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<p>Dynamic modeling of various aeroelastic control systems require at some point in the derivation of the model an application of Söngren's inversion formula for finite Hilbert transforms to obtain a desired representation for the solution of the airfoil equation. Conditions on initial data to guarantee well-posedness of the resulting model equations must be matched with those needed to justify the validity of the inversion formula. We showed that this compatibility can be achieved by assuming that the circulation history belongs to a weighted <math>L^2</math> space. The resulting state space formulation provides a suitable setting for control design for the aeroelastic system. Keywords: Mathematical models; Mathematical formulas; Airframe; Inversion; Dynamic response.</p>					
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TECHNICAL REPORT

on

STATE SPACE MODELS FOR  
AEROELASTIC AND VISCOELASTIC SYSTEMS

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1 January 1988 - 31 December 1988

by

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## TECHNICAL REPORT

Dynamic modeling of various aeroelastic control systems require at some point in the derivation of the model an application of Söhngen's inversion formula for finite Hilbert transforms to obtain a desired representation for the solution of the airfoil equation. Conditions on initial data to guarantee well-posedness of the resulting model equations must be matched with those needed to justify the validity of the inversion formula. During this period we showed that this compatibility can be achieved by assuming that the circulation history belongs to a weighted  $L_2$  space. The resulting state space formulation provides a suitable setting for control design for the aeroelastic system.

In recent years the feasibility and advantages of activating control surfaces to reduce maneuver, gust, and fatigue loads, and dampen vibration that contributed to flutter has been extensively studied. A systematic procedure for control design requires the development of a "realistic" mathematical model that predicts the dynamic behavior of the physical system. The development of state space models for aeroelastic systems, including unsteady aerodynamics, is potentially important for the design and development of highly maneuverable aircraft.

Burns, Cliff and Herdman formulated a complete dynamic model in terms of a functional differential equation of neutral-type for the elastic motions of a three-degree-of-freedom "typical" airfoil section, with flap, in a two-dimensional, incompressible flow (Theodorsen's problem). In subsequent papers the well-posedness of the modeling neutral equation was studied in a product space framework. The analysis showed that the dynamic model extends to a well-posed state-space model on the product spaces  $R^3 \times L_p$ ;  $p > 1$  if and only if  $p < 2$ . Since the ultimate goal of the modeling process is to generate a framework for the design of active control schemes for flutter suppression, the above results would suggest to consider circulation histories belonging to  $L_p$ ;  $p < 2$ . On the other hand, in the derivation of the evolution equation for the circulation on the airfoil, one has to assume that the circulation history belongs to  $L_p$  for  $p > 2$  in order to guarantee the application of Söhngen's inversion formula for obtaining a representation for the solution of the airfoil equation. In view of the above observations it is natural to ask if it is possible to find a state-space such that Söhngen's inversion formula is applicable and at the same time the resulting neutral equation is well-posed?

During this period we studied the derivation of the model equations assuming that the circulation history belongs to a weighted  $L_2$ -space. The motivation for this is

that recently Burns and Ito established the well-posedness of the finite delay version of the model equations established on the product space  $R^7 \times L_2, g$  ( $g$  denotes the weight-function). We have shown that  $L_2, g$  is appropriate for the derivation of the model (i.e., representation for the solution of the airfoil equation can be obtained by using Söhngen's formula). The most significant consequence of this result is that it provides a candidate for the state-space for the infinite delay neutral equation which then can be used to study the flutter-suppression problem. The well-posedness of the infinite delay neutral equation on the weighted product space  $R^7 \times L_2, g$  will be one phase of our continued effort.

Ito and Turi have obtained numerical results, including convergence, for a scalar finite delay nonatomic neutral equation. Their numerical scheme is based on the semigroup associated with the neutral equation. We propose to generalize their results to the complete mathematical model for the aeroelastic system discussed above.

We have begun our study of viscoelastically damped structures. As a first step in this effort we have studied an iterative method based on quasilinearization for the numerical approximation for such systems. T. Hammer, mathematics Ph.D. student, has studied the research papers of H. T. Banks, D. W. Brewer, J. A. Burns, E. M. Cliff and G. M. Groome which develop this technique for parameter identification and control for delay and distributed parameter systems. Our concentration on studying such quasilinearization algorithms was motivated by the idea of (as a first step) approximating the partial-differential system by a delay equation. We propose to generalize this approximation scheme to the complete partial-differential system appearing in the viscoelastic model.

In addition to the research described above, we have continued to interact with Air Force Laboratory personnel at the Air Force Flight Dynamics Laboratory, Wright-Patterson AFB. We see this as an important part of our research effort and propose to continue this interaction.

RESEARCH ARTICLES

Terry L. Herdman and Janos Turi, "An application of finite Hilbert transforms in the derivation of a state space model for an aeroelastic system," submitted.

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