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A new modelling and computational task for the integrated design of flexible structures undergoing large overall motions has been developed and analyzed. This new approaches utilizes geometrically exact structured models and have the advantage that these models can handle coupled rigid body-flexible appendage systems without resorting to the introduction of the so-called floating frames.

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1. Introduction

Over the past two years we have pursued actively new methodologies in the *mathematical modeling and numerical simulation of the dynamics of flexible structures undergoing large overall motions*. This two year research effort, sponsored by the AFOSR under grant DJA/AFOSR 86-0292 with Stanford University, has resulted in 15 publications, a similar number of publications in closely related fields (see Appendix II), and a strong collaboration (in a related research effort also sponsored by AFOSR) with the groups of J.E. Marsden at Berkeley (currently at Cornell) and P.S. Krishnaprasad at Maryland‡.

First, we outline in Section 2 the scope of the proposed research, and summarize the research goals as stated in the original proposal. In Section 3 we provide a brief summary of the objectives accomplished during the past two year effort. Explicit reference is made, were appropriate, to the work published under the present research contract. We remark that *all* of the proposed objectives (summarized in Section 2) have been accomplished. Section 4 is concerned with additional topics pursued in these past two years, which which fall within the subject area of this research and are not explicitly identified in the original proposal. The contents of these three sections is essentially a summary of the results reported in the final report submitted to AFOSR in spring of 1987. We refer to this latter document for further details and elaboration on the nature of these results.

2. Review of scope and objectives of the original AFOSR proposal

As stated in the abstract of our original AFOSR proposal, the ongoing research under the present AFOSR contract constitutes an effort towards the development of modeling and computational tools for the integrated design of flexible structures undergoing large overall motions Approaches in wide use at present rely crucially on the use of a floating reference frame that follows the overall rigid body motion of the system thus allowing the use of simplified structural models based on the assumption of "small strains." From a physical standpoint this approach precludes the modeling of physically important

‡ An example of this collaboration is the forthcoming AMS conference in Maine, jointly organized by P.S. Krishnaprasad, J.E. Marsden & J.C. Simo.

nonlinear effects, such as large deformations, nonlinear vibrations and structural damage that may arise, for instance, as a result of control system breakdown. Thus, worse case situations cannot be accounted for in the design process. From a computational stand point, approaches currently in use lead to discrete equations of motion that are ***highly coupled in the inertia terms*** as a result of inertia effects due to rotation of the floating frame.

2.1. Overview and general remarks

The central theme of the present work is the development of an a new alternative methodology that hinges on the use of *geometrically exact structural models* to mathematically characterize the flexibility properties of the structure. This approach enjoys the following advantages over traditional formulations

i. Geometrically exact models possess full $SO(3)$ invariance; hence, they can be used directly in the modeling of coupled rigid body-flexible appendage systems without resorting to the introduction of the so-called *floating frames*. This is of particular importance in applications concerned with the dynamics of orbiting satellites with flexible appendages. In fact, the entire dynamics of the coupled system can be referred directly to the inertial frame.

ii. One has the added capability of modeling important physical phenomena precluded by linearized theories; such as large deformation, bifurcation and loss of stability. In addition, complete inertia effects can be *exactly* accounted for without simplification. Such a generality is warranted in many engineering applications; for instance, helicopter and rotor blades, modern highly flexible robot arm manipulators, and space structures. Finally, nonlinear geometric effects, which play an important role in the analysis of rotating structures, are automatically accounted for. In this regard, it should be noted that use of linearized theories in the dynamic analysis of rotating structures may lead to completely erroneous, nonphysical, results.

iii. Computationally, by referring the motion of the system to the inertial frame, extensive coupling in the inertia operator due to Coriolis, centrifugal, and inertia effects due to rotation of the floating frame is avoided. A case in point concerns the dynamics of planar systems for which the inertia operator becomes linear. Furthermore, because of the full $SO(3)$-invariance property, the proposed formulation is specially well-suited for the numerical simulation of multi-body interconnected systems in complex configurations. This is the case, for instance, of closed-loop chains undergoing large overall motions.

2.2. Objectives of the investigation

The computational and physical considerations briefly summarized above led to the following research goals to be accomplished in the course of a two-year investigation

1. Development of a new class of covariant time stepping algorithms for the numerical simulation of the transient response of fully nonlinear flexible structures. For this purpose, concepts of modern differential geometry play a central role.
2. Application of the methodology proposed above to the dynamics of multi-body flexible systems.
3. Development of a new class of fully nonlinear formulations for the dynamics of structural elements such as plates and shells.
4. Extension to more general constitutive behavior that includes possibly nonlinear dissipative mechanisms, and structural damage. Formulation of general integration algorithms for numerical simulation of this constitutive behavior.
5. Development of effective numerical algorithms for the simulation of orbiting LSS composed of beam-like elements. An essential ingredient is the separation and effective treatment of the far field and the near field.
6. Simulation and design of flexible structures undergoing large overall motions. This includes fast slewing and precision pointing in a wide range of applications, and appropriate treatment of follower and non-conservative loading (e.g. gravity, actuator control forces on large space structures).

A summary of the work completed during the past two year effort, and its relation to the aforementioned research goals is given in the following section.

3. Summary of objectives proposed and accomplished in the two years

All of the tasks proposed in the original proposal to the AFOSR outlined above have been completed in the two-year period spanned by our investigation. An outline of our results follows.

