = t;\text{ element in } i'] = \frac{s}{\pi R_t^2}\,,\quad R_i \le s < R_{i-1};\ \theta \le t < \theta + \alpha_i.$

From this,

$$P[\,S_W = s \text{ and element is in } i'\,] \equiv f_{S_W;i'}(s) = \frac{s\,\alpha_i}{\pi R_t^2}\,, \qquad R_i \le s < R_{i-1}\,, \qquad [3\text{-}3]$$

which is the probability density for the distance from the AGZ of an element in a target unit receiving effect levels within Category *i* .

3-2. COMPUTING THE DISTRIBUTION OF THE EFFECTS

We approximate the nuclear effects of interest using the inverse square law -- that is, the level of the effect is inversely proportional to the square of the distance. Although there are other factors (scattering, air absorption, etc.) that will cause the actual effect to deviate from the inverse square ideal, it forms a reasonable first order approximation.

Let X = level of effect received by a target element. We wish to determine $f_X(x) \equiv P[\,X = x\,]$ for each Category *i*. From the inverse square law, there exists a constant k_i such that

$$X = \frac{k_i}{{S_W}^2}\,,\quad \text{for } R_i \le S_W < R_{i-1}\,;$$

these bounds are equivalent to $U_{i-1} \le X < U_i$, $i = 1, \ldots, 5$.

We know that $f_{S_W}(s) = \frac{s\,\alpha_i}{\pi R_t^2}\,,\quad R_i \le s < R_{i-1}\,,\ i = 1, \ldots, 5.$

$$f_X(x) = \left|\frac{\partial S_W}{\partial X}\right| f_{S_W}(x) = \frac{k_i\,\alpha_i}{\pi R_t^2}\,x^{-2}\,, \qquad U_{i-1} \le x < U_i\,,\ i = 1, \ldots, 5.$$

Since $P[\,U_{i-1} \le X < U_i\,] = P_i = \int_{x = U_{i-1}}^{U_i} \frac{k_i\,\alpha_i}{\pi R_t^2}\,x^{-2}\,dx = \frac{A_i}{\pi R_t^2}\,,$

$$k_i = \frac{\frac{1}{2}\,(\,R_{i-1}^2 - R_i^2\,)}{\frac{1}{U_{i-1}} - \frac{1}{U_i}}\,.$$

An easier expression to evaluate can be derived if we define $c_i \equiv \dfrac{k_i\,\alpha_i}{\pi R_t^2}$. Clearly,

$$c_i \left[\frac{1}{U_{i-1}} - \frac{1}{U_i} \right] = P_i \,. \qquad [3\text{-}4]$$

Finally, we let $P_0 \equiv P[\, X = 0 \,]$ = proportion of target area outside of the largest effects circle, where $P_0 = 1 - \sum_{i=1}^{5} P_i$. Alternatively, we can define P_0 as $P[\, X < U_0 \,]$, since we have assumed that exposures less than U_0 are equivalent to no exposure.

We now have a complete description of the probability density function of the level of effect X, derived from the proportion of target covered by a particular level of effect, P_i [which comes from standard effects codes], and the threshold levels for each category, U_i [which we define], for $i = 0, \ldots, 5$.

$$f_X(x) = \begin{cases} P_0 & x = 0\,, \\ c_i\, x^{-2} & U_{i-1} \le x < U_i\,,\ i = 1, \ldots, 5\,, \\ 0 & \text{otherwise.} \end{cases} \qquad [3\text{-}5]$$

We recognize that this is a *piecewise Pareto distribution* with a discrete component at 0 (c.f. Johnson and Kotz [1970], pp. 233 - 249).

This density establishes the distribution immediately after detonation of any nuclear effect modeled using the inverse square law. We record the values P_i for $i = 0,1,\ldots,5$ in our state vector for each target. If the number of personnel or equipment found in each category did not change over time, this density would hold at any later time given P_0 through P_5 and U_0 through U_5. This density may be useful in analyses which consider only the effects immediately after a nuclear pulse, where any given target receives effects from only one nuclear burst. Of interest here, however, is how we can update these effects over time, and how we can account for effects derived from multiple bursts.

3-3. COMPUTING THE DISTRIBUTION OF EFFECTS FROM MULTIPLE BURSTS

It is possible that personnel or equipment within a particular unit may receive effects from more than one nuclear detonation. This will occur when a unit is in the area of secondary or bonus effects from detonations intended for other targets, or when a unit is engaged more than once by nuclear weapons. These additional exposures may occur at points separated in time.

We begin by examining the effect of a second detonation occurring some time (however brief) after an initial engagement.

Let X_1 denote the effect received by an element from the first detonation.
Let X_2 denote the effect received by the same element from the second detonation.
Let P_{i1} denote the proportion of the target area overlapped by levels of effect within Category i produced by the first detonation.
Let P_{i2} denote the proportion of the target area overlapped by levels of effect within Category i produced by the second detonation.
Let U_0 through U_5 denote the upper level of categories 0 through 5 as before.

Let $c_{ij} = P_{ij} \,/\, \left[\dfrac{1}{U_{i-1}} - \dfrac{1}{U_i} \right]$, $i = 1, \ldots, 5;\ j = 1, 2$.

We assume that the effects of the first detonation are dispersed uniformly over the target area prior to the second detonation; that is, the density of X_1 is the same at any location within the target area. We also assume that X_1 and X_2 are independent given the values P_{ij} and U_i .

We are interested in the sum of X_1 and X_2. This sum is not distributed as Pareto, but we will show that we may closely approximate it as a Pareto random variable with certain parameters. This allows us to use the piecewise Pareto distribution to represent the levels of effect experienced by the target elements for multiple as well as for single bursts.

