

ASSESSMENT AND COMPARISON OF
TECHNIQUES FOR TRANSFORMING PARAMETERS TO A
COMMON METRIC IN ITEM RESPONSE THEORY

BY

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B.A., University of California, 1979

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THESIS

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University of Illinois at Urbana-Champaign, 1983

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CHAPTER 1

INTRODUCTION

Item Response Theory (IRT) enjoys several theoretical advantages over earlier theories of psychological measurement. One such characteristic, the property of "item parameter invariance," is the central focus of this study. As Lord (1980, p.35) states "The invariance of item parameters across groups is one of the most important characteristics of item response theory."

The property of "invariance" refers to the parameters of the item response function. The shape of the response function is usually described by either the logistic or normal ogive models (see Table 1). We can see from Table 1 that for any given item with parameters a , b , and c , the relation between the ability parameter (θ) and the probability of a correct answer is fully specified. The probability of a correct answer to a particular item, among examinees selected at random with a given ability level θ_0 , depends only on θ_0 , not on the number of people at θ_0 , or on the number of people at other ability levels. Even if two groups differ in their distributions of ability, their response functions will be the same. Examinees with θ_0 from one group have the same probability of passing the item as do examinees with θ_0 from the other group. Thus, the probability of a correct response among individuals with ability θ is independent of group membership, and depends only on θ and the item parameters (a , b , and c). The lower asymptote, the point of inflection, and the slope all remain invariant across groups.

Table 1

Three parameter logistic:

$$P = C + \frac{1 - C}{1 + e^{Da(\theta - b)}}$$

Two parameter logistic:

$$P = \frac{1}{1 + e^{Da(\theta - b)}}$$

Three parameter normal ogive:

$$P = C + (1 - C) \int_{-\infty}^{a(\theta - b)} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

where $D = -1.7$

The property of item parameter invariance has led to several important applications of item response theory. One such application has been in the area of item bias. An item is said to be biased when examinees with the same level of the trait from different subpopulations have different probabilities of answering the item correctly. The basic paradigm used in this problem area involves two independent estimates of item parameters, one from each of two groups. Because of the invariance property, any differences in corresponding item parameters between the two groups (beyond those expected by sampling error) would be an indication of bias.

Item parameter invariance has also suggested a powerful technique for the assessment of the quality of a translated item from one language to a new language. The paradigm is identical to that described under item bias, except here, if the response function is found to be group dependent, the quality of the translation is suspected.

A third important application of item response theory, made possible in part by item parameter invariance, is the development of large, pre-calibrated item pools. One of the most appealing procedures for developing these pools involves administering a number of overlapping tests to separate groups of individuals (McKinley & Reckase, 1981). These tests are overlapping in the sense that they have some items in common. These common items provide the link necessary to place all items on the same scale. The development of large pools of items is especially important for tailored testing.

There is one problem, however, that hampers the application of IRT as described above and which provides the impetus for this paper. The

parameters for the logistic and normal ogive models are invariant only up to a linear transformation of the scale of ability. This problem is caused by the indeterminacy of the origin and unit of measure of the ability scale. As Lord explains:

If a parameter value is in principle indeterminate even when we are given the entire population of observable values, then the parameter is called unidentifiable. Actually, all θ , a_i and b_i (but not c_i) are unidentifiable until we agree on some arbitrary choice of origin and unit of measurement. Once this choice is made, all θ and item parameters will ordinarily be identifiable in a suitable infinite population of examinees and infinite pool of test items. (Lord 1980, p.184-185)

Thus the origin and unit of measure of our ability scale is arbitrary. This causes difficulty when we wish to compare two sets of independently estimated parameters, as outlined in the examples above. Because the decision is arbitrary for each group, there is no assurance that the origin and unit were selected in such a way as to make the two sets of parameters comparable. The purpose of this paper is to investigate techniques that transform one test's metric to the metric of another test and thus permit the direct comparison of all item response functions between the two groups. In addition a new technique for transforming parameters to a common metric is introduced.

Transforming to a Common Metric:

Symptoms and Formalization

In the remainder of this chapter we will work through a hypothetical

example that will demonstrate some common symptoms of the equating problem. In the final section of this chapter, the problem will be formalized and the basic transformation equations presented.

Let us suppose in this example that we are dealing with a test of verbal skills and that our two hypothetical populations are all 5th grade and 6th grade students respectively. Let us further suppose that we are interested in examining a vocabulary test for bias between our 5th and 6th grade populations. That is, we are interested in identifying items that function differently for the two grade levels.

Our first step as indicated by Figure 1, would be to select two samples of 5th and 6th grade students. Next, we administer the test to each group to obtain our item responses. From these item responses we obtain independent estimates of item and person parameters, as indicated at the bottom of Figure 1. Let us suppose we used LOGIST (Wood, Wingersky, & Lord, 1976) to obtain our parameter estimates.

The next step, before comparing any parameters directly, would be to transform one set of parameters to the scale of the other set. That is, to transform the parameters to a common metric. For now, however, let us observe the consequences of ignoring the equating phase altogether. That is, let us observe symptoms of the equating problem when we attempt to compare parameters directly after estimation.