3.1. Time-stepping covariant algorithms for nonlinear structural dynamics.

The methodology for planar systems developed in Simo & Vu-Quoc [1986a,b], is first extended in Simo & Vu-Quoc [1986c], within the context of the static problem, to the full three-dimensional case. The general, fully coupled dynamic case is considered in Simo & Vu-Quoc [1988a]. A summary of the methodology is given in the invited contribution in Simo & Vu-Quoc [1987]. Some salient features of this work are the following:

i. A complete convergence, and accuracy analysis is given.

ii. Methods of differential geometry are rigorously used to exploit the underlying geometric structure of the model. In particular, configurations are updated by means of the discrete version of the *exponential map* in $\mathbb{R}^3 \times SO(3)$, whereas velocity and accelerations are updated by making use of the discrete *parallel transport* in $\mathbb{R}^3 \times SO(3)$.

iii. The algorithmic procedure can accommodate any degree of geometric nonlinearity. In particular, limit and bifurcation points are traverse by continuation procedures, as discussed in Simo, Wriggers, Schweizerhof & Taylor [1986].

iv. The formulation and numerical implementation includes actuator and follower forces of importance, for instance, in attitude control by means of thrusters.

v. A singularity free parametrizations of the rotation field based on the use of quaternion parameters is employed, which appears to be optimal for large-scale computations.

A numerical simulation that illustrates the performance of the methodology described above is contained in Figure 1.

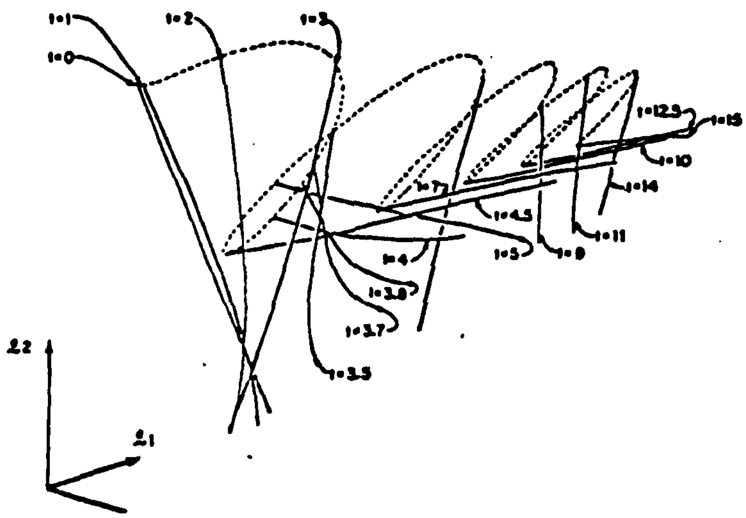

Figure 1. *Rod undergoing large amplitude vibration superposed onto three dimensional overall motions.*

3.2. Application to multibody-dynamics.

The numerical simulations presented in Simo & Vuoc [1987,1988a], and in Vu-Quoc and Simo [1987] demonstrate the applicability of the proposed methodology to multibody dynamics. As an illustration, Figure 2 shows the simulation of the dynamics of an *all-flexible closed loop* chain system undergoing large overall motions; an example of a class of problems to which the highest degree of difficulty is assigned in the well-known classification of Ho & Herber [1985]. Furthermore, the example exhibits an added degree of difficulty which appears to have no counterpart in the current literature on multibody dynamics: ***no restriction is placed on the degree of flexibility of the chain members.***

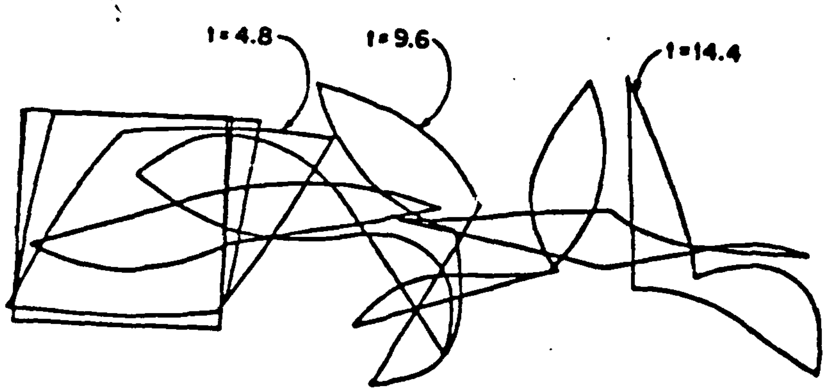

Figure 2. *All-flexible, closed loop chain undergoing finite vibration and large overall motions.*

The fact that classical methodologies relying on ad-hoc linearized models can result in completely erroneous non-physical results is demonstrated, both analytically and numerically, in Simo & Vu-Quoc [1988b]. This analysis also provides a systematic means of developing higher order theories from the geometrically exact theory by an asymptotic expansion procedure.