Let $W = X_1 + X_2$. We wish to determine $P[\ W \leq w\ ]$ for each Category i. It will be more convenient to work with the cumulative distribution function (cdf), so we derive $F_X(x) \equiv P[\ X \leq x\ ]$.

$$F_X(x) = \begin{cases} 0 & x < 0\,, \\ P_0 & x = 0\,, \\ \sum_{k=0}^{i-1} P_k + \frac{c_i}{U_{i-1}} - \frac{c_i}{x} & U_{i-1} \leq x < U_i\,,\ i = 1, \ldots, 5\,. \end{cases} \qquad [3\text{-}6]$$

We use the notation "$X \in H_i$" to denote $\{\ X \mid U_{i-1} \leq X < U_i\ \}$. For each Category i ($W \in H_i$), we can derive the cdf of W by conditioning on X_1 and X_2, recalling that X_1 and X_2 are independent.

$$P[\ W \leq w \text{ and } W \in H_i] = P[\ X_1 + X_2 \leq w\ ;\ W \in H_i\ ]$$

$$= \sum_{j=0}^{5} \sum_{k=0}^{5} P[\ X_1 + X_2 \leq w\ ;\ W \in H_i \mid X_1 \in H_k,\ X_2 \in H_j\ ] \cdot P[\ X_1 \in H_k\ ]\, P[\ X_2 \in H_j\ ]$$

$$= \sum_{j=0}^{i} \sum_{k=0}^{i} \int_{x_1} P[\ X_2 \leq w - x_1\ ;\ W \in H_i \mid X_1 = x_1,\ X_1 \in H_k,\ X_2 \in H_j\ ] \cdot P[\ X_1 = x_1 \mid X_1 \in H_k\ ]\, P[\ X_1 \in H_k\ ]\, P[\ X_2 \in H_j\ ]\ dx_1$$

For $W \in H_0$:

$$P[\ W \leq w \text{ and } W \in H_0\ ] = P[\ W \leq w \mid W \in H_0\ ]\, P[\ W \in H_0\ ]$$

$$= 1 \cdot P_{01}\, P_{02} \qquad w = 0$$

For $W \in H_1$:

$$P[\ W \leq w \text{ and } W \in H_1\ ] = P[\ X_1 + X_2 \leq w\ ;\ W \in H_1\ ]$$

$$= \sum_{j=0}^{1} \sum_{k=0}^{1} P[\ X_1 + X_2 \leq w\ ;\ W \in H_1 \mid X_1 \in H_k,\ X_2 \in H_j\ ] \cdot P[\ X_1 \in H_k\ ]\, P[\ X_2 \in H_j\ ]$$

$$= \left[\ \frac{c_{12}}{U_0} - \frac{c_{12}}{w}\ \right] \cdot P_{01} + \left[\ \frac{c_{11}}{U_0} - \frac{c_{11}}{w}\ \right] \cdot P_{02} \qquad U_0 \leq w < 2U_0$$

$$P[\,W \le w \text{ and } W \in H_1\,] = P[\,X_1 + X_2 \le w\,;\, W \in H_1\,]$$

$$= \left[\frac{c_{12}}{U_0} - \frac{c_{12}}{w}\right] \cdot P_{01} + \left[\frac{c_{11}}{U_0} - \frac{c_{11}}{w}\right] \cdot P_{02}$$

$$+ \frac{c_{11}}{U_0}\left[\frac{w - 2U_0}{w - U_0}\right]\left[\frac{c_{12}}{U_0} - \frac{c_{12}}{w}\right] - \frac{2\,c_{11}\,c_{12}}{w^2}\ln\left[\frac{w}{U_0} - 1\right] \qquad 2U_0 \le w < U_1 \qquad [3\text{-}7]$$

We recognize that all of the terms in [3-7], except the last two for $w \ge 2U_0$, are of the form $K_1 - K_2/w$ for some constants K_1, K_2. This is the form of the piecewise Pareto cdf. When w is large, the order of the last term is $1/w^2 \simeq 0$ and the ratio $[\,w - 2U_0\,] / [\,w - U_0\,] \simeq 1$. When w is small, $1/w^2$ is no longer negligible, but the term $\ln[\,w/U_0 - 1\,]$ is, and the error introduced into these calculations by assuming that the ratio $[\,w - 2U_0\,] / [\,w - U_0\,] \simeq 1$ will have little tactical significance. Therefore, a reasonable approximation to the cdf of W for $W \in H_1$ is piecewise Pareto:

$$P[\,W \le w \text{ and } W \in H_1] \doteq$$

$$\left[\frac{c_{12}}{U_0} - \frac{c_{12}}{w}\right] \cdot P_{01} + \left[\frac{c_{11}}{U_0} - \frac{c_{11}}{w}\right] \cdot P_{02} + \frac{c_{11}}{U_0}\left[\frac{c_{12}}{U_0} - \frac{c_{12}}{w}\right], \qquad U_0 \le w < U_1\,. \qquad [3\text{-}8]$$

<u>Example 1:</u>

Let us compute $P[\,W \le U_1$ and $W \in H_1]$, using the U_i categories found in IWFORCEM. $P[\,W \le U_1$ and $W \in H_1] \doteq$

$$\left[\frac{c_{12}}{U_0} - \frac{c_{12}}{U_1}\right] \cdot P_{01} + \left[\frac{c_{11}}{U_0} - \frac{c_{11}}{U_1}\right] \cdot P_{02} + \frac{c_{11}}{U_0}\left[\frac{c_{12}}{U_0} - \frac{c_{12}}{U_1}\right],$$

$$= P_{12} \cdot P_{01} + P_{11} \cdot P_{02} + \frac{c_{11}}{U_0} \cdot P_{12}$$

$$= P_{12} \cdot P_{01} + P_{11} \cdot P_{02} + \frac{75}{73} P_{11} \cdot P_{12}\,.$$

If we calculate the exact probability instead of the approximation, we get $P_{12} \cdot P_{01} + P_{11} \cdot P_{02} + P_{11} \cdot P_{12}$ instead of the above, for a difference of $2/73\ P_{11}\,P_{12}$.

Calculations for $W \in H_2$, H_3, etc. are carried out in a similar manner. For example,

$$P[\,X_2 \le w - X_1\,;\, W \in H_2 |\, X_1 \in H_1, X_2 \in H_2\,]\, P[\,X_1 \in H_1\,]\, P[\,X_2 \in H_2\,]$$

$$= \frac{c_{11}}{U_0}\left[\frac{w - (U_0 + U_1)}{w - U_1}\right]\left[\frac{c_{22}}{U_1} - \frac{c_{22}}{w}\right] + \frac{c_{11}\,c_{22}}{w^2}\ln\left[\frac{U_0\,U_1}{(w - U_0)(w - U_1)}\right],$$

$$U_0 + U_1 \le w < U_2.$$

Once again, if we approximate $w - [\,(U_0+U_1) \,/\, (w - U_1)\,]$ by 1 and ignore the order $1/w^2$ term, we have a piecewise Pareto distribution. If we continue for $W \in H_3$, $W \in H_4$, and $W \in H_5$, we find that the distribution continues to be approximately piecewise Pareto.

