COST IMPROVEMENT ANALYSIS

QMT 180

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This is the course text for the AFIT Cost Improvement Curve Analysis course. The text introduces the unit and cumulative average cost improvement curve formulations, addresses the theory of the cost improvement curve and focuses on how to use the cost improvement curve in an environment of change and instabilitly.

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This text has been prepared for your use here at the AFIT School of Systems and Logistics. Our intent is that it will serve you not only during your academic studies but equally well as a handbook on your job in the future. Any text is the culmination of the efforts of many people over a long period of time. This text is no exception and I would like to give recognition to those instrumental in its writing. The substance of Chapter 1 was written by Colonel Larry L. Smith. Chapters 2 and 3 are from materials originally developed in the text of Messrs Kroeker and Peterson (see Bibliography), with the section on changes written by Captain John D. Scherrer. Minor changes to Chapters 1, 2, and 3, were written by the undersigned, while Chapter 5 was written by Mr. Jack Hale (see Bibliography). A special thanks to Mrs. Betty Mash and Ms. Patty Beal who typed the draft manuscript and to Ms. Kim Crawford who patiently and expertly completed the final version of this textbook. Thank you one and all.

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Assistant Professor of Cost
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Appendices

CHAPTER I

THE COST IMPROVEMENT CURVE - CONCEPT AND FUNDAMENTALS

The cost improvement curve is a quantitative technique used to predict resource requirements in a manufacturing operation. As an estimating tool, it belongs to the parametric family of estimating as it depends on historical costs to forecast future costs through trend projection. The cost improvement curve theory has been used successfully to predict the direct engineering and manufacturing labor hours needed to produce a known quantity of a product. It has been used to predict the dollar costs of material and hardware items **AFTER ADJUSTMENT OF THE HISTORICAL COST DATA FOR INFLATION.** In this chapter, the theory of the cost improvement curve will be discussed under the following sub-topics: An overview of the history of the cost improvement curve, characteristics of the cost improvement curve process, the mechanics of using log-log paper, measuring the slope of a cost improvement curve, extending the line and using the Boeing Improvement Curve Tables.

AN OVERVIEW OF THE HISTORY OF THE COST IMPROVEMENT CURVE

The term "cost improvement curve" was adapted from observation that individuals performing repetitive tasks exhibit a rate of improvement due to increased manual dexterity. The mental and muscular adjustments made by an individual from the time the task is first performed to the time the task has been repeated a number of times result in a reduction in the time required for each repetition. Psychologists, teachers, personnel directors, manpower planners, and others have recognized and used this principle for a long time. When this improvement factor in a manufacturing process is subjected to further refinements of observation and analysis, an indication of the causes of improvement become apparent. Dexterity on the part of individual workers is only one of the reasons for improvement in the reduction of labor hours per unit of production. Changes in the worker's environment, changes in morale, changes in the flow process, work simplification, engineering changes, changes in work set-up all may contribute to improvement (or, conversely contribute to disimprovement). Such changes are nearly always induced by management functions. Thus not only the cost effects of changing manual dexterity, but also a broad group of factors which might be called management innovations (and the interactions among manual dexterity and the various management innovations) are measured and predicted by the cost improvement curve. Several other terms also describe the cost improvement curve: Learning curve, improvement curve, cost or time reduction curve, experience curve, cost-quantity curve, Wright curve (cumulative average theory), or Crawford Curve (unit theory). A cost improvement curve is the term currently being used by many DOD analysts. When referring to the cost improvement curve, one must understand that all the complexities of causal relationships are embodied in its meaning. In essence, it represents the learning of the firm and is not specifically isolated to the learning of individuals.

One person who has contributed much in establishing the cost improvement curve as a forecasting tool in the aircraft industry is **T.P. Wright**. His article which pioneered the idea was published in the Journal of Aeronautical Sciences, February, 1936, under the title, Factors Affecting the Cost of Airplanes. Wright's findings showed that, as the number of aircraft produced in sequence increased, the cumulative average direct labor input per airplane decreased in a regular pattern. The regularity of the pattern existed in a relationship which was exponential (see Figure 1-1). This pattern is becomes a linear function by taking the log of each of these ratios of changes (see Figure 1-1). Cost improvement curve theory and practice, as it is known today, received its initial impetus from this pioneering work.

Aircraft companies and the DOD became interested in the regular and predictable nature of the reduction of production costs because, among other considerations, the phenomena implied that a fixed application of labor and facilities could be expected to produce greater and greater quantities of defense products in each successive time period. Accordingly, the Government engaged the Stanford Research Institute to study the validity of the cost improvement curve concept. The method adopted for this study was a statistical analysis of nearly all World War II airframe direct labor data to determine whether there was sufficient conformity in the data to establish a cost estimating relationship (CER). The study confirmed the fact that direct labor cost (hours) declines by some constant percentage over successively doubled quantities of units produced. The Stanford study, headed by **J. R. Crawford**, also validated the concept of a model based on the World War II findings that could be used as a forecasting tool.

Since World War II, the cost improvement curve concept has been used by Government agencies to aid in estimating the cost of selected Government hardware items. Its application has been quite conspicuous in airframe production where conditions were most favorable for its use. More recently, the cost improvement curve has been used in such production industries as electronic systems, machine tools, ship building, missile systems and depot level maintenance of equipment.

CHARACTERISTICS OF THE COST IMPROVEMENT CURVE PROCESS

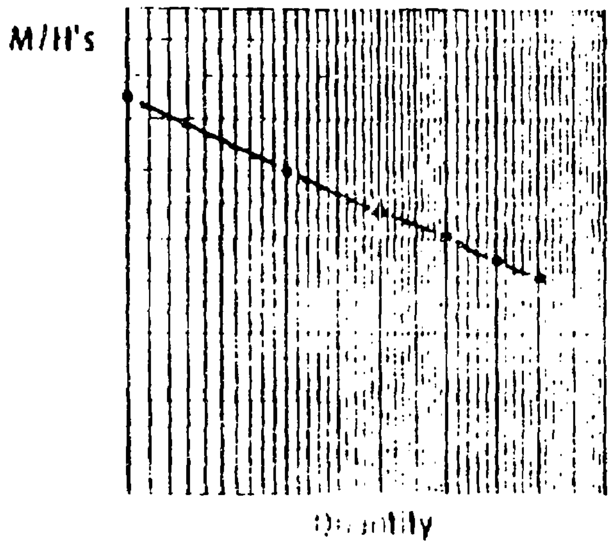
The cost improvement curve theory was developed from observations of cost behavior as a function of sequential aircraft produced. Factors associated with the airframe industry which seem to be necessary for the cost improvement curve theory to work are:

LEARNING CURVE: IMPROVEMENT CURVE
Model of a Common Phenomenon

In Repetitive Production, the Unit Cost Decreases As More and More Units Are Made. Simply Stated It Takes Less Time to do Something A Second Time, and Even Less The Third Time, And So On. Learning Curve Is A Tendency, Is Not Infallible

- **MATHEMATICALLY:**

$T_1 > T_2 > T_3 > T_4 > \cdots T_{N-1} > T_N$



- **REAL LIFE POSSIBILITY:**

$T_1 > T_2 > T_3 < T_4 > T_5$

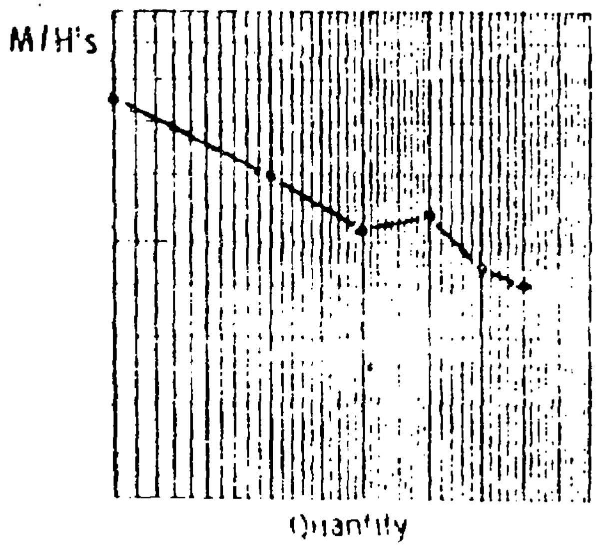


FIGURE 1-1

(1) The building of a sizable, complex end item which requires large numbers of direct labor hours. The many individual tasks associated with these hours provide myriad opportunities to learn.

(2) A production process in which non-mechanized assembly operations are predominant. If the operations were mechanized or machine paced as are many fabrication operations, the learning process would be inhibited.

(3) A continuous manufacturing process with constant pressure to reduce labor hours. If production breaks were common or long, the accrued improvement would be dispersed through reassignment of workers or even forgetfulness.

(4) The element of constant change in the product. The many engineering changes that are characteristic of a "state of the art" weapon system seem to contribute to the overall process of improvement that was observed (see Figure 1-2).

Noteworthy is the significant impact of major engineering changes or model changes. Airframe production is characterized by short model/series production runs. With each change in model, the cost improvement curve phenomena tends to repeat itself. That is, when a production program is completed for a particular airframe model and a new production is set up for a similar but new model, it cannot be expected that the first unit of the new model will continue where the old model left off. Rather, it can be expected that the labor hours to be used for the first unit of the new model will behave as unit one of a new production run and learning will begin anew.

It should be emphasized from the outset that while the cost improvement curve is essentially a trend concept, it is not a time-series trend form. Rather, the **independent variable** is taken to be the number of opportunities to learn while the **dependent variable** is cost input per constant unit of production. At first, the independent-dependent variable relationship may seem obscure. At best it is not likely to seem quite as straight forward as a simple cost per unit time-series. This relationship is one of the key concepts which make the cost improvement curve a useful device for measuring and predicting change in production cost input.

TOOLS OF THE COST IMPROVEMENT CURVE THEORY

DEFINITION:

The cost improvement curve theory is defined as follows:

> **As the total quantity of units produced doubles, the cost per unit decreases by some constant percentage.**

THE LEARNING CURVE PROCESS
CHARACTERISTICS

- BUILDING OF A ITEM WHICH IS
  - SIZEABLE
  - COMPLEX
  - SUBSTANTIAL LABOR HOURS
- NON-MECHANIZED ASSEMBLY OPERATIONS
- CONTINUOUS MANUFACTURING OPERATIONS
- PRESSURE TO REDUCE HOURS
- LIMITED CHANGE TO PRODUCT CHARACTERISTICS

FIGURE 1-2

Expressed in equation or model form, the cost improvement curve theory is:

$$Y_x = AX^b$$

where: Y represents the unit cost (usually expressed in hours) of the x^{th} unit and X represents some sequential unit number.

A is a coefficient (constant) that represents the theoretical cost (also usually expressed in hours) of the first unit, usually abbreviated as T_1.

b is a coefficient (constant) that is related to the slope and the rate of change of the cost improvement curve. It can be calculated from the relationship:

b = logarithm "slope"/logarithm 2.

In this equation, the slope must be expressed in decimal form rather than the percentage form for calculation purposes.

LOG-LOG PAPER:

One form of graph paper marked so that number values are expressed in terms of equal relative differences on both vertical and horizontal scales is called log-log paper and is illustrated in Figure 3. Log-Log paper is so constructed that the distances between numbers on the horizontal scale are equal percentage changes. The distance, for example, between 1 and 2 is the same as between 4 and 8; also, the distance between 3 and 4 is the same as between 60 and 80. In each set of distances the differences in numbers represents a 100% and a 33 1/3% increase respectively. The vertical scale has the same characteristics.

A straight line on log-log paper indicates that the rate of change between two variables is constant. Any two lines that are parallel on log-log paper indicate that the rate of change is equal for each of the two sets of relationships.

CHARACTERISTICS OF LOG-LOG PAPER

There are several characteristics which should be observed about log-log paper:

(1) There are no zeros. Values approach zero but never achieve it.

(2) This type of graph paper is drawn in terms of cycles. The first cycle, either vertical or horizontal, must be labeled .1, 1 or 10, or any integral power of 10. It is essential to observe that the first cycle values cannot be a number such as 5 or 6 (see Figure 1-3).

LOG-LOG PAPER

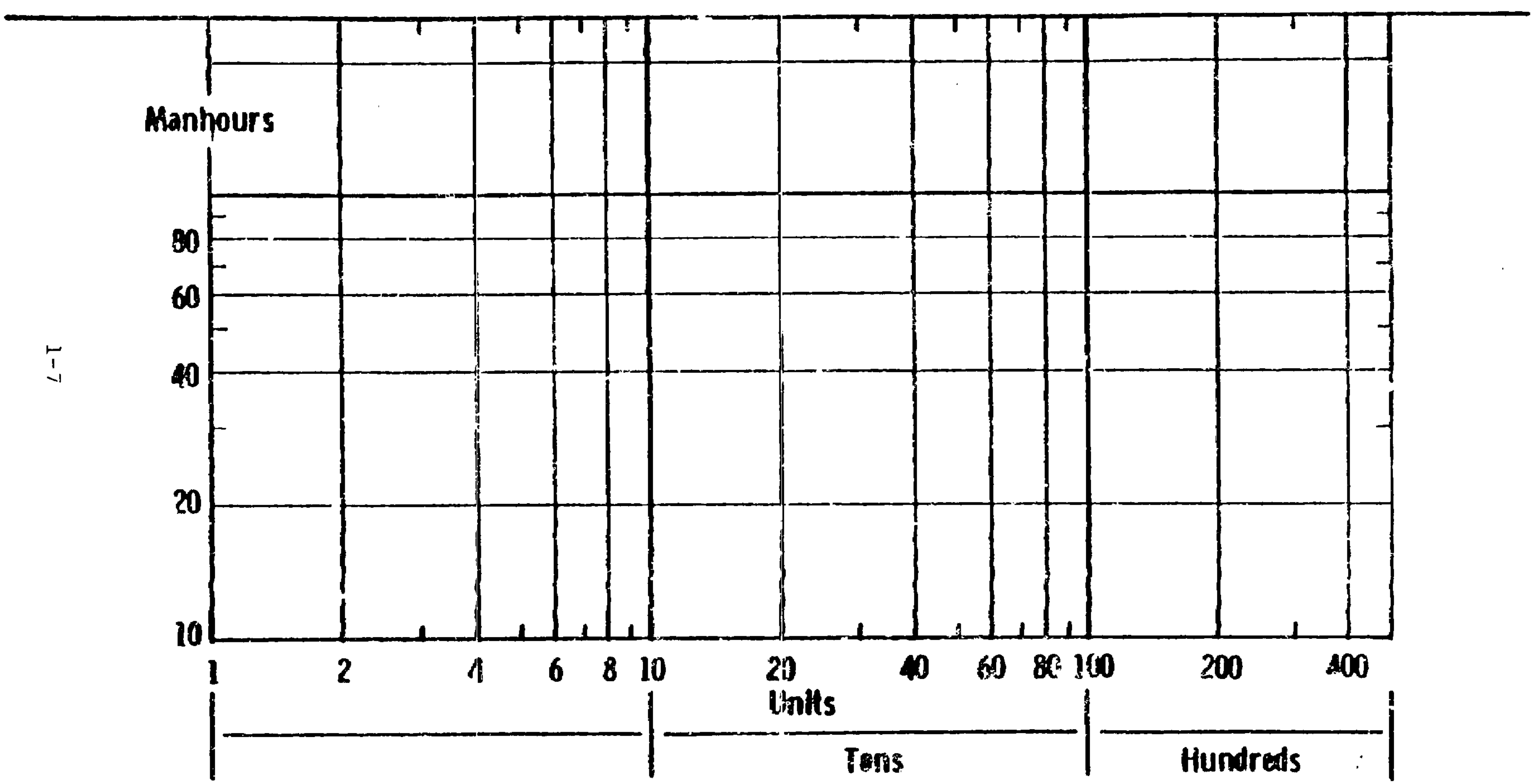


FIGURE 1-3

Each cycle has a definite starting point when designating values. But, if the first cycle starts with 1, the next cycle must start with 10, and the third cycle would start with 100. If the first cycle is 10, the value of subsequent cycles would be 100 and 1000. That is, a cycle need not always start with 1, but may start with .01 or 0.1 or 1.0 or 10.0 or 100.0, etc. **However, once an absolute value is assigned to a point on an axis, either horizontally or vertically, all other locations on that same axis have a fixed absolute value such that comparable locations in each successive cycle (to the right on the horizontal axis or above on the vertical axis) have an absolute value exactly ten times as great as in the preceding cycle.**

In all graphs, the horizontal axis is conventionally called the X axis and the vertical axis is called the Y axis. For purposes of this course, sequentially produced units will always be plotted on the X axis: labor hours, cost, pounds of material or whatever quantity varies as production proceeds will be plotted on the Y axis.

On the X axis, the first cycle starts with the first unit produced or T_1. The next cycle starts with the tenth unit produced; the third cycle the 100th unit produced and the fourth cycle, the 1000th unit produced. It is advisable to mark these cycles on the margin of the log-log paper before starting to plot points. Note that for the X-axis, the first cycle will always be labeled "1" because we always want to know the value of that first unit, or T_1. Therefore, the cycles on the X-axis will always be labeled 1, 10, 100, and 1000 (see Figure 1-4).

On the Y-axis, the scale is not always the same, but varies with the respective data set. It is important to specify the scale before beginning to plot points, otherwise it is easy to make errors in plotting or reading figures. To determine the scale to be used, first determine the largest figure to plot or read on the Y axis. This figure is probably the theoretical cost of the first unit (T_1). If this is 60,000 hours, determine the next integral power of ten above this figure. (An integral power of ten is ten multiplied or divided by itself a number of times.) The next integral power of 10 above 60,000 is 100,000, which is ten multiplied by itself four times. This value is given to the horizontal line at the top of the paper; the lower cycle must then represent 10,000 units and the bottom cycle, 1,000.

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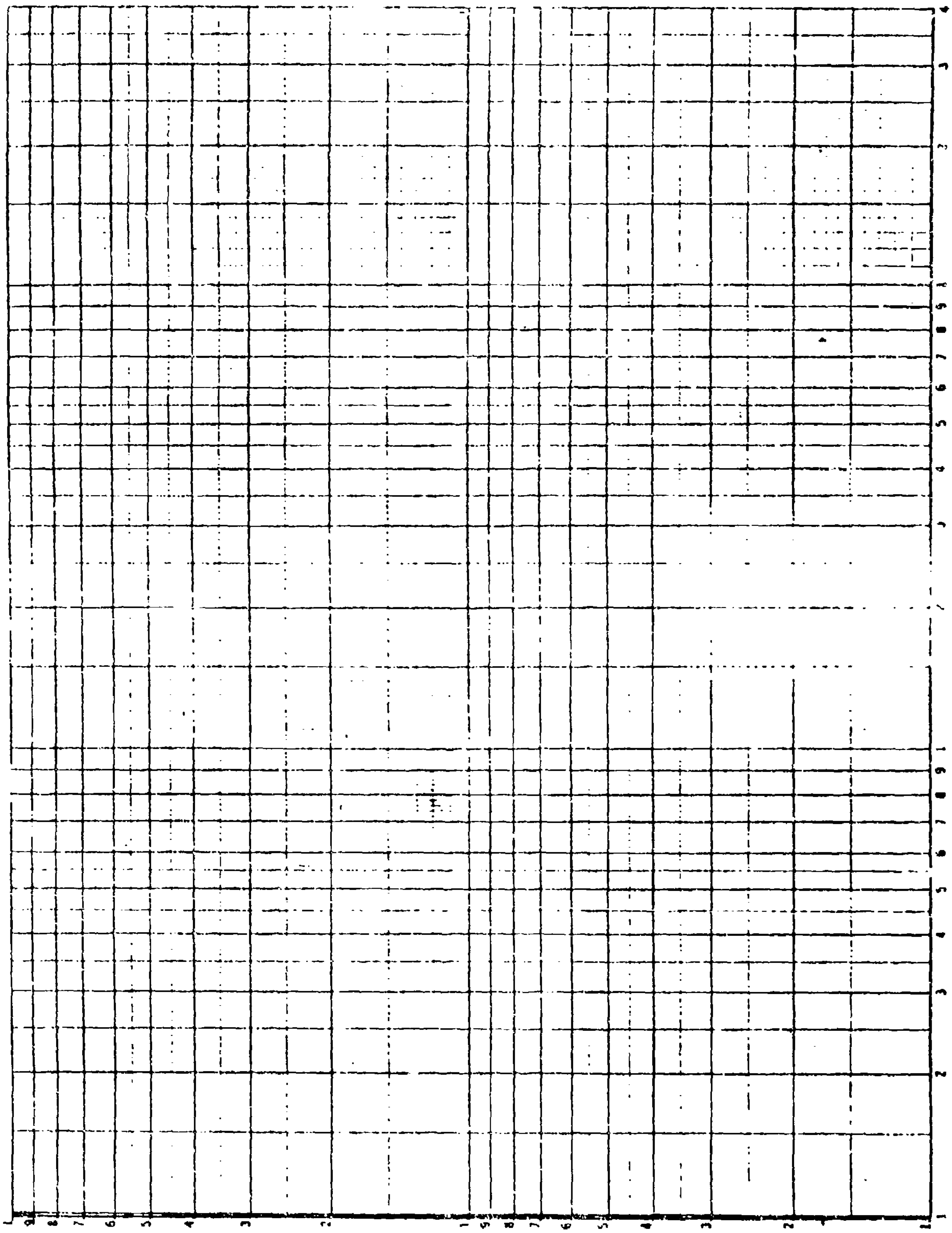

FIGURE 1-4

The accuracy of the results obtained from graphs depends greatly on the degree of refinement of the plotting technique. A sharp pencil should always be used. Points plotted on the paper should be as small as possible, lines as narrow as possible. When the smallest possible point has been marked on paper, it may be easily lost or confused with a blemish in the paper. To avoid this, draw a small ring around your plot point. Circles, triangles, and squares may also be used to identify points which belong to different sets of data. Great care should be exercised in drawing a line. If it is supposed to go through a point it should pass exactly through it, not merely close to it.

When plotting real production data on log-log paper, the data points will seldom all fall in a perfect straight line. In this situation, the analyst must "best fit" a straight line through the data points. The object of this "best fit" approach is to discern the trend of the data. **The usual approach is to attempt to locate a straight line on the log-log paper such that the sum of the distances of each of the data points from the line is as small as possible.** [Note: If one data point is a significant distance away from the "best fit" line, further analysis into the cause of the deviation is indicated. If this analysis so indicates, adjustment or eliminating of the errant data point might be in order.]

MEASURING THE SLOPE OF A COST IMPROVEMENT CURVE:

The slope of a cost improvement curve is a mathematical misnomer. Accordingly, it cannot be related to the definition of slope in a straight line model as discussed in the linear regression model. Because of this misnomer, one must specify "slope" of a cost improvement curve as distinguished from slope of a straight line (rectangular coordinates).

In the definition of the cost improvement curve it was stated that "as the total quantity of units produced doubles, the cost per unit decreases by some constant percentage." The slope of a cost improvement curve can be calculated by dividing the unit cost (hours) at some quantity X into the unit cost (hours) at twice the quantity and then multiplying the resulting ratio by 100.

$$\text{Slope} = 100\ (y_{2x}/y_x)$$

Therefore, one way to measure the slope of a cost improvement curve drawn on log-log paper is to read a y value at any quantity x, read a y value at any quantity two times x, divide the second value by the first and multiply by 100. For example, if the

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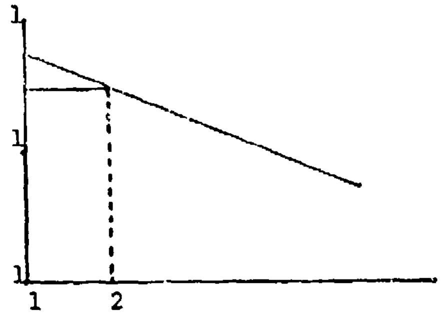
number of hours read from the graph for unit number 5 is 70 and the number of hours read from the graph for unit number 10 is 50, the slope of the cost improvement curve is --

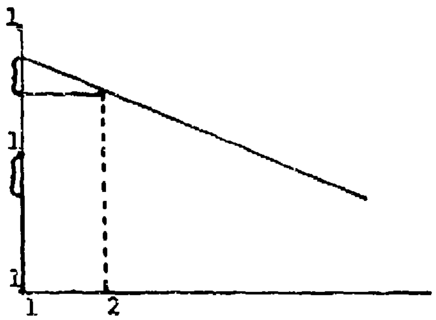
$$100(y_{10}/y_5) = 100\ (50/70) = 71.4\%.$$

Another approach to measuring the slope of a cost improvement curve drawn on log-log paper is the **measurement method.** The measurement method is applied by following these steps:

STEP 1. Locate unit 2 on the X-axis and draw a horizontal line from your best-fit cost improvement curve line at unit 2 back to the Y-axis--



STEP 2. Using a ruler, measure the distance from your horizontal line at unit 1 to your cost improvement curve line at unit 1; that is, measure the vertical distance between the two lines you have drawn at the Y-intercept.



STEP 3. Take the 'distance' you have measured and move to a new cycle on the Y-axis. Measure this 'distance' down from '1' and read the scaled value as your cost improvement curve percent. **This is your cost improvement curve slope in percent.**

For example, suppose you have drawn a best-fit cost improvement curve line and a horizontal line from unit 2 on the X-axis. (See Figure 1-5). The 'distance' measured is approximately 1/4 inch from T_1 to your horizontal line. Move to a new cycle at the top of the log-log paper and measure down from '1' 1/4 inch; the value you read is '8' which is interpreted as an 80% cost improvement curve slope.

The analyst needs to know the slope of the cost improvement curve for a number of reasons. One is to facilitate communication among analysts, as it is an important part of the language of cost improvement curve theory. The steeper the slope (lower the percent) the more rapidly the resource requirements (hours) will decline as production increases. The slope of the cost improvement curve is usually a significant issue in a negotiation. The slope of the cost improvement curve is also needed to project follow-on costs. Cost improvement curve slopes can be developed from actual experience on production programs. These historical slopes can then be useful in analyzing future contracts.

EXTENDING THE LINE:

The primary purpose for developing the cost improvement curve as a forecasting tool is to permit the analyst to predict costs. The prediction is based upon the assumption (not always true) that the future will behave as the past. In terms of the cost improvement curve theory, this assumption means that the cost (hours) of doubled quantities will continue to decrease by some constant percentage. Prediction can most easily be accomplished by drawing a straight line through the historical observed data on log-log paper and extending the straight line through some future quantity to be produced. The predicted cost per unit to produce any particular unit is read on the y axis for the corresponding X-unit.

For example, by referring to Figure 1-5 again, suppose the unit number one had a value of 3000 hours; No. 2, a value of 2400 hours; No. 4, a value of 1920 hours, and No. 8, a value of 1536 hours. By connecting the points, (when plotted on log-log paper), one will observe a straight line with an 80% slope. (Note that this is what we previously determined as the cost improvement curve slope with the measurement method.) If the line were extended sufficiently far beyond the eighth unit, one could estimate the value for the 100th unit. (The extended line should reveal a Y-value of approximately 680 hours, where X equals 100).

The cost improvement curve line can also be extended backward. This is especially important if the theoretical value of unit number one (T_1) is needed.

As with any method of projecting the future, the theory of the cost improvement curve falls short of perfection. Such a

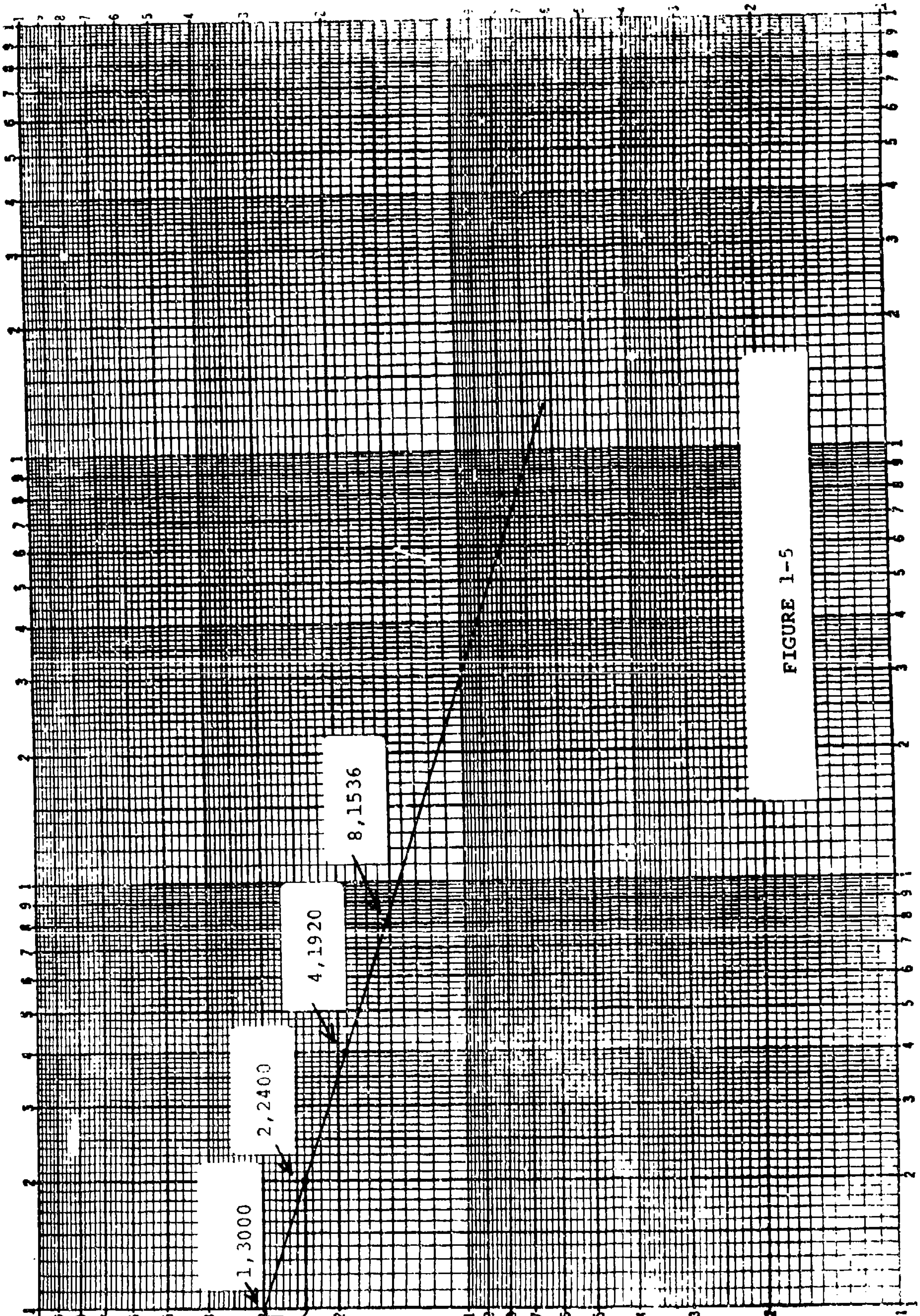


FIGURE 1-5

simple model of the real world cannot hope to cover all estimating situations. However, the method of extending straight lines on log-log paper as described provides a reasonable approach to predicting the future if the historical data behave in a straight line trend. Conversely, the further away historical data points lie from the selected trend line, the less confidence the analyst can place in the forecasted prediction.

USING THE BOEING IMPROVEMENT CURVE TABLE:

The Boeing Tables indicate values for every unit from 1 to 999 for every cost improvement curve from 51% to 99% in terms of its ratio relationship to the first unit.

The Tables have two major divisions: The unit progress curve table and the cumulative progress curve table. There are two pages devoted to every percent of slope curve from 51 to 99. The first major division with an 80% slope is illustrated in Table 1-1. Note here, that the unit digits for X units are listed across the top of the page (from 0 to 9) and the 10's and 100's digits are listed vertically from top to bottom in the extreme left column. Note too, that unit number 1 is, in every case, listed as a ratio value of 1.00000000. The ratio values for units 1 to 9 can be read from the top line across the page, but for units above nine, segregate the unit from the tens or hundreds. For example unit #12 is located in the "1" row and the "2" column. Where the two intersect, the ratio value will be given. Thus, the 80% slope ratio value for 12 would be .449346. Using the previous example, if unit 1 is 3000 hours, unit 12 is expected to require .449346 * 3000 = 1,348 hours. For unit #103, the ratio value would be found by segregating the 3 and reading down the "3" column and across the "10" row. Thus the ratio values for 103 would be .224911.

The ratios for cumulative values are found in the second section of the Tables and are designated by a "B" following the page numbers (see Table 1-2). The ratios throughout this section refer to cumulative total (CT) ratios and not to cumulative average (CA) ratios. Here, too, the value for unit #1 is 1.00000000. To find the ratio value for 103 units, for example, we follow the same procedure as before, and we read 33.327686. This means that the required number of hours to produce 103 units for an 80% curve is 33.327686 hours when unit one is 1 hour, or 33,327.686 hours when the value of unit 1 is 1,000 hours.