3.3. Well-conditioned methodologies for satellite dynamics

That flexibility plays a significant role in the stability of satellites is a well-established fact that was recognized as early as the start of the space program. † In Vu-Quoc & Simo [1987], by exploiting the full $SO(3)$-invariance of the proposed geometrically exact models, the dynamics of the orbiting satellite is referred directly to the inertial frame. This point of view represents a radical departure from traditional approaches, and offers the following advantages with respect to traditional formulations which rely, typically, on floating frame systems and make crucial use of the "small deformations" assumption (e.g., Canavin & Likins [1977]),

† As noted in the original proposal submitted to AFOSR, the instability of Explorer I, the first U.S. satellite, was caused by the flexibility of its small wire turnstile antennae.

i. From a computational point of view, the task of coupling rigid body and flexible appendage becomes remarkably simple.

ii. The structure of the inertia terms simplifies considerably.

iii. Large deformations, which are expected to take place in very flexible appendages and are associated with important physical phenomena such as loss of stability and buckling, can be predicted.

iii. Numerical ill-conditioning associated with the enormous differences in orders of magnitude between orbital radius and amplitude of deformation is avoided by referring the dynamics of the orbiting satellite to a *parallel translate of the inertial frame* located in the vicinity of the coupled system (*not necessarily at the center of mass of the system*).

iv. In addition to avoiding ill-conditioning, the aforementioned technique has the advantage of effectively *decoupling the dynamics of the system* into a *far field* and a *near field* problems.

v. We remark that the formulation can accommodate *configuration dependent* follower loads. This feature is of crucial practical importance in attitude control of orbiting satellites since "gravity gradient" and actuator or control forces; such as gas jets or ion thrusters, follow the deformation of the space structure

3.4. General nonlinear dynamic rod models

A main limitation of existing geometrically exact mathematical models for beam-like structures, see i.e., Antman [1974], Antman & Jordan [1975], Reissner [1973,1981], is their inability to accommodate the effect of *torsional warping distortion* of the cross section of the rod. This effect plays a dominant role in many situations of engineering interest; particularly those concerned with light-weight structures that employ thin-walled elements with open or closed channel sections. A case in point is the design of robot-arm manipulators where light-weight consideration preclude, almost always, the use of a solid cross-section. Partly because of these considerations, the problem of coupling bending-torsion has received considerable attention in recent years; i.e., Reissner [1979,1983a,1983b,1984], Hodges [1980], Krenk [1983a,b], Krenk, & Gunneskov, [1985]. No results, however, appear to exist in a geometrically exact context.

In Simo & Vu-Quoc [1988c], we have extended the model proposed in Simo [1985] to accommodate the effect *torsional warping distortions* of the cross section. In addition, our earlier computational framework has been extended to accommodate the effect of warping. A numerical simulation that illustrates the important effect of warping, and exhibits the full-nonlinear capabilities of the model is presented in Figure 3.

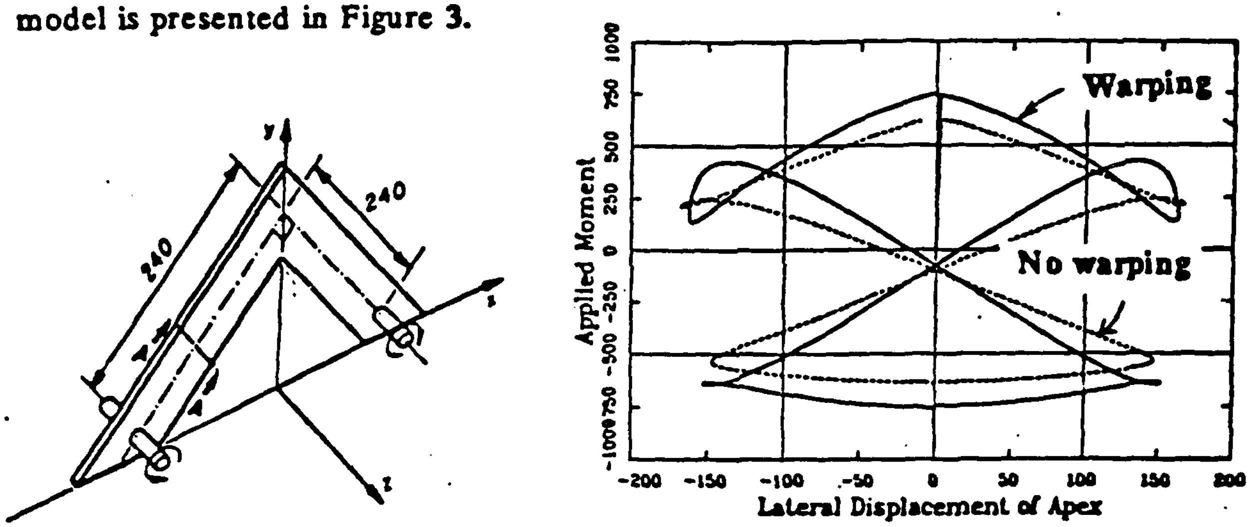

Figure 3. *Lateral loss of stability and bifurcation of a right-angle frame is presented illustrating the effect of coupled torsion-bending-warping in the bifurcation diagram.*

3.5. Formulation of geometrically exact plate and shell models

Over the past two decades engineering shell analysis has been dominated, to a large extent, by the so-called *degenerated solid approach*, which finds its point of departure in the paper of Ahmad, Irons & Zienkiewicz [1970]. The works of Ramm [1977], Parish [1981], Hughes & Liu [1981a,b], Hughes & Carnoy [1983], Bathe & Dvorkin [1984], Hallquist, Benson & Goudreau [1986], Parks & Stanley [1986], and Liu, Belytschko, Law & Lam [1987], among many others, constitute representative examples of this methodology. Our work in Simo & Fox [1988], on the other hand, constitutes a radical departure from the aforementioned methodology. In a sense, the proposed approach represents a return to the origins of classical nonlinear shell theory, which has its modern point of departure in the fundamental work of the Cosserats [1909], subsequently rediscovered by Ericksen & Truesdell [1958], and further elaborated upon by a number of authors; notably Green & Laws [1966], Green & Zerna [1960], Cohen & DaSilva [1966], Naghdi [1972,1980], Green & Naghdi [1974], and Antman [1976a,b].