Figure 2 displays the two hypothetical histograms for our 5th and 6th grade samples that we would expect to obtain from our LOGIST estimated thetas. The ordinate represents the proportion of examinees observed at each level of theta, on a basis of the LOGIST estimated thetas. (In reality we would not expect our observed histograms to be

Figure 1

Outline of Procedure of Hypothetical Example

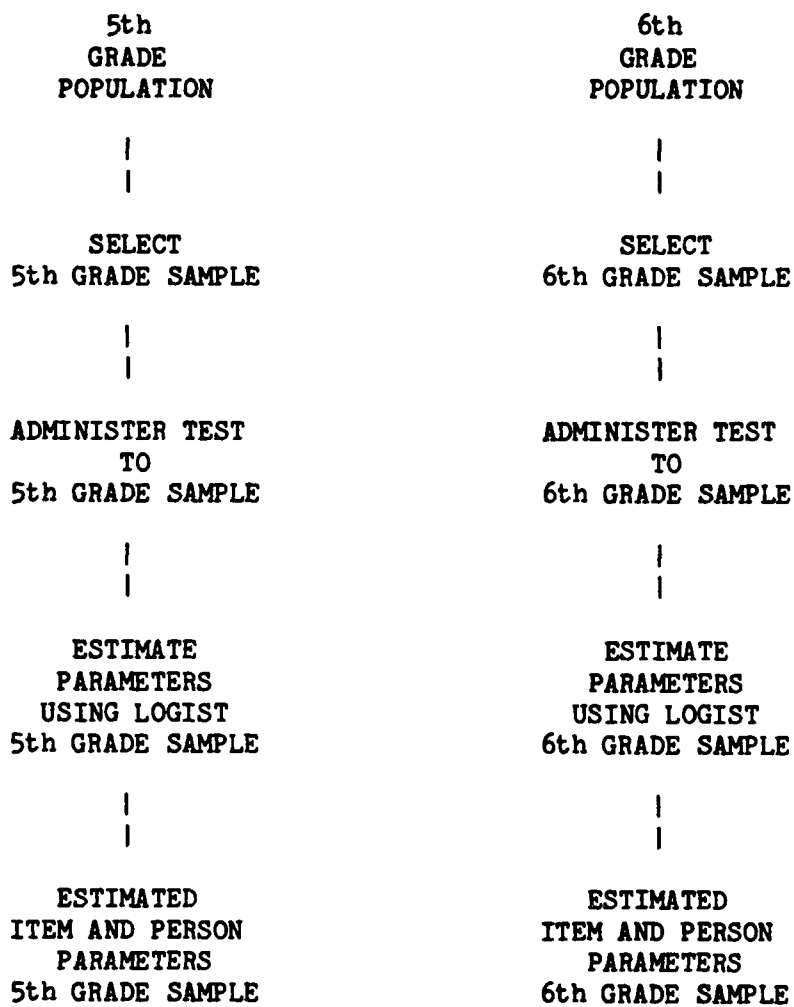
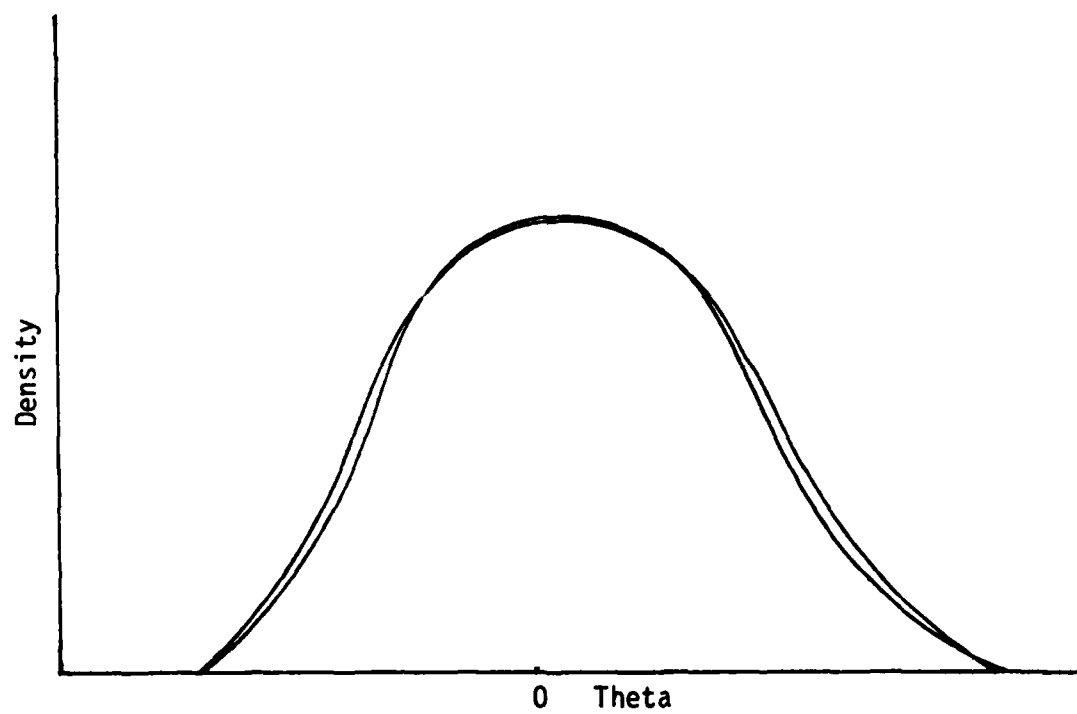


Figure 2

Histograms for 5th and 6th Grade
Ability Estimates from LOGIST
(Example)



as smooth as those in Figure 2) Notice that the two histograms for our two sets of independently estimated ability parameters overlap to a high degree. In fact, just by observing these two distributions, one might be easily convinced that their mean and standard deviations are identical. But this observation does not support what we know about the verbal ability of 5th versus 6th grade students. We would expect, with a great deal of certainty, that the mean of our estimated thetas for the 6th grade sample would be significantly higher than the mean of our 5th grade sample. But when we observe our independent estimates of the ability parameters for our two groups, their mean and standard deviation appear identical.

