AFOSR-TR- 88-1109

CENTER FOR STOCHASTIC PROCESSES

Department of Statistics
University of North Carolina
Chapel Hill, North Carolina

ADMISSIBLE TRANSLATES OF STABLE PROCESSES:
A SURVEY AND SOME NEW MODELS

by

Stamatis Cambanis

Technical Report No. 235

July 1988

164. P. Avram, On bilinear forms in Gaussian random variables, Toeplitz matrices and Parseval's relation, Nov. 86.

165. D.B.H. Cline, Joint stable attraction of two sums of products, Nov. 86. *J. Multivariate Anal.*, to appear.

166. R.J. Wilson, Model fields in crossing theory—a weak convergence perspective, Nov. 86.

167. D.B.H. Cline, Consistency for least squares regression estimators with infinite variance data, Dec. 86.

168. L.L. Campbell, Phase distribution in a digital frequency modulation receiver, Nov. 86.

169. B.G. Nguyen, Typical cluster size for 2-dim percolation processes, Dec. 86. *J. Statist. Physics*, to appear.

170. H. Oodaira, Freidlin–Wentzell type estimates for a class of self-similar processes represented by multiple Wiener integrals, Dec. 86.

171. J. Nolan, Local properties of index-β stable fields, Dec. 86. *Ann. Probability*, to appear.

172. R. Menich and R.F. Serfozo, Optimality of shortest queue routing for dependent service stations, Dec. 86.

173. F. Avram and M.S. Taqqu, Probability bounds for M–Skorohod oscillations, Dec. 86.

174. F. Moricz and R.L. Taylor, Strong laws of large numbers for arrays of orthogonal random variables, Dec. 86.

175. G. Kallianpur and V. Perez–Abreu, Stochastic evolution driven by nuclear space valued martingales, Apr. 87.

176. E. Merzbach, Point processes in the plane, Feb. 87.

177. Y. Kasahara, M. Maejima and W. Vervaat, Log fractional stable processes, March 87.

178. G. Kallianpur, A.G. Miamee and H. Niemi, On the prediction theory of two parameter stationary random fields, March 87.

179. R. Brigola, Remark on the multiple Wiener integral, Mar. 87.

180. R. Brigola, Stochastic filtering solutions for ill–posed linear problems and their extension to measurable transformations, Mar. 87.

181. G. Samorodnitsky, Maxima of symmetric stable processes, Mar. 87.

182. H.L. Hurd, Representation of harmonizable periodically correlated processes and their covariance, Apr. 87.

183. H.L. Hurd, Nonparametric time series analysis for periodically correlated processes, Apr. 87.

184. T. Mori and H. Oodaira, Freidlin–Wentzell estimates and the law of the iterated logarithm for a class of stochastic processes related to symmetric statistics, May 87.

185. R.F. Serfozo, Point processes, May 87. *Operations Research Handbook on Stochastic Processes*, to appear.

186. Z.D. Bai, W.Q. Liang and W. Vervaat, Strong representation of weak convergence, June 87.

187. O. Kallenberg, Decoupling identities and predictable transformations in exchangeability, June, 87.

188. O. Kallenberg, An elementary approach to the Daniell–Kolmogorov theorem and some related results, June 87. *Math. Narch.*, to appear.

189. G. Samorodnitsky, Extrema of skewed stable processes, June 87.

190. D. Nualart, M. Sanz and M. Zakai, On the relations between increasing functions associated with two–parameter continuous martingales, June 87.

191. F. Avram and M. Taqqu, Weak convergence of sums of moving averages in the α–stable domain of attraction, June 87.

192. M.R. Leadbetter, Harald Cramér (1893–1985), July 87. *ISI Review*, to appear.

193. R. LePage, Predicting transforms of stable noise, July 87.

194. R. LePage and B.M. Schreiber, Strategies based on maximizing expected log, July 87.

195. J. Rosinski, Series representations of infinitely divisible random vectors and a generalized shot noise in Banach spaces, July 87.

196. J. Szulga, On hypercontractivity of α–stable random variables, $0<\alpha<2$, July 87.

197. I. Kuznezova–Sholpo and S.T. Rachev, Explicit solutions of moment problems I, July 87. *Probability Math. Statist.*, 10, 1989, to appear.