In Simo & Fox [1987], we have initiated the mathematical formulation and optimal parametrization of a general class of nonlinear, geometrically exact plate and shell models, *which is amenable to numerical analysis and suitable for large-scale computation.* In Simo, Marsden & Krishnaprasad [1987] we have undertaken what appears to constitute the first mathematical analysis of the Hamiltonian structure underlying this important class of models. In contrast with classical treatments, the emphasis of this work is on the underlying geometric structure of the nonlinear shell theory. The understanding and exploitation of this structure leads to the following noteworthy results

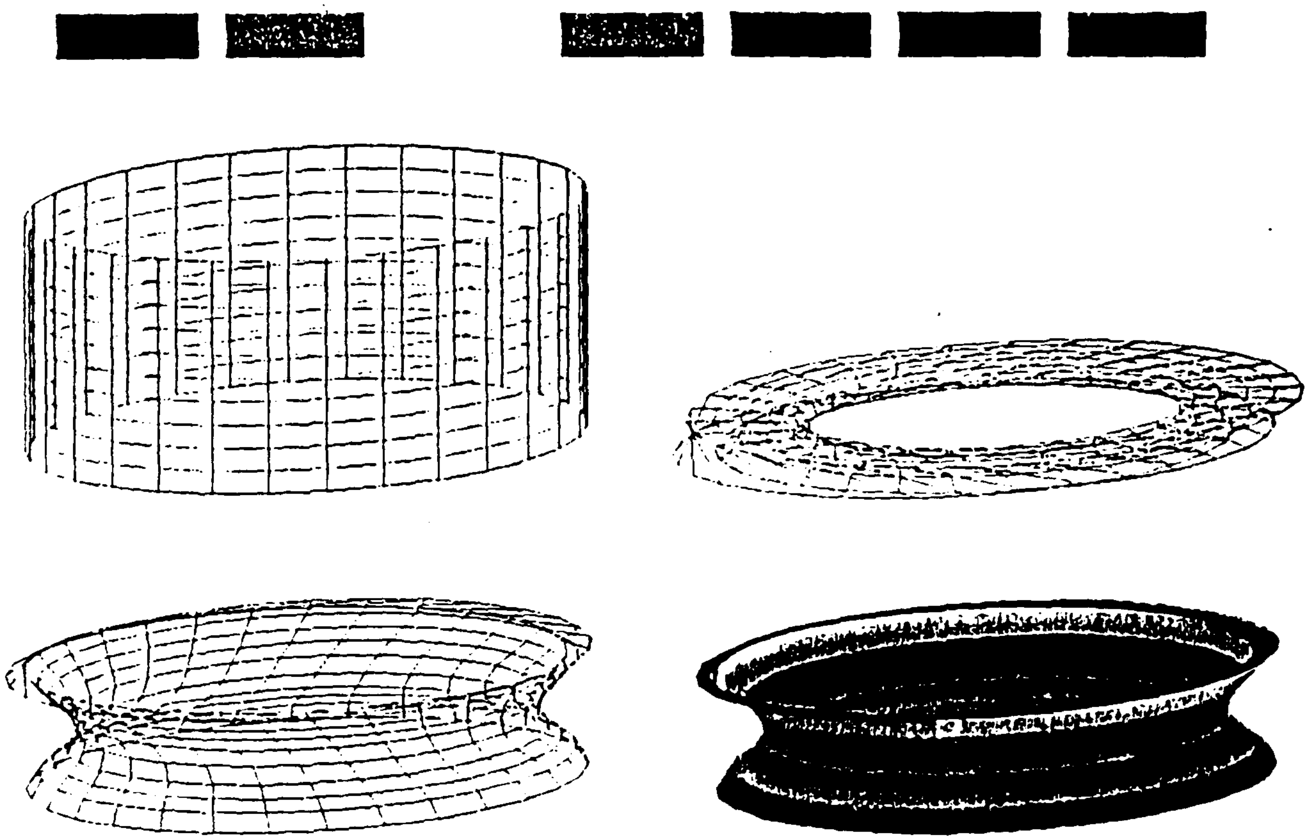

Figure 4. *Large deformation and torsional collapse of a cylindrical shell. Actual deformations, shown without magnification, are attained in three load steps.*

i. *Optimal singularity-free parametrizations* of the director (rotation) field are developed by identifying the geometric structure of the configuration space C; essentially a differentiable manifold of mappings modeled on the product manifold $S^2 \times \mathbb{R}^3$, where S^2 denotes the unit sphere.

ii. *Update procedures* for the director and associated rotation field, which are exact and preserve objectivity for any magnitude of the director and rotation increment, are constructed by exploiting the following two geometric objects: (a) A discrete version of the exponential map in the unit sphere S^2, and (b) The realization of the unit sphere as the non-trivial bundle $SO(3)/S^1$. This realization corresponds to a reduction of $SO(3)$ by the group of rotations in the plane S^1.

iii. *Exact enforcement of constraints such as inextensibility of the director field* is achieved exactly without resorting to ad-hoc procedures (or penalty methods) by exploiting the aforementioned realization of the unit sphere as the bundle $SO(3)/S^1$.

iv. A parameterization of the basic weak form of the momentum equations is given which avoids explicit appearance of the Riemannian connection on the mid-surface. Objects such as Christoffel symbols and covariant derivatives, which are not readily accessible in a computational context, never appear explicitly in our formulation. Consequently, a major drawback in the numerical treatment of of classical shell theory is entirely bypassed.