This latter occurrence is no accident. Remember, that because the origin and unit of measure are arbitrary in item response theory, our estimation procedure is free to select any values. LOGIST selects the origin and unit of the measurement scale such that they correspond to the mean and standard deviation (respectively) of the estimated person parameters. Thus, for each group of independent parameter estimates, the origin of our scale was set to the mean of our estimated thetas, and the unit was set equal to the standard deviation. It is not surprising that the first two moments of our estimated 5th and 6th grade distributions look identical. The scale of ability was selected using these moments as criteria. When these criteria are used to select the origin and unit, no matter to what extent the mean and standard deviation differ between two groups, they will always appear identical.

Now, let us observe the symptoms of the equating problem when we attempt to compare estimated item parameters, without first transforming

to a common metric.

Suppose for the first item of our vocabulary test we obtained the two items with parameter estimates given in Table 2. ICC's for these two curves are given in Figure 3. The two overlapping ability distributions are represented by the u-shaped curves along the base line. By examination of either Table 2 or Figure 3, we might conclude (if we were unaware of an equating problem), that the first item is biased against the 5th grade sample. That is, for most levels of ability, the probability of obtaining a correct response is larger for 6th graders than for the 5th graders. But, when we examine the item parameter estimates for the rest of the items in the test, we notice a similar pattern. The b_i 's for the 5th grade sample appear consistently higher than the 6th grade estimates, while the a_i 's are slightly smaller for the 5th grade group. Could every item on the test be biased? Not likely. These problems are all symptoms of side-stepping the equating stage before attempting to compare the two independent sets of parameters.

Let us now consider how to resolve the discrepancy between the two independent estimates of these item parameters through equating to a common metric. Remember that, in accordance with item response theory, the origin and unit of measure for each of our scales is arbitrary. Thus we can apply a linear transformation to either or both of our scales (base lines) and not violate any assumptions of IRT. A convenient terminology has been developed (Linn, Levine, Hastings & Wardrop, 1981) that helps clarify the basic issues in developing a common metric. This terminology involves the arbitrary designation of

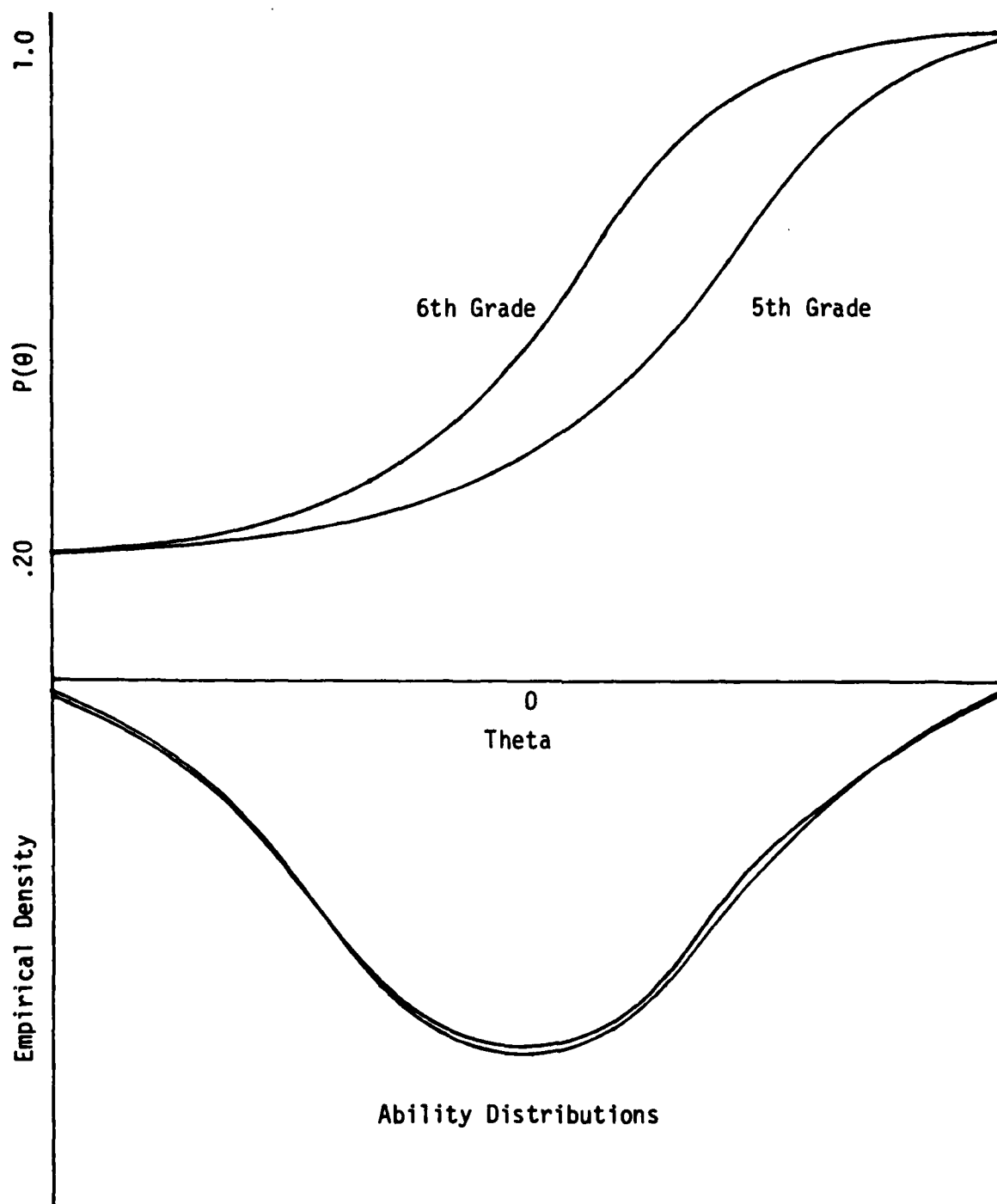
Table 2

Estimated Item Parameters for Hypothetical Example

FIFTH GRADE ITEM PARAMETERS		SIXTH GRADE ITEM PARAMETERS	
<u>a</u> 1	.95	<u>a</u> 1	1.00
<u>b</u> 1	.79	<u>b</u> 1	0.00
<u>c</u> 1	.20	<u>c</u> 1	.20

Figure 3

Item Parameter (first item) and Ability Distribution Estimates
from LOGIST (before equating)



one of our two groups as the "base group" and the other as the "comparison group." The scale and parameters of our base group are held fixed, while the parameters of our comparison group are transformed to the scale of the base group. Thus, after transformation of the comparison group parameters, the scale of our base group will be the scale on which all our estimated parameters will be measured.