198. T. Hsing, On the extreme order statistics for a stationary sequence, July 87.

199. T. Hsing, Characterization of certain point processes, Aug. 87. *Stochastic Proc. Appl.* 26, 1987, 297–316.

200. J.P. Nolan, Continuity of symmetric stable processes, Aug. 87.

201. M. Marques and S. Cambanis, Admissible and singular translates of stable processes, Aug. 87.

202. O. Kallenberg, One–dimensional uniqueness and convergence results for exchangeable processes, Aug. 87.

203. R.J. Adler, S. Cambanis and G. Samorodnitsky, On stable Markov processes, Sept. 87.

204. G. Kallianpur and V. Perez–Abreu, Stochastic evolution equations driven by nuclear space valued martingales, Sept. 87. *Appl. Math. Optimization*, 17, 1988, 237–272.

205. R.L. Smith, Approximations in extreme value theory, Sept. 87.

206. E. Willekens, Estimation of convolution tails, Sept. 87.

207. J. Rosinski, On path properties of certain infinitely divisible processes, Sept. 87.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

ADA200636

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY NA	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE NA	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Technical Report No. 235	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 88-1109
6a. NAME OF PERFORMING ORGANIZATION University of North Carolina 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
6c. ADDRESS (City, State and ZIP Code) Statistics Department CB #3260, Phillips Hall Chapel Hill, NC 27599-3260	7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F49620 85 C 0144
8c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 6.1102F PROJECT NO.: 2304 TASK NO.: A5 WORK UNIT NO.: leave blnk

11. TITLE (Include Security Classification)
Admissible translates of stable processes: a survey and some new models

12. PERSONAL AUTHOR(S)
Cambanis, S.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
preprint	FROM 9/87 TO 8/88	July 1988	8

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Key words and phrases: N/A
XXX	XXXXXXXXXX	XX	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

We survey some recent results on the admissible translates of stable processes and we contrast them with the analogs for Gaussian processes. Whereas Gaussian moving averages and Fourier transforms of independent increments processes have rich classes of admissible translates, their stable counterparts frequently have all translates singular. By removing the requirement of independence of the increments, we introduce stable processes that are generalized moving averages and harmonizable which can have rich classes of admissible translates. These are generally nonstationary processes but we also show a class of stationary generalized moving averages.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Major Brian W. Woodruff	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5026 22c. OFFICE SYMBOL AFOSR/NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

ADMISSIBLE TRANSLATES OF STABLE PROCESSES:

A SURVEY AND SOME NEW MODELS

Stamatis Cambanis

Department of Statistics
University of North Carolina
Chapel Hill, NC 27599-3260

ABSTRACT

We survey some recent results on the admissible translates of stable processes and we contrast them with the analogs for Gaussian processes. Whereas Gaussian moving averages and Fourier transforms of independent increments processes have rich classes of admissible translates, their stable counterparts frequently have all translates singular. By removing the requirement of independence of the increments, we introduce stable processes that are generalized moving averages and harmonizable which can have rich classes of admissible translates. These are generally nonstationary processes but we also show a class of stationary generalized moving averages.

Research supported by the Air Force Office of Scientific Research Grant Number F49620 85 C 0144.

1. INTRODUCTION

Throughout $X = \{X(t,\omega), -\infty < t < \infty\}$ is a stochastic process defined on a probability space $(\Omega,\mathcal{F},P)$, and $s = \{s(t), -\infty < t < \infty\}$ is a nonrandom function. The map $\omega \rightarrow X(\cdot\ , \omega)$ from the probability space to the space of all functions on the real line, induces a probability measure P_X on the σ-field of cylinder sets, the distribution of the process X. If the distribution of the translate of X by s, P_{s+X}, is absolutely continuous with respect to the distribution of X, P_X, then s is called an admissible translate of X, and if P_{s+X} and P_X are singular, then s is called a singular translate. The set of all admissible translates of X is denoted by AT(X).

In signal detection s is the signal, X is the random noise, and based on a sample observation (a function on the real line) one decides whether the observation is due to signal plus noise (s+X) or to noise alone (X). For a singular translate s, in principle, a correct decision can be made with probability one, i.e. signals that are singular translates of the noise can be perfectly detected. For an admissible translate s the decision is based on the Radon-Nikodym derivative of P_{s+X} with respect to P_X (likelihood ratio) according to the Neyman-Pearson rule and the resulting probability of detecting correctly the presence of the signal is always strictly less than one. Most real life signal detection problems correspond to admissible translates.