Some preliminary numerical simulations which illustrate the effectiveness of our methodology are given in Figure 4 and Figure 5.

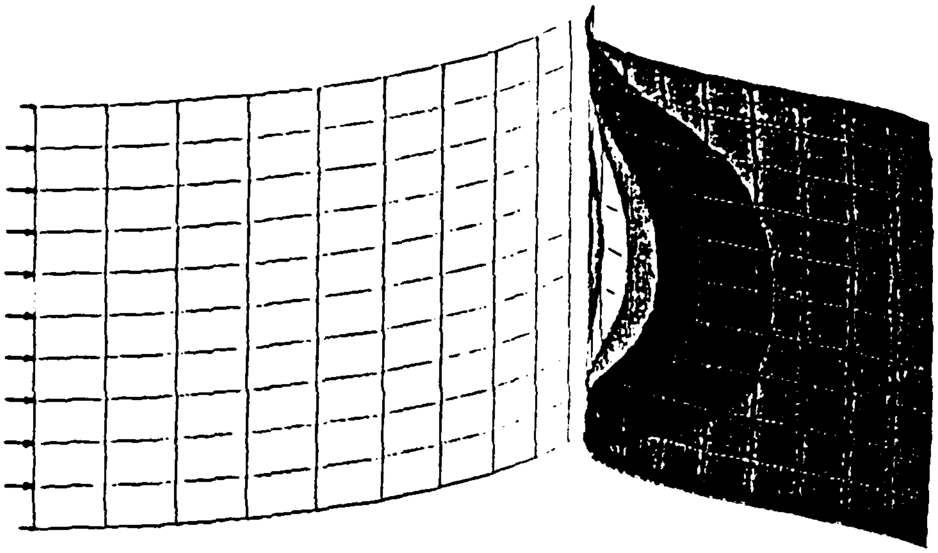

Figure 5. *Large deflection of a quarter of a cylindrical shell. The deformation shown (with no magnification) is obtained in one single-step loading.*

3.6. Numerical analysis of geometrically exact shell models

A point frequently made concerning the degenerated solid approach is that it avoids the mathematical complexities associated with classical shell theory, and hence is better conditioned for numerical implementation. In Simo, Fox & Rifai [1988a,b], on the other hand, we have demonstrated that stress-resultant geometrically exact shell models, phrased in the geometric setting discussed in Section 3.5 are indeed ideally suited for large scale numerical simulations, and amenable to numerical analysis. In order to evaluate the proposed finite element formulation and assess its performance in standard benchmark problems, we have initially restricted our attention to the linear theory which is obtained by consistent linearization of the full nonlinear formulation in Simo & Fox [1988]. This is a preliminary step which must be taken before addressing the numerical analysis aspects involved in the implementation of the full nonlinear theory. Some of the results emanating from our recent work are as follows

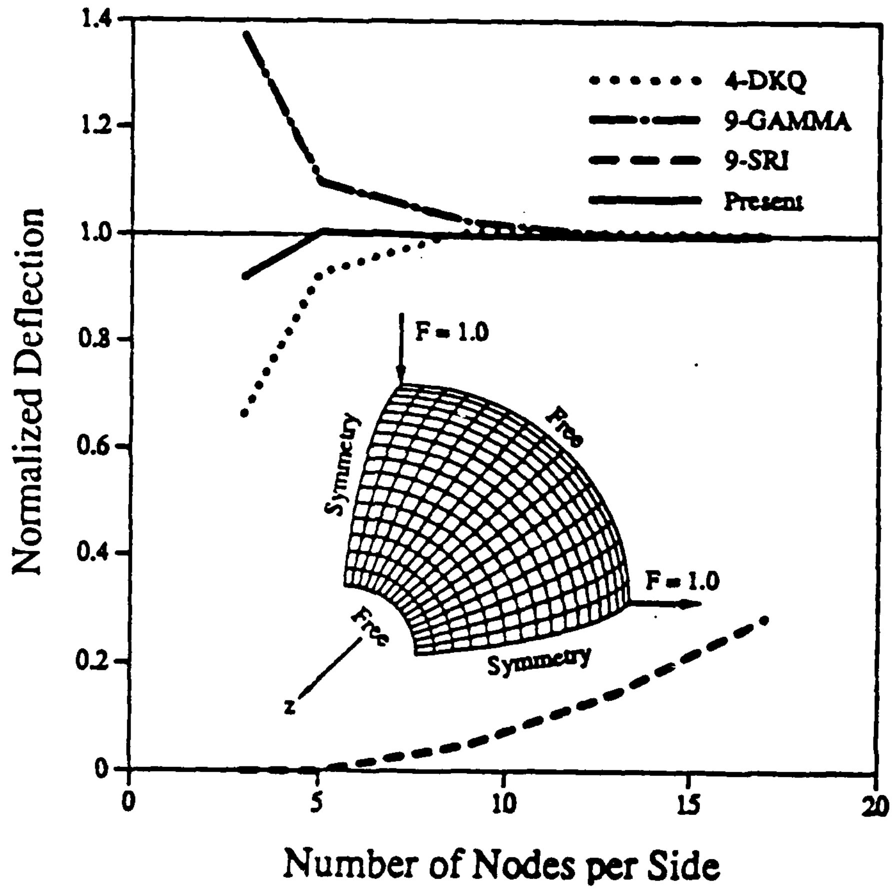

Figure 6. ***The pinched hemisphere: A standard benchmark problem for inextensional response and large rigid body motions.***

