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APPLICATIONS OF GRAPH THEORY AND SYSTEM THEORY TO LARGE SCALE SYSTEMS

July 25, 1980

Abstract **AFOSR-TR-89-0515**

In graph theory, the minimum cardinality (infimum measure) of set colorings, phasings, and intersection assignments has been studied for the important cases where there is no restriction on the sets, where each set is a real interval, and where each set is a consecutive set of integers. The maximum cardinality score (supremum measure score) has also been studied in these cases. Progress has included the development of general procedures, explicit formulas, and efficient algorithms. Related work has explored a series of ultimate numbers related to the n-chromatic numbers, and the structural characterization of certain classes of graphs of boxicity at most 2, which generalize the interval graphs.

In system theory, a computationally oriented approach to nonlinear system regulation has been developed, based on the notion of piecewise-linear systems. The necessary algebraic concepts had to be themselves developed during the course of the research. New designs and theoretical results were obtained also for the control and observation of parametrized families of systems, and for delay and other well-known types of systems. (KR)

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1. The Research in Graph Theory

1.1. Technical Results

The main emphasis has been on the computation of the chromatic numbers, phasing numbers, and intersection numbers for general classes of graphs, and on the structural characterizations of certain classes of graphs.

The main piece of work has been a major extension and generalization of a procedure developed by Karl Stoffers to deal with the phasing of traffic lights. The procedure applies to the case where each set is a real interval. The results show that for interval graphs, the optimal intersection numbers $i_{I \geq r_x}(G)$, $i^{I,N}(G)$, and $i_{I \geq r_x, N}^{I,N}(G)$ can always be computed by methods of linear programming, and hence can be computed in polynomial time. These general results are exploited to develop explicit formulas for the case where all $r_x = 1$. The results give explicit formulas for $i_{I \geq 1}(G)$ for G an interval graph and $\theta_{I \geq 1}(G)$ for G any graph, and show that in general the computation of the latter number is an NP-complete problem. The results are also exploited to give explicit formulas for the numbers $i^{I,N}(G)$ for G an interval graph and $\theta^{I,N}(G)$ for G arbitrary, and to show that in general, the problem of computing $\theta^{I,N}(G)$ is NP-hard. The results also yield general upper bounds on $i_{I \geq 1, N}^{I,N}(G)$ and $\theta_{I \geq 1, N}^{I,N}(G)$ for G an interval graph. A major reduction from the problem of arbitrary lengths to unit length restrictions is obtained. These results make up a long paper [1]. Also on the problem of computing $i_{I \geq 1, N}^{I,N}(G)$, an efficient algorithm has been developed for computing this number if $\omega(G) \leq 3$. This will appear in [5].

For the case where there is no restriction on the sets in a set assignment, we have obtained an explicit formula for $i_\phi(G)$ for G an interval graph,

and bounds on $i^{\phi, N}(G)$ for arbitrary G , which in turn lead to exact results in a number of very useful special cases. We have also obtained an exact formula for $\theta^{\phi, N}(G)$ for arbitrary G . These results appear in the paper [2].

For the case where each set in an assignment is a set of consecutive integers, we have obtained explicit formulas for $i_c(G)$ for G an interval graph, for $i^{c, N}(G)$ for G an interval graph, and for $\theta^{c, N}(G)$ for arbitrary G . The results appear in the paper [2].

The PI noticed analogies between the results about the Shannon capacity of a graph, a concept arising in connection with communications, and the results on the n -tuple chromatic numbers. This suggested defining several analogues of the Shannon capacity, and led to a whole series of new product numbers (ultimate numbers). The PI, in collaboration with Professor Pavol Hell, explicitly worked out the relations among these numbers. The results are being prepared for publication, in the paper [3].

The condition that each set in a set assignment be a rectangle in the plane, which is relevant to the aircraft maintenance problem, has been studied, in particular with regard to the computation of the boxicity of a graph G , the smallest k such that G is the intersection graph of boxes in k -space. The main results include a structural characterization of graphs of boxicity at most 2, the important case, under some special assumptions, about diameter. The computation of boxicity is also shown to be NP-complete, and algorithms for the computation of boxicity have been developed. These results and others to be obtained in the continuation of this grant will be written up in [4] and in a series of papers by the PI and Margaret Cozzens.

1.2. Doctoral Students, Travel, and Publications

During the period of this contract, two graduate students, Margaret Cozzens and Robert Opsut, were partially supported by the contract. The results of

their research will appear in their Ph.D. theses, which are expected to be completed by early 1981.

During the period of this contract, F. Roberts was invited to lecture on these topics at Bell Telephone Laboratories (July 1979), Allegheny College (October 1979-part of a series of 5 lectures on related topics), Cornell University (February 1980), Birmingham, Alabama (Southeast Atlantic Section of SIAM, annual meeting, March 1980, where he was a featured speaker), Exxon Research Laboratories (April 1980), and Kalamazoo, Michigan (International Conference on the Theory and Applications of Graphs, May 1980, where he was an invited speaker). Margaret Cozzens lectured on these topics in Boca Raton, Florida (XIth Southeastern Conference on Combinatorics, Graph Theory, and Computing, March 1980), and was invited to lecture on these topics at Union College (February 1980), Vassar College (February 1980), St. Lawrence University (March 1980), Beloit College (April 1980), Bard College (April 1980), York College (April 1980), and New York University (April 1980).