In Section 2 we describe the well-known complete results on the admissible and singular translates of Gaussian processes. In Section 3 we summarize some recent results for stable processes which are far from being complete. We refer to [1] and references therein.

It turns out that the widely used moving average and Fourier transform models typically have all translates singular in the non-Gaussian stable case, and thus do not provide realistic noise models for signal detection. We thus introduce in Section 3 some generalized moving averages and harmonizable processes which in the non-Gaussian stable case may have rich classes of admissible translates and therefore serve as stable noise models in signal detection.

2. GAUSSIAN PROCESSES

Let X be a Gaussian process with mean zero and covariance function R. The set of admissible translates of X is precisely the reproducing kernel Hilbert space of its kernel R, and all other translates are singular. More concrete representations follow.

Every Gaussian process X can be represented in terms of a Gaussian process ξ with independent increments as follows:

$$X(t) = \int_{-\infty}^{\infty} f(t,\lambda)\, d\xi(\lambda), \quad -\infty < t < \infty, \tag{1}$$

where $f(t,\cdot) \in L_2(\mu)$ and μ is the control measure of ξ : $d\mu(\lambda) = E\,|d\xi(\lambda)|^2$. The integral $\int g\, d\xi$ is defined for all $g \in L_2(\mu)$ and is a normal random variable with characteristic function

$$E \exp\{\, ir \int g\, d\xi \,\} = \exp\{\, -\tfrac{1}{2} r^2 \int |\, g\,|^2\, d\mu \,\}.$$

The covariance function of X is represented as $R(t,s) = \int_{-\infty}^{\infty} f(t,\lambda)\, \overline{f}(s,\lambda)\, d\mu(\lambda)$. Then

$$AT(X) = \{\, s(t) = \int_{-\infty}^{\infty} f(t,\lambda)\, g(\lambda)\, d\mu(\lambda), \quad g \in L_2(\mu) \,\}. \tag{2}$$

Stationary Gaussian processes (that are continuous in probability) have a spectral representation:

$$X(t) = \int_{-\infty}^{\infty} e^{it\lambda}\, d\xi(\lambda), \tag{3}$$

where ξ has independent increments and finite spectral (control) measure μ, and their covariance function is $R(t,s) = \int_{-\infty}^{\infty} e^{i(t-s)\lambda}\, d\mu(\lambda)$. Their admissible translates are Fourier transforms of finite signed measures that are absolutely continuous with respect to μ with μ-square integrale Radon-Nikodym derivative:

$$AT(X) = \{\, s(t) = \int_{-\infty}^{\infty} e^{it\lambda}\, d\nu(\lambda), \quad \nu << \mu, \ \frac{d\nu}{d\mu} \in L_2(\mu) \,\}. \tag{4}$$

When μ << Leb with spectral density $\phi(\lambda) = d\mu(\lambda)/d\lambda$ then

$$AT(X) = \{\, s(t) = \int_{-\infty}^{\infty} e^{it\lambda}\, S(\lambda)\, d\lambda, \quad \int_{-\infty}^{\infty} \frac{|S(\lambda)|^2}{\phi(\lambda)}\, d\lambda < \infty \,\},$$

and in addition to the spectral representation (3), X also has a moving average representation:

$$X(t) = \int_{-\infty}^{\infty} h(t-s)\, d\xi(s), \tag{5}$$

where in (5), ξ has stationary independent increments (i.e., μ = Leb) and $h \in L_2$; its covariance function is $R(t,s) = \int_{-\infty}^{\infty} e^{i(t-s)\lambda} |\hat{h}(\lambda)|^2\, d\lambda$; and its admissible translates are

$$AT(X) = \{\, s(t) = \int_{-\infty}^{\infty} h(t-s)\, g(s)\, ds, \quad g \in L_2 \,\}. \tag{6}$$

For nonstationary processes representations in terms of processes with dependent increments are useful. Every Gaussian process X can be represented in terms of a Gaussian process η with possibly dependent increments as follows:

$$X(t) = \int_{-\infty}^{\infty} f(t,u)\, d\eta(u), \quad -\infty < t < \infty. \tag{1'}$$