SLOPE 80%

UNIT PROGRESS CURVE TABLE 80%

	0	1	2	3	4	5	6	7	8	9
0		1.00000000	.80000000	.70210000	.64000000	.59563700	.56168300	.53450000	.51200000	.49295200
1	.47691000	.46231100	.44934400	.43759600	.42679700	.41019900	.40960000	.40185100	.39520700	.38882100
2	.38120800	.37524700	.36968000	.36449600	.35947700	.35478600	.35033700	.34617700	.34207300	.33823100
3	.33455900	.33104600	.32768000	.32445000	.32134700	.31835200	.31548800	.31271100	.31004400	.30746200
4	.30496400	.30159200	.30021600	.29704800	.29575100	.29361900	.29154900	.28955100	.28756200	.28563700
5	.28382700	.28207500	.28026800	.27855300	.27688100	.27525100	.27365900	.27210400	.27058500	.26910000
6	.26764700	.26627700	.26483700	.26351600	.26216400	.26083900	.25956000	.25830600	.25707700	.25587200
7	.25469000	.25357000	.25239000	.25121200	.25017400	.24929500	.24803300	.24699600	.24597000	.24496100
8	.24387500	.24284900	.24204100	.24109900	.24017100	.23975800	.23885800	.23797300	.23660100	.23574200
9	.23480500	.23400000	.23323900	.23242900	.23163000	.23084700	.23006500	.22929000	.22854100	.22779800
10	.22706700	.22621000	.22541900	.22491100	.22421300	.22352300	.22284200	.22218000	.22150300	.22086000
11	.22020100	.21954000	.21892700	.21830100	.21760300	.21707200	.21644800	.21581000	.21528000	.21464700
12	.21411000	.21354700	.21290700	.21247700	.21187000	.21132900	.21078100	.21024500	.20971500	.20919000
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14	.20375200	.20326500	.20282300	.20239400	.20191100	.20148700	.20101800	.20057600	.20011900	.19970000
15	.19927000	.19881000	.19842000	.19801000	.19759500	.19718400	.19677600	.19637100	.19597000	.19557100
16	.19517000	.19478700	.19439900	.19401500	.19363100	.19325500	.19287900	.19250800	.19213700	.19177000
17	.19140800	.19104900	.19068100	.19033100	.18997800	.18962800	.18928100	.18893600	.18859300	.18825400
18	.18791800	.18758100	.18724900	.18691900	.18659100	.18625600	.18594300	.18562200	.18530600	.18499600
19	.18467400	.18436200	.18405200	.18374900	.18343900	.18313600	.18283500	.18253500	.18223900	.18194300
20	.18164900	.18135900	.18106800	.18078100	.18049500	.18021100	.17992900	.17964900	.17937000	.17909500
21	.17881000	.17854900	.17827600	.17800400	.17773300	.17746900	.17720400	.17694100	.17667900	.17641900
22	.17616000	.17590300	.17564800	.17539400	.17514200	.17489100	.17464100	.17439300	.17414600	.17390100
23	.17365700	.17341500	.17317400	.17293400	.17269600	.17245400	.17222400	.17198300	.17175600	.17152500
24	.17129400	.17106500	.17083700	.17061100	.17038500	.17016100	.16993800	.16971600	.16949500	.16927600
25	.16905800	.16884100	.16862500	.16841000	.16819500	.16798400	.16777200	.16756200	.16735200	.16714400
26	.16693700	.16673100	.16652600	.16632100	.16611800	.16591400	.16571500	.16551300	.16531600	.16511800
27	.16492100	.16472900	.16453000	.16433500	.16414200	.16395000	.16375600	.16356400	.16337300	.16318400
28	.16300100	.16281400	.16262800	.16244300	.16225900	.16207500	.16189300	.16171100	.16153000	.16135000
29	.16117000	.16099200	.16081400	.16063700	.16046100	.16028600	.16011100	.15993700	.15976500	.15959200
30	.15942100	.15925000	.15908000	.15891100	.15874300	.15857500	.15840800	.15824100	.15807600	.15791100
31	.15774700	.15758300	.15742100	.15725900	.15709700	.15693400	.15677600	.15661700	.15645800	.15630000
32	.15614300	.15598800	.15583000	.15567400	.15552000	.15536500	.15521200	.15505900	.15490800	.15475500
33	.15460400	.15445300	.15430100	.15415400	.15400500	.15385700	.15370900	.15356200	.15341600	.15327000
34	.15312900	.15298000	.15283600	.15269100	.15254900	.15240700	.15226500	.15212400	.15198300	.15184200
35	.15170300	.15156300	.15142900	.15128800	.15114900	.15101100	.15087500	.15073900	.15060300	.15046800
36	.15033300	.15019800	.15006500	.14993200	.14979900	.14966700	.14953500	.14940400	.14927300	.14914300
37	.14901300	.14888300	.14875400	.14862600	.14849800	.14837000	.14824300	.14811600	.14799000	.14786400
38	.14773900	.14761400	.14749000	.14736600	.14724200	.14711900	.14699600	.14687300	.14675100	.14662900
39	.14650900	.14638800	.14626800	.14614800	.14602900	.14590900	.14578900	.14567200	.14555400	.14543700
40	.14531900	.14520300	.14508600	.14497000	.14485500	.14473900	.14462500	.14451000	.14439700	.14428300
41	.14416900	.14405400	.14394300	.14383100	.14371900	.14360700	.14349600	.14338500	.14327500	.14316500
42	.14305300	.14294500	.14283600	.14272700	.14261900	.14251100	.14240300	.14229600	.14218900	.14208200
43	.14197500	.14186800	.14176300	.14165800	.14155300	.14144800	.14134300	.14123900	.14113500	.14103200
44	.14092800	.14082700	.14072300	.14062000	.14051800	.14041700	.14031500	.14021400	.14011300	.14001300
45	.13991200	.13981300	.13971300	.13961400	.13951600	.13941600	.13931700	.13921900	.13912100	.13902300
46	.13892800	.13882900	.13873200	.13863600	.13853900	.13844300	.13834800	.13825200	.13815700	.13806200
47	.13796700	.13787300	.13777900	.13768500	.13759200	.13749800	.13740500	.13731200	.13722000	.13712800
48	.13703600	.13694400	.13685200	.13676100	.13667000	.13657800	.13648800	.13639800	.13630800	.13621800
49	.13612900	.13604000	.13595100	.13586200	.13577300	.13568500	.13559700	.13550900	.13542100	.13533400

TABLE 1-1

BOEING TABLES

SECTION 8

CUMULATIVE PROGRESS CURVE TABLE

SLOPE	80% 0	1	2	3	80% 4		80% 5	6	7	8	PER 5 IN 9	
		1.00000000	1.00000000	2.50210100	1.14110400		2.37770100	6.29867400	5.83382600	5.34592000	5.83007600	
1	6.31530400	6.77740500	7.22694100	7.88875700	8.09235900	1	8.51054800	8.92034800	9.37189600	9.71618100	10.10374900	1
2	10.48495407	15.86022107	11.22991000	11.92454800	11.95302300	2	12.30860100	12.65093900	13.00504100	13.34711800	13.68516500	2
3	14.04290400	14.35099000	14.67863000	15.60308200	15.32442700	3	15.64278900	15.95827700	16.27098400	16.58103800	16.88854000	3
4	17.18256600	17.42601800	17.74672200	18.39418000	18.38993100	4	18.68355000	18.97504900	19.26461600	19.55221800	19.83706700	4
5	20.12177400	20.40374700	20.68401100	20.96296800	21.23944700	5	21.51469800	21.78835700	22.06194100	22.33134800	22.60214600	5
6	22.66770900	21.12402000	23.37885700	23.66233300	23.92447700	6	24.18511800	24.44407600	24.70310700	24.96025900	25.21611100	6
7	25.41082100	25.72439200	25.97674000	26.22801200	26.47818600	7	26.72728100	26.97531600	27.22231000	27.46878300	27.71374100	7
8	27.75221600	28.20021900	28.44227800	28.80335300	28.92352000	8	29.16278400	29.40114200	29.63861500	29.87521600	30.11099500	8
9	30.34585300	30.51991400	30.81115300	31.04550200	31.27721200	9	31.50005400	31.73011900	31.96741800	32.19596100	32.42315000	9
10	32.65082100	32.87715600	33.10277500	33.32780400	33.55189900	10	33.77542200	33.99826400	34.22094100	34.44309800	34.66270100	10
11	34.88726000	35.10254000	35.32147500	35.53377600	35.75745900	11	35.97537100	36.19099900	36.40686700	36.62234900	36.83688500	11
12	37.03074300	37.28451000	37.47745200	37.68881500	37.90176900	12	38.11310300	38.32188600	38.53413400	38.74306300	38.95357300	12
13	39.16111000	39.36908700	39.57731500	39.78405900	39.98130400	13	40.19745500	40.40311700	40.60879400	40.81288200	41.01722100	13
14	41.22096800	41.42625100	41.62707400	41.82844000	42.03135200	14	42.23281500	42.43381100	42.63440900	42.83454800	43.03425600	14
15	43.23351000	43.45218000	43.63350800	43.82881000	44.02843700	15	44.22353700	44.42037100	44.61574400	44.81171400	45.00648100	15
16	45.20765500	45.39825200	45.59263100	45.78666600	45.96023900	16	46.17355400	46.36643300	46.55845900	46.75107600	46.94286400	16
17	47.13655700	[illegible]	47.51508400	47.70631500	47.89629100	17	48.08592100	48.27520700	48.46411800	48.65273100	48.84095000	17
18	[illegible]	49.21068200	49.40371100	49.59057000	49.77726400	18	49.96350700	50.14949000	50.33501200	50.52017600	50.70516400	18
19	[illegible]	51.02440000	51.29845200	51.44217700	51.62563500	19	51.80877200	51.99160700	52.17414300	52.35530000	52.53812700	19
20	52.71189700	52.90133000	53.05239000	53.26317900	53.44567600	20	53.62140500	53.80181400	53.98144100	54.16283500	54.34187800	20
21	54.52274400	54.40728900	54.87756300	55.05554100	55.23333300	21	55.41037200	55.58797500	55.76491700	55.94159600	56.11801500	21
22	56.29417300	56.47007000	56.64572600	56.82112000	56.99676200	22	57.17115100	57.34579400	57.52018700	57.69433100	57.86823400	22
23	58.24109100	58.21933600	58.38848000	58.56141600	58.73411000	23	58.90656200	59.07879100	59.25079200	59.42253800	59.59406100	23
24	59.76595700	59.93602200	60.10725900	60.27781000	60.44825900	24	60.61841600	60.78835400	60.95807000	61.12750000	61.29664200	24
25	61.45590000	61.61676100	61.80038600	61.91127800	62.13097200	25	62.30739600	62.47577800	62.64329000	62.81084200	62.97774600	25
26	63.10447300	63.28115400	63.47798000	63.64495100	63.81091900	26	63.97633500	64.14205000	64.30756500	64.47280100	64.63799000	26
27	64.60272400	64.95789300	65.13217500	65.29611000	65.46069200	27	65.62462000	65.78836000	65.95182600	66.11530000	66.27849500	27
28	66.44189000	66.60411000	66.75693000	66.92938100	67.05154700	28	67.25371500	67.41580000	67.57731900	67.73884300	67.90019600	28
29	68.04136900	68.22216100	68.38117500	68.54184200	68.70427100	29	68.86355900	69.02487000	69.18460700	69.34437200	69.50196300	29
30	69.60150900	69.87283500	69.98177700	70.14062000	70.20938900	30	70.45734400	70.61615200	70.77459100	70.93286000	71.09086000	30
31	71.21882700	71.40591000	71.56133100	71.72019000	71.87750700	31	72.03492100	72.19136900	72.34801800	72.50447600	72.66071600	31
32	72.81531700	72.97230300	73.12807100	73.28440700	73.43902700	32	73.59529200	73.75050400	73.90556300	74.06048000	74.21537400	32
33	74.40981800	74.57420100	74.67450400	74.83273800	74.98676300	33	75.14080000	75.29437000	75.44787100	75.60122700	75.75455700	33
34	75.90762700	76.06064700	76.21349000	76.36617000	76.51875000	34	76.67134600	76.82341100	76.97553300	77.12351000	77.27936000	34
35	77.43108300	77.58262000	77.73405100	77.88537700	78.03640600	35	78.18749700	78.33837200	78.48911100	78.63973400	78.79018200	35
36	78.94051900	79.09071400	79.24077900	79.39071100	79.54051000	36	79.68837700	79.83871200	79.98811600	80.13658000	80.28753200	36
37	80.43059500	80.58542800	80.73419200	80.88280000	81.03110600	37	81.17961800	81.32791000	81.47603500	81.62402500	81.77189900	37
38	81.91907000	82.06776200	82.21673200	82.36209700	82.50911500	38	82.65549800	82.80345400	82.95032100	83.09787000	83.24510800	38
39	83.39001700	83.53608100	83.68201100	83.82802100	83.97104900	39	84.12093900	84.26674900	84.41247900	84.55797400	84.70341100	39
40	84.84079000	84.98533300	85.13501900	85.28310000	85.42006400	40	85.57358300	85.71820900	85.86271800	86.00711400	86.15138000	40
41	86.29956500	86.41702100	86.58396400	86.72739300	86.87111400	41	87.01472100	87.15821700	87.30150700	87.44487700	87.58804200	41
42	87.73109700	87.87454200	88.01827700	88.15960100	88.13722400	42	88.44473500	88.58713800	88.72943000	88.87162200	89.01370400	42
43	89.15551900	89.29758000	89.43791100	89.58096700	89.72252200	43	89.86397000	90.00531300	90.14659200	90.28780100	90.42871300	43
44	90.56964700	90.71047700	90.85110900	90.99801100	91.11323500	44	91.27279000	91.41306500	91.55327700	91.69339200	91.83340500	44
45	91.97331700	92.11313000	92.25284300	92.39249800	92.53197000	45	92.67138500	92.81070300	92.94992800	93.08904300	93.22800000	45
46	93.36699700	93.50502100	93.64453300	93.78110900	93.92178000	46	94.06017100	94.19851900	94.33477700	94.47492800	94.61277000	46
47	94.75095700	94.99562000	95.02660000	95.16429400	95.30180000	47	95.43938400	95.57678900	95.71416100	95.85132100	95.98841000	47
48	96.12790700	96.26292900	96.39920100	96.53504200	96.67271200	48	96.80929100	96.94537000	97.08217000	97.21848000	97.35470400	48
49	97.50003500	97.63687300	97.76202400	97.89040600	98.02445700	49	98.17014400	98.30576100	98.44125000	98.57667200	98.71200500	49

TABLE 1-2

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With information thus obtained from the Tables, we can now fill in the values for the intervening units:

UNIT #	UNIT VALUE	CUMULATIVE TOTAL VALUE
	"A" PAGES	"B" PAGES
1	1,000	1,000
2	800	1,800
3	702	2,502
4	640	3,142
5	596	3,738
6	562	4,299
7	535	4,834
8	512	5,346

Note: The Boeing Tables are used principally with the Unit Curve Construction (Boeing Curve) and may be used exclusively with this construction, or if one has a calculator with an exponential function, for calculating the cumulative average construction (Northrop Curve).

PROBLEMS

1. On a sheet of arithmetic graph paper, label the X axis as "units produced" and Y axis as "labor hours/unit". Designate values for each axis as appropriate for the following set of values:

UNITS PRODUCED	LABOR HOURS/UNIT
10th	10,000
20th	7,000
40th	4,900
80th	3,430

a. Plot the relationships on arithmetic paper.
b. Connect the plot points.
c. Calculate the value of the first unit (T_1).
d. Calculate the labor-hour value for unit 5.
e. Estimate the labor-hour value of unit 70.

2. Plot the relationships of Problem 1 on log-log paper and connect the points. Note difference between this line and the one constructed in Problem 1. Explain the difference in concept.

3. Read the value of unit 70 from the log-log line and compare this reading and the estimate for this unit in Problem 1. Explain the difference, if any. What is the rate of learning?

4. Plot the Y value for Unit 1 at 75 and a Y value unit 10 at 36. Measure the slope of the line.

5. Measure the slope of the line when unit 3 has a value of 4,000 and unit 15 has a value of 1,300.

6. Illustrate two ways of measuring slopes on log-log graph paper.

7. Why are all 90% (as well as 80%, 70% etc.) slopes parallel on log-log graph paper?

8. Draw a 75% slope when unit one has a value of 15. When unit 22 has a value of 15.

9. What is the value of unit 1 if unit 5 has a value of 595 hours and unit 15 has a value of 418 hours?

CHAPTER II

THE UNIT COST IMPROVEMENT CURVE THEORY -- BOEING CONSTRUCTION

THE UNIT COST IMPROVEMENT CURVE THEORY

The Stanford Research Institute study conducted by J.R. Crawford validated a cost improvement curve cost model that is known as the "unit curve" or the "Boeing" Construction. This theory can be stated as follows:

> **As the total quantity of units produced doubles, the cost per unit decreases by some constant percentage.**

The constant percentage by which costs of doubled quantities decrease is called the rate of learning. Another useful term, the "slope" of the cost improvement curve is related to the rate of learning. The rate of learning is the difference between 100 percent and the slope of the cost improvement curve:

Rate of Learning = 100% - Slope of Cost Improvement Curve

BOEING CONSTRUCTION MODEL:

Companies using the Boeing Construction theoretically may be expected to exhibit certain significant production process characteristics. These production process characteristics relate especially to what happens during the early stages of production, for it is this stage which determines the appropriate cost improvement curve theory. A company applying the Boeing Construction may be expected to: (1) Have had previous experience in producing a similar item and can thus plan in detail for production runs from the start; (2) Have the major engineering problems well under control; and (3) Start with the same "hardness" of tools as required for the entire production process. The company may also be expected to maintain a constant cost improvement curve slope throughout the program.

The unit curve theory can be expressed in equation or model from as:

$$Y_x = AX^b$$

where **Y** represents the unit cost (usually expressed in hours) of the x^{th} unit,

x is a sequential unit number.

A is a coefficient (constant) that represents the theoretical cost (also usually expressed in hours) of the first unit, usually abbreviated T_1.

b is a coefficient (constant) that is related to the slope and the rate of change of the cost improvement curve. It is calculated from the relationship **b = logarithm "slope"/logarithm 2.**

In the equation, $Y_x = AX^b$, the slope must be expressed in decimal form rather than in percentage form.

APPLYING THE BOEING CONSTRUCTION MODEL:

The Boeing Construction states that observations (values of x and y) that are related by the model $Y_x = AX^b$ form a straight line when plotted on log-log paper. The fact that a cost improvement curve is a straight line on log-log paper has tremendous advantages for predicting future production costs. Future production costs can be predicted simply by placing a ruler against the line representing past experience and extending that line into the future.

To illustrate the Boeing Construction concept, assume that it takes 100,000 labor hours to produce unit one (T_1 is 100,000 hours). Assuming an 80% slope, the second unit would require 80,000 labor hours, (or 80% of 100,000 hours) the fourth 64,000 (or 80% of 80,000 hours), etc. In tabular form, the arrangement would appear as shown in Table 2-1.

Note that the difference in labor-hour reduction is not constant. Rather, it **declines by a continually diminishing amount as the quantities are doubled.** But the **rate of change is a constant percentage of prior hours** because the decline in the base figure is proportionate to the decline in the amount of change.

UNITS PRODUCED	HOURS PER UNIT	DIFF. IN HOURS PER UNIT AT DOUBLED QUANTITIES	RATE OF CHANGE(%)	SLOPE OF CURVE(%)
1	100,000	*	*	*
2	80,000	20,000	20	80
4	64,000	16,000	20	80
8	51,200	12,800	20	80
16	40,960	10,240	20	80
32	32,768	8,192	20	80

TABLE 2-1

When the labor-hour curve is drawn on ordinary graph paper (rectangular coordinates), it becomes a hyperbolic line as shown in Figure 2-1. The non-linear appearance of Figure 2-1 has the advantage of depicting a relationship between two variables: units produced in sequence (x) and labor hours per unit (y). This relationship is expressed in terms of an arithmetic graph in which equal spaces represent equal amounts of difference. When thinking of numbers in terms of their absolute values, the graphical picture presents an accurate description. But when numbers are intended to show rate of change as in the cost improvement curve case, they play conceptual tricks, for rectangular coordinates assign the same spacing to the difference of one unit between two large numbers as they do for the difference of one unit between two small numbers.

For example, the change from 100 to 101 is a difference of one; whereas the change from 1 to 2 is also a difference of one. Relatively speaking, the first difference is a 1% change in values whereas the latter is 100% change in values. What we need is a measure of "rate of change" rather than a measure of amount of change. Such a measurement would show the relative importance of changes regardless of where the changes occurred on the number scale. For example, Figure 2-1 shows that the distance between 4 and 8 on the horizontal scale is the same as the distance between 28 and 32. This is because the difference between both sets is 4. Relatively speaking, however, the difference between 4 and 8 should be the same as between 16 and 32 because both represent a 100% change. If the distances are not equal, the changes in labor hours occurring in the first set will appear to be more important because the numbers are spaced further apart even though the relative change may be the same.

When labor-hour figures which conform to the cost improvement process are plotted on log-log paper against the units of production to which they apply, the points lie on a straight line called a curve. Figure 2-2 shows the data of Table 2-1 plotted on log-log paper. Data which conforms to the theory of the cost improvement curve (the cost of doubled quantities decreases by some constant percentage) form a straight line when plotted on log-log paper. Not only can the analyst estimate future production hours by extending the straight line, but the analyst can also dispense with the mathematical models. With careful attention to detail, the graphical approach to cost improvement curve analysis will yield satisfactory estimates when a computer assisted estimate is not possible. Additionally, a graph of the data may reveal abnormalities not easily evident from a computer print. **DATA SHOULD ALWAYS BE GRAPHED INITIALLY, REGARDLESS OF THE ANALYSIS TO BE USED.**

80% IMPROVEMENT CURVE ON ARITHMETICAL PAPER

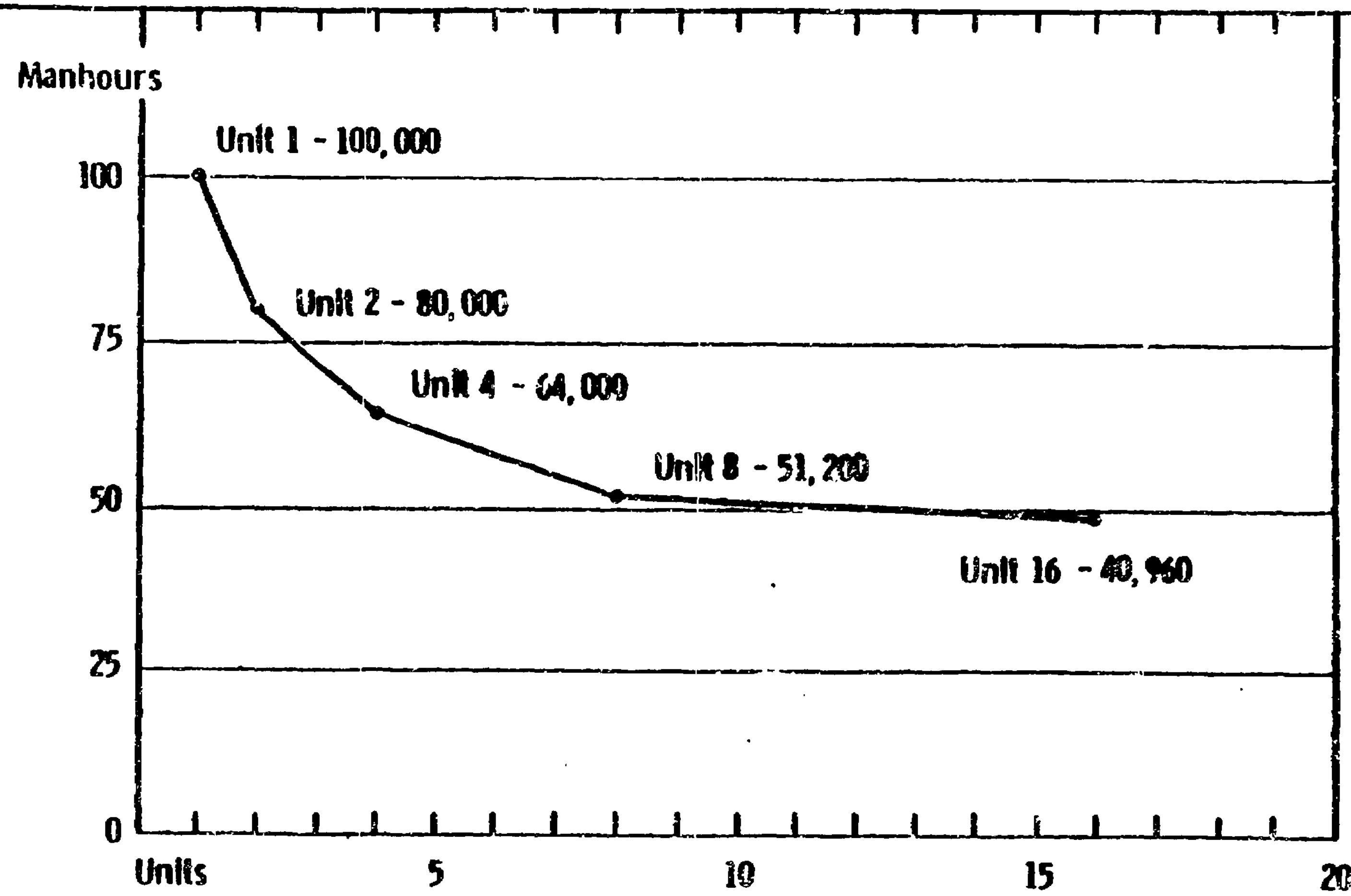


FIGURE 2-1

20% IMPROVEMENT CURVE PLOTTED ON LOG-LOG PAPER

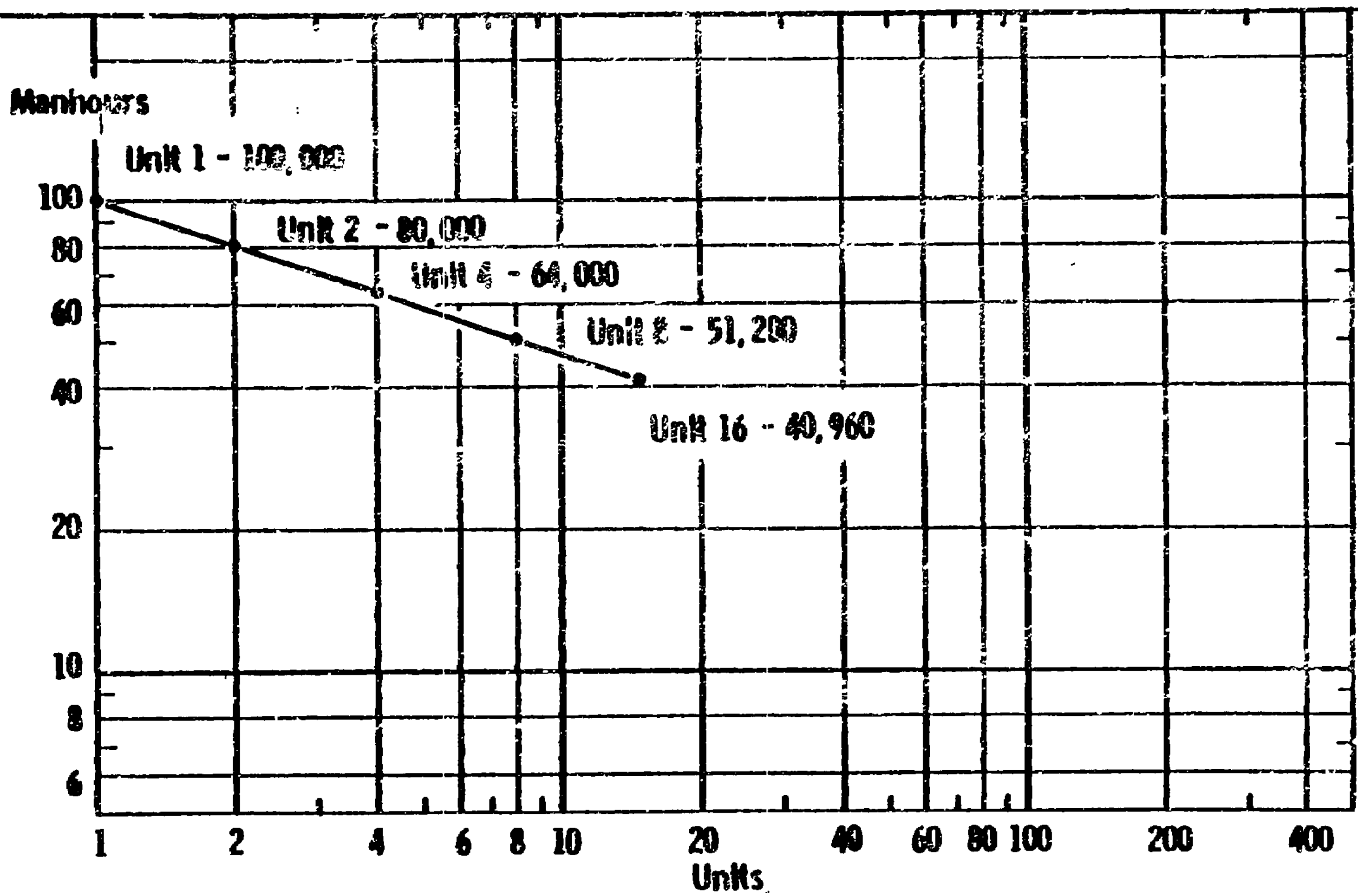


FIGURE 2-2

THE BOEING IMPROVEMENT TABLES:

Manufacturing companies whose cost experience typically forms a straight line on a log-log graph for unit labor hour cost use the Boeing formulation of the cost improvement curve. The concept states that theoretically the unit curve is linear on a log-log graph.

The unit curve is simple to construct since the unit values are merely some percentage (cost improvement curve slope) of each new base as 100% differences in units are observed. However, this calculation tells us nothing of the value for the 3rd unit or intervening units. Arithmetic interpolation will not yield the correct value since we are working with **geometric progressions.** It is possible to find the value of the third unit by a complex method of mathematical calculations, but it would be unnecessary to do so if we use the Boeing Improvement Curve Tables. (See Chapter 1, using the Boeing Improvement Curve Tables)

UNIT COST IMPROVEMENT CURVE FORMULAE:

The unit curve theory, or Boeing Construction, has five concept-peculiar formulae for calculating values. These formulae require the use of the Boeing Improvement Curve Tables or a computerized software package. To use the formula, you must know the cost improvement curve slope and the first unit value (T_1). The formulas are presented in Table 2-2. An introductory vocabulary is also provided as explanation of the concept terminology in Appendix F.

These five formulae will be applied in estimating lot values and total program costs. To apply them, the analyst must have either production program 'actuals' or analogous program data from which to derive a T_1 and cost improvement curve slope.

APPLICATION OF THE UNIT COST IMPROVEMENT CURVE THEORY FORMULAE

LOT COSTS AND USE OF MIDPOINTS: The use of the cost improvement curve is dependent on the methods of recording costs which companies employ. An accounting or statistical record system must be devised by a company so that data are available for cost improvement curve purposes; otherwise it may be impossible to construct a cost improvement curve. Costs, such as labor hours per unit or dollars per unit, must be identified with the unit of production. It is preferable to use labor-hours-per-unit rather than dollars-per-unit since dollars-per-unit contain an additional variable, the effect of inflation or deflation (wage rate changes). In any event, the record system must have definite cut-off points for such costs, thus permitting identification of the costs with the units involved. Most companies use a "lot release system" whereby costs are accumulated on a job order in which the number of units completed

FIGURE 2-2

CALCULATION FORMULAE FOR SELECTED COST IMPROVEMENT CURVE CONCEPTS

(Formula for calculation concept in stub when Formula in Head describes a straight line on log-log paper)

Formula Number	Concept	Unit Curve $Y_x = AX^b$
1	Cost of Unit X Y^x	AX^b
2	Cum Total Cost of N Units CTN	$A\left[\sum_{x=1}^{N} x^b\right]$
3	Cum Av Cost of N Units $\bar{Y}n$	$\frac{CTn}{N}$
4	Cost of Lot of F to L Units $TC_{F,L}$	$A\left[\sum_{x=1}^{L} x^b - \sum_{x=1}^{F-1} x^b\right]$
5	Lot Average Cost $\bar{Y}_{F,L}$	$\frac{TC_{F,L}}{[L-(F-1)]}$

A is Y_1, Cost of Unit One
b a constant such that 2^b * 100 = SLOPE
F is first unit in lot; L is last unit in lot
N and X are Unit Numbers

are specified and costs are cut off at the completion of the number of units in the lot.

Since the job system is commonly used, the unit cost is not the actual cost per unit for any particular unit in the lot. Rather, it is an average cost for all units in the lot. This means that when lots are plotted on graph paper the unit value corresponding with the average cost value must be found. In nearly all cases this unit value (x) is the median unit within the lot. Therefore, the plot point used to represent the first lot on log-log paper would be the x value and the average lot cost for the y value. For each succeeding lot the x value will be the median unit of the lot plus the total number of units produced up to that lot. The y values will be simply the average cost of the lot. Therefore, for plotting purposes:

x = lot plot point (median of the lot plus the total of all units produced up to the lot)

y = lot average cost

It is characteristic for the early units in the first lot to decline very rapidly (arithmetically speaking). Consequently, there may be some distortion when locating the representative value at the mid-point of 10 or more units for the first lot. This distortion is compensated for by a rule-of-thumb which states that when the first lot contains ten or more units, one-third the lot size should be chosen as the unit value estimate of the first lot plot-point. It is an arbitrary rule and applies to <u>first lot only.</u> True lot plot points can be calculated from a rather complicated formula, but in most instances the rule-of-thumb is sufficiently accurate.

This process of identifying the appropriate X and Y values for plotting the cost improvement curve relationship is called "editing the data." This process is summarized in the worksheet for the Boeing Construction, Table 2-3.

EDITING LOT DATA FOR THE BOEING CONSTRUCTION

A WORK SHEET

LOT DATA

(1) LOT NO	(2) LOT SIZE	(3) LOT VALUE	(4) CUM UNIT (CU)	(5) LOT MID POINT (LMP)	(X) (6) LOT PLOT POINT (LPP)	(Y) (7) AVG UNIT COST (AUC)

EDITING RULES:

STEP 1 Set up a worksheet with seven columns labeled as above.

STEP 2 Enter Lot No, Log Size, and Lot Values for each and every lot (columns 1-3).

STEP 3 Calculate CU (Column 4) -- Lot size plus cumulative units through previous lots

STEP 4 Calculate LMP (Column 5) --

(a) For first lot, apply rule-of-thumb: less than 10 units, divide lot size by 2 [<10; Lot Size/2]; 10 or more units divide lot size by 3 [$\geq$ 10; Lot Size/3].

(b) For all following lots, LMP = Lot Size/2.

STEP 5 Calculate LPP (Column 6) -- LMP plus cumulative units through previous lot.

STEP 6 Calculate AUC (Column 7) -- Lot Value divided by lot size.

STEP 7 Plot columns 6 and 7 on log-log graph paper.

TABLE 2-3

EXAMPLE:

Suppose we had the following historical unit cost improvement curve date:

LOT NO	LOT SIZE	LOT VALUE
1	8	2312
2	16	2672
3	26	3120
4	32	3040
5	40	3000

We could determine the representative X and Y values by applying steps 1 through 6. Our completed worksheet appears below:

WORKSHEET

LOT NO	LOT SIZE	LOT VALUE	CU	LMP	X LPP	Y AUC
1	8	2312	8	4	4	289
2	16	2672	24	8	16	167
3	26	3120	50	13	37	120
4	32	3040	82	16	66	95
5	40	3000	122	20	102	75
6	45	To be Est'd	167	22.5	144.5	

Step 7 would have us plot the X and Y values in columns 6 and 7. We could then fit a line through the plot points and make predictions about future unit costs.

Plotting cost improvement curve data should indicate a straight line relationship and, without a great deal of experience, an analyst can readily draw a line of best fit. Lines drawn by inspection or eye-sight are usually adequate, especially when there are small deviations of the actual points from the straight line. Such a line can be used for purpose of projection.

There are instances when more accuracy is desired than a line of best fit by inspection can produce. **When high values are involved, a difference of 1% in slope estimation can make a sizable difference in estimating dollar amounts on the contract.** A mathematical best fit line can be constructed by using the method of least squares to fit the line. **To find a least squares** best fit line, the formula for the cost improvement curve, $y = ax^b$ must be transformed into a linear form, or $\log y = \log a + b \log x$. To find the logarithmic least squares line requires that the following two simultaneous linear equations be solved:

$$\sum \log y = n \log a + b \sum \log x$$

$$\sum (\log y * \log x) = \log a \sum \log x + b \sum \log x^2$$

Where: x, y, a, and b retain the same definition as originally presented in Chapter 1 and 'n' is the number of data points in the data set.

As a practical matter, if this kind of precision is required, there are usually several cost improvement curve computer programs available that, among other things, will calculate a best-fit least-squares line.

Once the data has been plotted, the T_1 and slope can be determined. In the example above, the T_1 is a little less than 520 hours, (we'll use 516 hours); and the slope is approximately 75% (See figure 2-3 for log-log graph). We could predict lot 6 with 45 units, by using formula 4 from Table 1, the Boeing Improvement Tables for a 75% slope and our graphical T_1. Our formula would be:

$$TC_{F,L} = A \left[\sum_{X=1}^{L} x^b - \sum_{X=1}^{F-1} x^b \right]$$

Where: A = Value of the first unit (T_1)
F = First unit in the lot being estimated
L = Last unit in the lot being estimated

then,

$$TC = 516 \left[\sum_{X=1}^{167} x^b - \sum_{X=1}^{123-1} x^b \right]$$

Refer to the Boeing Tables, page 49B, for the sum of 167 units and the sum of 122 units:

$$TC = 516 \left[33.008677 - 27.291313 \right]$$

$$TC = 2950 \text{ hours}$$

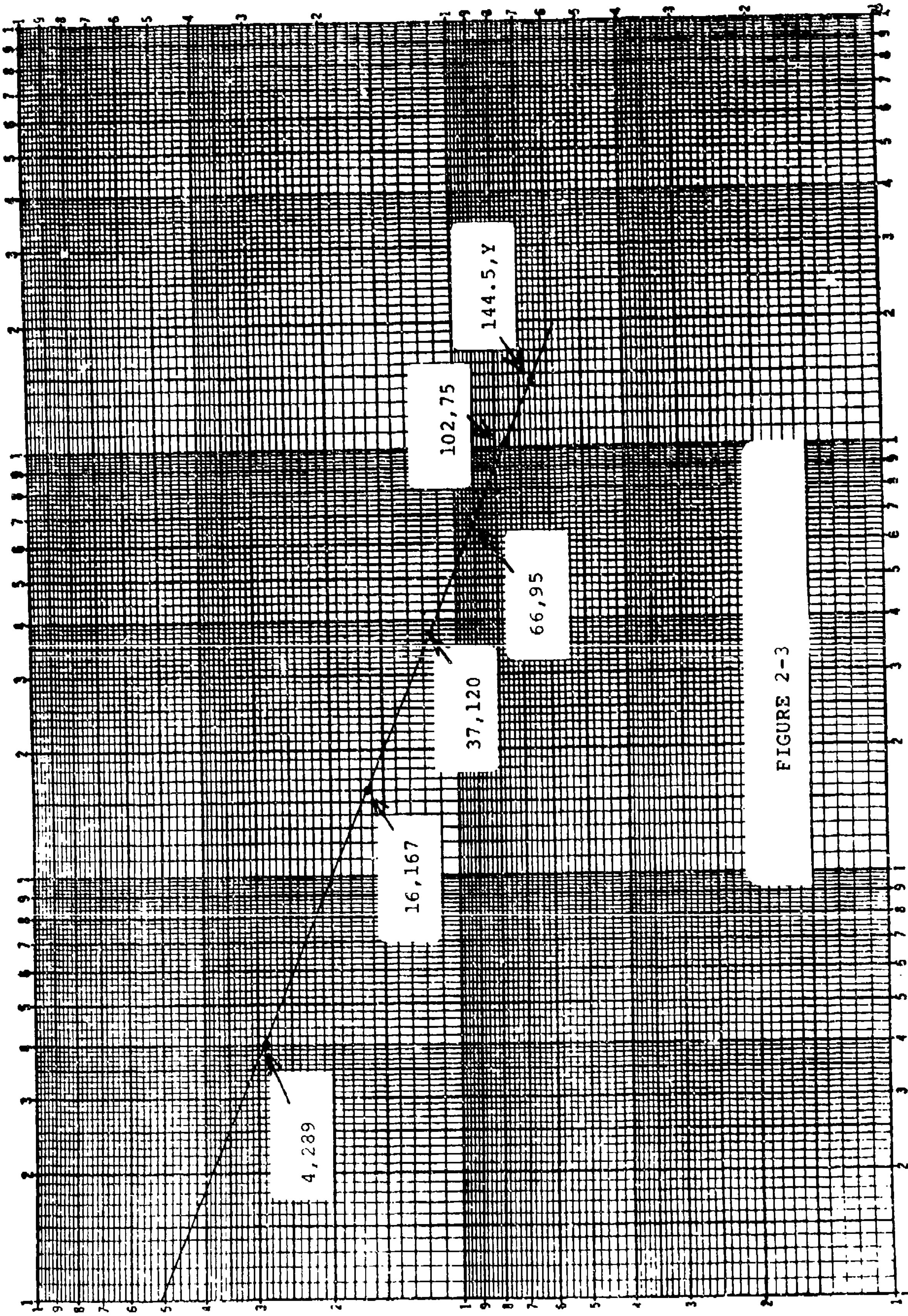


FIGURE 2-3

Our combined graphical and calculated estimate for lot 6 is 2950 labor hours. We could also make our estimate directly from the graph by reading the 'y' value which corresponds to the Lot 6 plot point. Where 'x' is 144.5, our graph shows a 'y' value of approximately 66 hours. This is the average hours to complete unit 144.5 or the unit in the middle of lot 6. We can calculate the lot value by multiplying the 66 hours by the 45 units in lot 6 -- 66 hours * 45 units = 2970 hours. This is close to our previous calculation. The difference of 20 labor hours is probably due to our inability to read exact values from the graph and/or the rounding of the slope to two decimal places when using the Boeing Improvement Tables.

ANALYZING MAJOR PROGRAM CHANGES USING THE BOEING CONSTRUCTION

Up to this point the cost improvement curve has been discussed under the assumptions of an uninterrupted production and a stable product design. However, change is an integral part of any process and the cost improvement curve phenomena is no exception. This section will discuss the effects of changes in product design.