The following publications were prepared by FSR under this contract.

- [1] 'I-Colorings, I-Phasings, and I-Intersection Assignments for Graphs, and their Applications,' (with R.J. Opsut), submitted to SIAM Journal on Algebraic and Discrete Methods.
- [2] 'On the Fleet Maintenance, Mobile Radio Frequency, Task Assignment, and Traffic Phasing Problems,' (with R.J. Opsut), accepted for publication in Proceedings of the Fourth International Conference on the Theory and Applications of Graphs, to be published by Wiley.
- [3] 'Analogues of the Shannon Capacity of a Graph,' (with P. Hell), in preparation, to be submitted to Journal of Graph Theory.

- [4] Doctoral dissertation by Margaret Cozzens, in preparation, expected completion early 1981.
- [5] Doctoral dissertation by Robert Opsut, in preparation, expected completion early 1981.

2. The Research in System Theory

2.1. Technical Results

The main objective of the system-theoretic component of this research was (and continues to be) the study of algebraic methods in nonlinear regulator design. The approach pursued was based on the notion of discrete-time piecewise linear (PL) systems. Such systems appear naturally in many engineering applications and also as global approximations of rather arbitrary nonlinear dynamics; they are characterized by piecewise linear transition (next-state) and measurement (output) maps. Real-time implementation of PL controllers requires a large amount of logic (deciding on which 'piece' of the state-space the state is at a given time) but simple arithmetic (linear operations); thus a strong motivation for the study of PL systems comes from the possibility of inexpensive digital control.

The first step in the development of the PL theory was to study the algebraic properties of PL maps. This gave rise to the theory reported in [1]. Results obtained include a characterization of morphisms, congruences, and isomorphism classes of PL objects. The theory proceeds in such a way that all constructions are in principle computational, and in fact most constructions are, as one may expect, related to linear programming problems. (A rigorous statement of this last fact was proved in [1], after the introduction of a suitable formal language.) Recent developments on the computational

complexity of linear programming should directly influence the development of algorithms for PL algebra, and this will be one of the foci of future research, since such algorithms should form the basis for the (offline) calculations needed to compute control laws.

The next step in the development of the tools from PL algebra was the study of basic problems in PL control: state-feedback, observers, i/o regulators. Fairly complete results were obtained in this area. (Example: under weak technical conditions, a controllable system admits a PL regulator; if the original system evolves in continuous time, the result is still true when the PL regulator is placed in a loop including constant-rate sampling.) Studying these problems in the PL context actually forced consideration of many fundamental issues of nonlinear regulation which were totally unexplored. A second paper [2], containing the control-theoretic results, was submitted to the IEEE Transactions in Automatic Control, and has already been accepted for publication. A great number of open problems, both theoretical and computational, have arisen from this research, and future efforts will address some of these.

Other areas mentioned in the original proposal also received attention. General problems of regulation, not necessarily of PL systems, led to the recent research [6], which emphasizes the fact that a logical, discrete, component is unavoidable in the design of controllers contrary to widely held views among theoretical researchers. Problems of constructing observers for families of systems, delay, and other types of systems were studied in [3], which is to appear as part of a book published by the American Mathematical Society. The results obtained there, together with current research along those lines by other authors, are finally settling basic questions of regulation of systems over rings which had been open for about 10 years.

Related to this are also the more theoretical results in [4] on pole-shifting, and also those in [5] on generalized inverses for polynomial matrices; in particular [5] provides answers to questions that had been posed by authors motivated by multidimensional and large-scale system problems.

2.2. Travel and Publications

During the period of this contract, E. Sontag lectured on these and other topics at Harvard University (Conference on Algebraic and Geometric methods, June 1979), Rice University (CAS Nonlinear Systems conference, Jan. 1980), University of Florida (where many useful interactions resulted on PL systems with the group working under Professor R.E. Kalman), Princeton (where a paper [7], basically just a summary of [2], was presented, and where the PI cochaired and co-organized the special session on nonlinear systems, as part of the 1980 Information Sciences and Systems conference in March), and Virginia Beach (invited survey talk at the Workshop on the Math. Theory of Networks and Systems). The PI also attended the 1979 CDC, and is currently organizing a special session for this year's CDC at Albuquerque.

The following papers were submitted by EDS under the present contract:

- [1] 'Remarks on Piecewise Linear Algebra,' submitted.
- [2] 'Nonlinear Regulation: The Piecewise Linear Approach,' to appear in IEEE Trans. on Autom. Control.
- [3] 'An Approach to Detectability and Observers,' (with M.L.J. Hautus), to appear in the Proc. of the AMS Summer Workshop at Harvard (June 1979).
- [4] 'Remarks on the Pole-Shifting Problem over Rings,' (with R. Bumby, H. Sussmann, and W. Vasconcelos), to appear in J. Pure Applied Algebra.
- [5] 'On Generalized Inverses of Polynomial and other Matrices,' IEEE Trans. on Autom. Control, June 1980.

[6] 'Remarks on Continuous Feedback,' (with H. Sussmann), submitted.

[7] Same title as in [2], Proc. Princeton Conference on Information Sciences and Systems, March 1980. (Summary of [2].)