Here η is an $L_2(\Omega,\mathcal{F},P)$-valued Gaussian measure generally not independently scattered, with bimeasure β: $\beta(B,C) = E\{\, \eta(B)\, \overline{\eta}(C) \,\}$, B,C : bounded Borel sets, and each $f(t,\cdot)$ is η-integrable. The covariance of X is $R(t,s) = \iint_{-\infty}^{\infty} f(t,u)\, \overline{f}(s,v)\, d\beta(u,v)$, and its admissible translates are

$$AT(X) = \{\, s(t) = \iint_{-\infty}^{\infty} f(t,u)\, g(v)\, d\beta(u,v) = \int_{-\infty}^{\infty} f(t,u)\, d\beta_g(u), \quad g : \eta\text{-integrable} \,\} \tag{2'}$$

where β_g is the measure $\beta_g(\cdot) = \int_{-\infty}^{\infty} g(v)\, \beta(\cdot\,, dv)$.

When $f(t,u) = e^{itu}$ in (1′) and η has bounded semi-variation, then X is harmonizable:

$$X(t) = \int_{-\infty}^{\infty} e^{itu}\, d\eta(u), \tag{3'}$$

with bispectral measure β and covariance $R(t,s) = \iint_{-\infty}^{\infty} e^{i(tu-sv)}\, d\beta(u,v)$. It is stationary only if its bispectral measure β is concentrated on the diagonal of the plane. Its admissible translates are:

$$AT(X) = \{\, s(t) = \int_{-\infty}^{\infty} e^{itu}\, d\beta_g(u), \quad g : \eta\text{-integrable} \,\}. \tag{4'}$$

When $f(t,u) = h(t-u)$ in (1′), X is a "generalized" moving average (of a dependent increments process):

(5′) $$X(t) = \int_{-\infty}^{\infty} h(t-u)\, d\eta(u),$$

whose admissible translates are

(6′) $$AT(X) = \{\ s(t) = \int_{-\infty}^{\infty} h(t-u)\, d\beta_g(u), \quad g : \eta\text{-integrable}\ \}.$$

The representations of the admissible translates of general nonstationary processes, (2′), (4′), and (6′), are similar to those of stationary processe, (2), (4) and (6), but not as explicit and simple because generally no explicit description of the η-integrable functions is available.

3. STABLE PROCESSES

In this section X is a symmetric α-stable (SαS) process ($0 < \alpha \leq 2$), i.e. all linear combinations $c_1X(t_1)+ \ldots +c_nX(t_n)$ are SαS random variables (with characteristic functions of the form $\exp\{ -\text{const.} \mid r \mid^{\alpha} \}$). When $\alpha = 2$, X is Gaussian. When $0 < \alpha < 2$, X is non-Gaussian stable and can be represented as in (1), where ξ has SαS independent increments and control measure μ. Here the integral $\int g\ d\xi$ is defined for all $g \in L_\alpha(\mu)$ and is a SαS random variable with characteristic function

$$E \exp\{\ ir \int g\, d\xi\ \} = \exp\{\ -\mid r \mid^{\alpha} \int \mid g \mid^{\alpha} d\mu\ \}.$$

A crucial difference in the representation (1) between the Gaussian (α=2) and non-Gaussian ($0 < \alpha < 2$) cases is that whereas in the Gaussian case the linear space of the increments of ξ can always be taken to be equal to the linear space of the process X, in the non-Gaussian case the former is generally larger than the latter (and generally infinite dimensional even when the linear space of X is finite dimensional).

The analog of (2) is no longer valid when $0 < \alpha < 2$ and all that can be said in general is that

(7) $$AT(X) \subset \{\ s(t) = \int_{-\infty}^{\infty} f(t,\lambda)\, g(\lambda)\, d\mu(\lambda), \quad g \in L_{\alpha^*}(\mu)\ \} \triangleq \mathbf{F}(X),$$

where $1/\alpha + 1/\alpha^* = 1$, and that translates outside the set $\mathbf{F}(X)$ are singular. AT(X) may be as large as $\mathbf{F}(X)$ (e.g. when X is sub-Gaussian) or as small as $\{0\}$ (e.g. when X is Lévy motion).