For now, let us hold the scale of our 6th grade group fixed (thus designating it as the base group) and apply a linear transformation only to the scale of our 5th grade sample (which now becomes our comparison group). Our new scale will be defined as:

$$\theta^* = A \times \theta + B \quad [1.1]$$

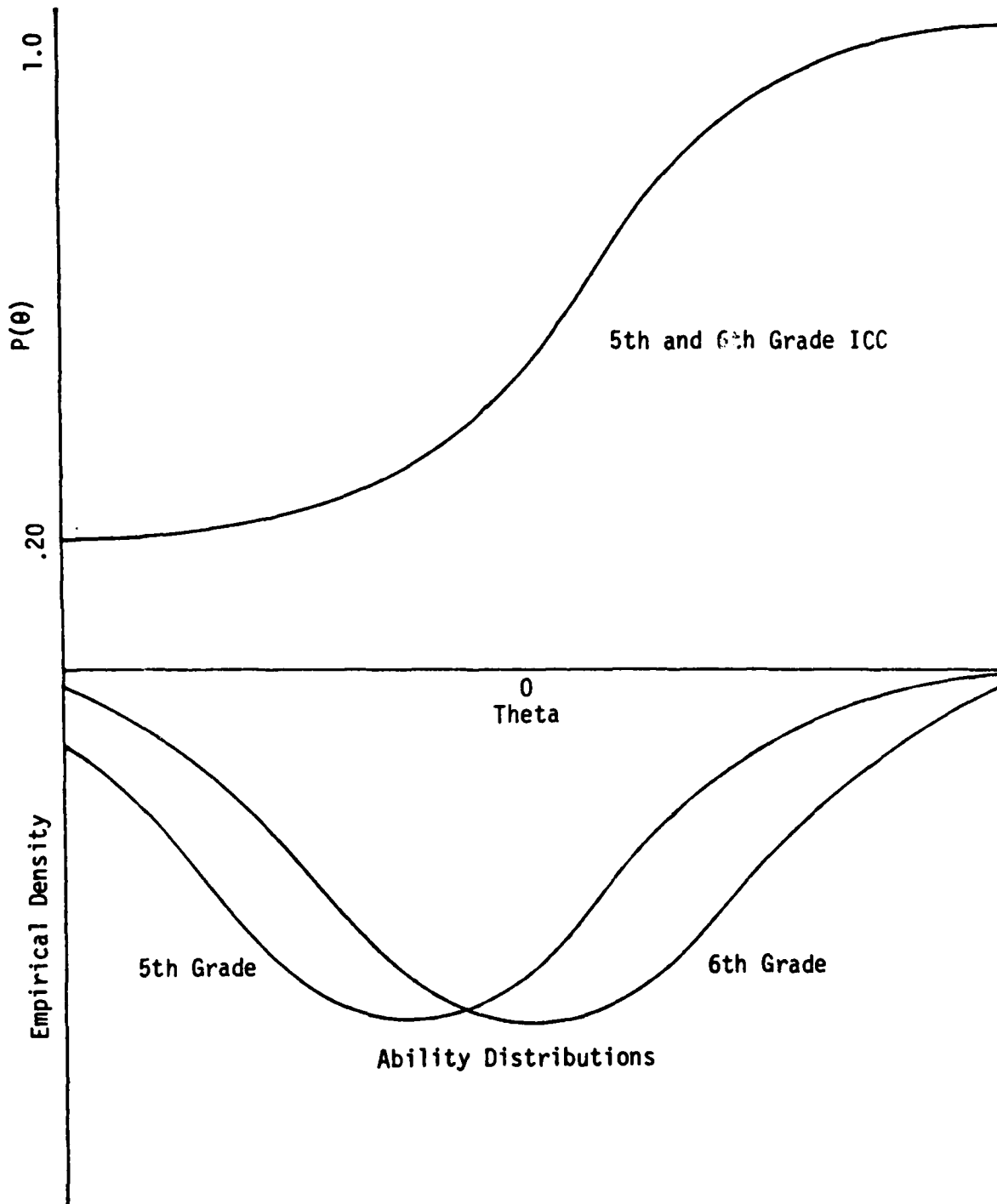
where A and B are the components of the linear transformation that transform points on the old comparison group scale (θ) to points on the new equated base group scale (θ^*).

Returning to the example in Figure 3, we can see that if we move the scale for our comparison group (5th graders) to the left, and then contract it slightly, the two item characteristic curves will line up exactly, as displayed in Figure 4. Notice that as we moved (transformed) the scale for the 5th grade ICC, the ability distribution for that group moved along with it. Thus after the transformation we observe an item characteristic curve that is identical for both groups. We also observe two distinct ability distributions, with our 5th grade sample scoring, on the average, lower than our 6th grade sample, as we would have expected on a basis of prior knowledge.

If this same exact transformation of scale were applied to all the other items of our comparison group, we would observe similar results.