Although the piecewise Pareto parameters for other combinations of X_1 and X_2 are easy to derive for all $w > 0$, we only need to determine the percentage of the target elements (P_i) in each category after the second detonation. To do this, we calculate the c_{ij}'s from the P_{i1} and P_{i2} values (the P_{i1}'s would be found in the state vector; the P_{i2}'s will be output from the standard effects algorithms at the time of the second detonation), and then calculate the piecewise Pareto distribution for W *only* at U_0, U_1, etc., to derive the new values for the state vector. For example, if P_i^* denotes the updated state vector after the second detonation, $P_0^* = P[\,W \le U_0$ and $W \in H_0\,]$, $P_1^* = P[\,W \le U_1$ and $W \in H_1\,]$, etc.

Complete formulas for updating the state vector given the P_i's from the first and second burst are given in Appendix A using the IWFORCEM personnel radiation exposure categories. The same formulas apply to any subsequent bursts.

Most effects calculations compute the areas A_i separately for each burst and simply add the resulting exposures; this is worse (in terms of accuracy) than using the assumption that the population is "mixed" between bursts, as it ignores the fact that, for example, two category 1 exposures may yield a category 2 exposure. If we make the approximation that the sum of two Pareto distributed variables is also distributed as Pareto, then we can use these techniques for any unit in our simulation, regardless of how many weapons have detonated near the unit. If the effects module can accurately add the effects of multiple, essentially simultaneous bursts on a single unit, then the net contribution of the bursts should be fitted to a piecewise Pareto to use this methodology. The fit will probably not be as good -- so it is a tradeoff between the accuracy of the $\mathbb{P}^{(0)}$ calculation and the goodness of fit for subsequent updates.

3-4. UPDATING NUCLEAR EFFECTS OVER TIME

Personnel and equipment categorized according to the impact of nuclear effects will tend to shift over time from one category to another as the processes of recovery or latent injury/damage manifest themselves. This may be regarded either as a decrease or increase in the effective level of effect experienced by an individual or piece of equipment over time, or it may be regarded as a shift in the category boundaries over time. In terms of determining the percentage shift from one category to another, the viewpoints are the same; the difference lies in interpretation. Because the categories are defined according to their impact on the current operational capability of personnel and equipment, we have chosen to regard the process of recovery and latent injury as equivalent to a decrease or increase in the level of effect with which the individual or piece of equipment is burdened. For example, a 1 percent biological recovery by an individual exposed to radiation is equivalent to reducing his exposure history by 1 percent. We represent the change in the current level of effect, X_t, after t time units have elapsed as a multiplier δ_t which may change over time. Thus

$$X_t = \delta_t X_0 \qquad [3\text{-}9]$$

where X_0 is the level of effect immediately after the detonation.

Example 2:

We will illustrate the method of updating $\mathbf{P} \equiv \{ P_0, P_1, \ldots, P_5 \}$ of our piecewise Pareto distribution for the unit with an example involving recovery to radiation exposure. Although biological repair to radiation is a subject of some controversy, several formulas have been proposed; we use one developed by Blair, referenced by SAIC [1984]:

$D_t = D_0 \cdot [\, \alpha + (1 - \alpha)\, e^{-\gamma t} \,]$, where
D_0 is the dose received by an individual on day 0,
D_t is the equivalent dose on day t,
α is the irreparable injury fraction (~ 0.1),
γ is the repair time constant (~ 0.024), and
t is the time in days.

After the first day, we would estimate that $D_t = 0.979\ D_0$. In this case, $\delta_t = D_t/D_0 = 0.979$. If we denote the percentages of personnel in each category i on day t, $t = 0,1$, as $P_i^{(t)}$, then for $i = 1$,

$$P_1^{(1)} = P[\, 2 \le \frac{D_t}{D_0} X < 75 \,] = P[\, \frac{2}{.979} \le X < \frac{75}{.979} \,]$$
$$= P[\, 2.04 \le X < 76.6 \,]$$
$$P_1^{(1)} = P[\, X < 75 \,] - P[\, X \le 2.04 \,] + P[\, 75 \le X < 76.6 \,]$$
$$= \left[\, P_0^{(0)} + P_1^{(0)} \,\right] - \left[\, P_0^{(0)} + \frac{c_1}{2} - \frac{c_1}{2.04} \,\right] + \left[\, \frac{c_2}{75} - \frac{c_2}{76.6} \,\right]$$
$$= P_1^{(0)} - 0.0098\ c_1 + 0.00028\ c_2 \,.$$

Since $c_i = P_i^{(0)} \Big/ \left[\, \frac{1}{U_{i-1}} - \frac{1}{U_i} \,\right]$,

$0.0098\ c_1 = 0.02\ P_1^{(0)}$ and $0.00028\ c_2 = 0.042\ P_2^{(0)}$. Thus

$$P_1^{(1)} = 0.98\ P_1^{(0)} + 0.042\ P_2^{(0)}.$$

Other values of $P_i^{(1)}$ would be calculated similarly.

To calculate $P_i^{(2)}$, we need to calculate the recovery over a 2-day period. From above, we see that $\delta_t = D_2/D_0 \doteq 0.958$. Thus

$$P_1^{(2)} = P[\, \frac{2}{.958} \le X < \frac{75}{.958} \,] = P[\, 2.08 \le X < 78.3 \,] \,.$$
$$= \left[\, P_0^{(0)} + P_1^{(0)} \,\right] - \left[\, P_0^{(0)} + \frac{c_1}{2} - \frac{c_1}{2.08} \,\right] + \left[\, \frac{c_2}{75} - \frac{c_2}{78.3} \,\right]$$

After some algebra, we see that $P_1^{(2)} = 0.96\ P_1^{(0)} + 0.084\ P_2^{(0)}$.

Alternatively, we could note that $\frac{D_2}{D_1} \cdot \frac{D_1}{D_0} = \frac{D_2}{D_0}$. Thus

$$P_1^{(2)} = P[\, 2 \le \frac{D_2}{D_0} X < 75 \,] = P[\, 2 \le \frac{D_2}{D_1} \cdot \frac{D_1}{D_0} X < 75 \,]$$

Since $\frac{D_2}{D_1} = \frac{.958}{.979} = 0.9785$, $\quad P_1^{(2)} = P[\, \frac{2}{.9785} \le \frac{D_1}{D_0} X < \frac{75}{.9785} \,]$

$$P_1^{(2)} = \left[P_0^{(1)} + P_1^{(1)} \right] - \left[P_0^{(1)} + \frac{c_1^{(1)}}{2} - \frac{c_1^{(1)}}{2.04} \right] + \left[\frac{c_2^{(1)}}{75} - \frac{c_2^{(1)}}{76.6} \right]$$

$$= 0.98\ P_1^{(1)} + 0.042\ P_2^{(1)}.$$

Ideally, these two calculations would yield identical results. In actuality, each time we compute $\mathbb{P}^{(t)}$, we approximate a fit to the actual distribution of effect X by recalculating $c_i^{(t)}$ for $i = 0,1,\ldots,4$. The second method repeats this approximation at each time step, losing a little accuracy each time. Thus, for small t, the results are approximately the same, but for large t, the first method yields more nearly accurate results.