CHANGES TO PRODUCT DESIGN

As you will recall, part of the cost improvement curve phenomena is attributed to **minor changes** which are made to improve the efficiency of the production process. These types of changes are usually management initiatives which have insignificant, if any, impact upon the design of the item being produced. However, the changes that will be discussed here are changes which have a **substantial impact** on the design (form, fit, function) of the item being produced. Thus, we are talking about changes introduced into an on-going process.

The problem of changes requires a clarification in terms of the nature of the change. That is, a change can be:

1. The **addition** of a component or components
   or
2. The **deletion** of a component or components
   or
3. The **substitution** of one component for another component.

Thus, there are three types of changes that can occur. The first two (addition or deletion) are readily apparent and the third (substitution) is merely combining the first two, i.e., a substitution is made up of the deletion of one component and its replacement by a new component. If an analyst knows how to handle a deletion and an addition, then the analyst knows how to handle a substitution. In the following sections the analysis of changes will be developed accordingly -- first deletions, then additions, and finally substitutions. As you have already learned, cost improvement curve problems can be solved either graphically or by use of the Boeing tables. For each type of change both the graphic and formula solution will be shown. While this may appear to be redundant, it is always a useful cross check to assure mechanical errors have not been made.

Deletions. A deletion is the removal of a component from an item that is being produced. To analyze the effects of a deletion, certain questions must be answered. Suppose a unit had been in production for some time and a decision was made that a particular component in the unit was not required for future lots of production.

Question 1. Would you expect the rate of learning to be different after the deletion than before? No. The expectation would be that the process experience would continue as before because, for the most part, no new learning is required.

Question 2. Would you expect the future cost to be less than had the deletion not been made? Certainly. Because there is not as much work to be performed on each unit as there was before the deletion occurred.

The answers to these two questions are the crux of the solution for analyzing the effect of a deletion -- that is, we are assuming that the rate of learning will persist but the amount of work to be performed will be less. An example should illustrate the approach and the solution.

EXAMPLE:

Theoretical value of unit 1 = 1000

Slope = 80%

Units produced before deletion = 50

A component representing 10% of the effort to produce the original unit will be deleted starting with 51.

Estimate the cost of units 51-75.

Graphic Solution. Figure 2-4 shows this example. Notice that the cost improvement curve drops down at unit 50. A closer inspection will also show that the segment of the curve after 50 is parallel to the segment of the curve before unit 50. Thus, the same rate of learning persists which is consistent with the answer to the first question we asked previously. Now, if the second line segment were extended back to the "y" axis, as shown by the dotted line, it would intersect at a value of 900 or 10%

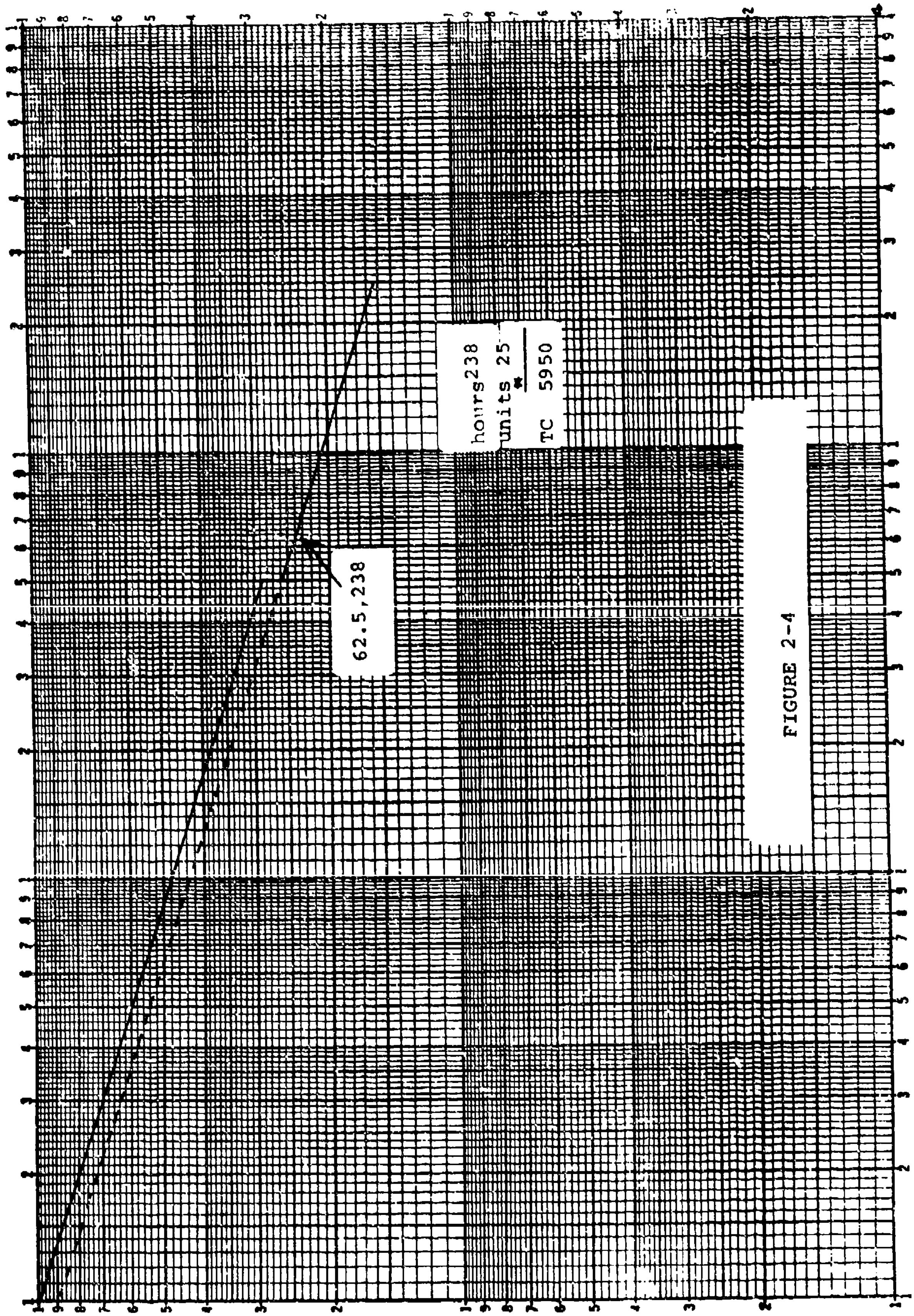


FIGURE 2-4

less than the original effort. In fact the vertical distance between the dotted line and the solid line is 10% for each unit. To put it another way, the dotted line represents the cost improvement curve for the first 50 units had the deleted component not been included in those units. Thus, to estimate the cost of units 51 through 75, we would read the lot average value off the second curve segment at lot plot point 62.5:

Estimated lot average cost	238
* Nr of units in lot	* 25
= cost of units 51-75	5950

Before going to the formula solution for this problem we need to discuss the notation and the various ways of stating deletions.

Expressing Deletions. Essentially deletions can be expressed in one of three ways:

1. A percent of original effort
2. Hours of current effort
3. A percent of current effort

Regardless of which method is used, it must be translated to the theoretical value of unit 1 of the deletion since it is this value which is used in all of the formula calculations. Moreover, this translation facilitates the graphic portrayal of the deletion.

Refer to Figure 2-4. Recall that the dashed line is parallel to the original line but is below it; and, in essence, reflects a line with the same slope but a lower theoretical first unit value of unit 1. It is this new value of **A** that we are looking for. To distinguish it from the original **A**, the notation A_- will be used.

The minus sign in the subscript is used to connote a deletion and has **no algebraic meaning.** To illustrate how A_- is determined, given the three methods discussed above, we will use the following situation:

A = 1000
Slope = 80%
Deletion effective with unit 51.

Deletion Expressed as a Percent of Original Effort. This approach means that the engineers will go back to unit #1 and determine the percent of the labor hours expended on the item being deleted. Thus, given our example of A = 1000 if the deleted component's share of the original unit were 10% then:

$$
\begin{aligned}
A\_ &= A(1-.1) \\
&= A(.9) \\
&= 1000(.9) \\
&= 900
\end{aligned}
$$

Deletion Expressed as Hours of Current Effort. Here the engineers estimate the percent of the labor hours expended on the deleted item for current production. Thus if it were estimated that the deleted item represented 70 labor hours at unit #50, it must be determined what this 70 hours is equivalent to at unit # since the 70 hours reflect the effect of the improvement on units 1 through 50. The simplest approach is to calculate the unit value for the 50th unit from the formula as follows:

$$Y_{50} = a(X^b)$$

$$70 = a(.283827)$$

$$70/.283827 = a$$

$$246.6291086 = a$$

This procedure takes the deletion and "grows" it back to what it would have been at unit 1 or the amount by which **A** must be reduced to accommodate the deletion. Thus we have:

$$A\_ = A - Y_{50}/50^b$$

$$A\_ = 1000 - \frac{70}{.283827}$$

$$A\_ = 1000 - 246.6291086$$

$$A\_ = 753.3708914$$

And, of course, we could now state that the deletion represented 24.66% of the original effort. In fact we could take any value along the original curve and reduce it by 24.66% and we would have the equivalent unit value for the deletion line, which leads us to our third approach.

45

Deletion Expressed as a Percent of Current Effort. In this approach, the engineers state the deletion as a percent of current experience such as: The deleted effort represented 10% of the hours expanded on unit 50. If the labor hours on unit #50 were 284 hours, the deleted effort would have been 28.4 hours. This puts us back to the procedure just discussed, i.e., hours of current effort, or:

$$A_{-} = A - [10\%(Y_{50}/50^{b})]$$

$$A_{-} = 1000 - \frac{.1(284)}{.283827}$$

$$A_{-} = 1000 - 100.0609526$$

$$A_{-} = 899.9390474$$

$$A_{-} = 900$$

A more straight-forward approach is simply to reduce the original A by the percent deletion since the percent difference between the original line and the deletion line is the same for both.

Now that we have covered the notation and expression of changes, we are in a position to return to the problem we were working and estimate the impact of a deletion using the formulae. The problem was:

Given A = 1000
Slope = 80%
Units produced before deletion = 50
A component representing 10% of the original effort will be deleted starting with unit 51,

Estimate the cost of units 51-75

Our graphic estimate of units 51-75 was 5950 labor hours.

Formulae Solution. Recognizing the graphic estimate may include visual errors, a more refined estimate can be made using Boeing Improvement Curve Tables and Formula. In using the tables, the analyst will find it more convenient to use Part B - Cumulative Total Values because it eliminates the problem of having to find the table value for a fractional unit such as occurs in this problem, i.e., finding the table value for 62.5, for the lot mid-point. Therefore, the cost of units 51-75 from the cumulative tables would be:

$$TC = (A\_)(\sum X_{75}^{b} - \sum X_{50}^{b})$$

where: $A\_ = (1 - .1)A$

$= .9(1000)$

$= 900$

X^{b}_{75} = The cumulative total ratio value for 75 units, found in Boeing Improvement Curve Tables, 80% slope, "'B' pages."

X^{b}_{50} = The cumulative total ratio value for 50 units, found in Boeing Improvement Curve Tables, 80% slope, "'B' pages."

substituting we have:

= (900) [26.727271 - 20.121714]
= (900) (6.605557)
= 5945.0013 labor hours for units 51-75 with a 10% deletion starting at unit 51.

Additions. When a new component is added to the production of an on-going unit, additional labor hours will be required to accommodate the new component. As with deletions the answers to a few questions hold the key for handling an addition. Again, suppose a particular unit had been in production for some time and a decision has been made to add a component to the unit to increase the unit performance.

Question 1. If by adding a component would you expect the rate of learning to be different? Generally, No. Because for most situations the items and units produced are similar and the work environment (company policy, management attitudes, etc.) is sufficiently stable that we expect the same rate of learning. However, if we are introducing a new component to be built by a new subcontractor, the rate of learning may well change. The analyst will need to examine this aspect closely to determine the appropriate rate of learning.

Question 2. Would you expect a relative increase in the cost of units which include the new component? Yes, because effort not previously expended will be required to accommodate the new component.

Question 3. Will the previous production experience apply to the new component? No. With respect to the original unit process, substantial improvement has taken place, but with respect to the new component there has been no improvement.

The answers on the previous page suggest that an addition is treated as a new cost improvement curve having the same slope or rate of learning as the original unit. An example should clarify the treatment of the addition:

EXAMPLE:

Theoretical value of unit 1 = 1000

Slope = 80

Units produced before the addition = 50

A new component will be added to the line starting with unit 51. Engineers and production personnel estimate the additional labor hours to include the new component in unit 51 will be = 100 hours

* Estimate the cost of units 51-75.

__Graphic Solution.__ Figure 2-5 shows the effect on the cost improvement curve of adding the component. The original cost improvement curve for the first fifty units is shown at the top of the figure. At unit 51 there is a vertical increase of 100 labor hours representing the added effort for the new component. The hatched area above the original line shows the respective additional labor hour costs for each unit produced which includes the new component. The curve at the top of the hatched area reflects the new improvement for the production process. As can be seen, the curve is not a straight line, which makes graphic projection somewhat difficult. **The reason for this curvature is that, with respect to the added component, improvement starts all over and this new improvement (smaller doubled quantities) is being compressed into the interval from 50-100. This can be readily seen in the lower** portion of figure 10 which shows the additional component by itself with its own first unit value of 100 and a slope of 80%. It should be recognized that the **AREA AT THE BOTTOM OF THE FIGURE IS THE SAME AS THE HATCHED AREA IN THE TOP PART OF THE FIGURE.** It is exactly this procedure which allows us to estimate the cost of units 50-75.

__**Disregarding the effect of the addition:**__

$LPP_{62.5}$	265
* Nr of units	* 25
	6625

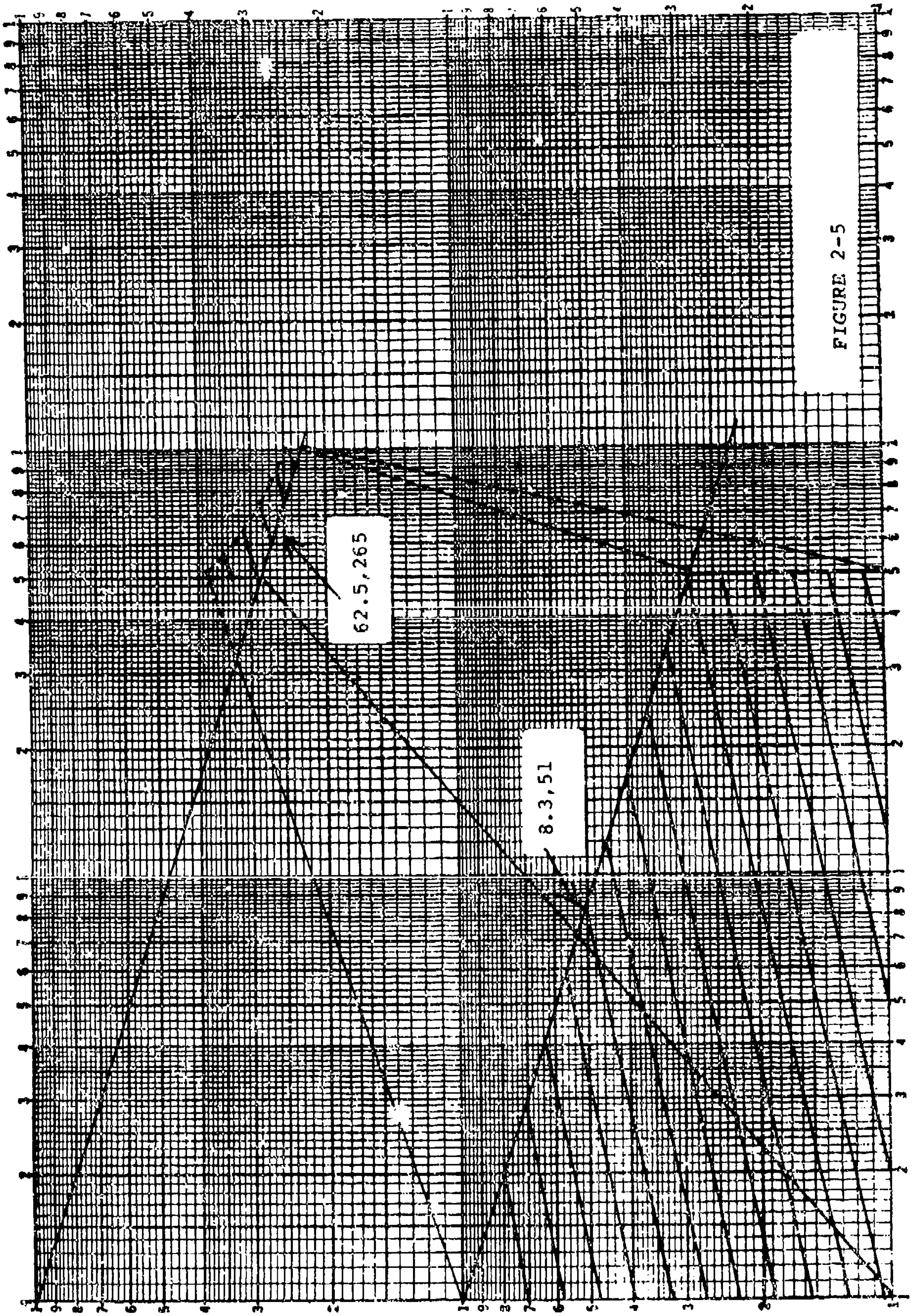


FIGURE 2-5

and the cost of the additional components would be:

$LPP_{8.3}$	51	[Note: $LPP_{8.3}$ used because this is a 1st Lot with respect to the "new" curve for the addition and therefore is subject to 1st Lot rule-of-thumb mid-point.]
* Nr of units	* 25	
	326	

Thus, the total cost of units 51-75:

Original	6625
Addition	+ 1275
	7900 labor hours

Computationally a more convenient method would be:

$LPP_{62.5}$	265
$LPP_{8.3}$	51
	316
* Nr of units	25
	7900 labor hours

Formulating Additions. The procedure for formulating additions is analogous to the previous discussion of formulating deletions. The symbol used to represent the first unit value of the addition is A_+. The + subscript has no algebraic meaning.

Formula Solution. For a formula solution, the logic is the same as the graphic, i.e., essentially dealing with two different curves. Again the cumulative portion of the Boeing Improvement Curve tables will be used.

$$\text{Basic Units (51-75)} = A\,[\textstyle\sum x^b{}_{75} - \sum x^b{}_{50}]$$

$$\underline{\text{Additional Components (1-25)}} = A_+\,[\textstyle\sum x^b{}_{25} - \sum x^b{}_{0}]$$

Total Cost of Units 51-75

Specifically:	1000 (26.72781 - 20.121724)	6605.557
	100(12.308607 - 0)	+1230.8607
	Total Cost of Units 51-75	7836.4177

It should be noted that the differences between the graphic estimate and the formula estimate is due to:

1. Not using the true lot mid-point.
2. Inaccuracies in plotting and reading values on the graph.

Substitutions. Previously it was stated that a substitution is, in actuality, a simultaneous deletion and addition. An example should illustrate how a substitution is handled.

EXAMPLE:

Given: Theoretical value of unit Nr 1 = 1000

"Slope" = 80%

Units produced prior to substitution = 50, and a substitution of components will be made starting with unit 51. The component being deleted represents 10 percent of the existing effort and a new component will represent 150 hours for unit 51:

Estimate the cost of units 51-75.

Graphic Solution. Figure 2-6 shows the net effect of the substitution on the cost improvement curve at the top of the figure and, as in the previous discussion, the added portion is shown at the bottom of the figure. Using estimates from the graph, the estimate would be:

Basic unit less the deletion $LPP_{62.5}$	242
Addition portion of substitution $LPP_{8.3}$	+76
Estimated unit cost	318
Number of units in lot	*25
Estimated cost of units 51-75	7950

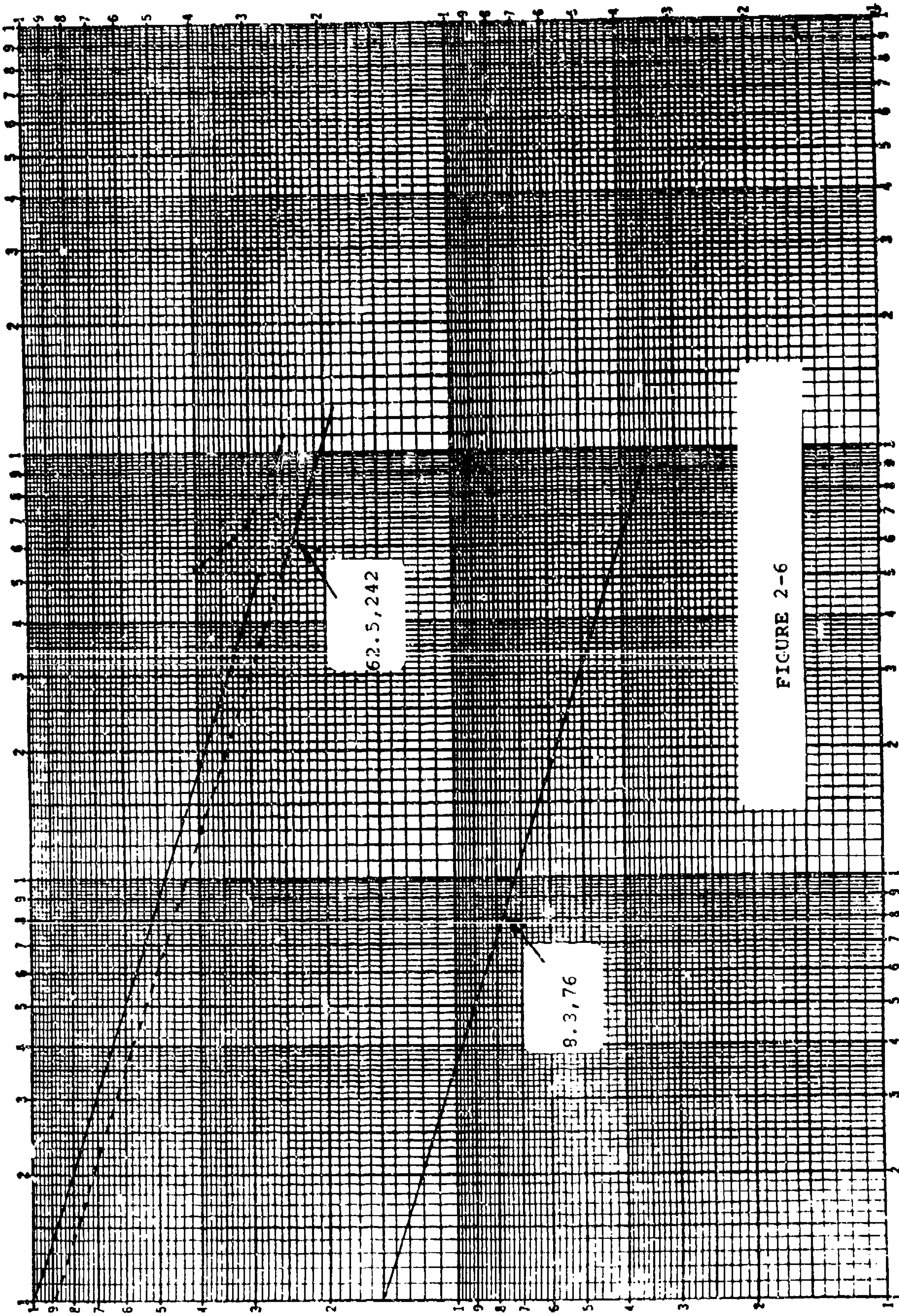


FIGURE 2-6

Formula Solution.

$$\begin{array}{ccc} \text{Deletion} & \text{Addition} & \text{Total} \end{array}$$

$$(A-) \left[\sum x^b_{75} - \sum x^b_{50} \right] + A_+ \left[\sum x^b_{25} - \sum x^b_{0} \right] = TC$$

$$(900)(26.727281 - 20.121724) + 150\ (12.308607 - 0) = TC$$

$$900\ (6.605557) + 150(12.308607) = TC$$

$$5945.0013 + 1846.2910 = 7791.2923$$

Again, the difference in the answers is attributed to inaccuracies in reading the exact value from the graphic depiction.

The methodology for handling all types of changes has been covered. As long as you remember the logic and methodology, you should be able to handle most any problem. A final example should demonstrate how the methods that have been developed can be used to handle a somewhat more complicated problem.

A manufacturer has been producing a piece of equipment as shown below:

LOT #	UNITS	LOT COST
1	12	14454
2	10	8006
3	10	6897
4	18	10886

The theoretical value of unit 1 = 2000. The cost improvement curve slope has been 80%. Because of safety requirements, the manufacturer is being required to add an additional component. The engineers (design, industrial and production) have estimated that the labor hours required to construct and install the first new component in the main assembly will be 500 labor hours. Also, they have determined, through analysis of previous similar components, that the slope on the new component will be 75%. Estimate the labor hour cost of lot 5 (units 51-75).

Graphic Solution. Figure 2-7 shows the plot of the main unit and the additional component. From the graph, the estimated labor hours would be:

Basic unit cost $LPP_{62.5}$	530
Additional Component $LPP_{8.3}$	+ 213
Estimated average unit cost	743
Number of units in lot	* 25
Estimated cost of units 51-75	18575

Formula Solution.

Basic Unit + Addition = Total

$$A[\sum x^b{}_{75} - \sum x^b{}_{50}) + A_+ [\sum x^b{}_{25} - \sum x^b{}_0] = TC$$

$$2000(26.727281 - 20.121724) + 500\ (10.190694) = TC$$

$$2000(6.605557) + 500(10.190694) = TC$$

$$13211.114 + 5095.3465 = 18306.4605$$

After lot 5 production is in process, the manufacturer decides to substitute a new mechanism in the product for an old one. This new mechanism begins with lot 6. The engineers have estimated that the deletion of the old mechanism will reduce labor hour requirements by 10% and the additional labor hours required for the new mechanism will be 400 hours for the first unit with the same rate of learning. Estimate the labor hours required to produce lot 6 (units 76-87). The only difference in this problem is that the safety component added in lot 5 must be carried forward into lot 6 where the safety component is still being estimated on a 75% cost improvement curve slope.

Graphic Solution. Again Figure 2-7 shows the impact of the substitution. To estimate the cost of lot 6 we have:

Basic unit less 10% at LPP_{81}	444
Component added in lot 5 at LPP_{31}	+ 124
The new mechanism at LPP4	+ 255
Estimated average unit cost	823
Number of units in lot	* 12
Estimated cost of units 76-87	9876

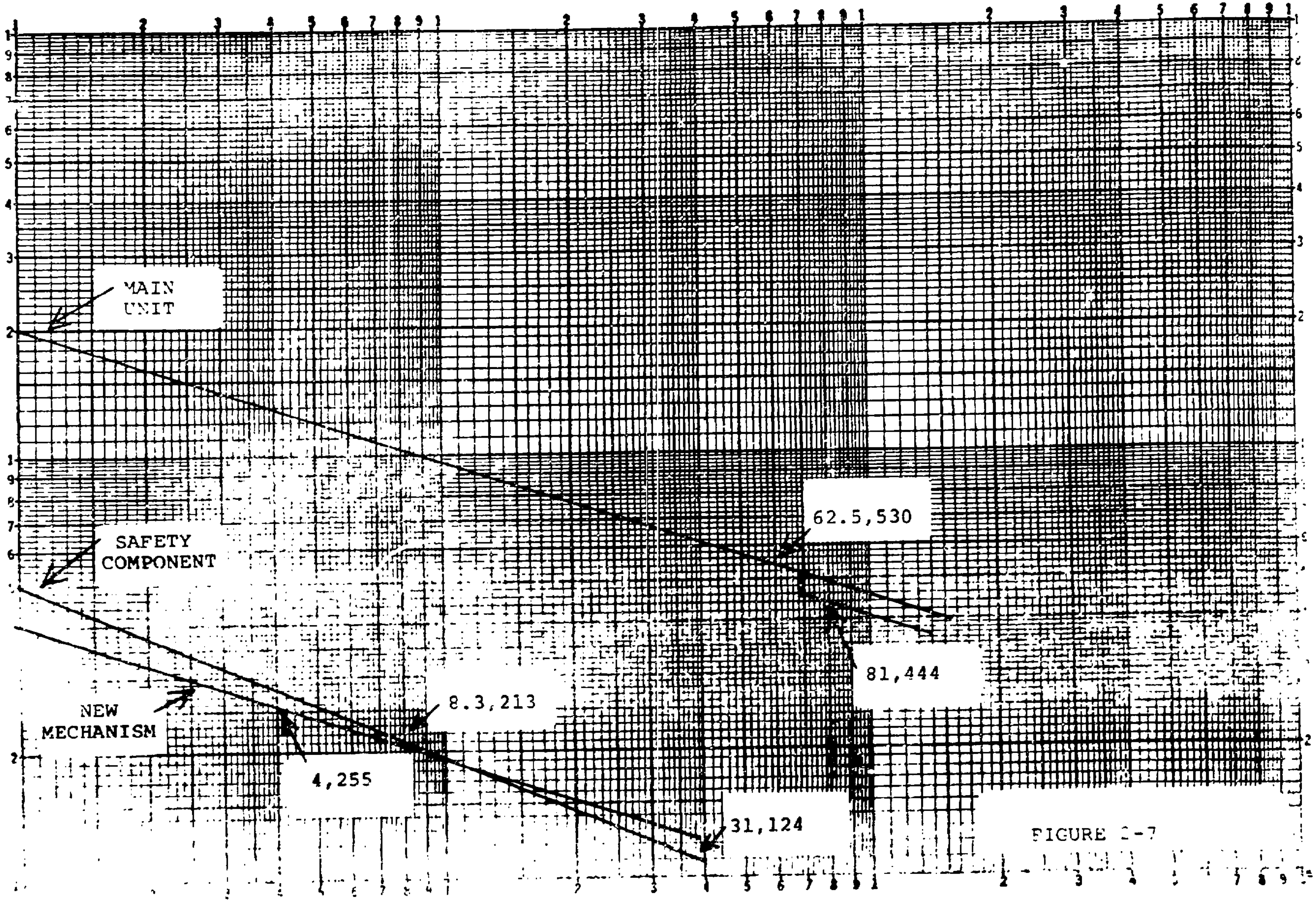

MAIN
UNIT
SAFETY
COMPONENT
NEW
MECHANISM
62.5,530
81,444
8.3,213
4,255
31,124

FIGURE 2-7

Formula Solution.

Basic Unit less 10% on 80% slope

$(A_-)(\sum x^b{}_{87} - \sum x^b{}_{75}) = TC$

$1800(29.638615 - 26.727281) = TC$

$1800(2.911334) = 5240.4012$

Component added in lot 5 on 75% slope

$A_+(\sum x^b{}_{37} - \sum x^b{}_{25}) = TC$

$500(13.067284 - 10.190694) = TC$

$500(2.876590) = 1438.295$

New mechanism this lot

$A_{++}(\sum x^b{}_{12} - \sum x^b{}_0) = TC$

$400(7.226841) = \underline{2890.7364}$

Labor-hour cost of Lot 6 = 9569.4326

Once again, the disparity between the graphic and formula solution can be attributed to true lot midpoints and inaccurate reading of the graph. For instance, the rule of thumb for the added mechanism would indicate an LPP, but in fact the true-lot midpoint is 4.83 which would equate to a reading of about 235 labor hours versus 255. This disparity, when multiplied by the twelve units, equals 240 labor hours which accounts for a substantial portion of the discrepancy.

PROBLEMS

1. Determine lot midpoints for the lots and quantities on the following:

LOT	LOT SIZE	TOTAL LABOR HOURS PER LOT	AVERAGE LABOR HOURS FOR LOT
1	8	2312	289
2	16	2672	167
3	26	3120	120
4	32	3040	95
5	40	3000	75
6	50	3500	70
7	60	3660	61

2. Plot the labor hour values for the lots in the above problem.

3. If the company in the above problem contemplated continuous production beyond the 232 units, for an addition of 140 units, what average and total labor hour values would you estimate for the additional units?

4. What are the estimated labor hour requirements for the 372-unit production?

5. Plot the following value on log-log paper:

X	Y
6	34
14	32
30	26
60	20
80	21
110	18
150	19

a. Fit a line of best fit by inspection

b. Calculate the slope of curve

PRACTICE PROBLEMS CONCERNING CHANGES

In the following problems make at least two estimates, one from graphs and at least one from tables.

1. 150 units will be made

   Experience shows
   Unit 1 COST: 1600 LABOR HOURS
   SLOPE 82%
   BOEING CONSTRUCTION

   A component equivalent to 5% of current unit cost will be removed, effective with unit 151.

   Estimate cost of follow-on lot of 40 units.
   Answer: Near 14,000 Hours

2. It is planned to make 200 units

   Experience shows
   VALUE OF UNIT 1: 1800 LABOR HOURS
   "SLOPE" 78%
   BOEING CONSTRUCTION

   At unit 130 it is decided to add a part effective with unit 151.

   Engineering estimates indicate the cost of unit one of the addition will be 225 Hours.

   Estimate cost of a lot of 50 containing units 151-200.
   Answer: Near 18,000 Hours.

3. Plans call for the manufacture of 100 units.

   Experience shows:
   UNIT 1 COST: 1200 LABOR HOURS
   "SLOPE" 75%
   BOEING CONSTRUCTION

   After unit 74, it is decided to replace a component effective with unit 101 and produce 50 more units.
   Engineers estimate the replaced part is 5% of current effort (effort on unit 75).
   First unit cost of replacement will be 180 LABOR HOURS (A_+)

   Estimate cost of additional lot 50 units; unit numbers 101-150.
   Answer: Near 10,500 LABOR HOURS

CHAPTER III

THE CUMULATIVE AVERAGE COST IMPROVEMENT CURVE THEORY

When companies experience costs or labor hours so that the improvements are linear on log-log paper when the cumulative average is taken, this is known as the Northrop Construction or the cumulative average cost improvement curve theory. By definition the Northrop curve is one in which the cumulative average curve is the more nearly straight line, which suggests that the unit cost improvement curve theory is bowed. To see the distinction between this and the Boeing curve construction, refer to Figure 3-1.

COMPARISON OF THE TWO BASIC CONSTRUCTIONS OF THE CURVE

In analyzing a company's cost improvement curve proposal, it is very important to understand the basic construction of its curve. Negotiators may differ drastically merely because the analyst considered a different construction than the company considered.

It is not true that one construction is better than the other; nor should an analyst insist on only one construction for all instances. The basic question we need to consider is what happens during the early stages of production, for it is this stage which determines the basic approach. Companies using the Northrop Construction theoretically may be expected to:

(1) experience rapid declines in cost per unit at first due to having started production with a large number of engineering problems still to be worked;

(2) use tooling which may not be adequate for the entire production run; and

(3) have insufficient detail in the production planning.

Whatever the reason, companies at times experience high initial costs but decrease them rapidly and settle down to a constant rate of improvement. If this is the case, the Northrop Construction is theoretically the more appropriate construction.

The conclusion should not be reached that companies following the Northrop curve are more efficient or lower-cost companies. It is reasonable to believe in the case of a firm following the Northrop curve, that its unit number 1 cost would have been approximately 700 in Figure 3-1, if they had not experienced the initial difficulty as described above (See Figure 3-1). Nor should the conclusion be reached that companies using

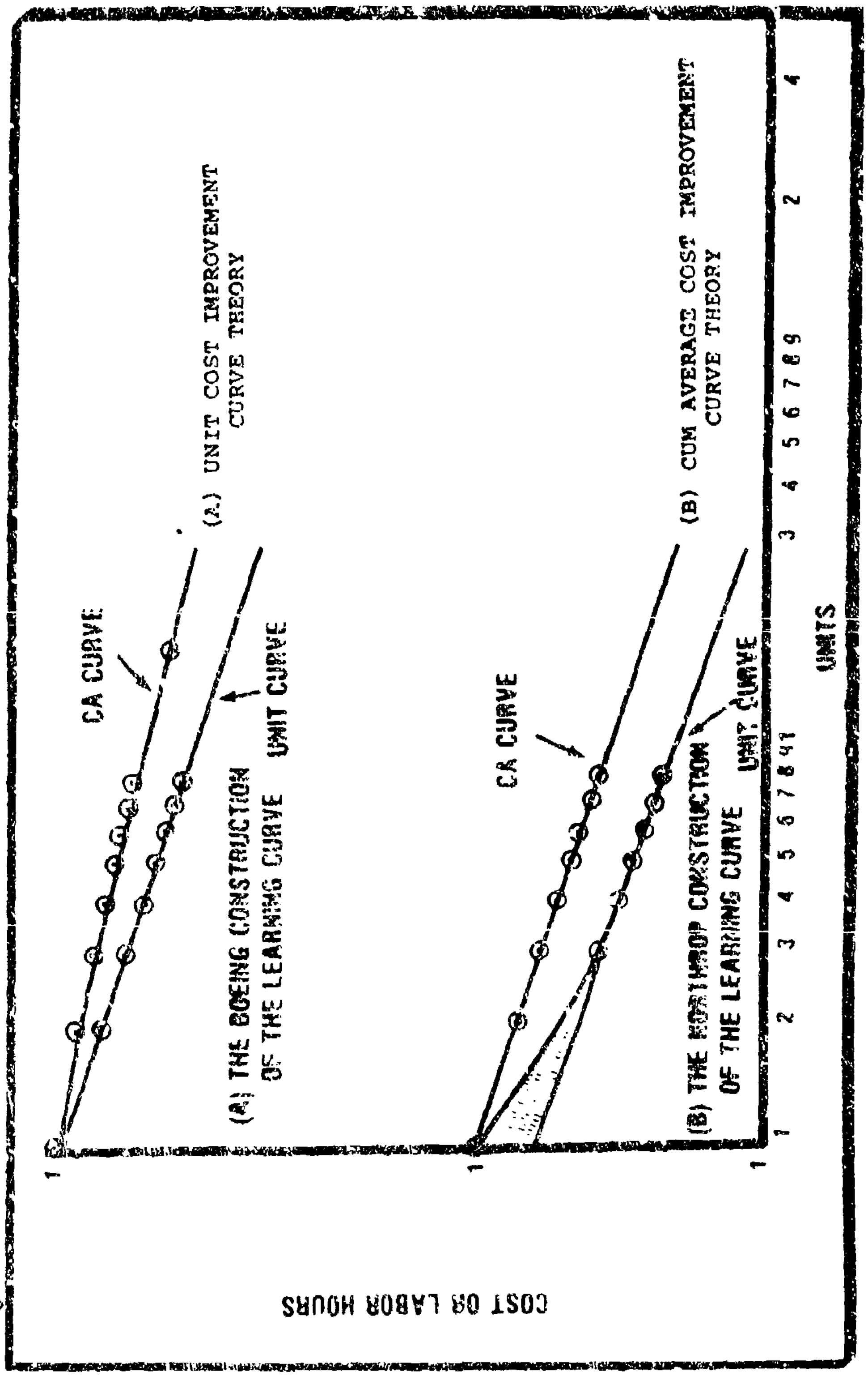

COST OR LABOR HOURS
(A) UNIT COST IMPROVEMENT CURVE THEORY
CA CURVE
UNIT CURVE
(A) THE BOEING CONSTRUCTION OF THE LEARNING CURVE
(B) CUM AVERAGE COST IMPROVEMENT CURVE THEORY
CA CURVE
UNIT CURVE
(B) THE NORTHROP CONSTRUCTION OF THE LEARNING CURVE
UNITS
1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 2 4

FIGURE 3-1

the Northrop curve are high-cost producers. The shaded area in Figure 3-1 does not necessarily represent additional cost due to high initial cost, because this cost might have been absorbed in additional engineering or tooling cost.