i. The underlying geometric structure of the continuum theory is preserved exactly by our discrete finite element interpolation of the director and the linearized director fields. A discrete singularity-free mapping between the five and the six degree of freedom formulation, which appears to be canonical, is constructed again by exploiting the realization of S^2 as the bundle $SO(3)/S^1$. As a result, the proposed numerical formulation exhibits excellent performance under large rigid body motions. This is illustrated in Figure 6 in a classical benchmark problem. In fact, our bilinear finite element formulation exhibits performance superior to current *higher order* state-of-the-art finite element formulations based on the degenerated solid concept.

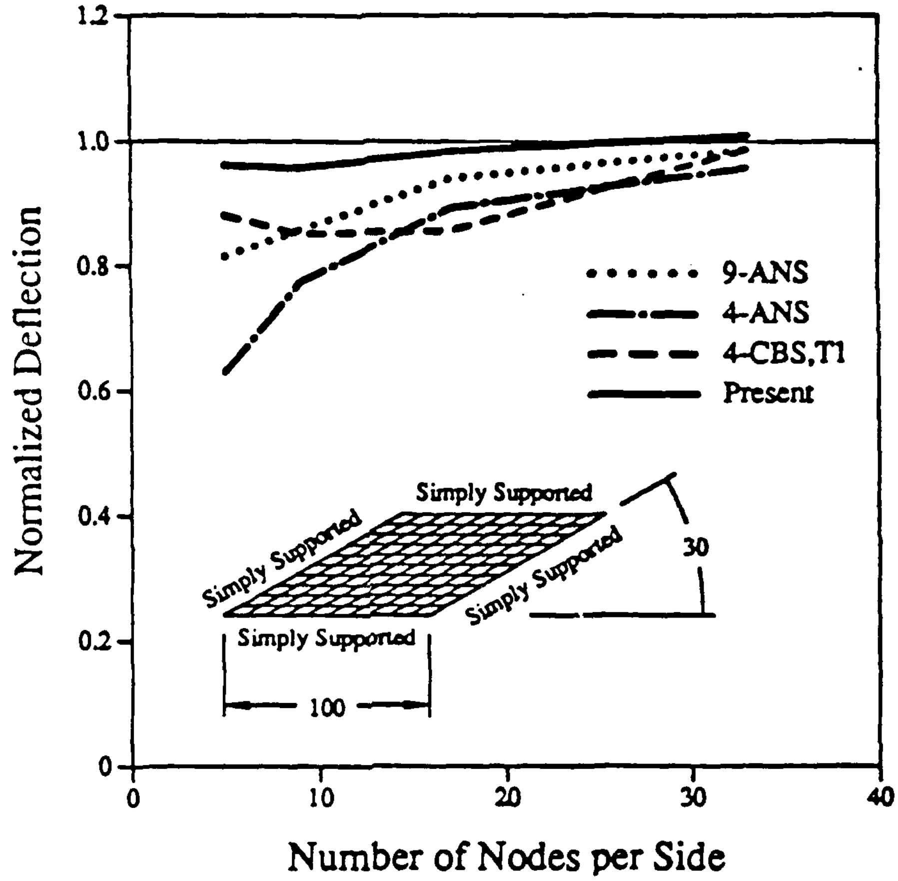

Figure 7. *Convergence results for the rhombic plate: A severe test of the performance of plate bending elements.*

ii. The proposed mixed interpolation for the bending field, which emanates from a Hellinger-Reissner variational formulation, leads to a finite element formulation which is free from spurious zero energy modes, and appears to yield optimal results in standard benchmark problems, (i.e., MacNeal & Harder [1984]). For the classical rhombic plate test (a singularity dominated problem) for instance (see Figure 7) the performance of the element is clearly superior to existing four-node and even nine-node elements; including T1 (Hughes & Tezduyar [1981]), the equivalent element of Bathe & Dvorkin [1984], and the four and nine-node ANS formulations of Parks & Stanley [1986].