When our updating factor depends on time, we need to store one additional bit of information: the base time t_0 of the update (the time of the most recent detonation when we updated our vector $\mathbb{P}$), or the most recent updating factor. If we wish to retain $\mathbb{P}^{(0)}$ and compute $\delta_t = D_t/D_0$ at each time t to get $\mathbb{P}^{(t)}$, then we store the base time t_0. Alternatively, we can calculate D_t/D_{t-1} at each time t to get $\mathbb{P}^{(t)}$ from $\mathbb{P}^{(t-1)}$ if we store either D_{t-1} or the base time for t, whichever is more convenient. The method chosen will depend upon the model being used to represent the recovery and latent injury processes, and the degree of accuracy desired.

If we need to account for the effects of a second nuclear burst after recovery or delayed damage from the first, we use the adjusted vector $\mathbb{P}^{(t)}$ for the first burst rather than the original vector $\mathbb{P}^{(0)}$ in our computations. We assume that, for purposes of updating the effects over time, the process of delayed injury or recovery attributable to the initial burst is interrupted at the time of the second or subsequent bursts. Thus, delayed injury or recovery calculations subsequent to the second burst are performed based on the distribution of the sum of the effects of the bursts (W), with a new base time equal to the time of the most recent burst. It can be easily shown that for any multiplying factor $\delta > 0$, if X is piecewise Pareto, then so is δX.

If the ratio D_t/D_{t-1} is approximately constant (not dependent upon t), we need not store the base time. A moment's consideration will show that in this case, the vectors $\mathbb{P}$ define a Markov chain (for fixed time interval updates) or a semi-Markov process (for updates that occur stochastically), using a single transition matrix. This simplifies the representation and permits some analytic results; the utility of approximating D_t/D_{t-1} with a constant depends upon the physical model and the degree of accuracy desired.

Complete formulas for updating the state vector given $\mathbb{P}^{(0)}$ and a general multiplying factor δ_t are given in Appendix A for the IWFORCEM personnel radiation exposure categories.

CHAPTER 4

ADDITIONS AND VARIATIONS TO THE METHODOLOGY

4-1. ACCOUNTING FOR PROTECTION FROM NUCLEAR EFFECTS

Typically, many individuals and some equipment are within some type of protection at the time of detonation, which reduces the level of effects experienced. A particular protective status, such as being within an armored vehicle, will afford different amounts of protection to different effects (blast, thermal, and nuclear radiation). We characterize the protection provided as a factor between 0 and 1 that acts to reduce the effect received. That is, if X is a level of effect (e.g., blast) that would be received by an unprotected target element (e.g., a soldier), and F_j is the factor pertaining to the jth type of protection (e.g., a foxhole) for this effect, the target element receives a level of effect equal to F_jX.

Let F_j be defined as above, and let $Y_j \equiv F_jX$. Then

$$P[\,Y_j \in H_i\,] = P[\,U_{i-1} \leq Y_j < U_i\,] = P[\,U_{i-1} \leq F_j\,X < U_i\,]$$

$$P[\,Y_j \in H_i\,] = P[\,X < \frac{U_i}{F_j}\,] - P[\,X < \frac{U_{i-1}}{F_j}\,]\,. \tag{4-1}$$

As a result, for any target element provided protection with a factor F_j, we can compute the probability that any given target element receives a level of effect Y_j within category i, $i = 1, \ldots, 5$, based on the distribution of unprotected elements, X.

<u>Example 3:</u>

An example using our radiation criteria will help to clarify this point. Let $P_{yij} \equiv P[\,Y_j \in H_i\,]$. Let $i = 2$, and let $F_j = 0.7$ for some j. Then

$$P_{y2j} = P[\,X < \frac{150}{0.7}\,] - P[\,X < \frac{75}{0.7}\,]$$

$$P_{y2j} = P[\,X < 214.3\,] - P[\,X < 107.1\,]$$

$$= \left[P_0 + P_1 + P_2 + \frac{c_3}{150} - \frac{c_3}{214.3}\right] - \left[P_0 + P_1 + \frac{c_2}{75} - \frac{c_2}{107.1}\right]$$

$$= P_2 - 0.0039\,c_2 + 0.002\,c_3\,.$$

Recalling the relationship between c_i and P_i, we can restate this equation as:

$$P_{y2j} = 0.4\,P_2 + 0.45\,P_3\ ,$$

where P_1 and P_3 refer to the total percentages found in categories 2 and 3, as before.

We can combine the different values of P_{yij} for different j simply by summing the P_i coefficients, weighted by the proportion of elements provided the jth level of protection. For example, if PF_j denotes the proportion of elements provided protection with a factor F_j (an input to our simulation), then $P_i = \sum_j PF_j \cdot$ (the ith coefficient of P_{yij}).

4-2. REPRESENTING BLAST, THERMAL RADIATION, AND RESIDUAL NUCLEAR RADIATION

Until now, we have used nuclear radiation for our examples. However, we can characterize the effects of blast and thermal radiation using the same techniques. The P_i's for each effect will be different, reflecting a difference in category definitions and the distance at which a particular effect will be realized. The inverse square law will actually provide a closer approximation to the effects of blast and thermal radiation than to immediate nuclear radiation.

Exposure to residual nuclear radiation can be treated through an additive factor γ to the original or current dose.

Example 4:

> Suppose all members of a unit are exposed to 30 rads while crossing a contaminated area (outside dose -- the actual dose received may be less when adjusted for protection). A new value for P_1 at a time t after the most recent detonation is calculated as follows: let X_t^- denote the previous exposure to radiation, X_t the new exposure level, and γ_t the additional dosage. Clearly, $X_t = X_t^- + \gamma_t$. Thus

$$
\begin{aligned}
P[\, U_0 \le X_t < U_1 \,] &= P[\, U_0 \le X_t^- + \gamma_t < U_1 \,] \\
&= P[\, X_t^- = 0 \,] + P[\, X_t^- < 75 - 30 \,] \\
&= P_0^{(t)} + \frac{c_1}{2} - \frac{c_1}{45} = P_0^{(t)} + 0.98\, P_1^{(t)}.
\end{aligned}
$$

Combining this offset γ_t with the multiplying factor δ_t (for example, $\delta_t = D_t/D_0$) obtained from modeling the processes of recovery and delayed injury, we get the following expression for the level of effect at time t after the most recent detonation, where X_0 denotes the level of effect calculated at the time of that detonation:

$$X_t = \delta_t X_0 + \gamma_t \, . \qquad [4\text{-}2]$$

X_t is no longer distributed as piecewise Pareto, but is distributed according to a two-parameter or *offset piecewise Pareto* distribution, where γ_t is the offset parameter. The best way to handle this is to store the offset parameter γ_t and update it each time by δ_t (if γ_t is received some time after the process of recovery or delayed injury is underway, we initialize γ_t to the equivalent dose at the base time $\gamma_0 = \gamma_t/\delta_t$ and store γ_0). In the most general case, then, our piecewise Pareto distribution requires the storage of two additional values, δ_t (or t_0, the base time for t) and γ_0. The most convenient way to store the information is to expand the dimension of the vector $\mathbb{P}$ by 2.