Figure 4

Item Parameter (first item) and Ability Distribution Estimates
from LOGIST (after equating)



That is, our ICC's should match up, just as they did in the above example.

As implied by equation [1.1] we can quantify the transformation required to convert the scale of our comparison group parameters (θ) to the metric (θ^*) of our arbitrarily designated base group. In the example above, we found the transformation for our 5th grade sample that placed it on the same metric as our 6th grade sample. Finding the "correct" transformation involves finding the correct values of A and B in equation [1.1]. These particular values will depend on several factors. First, the values of A and B will depend on the rule that our estimation procedure uses for assigning values to the origin and unit of the scale for each set of parameter estimates. If this rule is consistent across data sets, then indirectly this transformation will also depend on the differences in the distributions of ability of the two groups from which our independent estimates were calculated.

Once we have found the correct values for A and B, we can apply the same scale transformation to every item in the test for the group requiring the transformation (comparison group). The point to be stressed here, is that the scale transformation is identical for every item of the test. That is, the values of A and B are not item specific. Rather, they (and the transformation they represent) are constant across items. Because the values of A and B are constant across items, they are often referred to as "equating constants".

After the values of our equating constants (A and B) have been identified, the values of all the item parameters for the comparison group may be transformed to the new scale using the following equations:

$$\underline{a_i^*} = \underline{a_i} / A \quad [1.2a]$$

$$\underline{b_i^*} = A \times \underline{b_i} + B \quad [1.2b]$$

$$\underline{c_i^*} = \underline{c_i} \quad [1.2c]$$

$$\underline{Qa^*} = A \times \underline{Qa} + B \quad [1.2d]$$

These equations represent the new values of the parameters for the comparison group that place them on a common metric with the base group. It is important to remember that under this paradigm, we are transforming the parameters for only one of our two groups (the comparison group). The parameters for the base group remain completely unaffected by these transformations. That is, their values go unaltered throughout the entire equating process. The asterisk in equations [1.2a-d] represent the values of the comparison group parameters transformed to the base group scale. The subscript *i* refers to item *i*; the *A* and *B* are the equating constants introduced in equation [1.1]; and the parameters without the asterisk are the comparison group parameters before transforming to the base group metric.

At this point, a few comments about equations [1.2a-d] would be in order. Let us begin with the transformation of $\underline{c_i}$. Why does this parameter remain unaffected by a linear transformation of the theta scale? If we refer back to Table 1, we see that for both the three parameter logistic and the normal ogive models, the $\underline{c_i}$ represents the probability of making a correct response to item *i* by a randomly sampled individual with a theta of minus infinity. It is readily apparent that any linear transformation of the theta scale is not going to alter the position of minus infinity, and therefore will not effect the value of $\underline{c_i}$. Next, if our scale transformation is given by [1.1] then our \underline{Qa} and

$\underline{b_i}$ must follow this same transformation because they are both based on this same metric. Finally, we must remember that any transformation of our parameters must not change the probability of a correct response to an item given a particular level of ability. That is, a change of the numerical scale value attached to a particular level of ability does not alter the conditional probability of passing the item. In typical models of the type with which we are concerned here, this probability is a function of $a_i(Q_a - b_i)$. The item discrimination transformation is given by: $a_i^* = a_i / A$. The reason for this form is that for any admissible values of A , B , $\underline{a_i}$, $\underline{b_i}$, $\underline{c_i}$,

$$a_i^*(Q_a^* - b_i^*) = a_i(Q_a - b_i).$$

Because our probability values are functions of these quantities, they too remain unaltered.

In summary, the entire problem of converting item parameters to a common metric hinges on identifying the correct linear transformation of our comparison group scale. If the parameter estimates were error free, as in the example given in this chapter, the problem would have a simple solution: for any item find the linear transformation of the scale for one group that causes the ICC for that item to match the ICC for the same item in the other group. Once this transformation has been identified, all parameters could be transformed according to equations [1.2a-d].

Unfortunately the above procedure is not generally applicable. This is because our parameter estimates are not error free. Some of the difference between corresponding item parameters estimated from

independent samples may be explained in terms of differences in metric, while another portion of this difference may be due to error in parameter estimation. For any given problem, the exact contribution from each source is difficult to determine. Several approaches, however, have been suggested to identify the appropriate linear transformation when error of estimation is present. These techniques are described in the following Chapter.

CHAPTER 2
TECHNIQUES FOR TRANSFORMING PARAMETERS TO A COMMON METRIC
IN ITEM RESPONSE THEORY

Having identified the basic problem in the previous chapter, this chapter will examine seven approaches that have been proposed to find the appropriate scale transformation that places two or more independently estimated sets of parameters on a common metric. A theoretical presentation, along with a discussion of criticisms of each technique is given. These techniques can be roughly classified into three categories. The first category involves three approaches that rely on information supplied by the estimated b-parameters from each group. These approaches all find the transformation that equates the first two moments of the distribution of estimated b_i 's between groups. They differ in the way poorly estimated difficulty parameters are treated.

The second class of techniques incorporates test and item characteristic curves to estimate the equating constants necessary for transforming to a common metric. Stocking and Lord (1982) suggest a method that examines a weighted test characteristic curve, while the two methods suggested by Haebara (1980), and Segall & Levine (1983) examine weighted sums of squared differences between corresponding ICC's.

Finally, the last technique, suggested by Segall examines vectors of estimated item parameter differences for corresponding items from the

two groups. This technique finds the values of the equating constants that maximizes a criterion related to the likelihood of observing these vectors of item parameter differences.