THE NORTHROP CONSTRUCTION MODEL:

The Northrop Construction theory can be stated as follows:

As the total quantity of units produced doubles, the average cost per unit decreases by some constant percentage.

Expressed in equation or model form, the cumulative average theory is:

$$Y_x = AX^b$$

where:

Y_x represents the cumulative average cost of X units

X represents cumulative units

A is a coefficient (constant) representing the theoretical first unit cost (T_1)

b relates to the slope and the rate of change of the cost improvement curve

CUMULATIVE AVERAGE CURVE FORMULAE:

As with the Boeing Construction, the Cum Average Curve theory or Northrop Construction has its own five concept-peculiar formulae for calculating values. These formulae may be used with any hand-held calculator having a power function and a logarithmic function (or natural log function). **No Boeing Improvement Curve Tables are required.** A computerized software package may also be used to calculate cum average curve value.

As with the Boeing Construction, to use the formulae you must know the cost improvement curve slope and the first unit value (T_1). The formulae are presented in Table 3-1. An introductory vocabulary is also provided as explanation of the concept terminology in Appendix F.

These five formulae will be applied in estimating lot values and total program costs. To apply them the analyst must have either production program 'actuals' or analogous program data from which to derive a T_1 and cost improvement curve slope.

CALCULATION FORMULAE FOR SELECTED COST IMPROVEMENT CURVE CONCEPTS

(Formula for calculation concept in stub when Formula in Head describes a straight line on log-log paper)

Formula Number	Concept	Cu Av Curve $\bar{Y}_X = AX^b$
1	Cost of Unit X Y^X	$A[X^{b+1}-(X-1)^{b+1}]$
2	Cum Total Cost of N Units CTN	AN^bN or AN^{b+1}
3	Cum Av Cost of N Units $\bar{Y}_N$	AN^b
4	Cost of Lot of F to L Units $TC_{F,L}$	$A[L^{b+1}-(F-1)^{b+1}]$
5	Lot Average Cost $\bar{Y}_{F,L}$	$\frac{TC_{F,L}}{[L-(F-1)]}$

A is Y_1, Cost of Unit One

b a constant such that $2^b * 100$ = SLOPE

F, L, N, X Unit Numbers

TABLE 3-1

APPLICATION OF THE CUM AVERAGE COST IMPROVEMENT CURVE THEORY FORMULAE:

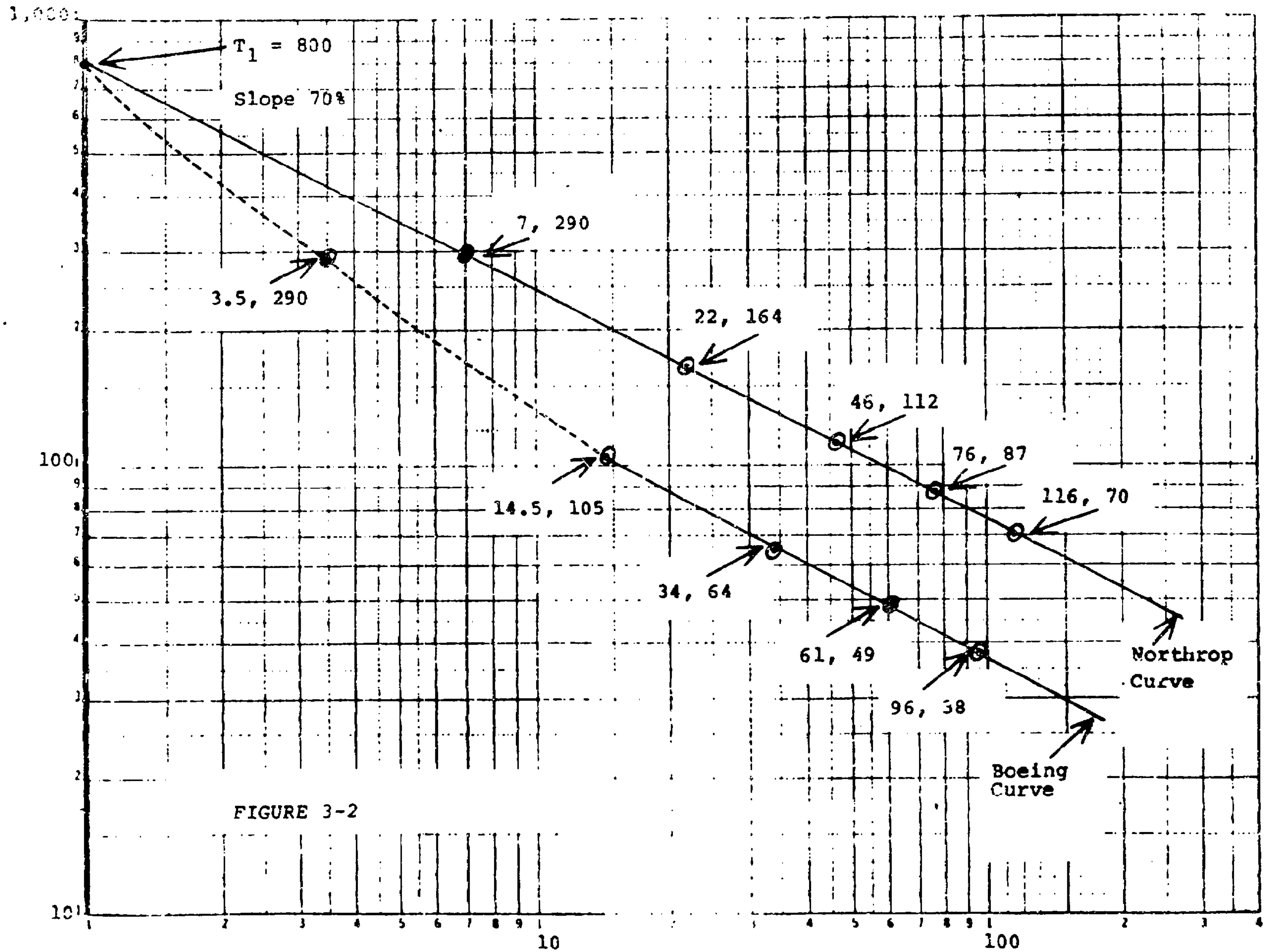
To analyze costs using the cum average theory, the analyst must develop a new editing routine. For plotting purposes, the cumulative averages (CA) for each lot are taken, but here emphasis must be given to the fact that each cumulative average is an average for all units from the first unit in the entire production sequence to the last unit of the last lot included. The problem presented below in Table 3-2 will be edited using the cumulative average theory:

LOT NR	LOT SIZE	TOTAL COST OF LOT	CUM TOTAL	(X) CUM UNITS	(Y) C.A.
1	7	2030	2030	7	290
2	15	1578	3608	22	164
3	24	1544	5152	46	112
4	30	1460	6612	76	87
5	40	1508	8120	116	70
6	40	1240	9360	156	60
7	50	To be Est'd		206	

TABLE 3-2

The first lot would be plotted at 290 for the average of 7 units, the second lot at 164 for the average of 22 units, the third at 112 for the average of 46 units, the fourth lot at 87 for the average of 76 units, the fifth lot at 70 for the average of 116 units and the sixth lot at 60 for the average of 156 units. After plotting the values on log-log paper, the reader will notice that the points are following a linear progression and projecting for the seventh lot is fairly easy. Our projection for the average of 206 units is approximately 52 hours (see Figure 3-2). However, if we plot the data in Table 3-2 under the Unit Theory, we find that this curve is not linear; rather when the data in Table 3-2 is plotted under the Unit Theory, we get a curvi-linear relationship. Therefore it would be difficult to project from the data plotted under the Unit Theory. We note, however, that unit curves have a tendency to straighten out beyond the 20th or 30th unit and become nearly parallel with cumulative average curves when the cumulative average curve is a straight line from the start. However, **parallelism does not occur until somewhere around unit 300.**

Assume, for the moment, that the labor hour cost for lots 5 and 6 of Figure 3-2 is known as shown in the table 3-2 and that we wish to project beyond the 156th unit an additional 50 units. In this case we might connect the points at cumulative unit numbers 116 and 156 and then extend to 206. The new lot values of 50 units from 157 to 206 inclusive can be measured on the cumulative average curve by calculating the cumulative total (CT) at 156 units (which is 9360 according to Figure 3-2) and then



1,000
100
10
10
100
T1 = 800
Slope 70%
7, 290
3.5, 290
22, 164
46, 112
76, 87
116, 70
14.5, 105
34, 64
61, 49
96, 38
Northrop Curve
Boeing Curve

FIGURE 3-2

taking a second reading from the log-log graph at 206 units (which should be approximately 52 hours). To obtain the cumulative total calculation at 206 units, it is next necessary to multiply 52 (CA hours) by 206 (the cum units) which is 10,712. By subtracting the two cum totals (9360 from 10,712) we get a difference of 1352 hours. This represents the total labor hours needed for the 50 units from unit 157 through unit 206.

If we now make the same assumptions as for the Unit Theory, our results should be approximately the same. Plot the unit theory to include the first 156 units, then extend the line to the 206th unit. Taking the mid-point at 181 (156 + 25 = 181) we get a reading of 27. The product of 27 X 50 is 1350, which is the projected total hours expected for the 50 units. Compare this with the 1352 when projecting from the cumulative average theory. The difference of 2 labor hours is due to the lack of precise parallelism between the two lines.

When working with both the cum average and the unit theories, there are several characteristics which should be noted:

(1) when both are plotted on the same scale and the same basic data is used, the unit curve is lower on the scale than the cumulative average curve as long as there is learning;

(2) when one is linear, the other is curvi-linear;

(3) one is most drastically curvi-linear only during the early units of production such as the first 20 or 30 units;

(4) the curvi-linear line tends to become a straight line and tends to parallel the other beyond approximately the 30th unit, although, theoretically, it is never quite a straight line;

(5) the slopes of the line are approximately the same beyond a certain point; and

(6) the labor hour calculations from either theory, beyond approximately 30 units, should produce about the same results.

To visualize these statements, refer to the top half of Figure 3-1.

AN EXAMPLE:

Assuming an 80% slope, values are as follows for the Northrop Construction:

UNIT	C.A.	CT	Y_x
1	1,000	1,000	1,000
2	800	1,600	600
3	702	2,106	506
4	640	2,506	454
5	596	2,980	420
6	562	3,372	392
7	535	3,745	373
8	512	4,096	351

TABLE 3-2

In Table 3-2, note that the cumulative total values are obtained by a simple multiplication: The cumulative average value for a unit times the unit. Thus, the cumulative total value for 5 units is 2,980, i.e., the product of 596 and 5. The unit value for the 5th unit is 420, the difference between the cumulative total value of five units and that of four units (2,980 - 2,560). **The unit value, then, is always the difference in cumulative** ~~totals which one unit makes.~~ Expressing the same thing in formula, we have:

$$Y_x = A[x^{b+1} - (x-1)^{b+1}]$$

where:

x^{b+1} = Cum total value for all units through the last unit considered.

$(x-1)^{b+1}$ = the cum total value for all units through the last unit considered less one.

Y_x = Unit value for a specific unit.

This formula will enable us to find the hours required for any single unit by simple calculation. Suppose we wish to know the Northrop unit value for the 100th unit with an 80% curve and a first unit value of 1000 hours.

We proceed by calculating the total value for 100 units. It is 22.706166; and the cum total for 99 units; it is 22.551953. Then by formula, we have 1000[22.706166 - 22.551953] = 154.2, the unit value for the 100th unit. Note also that this value assumes a T_1 value of 1000 hours.

The editing procedure for analyzing data when using the Cum Average Theory differs from the Unit Theory. With the Unit Theory we were concerned with lot mid-points and an associated average cost for each lot. With the Cumulative Average Theory, we are concerned with cumulative totals--cumulative totals for sequential units and cumulative averages for sequential lot values. Hence, our worksheet for the Cumulative Average Theory is unique to the cum average theory. This new worksheet construction is summarized in Table 3-3 for the Northrop construction.

EDITING LOT DATA FOR THE NORTHROP CONSTRUCTION

A WORKSHEET

LOT DATA

(1) LOT NO	(2) LOT SIZE	(3) LOT VALUE	(4) CT	(X) (5) CU	(Y) (6) AUC

EDITING RULES:

STEP 1 Set up worksheet with six columns labeled as above

STEP 2 Enter Lot No, Lot Size, and Lot Values for each and every lot (Column 1-3)

STEP 3 Calculate CT (Column 4)--Lot value plus cumulative lot values through previous lots

STEP 4 Calculate CU (Column 5)--Lot size plus cumulative units through previous lots

STEP 5 Calculate AUC (Column 6)--Cumulative total (CT) divided by cumulative units (CU)

STEP 6 Plot columns 5 and 6 on log-log graph paper

TABLE 3-3

EXAMPLE:

The data in Table 3-1 is reproduced below. We will apply the editing rules from Table 3-3 to forecast the value of Lot 7:

LOT NO	LOT SIZE	LOT VALUE
1	7	2030
2	15	1578
3	24	1544
4	30	1460
5	40	1508
6	40	1240
7	50	To be estimated

Our completed worksheet appears below:

(1) LOT NO	(2) LOT SIZE	(3) LOT VALUE	(4) CT	(X) (5) CU	(Y) (6) AUC
1	7	2030	2030	7	290
2	15	1578	3608	22	164
3	24	1544	5152	46	112
4	30	1460	6612	76	87
5	40	1508	8120	116	70
6	40	1240	9360	156	60
7	50	To be Estimated		206	

TABLE 3-4

Step 6 would have us plot the X and Y values in columns 5 and 6. We could then fit a line through the plot points and make predictions about future cumulative average costs. Note that with the Northrop Construction, when we read the Y-value which corresponds to our "X" of 206, in lot 7, we are reading the cumulative average cost of 206 units. With the Northrop Construction, we are plotting cumulative totals or lot end points rather then lot mid-points as in the Boeing Construction. "This difference is important to recognize in arriving at an estimate of the lot hours required" for lot 7 with 50 units. To estimate lot 7 graphically, we must first calculate the cumulative total of 206 units by multiplying the cumulative average hours for 206 units by 206 units; i.e., where 'X' is 206, 'Y' is approximately 52; cumulative total then is 206 * 52 = 10,712 cumulative total hours. Next we must read the 'Y' value where 'X' is 156 and multiply this value by 156 units to determine the cumulative total of 156 units. This value is approximately 9360 hours. Finally, we can determine the hours for lot 7 by subtracting the cumulative total for 156 units from the cumulative total for 206 units; or 10,712 - 9360 = 1352 hours to build the 50 units in lot 7.

As with Boeing Construction, we can estimate the hours required for lot 7 with a formula given the slope and first unit value (T_1). These are determined using the same procedure as we used with the Boeing Construction. In this example, the T_1 is approximately 800 hours and the slope is 70%. (See Figure 3-2). Our formula would be:

$$TC_{F,L} = A[L^{b+1} - (F-1)^{b+1}]$$

then

$$TC = 800\ [206^{b+1} - (157-1)^{b+1}]$$

The Boeing Improvement Curve Tables are not used with the Northrop Construction. Instead, we calculate 'b+1' as follows:

STEP 1 Calculate the 'b' value for a cost improvement curve slope of 70% $b = \frac{\log .70}{\log 2}$

$$b = -.5145732$$

STEP 2 Calculate the 'b+1' value by adding 1 to the 'b' value calculated in Step 2.

$$\begin{array}{rcr} b & = & -.5145732 \\ & + & \underline{1.0000000} \\ b+1 & = & +\ 0.4854268 \end{array}$$

Using the 'b+1' value in our formula, we calculate lot 7 as follows:

$$TC = 800[13.280463 - 11.603834]$$

$$TC = 1341 \text{ hours}$$

Our estimate using the formula differs slightly from our graphical estimate. As with the Boeing Construction, this difference is due to lack of precision in reading graph values and/or rounding the slope to two decimal places.

ANALYZING MAJOR PROGRAM CHANGES USING THE NORTHROP CONSTRUCTION

The logic used for analyzing changes for the Northrop Construction is the same as we used for the Boeing Construction. Thus, there are three types of changes that can occur.

1. **The addition of a component or components**
   or
2. **The deletion of a component or components**
   or
3. **The substitution of one component for another component**

THE CUMULATIVE AVERAGE THEORY AND CHANGES

ADDITIONS. Handling an addition is exactly the same as the procedure using the unit theory. For instance, suppose a manufacturer has a process in operation and the following data reflects the production experience:

Value of 1st unit = 2500

Units produced to date = 65

Slope = 85%

Beginning with unit 66, an additional component is being added which will increase the sensitivity of the unit. The engineering estimate is that the additional component will require 600 additional hours for the first unit. Estimate units 66-85.

GRAPHIC SOLUTION. See Figure 3-3. From the graph we have Basic unit:

CT through 85 units. . . . . .895 * 85 = 76075

CT through 65 units. . . . . .947 * 65 = 61555

Cost of basic units 66-85. . . . . = 14520

Cost of Additional Units 1-20. . .304 * 20 = 6080

Total cost of units 66-85. . . . . = 20600

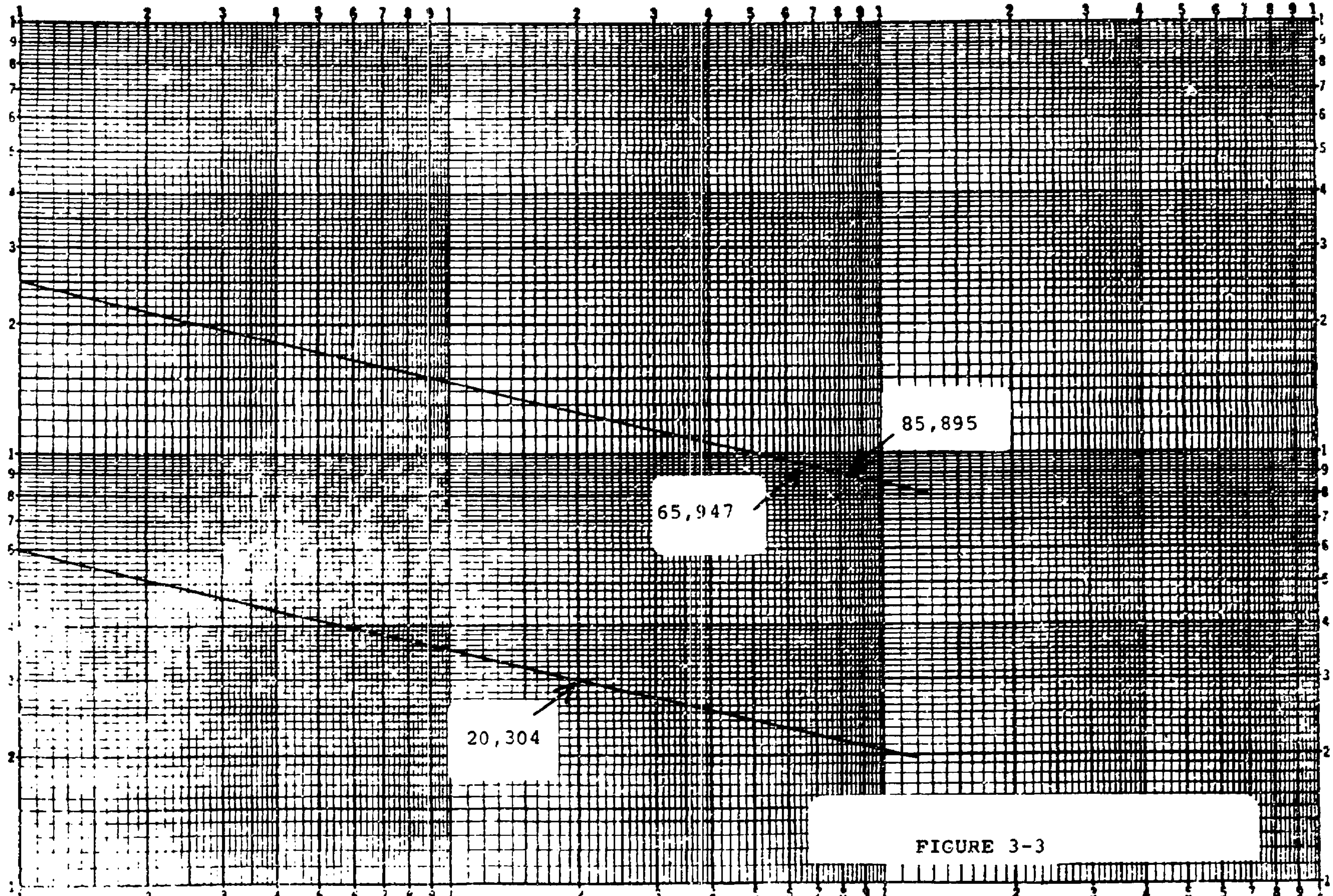


FIGURE 3-3

FORMULA SOLUTION.

Basic unit + New Component = TC

$A[85^{b+1} - 65^{b+1}] + A_+ [20^{b+1} - 0] = TC$

$2500 [29.994205 - 24.425765] + 600[9.90794] = TC$

$2500[5.56844] + 5944.764 = TC$

$13921.1 + 5944.764 = TC$

$TC = 19865.864$

DELETIONS

When estimating the impact of a deletion using the cumulative average theory, we must be very careful how we measure the amount of the deletion. If the deletion is measured as a change in the cumulative average value starting with the first unit affected, we would simply shift the curve downward by the amount of the deletion. Suppose, for example, that we have a 75% cumulative average cost improvement curve with a first unit value of 10,000 and a deletion representing 30% of the cumulative average occurred at unit 31. Our approach would be to drop the cost improvement curve line by 30% and proceed to estimate off the new line. (See Figure 3-4). To estimate units 31 through 60 we would read the $CT_{60} = 1320 \times 60 = 79200$ and subtract from that $CT_{30} = 1750 \times 30 = 52500$. The estimated cost of units 31 through 60 would be 26,700 labor hours.

FORMULA SOLUTION:

$(A_-) = .70(A) = .70(10,000) = 7000$

$(A_-)[60^{b+1} - 30^{b+1}] = TC$

$(7000)[10.9686 - 7.31241] = TC$

$7000[3.65619] = 25593.33$

If the deletion is expressed in terms of its impact on the last unit produced before the change, the procedure for estimating future lots under the cumulative average theory becomes more difficult. We are reading cumulative average values from our line, but the deletion is expressed as a unit value. Therefore, we need to find the location of a new cumulative average line with a corresponding unit curve that would give us our projected unit value with the deletion.

Suppose we had a first unit value of 5000 labor hours and a cumulative average cost improvement curve of 82%. (See Figure 3-5). Beginning with unit 31, there is to be a deletion of 300

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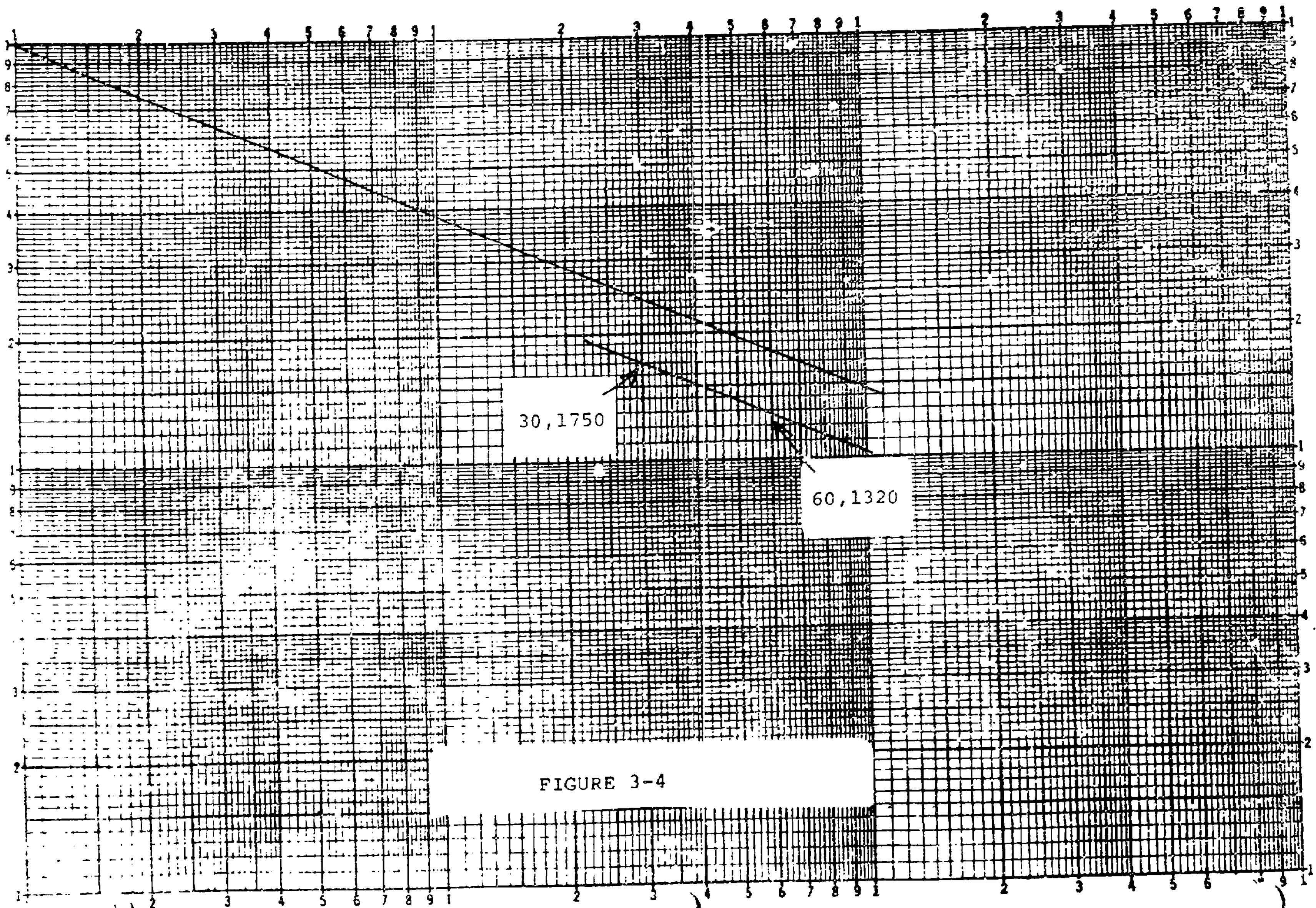

FIGURE 3-4

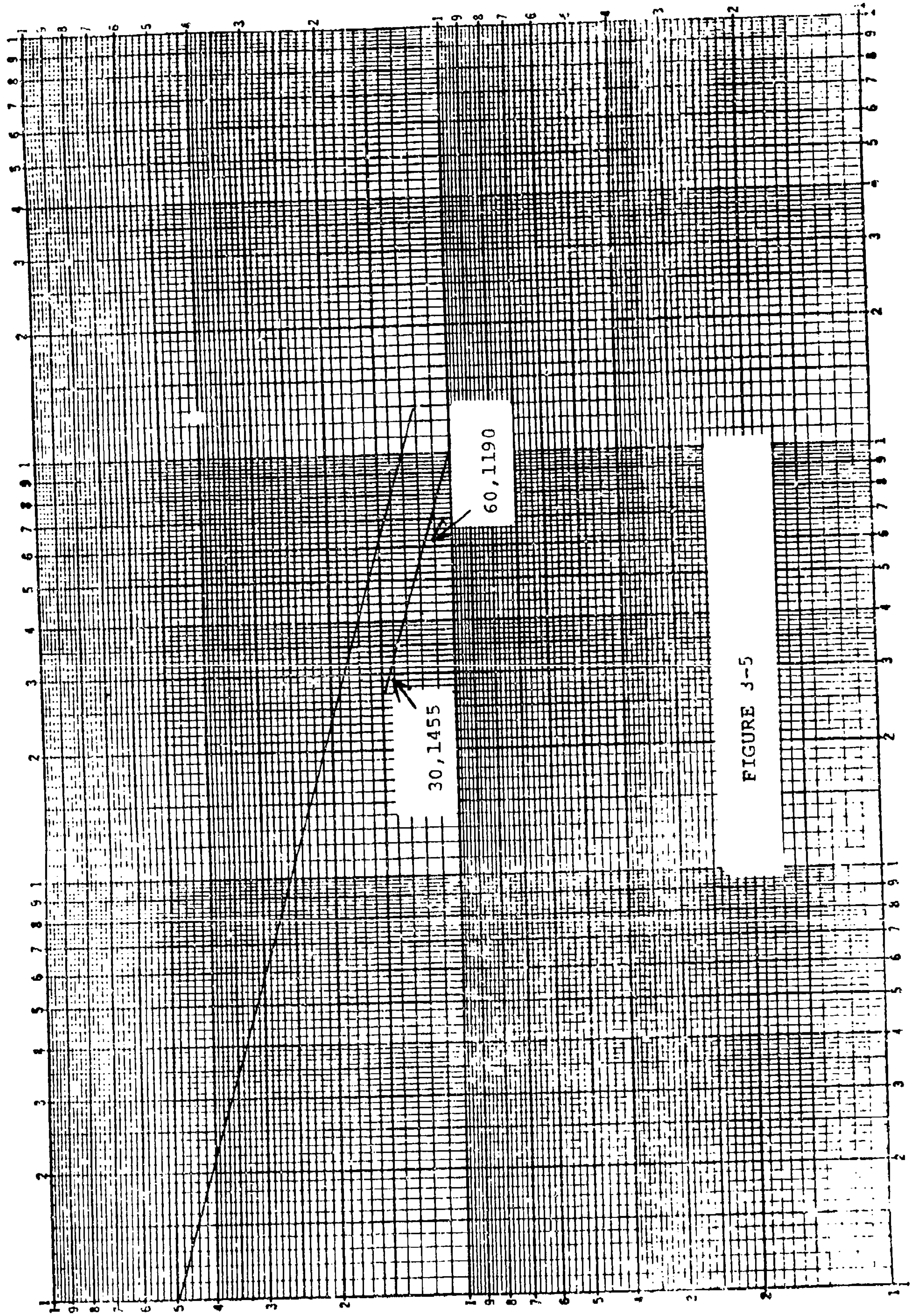

FIGURE 3-5

labor hours. How would we go about projecting the number of hours required for lot 31-60? We cannot merely drop our cumulative average line by 300 hours because this deletion is expressed in unit hours, not cumulative average hours. Therefore, we must find the unit value that corresponds to the cumulative average value at unit 31. From our graph, we can determine thecumulative average values at unit 30 and unit 31, from which we can calculate the number of labor hours to produce unit 31 as follows:

CT_{31}	(1880)(31)	= 58,280
CT_{30}	(1900)(30)	= 57,000
Unit 31		1,280

Based upon the labor hours required to produce unit 31, we can see that the 300 labor hour deletion represents 23.4375% (300/1280) of the unit labor hours. **Therefore, our cumulative average curve needs to be dropped 23.4375% to trace the impact of the deletion on future units of production.**

Based on our new curve, we can read the cumulative average values of unit 30 and unit 60 and calculate the labor hours required to produce lot 31-60, as follows:

$(CA_{60})(60)$	(1190)(60)	= 71,400
$(CA_{30})(30)$	(1455)(30)	= 43,650
Lot 31-60		27,750

FORMULA SOLUTION:

Labor hours to produce unit 31:

$$A\ [31^{b+1} - 30^{b+1}] = Y_{31}$$

$$(5000)[11.597875] - (11.32962)] = Y_{31}$$

$$(5000)(.268255) = 1341.275$$

The % deletion at unit 31 equals:

$$(300/1341.275) \times 100 = 22.366778$$

Adjusting our first unit value for the deletion:

$$A_{\_} = A(1 - .2236677)$$

$$A_{\_} = (5000)(.7763323) = 3881.6615$$

Using our adjusted first unit value, we can calculate the labor hours required to produce lot 31-60:

$$A_{-}\,[60^{b+1} - 30^{b+1}] = TC$$

$$(3881.6615)[(18.58056) - (11.32962)] = TC$$

$$(3881.6615)(7.25094) = 28{,}145.694$$

The difference between the two estimates (approximately 1.4%) is due to errors in plotting and in reading values from the graph.

In evaluating the effect of a deletion when labor hours are following a cumulative average cost improvement curve, three situations can prevail:

(1) If the deletion can be estimated in terms of its impact on the cumulative average labor hours at the unit affected, the adjustments necessary to locate the new cost improvement curve are very similar to the procedure followed under the unit cost improvement curve construction. This is the simplest case. However, it is also the most difficult estimate to derive.

(2) If the deletion can be estimated in terms of a percentage change in the total labor hours for the unit affected, the adjusted cumulative average cost improvement curve can be located by applying the same percentage change. This is the second simplest case.

(3) The most complicated case, and probably the most likely, is when a deletion is estimated as a reduction in labor hours for the first unit affected by the change. When this occurs, the hourly reductions must be converted to a percentage reduction in terms of unit values, and then the cumulative average cost improvement curve must be adjusted by the percentage change.

<u>SUBSTITUTIONS:</u> As before, a substitution is merely the joint effort of a deletion and an addition.

Example:

Theoretical value of first unit = 10,000

Slope (cumulative average) = 80%

Units produced to date = 70

Substitution effective at unit 71:

(a) Deleted effort 500 hours
(b) Added effort 600 hours

Estimate the cost of 71-90:

GRAPHIC SOLUTION:

See Figure 3-6. From the graphs we would estimate t deleted portion of the substitute.