Combining effects remains a thorny problem. We know that an individual exposed to one effect will often be exposed to all three, and their combined effect exceeds any single one of them. Nevertheless, proposed ways of combining effects have not been generally accepted. We recommend using the standard (albeit less satisfying) methods of accounting for multiple effects -- either compute and carry forward each effect separately (increasing the dimension of $\mathbb{P}$), or assume a dominant effect for each type of target element ($\mathbb{P}$ thus referring to the dominant effect).

4-3. IMPROVING THE MODEL ACCURACY FOR LOW LEVELS OF EFFECT

The offset parameter γ_t can also be used to improve the accuracy of the distribution at the lower levels of effect, if desired. If the weapon radii of effect completely overlap the unit area, every target element is exposed to some level of effect and there exists some level $\eta > U_0$ such that $P[X < \eta] = 0$. For example, all personnel might receive at least 30 rad of radiation exposure, thus $\eta = 30$. When we fit our Pareto distribution to the proportion of elements within category 1, P_1, the distribution has positive density over the entire category; thus we predict that $P[2 \le X < 30] > 0$. In many cases, this is unimportant; the simulation treats all elements in a given category alike by definition, and there is no transition from category 1 (exposed) to category 0 (unexposed). In other cases, such as having a significant delayed injury rate or a high value of η, it may be useful to correct this problem.

We correct this by setting our offset parameter γ_0 at time 0 (when the exposures are calculated) equal to η - U_0. If X_0" denotes the *actual* level of effect realized at time zero, we let $X_0" = X_0 + \gamma_0$. We then calculate our initial parameters to store in the model, P_0 through P_5, from the actual proportions calculated by our effects model, P_0" through P_5", using the initial offset parameter γ_0.

Example 5:

This example uses the data developed for the first burst of our example in Appendix B. Let $\eta = 67$ (all personnel were exposed to at least 67 rad). Then $\gamma_0 = \eta - U_0 = 67 - 2 = 65$. The actual proportions of personnel exposed in categories 0, 1, and 2 are:

$$P_0" = 0, \quad P_1" = 0.009, \text{ and } \quad P_2" = 0.146 .$$

$$P_0 = P[X_0 < 2] = P[X_0" - \gamma_0 < 2] = P[X_0" < 67] = 0$$

$$\begin{aligned} P_1 &= P[2 \le X_0 < 75] = P[2 \le X_0" - \gamma_0 < 75] \\ &= P[67 \le X_0" < 140] \\ &= P[67 \le X_0" < 75] + P[75 \le X_0" < 140] = P_1" + \frac{c_1"}{75} - \frac{c_1"}{140} \\ &= P_1" + 0.929\ P_2" = 0.009 + 0.136 = 0.145 . \end{aligned}$$

When we calculate our current level of effect at time t, X_t, we use our formula $X_t = \delta_t X_0 + \gamma_t$ as given in the previous section.

$$P_i^{(t)} = P[U_{i-1} \le X_t < U_i] = P[U_{i-1} \le \delta_t X_0 + \gamma_t < U_i] =$$

$$P_i^{(t)} = P[\frac{U_{i-1}}{\delta_t} - \gamma_t \le X_0 < \frac{U_{i-1}}{\delta_t} - \gamma_t] , \qquad [4\text{-}3]$$

which is a function of $P^{(0)}$, δ_t, and γ_t.

4-4. ACCOUNTING FOR CONVENTIONAL ATTRITION AND TROOP REPLACEMENT

We can account for conventional attrition easily by assuming that all target elements are equally at risk -- thus the conventional attrition is uniform across all categories. As a result, we can multiply $P_i^{(t)}$ or $P_i^{(0)}$ by 1 minus the percent attrition for $i = 0,1,\ldots,4$ and add the difference to $P_5^{(t)}$ or to $P_5^{(0)}$. Replacements are assumed to have zero previous exposure; they are added to $P_0^{(t)}$ or to $P_0^{(0)}$ and the unit totals and the remaining P_i's are normalized accordingly.

4-5. AGGREGATING UNIT DISTRIBUTIONS

Until now, we have used the term target or target unit to mean any type of tactical formation that may be engaged by or affected by nuclear weapons. Depending upon the scenario, nuclear weapons may be targeted against units as small as a maneuver company or firing section. However, in large-scale, low-resolution corps or theater models, we generally do not represent units at levels below divisions or brigades. We therefore must aggregate the effects of nuclear weapons against small units into totals for the larger units represented in the model.

Fortunately, aggregation is straightforward. Suppose that we wish to determine the distribution of the levels of a nuclear effect in an aggregated unit which has K subunits. Let $X^{\underline{k}}$ denote the exposure to this effect of an element in subunit k, $k = 1, 2, \ldots, K$. Let $N^{\underline{k}}$ denote the number of such elements in the kth subunit, and N denote the total number of elements in the aggregated unit; clearly,

$$N = \sum_{k=1}^{K} N^{\underline{k}}.$$

Let $P_i^{\underline{k}}$ denote $P[\, U_{i-1} \leq X^{\underline{k}} < U_i \,]$; these numbers will be the result of calculating the coverage of a weapon targeted on unit k. If P_i denotes the probability that any element in the larger aggregated unit receives an effect within category i, then

$$\begin{aligned} P_i &= P[\, U_{i-1} \leq X < U_i \,] \\ &= \sum_{k=1}^{K} P[\, U_{i-1} \leq X < U_i \mid \text{element} \in k \,]\; P[\, \text{element} \in k \,] \\ P_i &= \sum_{k=1}^{K} P_i^{\underline{k}} \cdot \frac{N^{\underline{k}}}{N}. \end{aligned} \qquad [4\text{-}4]$$

Thus we simply sum the individual $P_i^{\underline{k}}$'s from each subunit k, $k = 1,2,\ldots,K$, weighted by the number of such elements in subunit k divided by the total in the aggregated unit.