Equating Metrics using the First Two Moments of the Distributions of Estimated Item Difficulties

If the difficulty parameters for our two groups of independent parameter estimates were measured without error, the task of finding a common metric would be greatly simplified. Remember that from eq. [1.2b] we have:

$$\underline{b_i^*} = A \times \underline{b_i} + B. \quad [1.2b]$$

If we examined the $\underline{b_i}$ values for any two items (say items 1 and 2), we would have a system of two linear equations with only two unknowns (A and B):

$$b1^* = A \times b1 + B$$

$$b2^* = A \times b2 + B$$

Solving these equations for our equating constants A and B would be a simple task. Unfortunately our $\underline{b_i}$'s are not measured without error, so this approach would very likely produce poor estimates of our A and B.

Instead, let us examine an approach that incorporates information supplied by all the estimated $\underline{b_i}$'s from both groups. The basic motivation for this approach stems from the premise that if both our comparison and base groups are on equivalent scales, then the mean and standard deviation (SD) of the estimated $\underline{b_i}$'s should also be equivalent

across groups. Intuitively this approach has some appeal, for both, the mean and SD aggregate over individual $\underline{b_i}$'s, allowing for errors of measurement contained in these estimates to cancel out (or so it is hoped). On the other hand, if our two sets of parameter estimates are not on equivalent metrics, we would not expect to observe equivalent means and SD's of the estimated $\underline{b_i}$'s for the two groups. In this case, however, there exists a linear transformation of the theta scale for one group, that will equate the mean and SD of our estimated $\underline{b_i}$'s for the two groups. Once we find this linear transformation, we can then apply the components (A and B) of this transformation to all the parameters of the comparison group (as indicated by eq. [1.2a-d]).

We may now formalize the problem under this approach as one of finding the linear transformation of the theta scale for the comparison group by equating the first two moments of the difficulty parameters across groups. We may further elaborate our goal as one of finding the values of A and B such that:

$$\text{and} \quad \bar{b}^*(\text{comp}) = \bar{b}(\text{base}) \quad [2.1a]$$

$$SD^*(\text{comp}) = SD(\text{base}) \quad [2.1b]$$

where

$$\bar{b}^*(\text{comp}) = \frac{\sum_{i=1}^n [A \underline{b_i}(\text{comp}) + B]}{n} \quad [2.1c]$$

and

$$SD^*(comp) = \sqrt{\frac{\sum_{i=1}^n [Ab_i(comp) + B - \bar{b}^*(comp)]^2}{n}} \quad [2.1d]$$

and where $\bar{b}(base)$ = mean of base group b-parameters, and

$SD(base)$ = the standard deviation of the base group b-parameters.

We may simplify [2.1c] to obtain:

$$\bar{b}^*(comp) = A \times \bar{b}(comp) + B \quad [2.3]$$

and by substituting [2.3] into [2.1d], we can simplify and obtain:

$$SD^*(comp) = A \times SD(comp) \quad [2.4]$$

Now from eq [2.1b] and eq [2.4] we have:

$$A \times SD(comp) = SD(base) \quad [2.5]$$

Solving eq [2.5] for A, we obtain:

$$A = SD(base) / SD(comp) \quad [2.6]$$

And now, from eq [2.1a] and eq [2.3] we obtain:

$$\bar{b}(base) = A \times \bar{b}(comp) + B \quad [2.7]$$

Solving this equation for B:

$$B = \bar{b}(base) - A \times \bar{b}(comp) \quad [2.8]$$

and substituting eq [2.6] for A:

$$B = \bar{b}(base) - [SD(base)/SD(comp)] \times \bar{b}(comp) \quad [2.9]$$

Thus equations [2.6] and [2.9] specify the expressions for our equating constants. When these constants are applied to the scale of

our comparison group, the first two moments of the distributions of estimated item difficulties are equal to those of the base group. Once we obtain the values of our equating constants (A and B) from eq [2.6] and [2.9] respectively, we can transform all the parameters of the comparison group to the metric of the base group by use of equations [1.2a-d].

There is, however, a serious shortcoming with the procedure as outlined above. The shortcoming centers around our error of estimate for the b-parameters used to find our A and B. As pointed out previously, by basing our A and B on the mean and SD of our estimated item difficulties, we hope that errors of measurement contained in the difficulty parameter estimates cancel out. However, this may not happen. Poorly estimated difficulties may have a large influence on the sample moments, producing equating constants that are poor indicators of the transformation necessary to equate the two groups of parameters. Several "fix-ups" have been proposed to deal with the problem of the effect of poorly estimated difficulties on the sample moments. Two of these "fix-ups" are described in the following sections.

Difficulty Parameter Equating with Restricted Range of Discrimination and Difficulty Parameter Values

One way to reduce the effect of poorly estimated b_i 's on the computation of sample moments is to exclude items with extreme difficulty values (eg. $|b_i| > 3$). The error of estimate of the b_i 's

for these items is high relative to items with moderate difficulty values. Also, items with low discrimination values (a_i 's) should be excluded from the computation of the sample moments (eg. $|a_i| < .15$). These items also have large sampling variances for the b_i 's. The goal is to obtain a smaller set of better estimated b_i 's for each group from which to compute our sample moments, as outlined in the previous section.

As might be anticipated, one of the major drawbacks of this approach is that it is heuristic in nature, offering no firm statement as to which items to exclude. Rather, we have only a few rules of thumb to follow. In an attempt to remedy this condition, and to control for the effects of the poorly estimated b_i 's on the sample moments in a more systematic manner, Linn, Levine, Hastings and Wordrop (1980) suggest the procedure outlined in the following section.