(a) The cost of unit 71:

$(CA_{71})(71) - (CA_{70})(70) = Y_{71}$

$(2490)(71) - (2500)(70) = Y_{71}$

$176790 - 175000 = 1790$

(b) The percent reduction in unit 71:

$500/1790 - 27.932968$

(c) The adjusted first unit value:

$A_{-} = A(1 - .2793296)$

$A_{-} = 10,000\ (.7206704) = 7206.704$

(d) The cost of lot 71-90 with the deletion:

$(CA_{90})(90) - (CA_{70})(70) = TC$

$(1650)(90) - (1800)(70) = TC$

$148500 - 126000 = 22,500$

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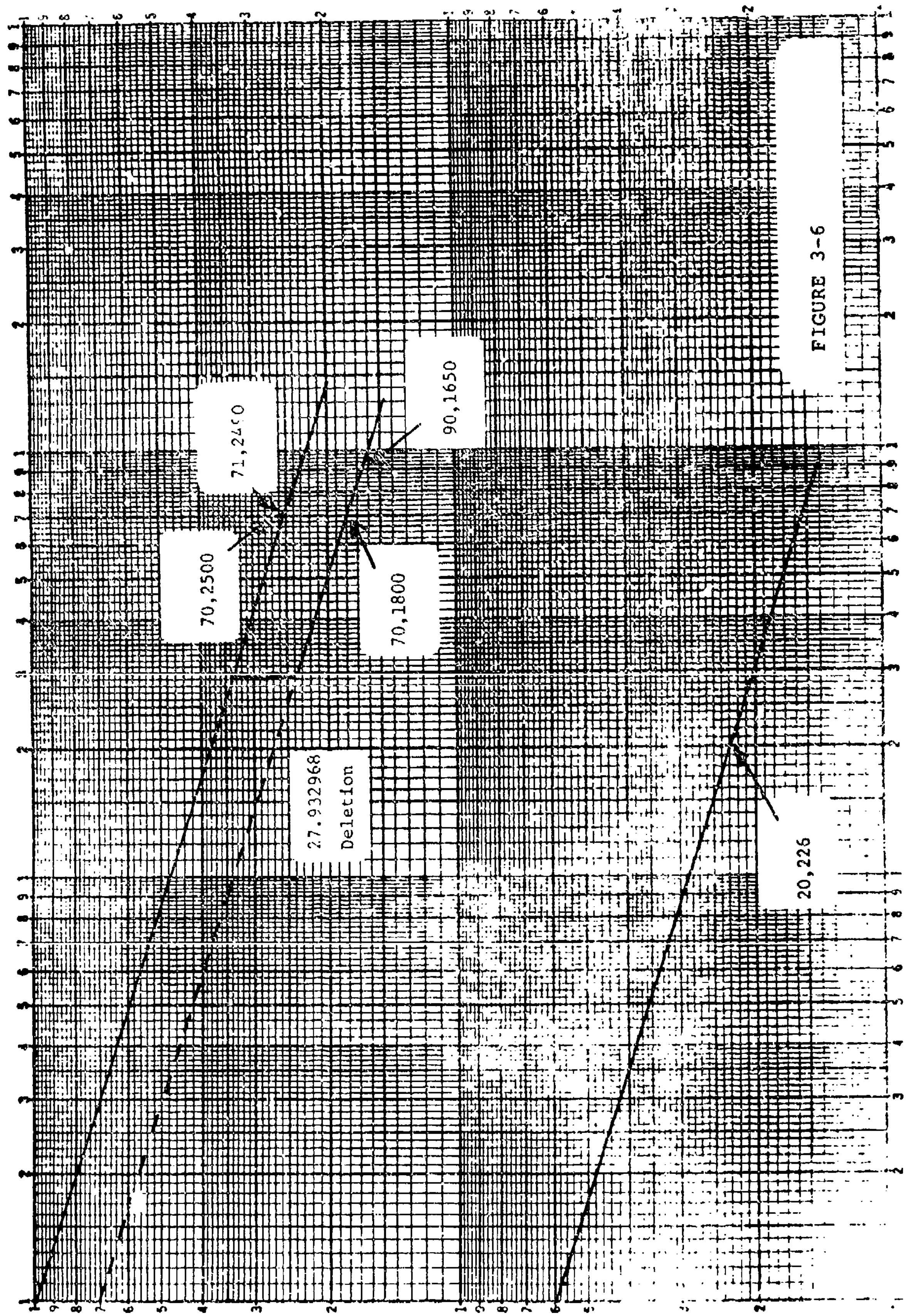


FIGURE 3-6

Plus the added portion of the substitute:

$$(CA_{20})(20) = (226)(20) = 4520$$

and add the two together for our total lot estimate:

Lot 71-90 with deletion		22,500
Lot 1-20 of the addition	+	4,520
Lot 71-90 with the substitution:		27,020

FORMULA SOLUTION:

(a) The cost of unit 71:

$(10,000)[71^{b+1} - 70^{b+1})] = Y_{71}$

$(10,000)[(18.000559) - (17.8283)] = Y_{71}$

$(10,000)\ (0.172259) = 1722.59$

(b) The percentage reduction in unit 71:

$500/1722.59 = 29.026058\%$

(c) The adjusted first unit value:

$A_- = A(1- .2902605)$

$A_- = (10,000)(.7097395) = 7097.395$

(d) The cost of lot 71-90 with the deletion:

$A_-[(90^{b+1}) - (70^{b+1})] = TC$

$(7097.395)[(21.14055) - (17.8283)] = TC$

$(7097.395)(3.31225) = 23,508.346$

(e) The cost of the addition:

$A_+[(20^{b+1})] = TC$

$(600)(7.62416) = 4574.496$

The estimate for the substitution using the formulae is:

Lot 71-90 with the deletion	23,508.346
Lot 1-20 of the addition	+4,574.496
Lot 71-90 with the substitution	28,082.842

The difference between the two solutions (approximately 3.8%) is due to plotting and reading values of the graph. This error is especially prevalent when cumulative average cost improvement curves are used because the graphic solution requires that you read the cumulative average values at two points which are only one unit apart.

IDENTIFYING CHANGES

The preceding discussion of changes has covered the fundamental approaches an analyst would use in assessing and predicting their impact on the cost improvement curve. However, there may be situations where previous production history is provided which includes changes. The analyst may well be tempted to simply plot the data and fit a line through the data. Such an approach had some rather serious implications. For instance, the following data has been plotted using the cumulative average theory.

LOT	UNITS IN LOT	LOT COST	CUM AVG
1	10	4765	476.5
2	10	2859	381.2
3	10	2413	334.6
4	15	4117	314.5
5	15	3411	267.6

In Figure 3-7a, the analyst fits a line through the data trying to equalize the distance above and below the line. The approximate results are an A = 875 and a slope = 82%. In Figure 3-7b, the analyst recognizes that the cumulative average line should be a smooth line and upon inspection, decides to fit the line through the first three points. The last two points do not make sense--something appears to have happened in lot 4. Or perhaps the curve is not a cumulative average curve. Figure 3-8 shows the same data plotted for both theories (x's represent unit theory and •'s represent cumulative average theory). The last two unit points dramatize the anomalies in lots 4 and 5. The fact that both curves show an aberration strongly suggests that some type of change has occurred. The point thus far is that the cumulative average curve tends to <u>hide changes</u> because of the averaging process. Thus, regardless of what theory is presumed, a good analyst will always plot both theories and inspect both curves to determine if there are or were any problems. A question raised previously was whether the data was cumulative average or not. Take another look at Figure 3-8. Placing a straight edge against the first three points of each theory is inconclusive--both look straight! Recall that when this problem was introduced it was presumed to be cumulative average. If this presumption is based upon the past experience of the producer for similar type items it should not be lightly discarded. This point is an important one--an analyst, whenever possible, should always analyze past experience to assist in making the correct judgment. Thus, in this problem there remains a strong

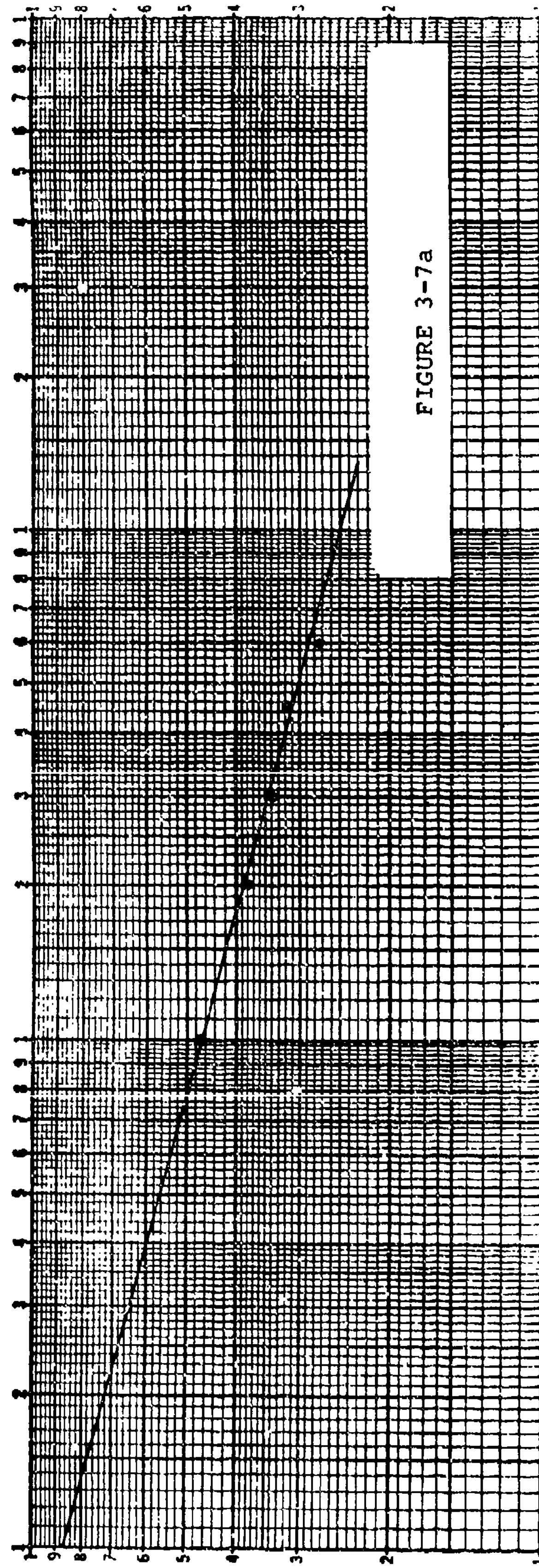


FIGURE 3-7a

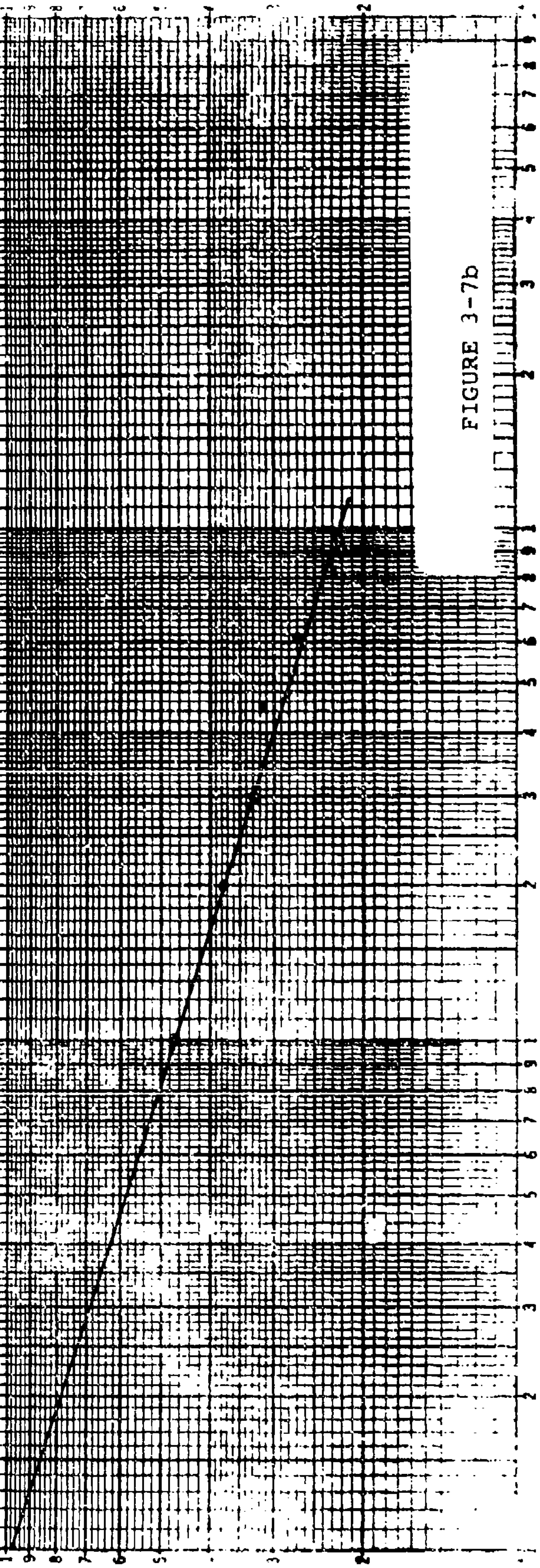
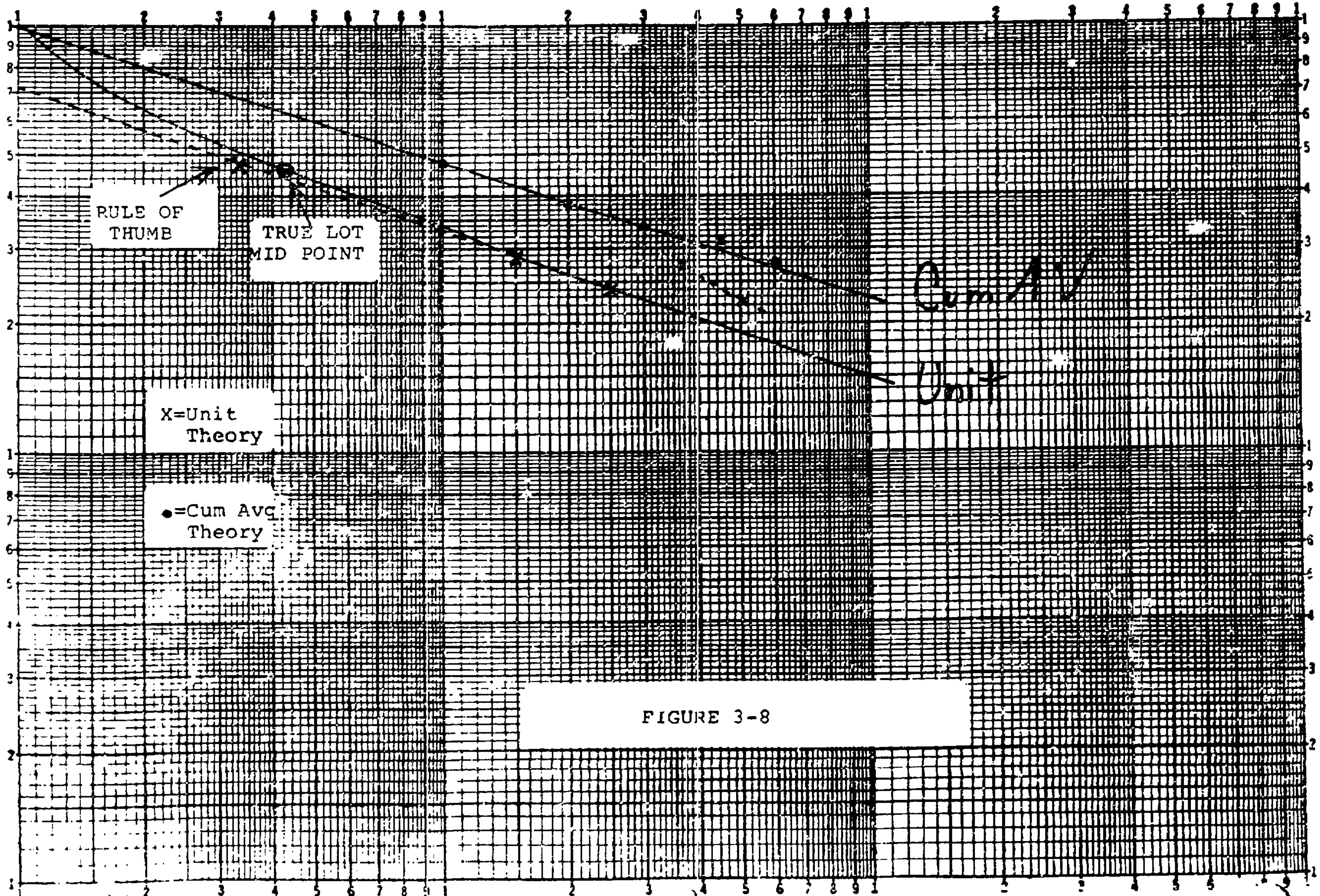


FIGURE 3-7b


RULE OF THUMB
TRUE LOT MID POINT
Cum AV
Unit
X=Unit Theory
●=Cum Avg Theory
FIGURE 3-8

suggestion that the curve is cumulative average. I the curve is cumulative average, then the analyst would expect the unit curve to bend upward moving from right to left--this upward bend should be most pronounced between units 1 and 30. Is there an error in plotting the unit data? Yes. The first data point was plotted using the rule-of-thumb lot midpoint. The true-lot mid-point is 4.17 and is shown in Figure 3-8 as an x. Using the true-lot mid-point and the second and third points on the unit curve, the curve appears to bend upward toward the "y" axis. **This curvature combined with past history, although not totally conclusive, appears sufficient to substantiate the cumulative average theory.** Our attention now turns to trying to determine the nature and magnitude of the change. First, the behavior in lots 4 and 5 indicates the change is either an addition or a substitution. The easiest way to find out is to ask the producer. It would seem that even if the producer cannot recall the magnitude the producer could, as a minimum, determine whether the change was an addition or a substitution. If the change were an addition, its magnitude can be approximated. For the moment, the assumption will be that the change was an addition. To approximate the magnitude, we must assess what we know and what our logic tells us. We have:

- -- cumulative average theory
- -- theoretical value of A = 1000
- -- slope = 80%
- -- an addition which took place with lot 4

also, in the absence of information to the contrary, we will assume:

- -- same rate of learning on the addition
- -- the addition took effect with the first unit of lot 4

If there had been no addition the estimated cost of the lot would have been:

$$TC = 100[(45^{b+1}) - (30^{b+1})]$$

$$TC = 1000[(13.121855) - (10.03677)]$$

$$TC = 1000(3.176085)$$

$$TC = 3176.085$$

However, the actual cost of the lot was 4117. Or a difference of 941 labor hours which represents the total impact of the first 15 units of an addition. To estimate the cost of a first lot of 15 units we would use:

$$\text{Total lot cost} = A_+[(15^{b+1} - (0^{b1})]$$

by substitution we have:

$$941 = A+[6.272985]$$

$$941/6.272985 = A_+$$

$$150 = A_+$$

Therefore, we have an addition which took place starting with unit 31 with a first unit value of 150. The logic applied to this problem is equally applicable to the unit theory.

When the producer states that the aberration or anomaly is due to substitution, the analyst is confronted with a more complicated dilemma because of the simultaneous impact of a deletion and an addition. A substitution can only be analyzed when two data points are given, i.e., two lots in which the substitution was effective or two unit values in a lot. Take the following example:

LOT	LOT SIZE	LOT COST
1	10	8315
2	10	4170
3	10	3535
4	10	3488
5	10	3052

Slope = 80%
$A\ (T_1)$ = 1000
Theory = unit

ASSUMPTIONS:

-- The substitution would take effect with the first unit of the lot

-- The slope for the substitution would remain the same

The above data has been plotted and is shown in Figure 3-9. Lots 4 and 5 show an increase and since we are assuming the producer has told us there was a substitution we can conclude that the addition portion had more impact than the deletion. The task now becomes finding A_+ and A_-. If A_+ and A_- were known, the estimates of lots 4 and 5 would be:

$$\text{Lot } 4 = A_-\ (40^{b+1} - 30^{b+1}) + A_+(10^{b+1} - 0^{b+1}) = 3488$$
$$\text{Lot } 5 = A_-\ (50^{b+1} - 40^{b+1}) + A_+(20^{b+1} - 10^{b+1}) = 3052$$

84

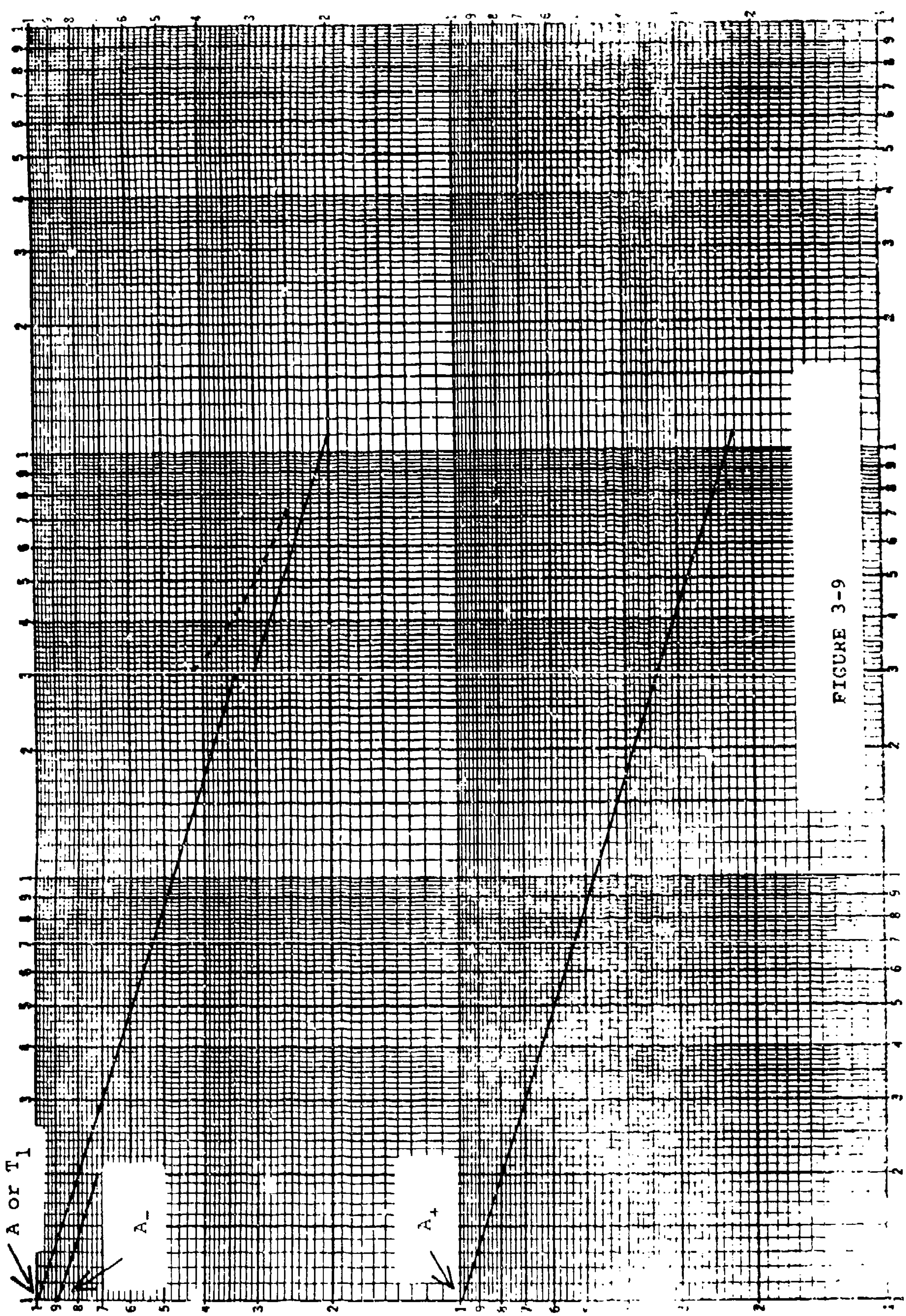


FIGURE 3-9

by substitution:

Lot 4 A_- (17.193456 - 14.019893) + A_+(6.315373 - 0) = 3488
Lot 5 A_- (29.121704 - 17.193456) + A_+(10.484943 - 6.315373)
TC of Lot 5 = 3052

and

Lot 4 A_- (3.173563) + A_+(6.315373) = 3488

Lot 5 A_- (2.928258) + A_+(4.16957) = 3052

multiplying Lot 4 by 2.928258 and

Lot 5 by -3.173563

we have:

$$A_-(9.293011243) + A_+(18.49304151) = 10213.763900$$

$$A_-(9.293011234) + A_+(13.23239308) = -9685.714276$$

$$A_+(5.260648432) = 528.049624$$

$$A_+ = 100.3772882$$

substituting into Lot 4:

$$A_-(3.173563) + 100.3772882(6.315373) = 3488$$

$$A_-(3.173563) + 633.920016 = 3488$$

$$A_-(3.173563) = 2854.079984$$

$$A_- = 899.3298649$$

This is very close to the actual change which had a 10% deletion (A_- = 900) and an addition of 100 hours (A_+) for the first unit. Rounding in the lot total cost values caused the slight error.

The logic discussed here applies to either cost improvement curve theory provided that two data points are known.

PROBLEMS

1. The following data was obtained for six operators, each of whom repeated an identical task ten times and with each starting from a completely inexperienced stage. They worked independently and were given a minimum of instruction.

OPERATOR (Time in Minutes)

UNIT	A	B	C	D	E	F	GROUP UNIT COST	GROUP CUM TOTAL	GROUP CUM AVG
1	12.3	13.3	14.6	14.2	9.9	15.7	80.0	80.0	80.0
2	10.8	4.5	3.2	11.2	4.4	2.8	36.9	116.9	58.4
3	3.0	3.5	3.0	6.8	4.0	1.5	21.8	138.7	46.2
4	3.2	2.1	2.5	4.1	5.3	2.5	19.7	158.4	39.6
5	2.1	3.4	1.5	2.2	3.1	1.8	14.1	172.5	34.5
6	2.1	2.5	1.7	2.0	2.4	1.7	12.4	184.9	30.8
7	2.9	3.8	1.9	2.0	2.9	1.8	15.3	200.2	28.6
8	1.8	2.5	2.0	1.5	1.4	1.7	10.9	211.1	26.3
9	1.5	3.8	1.9	1.7	2.3	1.5	12.7	223.8	24.8
10	3.1	2.1	1.8	1.4	2.5	1.3	12.2	236.0	23.6

Plot the group unit cost curve and the group cum average cost curve. Which basic construction of the curve is indicated? Why? Where would you measure slope of curve? What is the slope?

2. Plot the cum average and the unit curves for Operator E and for Operator F. Which basic construction is evident for each?

3. Does the fact that one operator started at a higher unit number 1 value as compared to the other change the basic construction pattern?

4. When the six operators are taken as a group, what prediction would you make for the time it would take to do unit 20 if they continued their operations from 10 without interruption? What prediction for Operator E? For Operator F?

5. Plot the following values for given units to construct both the unit and CA curve: (Note: the values are given for every 5th unit, but the values for units between have been used to obtain the CT).

UNIT	UNIT VALUE	CT	CA
1	370	370	370
5	235	1,438	287
10	205	2,512	251
15	186	3,435	229
20	160	4,290	214
25	153	5,078	203
30	140	5,818	194
35	147	6,542	187
40	126	7,205	180
45	130	7,880	175
50	128	8,523	170

What basic construction of the cost improvement curve do these data follow?

6. In problem 5, does the line of best fit (by inspection) have the same unit number 1 value as the stated actual? By using the Boeing Improvement Curve Tables, find the value for the 20th unit. Is this value different from the stated value for the 20th unit? Predict the value for the 70th unit.

7. By using the following tabulation, find the predicted hours required for a follow-on of 48 units, assuming no interruption in production. Can you use the Tables for your answer?

LOT Nr	LOT SIZE	TOTAL LABOR HRS FOR LOT	LOT AVG HOURS	CT HOURS	CA HOURS
1	12	4,080	340	4,080	340.0
2	20	5,600	280	9,680	302.5
3	28	6,300	225	15,980	266.3
4	36	7,920	220	23,900	248.9
5	36	7,200	200	31,100	235.6

Did you use the unit or the CA curve to find your answer?

Why?

8. A certain manufacturer had the following labor-hour experience in a production program:

LOT Nr	LOT SIZE	TOTAL LABOR HRS FOR LOT	LOT AVG HOURS	CT HOURS	CA HOURS
1	7	2,030	290.0	2,030	290
2	15	1,578	105.2	3,608	164
3	24	1,544	64.3	5,152	112
4	30	1,460	48.7	6,612	87
5	40	1,508	37.7	8,120	70
6	40	1,240	31.0	9,360	60

With the use of the Tables, predict the labor hours needed for an additional 50 units.

9. If a manufacturer produced a product identical to the one in Problem 8 and had an identifical lot set-up, but the lot average direct labor hours were as follows:

LOT Nr	LOT SIZE	LOT AVERAGE HOURS
1	7	182
2	15	105
3	24	64
4	30	49
5	40	38
6	40	31

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Would you conclude that this manufacturer is more efficient than the one in Problem 8? Why or why not?

In the following problems make at least two estimates, one from graphs and at least one from tables.

1. 150 units will be made
   Experience shows:

   UNIT 1 COST: 1600 LABOR HOURS
   SLOPE: 82%
   CONSTRUCTION: BOEING

   A component equivalent to 5% of current unit cost will be removed effective with unit 151.

   Estimate cost of follow-on lot of 40 units.
   ANSWER: Near 14,000 Hours

2. 150 Units will be made
   Experience shows:

   UNIT 1 COST: 1600 LABOR HOURS
   SLOPE: 82%
   CONSTRUCTION: NORTHROP

   A component that consisted of 5% of original effort will be removed effective with unit 151.

   Estimate cost of follow-on lot of 40 units.
   ANSWER: Near 10,000 Hours

3. It is planned to make 200 units
   Experience shows:

   VALUE OF UNIT 1: 1800 LABOR HOURS
   SLOPE: 78%
   CONSTRUCTION: BOEING

   At unit 130 it is decided to add a part, effective with unit 151.

   Engineering estimates indicate the cost of unit one of the addition will be 225 Hours

   Estimate cost of lot of 50 containing units 151-200.
   ANSWER: Near 18,000 Hours

4. It is planned to make 300 units
   Experience shows:

   VALUE OF UNIT 1: 2400
   SLOPE: 78%
   CONSTRUCTION: NORTHROP

At unit 175 it is decided to add a component effective with unit 201.

Engineers estimate first unit cost will be equivalent to 103.3% of labor hours used in unit 175.

Estimate cost of lot containing units 201-300.
ANSWER: Near 26,000 Hours

5. Plans call for the manufacture of 100 units.
Experience shows:

UNIT 1 COST: 1200 LABOR HOURS
SLOPE: 75%
CONSTRUCTION: BOEING

After unit 74 it is decided to replace a component effective with unit 101 and produce 50 more units.

Engineers estimate:

Replaced part is 5% of current effort (effort on unit 75) First unit cost of replacement will be 180 labor hours

Estimate cost of additional lot 50 units; unit numbers 101-150.
ANSWER: Near 10,500 labor hours.

6. Material costs have been kept in 19X7 dollars. Planned production 200 units.

Experience:

UNIT ONE MATERIAL COST: $3600 (19X7 dollars)
SLOPE: 93%
CONSTRUCTION: NORTHROP

At unit 175 it is decided to substitute for a component and extend production 30 units, to unit 230.

The statistical accounts department tells us the material cost for unit one of the removed component was $360 in 19X7 dollars.

The same department estimates the material for the replacement component will cost $500 in 19X7 dollars.

Estimate the material cost of the follow-on lot of 30 units in 19X7 dollars.
ANSWER: Near $60,000 in 19X7 dollars.

7. The Splat Company has the following experience with an item for the U.S. Army:

LOT Nr	LOT SIZE	LOT VALUE (Labor Hours)
1	15	14367
2	15	10284
3	10	6118
4	20	11209
5	25	14956
6	30	15947

The last lot was finished 30 June 19X7. The Splat Company closes for vacations 1 July to 16 July inclusive. Experience has shown this well planned event does not affect the cost improvement curve.

The Army wants 40 more units but they wish to replace the snifter with a new one. The original snifter was estimated to be 5% of the original effort; its substitute will have a unit one cost of 150 labor hours.

Your job is to estimate the cost of the follow-on lot of 40 units and do it now so a contract can be negotiated before the company resumes production; hence, there will be no interruption of the line.

Lots 5 and 6 are interesting. The Splat Company added a component (at the Army's direction). This component is subcontracted and the subcontractor has an 85% cost improvement curve, Northrop construction. The subcontractor states his first unit cost was 150 labor hours. The Splat Company's engineers estimate that adding the component added the equivalent of 2% of the original effort to their labor hour costs.

Answer: about 22,000 labor hours.

Adjusting our first unit value for the deletion:

$$A_{-} = A(1 - .2236677)$$

$$A_{-} = (5000)(.7763323) = 3881.6615$$

Using our adjusted first unit value, we can calculate the labor hours required to produce lot 31-60:

$$A_{-}\,[60^{b+1} - 30^{b+1}] = TC$$

$$(3881.6615)[(18.58056) - (11.32962)] = TC$$

$$(3881.6615)(7.25094) = 28,145.694$$

The difference between the two estimates (approximately 1.4%) is due to errors in plotting and in reading values from the graph.

CHAPTER IV

NORMALIZING DOLLARS FOR COST IMPROVEMENT CURVE ANALYSIS

When to Use Inflation Indices

Wherever possible, the analyst should always apply the cost improvement curve theory against historical hours. Hours are not influenced by outside economic factors; hence analysis of hours allows the analyst to concentrate on the mechanics of the production process. Occasionally, however, the analyst may find that the history is available in dollar values only. If the analyst wants to use those historical dollars for forecasting purposes, it is extremely critical that the effects of the economic environment be normalized. **To use historical dollar values with the cost improvement curve theory, the analyst must first remove the effects of inflation.**

Inflation must be removed from the historical dollar values before the worksheet can be constructed and before a first unit value (T_1) and slope are determined. This must be accomplished first so that the analysis will reflect only the reduction in labor costs associated with production process. If the cost data were not normalized, the analyst would be plotting not only the reduction in labor costs due to repetition but also the impact of inflation over the time span reflected in the history. The analysis would be meaningless because the analyst would be unable to determine a valid rate of improvement. Therefore, the point of normalizing cost data for the effects of inflation cannot be overemphasized--**NEVER, NEVER, NEVER, apply the cost improvement curve theory to "then year" dollars --ALWAYS work with constant dollars.**

The first step when working with historical cost data (history depicted in dollar values) is to secure an appropriate set of inflation indices for normalizing the cost data. This requires an appreciation of just what inflation is, what the historical cost data you have represents, and identification of the correct set of indices. To do this, you must consult formal guidance.

The availability of formal guidance on inflation indices is fairly extensive. Your organization should have a library of these documents required for your analysis needs. Therefore, the recommended procedure for securing the appropriate formal guidance to analyze your historical cost data is to consult your organization's cost library or technical documents files.

As a initial exposure to the terminology and mechanics of inflation indices, an overview is provided as extracted from DOD 7000.3-G, Chapter 4, and supplemental handbooks (See Bibliography for Sources).

Understanding Inflation Indices

DEFINITIONS

a. **INFLATION:** Inflation is defined as a rise in the general price level of goods and services produced in the economy. Inflation is measured by the rate of rise of some general product-price index in percent per year. The definition involves rising prices for current output. Rising prices for bonds, equity claims (stocks), existing durable goods, and land may accompany inflation but they do not constitute inflation. Also, the price increases must occur across many lines of goods and services. For example, if the price of a particular machine tool is increasing but comprehensive indices, such as the implicit GNP price index, are relatively stable, the increase probably cannot be attributed to inflation. A supply and demand imbalance or declining productivity at the plant or in the industry may be responsible.

(1) The terms inflation and escalation in this text are considered to be synonymous. However, the following distinctions may occasionally be encountered:

(a) Inflation is sometimes used in connection with historical price level changes only (that is, those that have already occurred).

(b) Escalation is then defined as those price level changes that are predicted to occur.

b. INDICES: An index number is a number that expresses the relative relationship between two or more figures, where one of the figures is used as a base. If there is a time series of prices for a particular item, an index is established by dividing each price by the base period price. The single commodity index just described is called a **simple index.** If we combine the simple indices for several commodities into a single summary figure, the result is a **composite index.** In common practice, no distinction is made between simple and composite indices.

(1) **RAW INFLATION INDEX:** A raw inflation index is used to convert constant dollars in one year to constant dollars in another year. Raw inflation indexes are used when all dollars are to be expended in a single year.

(2) **OUTLAY PROFILE (expenditure pattern):** The outlay profile reflects the rate at which dollars are expected to be expended. For example, if budget dollars are expected to be spent over a four-year period, the outlay profile might be 30% in

the first year, 30% in the second year, 20% in the third and fourth years. The outlay profile is used with the raw inflation rate to develop weighted inflation indexes (see definition b. (3) below).

(3) **WEIGHTED INFLATION INDEX:** Weighted inflation indexes reflect the amount of inflation expected to occur over the period in which the dollars will be spent. Weighted inflation indexes combine the raw inflation indexes with the outlay profile to generate the weighted indexes. Weighted indexes are used whenever dollars will be spent over more than a single year.

c. **CURRENT DOLLARS**: Dollars that are current to the year in which the cost is incurred. When incurred costs are stated in current year dollars, the figures given are the actual amounts paid out or owed. When future costs are stated in current dollars, the figures given are the actual amounts that will be or are expected to be paid, including any amount due to future price changes. The word current in current dollars does not refer to the year in which the estimate is made or to any other single year. The terms "current", "then-year", and "escalated dollars" are synonymous.

d. **CONSTANT DOLLARS**: Dollars that are always associated with a given base year (e.g., FY87, constant dollars). The terms "constant", "constant year", and "base year" dollars are synonymous. An estimate is said to be in constant dollars if costs for all work contemplated in each year of a multiyear program are adjusted so that they reflect the average level of prices prevailing in the base year. An average can be calculated from monthly or quarterly data, but the precision is probably not worth the effort. Common practice is to assume the average level of prices to be the prices prevailing at the midpoint of the fiscal year.

(1) The phrase "program base year constant dollars" references the purchasing power year that is held constant, or the program base year. The phrase is redundant unless the program base year is identified in context. For clarity, it is better to use terminology that is self-explanatory such as "constant FY87 dollars."

Source of Standard Inflation Rates and Outlay Profiles.

The Office of Management and Budget (OMB) provides all federal agencies/departments with projected inflation rates. OSD/PA&E publishes in the POM Preparation Instructions (PPI) outlay profiles and inflation rates (both by appropriation) that have been obtained from OASD(C) and are based on the OMB rates. Normally, inflation rates are revised again and issued by OSD(C) during the preparation of the President's Budget. These rates are not directly usable for budgeting purposes. AF/ACCC uses these rates to produce raw inflation index tables which are then weighted using the appropriation outlay profiles to produce weighted inflations index tables. AF/ACCC issues these tables to various Air Staff offices and major commands/special operating agencies for use in all Programming, Planning and Budgeting System (PPBS) documents and for cost analysis purposes.

Separate Indices for All Appropriations. Raw inflation rates and outlay profiles are not the same for all appropriations so separate sets of indices are issued for each. OSD Comptroller also provides separate rates for major items or categories in the industrial and stock funds, in the customer accounts, and for various Elements of Expense Investment Code (EEIC) categories in the Operation and Maintenance (O&M) appropriation (3400). AF/ACCC does not include these separate inflation rates in the raw or weighted inflation index tables; they are received in the Budget Instructions and applied by AF/ACB. The sum of the inflation amounts for each EEIC, however, must equal the amount that would be computed using the aggregate O&M rate.

Exceptions to Use of the Standard Inflation Indices.

In addition to the EEIC rates used in O&M, AFSC/AC and AFLC/AC prepare inflation data sheets for major weapon systems which are updated when OSD rates and outlay profiles are revised and which are approved by AF/ACC prior to distribution to Air Staff PEMs and AF/ACBI. In the past, exemptions were granted based on unique, well-documented contractual arrangements between a program office and a prime contractor, or the U.S. and foreign governments co-producing a weapon system, such as the F-16 aircraft. Currently, the only exemptions being approved are for unique historical rates--not projections of inflation. Documentation for unique historical rates must include:

a. Justification on why OSD rates should not apply to the program.

b. Presentation of the proposed rates and methods used to develop them.

c. Comparison of OSD rates with proposed rates to show the dollar impact of the difference in costs for the approved program.