In order to describe better the contrasts and similarities between the Gaussian and other stable cases we first concentrate on the independently scattered SαS measure $\xi = \{ \xi(B), B \in \mathcal{B}_b\}$ in (1), where $\mathcal{B}_b$ denotes the bounded Borel sets of the real line. When $\alpha = 2$, AT(ξ) consists of all signed measures which are absolutely continuous with respect to μ with Radon-Nikodym derivative in $L_2(\mu)$. In sharp contrast when $0 < \alpha < 2$ and μ is nonatomic, then ξ has no admissible translates and in fact all translates are singular. On the other hand, when $0 < \alpha < 2$ and μ is purely atomic with atoms $\{ a_n\}_{n=1}^N$, $1 \leq N \leq \infty$, then AT(ξ) consists of all measures s concentrated on $\{a_n\}_{n=1}^N$ with

$$\sum_{n=1}^{N} |s(\{a_n\})|^2 / \mu^{2/\alpha}(\{a_n\}) < \infty ;$$

this result is due to Shepp [2] and is valid for all $0 < \alpha \leq 2$. For a general control measure μ, AT(ξ) $= \{0\}$ when μ has no atoms, and when μ has atoms $\{a_n\}_{n=1}^N$ then AT(ξ) is as above.

We now consider a general SαS process with representation (1) and $0 < \alpha < 2$. X is called invertible if its representors $\{f(t, \cdot\,), -\infty < t < \infty \}$ are complete in $L_\alpha(\mu)$, i.e. if the linear spaces of X and of the increments of ξ are equal. When X is invertible then every translate is either admissible or singular and AT(X) = $\{0\}$ when μ has no atoms and if μ has atoms $\{a_n\}_{n=1}^N$, $1 \leq N \leq \infty$, then

$$AT(X) = \{ s(t) = \sum_{n=1}^{N} s_n \, \mu^{1/\alpha}(\{a_n\}) \, f(t,a_n) \, , \quad \sum_{n=1}^{N} | s_n|^2 < \infty \} .$$

Here AT(X) is a proper subset of $\mathbf{F}(X)$ even when μ is purely atomic.

When X is stationary and has the spectral representation (3), where ξ has independent SαS increments and finite spectral (control) measure μ, then X is invertible. In the Gaussian case its admissible translates are described in (4). In the non-Gaussian case $0 < \alpha < 2$, if the spectral measure μ has no atoms all translates of X are singular and if μ has atoms $\{a_n\}_{n=1}^N$, $1 \leq N \leq \infty$, then

$$AT(X) = \{ s(t) = \sum_{n=1}^{N} s_n \, \mu^{1/\alpha}(\{a_n\}) \, e^{ita_n} \, , \quad \sum_{n=1}^{N} | s_n|^2 < \infty \}.$$

When X is stationary and has a moving average representation (5), where ξ has stationary independent SαS increments, then all translates of X are singular when $0 < \alpha < 2$, provided the translates of its kernel h are complete in L_α. In the Gaussian case the admissible translates of X are described in (6).

The processes with spectral (3) and those with moving average (5) representation are the most widely studied classes of stationary stable processes, and are actually disjoint when $0 < \alpha < 2$, while in the Gaussian case $\alpha = 2$ the latter is a subset of the former. Since when $0 < \alpha < 2$ in most cases they have no admissible translates and all their translates are singular, they do not provide reasonable models of stable noise in signal detection, as every signal can be detected perfectly in principle in the presence of such noise. One way to introduce more realistic stable noise models is to allow the increments of ξ in (3) and (5) to have some dependence, i.e. to consider the stable counterparts of (3$'$) and (5$'$).

Thus let X have representation (1$'$), where η has dependent SαS increments with $0 < \alpha < 2$. Here η is a SαS measure which is not independently scattered and can be represented as

$$\eta(\cdot) = \int_{-\infty}^{\infty} \nu(\cdot\ ,\lambda)\ d\xi(\lambda),$$

where ξ has independent SαS increments and control measure μ, and ν is an $L_\alpha(\mu)$-valued measure. If g is ν-integrable then $\int g\ d\nu \in L_\alpha(\mu)$ and

$$\int_{-\infty}^{\infty} g\ d\eta = \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} g\ d\nu \}\ d\xi.$$

Thus with each $f(t,\cdot)$ being ν-integrable we have

$$X(t) = \int_{-\infty}^{\infty} f(t,u)\ d\eta(u) = \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} f(t,u)\ \nu(du,\lambda) \}\ d\xi(\lambda), \quad -\infty < t < \infty. \tag{8}$$