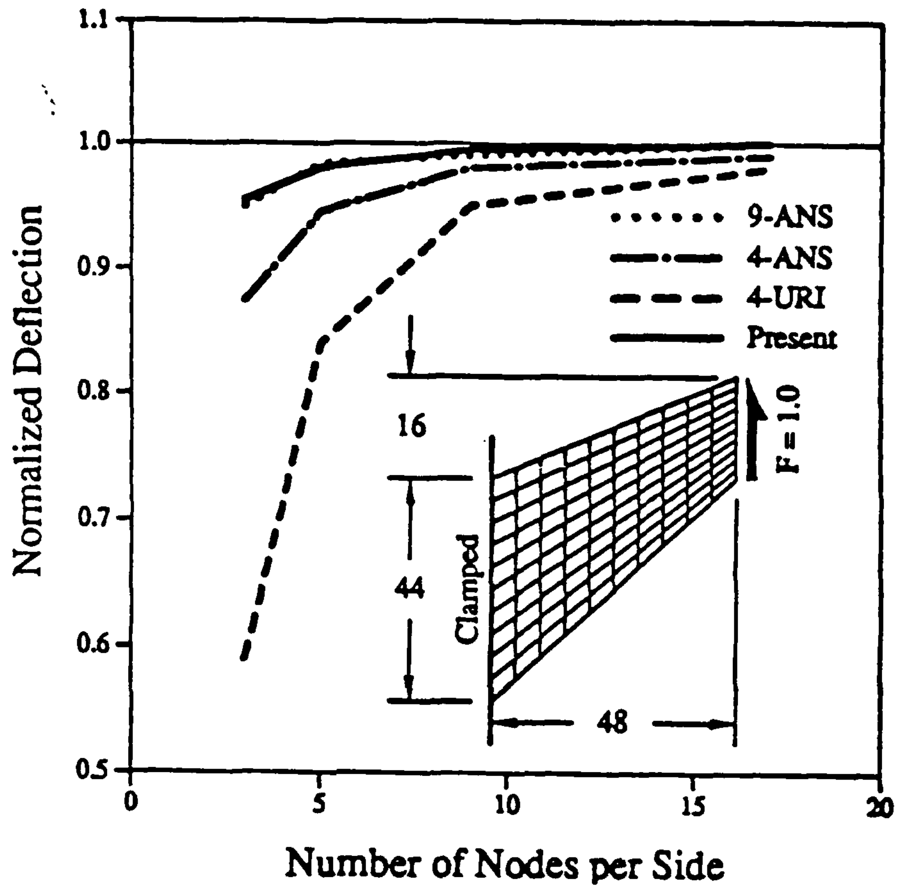

Figure 8. ***Convergence results for Cook's membrane problem: A classical benchmark for membrane dominated problems.***

iii. The proposed interpolation for the membrane field, also based on a mixed Hellinger-Reissner formulation, appears to yield optimal results in standard benchmark tests for membrane dominated problems; such as ***Cook's membrane test***, see Figure 8.

iv. Although our finite element interpolations satisfy exactly all the standard engineering test for convergence; in particular the modified version of the patch test (Taylor, Simo, Zienkiewicz & Chan [1986]) no rigorous mathematical proof of convergence is yet available. Nevertheless, we note that with the exception of linear analysis of Koiter's model by Bernadou & Boisserie [1982], and the recent work of Brezzi & Fortin [1985] and Arnold & Falk [1987] for the *plate problem*, no rigorous convergence results for the discrete shell problem appear to be available in the numerical analysis literature.

4. Additional research accomplished in the two year effort

In addition to the research objectives outlined in the preceding section, two complementary lines of investigation have been explored the results of which are summarized below. The theoretical results obtained in the first area have proven essential in the formulation and numerical analysis aspects of the geometrically exact shell models described above.

4.1. Hamiltonian structure of elasticity, rods and plates

It is now widely accepted that a thorough understanding of the mathematical underpinnings of elasticity is crucial to its analytical and computational implementation. In particular, in the work reported above, a geometric understanding of elasticity has proven essential in the formulation of efficient algorithms for geometrically exact rod models. Underlying this geometric view of elasticity there exists a remarkably simple *non-canonical* Hamiltonian structure which is explored and developed in detail in Simo, Marsden & Krishnaprasad [1987]. Basic motivations for pursuing this line of investigation are the following

1. From a fundamental standpoint, a geometric formulation of the equations of elasticity in Hamiltonian form is extremely useful in the analysis of a wide range problems of physical interest. Typical examples include the study of nonlinear stability of solids, fluids, and plasmas; see i.e., the comprehensive survey of Holm, Marsden, Ratiu & Weinstein [1986]; and the analysis of bifurcation phenomena; i.e., Golubitsky & Stewart [1986], and Marsden & Ratiu [1986].
2. From a computational point of view, geometric methods play an important role in the the development of numerical schemes in nonlinear elasticity. For instance, as already noted, the algorithmic counterparts of the exponential map and the parallel transport are the essential tools in the formulation of algorithms discussed in Section 3.1. Similar techniques are useful in nonlinear inelasticity; i.e., Simo [1987a,b,c].
3. A geometric approach along with the use of Hamiltonian formalism has proven useful in successfully tackling problems involving coupled rigid bodies/flexible attachments. An example of this is the rigorous stability analysis of Krishnaprasad & Marsden [1987] for a restricted class of flexible appendages coupled with a rigid body.

With this motivation at hand, the results obtained in Simo, Marsden & Krishnaprasad [1988] may be briefly summarized as follows *i.* A complete and systematic development of the Hamiltonian structure of nonlinear elasticity, and geometrically exact rods and shells models is given. An interesting point of this development is the central role played by the convective representation which, for rigid bodies, collapses to the familiar description of the motion in *body coordinates*.

ii. Identification of an underlying non-canonical Lie-Poisson structure common to all geometrically exact models, and emanating from the convective representation is given. A complete derivation of the corresponding Lie-Poisson brackets is also given. These Lie-Poisson structures will become essential in future stability calculations of coupled rigid bodied with flexible appendages. A systematic analysis of the dynamics of geometrically exact models is already in progress and the results are reported in Simo, Marsden & Posbergh [1988].

iv. For the case of geometrically exact shell models, a novel geometric picture emanates from this analysis which is currently perceived as playing a central role in future algorithmic treatments. In particular, as the discussion of Section 3.5 and Section 5.6 suggests, the evolution of the director field in a one-director Cosserat surface is intimately connected to the geometric structure associated with the group of mappings from an open set $\Omega \subset \mathbb{R}^2$ onto the unit S^2-sphere.