CONSTRUCTING INFLATION INDICES

1. Calculating A Raw Inflation Index:

a. Designate a base year and assign that year an index of

$$i = 1.00$$

b. Obtain the current table of raw inflation rates and let

r = **inflation rate** from one year to the following year.

c. Compute the raw inflation index (R) using the following formula where n = year of desired index. Therefore,

$$R_n = (i) * (i + r_1) * (i + r_2) * \cdots * (i + r_n)$$

For example, suppose you had the following table of raw inflation rates for the RDT&E appropriation.

FISCAL YEAR	RAW INFLATION RATE, r_n (Percent)
78-79	6.2%
79-80	6.3
80-81	5.8
81-82	5.5
82-83	5.5
83-84	5.5

Then the following computations would be necessary to construct a table of raw inflation indices where FY78 is the base year and percentages are expressed as decimals.

FISCAL YEAR	FORMULA	RAW INFLATION INDEX
78	1.000	1.000
79	1.000 X (1.00 + .062)	1.062
80	1.000 X (1.00 + .062) X (1.00+.063)	1.129
81		1.194
82		1.260
83		1.329
84		1.402

1.000 * (1.00 + .062) * (1.00 + .063)
*(1.00 + .058) * (1.00 + .055) *
1.00 + .055) * (1.00 + .055)

2. Determining the Raw Inflation Rate Using a Table of Raw Inflation Indices.

The following formula is used:

$$\text{Raw Infl ion Rate (n to n+1)} = 100 \; \frac{\text{Raw Inflation Index}_{n+1}}{\text{Raw Inflation Index}_{n}} - 1$$

$$r_{(n \text{ to } n+1)} = 100 \; \frac{R_{n+1}}{(R_n)} - 1$$

For example, the raw inflation rate for '82 to '83 would be computed as follows using the above table:

$$r_{(82 \text{ to } 83)} = 100 \left[\frac{R83}{R_{82}}\right] - 1 = 100\left[\frac{1.329}{1.260}\right] - 1 = 100\ (1.0548 - 1) = 5.5\%$$

3. Calculation of Weighted Inflation Index. The raw inflation index, the outlay profile, and the following formulas are needed:

$$WI_n = 1 \div \left[\frac{OR_i}{r_i} + \frac{OR_{i+1}}{r_{i+1}} + \cdots + \frac{OR_{ik}}{r_{i+k}}\right]$$

Where WI_n = Weighted Index for n^{th} year

OR_i = Outlay Rate for initial year expressed as a decimal

OR_{i+k} = Outlay Rate for last year expressed as a decimal

r_i = Raw inflation index expressed as a decimal with a base of 1.000

r_{i+k} = Raw inflation index for the last year in the outlay profile expressed as a decimal

For example, suppose we had the following raw inflation index and outlay profile.

FISCAL YEAR	AIRCRAFT PROCUREMENT RAW INFLATION INDEX(r_n)	AIRCRAFT PROCUREMENT OUTLAY RATES (OR_i-OR_{i+k})
79	1.000	.10
80	1.062	.40
81	1.121	.30
82	1.182	.12
83	1.246	.05
84	1.313	.03

The weighted index for FY79 therefore would be:

$$WI_{79} = 1 \div \left[\frac{OR_{79}}{r_{79}} + \frac{OR_{80}}{r_{80}} + \frac{OR_{81}}{r_{81}} + \frac{OR_{82}}{r_{82}} + \frac{OR_{83}}{r_{83}} + \frac{OR_{84}}{r_{84}} \right]$$

$$WI_{79} = 1 \div \left[\frac{.10}{1.000} + \frac{.40}{1.062} + \frac{.30}{1.121} + \frac{.12}{1.182} + \frac{.05}{1.246} + \frac{.03}{1.313} \right]$$

$$WI_{79} = 1 \div [.10 + .3766 + .1015 + .0401 + .0228]$$

$$WI_{79} = 1 \div [.9086] = 1.1006 \text{ (or a rate of } 10.06\%)$$

APPLICATION OF INFLATION INDICES

USES OF INFLATION INDICES

1. **Inflation Indices to Used to Extend Program One Year With No Change in Level of Effort.** If you are updating the POM or BES, remember that your program is already priced in then-year dollars for which you always use weighted inflation indices. Therefore, if you need to extend a program one year with the constant dollar amount remaining the same, simply multiply by a ratio of the weighted indices. Use the weighted index table for the base year in which your program is currently priced, and the following formula:

$$\text{Year}_{n+1}\ \text{Then-Year \$} = \left[\frac{\text{Year}_{n+1}\ \text{Weighted Index}}{\text{Year}_{n}\ \text{Weighted Index}}\right] * \left[\text{Year}_{n}\ \text{Then-Yr\$}\right]$$

Suppose FY87 Then-Yr$ = $500 million

FY88 Weighted Index = 1.649

FY87 Weighted Index = 1.563

Then your FY88 Then-Year $ requirement is calculated to be:

$$\text{FY88 Then-Year \$} = \frac{1.649}{1.563} \times \text{\$500 million}$$

FY88 Then-Year $ = $527.5 million

Therefore, to extend your program into FY88 at the current level of effort, you would need $527.5 million; this increase of $27.5 million over FY87 is the amount of funding needed simply for inflation.

2. **Inflation Indices to Use to Price a New Program.** Since you need then-year dollars to update your PPBS documents, use the weighted index table for the base year in which a program was initially funded. Develop the then-year dollar profile by multiplying the constant dollar amount for each year by the weighted index for that year. Use the following formula:

Then-Year $ Amt (Base Year n) = [Constant $ Amt Base Yr] x [Wtd Index for Year of Program (Base Yr n)].

For example, a then-year dollar profile for a new program requiring $100 million per year in Base Year 82 dollars would be computed as follows:

FY	[BASE YR 82 AMT]	x	[WTD INDEX BASE YR 82]	=	THEN YR AMT
1982	$100 MIL		1.108		$110.8 MIL
1983	$100 MIL		1.174		$117.4 MIL
1984	$100 MIL		1.235		$123/5 MIL
1985	$100 MIL		1.294		$129.4 MIL
1986	$100 MIL		1.353		$135.3 MIL
1987	$100 MIL		1.414		$1.414 MIL
1988	$100 MIL		1.477		$1.47.7 MIL

Amounts may vary from one year to the next, but the same formula would still be used. The important point to remember is to **make sure your dollars are all in the same base year.** The following table might be constructed for a program with different levels of effort or program in each year (again all dollars are in the same base year).

FY	[BASE YR 82 AMT]	x	[WTD INDEX BASE YR 82]	=	THEN YR AMT
1982	$ 50 MIL		1.108		$ 55.4 MIL
1983	$ 80 MIL		1.174		$ 93.9 MIL
1984	$100 MIL		1.235		$123.5 MIL
1985	$200 MIL		1.294		$258.8 MIL
1986	$150 MIL		1.353		$203.0 MIL
1987	$ 70 MIL		1.414		$ 99.0 MIL
1988	$ 10 MIL		1.477		$ 14.8 MIL

3. **Revising a Then-Year Dollar Profile for Program Changes.** Programs may be changed during POM reviews (slipped a year, quantities changed, etc.). If your program is priced in then-year dollars, convert back to base year constant dollars, make your necessary program adjustments, and reapply the weighted indices for the base year in which your program is expressed. The formula for converting back to base year constant dollars is as follows:

$$\text{CONSTANT \$ (Base Yr n)} = \frac{\text{Then-Yr \$ (Base Yr n) For Yr of Change}}{\text{Wtd Index (Base Yr n) For Yr of Change}}$$

After calculating your base year constant dollar amounts and making your program adjustments, simply reapply your weighted indices for the base year in which your program was expressed to develop a new then-year dollar profile. For example, suppose a decision was made to change your program from four to five years with a then-year dollar profile and weighted indices in Base Year 82 as follows:

	1981	1982	1983	1984	TOTAL
THEN-YEAR $ PROFILE (Base Yr 81)	104.8	112.8	120.4	127.5	465.5
WTD INDICES (Base Yr 81)	1.048	1.128	1.204	1.275	

The constant dollar amounts are calculated by dividing then-year dollar amounts by weighted indices for each year and yields the following:

	1981	1982	1983	1984	TOTAL
CONSTANT $ PROFILE (Base Yr 81)	$100	$100	$100	$100	$400

Therefore, in constant dollars (Base Yr 81), your program will cost $400 million. But you have been directed to change your program to be completed over a five year period with an equal level of expenditure in each year. In other words, a constant dollar profile of $80 million per year would be needed and you would multiply that amount by the weighted index (Base Year 81) to produce a revised then-year dollar profile for your program as follows:

	1981	1982	1983	1984	1985	TOTAL
CONSTANT $ PROFILE (Base Yr 81)	$80	$80	$80	$80	$80	$400
WTD INDICES (Base Yr 81)	1.048	1.128	1.204	1.275	1.341	
THEN-YR $ PROFILE (Base Yr 81)	$83.8	$90.2	$96.3	$102.0	$107.3	$479.6

Your total then-year program costs are now $479.6 million vice the $465.5 million that was needed before your program was changed. The difference of $14.1 million is the additional inflation your program experiences by stretching out the program an extra year.

PROBLEMS:

APPROVED INFLATION INDICES			PROGRAM COST ESTIMATES	
	COMPOUND (RAW)*	WEIGHTED (COMPOSITE)**	ESTIMATE A (CONSTANT) FY1 $	ESTIMATE B (CONSTANT) FY3 $
FY 1	1.00	1.05	$20.0M	$23.4M
FY 2	1.08	1.14	50.0M	58.8M
FY 3	1.17	1.23	70.0M	82.0M
FY 4	1.26	1.33	60.0M	70.0M
FY 5	1.36	1.43	10.0M	11.7M

*8 percent per annum inflation
** assumes an outlay/expenditure ration of .5, .3, .2

Problem 1: Using the data above, develop both FY1 and then year budgets for Estimate A.

Problem 1 Solution:

	FY 1	FY 2	FY 3	FY 4	FY 5
BASE YR BUDGET	$20M *	$50M *	$70M *	$60M *	$10M *
WTD INDICES (FY 1 = 1.0000)	1.05	1.14	1.23	1.33	1.43
THEN YR BUDGET	$21.0M	$57.0M	$86.1M	$79.8M	$14.3M

Problem 2: Using the data above, develop both constant FY 1 and then-year budgets for estimate B.

Problem 2 Solution:

Step 1: Weighted indices must be rebased to FY 3 by dividing each FY weighted index by the compound raw index for the change year (FY 3)

FY 1 weighted indices---FY 3 compound (1.17)

	FY 1	FY 2	FY 3	FY 4	FY 5
WEIGHTED INDICES	1.05	1.14	1.23	1.33	1.43
FY 3 COMP RAW INDEX	1.17	1.17	1.17	1.17	1.17
WEIGHTED INDICES (FY 3=1.00)	.90	.97	1.05	1.14	1.22

Step 2: Next, calculate then year budget using FY 3 weighted indices. Finally, calculate FY 1 weighted indices.

	FY 1	FY 2	FY 3	FY 4	FY 5
FY 3 CONSTANT BUDGET	$23.4	$58.8	$82.0	$70.0	$11.7
FY 3 WEIGHTED INDEX	.90	.97	1.05	1.14	1.22
THEN-YR BUDGET	$21M	$57.0M	$86.1M	$39.8M	$14.3M
FY 1 WEIGHTED INDEX	1.05	1.14	1.23	1.33	1.43
FY 1 CONSTANT BUDGET	$20M	$50M	$70M	$60M	$10M

Problem 3: Compare your solution to Problem 2 with Estimate A given in the Program Cost Estimates section; why are both Estimate A and Estimate B the same in FY 1 constant dollars?

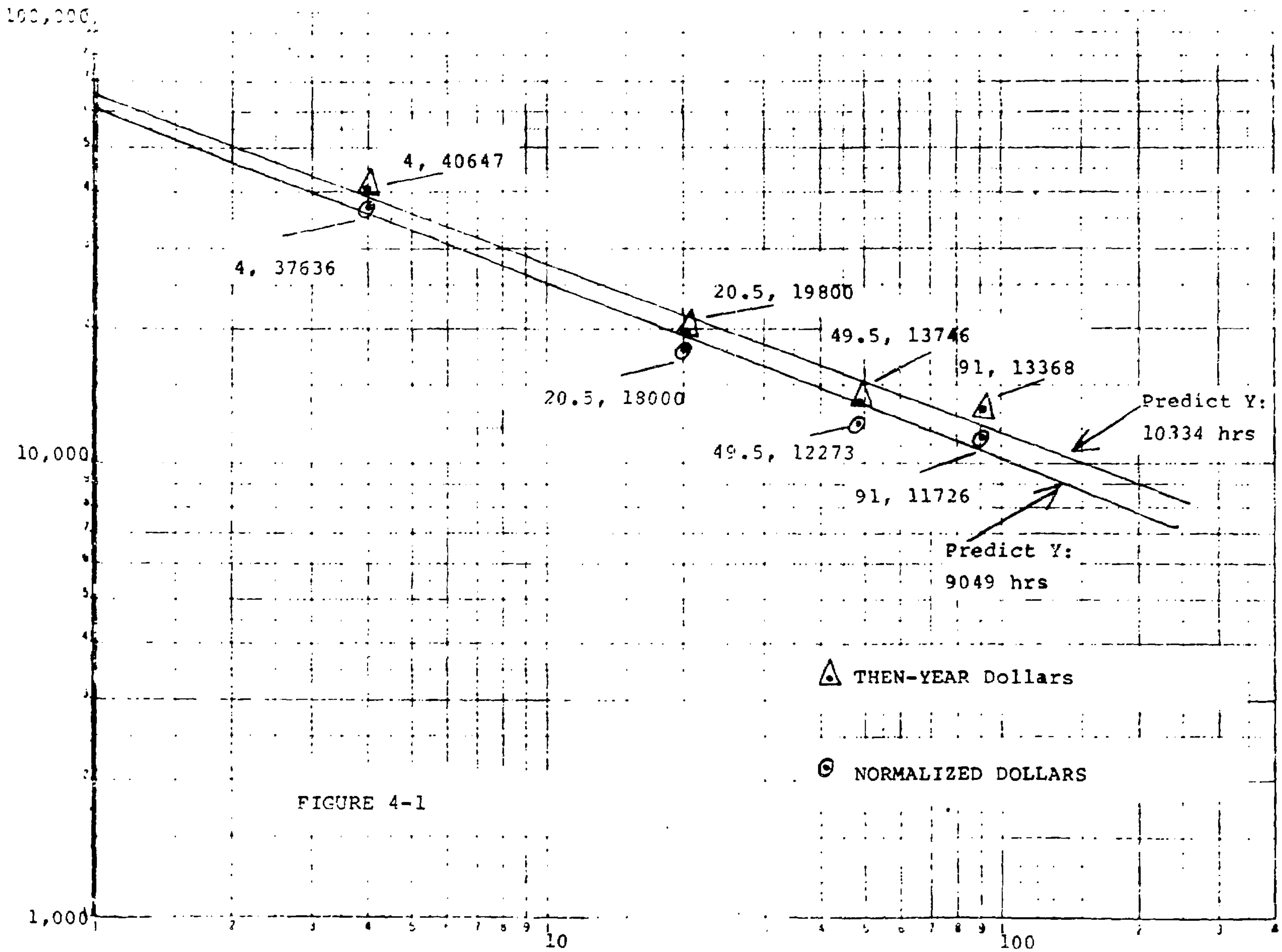
Problem 3 Solution: They are really the SAME estimate!

Problem 4: Using the historical cost data below, construct a cost improvement curve worksheet using the Boeing Construction. Remember, you must first normalize the cost data to remove the effects of inflation!

LOT NO	LOT SIZE	LOT COST($)	WEIGHTED COMPOSITE INDEX NUMBER
1	8	$325,176	108.0%
2	25	495,000	110.0%
3	33	453,618	112.0%
4	50	668,400	114.0%
5	40	To be est'd	115.0%

SOLUTION: Figure 4-1 displays the data as it would appear if it were plotted without normalizing the data and as it should appear with the data normalized for inflation. Notice that your estimated cost of lot 5 for 40 units is approximately $475,364 where the data is NOT normalized and approximately $416,254 where the data is normalized. The estimated cost difference of $59,110 is significant and points up why you should **ALWAYS NORMALIZE DOLLARS FIRST BEFORE FORECASTING COSTS.**

(NOTE: The sum of the normalized deflated cost for lots 1 through 4 is approximately $1.7 million)



100,000
10,000
1,000
10
100
4, 40647
4, 37636
20.5, 19800
20.5, 18000
49.5, 13746
49.5, 12273
91, 13368
91, 11726
Predict Y:
10334 hrs
Predict Y:
9049 hrs
THEN-YEAR Dollars
NORMALIZED DOLLARS

FIGURE 4-1

CHAPTER V

PROBLEMS WITH INTERRUPTIONS IN PRODUCTION SCHEDULES

There are a number of basic assumptions in the model which describes costs as declining exponentially according to a "cost improvement curve". Some of these assumptions are:

1. Constant work force
2. Constant engineering change makeup
3. Unchanged working conditions
4. Uninterrupted production of sequential units

When the production of an end item is interrupted, changes usually occur in the work force, in supervisory personnel, in the tooling and quite often in the support areas of working drawings, blueprints, shop layout and other important areas.

This chapter will deal with some questions of how to handle the extra costs of an interruption in the production line.

There are at least three approaches which have been suggested and which are used.

1. Estimate a cost of interruption, add that to the basic cost and work it out as a major change.

2. Estimate the cost of the first unit produced after the interruption (X_1), find the cost of that unit on the cost improvement curve already established (X_D), find the units of retrogression $X_0 = X_i - X_D$, and renumber all subsequent units as $(X - X_0)$.

3. Proceed as in (2) above but use some device to accelerate ending of the premium, i.e., the first unit after the interruption has a large premium (corresponding to the cost of unit X_D) but the subtracted factor (X_0) tends to diminish and at some subsequent unit the work will proceed almost as though no interruption had occurred.

While each of these three techniques has certain weaknesses, one is inclined to say that any systematic way of treating interruptions in the production process is superior to having no way of treating the added costs.

A SIMPLE TIME DETERMINED PENALTY One way to handle the problem would be to continue the already determined cost improvement curve and treat some percent of the difference between the current position and the cost of unit 1 as a major change. To illustrate this model, 8.33% per month of interruption will be used. **This 8.33% per month has not been scientifically developed;** it simply represents 1/12 of the year. Hence, if the interruption **is a full year, this method says start over at unit one. To use this model, the analyst needs to identify an appropriate penalty percentage.**

The really serious weakness of this approach is determining an appropriate percentage penalty. Additionally, the basic philosophy of the model implies that by applying a flat percent of difference per month, the duration of the interruption, the less serious a given interval is. Table 5-1 illustrates the model using a cost improvement curve with an 85% slope and a unit 1 cost of 1000 labor hours. A 3 month interruption is costed for five succeeding lots of 10 units each. The cost premium paid is shown in terms of percent of cost on original curve. Note that, if the interruption occurs after unit 10, the premium is on the order of 14 percent, after 100 units the premium is on the order of 35 percent. The significance of these premiums is best visualized graphically on arithmetic graph paper (see Figure 5-1). Note that a very small set back early in the program has a very significant cost impact and that a fairly large setback later in the program has a modest cost impact. This is due to the hyperbolic shape of the curve on the arithmetic graph paper. **Ease of calculating the cost of the interruption is the main advantage** of this system, acknowledging that this calculation is dependent upon how good the percentage penalty is and general agreement with the model's basic philosophy. Use of this model should be restricted to those instances where little information is available and ROM-type estimates are acceptable. **If refinement in the estimate is desired, this model should not be used.**

TABLE 5-1

Illustration of Cost of Interruption of Three Months on a Program with Unit One Cost of 1000 Manhours and an 85 percent Slope.

CHANGE AT 10

	ORIGINAL	ADDITION	TOTAL	PREMIUM %
	5286.21	742.21	6028.42	14.04
	4688.38	551.35	5239.73	11.76
	4334.50	489.00	4823.50	11.23
	4087.95	452.09	4540.04	11.06
	3901.15	426.37	4327.52	10.95
TOTAL*	22298.19	2661.02	24959.21	11.93

*LOTS MAY NOT SUM TO TOTAL BECAUSE OF ROUNDING

CHANGE AT 100

	ORIGINAL	ADDITION	TOTAL	PREMIUM %
	3354.78	1174.72	4529.50	35.02
	3284.24	872.65	4156.89	26.57
	3220.87	773.96	3994.83	24.03
	3163.46	715.54	3879.80	22.62
	3111.06	674.84	3785.90	21.69
TOTAL*	16134.42	4211.70	20346.12	26.10

*LOTS MAY NOT SUM TO TOTAL BECAUSEE OF ROUNDING

Costs are shown as they would occur if no interruption took place, the cost added by the interruption for each of five 10 unit lots and for a total of 50 units made after the interruption.

CALCULATING COST OF INTERRUPTION BY RETROGRADE METHOD

In principle this is a simple way of calculating the cost of an interruption. The technique is to estimate the cost of the first unit in the new lot, find that cost on the cost improvement curve established for the porduction run before the interruption. Then simply continue down the same curve renumbering all units. A simple example will show how to do it.

TIME DETERMINED PENALTY MODEL

Break in Production

Units	@Unit 10	@Unit 100
150	65,810	64,100
200	82,786	82,011
250	97,811	98,719

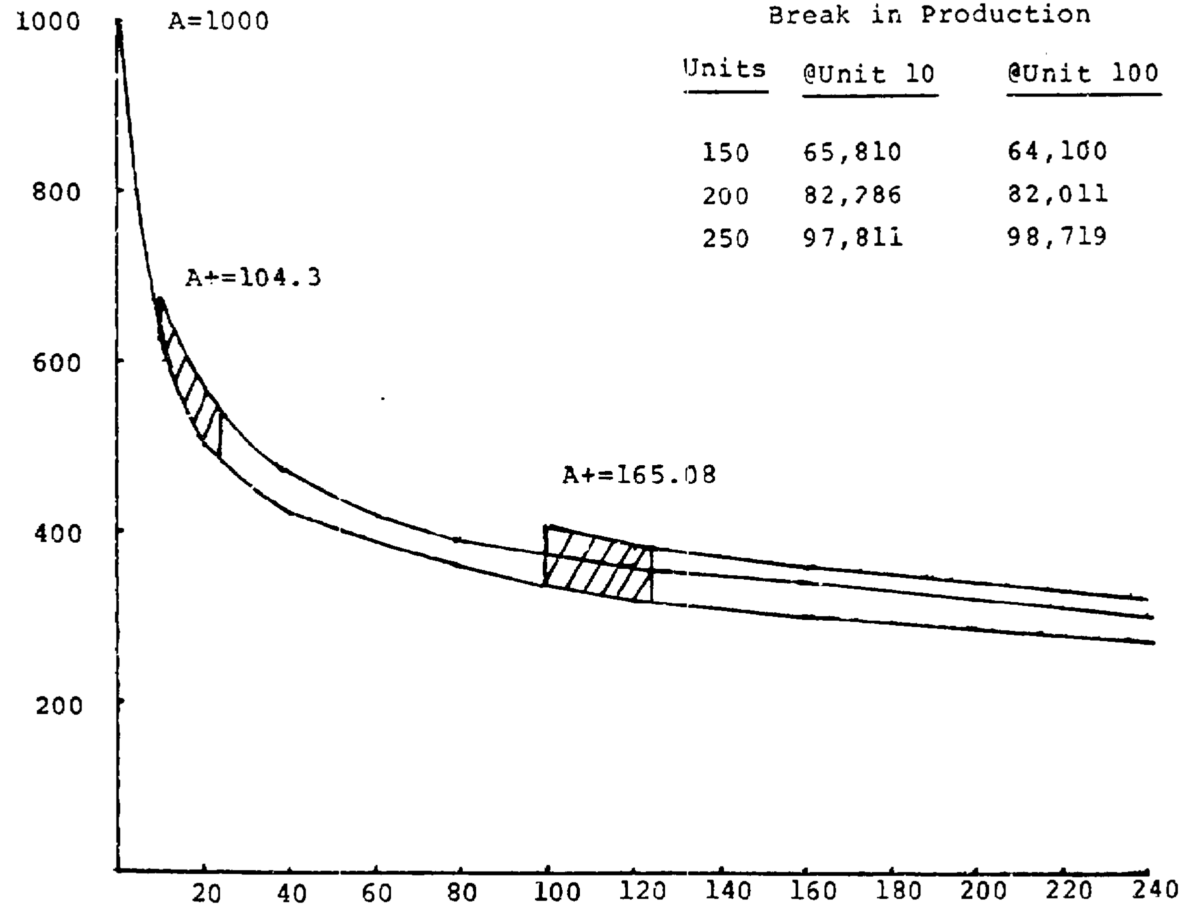


FIGURE 5-1

A firm has experience making an end item. They have produced 200 units, with a unit one cost 1000 labor hours and an 85% slope on the cost improvement curve. An interruption occurs. It is estimated the cost of unit 201 will be 308 labor hours. Unit 151 on the firm's cost improvement curve costs 308 labor hours. 201 - 151 = 50, hence for the production after the interruption all calculations will be based on actual unit number less 50 units. We can estimate the cost of unit 250 to be:

$$1000(\sum x^b{}_{250-50}) = 1000(\sum x^b{}_{200})$$

or, from the tables, 1000(.288728) = 289 labor hours. The total cost of units 201 - 300 will be:

$$1000(\sum x^b{}_{300-50} - \sum x^b{}_{200-50}) =$$

$$1000(\sum x^b{}_{250} - \sum x^b{}_{150}) =$$

$$1000(88.832736 - 59.983900) =$$

$$1000(28.949836) = 28950 \text{ labor hours.}$$

If there had been no interuption, the cost of units 201 to 300 would have been 27,440 labor hours. The interruption added 1,510 labor hours or about 6% to the cost. Figure 5-2 is a graph of this case. Obviously the "true" cost line is approaching the base line, the effect of the interruption is disappearing and the cost premium will be less than two percent at unit 618.

If we had been treating interruptions as a change and if we had estimated the extra cost due to the interruption to be 20 labor hours, we would have figured the cost of units 201-300 as:

$$1000(\sum x^b{}_{300} - \sum x^b{}_{200}) + 20(\sum x^b{}_{100}) = TC$$

$$27442 + 875 = 28318$$

Thus, treating the interruption as a change gives a cost due to interruption as 875 labor hours and retrogressing 50 units on the cost improvement curve gives a cost due to the interruption of 1510. **How you calculate the cost of an interruption does make a difference.**

The major problem with the retrogression method is how to estimate the number of units to retrogress on the cost improvement curve. Cochran (Reference 2) says there are a number of relevant points to consider - time period involved, loss of crew and loss of skill of the remaining crew members and how many units had been produced before the interruption. According to Cochran, judgments regarding the severity and importance of each of these is a matter of experience with the particular firm involved.

Anderlohr (Reference 1), takes a more quantitative approach. Anderlohr weighs five factors in estimating the loss of learning.

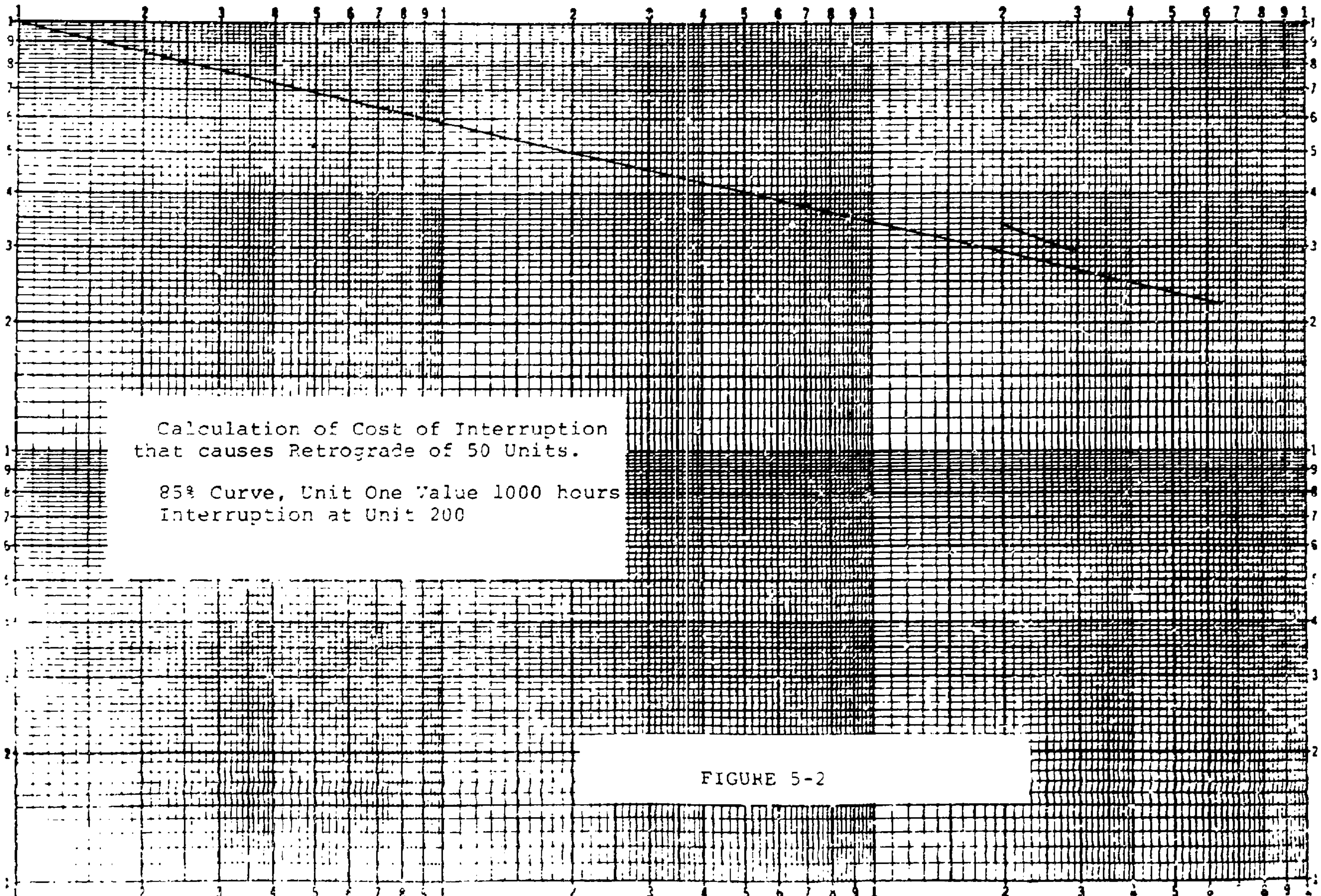

Calculation of Cost of Interruption
that causes Retrograde of 50 Units.
85% Curve, Unit One Value 1000 hours
Interruption at Unit 200

FIGURE 5-2

Anderlohr's factors are:

1. Production line labor (numbers and skill)

2. Supervisory Personnel (numbers and skills)

3. Continuity of Production (the production line itself)

4. Methods (drawings and operation orders for shop work)

5. Special tooling (physical condition of the tools)

Anderlohr also leans heavily on experience with the company. He suggests equal weights for each of these factors but, once experience has been gained different weights might be employed.

According to Anderlohr, one can check company personnel records as to numbers of production line people who have experience on the line. Plans for restarting the line indicate who will be on that line and personnel records will show whether or not they were previously on the line.

Once numbers are known, the analyst must make an estimate as to how much skill (manual dexterity, experience in placing equipment and material, etc.) has been lost. The latter, i.e., loss of skill is a rather hazy area. Again, the analyst's experience with the company and cost improvement curves will make a difference. The more times a job has been done, the greater the retention (you never forget how to ride a bicycle assuming you were once a skilled rider) so, if the interruption occurs after many units have been produced, less skill will be lost than if only a few units had been produced before the interruption.

Supervisory personnel calculations are the same as production line labor calculations. The firm will make more effort to hold supervisory people than it does to hold production line labor but, there may be spillovers here. If supervisory personnel are otherwise employed when the line restarts then there will be a cost associated with pulling them off their current task to supervise the restarting line. Again, company personnel records will be used to calculate the percent of supervisory personnel who have previous experience with this line.

Supervisory personnel lose their skill also (how <u>did</u> I work with Joe? - How <u>did</u> we solve that problem?) and an allowance must be made for this.

Continuity of production refers to the physical condition of the line, light, parts bin, work stations, tool bins, etc. **According to Anderlohr, interruption in the continuity of the production process accounts for the greatest loss of learning.**

Methods refers to machine operations, orders, and drawings. Loss of learning will be small in this area. The methods sheets, drawings and prints are generally kept on file and their reproduction for reissue has a negligible cost.

Special tooling is a physical area. Are the tools available? Has breakage, age or wear made the special tool useless? It could well be that restarting the line will call for a change from "soft" to "hard" tooling or possibly reacquisitions of "soft" tools. These costs should be considered by the analyst.

Let us work through an example using Anderlohrs method. Again, assume 200 units have been produced on an 85% cost improvement curve, unit one cost 1000 labor hours. Assume a six months interruption, then find the cost of a one-hundred unit lot, units 201-300.

Example: Ander Labor Model

The weights are: Production..................20%
Supervisory Personnel.......20%
Continuity of Production....20%
Methods.....................20%
Special Tooling.............20%

	WEIGHT RETAINED	WEIGHT LOST
FOR LABOR:		
For Personnel Remaining.....55%		
For Skill Remaining.........50%		
Calculation of retained weight:		
.20*.55*.50	.055	.145
FOR SUPERVISORY PERSONNEL:		
Personnel remaining.............60%		
Skill remaining.................75%		
Calculation of retained weight:		
.20*.60*.75	.090	.110
FOR CONTINUITY OF PRODUCTION:		
After six months the line is gone, retained 0		
Calculation of retained weight:		
.20*0	.000	.200
FOR METHODS:		
Methods sheets available 90%		
Calculation of retained methods:		
.20*.90	.18	.02
FOR TOOLING:		
92% of the tooling is available		
Calculation of the retained tooling		
.20*.92	.184	.016
TOTALS	.509	.491

FIGURE 5-2

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It is estimated 49.1% of the learning is lost.

Unit 1 cost	1000
Unit 200 cost	289
Learning Achieved	711

Estimated cost of unit 201:

$$288.39 + 49.1\% \text{ of } 711 =$$

$$288.39 + .491 * 711 = 288.39 + 349.10 = 637.49$$

If one examines the tables (or uses a computer) one finds the cost 637.49 associated with unit 6.82. Theoretically one should then retrogress to unit 6.82 and estimate cost from that point. The total cost of units 201 to 300 will then be 41,143 labor hours. In practice it is probably just as well to use the nearest whole unit. In this case, go back to unit 7, so retrogress 194 units, and make each set of calculations from the sequential unit less 194 units. Using this method one finds the cost of units 201 to 300 as:

$$1000(x^b_{300-194} - x^b_{200-194}) \text{ or } 1000(x^b_{106} - x^b_6)$$

and gets 41,086 as the total hours for units 201 to 300, the difference is only 57 labor hours.

Without the interruption units 201 to 300 would have cost 27,442 hours so the cost of the interruption is 13,644 labor hours. Six months is a serious interruption. The cost premium due to an interruption that causes a 194-unit retrogression will not become trivial (less than 2%) until about unit 2532.

It is not really reasonable to assume each of the five factors suggested by Anderlohr are equal. When one gains experience with a company, more reasonable weights can be assigned. Furthermore, experience with a given company will give insights into size of losses. Table 5-3, taken from Anderlohr's paper gives the weights and losses actually negotiated in one case. **Table 5-3 is meant only as a guide, your experience will indicate how it should be refined.**

The effect of going back (retrogressing) so many units will eventually "wearout". The new costs will form a curve asymptotic to the original cost improvement curve. The affects of an interruption will depend upon the severity of the interruption, how many units one had to go back and the slope of the cost improvement curve. The steeper the slope of the curve, the more severe the interruption.

TABLE 5-3 BREAK-IN PRODUCTION

LOSS OF LEARNING WEIGHT	ELEMENT	DESCRIPTION	BREAK TIME DAYS 10-30	31-90	MONTHS 3-6	6-12	12 OR MORE
30%	Employee Learning	(Loss of Personnel)	10%	20%	40%	50%	100%
	Retained Personnel	(Loss of Talent)	10%	25%	45%	70%	100%
20%	Supervisory Learning	(Loss of Personnel)	0%	10%	25%	40%	65%
	Retained Personnel	(Loss of Talent)	5%	10%	20%	30%	40%
20%	Continuity of Production	Work Station Layout	50%	75%	100%	100%	100%
15%	Tooling	Hard Tooling	0%	0%	10%	20%	30%
		Soft Tooling	10%	20%	35%	50%	75%
		From Soft to Hard	50%	50%	50%	50%	50%
15%	Methods	Hard Tooling	0%	5%	10%	20%	20%
		Soft Tooling	5%	10%	20%	25%	25%
		From Soft to Hard	50%	50%	50%	50%	50%

Example of 30-90 days Production Break Calculations

Employee Learning (Loss of Personnel)	Weight 30% x 20% Loss	= 6%	12%
(Retained Personnel Loss of Talent	Retained Weight 24% x 25% Loss	= 6%	
Supervisory Learning (Loss of Personnel)	Weight 20% x 10% Loss	= 2%	3.8%
(Retained Personnel Loss of Talent)	Retained Weight 18% x 10%	= 1.8%	
Continuity of Production	Weight 20% x 75% Loss	=	15%
Tooling	Weight 15% x 20% Loss	=	3%
Methods	Weight 15% x 10% Loss	=	1.5%
Total Loss of Learning	Weight 15% x 10% Loss		35.3%

Table 5-4 shows the factors needed to calculate the point of indifference. The point of indifference is defined as the (X_R, Y_R) set of coordinates on the cost improvement curve at which the Y_R value is less than some specified percent of the value of Y at (X,Y). X_R is X minus the number of units of retrogression and X is the sequential number without regard for the interruption. Y_R is the value at unit X_R. Y is the value from the original cost improvement curve. For a given curve, the X_R value depends on the units of retrogression and the exponent of the curve. At indifference, if X_0 is the number of units of retrogression,

$$\frac{A(X-X_0)^b}{A(X)^b} < r$$

where r is some specified percent expressed as a ratio. The error from such calculations will always be less than the specified premium.