Difficulty Parameter Equating using Weighted Moments

Linn, Levine, Hastings and Wardrop (1980) controlled the effects of poorly estimated b_i 's by the use of weights that are inversely proportional to the estimated variance of the estimated item difficulties. (See equations 2.11a-b.) The weights are applied to the b-parameters for each group, producing weighted means and standard deviations from which our A and B are derived as outlined in the previous section. Thus, items with large standard errors of their b_i , would receive less weight in the computation of the means and SD's

relative to items with small standard errors of their $\underline{b_i}$.

The weighted indices are computed as follows:

STEP 1: First item covariance matrices are computed by inverting the 3x3 information matrix for each item, for each group. Formulas for the elements of the information matrix are given in Lord (1980, p.191), for the three parameter logistic model. Thus, for each item, two covariance matrices are computed, one from parameter estimates of the base group, and the other from the comparison group parameter estimates.

STEP 2: Next, the diagonal element for the variance of the difficulty parameter is extracted from each pair of covariance matrices for corresponding items; one variance term coming from the base group covariance matrix, and the other term from the comparison group covariance matrix. The larger of the two variance estimates is used in computing the weight for that item. If we let $V_i(\text{base})$ and $V_i(\text{comp})$ be the estimated sampling variances of $\underline{b_i}(\text{base})$ and $\underline{b_i}(\text{comp})$ respectively, the weight for item i is:

$$W_i = \begin{cases} 1 / V_i(\text{base}) & \text{if } V_i(\text{base}) \geq V_i(\text{comp}) \\ 1 / V_i(\text{comp}) & \text{if } V_i(\text{comp}) \geq V_i(\text{base}) \end{cases} \quad [2.10]$$

The effect of selecting the larger of the two variances was to give the greatest weight to those items that possessed relatively small estimated sampling variances in both groups. If the difficulty parameter was poorly estimated in either sample (base or comparison), then it would receive a small weight relative to a $\underline{b_i}$ that was well estimated in both samples.

Notice, however that there may be problems with comparing the two estimates of the sampling variances $V_i(\text{base})$ and $V_i(\text{comp})$ at this point.

The value of the V_i 's is dependent (in part) on the unit of the scale on which the b_i 's are measured. That is, we would expect that the choice of unit for either our base or comparison groups would effect the respective values of these estimated variances. (The exact nature of this relationship is given by eq [2.31b]) Since the two scales for our base and comparison groups are not on equivalent metrics at this point, comparisons of these sampling variances across groups may not be appropriate.

STEP 3: The next step involves using these weights to compute the weighted means and SD of the b_i 's for the base and comparison groups. The weighted means of the b_i 's are computed for each group as:

$$\bar{b}_w(\text{base}) = \frac{\sum_{i=1}^n [W_i \times b_i(\text{base})]}{k} \quad [2.11a]$$

$$\bar{b}_w(\text{comp}) = \frac{\sum_{i=1}^n [W_i \times b_i(\text{comp})]}{k} \quad [2.11b]$$

The weighted SD for each group is computed as:

$$SD_w(\text{base}) = \sqrt{\frac{\sum_{i=1}^n W_i [b_i(\text{base}) - \bar{b}(\text{base})]^2}{k}} \quad [2.12a]$$

$$SD_w(\text{comp}) = \sqrt{\frac{\sum_{i=1}^n W_i [b_i(\text{comp}) - \bar{b}(\text{comp})]^2}{k}} \quad [2.12b]$$

where

$$k = \sum_{i=1}^n W_i$$

STEP 4: Once the weighted means and SD's have been computed for the comparison and base groups using the above formulas, they can be

incorporated into formulas [2.6] and [2.9] just as their unweighted counterparts to find the values of our equating constants A and B.

The procedure described above attempts to control the influence of the sampling error of the \hat{b}_i 's. It is interesting to note that although the error of estimate for the difficulty parameter for a particular item may be relatively high, we may still know a great deal about the shape of the item characteristic curve. Because the shape of the ICC is determined by the values of all three item parameters, a procedure that relies on the shape of the ICC's may be more informative than these procedures that examine only the \hat{b}_i 's. In the next section, we turn to a class of techniques that use all three item parameters (\hat{a}_i , \hat{b}_i , and \hat{c}_i) in an attempt to find the linear transformation necessary to develop a common metric.

Sums of Squared Differences Between Estimated True Scores

Stocking and Lord (1982) suggest a technique that uses true scores to find the comparison group scale transformation. Each member of an arbitrarily selected group possesses an estimated true score. That is, an examinee, a, with ability θ_a , has an estimated true score $\hat{\xi}_a$ defined by:

$$\hat{\xi}_a = \frac{1}{n} \sum [P_i(\theta_a; \hat{a}_i, \hat{b}_i, \hat{c}_i)] \quad [2.13]$$

If two different calibrations of the same test resulted in parameter estimates that were based on comparable metrics, then we would expect the difference in true scores for examinee a, from these two calibrations to be small. If on the other hand, the parameter estimates for the two calibrations were not based on equivalent metrics, then we would expect to see a larger discrepancy in the two true score estimates, for examinee a, based on the two sets of item parameter estimates. These observations suggest the following approach. Utilizing our familiar terminology, we may represent the true score estimate for a member of our base group as:

$$\hat{\xi}_a(\text{base}) = \sum_{i=1}^n [P_i(\theta_a(\text{base}); a_i\text{-hat}(\text{base}), b_i\text{-hat}(\text{base}), \\ c_i\text{-hat}(\text{base}))] \quad [2.14a]$$

