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# REPORT DOCUMENTATION PAGE

| | |
|---|---|
| 1a. REPORT SECURITY CLASSIFICATION<br>Unclassified | 1b. RESTRICTIVE MARKINGS |
| 2a. SECURITY CLASSIFICATION AUTHORITY | 3. DISTRIBUTION/AVAILABILITY OF REPORT<br>Approved for public release; distribution unlimited. |
| AD-A202 273 | 5. MONITORING ORGANIZATION REPORT NUMBER(S)<br>ARO 22729.6-MA |
| 6a. NAME OF PERFORMING ORGANIZATION<br>Standord University | 6b. OFFICE SYMBOL (If applicable) |
| 6c. ADDRESS (City, State, and ZIP Code)<br>Dept. of Statistics<br>Stanford, CA 94305 | 7a. NAME OF MONITORING ORGANIZATION<br>U. S. Army Research Office |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION<br>U. S. Army Research Office | 8b. OFFICE SYMBOL (If applicable) |
| 8c. ADDRESS (City, State, and ZIP Code)<br>P. O. Box 12211<br>Research Triangle Park, NC 27709-2211 | 7b. ADDRESS (City, State, and ZIP Code)<br>P. O. Box 12211<br>Research Triangle Park, NC 27709-2211 |
| | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER<br>DAAG29-85-K-0239 |
| | 10. SOURCE OF FUNDING NUMBERS<br>PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO. |

DTIC ELECTE NOV 23 1988 S H

11. TITLE (Include Security Classification)
Time Series Analysis and Multivariate Statistical Analysis

12. PERSONAL AUTHOR(S)
Theorore W. Anderson

| 13a. TYPE OF REPORT | 13b. TIME COVERED | 14. DATE OF REPORT (Year, Month, Day) | 15. PAGE COUNT |
|---|---|---|---|
| Final | FROM 9/23/85 TO 9/22/88 | November 1988 | 3 |

16. SUPPLEMENTARY NOTATION
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

| 17. COSATI CODES | | |
|---|---|---|
| FIELD | GROUP | SUB-GROUP |
| | | |
| | | |

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Time Series Analysis, Multivariate Statistical Analysis, Autoregressive Processes, Elliptically Contoured Distributions

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research was carried out mainly in the areas of time series analysis and multivariate statistical analysis. The most important results in the first area apply to autoregressive and moving average processes. In the second area emphasis was on parameter consistency and elliptically contoured distributions.

To estimate the parameters of the moving average model 16 different iterative procedures have been devised. These involve alternative parametrizations, time and frequency domain representations, Newton-Raphson and scoring approaches, and use of likelihoods and concentrated likelihoods. Properties of the likelihood function, as well as the estimates, have been derived.

| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT | 21. ABSTRACT SECURITY CLASSIFICATION |
|---|---|
| ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS | Unclassified |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL | 22b. TELEPHONE (Include Area Code) / 22c. OFFICE SYMBOL |

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
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# TIME SERIES ANALYSIS AND MULTIVARIATE STATISTICAL ANALYSIS

FINAL REPORT

Theodore W. Anderson
Principal Investigator

September 23, 1985, to September 22, 1988

U. S. Army Research Office
Contract DAAG29-85-K-0239

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

Research was carried out mainly in the areas of time series analysis and multivariate statistical analysis. The most important results in the first area apply to autoregressive and moving average processes. In the second area emphasis was on parameter consistency and elliptically contoured distributions.

The autoregressive model has been developed in a simple, rigorous, and coherent way, with explicit expressions for the coefficients of the moving average representation and the autocovariance sequence. The conditions on the roots of the associated polynomial equation and properties of the disturbances are implied by stationarity and an independence assumption. A general martingale limit theorem was obtained and applied to autoregressive processes, giving asymptotic results stronger than previous results. The Kalman filter has been used to obtain a sequence of estimates of the parameters of an autoregressive model available as the data are collected; the technique is applicable to panel studies as well as to single time series. Bayes estimates for a prior exchangeable distribution of parameters and for a prior autoregressive distribution of parameters were found. The multivariate autoregressive model has been used to study causality.

To estimate the parameters of the moving average model 16 different iterative procedures have been devised. These involve alternative parametrizations, time and frequency domain representations, Newton–Raphson and scoring approaches, and use of likelihoods and concentrated likelihoods. Properties of the likelihood function, as well as the estimates, have been derived.

"Parameter consistency" of a test means that the power of the test tends towards 1 as the true value of the parameter moves away from the hypothesized value. A complete analysis of this property has been carried out in the multivariate analysis of variance and applied to the conventional tests. They are all parameter consistent except for the Bartlett–Nanda–Pillai trace test; in that case consistency depends on the significance level, dimensionality, and numbers of degrees of freedom. The relationship to the usual notation of consistency has been carried out.

It has been shown that in a class of elliptically contoured distributions $F$–tests in the analysis of variance are valid even though the quadratic forms in the numerator and denominator of the $F$–ratio are not independent; see discussion of "What is an analysis of variance?" by T. P. Speed. Conditions for Cochran's theorem have been relaxed. The normal distribution within the class of elliptically contoured distributions has been characterized by the distribution of a quadratic form.

## Scientific Personnel

T. W. Anderson, Principal Investigator

James Koehler

Chitra Lele

Bruce Newell

Hang P. Zhang

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19. Estimation of a Multivariate Continuous Variable Panel Model, Lawrence S. Mayer and Kun Mao Chen, June 1987.
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