A simple example will illustrate the use of Table 5-4. Assume 300 units have been produced on an 80% cost improvement curve, the unit 1 value is 1000 hours and an interruption occurs. It is estimated the cost of unit 301 will be 181.36 hours. This is the cost of unit 201 so X_0 = 100, there will be a one hundred unit retrogression. If one decides a 2% premium is small enough to ignore, the point of indifference will be 16.762 (from Table 5-4) times 100 or 1673 (we always round up to next unit, this makes calculation so much easier) so we must continue to take the retrogression into account until unit 1673.

If a 5% premium had been the amount we chose to ignore, then we would have used the factor for an 80% curve and 105% or 7.111 and 7.111 times 100 or unit 712 would have been the point of indifférence.

If we needed more accuracy and decided to use a 1% premium as the point of indifference, then unit 3286 (32.857 times 100) would have been the unit after which we ignore the retrogression.

For slopes between those given in Table 5-4, linear interpolation is close enough, slightly greater accuracy can be obtained by interpolating based on the exponent of the cost improvement curve, but the differences are not worth the extra troubles unless the retrogression is many units more than one is apt to encounter except, perhaps, in the case of small missiles where buys of several thousand are not uncommon. For those who have forgotten how to interpolate, there is a brief review in Appendix G.

SUMMARY OF GOING BACK TO UNIT X-NO If we know the slope and can calculate a value expected for the first unit after the interruption, this method is one way to handle the cost of an interruption.

FACTORS FOR INDIFFERENCE OR
X VALUE AT WHICH UNIT (X - NO) IS LESS THAN SPECIFIED PERCENT OF UNIT X

NO IS NUMBER OF UNITS OF RETROGRESSION CAUSED BY INTERRUPTION OF PRODUCTION

X VALUE IS FACTOR MULTIPLIED BY NO

SLOPE	SPECIFIED PERCENT			SLOPE	SPECIFIED PERCENT		
(PCT)	101	102	105	(PCT)	101	102	105
60	74.566	37.718	15.611	80	32.857	16.762	7.111
61	72.169	36.514	15.122	81	31.056	15.858	6.745
62	69.812	35.330	14.642	82	29.277	14.964	6.383
63	67.492	34.164	14.169	83	27.519	14.081	6.025
64	65.209	33.017	13.703	84	25.783	13.209	5.672
65	62.961	31.887	13.245	85	24.068	12.348	5.323
66	60.747	30.775	12.794	86	22.372	11.496	4.979
67	58.567	29.680	12.349	87	20.696	10.654	4.639
68	56.419	28.600	11.912	88	19.040	9.823	4.302
69	54.302	27.537	11.480	89	17.402	9.000	3.970
70	52.216	26.489	11.055	90	15.782	8.187	3.643
71	50.160	25.456	10.636	91	14.181	7.384	3.319
72	48.132	24.437	10.223	92	12.597	6.585	3.000
73	46.132	23.432	9.815	93	11.030	5.803	2.685
74	44.160	22.441	9.413	94	9.481	5.027	2.375
75	42.213	21.463	9.017	95	7.949	4.260	2.072
76	40.293	20.498	8.626	96	6.433	3.503	1.776
77	38.398	19.546	8.240	97	4.936	2.757	1.492
78	36.527	18.606	7.859	98	3.458	2.029	1.231
79	34.680	17.679	7.483	99	2.014	1.343	1.036

TABLE 5-4

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The problem of determining the number of units of retrogression is the really serious problem and only experience with the company or plant involved will be of much help with this basic part of the problem.

This particular method is probably a bit conservative, it may overstate the cost of an interruption.

ACCELERATED RECOVERY FROM AN INTERRUPTION It is quite likely the above method of retrogression is wrong. The firm is reexperiencing, not experiencing, they are going down a cost improvement curve they have been over before and should be better equipped to solve the problems the second time around so some method of accelerating recovery from an interruption may be useful. The academic problem posed is easy to solve but the particulars of the solution will depend upon the firm involved.

Cochran suggests an acceleration.

Let X_0 be the units of retrogression - as determined by Anderlohrs method for instance - then the X coordinate can be determined:

$$\left[X - \left(\frac{D}{D + X - X_1}\right) X_0\right]$$

where:

D is any positive number greater than 1 but usually 20 or more.

The value of each unit, X, will then be:

$$Y = A\left[X - \left(\frac{D}{D + X - X_1}\right) X_0\right]^b$$

where:

Y is the cost of unit X

A is the cost of unit 1

b is the exponent of the cost improvement curve

The larger D, the slower the recovery from the interruption. The specific value of D depends upon experience and it may be larger if the interruption occurs after only a few units. **There is no substitute for experience with a firm's behavior when the analyst is working on the cost of interruption.**

The accelerated recovery approach is the most difficult of the three we have discussed but it is probably the most accurate description of what happens on a production line. There are four problem areas encountered in using this approach:

1. Determining the units of retrogression when the line is restarted.

2. Determining the value of D

3. Determining the point of indifference

4. Calculation of the total cost of the lot

The first two are a matter of experience with the firm, knowing how serious interruptions affect that firm and how fast they recover. The third is a simple matter of arithmetic. Calculating the cost of the lot is also a matter of arithmetic but a computer is a very useful instrument for doing this kind of arithmetic.

Assume your experience is adequate and you have determined X_0 in the usual manner, i.e., you have estimated the cost of unit X_1, followed the cost improvement curve back to the X for that cost and substracted, giving the units fo retrogression. also, assume your experience with the firm gives you a good estimate of D. Then the point of indifference is the positive root of a quadratic equation and will be:

$$\frac{[(X_1 - D) + \sqrt{(X_1 - D)^2 + 4(X_0)(D)(F)}]}{2} = \text{Point of Indifference}$$

where:

X_1 first unit made when restarting the line (i.e., one more than the last unit made before the interruption)

D is the D used in the acceleration formula

X_0 the number of units of retrogression

F the factor for the appropriate curve with the desired indifference level from Table 5-3.

An example:

Assume a firm has made 200 units when production is interrupted, with a cost improvement curve slope of 78%. The analyst estimates a retrogression of 50 units. From experience the analyst knows D is 60. Then the unit where the cost premium due to the interruption is less than 2% is:

$$\frac{(X_1 - D) + \sqrt{(X_1 - D)^2 + 4(X_0)(D)(F)}}{2} = \text{Point of Indifference}$$

where:

X_1 = 201 (from the problem)

D = 60 (from experience with the firm)

X_0 = 50 (the calculated retrogression)

F = 26.489 (from Table 8, 70% slope, 2% level of indifference

$$\frac{(201 - 60) + \sqrt{(201-60)^2 + 4(50)(60)(26.489)}}{2} = PI$$

$$\frac{141 + \sqrt{(141)^2 + 12000(26.489)}}{2} = PI$$

$$\frac{141 + \sqrt{19881 + 317868}}{2} = \frac{141 + \sqrt{337749}}{2} = PI$$

$$\frac{141 + 581.16}{2} = \frac{722.16}{2} = 361.08$$

Unit = 362

We round up to the next unit; it is more convenient and a trifle conservative.

The effects of the interruption will be less than a 2% premium at unit 362. Without the accelerated recovery we would have estimated the effects to last until unit (50*26.489) or unit 1325.

If your calculation results in a number less than X_1 this means the interruption will not affect the costs enough to bother with, (premium will be less than the specified percent for indifference).

TOTAL COST OF A LOT UNDER ACCELERATED RECOVERY

From the formula:

$$Y_y = A\left[X - \left(\frac{D}{D + X - X_1}\right)X_0\right]^b$$

It is obvious that calculation of lot costs can be a problem. Each unit cost is affected by X, X_1, X_0 and D.

Four calculations based on the previous example were calculated for lot values, assuming a unit one cost of 1000 labor hours:

Cost of a lot of 100 units	201 - 300
Cost of a lot of 62 units	301 - 362
Cost of a lot of 162 units	201 - 362
Cost of a lot of 300 units	201 - 500

The lot of 100 cost 6292 labor hours.

Without accelerated recovery we would have calculated cost of this lot as:

$$1000(\ X^{b}{}_{3}{}^{b}{}_{00-50} - X_{2}{}^{b}{}_{00-500}) = 1000(\ X^{b}{}_{250} - X^{b}{}_{150})$$

$$1000(28.56378440 - 21.97224700) = 6592 \text{ labor hours.}$$

Without taking the interruption into account, we would have calculated cost of this lot as 5860 labor hours.

The cost of the lot of 62 units is 3214 labor hours.

The cost of the lot of 162 units is 9506 labor hours.

This is the same as the sum of the first two lots which is as it is supposed to be.

If we stopped at the point of indifference and went back to the normal curve we would have calculated the cost of the 300 unit lot as:

9506 + 6102 = 15608 labor hours

Using the accelerated recovery program, the cost of the lot of 300, units 201 - 500, was 15689, a difference of only 81 labor hours. This says that once we use accelerated recovery we might as well finish that lot but after the point of indifference we might as well use standard programs or the tables.

SUMMARY OF ACCELERATED RECOVERY This is the most difficult of the three methods. You have the problem of estimating units of retrogression, then, from experience with the company you must have a good feel for D. Even with these two solved there is no reasonable way to construct tables for standardized estimates.

The total cost of a lot should be calculated with a computer. It could be done by estimating the cost of several units in the lot and getting an idea of a "typical" value but that is rather sloppy.

Remember, you are not playing a game of fit the curve, you are trying for an accurate description of a production process that has been interrupted - it is not easy. But, even rather unprecise methods, used systematically, should give better results than trying to ignore the cost of an interruption.

Bibliography:

Selected Production Rate Change Models

The quest for a viable model to estimate the effects of rate change on cost is challenging. The models discussed above, while useful, have serious weaknesses--primarily because key variables depending upon estimator judgment. To date, no generic rate change model has been endorsed by the estimating community; however, much interest and study is on-going to develop such a model. Listed below are some of the more noteworthy and/or currently popular models. The list is provided for those analysts who desire more information on rate change models. While this list is by no means exhaustive, it should provide the analyst with a good initial exposure to this area of endeavor.

PRODUCTION RATE CHANGE

Bibliography:

1. Anderlohr, George, "Determining the Cost of Production Breaks." Management Review, Vol 58, No 12 (1969), pp 16-19.

2. Anderlohr, George, "What Production Breaks Cost." Industrial Engineering, September 1969, pp 34-36.

3. Asher, H., Project Rand, Cost-Quantity Relationships in the Airframe Industry, July 1956.

4. Balut, Steve J., Commander, USN, "Redistributing Fixed Overhead Costs," CONCEPTS, Journal of Defense Systems Acquisition Management, Spring 1981, Vol 2, No 2, pp 63-76.

5. Balut, Stephen J., Institute for Defense Analysis, Alexandria, Virginia; Thomas R. Gulledge Jr., Louisiana State University, Baton Rouge, Louisiana; Norman Keith Womer, University of Mississippi, University, Mississippi; "A method for Repricing Aircraft Procurement Programs," not dated, pp. 33.

6. Bemis, John C., "A Model for Examining the Cost Implications of Production Rate," CONCEPTS, Journal of Defense Systems Acquisition Management, Spring 1981, Vol 2, No 2, pp 84-92.

7. Bohn, M. and Kratz, L. A., "Analysis of Production Rate Effects on Unit Costs," TASC, EM-228-WA, 31 January 1984.

8. Cochran, E. B., Planning Production Costs: Using the Improvement Curve, Chandler Publ, 1968.

9. Congleton, D. E., Kinton, D.W., "An Empirical Study of the Impact of a Production Rate Change on the Direct Labor Requirements for An Airframe Manufacturing Program," AFIT Thesis, April 1978.

10. Croxier, M. W., McCann, E. J., Jr., "An Investigation of Changes in Direct Labor Requirements Resulting from Changes in Aircraft Engine Production Rate," AFIT Thesis, December 1979.

11. Gulledge, Thomas R. Jr., & Jeffrey D. Camm, "Notes on the AFSC Rate Analysis Tool and Estimator: Definitions, Assumptions, and Modeling Issuess," Working Paper 860104, Louisiana State University, Baton Rouge, Louisiana 70803, August 1986.

12. Hale, Jack R. "Learning Curve Problems with Interruptions in Production Schedules. Dayton, Ohio: AFIT, November 1972.

13. Kankey, R. D., "Learning Curves: A Review," Unpublished, 1982.

14. Kugel, W. H., "Learning Curves and Production Interruptions," Unpublished, 1982.

15. Large, J. P., Hoffmayer, K., Kontrovich, F., Rand, R-1609-PA&E, Production Rate and Production Cost, December 1974.

16. Martin, S. F., "Effective Pricing Techniques for Program Perturbation and/or Stretch-Out," Analytics Incorp, NES Presentation, November 1982.

17. Smith, C. H., Production Rate and Weapon System Cost: Reaearch Review, Case Studies, and Planning Model, AFIT Thesis, November 1980.

18. Smith, L. L., An Investigation of Changes in Direct Labor Requirements Resulting from Changes in Airframe Production Rate, Doctoral Thesis, University of Oregon, July 1976.

19. Wright, T. P., "Factors Affecting the Costs of Airplanes." Journal of Aeronautical Sciences, Vol 3, No 4 (1936), pp 122-128.

APPENDIX

APPENDIX A

FITTING A LEAST-SQUARES LINE to $Y=AX^b$

Fitting a least squares line to a linear situation involves finding the constants "a" and "b" for the equation $Y = a+bx$. To do this requires solving two simultaneous normal equations:

$$\sum Y = Na + b\sum x$$

$$\sum XY = a\sum X + b\sum x^2$$

In finding the least square line for $Y = ax^b$, we change this equation to a linear form by using logarithms (L) =

$$\text{Log } Y = \text{Log } a + b \text{ Log } X$$

and the two simultaneous normal equations become:

$$\sum \log Y = N \log a + b \sum \log X$$

$$\sum (\log X \times \log Y) = \log a \sum \log x + b \sum (\log x)^2$$

Let us take an illustration to show how such calculations can give us a greater degree of accuracy. In the following demonstration the line of best fit will be established by a calculation by using the following data set where:

X = the number of the unit produced in sequence, and

Y = value in terms of labor hours needed to produce the corresponding xth unit.

X	Y	LX	LY
7	660	.845098	2.819344
10	630	1.000000	2.799341
15	480	1.176091	2.681241
25	440	1.397940	2.643453
40	320	1.602060	2.505150
		$\sum$ 6.021189	13.448529

The first of the two normal equations would be:

$$13.448529 = 5[\text{Log}(a)] + 6.021189(b)$$

However, to set up the second normal equation additional information is necessary:

LogX LogY	$LogX^2$
2.382622	.714191
2.799341	1.000000
3.153383	1.383190
3.695389	1.954236
4.013401	2.566596
16.044136	7.618213

The second normal equation would read:

$$16.044136 = 6.02.89\,[\mathrm{Log}(a)] + 7.618213(b)$$

Setting the two up as simultaneous equations we have:

$$13.448529 = 5[\mathrm{Log}(a)] + 6.021189(b) \qquad (1)$$
$$16.044136 = 6.021189[\mathrm{Log}(a)] + 7.618213 \qquad (2)$$

By dividing each equation by its own coefficient of b we have:

$$2.233534 = .830401[\mathrm{Log}(a)] + b \qquad (3)$$
$$2.106024 = .790368[\mathrm{Log}(a)] + b \qquad (4)$$

By subtracting the second equation from the first we have:

$$.127511 = .04033\,[\mathrm{Log}(a)] \qquad (5)$$
$$\text{Therefore: } \mathrm{Log}(a) = 3.185147 \qquad (6)$$
$$\text{and: } a = 1.532 \qquad (7)$$

By substituting the Log(a) value in equation (1) we have:

$$\begin{aligned} 13.448529 &= (5)(3.185147) + 6.021189\,b \\ &= 15.925735 + 6.021189\,b \\ -2.477206 &= 6.021189\,b \\ b &= -.411415 \end{aligned}$$

Having calculated our constants, 'a' and 'b', our linear formula is:

$$Y_c = aX^b$$

or

$$Y_c = 1532\,X^{(-.411415)}$$

or

$$\mathrm{Log}Y_c = \mathrm{Log}(a) + b[\mathrm{Log}X)]$$

or

$$\mathrm{Log}Y_c = 3.185147 + (-.411415)\mathrm{Log}(X) \qquad (8)$$

We know that when X = 1, Y = a or Y = 1,532. This, then becomes our first plot point on the log-log graph scale. To obtain a straight line, it would be necessary to get only one more plot point which, when connected with the first, would represent the straight line of best fit. A convenient calculating point would be when X = 100. Therefore, when X = 100, what is the value of Y? To obtain the answer we must return to our predicting formula (8):

$$\text{Log}Y_c = 3.185147 + (-.411415)\text{Log}(X)$$

and substituting the log of 100 for Log(X) we have:

$$\begin{aligned}\text{Log}Y_c &= 3.185147 + (-.411415)2 \\ &= 3.185147 - .822830 \\ \text{Log}Y_c &= 2.362317\end{aligned}$$

Therefore:

$$Y = 230.3$$

This, then, is the Y value when X = 100. Plot this point and connect with the first plot point. The result should be a best fit straight line by least-squares calculation.

When a "b" value has been calculated as above, it is simple to compute the slope by formula - thus obtaining a more accurate slope calculation. The formula is as follows:

$$\text{Log}(R) = b * \text{Log}(2)$$

where Log(R) is the log of the ratio of the slope or the slope of the cost improvement curve in percent (%). Using the figures from the above example, our formula would read:

$$\begin{aligned}\text{Log}(R) &= (-.41415)(.301030) \\ &= -.124672 \\ R &= .7505 \text{ or } 75\% \text{ Slope}\end{aligned}$$

APPENDIX B

PART I. STATISTICAL MEASURES OF COST IMPROVEMENT CURVE REGRESSION AND CORRELATION

The Requirement for Measurement of Cost improvement Curve Regressions

Both the unit and cumulative average curves can be regressed as a straight line to fit production data. A choice between use of the cumulative average or unit curve can usually be made by: 1) Consideration of the production company's historical tendency to follow either of the curves; or 2) By visual examination of the scatter diagrams of the unit and cumulative average lot points in the initial production states (one of the scatters should follow a curved line). Where the initial contract lot sizes are large (thereby making visual determination of curvature in the initial unit or cumulative average values difficult) and the tendency of the company's data to follow one type of curve is not clearly established, some mathematical measurement of the fit of both curves to the data is useful in order to make a determination of which curve to use.

Where a unit or cumulative average line has already been fitted to data, statistical techniques may be used to measure the "goodness of fit". In this case, statistical measures are used to determine the degree of correlation between the data and the line and to give an indication of the reliability of values obtained by extrapolating the line beyond the range of the data.

A third use of statistical measures is to determine the relationships between different independent variables (e.g., airframe weight, design speed, engine weight) and dependent variables (e.g., airframe cost components). Use of these measures is particularly important in multiple regression, where it is necessary to identify the independent variables which account for most of the variation in the cost component being estimated.

Statistical measures most frequently cited in airframe literature are standard error of estimate, the coefficient of determination, and the coefficient of correlation. The two coefficients, correlation and determination, are measures of correlation and differ primarily in the degree of correlation they express. The standard error of estimate is a measure of the "goodness of fit" between the data and the regressed line.

The Standard Error of Estimates

Definition. The standard error of estimate (s_y for simple regression, S_y for multiple regression) measures the deviation of the scatter points about the line of regression within the range of the data. It is a measure of the variation between computed values of the dependent variable and observed values. s_y is expressed in the same units as the dependent variable. For a perfect fit (i.e., every plot point falling exactly on the regressed line), s_y would be 0; there is no upper limit for s_y.

Data Assumptions to give s_y Mathematical Significance.

In order to have mathematical certainty that the computed value of s_y actually represents the deviation of the observed values about the regression line, certain assumptions about the data (observed values) must be made: 1) The true relationship between Y and X is linear; 2) The mean of all possible Y values for any given X lies on the regression line; 3) The variances of the Y values are constant for any given X; and 4) The distribution of Y values about a given X is normal.[1]

Computation of s_y. s_y was previously defined as a measure of the deviation of computed values of Y (Y_C) from observed values of Y (Y_O). In order to measure this deviation, it is necessary to compute Y values for each X at which an observed value of Y occurs. Since the difference between Y_C and Y_O may be positive or negative, the differences are squared in order to prevent one difference from offsetting another when the differences are summed. The squared differences are summed and divided by the number of observations (n) in order to get the average difference; the square root is then taken so that the deviation is expressed in linear units.[2] The equation for s_y is:

$$s_y = \left[\frac{(Y_C - Y_O)^2}{N}\right]^{1/2} \qquad \text{(Eq. B-1)}$$

where the values of Y are expressed in logarithms, the equation becomes:

$$s_y = \left[\frac{(\log Y_C - \log Y_O)^2}{N}\right]^{1/2} \qquad \text{(Eq. B-1a)}$$

s_y is biased downward (appears to be smaller than it actually is) for small numbers of observations; therefore, a correction factor is applied to adjust for small sample sizes:

$$s_y \text{ (true)} = s_y \text{ (biased)} \left[\frac{N}{N-2}\right]^{1/2}$$

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or,

$$s_y \text{ (true)} = \left[\frac{\sum(Y_C - Y_O)^2}{N-2}\right]^{1/2} \qquad \text{(Eq. B-2)}$$

or logarithmically,

$$s_y = \left[\frac{\sum(\log Y_C - \log Y_O)^2}{N-2}\right]^{1/2} \qquad \text{(Eq. B-2a)}$$

As an example of the use of these equations, s_y is computed for the values of Y are shown in Table B-1:

TABLE B-1

OBSERVED AND ACTUAL VALUES OF Y FOR $Y = 22.865\ X^{-.44957}$

X	$\log Y_O$	$\log Y_C$
1.68	1.25527	1.25789
7.59	0.95424	0.96345
22.43	0.77815	0.75189
47.40	0.60206	0.60580
86.75	0.47712	0.48779

The Y_C values are computed in the standard manner. As an example:

For $X = 1.68$ with $Y = 22.865\ X^{-.44957}$

$$\log Y_C = \log A + B \log X$$

$$\log Y_C = 1.35918 + (-0.44957)(0.22531)$$

$$\log Y_C = 1.35918 - (0.10129) = 1.25789$$

The other Y_C values are computed in the same manner. To compute s_y (biased) using Eq. B-1a:

Y_O	Y_C	$Y_O - Y_C$	$(Y_O - Y_C)^2$
1.25527	1.25789	0.00262	0.000007
0.95424	0.96345	-0.00921	0.000085
0.77815	0.75189	0.02626	0.000689
0.60206	0.60580	0.00374	0.000014
0.47712	0.48779	-0.01067	0.000114
			0.000910

$$\frac{\sum(Y_0 - Y_C)^2}{N} = \frac{0.000910}{5} = 0.000182$$

$$(s_y)^2 = 0.000182 \qquad s_y = 0.01349$$

Note that s_y is expressed as <u>logarithmic</u> labor hours per pound of airframe. By finding the antilogarithm of each value of Y_C and using the arithmetic values of Y_0 (i.e., Y = 18.0, 9.0, etc.), the arithmetic standard error of estimate (s_y') can be computed. This was done using Eq. B-1 and following the above procedures; s_y' expresses the deviation of observed Y values about the <u>curvilinear</u> (or arithmetic) regression line $Y = 22.865\ X^{-.44957}$, whereas, s_y expresses the deviation about the log-log <u>straight</u> line. Since s_y measures the logarithmic deviation about the logarithmic straight line, it is the <u>proportional</u> deviation of the arithmetic regression line (just as the logarithmic straight line shows the proportional decrease in Y for proportional increases in X). s_y is roughly equivalent to s_y' expressed as a proportion of the mean value of Y. As stated earlier, s_y is biased downward for small sample sizes. In order to adjust s_y for the number of observed Y values, from Eq. B-2a:

$$s_y \text{ (unbiased)} = s_y \left[\frac{N}{N-2}\right]^{1/2}$$

$$s_y \text{ (unbiased} = s_y \left[\frac{5}{3}\right]^{1/2} = s_y\ (1.291)$$

$$s_y \text{ (unbiased)} = (.01349)(1.291) = 0.01742$$

<u>Uses of the Standard Error of Estimate</u>. The standard error of estimate is used in much the same manner as the standard deviation.[3] If all of the assumptions previously cited are met, s_y can be used to measure the probability of occurrence of Y values for any X value <u>within the range of the data</u>. From elementary statistics, there is a probability of 68.3% that the true value of Y will fall within the range defined by the computed value of Y plus or minus s_y; 95.5% for plus or minus 2 (s_y), and 99.7% for plus or minus 3 (s_y). As an example (if it is assumed that the data used in the example in Table B-1 meets all the necessary assumptions), to estimate Y for X = 50 with a confidence in the estimate of 95.5% (using the unbiased s_y):

$$Y \text{ (true)} = Y \text{ (computed} \mp 2 \, (s_y)$$

$$\log Y \text{ (true)} = \log Y_C \mp 2 \, (.01742)$$

but,

$$\log Y_C = \log 22.865 - (.44957)(\log 50) \mp 2 \, (.01742)$$

$$\log Y_C = 0.59537 \mp .03484 = 0.63021 \text{ \& } 0.56453$$

$$\text{antilog } 0.63021 = 4.268 \text{ and antilog } 0.56453 = 3.669$$

Therefore it can be stated that for X = 50, there is a 95.5% probability that Y will lie between 3.67 and 4.27. This same technique could be used to predict the probability limits for Y values for any X between 1.68 and 86.75 (the range of the data). By finding the 95.5% "probability range" for a few values of Y, plotting the outer extremes of the ranges, and connecting the plot points with straight lines, a 95.5% "confidence interval" could be graphed. These lines, generally parallel to the $Y = AX^B$ line and 2 (s_y) units above and below it, would indicate the range in which the true Y value would fall 95.5% of the time. s_y, then, is used to determine the realiability of estimated dependent variable values within the range of the data. s_y also gives an indication of the closeness of fit of the regressed line to the data. A small s_y in proportion to the mean value of all observed Y's indicates that the scatter points are closely grouped about the regressed line. The mean (or average) of the arithmetic Y values was computed to be 8.0 (Y_O/N) and s_y' was computed to be 0.190 (biased). Therefore, the arithmetic standard error of estimate indicates that the average deviation of the observed data from the regressed line is 0.190/8.0 (100) or 2.24%; an excellent fit.

The Coefficient of Determination

__Definition.__ The coefficient of determination (r^2 for simple or linear relationships and R^2 for multivariate relationships) measures the proportion of variance in the dependent variable (Y) that is explained by the variance of the independent variable (X). Mathematically, r^2 is the ratio obtained by dividing the explained variation of Y values from the mean of Y (caused by conformance to the regression line) by the total variation of the Y values from the mean of Y. r^2, then, is the ratio of the explained variation of Y divided by the total variation of Y. As opposed to s_y which is an absolute measure (being expressed in the units of Y), r^2 is a relative variable and has no units. r^2 ranges in value from 0 to +1, with a value of 1 showing that all the variation in Y is explained by X and a value of 0 indicating that X explains none of the variation in Y.

__Data Assumptions to Give r^2 Mathematical Significance.__ For r^2 to give an accurate mathematical measure of the correlation between X and Y, certain assumptions about the properties of both X and Y values must be made; 1) The Y values for each X value are normally distributed about the regression line with a constant variance; 2) X may be expressed as a linear function of Y (i.e., X + P + QY); and 3) The X values for each Y are normally distributed about the regression line X = P + QY with a constant variance.

Computation of r^2. It was stated that r^2 is the ratio of the explained variation of Y divided by the total variation of Y. The explained variation of Y is the summation of the squares of the differences between the computed Y's (Y_C) and the mean of Y (Y_M), $\sum(Y_C - Y_M)^2$. The total variation in Y is the summation of the squares of the differences between the observed Y's (Y_O) and the mean of Y, $\sum(Y_O - Y_M)^2$. Therefore:

$$r^2 = \frac{\sum(Y_C - Y_M)^2}{\sum(Y_O - Y_M)^2} \qquad \text{(Eq. B-3)}$$

Equation B-3 can be converted to a more usable form algebraically; however, the conversion is too lengthy for presentation here. An equation is given for r^2 for a straight line (Y = A + BX) that is adaptable for calculator operations; other equations for r^2 with different terms are frequently seen.

$$r^2 = \left[\frac{a\sum Y + b\sum XY - n\bar{Y}^2}{\sum Y^2 - n\bar{Y}^2}\right] \qquad \text{(Eq. B-4)}$$

for $Y = AX^B$ or $\log Y = \log A + B \log X$

$$r^2 = \frac{[\log(a)](\sum \log Y) + b(\sum \log X \log Y) - n(\overline{\log Y})^2}{(\sum \log Y^2) - n(\overline{\log Y})^2} \qquad \text{(Eq. B-4a)}$$

By referring to the regression example given below, Eq. B-4a can be computed:

REGRESSION EXAMPLE

X	Y	log X	$(\log X)^2$	log Y	$(\log Y)^2$	logX logY
1.4	18.0	0.14613	0.02135	1.25527	1.57570	0.18343
8.0	9.0	0.90309	0.81557	0.95424	0.91057	0.86176
23.0	6.0	1.36173	1.85431	0.77815	0.60552	1.05963
48.0	4.0	1.68124	2.82657	0.60206	0.36248	1.01221
88.0	3.0	1.9448	3.78100	0.47712	0.22764	0.92775
168.4	40	6.03667	9.29880	4.06684	3.68191	4.04479

$\overline{X} = 33.68 \quad \overline{Y} = 8 \quad n = 5 \quad \overline{\log Y} = .813368$

To compute r^2, a and b from the least squares best fit line must be determined. To calculate these variables, the following equations may be used:

Eq B - 4b
$$b = \frac{n(\sum \log X \log Y) - (\sum \log X)(\sum \log Y)}{n \sum (\log X)^2 - (\sum \log X)^2}$$

$$b = \frac{5(4.04479) - (6.03667)(4.06604)}{5(9.29880) - (6.03667)^2}$$

$$b = \frac{20.22395 - 24.550171}{46.49400 - 36.44139}$$

$$b = \frac{-4.326221}{10.05261}$$

$$b = -0.4303595$$

Eq B - 4c
$$\log a = \frac{\sum \log Y - B \sum \log X}{n}$$

$$\log a = \frac{4.06684-(-0.43036)(6.03667)}{5}$$

$$\log a = \frac{4.06684-(-2.59794)}{5}$$

$$\log a = \frac{6.66478}{5}$$

$$\log a = 1.3329572$$

$$a = 21.525694$$

Thus, the least squares best fit equation for the regression example is:

$$Y_C = 21.525694X - .4303595$$

Using Eq B - 4a to compute r^2:

$$r^2 = \frac{(1.3329572)(4.06684)+(-0.4303595)(4.04479)-5(.813368)^2}{3.68191-5(.813368)^2}$$

$$r^2 = \frac{5.4209237+(-1.7407138)-5(.6615675)}{3.68191-5(.6615675)}$$

$$r^2 = \frac{5.4209237-1.7407138-3.3078375}{3.68191-3.3078375}$$

$$r^2 = \frac{0.3723724}{0.3740725}$$

$$r^2 = .9954552$$

$$r^2 = 99.55\%$$

As with s_Y, r^2 is biased by being computed from a small number of observations. A mathematical formula for adjusting r^2 for small sample sizes is:

$$r^2 \text{ (unbiased)} = 1 - [1 - r^2 \text{ (biased)}] \frac{N-1}{N-2}$$

or,

$$r^2 \text{ (unbiased)} = r^2 \text{ (biased)} \left[\frac{N-1}{N-2}\right] - \frac{1}{N-2} \qquad \text{(Eq. b-5)}$$

from the sample,

$$r^2 = (.9956) \left[\frac{(5-1)}{(5-2)}\right] - \frac{1}{(5-2)}$$

$$r^2 = (.9956) \left[\frac{4}{3}\right] - \frac{1}{3} = .9941$$

Use of r^2. r^2 was defined as measuring the proportion of the variation in Y explained by the variation in X. This statement is best demonstrated by use of the example presented above. If the Regression Example data used to compute r^2 could be assumed to meet the requirements previously cited, the computed r^2 of 0.9941 indicates that, within the range of the data, 99.41% of the variation in unit direct labor hours per pound of airframe is explained by the variation in cumulative numbers of units produced. This, obviously, is a very high figure and must be viewed with caution because of the astringency of the assumptions necessary to compute r^2 with mathematical certainty. Nevertheless, where the basic data does meet the statistical requirements, r^2 gives a mathematical measure of the degree of the relationship between the dependent and the independent variables, **within the range of the data**.

The Coefficient of Correlation

The coefficient of correlation (r for simple or linear relationships and R for multivariate relationships) measures the strength and the direction of the relationship between the dependent variable (Y) and the independent variable (X). The sign of r indicates the direction of the relationship. If r is positive, there is a direct relationship. If r is negative, there is an inverse relationship. r takes the same sign as the slope; if b is positive, r takes the positive root of r^2 and if b is negative, r takes the negative root of r^2. The coefficient of correlation is the square root of the coefficient of determination or, logically, $r = \sqrt{r^2}$. Note that since r^2 varies between 0 and +1, r varies between -1 and +1. r values of +1 indicate perfect correlation; 0, no correlation between variables. Inasmuch as r is the square root of r^2, the same assumptions are necessary for the coefficient of correlation as for the coefficient of determination. If these conditions can be assumed to have been met in the Regression Example data, from the preceding example:

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$$r^2 = 0.9941$$
$$r = \sqrt{0.9941} = -.9970$$

This means that there is an inverse or negative relationship between the dependent and independent variables as indicated by the negative sign of b.

When discussing the relationship between X and Y in terms of variation, r^2 is commonly used (since it represents variation) in preference to r (the measure of the direction of correlation). It can be seen that in the range of r = .70, r^2 is approximately .50. The choice of terms to describe the degree of correlation can be confusing and misleading as to the true relationship between the variables unless the distinction between r and r^2 is made. r does not measure variation in Y as explained by the regression line. r is only valuable in determining whether the relationship is direct or inverse and as an indicator of the strength of the association.

Applications of the Statistical Measures to the Cost Improvement Curve

The Standard Error of Estimate. The standard error of estimate is a useful measure of the cost improvement curve; however, the extent of its usefulness is dependent upon recognition of its statistical limitations. The nature of the derivation and computation of s_y restrict mathematical measurement of its application to within the range of the originating data. **There is no mathematical basis for using s_y to measure the reliability of estimates extrapolated beyond the data range.** But, since s_y does give an absolute measure of the deviation of the data from the line, it follows that more confidence can be placed in an estimate based on a regression with a proportionally high s_y. Using that logic, the practice is sometimes made of establishing "confidence intervals" about extrapolations of cost improvement curves. Such a "confidence interval" (better termed prediction interval) is shown in Figure B-1 wherein the range spanned by + 2 s_y is shown for the extension of $21.52569^{-.4303595}$ from X = 100 to X = 500. There is, in fact no mathematical method of certifying that 95 out of every 100 Y values between X = 100 and X = 500 will fall in this range. The range is established based on the assumptions that: 1) The exact conditions that prevailed during the production of the first 113 units of the Regression Example will remain unchanged for the production of the next 400 units; and 2) all future Y values have the same characteristics that were required for the Y values on which calculation of s_y was based. The standard error of estimate, then, can be used to estimate the reliability of projection of a cost improvement curve. When the adjusted s_y was computed for the unit straight line for the Regression Example, it was found to be .01742 (logarithmic) and .245 (arithmetic) (all expressed in labor hours per pound). It was also shown that the standard error of estimate was only 2.24% of the mean Y value; therefore, it may be concluded that the curve fits the data very well and that a high degree of confidence might be placed in extrapolation of the curve. (Caution: The logarithmic transformation of the data generates a bias in all statistical measures. Thus, the analyst should not become overly confident just because the statistics look "good".)

The Coefficients of Determination and Correlation. Where X, the independent variable, is expressed in cumulative production numbers, the measures of correlation give a mathematical indication of the degree that Y, the dependent variable is following the cost improvement theory. For the cumulative average regression of the Regression Example, r^2 (adjusted) was found to be in excess of .99 as was r^2 for the unit curve. The unit curve was observed to follow cost improvement theory slightly better than the cum average.

It is already historically established that both unit and cumulative average labor costs do follow cost improvement theory closely and, further, **choice of the use of the cum ave or unit curves is not normally based on statistical measures.** Therefore, r^2 is most frequently used to measure the degree of association between airframe costs and different independent variables. This use is particularly important in multivariate regression, where R^2 is used to measure the tendency of airframe costs to vary in accordance with combinations of airframe characteristics (e.g., airframe weight, design speed, and wing surface area). An example of the use of R^2 in multiple regression is that determination of the degree of correlation between engineering costs and electronics complexity, ratio of installed equipment, and design speed led to establishment of empirical equations for computation of points on an engineering cost improvement curve (previously considered not to follow cost improvement theory). The detailed techniques for use of the coefficients of correlation and determination in simple and multiple regressions are too extensive for presentation in this paper. A RAND report by G. H. Fisher offers comprehensive examples of the use of these statistical measures. The Advanced Quantitative Methods and Cost Analysis Course (QMT 550) presents and develops the concept of multivariate regression and the application of statistical measures.