Example 6:

For example, suppose we had a brigade formed of three battalions; each battalion has an initial strength of 300 men, and we have the following exposures to radiation:

Unit	P_0	P_1	P_2	P_3	P_4	P_5
Bn 1	1.0	0	0	0	0	0
Bn 2	.35	.30	.20	.10	.05	0
Bn 3	0	.25	.25	.20	.15	.15

Then the brigade totals are:

Unit	P_0	P_1	P_2	P_3	P_4	P_5
Brigade	.45	.18	.15	.10	.07	.05

CHAPTER 5

CONCLUSIONS

5-1. IMPLEMENTATION

To implement the nuclear effects model into a low-resolution simulation:

- For each unit, define a vector $\mathbb{P} \equiv \{ P_0, P_1, \ldots, P_k, t_0, \gamma_0 \}$ for storing the parameters of the distribution. k depends upon the number of categories tracked in the simulation; associated with each category i is a threshold U_i.

- At the time of the first detonation affecting the unit, determine the area A_i of a target covered by threshold radius R_i (from any standard effects model); compute $P_i = A_i / \pi R_t$. If desired, compute an offset γ_0 and adjust the parameters P_i accordingly. Store the results in $\mathbb{P}$.

- Determine the processes of delayed injury and recovery to be modeled; express their effect as a factor δ_t for any time t. Store δ_t or t_0 in $\mathbb{P}$.

- If subsequent bursts may occur that will affect the unit, compute the transition matrix to convert P_i to P_i^* for $i = 1, \ldots, k$, updating to account for the time elapsed between bursts. This is simply a function of $\mathbb{P}$, the areas A_i of the later burst, and the time of the later burst. Store each P_i^* in $\mathbb{P}$. The time of the most recent detonation will be the base time t_0 for the future determination of δ_t.

- For any future time t after the most recent detonation, the current proportion of target elements in category i is:

$$P[\, U_{i-1} \le X_t < U_i \,] = P[\, \frac{U_{i-1}}{\delta_t} - \gamma_t \le X_0 < \frac{U_{i-1}}{\delta_t} - \gamma_t \,],$$

which is a function of the current $\mathbb{P}^{(0)}$.

- Adjustments can be made, if desired, to account for protection, residual radiation, aggregation, and conventional effects.

5-2. SYNOPSIS

The techniques outlined in this paper allow the analyst to model the processes of delayed injury (damage) and recovery (repair) over time resulting from nuclear warfare, as well as the effect of subsequent nuclear bursts. We accomplish this by defining a state vector $\mathbb{P}$ which contains the proportion of elements (personnel and equipment) that can be found in each of several categories of effect. This vector $\mathbb{P}$ is used by the simulation to determine the combat effectiveness of a unit, requirements for medical and maintenance support, etc. As discussed in this paper, we can regard $\mathbb{P}$ as a vector of coefficients of a piecewise Pareto distribution. This representation of $\mathbb{P}$ allows us to accurately update the effects of nuclear weapons against units simulated in low-resolution (corps and theater-level) models.

These techniques are limited by some of the same assumptions that are found in alternative updating techniques. We assume a circular target area with a uniform dispersion of target elements and account for the impact of multiple injuries (damages) caused by several effects by determining each effect separately or by assuming *a priori* a dominant effect. We also use the inverse square law to approximate the decrease in effect due to range. Nevertheless, these techniques provide a significant improvement in the procedures used to update nuclear effects in theater-level models. An example comparing the techniques derived in this paper to a high-resolution simulation and the uniform distribution updating technique is provided in Appendix B.

The research presented in this paper is intended to be a stepping stone toward a larger goal. Currently, we resort to large simulation models to determine the effect of a nuclear laydown on a target set, taking into account probabilities of acquisition, target location error, and weapon delivery errors, as well as target aggregation doctrine. However, the physical factors generally affect only the range from weapon detonation to target. Once we have a general form for the probability density of nuclear effects as a function of range, such as the piecewise Pareto, we can work toward deriving distributions for detonation locations and an unconditional distribution for the probability of achieving a given level of effect against a type target.

APPENDIX A

FORMULAS FOR IWFORCEM APPLICATIONS

A-1. FORMULAS FOR UPDATING THE STATE VECTOR **P**

Valid for the following radiation category thresholds in rads:

$U_0 : 2$	(category 0 : 0 - 2 rad)
$U_1 : 75$	(category 1 : 2 - 75 rad)
$U_2 : 150$	(category 2 : 75 - 150 rad)
$U_3 : 450$	(category 3 : 150 - 450 rad)
$U_4 : 8000$	(category 4 : 450 - 8000 rad)
$U_5 : \infty$	(category 5 : 8000+ rad)

Let X_1 be the exposure from the first burst and X_2 the exposure from the second burst. After the second burst, we are interested in the cumulative effect $W = X_1 + X_2$.

Let $P_i^* = P[\,U_{i-1} \leq W < U_i\,]$,
$P_{i1} = P[\,U_{i-1} \leq X_1 < U_i\,]$, and
$P_{i2} = P[\,U_{i-1} \leq X_2 < U_i\,]$.

$$P_0^* = P_{01}\,P_{02}$$

$$P_1^* = P_{01}\,P_{12} + P_{11}\,P_{02} + P_{11}\,P_{12}$$

$$P_2^* = P_{21}\,(\,P_{02} + P_{12}\,) + P_{22}\,(\,P_{01} + 0.941\,P_{11}\,)$$

$$\begin{aligned} P_3^* = {} & P_{22}\,(\,0.059\,P_{11} + P_{21}\,) \\ & + P_{31}\,(\,P_{02} + 0.991\,P_{12} + 0.848\,P_{22} + 0.403\,P_{32}\,) \\ & + P_{32}\,(\,P_{01} + 0.991\,P_{11} + 0.848\,P_{21}\,) \end{aligned}$$

$$\begin{aligned} P_4^* = {} & P_{31}\,(\,0.009\,P_{12} + 0.152\,P_{22}\,) \\ & + P_{32}\,(\,0.009\,P_{11} + 0.152\,P_{21} + 0.597\,P_{31}\,) \\ & + P_{41}\,(\,P_{02} + P_{12} + P_{22} + P_{32} + 0.976\,P_{42}\,) \\ & + P_{42}\,(\,P_{01} + P_{11} + P_{21} + P_{31}\,) \end{aligned}$$

$$\begin{aligned} P_5^* = {} & 0.024\,P_{41}\,P_{42} \\ & + P_{51}\,(\,P_{02} + P_{12} + P_{22} + P_{32} + P_{42} + P_{52}\,) \\ & + P_{52}\,(\,P_{01} + P_{11} + P_{21} + P_{31} + P_{41}\,) \end{aligned}$$

A-2. ADJUSTMENT FORMULAS FOR δ_t X

These equations allow us to update the percent of the unit assets that fall into each operational category. We assume that the current level of effect present at any time t after the last detonation, X_t, can be represented as $\delta_t \cdot X_0$, where X_0 is the level of effect present at the time of detonation.