To ensure that η is not independently scattered we assume that for some disjoint Borel sets B_1 and B_2, $\mu\{ \lambda : \nu(B_1,\lambda)\ \nu(B_2,\lambda) \neq 0 \} > 0$, since $\eta(B_1)$ and $\eta(B_2)$ are independent iff $\nu(B_1,\lambda)\ \nu(B_2,\lambda) = 0$ a.e. $[\mu]$. Note that $\eta(\{a\}) = 0$ iff $\int_{-\infty}^{\infty} |\nu(\{a\},u)|^2\ d\mu(u) = 0$, so that η may have no atoms even when ξ does, e.g. if ν has no atoms: $\nu(\{a\},u) = 0$ a.e. $[\mu]$. It is therefore possible for X to have admissible translates even when η has no atoms.

We further require X to be invertible, i.e. $\{ \int_{-\infty}^{\infty} f(t,u)\ \nu(du, \cdot),\ -\infty < t < \infty\}$ to be complete in $L_\alpha(\mu)$, and μ to have atoms $\{a_n\}_{n=1}^{N}$, $1 \leq N \leq \infty$. It then follows that

$$AT(X) = \{ s(t) = \sum_{n=1}^{N} s_n\ \mu^{1/\alpha}(\{a_n\}) \int_{-\infty}^{\infty} f(t,u)\ \nu(du,a_n)\ ,\quad \sum_{n=1}^{N} |s_n|^2 < \infty \}.$$

When $N < \infty$ the representation of an admissible translate can be written as in (2′): $s(t) = \int_{-\infty}^{\infty} f(t,u)\ d\nu_N(u)$ where the measure ν_N is

$$\nu_N(\cdot) = \sum_{n=1}^{N} s_n\ \mu^{1/\alpha}(\{a_n\})\ \nu(\cdot, a_n). \tag{9}$$

When $N = \infty$ the infinite series in (9) does not generally define a measure and we can only write $s(t) = \lim_{k\to\infty}\int_{-\infty}^{\infty} f(t,u)\ d\nu_k(u)$, where ν_k is defined as in (9).

With $f(t,u) = e^{itu}$ in (8) and η of bounded semi-variation then X is harmonizable:

$$X(t) = \int_{-\infty}^{\infty} e^{itu}\ d\eta(u) = \int_{-\infty}^{\infty} \hat{\nu}(t,\lambda)\ d\xi(\lambda),$$

where $\hat{\nu}(t,\lambda) = \int_{-\infty}^{\infty} e^{itu}\ \nu(du,\lambda)$, with admissible translates all functions of the form (as in (4′))

$$s(t) = \int_{-\infty}^{\infty} e^{itu}\ d\nu_N(u) \quad \text{if } N < \infty, \quad = \lim_{k\to\infty} \int_{-\infty}^{\infty} e^{itu}\ d\nu_k(u) \quad \text{if } N = \infty .$$

Harmonizable processes are nonstationary when η has dependent increments.

When $f(t,u) = h(t-u)$ in (8), X is a "generalized" moving average (of a dependent increments process):

$$X(t) = \int_{-\infty}^{\infty} h(t-u)\ d\eta(u) = \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} h(t-u)\ \nu(du,\lambda)\ \}\ d\xi(\lambda) \tag{10}$$

with admissible translates are all functions of the form (as in (6′))

$$s(t) = \int_{-\infty}^{\infty} h(t-u)\ d\nu_N(u) \quad \text{if } N < \infty, \quad = \lim_{k\to\infty} \int_{-\infty}^{\infty} h(t-u)\ d\nu_k(u) \quad \text{if } N = \infty. \tag{11}$$

It is easy to construct examples of nonstationary generalized moving averages, but there exist stationary ones as well. Indeed, taking

$$\nu(B,\lambda) = \phi(\lambda) \int_B e^{ix\theta(\lambda)}\, dx$$

for bounded Borel sets B where $\phi \in L_\alpha(\mu)$ we have for $g \in L_1$, $\int_{-\infty}^{\infty} g(x)\, \nu(dx,\lambda) = \phi(\lambda)\, \hat{g}[\theta(\lambda)]$ and from (10),

$$X(t) = \int_{-\infty}^{\infty} e^{it\theta(\lambda)}\, \phi(\lambda)\, \hat{h}[-\theta(\lambda)]\, d\xi(\lambda).$$

Therefore X is stationary and invertible with admissible translates all functions of the form (as in (6) and (6′))

$$s(t) = \int_{-\infty}^{\infty} h(t-u)\, g(u)\, du,$$

where $g(u) = \sum_{n=1}^{N} s_n\, \mu^{1/\alpha}(\{a_n\})\, \phi(a_n)\, e^{iu\theta(a_n)}$ and $\sum_{1}^{N} |s_n|^2 < \infty$.