SUMMARY: PART I

Statistical measures may be used to evaluate the fit of cost improvement curves to production data and the degree of correlation between the variables in the cost improvement curve. These measures may indicate the reliability of curve extrapolations and provide a means of choosing the type of curve to regress through production data.

The standard error of estimate (s_y) was defined as an absolute measure of the deviation of data from the regression line. Expressed in the same units as the dependent variable, the standard error of estimate has a minimum value of 0 which indicates a perfect fit between the data and the regressed line. Eqs. B-1a and B-2a were presented for computing s_y from airframe production data. Low values of s_y in proportion to the mean of the dependent variable values indicate that a reasonable degree of confidence may be placed in extrapolations of the regressed line.

The coefficients of determination (r^2) and correlation (r) were defined as the measures of the variation and direction, respectively, of the dependent variable as explained by the independent variable. Eqs. B-4 and B-5 were presented for computation of r^2 and r from production data. The measures of correlation are primarily used to establish the relationships between airframe cost components and airframe characteristics.

The statistical measures require stringent assumptions to be made about the originating data before mathematical certainty can be expressed in their values. Further, s_y, r^2, and r are all biased by small sample sizes (limited numbers of observations) and the logarithmic transformation fo the data.

PART II

Reliability of Statistical Measures of the Cost Improvement Curve

Difficulties in Applying Statistical Measures to Production Data

Statistical analyses are widely used to measure the accuracy and predictability of the cost improvement curve and to establish empirical equations for development of cost improvement curves. In using statistics for this purpose, it is necessary to recognize that statistically derived results may be misleading. Several factors make application of statistical techniques to the cost improvement curve difficult; failure to recognize and compensate for these factors can lead to misinterpretation of cost improvement curve results.

Assumptions Necessary to Apply Statistical Measures with Mathematical Certainty.

The statistical measures presented in Part 1, Appendix B, i.e., s_y, r^2, and r may be applied with mathematical certainty only where the production data from which the measures are computed meets stringent criteria. Generally, the observed values of the dependent variable (Y) for any independent variable (X) must represent random samples from a total group of values that are distributed "normally" about a mean value of Y which lies on the true regression line.

Further, the "populations" of Y values at each X must have a deviation that is constant and measurable for all values of X considered. Basically, the preceding statement means that the component production cost for a given unit of production that is used in a regression must be considered a random selection from a series of possible costs evenly distributed about some "true" cost for each unit. The distribution of the "possible costs" for each X may be closely grouped or widely dispersed but the distribution must be uniform and identical for each X. For application of r^2 with mathematical certainty, essentially the same assumptions must be made about the X (units of production) values. Where production costs from several airframes are used in order to develop an average or standard cost improvement curve, the cost values of all the airframes at a particular unit of production are assumed to represent random selections from a uniformly distributed totality of costs.

Generally, the assumptions necessitated for mathematical certainty of the values of the standard error of estimate are not so stringent as those required for the coefficients of determination and correlation, since the latter require X values to be random and normally distributed. In either case, the assumption that component costs per unit not only are normally distributed but have a constant variance within that distribution for all values of cumulative production is a broad assumption to make. It is almost superfluous to note that large errors can result from using equations based on the above assumptions when, in fact, the populations of Y values are not normal or have different variances.

Small Sample Sizes. A particularly vexing problem in applying statistical measures to airframe production data is the difficulty of obtaining sufficiently large numbers of observed costs that are applicable to the relationship being measured or established. When statistically measuring the characteristics of a "population" (airframe production costs), the probability that the values of statistical measures are accurate goes down sharply as the sample size decreases. Ezekiel and Fox present an excellent graphical comparison of the reliabilities of values of r for sample sizes ranging from 5 to 100. As an example, for a sample size of 5, where the indicated value of r (adjusted for sample size) is .90, one time in 20 the true value of r may be as low as .50. The standard error of estimate is equally unreliable for small sample sizes. Where a cost improvement curve is fitted to production data for a specific airframe at a point early in the production run, small sample sizes are unavoidable and the risk of misleading statistical values must be evaluated and accepted. Where empirical equations are being developed from historical data for establishment of predictive cost improvement curves, the sample size may be increased by including data from aircraft that do not necessarily have the same characteristics as the aircraft to be estimated. In establishing empirical equations in the Planning Research Corporation report (to be used for estimating future airframe costs), in order to obtain an adequate sample size, it was necessary to include production data from such aircraft as the B-50 and F-86. Further, the data base includes all types of aircraft (i.e., cargo, bombers, fighters, trainers, reciprocating engine, jet, subsonic and supersonic), primarily for this same reason. It is recognized in the PRC report that **inclusion of data from obsolete aircraft is not desirable** and that equations for estimating bombers, for example, would be more reliable if based on bomber production data only. PRC concludes, however, that these defects are outweighed by the advantages of a larger sample size. As in the case of assuming normality of distribution, use of small sample sizes or samples swelled to adequate size by the inclusion of data that may represent another "population" of values can lead to erroneous or misleading values for statistical measures.

Use of Logarithmic Equations. As shown in the example in Part I, the arithmetic standard error of estimate (s_y') differs in value from the **logarithmic standard error of estimate (s_y)**, since the latter **measures the proportionate deviation of the data from the regression line.** When the regressed unit line was extrapolated to the 500th unit, s_y remained at approximately 2.25% of the Y value or approximately 0.03 labor hours per pound; whereas, the absolute value of s_y' (adjusted) was approximately 0.25 labor hours per pound or 800% greater. **It is recognized that transforming data logarithmically sometimes has the effect of causing statistical measures to "look good".** This is particularly true in the cost improvement curve where extrapolated Y values are usually small and application of the logarithmic standard error of estimate to the extrapolated values results in a small "prediction interval". **It is important to note that statistical measures based on logarithmic data may have different characteristics than the same measures based on arithmetic values of the same data.**

Projection of the Cost improvement Curve, Predictability and Reliability

The factors affecting application of statistical measures to airframe production data represent difficulties encountered when using statistical techniques within the range of the data. Statistical techniques can be used to evaluate extrapolated curves but there is no mathematical certainty in the results. **It is almost universally recommended by statisticians that curves should never be extrapolated beyond the range of the data and, if done, should be done only with the utmost caution.** Nevertheless, for a variety of obvious reasons, estimates into the future are vital to every major industry; particularly the airframe industry where the majority of production is done under long term contract with the Department of Defense. The cost improvement curve is used by both the airframe companies and the Department of Defense as a tool for estimating future airframe costs; therefore, some criteria for determining its reliability as an estimation of future airframe costs is necessary and the statistical measures discussed (s_y, r^2, and r) are frequently used.

Where s_y and r^2 are to be used to measure the reliability of cost improvement curve projections, all of the assumptions previously cited concerning the characteristics of the production data used to compute s_y and r^2 must be considered. In addition, it must be assumed that all the conditions that existed at the time the cost improvement curve was established from the production data remain unchanged throughout the range of the curve extension. Some of the factors that affect the shape and position of the cost improvement curve have been presented. Many of these factors are the result of changed conditions occurring after the airframe production was in progress (e.g., engineering changes). Changes in the rate of production, significant technological advances, and changes in price levels are additional factors that represent changed conditions and serve to invalidate estimates based on projected values. If, however, conditions are assumed to remain unchanged, s_y and r^2 can be used effectively to indicate the reliability of estimates based on cost improvement curves.

The uses of s_y and r^2 depend upon the type of estimate to be made. For long range systems cost estimates and estimates of changes in force mix, the statistical measures are used to indicate the validity of the equations which establish the cost improvement curve as the basis of the estimate. Where the data obtained in the initial stages of a production run are used to form a cost improvement curve for estimating the cost of future units, s_y and r^2 are used to estimate the "goodness of fit" of the curve and thereby indicate the reliability of the curve extension.

Once the decision has been made to extrapolate from a data base, it is desirable to determine some prediction interval or range of Y values for the projected curve. It is sometimes held that prediction intervals should increase in range as the extrapolated X values increase in distance from the mean X value of the data base. However, it is also recognized that extrapolations are more reliable where there is a historical basis for establishment of a trend. In the airframe industry, it is firmly established that such a trend exists in the unit cost decrease for increases in cumulative units produced. This approach offers justification for a fixed prediction interval based on unchanged conditions. Were the expanding interval used for extensive projections, the interval would become so large as to be meaningless for cost estimation purposes. The arithmetical standard error of deviation is suspect for the same reason.

Some studies have been made on the reliability of the cost improvement curve for airframe estimation. In a RAND study based on World War II airframes, the average absolute error in cost estimates was found to be 25%; however, the estimates were made using industry average slopes rather than individual slopes for each airframe. In the PRC report, using the multivariate equations developed, 16 airframes were estimated and the estimates compared against actual cost. The mean of the actual total costs was found to be 97.8% of the mean of the estimated total costs; and the average error per total cost estimate approximately 8%. The total cost estimates benefited from compensating error in the cost component estimates (which had greater average errors per estimate); nonetheless, the results indicate that the cost improvement curve is a useful estimating tool.

The most important requirement in using the cost improvement curve as a tool for airframe cost estimation is that the estimator (or cost analyst) be thoroughly familiar with not only the curve, but all pertinent aspects of the airframe industry. As with most other estimating equations and techniques, the potential for error is great, and the predictability and reliability of the cost improvement curve is, in large part, dependent upon the judgment with which it is used.

SUMMARY: PART II

One of the primary uses of the cost improvement curve in the airframe industry is as a tool for cost estimating. Statistical measures provide a means of measuring the reliability of that tool. In applying statistical measures to cost improvement curves, certain restrictions must be recognized. The data on which the curve is based must conform to requirements for a "normal" population, small numbers of observations tend to introduce error, logarithmic equations tend to make curve-data fits look good, and the observations may be distorted by being correlated with each other.

Where cost improvement curves are projected, no mathematical certainty can be placed in the values obtained from statistical measures. If conditions in effect at the time the data was observed remain constant, however, the statistical measures provide an indication of the confidence that can be placed in cost improvement curve extrapolations. Nevertheless, the most important requirement in use of the cost improvement curve is that the user be familiar with the complexities of airframe production, as, in the final analysis, the judgment of the estimator determines the reliability of the tool.

APPENDIX C

'B' VALUES AND CONVERSION FACTORS FOR SLOPES BETWEEN 60 AND 99

PER CENT SLOPE	"b" VALUE	"b+1 (CONVERS FACTOR)
60	-.736966	.263034
61	-.713118	.286882
62	-.689659	.310341
63	-.666575	.333425
64	-.643856	.356144
65	-.621490	.378510
66	-.599462	.400538
67	-.577766	.422234
68	-.556393	.443607
69	-.535332	.464668
70	-.514573	.485427
71	-.494110	.505890
72	-.473933	.526067
73	-.454031	.545969
74	-.434402	.565598
75	-.415038	.584962
76	-.395927	.604073
77	-.377069	.622931
78	-.358453	.641547
79	-.340076	.659924
80	-.321928	.678072
81	-.304006	.695994
82	-.286304	.713695
83	-.268817	.731183
84	-.251540	.748460
85	-.234465	.765535
86	-.217593	.782407
87	-.200914	.799086
88	-.184423	.815577
89	-.168123	.831877
90	-.152001	.847999
91	-.136063	.863937
92	-.120294	.879706
93	-.104697	.895303
94	-.089267	.910733
95	-.073996	.926004
96	-.058894	.941106
97	-.043942	.956058
98	-.029147	.970853
99	-.014500	.985500

APPENDIX D

TRUE LOT MID-POINTS

Rule-of-thumb mid-points may be sufficiently accurate for most cost analysis forecasts, but when a greater degree of accuracy is demanded, the true-lot-mid-point calculation permits more precision. The rule-of-thumb provides that the mid-point chosen should be "half the way" if the first lot is less than 10 units, but "1/3 the way" if it is composed of 10 or more units. Actually, neither is exactly right. If the ratio values for the first 10 units for an 80% curve are added, the total is 6.315384. When this is divided by 10, the average is .6315384, which is close to the Boeing Table ratio value for the 4th unit (.64). Since the value of .631538 falls between the 4th and the 5th unit (which has a ratio value of .595637), interpolation* will yield a unit of 4.2 as having a ratio value of .631538. The true lot mid-point in our illustration would therefore be at 4.2 units when the lot is composed of 10 units. When using the 1/3 rule, the mid-point would have been at 3.3; while the 1/2 rule would have produced 5.

Consider the first lot as having 20 units. In this case the true-lot-mid-point would be established at 7.45 units, calculated as in the previous case. This represents 37% of the 20 units; whereas in the previous case of 10 units for the first lot, the true lot mid-point was 4.2 or 42%. The conclusion to be reached is that the larger the quantity in the first lot, the closer to the 1/3 point will be the true lot mid-point.

Instead of adding the unit values to determine the true-lot-mid-points, it would be easier to look up the cumulative total in Part B of the Improvement Curve Tables. For example, if a quantity of 15 were considered for the first lot and 70% curve the Tables state a cumulative total ratio value of 6.273895. Dividing this by 15, we have an average lot value of .418260. Turn to Part A of the Improvement Curve for the 70% curve and note that this value falls between units 5 and 6. By interpolation we would get a more exact answer of 5.48* units as representing the true lot mid-point.

True-lot-mid-points can be obtained as well for other than first lots by a similar process of calculation. Take, for example, a lot representing units 81 through 100, a lot of 20 units and a 78% curve. Find first the cumulative total value for all units through the end of the preceding lot, i.e., the cumulative total value for X = 80, which is 24.997156. Next find the cumulative total value for all the units thru the last unit in the lot under consideration, i.e., the 100th unit, which is 28.979078. Now subtract 24.997156 from 28.979078, which is 3.981922 (the cumulative total ratio value of the 20 units under consideration). Divide the difference by 20, which is .199096.

Finally, look in Part A of the Improvement Curve Tables for the 78% curve and note that the ratio value above falls between the 90th and the 91st unit. Interpolation yields a unit number of 90.25. Note that this is very close to the rule-of-thumb mid-point of 90. *This is arithmetic interpolation and is inexact but the difference between two successive units is usually not significant.

The added degree of exactness which true-lot-mid-points provide may be of real significance only for the consideration of the first lot. However, there are occasions when exactness is demanded. Such occasions often occur during negotiations when differences of opinions due to rule-of-thumb measurements can be solved only by exact calculations. It should be remembered, though, that **exactness in mid-point calculations may be cancelled by inexactness in slope and first unit value calculations.** When an exact slope is determined by formula and an exact "a" value is obtained, it is then appropriate to use a true-lot-mid-point to achieve over-all exactness.

Consider again the example above which yielded a tree-lot-mid-point of 89.62. If the value for the first unit were 860, the true-lot-mid-point calculation produces a total value for the lot of 3,433; whereas the rule-of-thumb mid-point would produce a lot value of 3,428. This is only a slight difference, but if we take one of the preceding examples of 20 units in lot number 1 and an 80% curve, the true-lot-mid-point was determined at 7.45 units. If the first unit had a value of 860, as above, the true-lot-mid-point calculations would result in a lot value of 9,017; whereas the rule-of-thumb result would be 9,348. In this case the difference is quite significant, and when a whole program of production is evaluated lot by lot, the sum of differences may be substantial.

There are several computer programs available which will calculate a least-squares line. Nonetheless, in plotting data, knowing the true-lot-mid-point may substantially assist the analyst in determining the type of curve involved--especially when the slope is in the high 80's.

Listed below in Table D-1 are the true-lot-mid-points for first lots of 1-25 units and slopes of 70-90.

TABLE D-1

TRUE LOT MIDPOINTS - UNIT THEORY

UNIT	PERCENT 70	71	72	73	74	75
1	1.00	1.00	1.00	1.00	1.00	1.00
2	1.37	1.37	1.37	1.38	1.38	1.38
3	1.72	1.73	1.73	1.73	1.74	1.74
4	2.06	2.07	2.07	2.08	2.08	2.09
5	2.39	2.40	2.40	2.41	2.42	2.43
6	2.71	2.72	2.73	2.74	2.75	2.76
7	3.03	3.04	3.05	3.07	3.08	3.09
8	3.34	3.35	3.37	3.39	3.40	3.42
9	3.65	3.67	3.69	3.70	3.72	3.74
10	3.95	3.97	4.00	4.02	4.04	4.06
11	4.25	4.28	4.30	4.33	4.36	4.38
12	4.45	4.58	4.61	4.64	4.67	4.70
13	4.85	4.88	4.92	4.95	4.98	5.01
14	5.15	5.18	5.22	5.25	5.29	5.32
15	5.44	5.48	5.52	5.56	5.60	5.63
16	5.73	5.78	5.82	5.86	5.90	5.94
17	6.03	6.07	6.12	6.16	6.21	6.25
18	6.32	6.37	6.42	6.46	6.51	6.56
19	6.60	6.66	6.71	6.76	6.82	6.87
20	6.89	6.95	7.01	7.06	7.12	7.17
21	7.18	7.24	7.30	7.36	7.42	7.48
22	7.47	7.53	7.60	7.66	7.72	7.78
23	7.75	7.82	7.89	7.95	8.02	8.09
24	8.04	8.11	8.18	8.25	8.32	8.39
25	8.32	8.40	8.47	8.55	8.62	8.69

TABLE D-1 (Cont'd)

TRUE LOT MIDPOINTS - UNIT THEORY (Cont'd)

UNIT	PERCENT 76	77	78	79	80	81
1	1.00	1.00	1.00	1.00	1.00	1.00
2	1.38	1.38	1.38	1.39	1.39	1.39
3	1.74	1.74	1.75	1.75	1.76	1.76
4	2.09	2.10	2.11	2.11	2.12	2.12
5	2.44	2.45	2.45	2.46	2.47	2.48
6	2.77	2.79	2.80	2.81	2.62	2.83
7	3.11	3.12	3.13	3.15	3.16	3.17
8	3.44	3.45	3.47	3.48	3.50	3.51
9	3.76	3.78	3.80	3.82	3.83	3.85
10	4.08	4.11	4.13	4.15	4.17	4.19
11	4.41	4.43	4.45	4.48	4.50	4.52
12	4.72	4.75	4.78	4.81	4.83	4.66
13	5.04	5.07	5.10	5.13	5.16	5.19
14	5.36	5.39	5.42	5.46	5.49	5.52
15	5.67	5.71	5.74	5.78	5.82	5.85
16	5.98	6.02	6.06	6.10	6.14	6.18
17	6.30	6.34	6.38	6.42	6.47	6.51
18	6.61	6.65	6.70	6.74	6.79	6.83
19	6.92	6.97	7.02	7.06	7.11	7.16
20	7.23	7.28	7.33	7.38	7.43	7.48
21	7.54	7.59	7.65	7.70	7.75	7.81
22	7.84	7.90	7.96	8.02	8.08	8.13
23	8.15	8.21	8.27	8.34	8.40	8.46
24	8.46	8.52	8.59	8.65	8.72	8.78
25	8.76	8.83	8.90	8.97	9.03	9.10

TABLE D-1 (Cont'd)

TRUE LOT MIDPOINTS - UNIT THEORY (Cont'd)

UNIT	PERCENT 82	83	84	85	86	87
1	1.00	1.00	1.00	1.00	1.00	1.00
2	1.39	1.39	1.39	1.39	1.40	1.40
3	1.76	1.77	1.77	1.77	1.78	1.78
4	2.13	2.13	2.14	2.14	2.15	2.15
5	2.48	2.49	2.50	2.51	2.51	2.52
6	2.84	2.85	2.85	2.86	2.87	2.88
7	3.18	3.20	3.21	3.22	3.23	3.24
8	3.53	3.54	3.56	3.57	3.58	3.60
9	3.87	3.89	3.90	3.92	3.94	3.95
10	4.21	4.23	4.25	4.27	4.29	4.31
11	4.55	4.57	4.59	4.61	4.64	4.66
12	4.88	4.91	4.93	4.96	4.98	5.01
13	5.22	5.25	5.27	5.30	5.33	5.35
14	5.55	5.58	5.61	5.64	5.67	5.70
15	5.88	5.92	5.95	5.98	6.02	6.05
16	6.22	6.25	6.26	6.32	6.36	6.39
17	6.55	6.59	6.62	6.66	6.70	6.74
18	6.88	6.92	6.96	7.00	7.04	7.08
19	7.20	7.25	7.29	7.34	7.38	7.42
20	7.53	7.58	7.63	7.68	7.72	7.77
21	7.86	7.91	7.96	8.01	8.06	8.11
22	8.19	8.24	8.30	8.35	8.40	8.45
23	8.51	8.57	8.63	8.68	8.74	8.79
24	8.84	8.90	8.96	9.02	9.08	9.13
25	9.16	9.2[illegible]	9.29	9.35	9.41	9.47

TABLE D-1 (Cont'd)

TRUE LOT MIDPOINTS - UNIT THEORY (Cont'd)

UNIT	PERCENT		
	88	89	90
1	1.00	1.00	1.00
2	1.40	1.40	1.40
3	1.78	1.79	1.79
4	2.16	2.16	2.17
5	2.53	2.53	2.54
6	2.89	2.90	2.91
7	3.25	3.26	3.28
8	3.61	3.63	3.64
9	3.97	3.99	4.00
10	4.32	4.34	4.36
11	4.68	4.70	4.72
12	5.03	5.05	5.08
13	5.38	5.41	5.43
14	5.73	5.76	5.79
15	6.08	6.11	6.14
16	6.43	6.46	6.49
17	6.77	6.81	6.85
18	7.12	7.16	7.20
19	7.47	7.51	7.55
20	7.81	7.86	7.90
21	8.16	8.20	8.25
22	8.50	8.55	8.60
23	8.85	8.90	8.95
24	9.19	9.25	9.30
25	9.53	9.59	9.65

APPENDIX E

IMPERFECT CURVES

Academic exercises tend to present data where the points of a relationship fall on a straight line without deviation. Such a situation exists only in the ideal, and since the ideal rarely exists, it is necessary to consider situations where the points of relationship are less than the ideal. Even though companies may use a straight line for planning manpower usage, the outcome usually is a broken line when connecting points. In fact, a company would be suspect if it showed its labor hour costs to fall on a straight line without deviation. However, since deviations are expected, this fact means that an analyst must use a concept of normal relationship. This means that an imperfect curve is not as accurate a predicting device as a perfect line of relationship. Thus the analyst must establish a line of best fit.

After the historical data have been plotted, the analyst should observe whether a straight line would reasonably fit the data. For example, in the following graph it is obvious, when connecting the points in succession that the single straight line is not the best fit line:

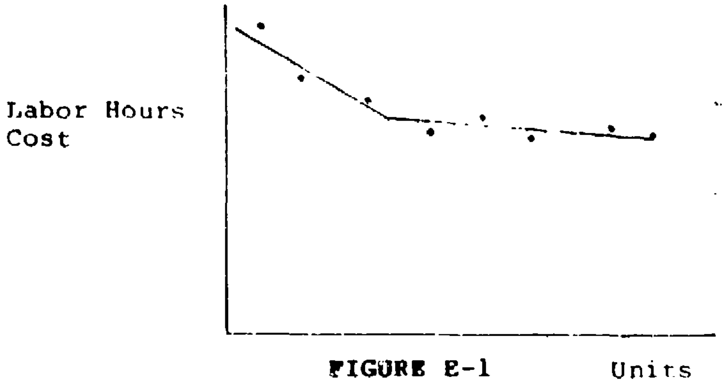

FIGURE E-1

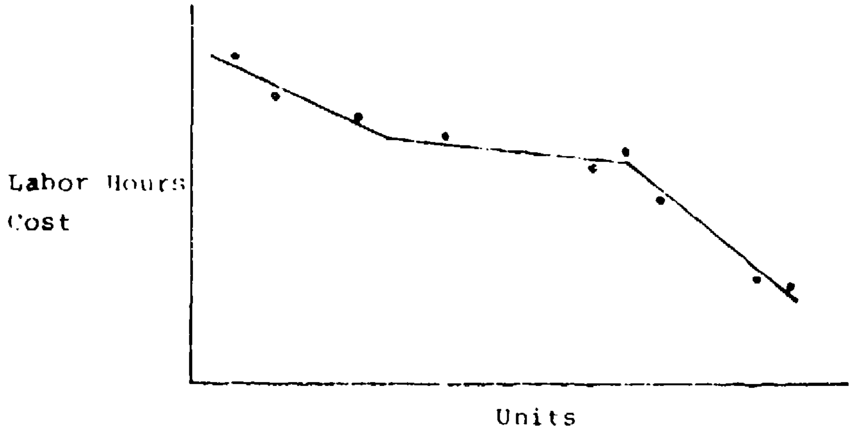

FIGURE E-2

It appears in the preceding two figures that segmented lines fit the data best. These situations may arise due to unusual circumstances and must be recognized as such when they occur. In such an event, a singl straight line would be misleading. However, our theory c. he cost improvement curve states that there is a constant relative decline for a production run. This means a line of best fit must be drawn to establish the constant relationship. In most cases, an over-all straight line will fit the data adequately, but in a few instances it may be necessary to construct segmented lines.

Even though actual data, when plotted on log-log paper, do not fall on a straight line, a fitted line can be constructed so that the deviations are minimized both above and below such a line.

For example, plot the following on log-log paper:

HOURS PER UNIT	UNIT NUMBER
660	7
630	10
480	15
440	25
320	40

FIGURE E-3

A straight line of best fit can be drawn quite easily by drawing a line through the midpoints of straight line segments between successive points. This can be done when the horizontal distances between points are approximately equal. Even though points do not alternate above and below a straight line, there must not be many continuing points either below or above to be a usable straight line.

As another example, plot the following values on log-log paper:

HOURS PER UNIT	UNIT NUMBER
9,200	9
8,600	11
8,600	13
8,400	17
7,800	20
7,600	35
6,800	40

FIGURE E-4

The plotting should indicate a straight line relationship, and, without a great deal of experience, an analyst could readily draw a line of best fit. As a check on accuracy, the student should have drawn a line of best fit passing through 9,000 labor hours at unit 9 and 8,000 at unit 18.

Lines drawn by inspection or eye-sight are usually adequate, especially when there are small deviations of the actual points from the straight line. Such a line can be used for purpose of projection. In the example above, the prediction for the 60th unit would be approximately 6,600 hours per unit.

However, suppose that the actual production continued uninterrupted beyond the 40th unit and that the actual hours per unit shown on the records were 6,400 hours for the 50th unit and 6,000 for the 60th unit. In this case the analyst might well consider the effect of the more immediate experience when projecting the more immediate future. Since the last 3 points were all below the average line, the analyst might conclude that if there were 10 or 20 (or as many as 50) more units to follow that they would continue the new trend. But if in predicting 400 units to follow the 60th, the analyst would consider the over-all average line as being more conclusive.

There are instances when more accuracy is desired than a line of best fit by inspection can produce. When high values are involved, a difference of 1% in slope estimation may make a sizable difference in estimating program costs. A mathematical best fit line can be constructed by using the method of least squares to fit the line. To find a least squares line of best fit, the formula for the cost improvement curve, $Y = AX^b$, must be transformed into a linear form, or:

$$\log y = \log a + b \log X$$

To find the logarithmic least squares line requires that the following two simultaneous linear equations be solved:

$$\sum \log y = n \log a + b \sum \log x$$
$$\sum (\log y \times \log x) = \log a \sum \log x + b \sum \log x^2$$

As a practical matter, if this kind of precision is required, there are usually several cost improvement curve computer programs available that, among other things, will calculate a best-fit, least squares line. The mathematical solution to the above simultaneous linear equations is included in Appendix A.

The idea of slope is somewhat of a misnomer. The solution for "b" in the above equation relates to the exponent of the equation $y = ax^b$, while slope relates to the rate of learning for doubled quantities. The relationship between these two is shown in Appendix C. The first column gives the slope and the second column gives the corresponding "b" value. The third column is a factor used in converting from one type of curve to another. The third column is also known as "b+1" when working with the Northrop Construction.

APPENDIX F

An Introductory Vocabulary: Words and symbols that cause trouble in cost improvement curve discussions.

WORD

Unit Number: Symbol X. In general a simple idea--the sequential number of the unit through the production process. Generally the first unit through is unit 1, the second, unit 2, etc. Sometimes if X would be ambiguous, some other letter is substituted, say N.

Lot: No commonly accepted symbol. A group of units that go through the production process as a set -- costs are generally accumulated to the lot, not to the unit.

Cumulative: No generally accepted symbol. Cumulative implies all units through the production process from units 1 to X (or 1 to N) inclusive.

Cumulative Units: Sometimes the written abbreviation CU is used. The oral abbreviation (jargon or idiomatic phrase) cum units is practically universal. It means the number of units that have been made or, in predictions, the number of units that will have been made by the end of some specified lot.

Cost: Symbol Y. In cost improvement curves always a direct cost, frequently in real terms (hours, pounds of material), if in dollars or monetary units, it must be in constant dollars.

Unit Cost: Symbol Yx. The cost of a specified unit - the resources used to make that unit. Y_{10} means the cost of unit 10.

Cumulative Total: Symbol CTx. Means the total cost for all units from 1 to X inclusive. CT_5 means the cost of the first five units, $Y_1 + Y_2 + Y_3 + Y_4 + Y_5$.

Sometimes written CT_n where N means the same as X, simply a unit number which limits the size of the problem.

Cumulative Average: Symbol Yx. Means the average cost for the first X units. The cumulative total divided by the number of units - Y_5 means $(Y_1+Y_2+Y_3+Y_4+Y_5)/5$. Y_{20} means the cumulative total cost of the first 20 units divided by 20.

Lot Cost: No generally accepted symbol. Usually implies the total cost of a lot, some specified set of units, and only occasionally means the first X units.

Lot Total Cost: Symbol $TC_{f,l}$. Means the cost of units 'f' to 'l' inclusive, and only in the case of a first production lot will 'f' be 1. $TC_{6,10}$ means the cost of a five unit lot, units 6 to 10 inclusive, $Y_6+Y_7+Y_8+Y_9+Y_{10}$.

Lot Average Cost: Symbol $Y_{F,L}$. Means the total cost of the lot divided by the lot size, in general $Y_{F,L} = TC_{F,L}/(L-(F-1))$. $Y_{6,10}/5$.

Slope: Sometimes abbreviated S or SL. The percent by which the cost at unit X must be multiplied to get the cost at unit 2X.

Rate of Learning: No generally accepted symbol. Rate of learning is 100-slope, the percent reduction from Y_x to Y_{2x}, (or Y_x to Y_{2x}).

$$\text{Rate of learning} = \frac{Yx - Y2x}{Yx} \text{ or } \frac{Yx - Y2x}{Yx}$$

APPENDIX G

NOTE ON INTERPOLATION

Interpolation is simply using some system of ratios to find values between (never beyond) the values in a table. Usually linear interpolation is adequate, nonlinear interpolation is at best left to mathematicians. Linear interpolation involves finding a correction factor from a ratio then applying that correction factor to a tabular value and getting the desired value between the tabular values.

Two bases are explained using Table 5-3 (Chapter 5) as a source of the problem. Assume we need the indifference factor for a cost improvement curve with an 82.3 percent slope, 2% premium is the level. From Table 5-3 we find a 2% premium:

82% slope	14.964 @ 2% premium
83% slope	14.081 @ 2% premium

Arithmetic interpolation is simple, 82.3 is 3 tenths of the difference between 82 and 83 so we take .3 of the difference between 14.964 and 14.081 or .3(14.964-14.081) = .3(+.833)=+.265.

This is the correction factor and we apply it to 14.964 and get 14.964 - .265 = 14.699 as the factor we use. Technically we have solved the ratio problem:

$$\frac{A}{B} = \frac{C}{D}$$

where A = 82 - 82.3; B = 82 - 83; C is our factor and D = 14.964 - 14.081:

$$\frac{-.3}{-1} = \frac{C}{.883} \qquad C = AD/B$$

$$= -.3(.833)/-1$$

$$= -.2649/-1 = .2649$$

and with the factor we got 14.699. We subtracted because the factor is decreasing.

If, instead of arithmetic we had based our interpolation on the exponents for 82%, 82.3% and 83% slopes and proceeded as follows:

b	FACTOR (from Table 5-3)
-.286304	14.964
-.281036	C
-.268817	14.081

From the ratios $\frac{A}{B}$ $\frac{C}{D}$

we get, A = -.286304 - (-.281036) = -.005268

B = -.286304 - (-.268317) = -.017487

D = 14.964 - 14.801 = .883

so the problem is:

$$\frac{-.005268}{-.017487} = \frac{C}{.833} \quad \text{or } C = .005268\ (.883)/-.017487$$

$$C = .266$$

hence, 14.964 - .266 (again the factor is decreasing so we subtract C, had factor been increasing, C would have been negative) or the factor for 82.3% slope is 14.698 as predicted in the test. The difference is trivial, 1 unit in a thousand.

[NOTE: Cost Improvement Curve slopes are typically presented and/or applied to two decimal places only; i.e., 82% slope, not 82.3% slope. This is because cost data is typically not available to validate accuracy of slope computations beyond 2 decimal places.

APPENDIX H

CALCULATION OF COST IMPROVEMENT CURVE WITHOUT TABLES

When Improvement Curve Tables are unavailable, it may be well to understand the basis of their construction. This will enable one to calculate any "Y" value for any given "X" value by formula. The basic formula for this purpose is $Y=X^b$. For the cost improvement curve application, we know that "b" will have a negative value since it is a downward sloping line. Since Improvement Curve Tables are ratio values, we let "a" equal 1. We now have a formula which states $Y = (1)\ (X^{-b})$ or $Y = X^{-b}$. When X = 1; Y = 1, but when X=2, Y=2-b. Since b is exponential, the formula must be expressed in logarithms: LY = (LX)(-b). If we measure at unit number 2, the Y value will depend upon the slope. For example, for an 80% curve Y = .8 at unit 2; for a 70% curve Y = .7 at unit 2, etc. Therefore, we could say L.8 = (L2)(-b) for an 80% curve. So, -b = L.8/L2 (or, generalizing we may say b=LS/L2 when LS is the log of the slope we have in mind). For an 80% curve, then we can calculate for the b value.

L.8 is 0.0969101 and L2 is 0.301030. Substituting in the formula we have $b = \frac{-.096910}{.301030}$ which gives us a value of -.32192 for b. We can now return to our earlier formula, $Y = X^{-b}$ and find our ratio value for any X for a particular slope.

As an example, suppose we want to find the ratio value for X=15 for an 80% curve. We know the b value for 80% slope is .32192. Our formula would then be:

$$y + X^{-.32192} \quad \text{or} \quad LY = (LX)(-.32192).$$

Since X = 15, we look up the log of 15 which is 1.176091. Substituting, we have: LY = (1.176091)(-.32192) = .378617. (-.32192) + -.378617. Since this is a negative value we obtain the compliment which is .621383. The natural number is therefore .4182 which is the ratio value for unit 15.

Appendix C contains a table of "b" values for each per cent slope from 60 to 99, thus simplifying the calculation by merely referring to the applicable "b" value for the corresponding slope. The only calculation remaining is the multiplication of this value by the log of any X and converting to the natural number.

APPENDIX I

There are a number of Cost Improvement Curve computer software packages available to the analyst. Several of these packages are available on the time share system, the Boeing Computer System (BCS). Others have been developed in-house. Still others have been developed by individual organizations for use on the mini-computers. Listed below are several packages locally available along with the OPR and a telephone number should the reader desire additional information on these packages. Please note: **COMPUTER SOFTWARE PACKAGES DO NOT ANALYZE YOUR DATA -- YOU, THE ANALYST, MUST ANALYZE THE DATA.** Only after a thorough understanding of what your data represents should you select and use a software package. Do not merely respond to computer prompts and blindly accept the computer output.

SOURCE	**OPR**	**TELEPHONE**
BCS	AFIT/LSQ	AV785-6280
Burroughs Cost Curve Programs	AFIT/LSQ Prof. Jeff Daneman	AV785-6280
Z-100 Cost Curve Programs	ASD/ACCR Capt Arthur Mills	AV785-8583

PROGRAMS	**CONCEPT**
ICLOT	Program fits a Unit Cost Improvement Curve to average labor hours or cost for up to 200 lots.
ICPRO	Program computes projected values on a Unit Cost Improvement Curve when the slope and the value of one lot or one unit are known.
CALOT	Program fits a Cumulative Average Cost Improvement Curve to average labor hours or cost for up to 200 lots.
CAPRO	Program computes projected values on a Cumulative Average Cost Improvement Curve when the slope and the value of one lot or one unit are known.

NOTE: Some of these programs use true-lot-mid-point, some use rule-of-thumb; some of these programs use weighted regression (weighted by lot size), some do not. Be sure to read the software documentation to insure what algorythms are being used.

BIBLIOGRAPHY

Andelohr, G., Production Break and Related Learning Loss (No publishing information available)

Asher, Harold, Cost Quantity Relationships in the Airframe Industry, R-291, The Rand Corporation, Santa Monica, California, 1956

Brewer, Glenn M., The Learning Curve in the Airframe Industry; A student thesis, AFIT/LS, Wright-Patterson AFB OH, 1965, pp 89-104, 117-128

Cathcart, G. R., LCDR, and Daneman, Jeffrey; AFIT Remote Terminal System, Draft, pp 27-41

Cochran, E. B., Planning Production Costs: Using the Improvement Curve, 1968, Chandler Publishing Company, San Francisco, California

Hale, J. R., Analyzing Program Changes, AFIT School of Systems and Logistics, Wright-Patterson AFB OH 45433-6583, 1972

-------- Problems with Interruptions in Production Schedules, AFIT School of Systems and Logistics, Wright-Patterson AFB OH 45433-6583, 1961

Kroeker, Herbert R. and Peterson, Robert, A Handbook of Learning Curve Techniques, AFIT School of Systems and Logistics, Wright-Patterson AFB OH 45433-6583, 1961

Wall, Richard L., Major, USAFRs, Orientation Brochure, Financial Management, "An Overview", AFSC/ASD, no date, pp 62-69

DOD 7000.3-G, Preparation and Review of Selected Acquisition Reports, May 1980, pp 4-1 - 4-14