NOTE: δ_t formulas are given for $\frac{1}{2} < \delta_t < 2$ due to the fact that $U_2 = 2\ U_1$. Factors outside this range can easily be derived but will require different formulas.

Let $P_i^{(t)} = P[\ U_{i-1} \leq \delta_t\ X_0 < U_i\ ]$.

$\frac{1}{2} < \delta_t \leq 1$ (decreasing effect over time):

$$P_0^{(t)} = P_0 + \tfrac{75}{73}(1 - \delta_t)P_1$$
$$P_1^{(t)} = P_1 - \tfrac{75}{73}(1 - \delta_t)P_1 + 2(1 - \delta_t)P_2$$
$$P_2^{(t)} = P_2 - 2(1 - \delta_t)P_2 + \tfrac{3}{2}(1 - \delta_t)P_3$$
$$P_3^{(t)} = P_3 - \tfrac{3}{2}(1 - \delta_t)P_3 + \tfrac{8000}{7550}(1 - \delta_t)P_4$$
$$P_4^{(t)} = P_4 - \tfrac{8000}{7550}(1 - \delta_t)P_4 + (1 - \delta_t)P_5$$
$$P_5^{(t)} = P_5 - (1 - \delta_t)P_5$$

$1 \leq \delta_t < 2$ (increasing effect over time) :

$$P_0^{(t)} = P_0$$
$$P_1^{(t)} = P_1 - \tfrac{2}{73}(\delta_t - 1)P_1$$
$$P_2^{(t)} = P_2 - (\delta_t - 1)P_2 + \tfrac{2}{73}(\delta_t - 1)P_1$$
$$P_3^{(t)} = P_3 - \tfrac{1}{2}(\delta_t - 1)P_3 + (\delta_t - 1)P_2$$
$$P_4^{(t)} = P_4 - \tfrac{9}{151}(\delta_t - 1)P_4 + \tfrac{1}{2}(\delta_t - 1)P_3$$
$$P_5^{(t)} = P_5 + \tfrac{9}{151}(\delta_t - 1)P_4$$

APPENDIX B

AN EXAMPLE SHOWING TWO NUCLEAR BURSTS AND RECOVERY FROM RADIATION

The following simple example has been prepared to illustrate some of the techniques described in this paper. Comparable results have been included using a simple unclassified nuclear effects algorithm (the "actual" results) and using a uniform distributional assumption.

We assume that we have a circular area target with a radius of 1,300 meters, with previously unexposed, unprotected personnel distributed uniformly within the target area. The unit personnel are exposed to the effects of a 1-kiloton weapon aimed at the unit detonating 100 meters from the center of the target area. For convenience, we designate the day that the detonation occurs as day zero.

An unclassified radiation dose algorithm yields the following results (Table B-1):

Table B-1. Ranges R_i for 1-kt Weapon

Threshold U_i	Threshold dose (rads)	Distance from AGZ (meters)	Range R_i
U_0	2	3,495	R_0
U_1	75	1,368	R_1
U_2	150	1,195	R_2
U_3	450	964	R_3
U_4	8,000	515	R_4

Given these ranges, the target radius, and the displacement distance, standard disk-on-disk area calculations yield the areas covered by each range band (A_i's in the paper). Using the equality $P_i = P[\ U_{i-1} \leq X < U_i\ ] = A_i\ /\ \pi R_T^2$ yields the values in Table B-2. For simplicity, we do not use an offset parameter γ.

Table B-2. Parameters P_i for the First Nuclear Burst, Day 0

P_0	P_1	P_2	P_3	P_4	P_5
0	.009	.146	.295	.393	.157

We assume for this example that the only shift in personnel from one category to another is due to biological recovery from radiation, using equation [B-1] below. In reality, there would also be some latent injury, causing a countervailing tendency to increase the effective exposure over time. Our multiplying factor for this example, accounting only for the biological recovery from radiation, will be:

$$\delta_t = 0.1 + 0.9\, e^{-0.024\,t},\ t \geq 0. \qquad \text{[B-1]}$$

From [B-1], $\delta_3 = 0.937$. Using the equations in Appendix A,

$$\begin{aligned} P_0^{(3)} &= P_0^{(0)} + \tfrac{75}{73}\,(1 - \delta_3)\,P_1^{(0)} \\ &= 0 + \tfrac{75}{73}\,(1 - 0.937)\,0.009 = 0.0006 \simeq 0.001\text{, etc.} \end{aligned}$$

We apply these equations with the exception of the transfer from Category 5 to Category 4 (which would be equal to $[\,9P_4/151\,]\cdot[\,\delta_t - 1\,]$), because Category 5 personnel are fatalities without any biological recovery. This yields (Table B-3):

Table B-3. Parameters $P_i^{(3)}$ for the First Nuclear Burst, Day 3

$P_0^{(3)}$	$P_1^{(3)}$	$P_2^{(3)}$	$P_3^{(3)}$	$P_4^{(3)}$	$P_5^{(3)}$
.001	.027	.155	.293	.367	.157

We continue our example by assuming that the unit personnel are exposed on day 3 to the effects of a 1-kiloton weapon intended for a sister unit; the weapon detonates at a distance of 2.3 kilometers away from the center of the target area. The results for this burst only are displayed in Table B-4.

Table B-4. Parameters P_i for the Second Nuclear Burst

P_0	P_1	P_2	P_3	P_4	P_5
.017	.920	.039	.024	0	0

We use the formulas provided in Appendix A to combine the effects of the two bursts. We denote the combined values using the notation $P_i^{(0)*}$, adjusting the baseline time to the time of the second detonation (day 3) -- thus $t_0 = 3$. $P_i^{(0)*}$ is formed from the P_i values from the first burst, adjusted to day 3, and the P_i values from the second burst. We use the notation $P_{ij}^{(t)}$ to denote the $P[\,U_{i-1} \leq X_t < U_i\,]$ for the jth burst.

$$P_0^{(0)*} = P_{01}^{(3)}\,P_{02}^{(0)} = (.001)\,(.017) \simeq 0$$

$$\begin{aligned} P_1^{(0)*} &= P_{01}^{(3)}\,P_{12}^{(0)} + P_{11}^{(3)}\,P_{02}^{(0)} + P_{11}^{(3)}\,P_{12}^{(0)} \\ &= (.001)\,(.92) + (.027)\,(.017) + (.027)\,(.92) = 0.026\text{, etc.} \end{aligned}$$

This yields the results in Table B-5.