Thus harmonizable and generalized moving averaged SαS processes can be constructed with rich classes of admissible translates which may serve as realistic models of noise in signal detection.

REFERENCES

1. M. Marques and S. Cambanis, Admissible and singular translates of stable processes, Center for Stochastic Processes Tech. Rept. No. 201, University of North Carolina, 1987.
2. L. A. Shepp, Distinguishing a sequence of random variables from a translate of itself, Ann. Math. Statist. 36, 1965, 1107-1112.

208. A.H. Korezlioglu, Computation of filters by sampling and quantization, Sept. 87.

209. J. Bather, Stopping rules and observed significance levels, Sept. 87.

210. S.T. Rachev and J.E. Yukich, Convolution metrics and rates of convergence in the central limit theorem, Sept. 87.

211. M. Fujisaki, Normed Bellman equation with degenerate diffusion coefficients and its applications to differential equations, Oct. 87.

212. G. Simons, Y.C. Yao and X. Wu, Sequential tests for the drift of a Wiener process with a smooth prior, and the heat equation, Oct. 87.

213. R.L. Smith, Extreme value theory for dependent sequences via the Stein–Chen method of Poisson approximation, Oct. 87.

214. C. Houdré, A note on vector bimeasures, Nov. 87.

215. M.R. Leadbetter, On the exceedance random measures for stationary processes, Nov. 87.

216. M. Marques, A study on Lebesgue decomposition of measures induced by stable processes, Nov. 87 (*Dissertation*).

217. M.T. Alpuim, High level exceedances in stationary sequences with extremal index, Dec. 87. *Stochastic Proc. Appl.*, to appear.

218. R.F. Serfozo, Poisson functionals of Markov processes and queueing networks, Dec. 87.

219. J. Bather, Stopping rules and ordered families of distributions, Dec. 87.

220. S. Cambanis and M. Maejima, Two classes of self-similar stable processes with stationary increments, Jan. 88.

221. H.P. Hucke, G. Kallianpur and R.L. Karandikar, Smoothness properties of the conditional expectation in finitely additive white noise filtering, Jan. 88.

222. I. Mitoma, Weak solution of the Langevin equation on a generalized functional space, Feb. 88.

223. L. de Haan, S.I. Resnick, H. Rootzén and C. de Vries, Extremal behaviour of solutions to a stochastic difference equation with applications to arch–processes, Feb. 88.

224. O. Kallenberg and J. Szulga, Multiple integration with respect to Poisson and Lévy processes, Feb. 88.

225. D.A. Dawson and L.G. Gorostiza, Generalized solutions of a class of nuclear space valued stochastic evolution equations, Feb. 88.

226. G. Samorodnitsky and J. Szulga, An asymptotic evaluation of the tail of a multiple symmetric α–stable integral, Feb. 88.

227. J.J. Hunter, The computation of stationary distributions of Markov chains through perturbations, Mar. 88.

228. H.C. Ho and T.C. Sun, Limiting distribution of nonlinear vector functions of stationary Gaussian processes, Mar. 88.

229. R. Brigola, On functional estimates for ill–posed linear problems, Apr. 88.

230. M.R. Leadbetter and S. Nandagopalan, On exceedance point processes for stationary sequences under mild oscillation restrictions, Apr. 88.

231. S. Cambanis, J. P. Nolan and J. Rosinski, On the oscillation of infinitely divisible processes, Apr. 88.

232. G. Hardy, G. Kallianpur and S. Ramasubramanian, A nuclear space–valued stochastic differential equation driven by Poisson random measures, Apr. 88.

233. D. J. Daley, T. Rolski, Light traffic approximations in queues (*II*), May 88.

234. G. Kallianpur, I. Mitoma, R. L. Wolpert, Diffusion equations in duals of nuclear spaces, July 88.

235. S. Cambanis, Admissible translates of stable processes: A survey and some new models, July 88.