4.2. Improved material modeling. Material damage and inelasticity

As pointed out in our AFOSR proposal, future generations of space structures are expected to exhibit highly nonlinear constitutive response. Weight and strength considerations motivate a growing use of composite materials, such as the traditional graphite/epoxi composites, or the relatively recent technology of metal-matrix composites. Effective use of this latter class of materials often requires the matrix to operate in a semi-plastic regime, leading to a highly nonlinear response. By contrast, the nonlinearity in the response of graphite-epoxi composites emanates from a physically different mechanism related to damage degradation of the elastic properties of the material.

Because of the complexity of the physical processes involved, which take place at the micro-structural level, characterization of damage degradation and plastic flow is often made by means of phenomenological constitutive models. This has lead to the expanding field of *damage mechanics*. In its present stage, however, a solid mathematical foundation appears appears to be lacking.

Our research efforts in this direction aim at the development of mathematically tractable constitutive models for inelasticity and damage response which are suited for large scale computation. The work in Simo [1987a,b,c], and Simo & Ju [1987a,b,c] constitutes an effort in this direction for plastic, viscoelastic-damage, and elastic-damage response. The work in Simo & Honein [1988], constitutes a systematic development of a variational formulation of elastoplasticity along with conservation laws within the context of Noether's theorems. Relevant numerical analysis and computational aspects are addressed in Simo & Taylor [1986], Simo & Hughes [1986,1987], Simo Kennedy & Govindjee [1988], Simo, Kennedy & Taylor [1988], and Simo [1987d].

Appendix I

Presentations at Meetings and Conferences in 1986-1988.

1. IFAC conference, July 1986.
2. World congress on Computational Mechanics, October 1986.
3. ASME Winter Anual Meeting, December, 1986.
4. International Conference in Numerical Methods Theory and Applications (NUMETA), July, 1987
5. Workshop on Dynamics and Controls, (Sponsored by NASA-SDIO) *Jet Propulsion Laboratory*, September 1987.
6. IFAC conference, December 1987.
7. International Conference on Computational Engineering Science, April, 1988. Key-Note address.
8. Instituto per le Applicazioni del Calcolo, Rome, Italy, May 1988. (2) Invited Lectures.
9. Work-Shop on Symmetry Groups and Invariance in Continuum Mechanics. Cornell University, June 1988. Key-Note address.

10. International Meeting on Engineering Science, Berkeley, June 1988.
11. Summer Conference on Dynamics and Multibody Systems, August, 1988, Organized by P.S. Krishnaprasad, J. Marsden and J.C. Simo. Sponsored by AMS and SIAM. Key-Note address.
12. IFAC Conference, December, 1988.

Appendix II

Publications under AFOSR Support

[1] Simo, J.C., T. Posbergh, and J.E. Marsden, [1988], "Stability Analysis of Coupled Flexible Structures-Rigid Bodies. The Energy-Momentum Method," Preprint, To be submitted to *Archive for Rational Mechanics and Analysis*,.

[2] Simo, J.C., D.D. Fox, and M.S. Rifai, [1988c], "On a stress resultant geometrically exact shell model. Part III: Computational aspects of the nonlinear theory," *Comp. Meth. Appl. Mech. Engng.* Accepted for Publication.

[3] Simo, J.C., and L. Vu-Quoc, [1988c], "A beam model including shear and torsional warping distorsions based on an exact geometric description of nonlinear deformations," *Int. J. Solids Structures*, Submitted for publication.

[4] Simo, J.C., D.D. Fox, and M.S. Rifai, [1988a], "On a stress resultant geometrically exact shell model. Part II: The linear theory; Computational aspects," *Comp. Meth. Appl. Mech. Engng.* Accepted for Publication.

[5] Simo, J.C. and D.D. Fox, [1988], "On a stress resultant geometrically exact shell model. Part I: Formulation and optimal parametrization," *Comp. Meth. Appl. Mech. Engng.* Accepted for Publication.

[6] Simo, J.C., D.D. Fox, and M.S. Rifai, [1988b], "Formulation and computational aspects of a stress resultant geometrically exact shell model," in *Computational Mechanics '88, Theory and Applications*, S.N. Atluri ans G. Yagawa Editors, Springer-Verlag.

[7] Simo, J.C., J.E. Marsden and P.S. Krishnaprasad, [1988], "The Hamiltonian structure of elasticity. The convective representation of solids, rods and plates," *Arch. Rat. Mech. Anal.*, Accepted for publication

[8] Simo, J.C. and L. Vu-Quoc, [1988a], "On the dynamics in Space of rods undergoing large motions -A geometrically exact approach," *Comp. Meth. Appl. Mech. Engng.*, **66**, *125-161*.

[9] Simo, J.C., and L. Vu-Quoc, [1988b], "The role of nonlinear theories in transient dynamic analysis of flexible structure," *J. Sound and Vibration*, **119**(3), 437-508.

[10] Vu-Quoc, L. and J.C. Simo, [1987], "A Novel Approach to the Dynamics of Multicomponent Flexible Satellites," *J. Guidance, Dynamics & Controls,*, 10(6), 549-558.

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