Table B-5. Parameters $P_i^{(0)*}$ for the Combined Bursts, Day 3 ($t_0 = 3$)

$P_0^{(0)*}$	$P_1^{(0)*}$	$P_2^{(0)*}$	$P_3^{(0)*}$	$P_4^{(0)*}$	$P_5^{(0)*}$
.000	.026	.146	.295	.376	.157

Computation of the actual results is very messy (which is why we recommend the Pareto approximation). In order to form a basis for comparison, a simple statistical simulation was conducted, drawing 10,000 random coordinates from the target area independently for each burst. From this, we computed the empirical distribution of the probability density function for each burst (adjusted to day 3 using equation B-1 and the updating formulas in Appendix A), and the combined density for both bursts. The simulated values for the parameters P_i are provided in Table B-6.

For comparison, we also calculated the results that would have been obtained if a simple uniform distribution within each category was assumed. The uniform single burst parameters were combined using the following rule: any previously injured (exposed) individual who receives a second, equal exposure (i.e., exposure within the same category) is advanced by one exposure category. Otherwise, the individual is placed in the higher of his first or second exposure category. This forms a reasonably good estimate of $P_i^{(0)*}$, provided not too many individuals receive the same category of effect from both bursts (i.e., the decision rule is reasonably good provided $P_{i1}^{(t)}$ and $P_{i2}^{(0)}$ are reasonably dissimilar for each i). Different rules may provide a better fit (although we are not aware of any simple ones that are better); however, when we adjust the totals within each category to account for the processes of delayed injury and recovery, the uniform assumption is poor, regardless of the combination rules used.

Up to this point, the uniform distribution assumption provides a reasonable fit to the actual $P_i^{(t)}$'s because we do not rely very much on the actual distribution of doses received *within* each category. The combined parameters $P_i^{(t)}$ for all three methods are shown in Table B-6:

Table B-6. Parameters $P_i^{(0)*}$ for the Combined Bursts, Day 3

Type	P_0	P_1	P_2	P_3	P_4	P_5
Simulated	.000	.005	.134	.311	.391	.159
Pareto	.000	.025	.154	.292	.383	.157
Uniform	.000	.000	.155	.290	.348	.157

We next compute an empirical probability density function (pdf) for the combined bursts after 30 days of simulation (27 days after the second burst). At this point, the Pareto approximation still provides a reasonable fit to the pdf, while the uniform distributional assumption provides a very poor fit to the pdf. The results are illustrated in Figure B-1.

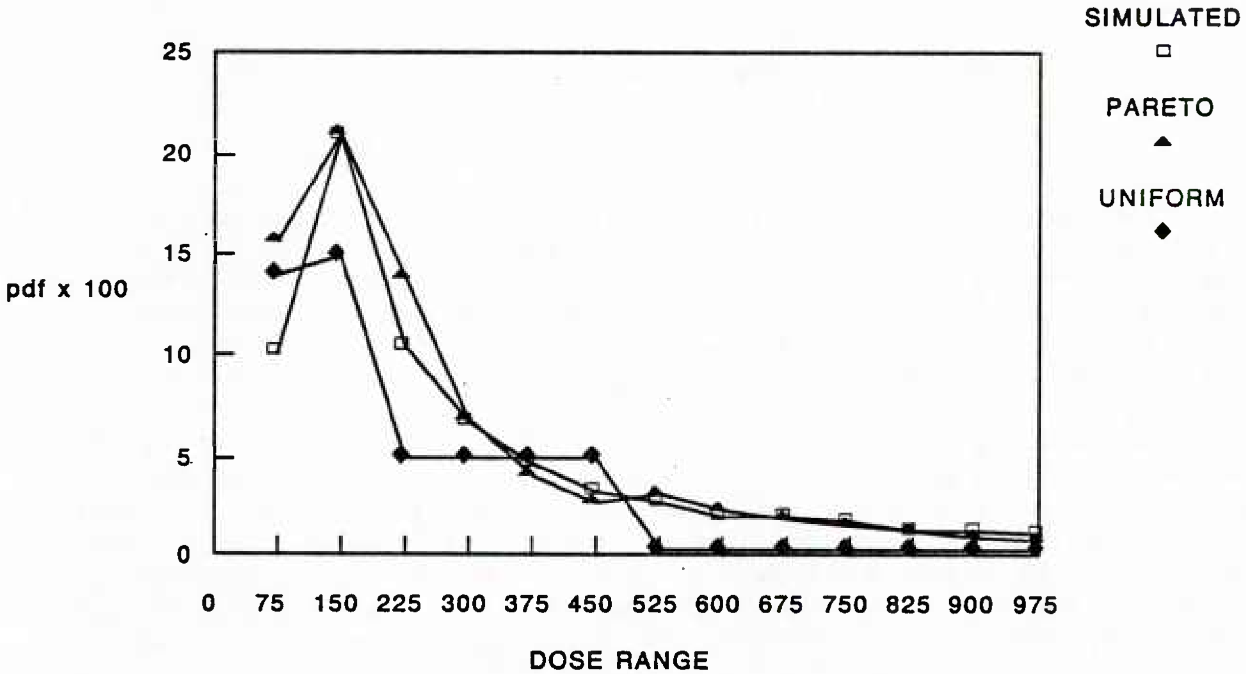

Figure B-1. Simulated and Calculated Segments of the Radiation Exposure Probability Density Function (pdf) 27 Days After the Second Nuclear Burst

This example demonstrates how some of the techniques discussed in this paper would be implemented. In actual simulations, we would want to include the process of latent injury as well as recovery and would provide algorithms for other effects, representing (at least) the dominant effect for each type of element (personnel and equipment) represented in the model. We would also adjust for protection factors and aggregate these results into higher level unit totals. The effort involved in implementing these techniques lies in establishing the initial probability model, which primarily involves evaluating a few simple integrals. Once this probability model is established for the categories maintained in a particular model, the transition equations (such as found in Appendix A) can be coded into the simulation. The result should be an improvement in the representation of the status, over time, of elements in a theater simulation model; this improved representation should improve the fidelity of simulation results which depend upon the status of elements within each modeled unit.

APPENDIX C

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