Notice that this estimate incorporates parameter estimates (ai-hat, bi-hat, ci-hat) obtained from our base group calibration. We may specify an alternative true score estimate for members of our base group as:

$$\hat{\xi}_a^*(\text{base}) = \sum_{i=1}^n [P_i(\theta_a(\text{base}); a_i\text{-hat}^*(\text{comp}), b_i\text{-hat}^*(\text{comp}), \\ c_i\text{-hat}(\text{comp}))] \quad [2.14b]$$

where this estimate is computed from comparison group parameter estimates, that are transformed to the base group scale. (These transformations are given in eq [1.2a-b].) Thus, we would like to find the values of A and B that would minimize the difference $[\hat{\xi}_a(\text{base}) - \hat{\xi}_a^*(\text{base})]$. Stocking and Lord propose the following function to be

minimized:

$$F = (1/N) \times \sum_a^N [\hat{\xi}_a^-(\text{base}) - \hat{\xi}_a^*(\text{base})]^2 \quad [2.15]$$

We wish to find values of A and B such that the average squared difference between the two true score estimates for members of our base group is a minimum. Ideally, to find the value of A and B that minimize [2.15] we would take the partial derivatives with respect to A and B, set these expressions equal to zero, and then solve for A and B. This approach, however is not possible in this instance because there is no closed form solution for A and B once the partials are set equal to zero. Thus, to find the values of our equating constants, an iterative numerical procedure must be used.

There are several observations that may help clarify this procedure. First, we should note that we are only dealing with true score estimates from members of our base group. True score estimates from members of the comparison group do not enter into the computations. Of course, the decision of which group is base versus comparison is arbitrary. Accordingly, the decision as to which group of true scores will be examined by this procedure is also arbitrary. The point to be stressed, however, is that only the true score estimates from one of the two groups are used.

A second observation worth noting is that in eq [2.14a-b] we acted as if we were using the true theta values rather than their estimated values. In practice, the true thetas are never known and the estimated ability parameters are used in their place.

A final observation might serve to clarify the role of our A and B in the minimization of F (eq [2.15]). Our A and B equating constants influence the values of $\hat{a}_i(\text{comp})$ and $\hat{b}_i(\text{comp})$ in expression [2.14b]. Their exact influence is specified by eq [1.2a-b].

Another way to conceptualize the function to be minimized by the current procedure is by: the squared difference between the test characteristic function for the base group, and the "transformed" test characteristic function for the comparison group, where this squared difference is weighted by the number of base group examinees occurring at each value of θ . This conceptual approach is easily reconciled with the concept of squared differences between true scores by remembering that both true score, and a point along the test characteristic curve (TCC), may be expressed as:

$$\hat{\xi}_a = \text{True-Score}(\theta_a) = n \times \text{TCC}(\theta_a) = \sum_{i=1}^n P_i(\theta_a) \quad [2.16]$$

(Where n represents the number of items in the test.) We can think of each θ_a along the test characteristic curve as being weighted by the number of base group examinees with ability " θ_a ".

This latter conceptualization of the current approach helps bridge the gap between this technique and the next two methods described in the following section. In an attempt to find the proper scale transformation for the comparison group, the two techniques described in the following section examine the squared differences between corresponding item characteristic curves, rather than the squared

differences of the test characteristic curve. An additional distinguishing feature of the current method and the two that follow, is in the manner by which estimated θ_a 's are used to weight these "squared differences." Whereas Stocking and Lord use estimated θ 's from only the base group, both of the following techniques use information supplied by both groups, base and comparison, in deriving weights for developing the linear transformation necessary to form a common metric.

Squared Difference Between Corresponding Item Characteristic Curves

Haebara's Method:

As in each of the previous techniques, our task is to find the linear transformation of the comparison group scale that places the estimated parameters of this group and those of our base group on a common metric. Haebara (1980) suggests a method that finds this transformation by examining the sum (across items) of the weighted sum of squared differences between corresponding ICC's.

If our item parameters were measured without error, then using any item i , we could find the values of A and B , such that for every value of θ :

$$P_i(\theta(\text{comp}); a_i(\text{comp}), b_i(\text{comp}), c_i(\text{comp})) = \\ P_i(\theta(\text{comp}) \times A + B; a_i(\text{base}); b_i(\text{base}), c_i(\text{base})) \quad [2.17]$$

Notice that [2.17] implies perfect equating. Since our item parameters

are not measured without error, we would not expect this relationship to hold exactly. Instead, we would like to find the values of A and B such that the equality in [2.17] holds as closely as possible across items in our test. This objective motivates the following specification for the criterion function. Let:

$$\begin{aligned} \text{ERia}(\text{comp}) &= P(\text{Oa}(\text{comp}); \text{ai}(\text{comp}), \text{bi}(\text{comp}), \text{ci}(\text{comp})) \\ &- P(\text{Oa}(\text{comp}) \times A + B; \text{ai}(\text{base}), \text{bi}(\text{base}), \text{ci}(\text{base})) \quad [2.18] \end{aligned}$$

Then one candidate for our criterion function would be:

$$Q(\text{comp}) = \sum_i^n \sum_a^{Nc} [\text{ERia}(\text{comp})]^2 \quad [2.19]$$

That is, for each examinee in our comparison group, on each item, we examine the squared difference between two probabilities. One probability is obtained from the estimated item parameters for our comparison group (by way of the logistic or normal ogive models). The other probability is obtained from transforming Oa to the base group metric and employing our estimated base group parameters for that item. Finally, we sum the squared differences of corresponding probabilities across people in our comparison group, and then across items.

For practical reasons (dealing with computation time and computer storage), Haebara uses an approximation to the quantity in [2.19] that incorporates a relative frequency distribution of $\theta(\text{comp})$ rather than using each individual value. This relative frequency distribution $h(\text{comp})$ of $\theta(\text{comp})$ is constructed by dividing the range of $\theta(\text{comp})$ into

k small intervals with the midpoints $Q_j(\text{comp})$ (where $j=1,2,\dots,k$). Then minimizing $Q(\text{comp})$ is approximately equal to minimizing $Q_h(\text{comp})$, that is define as:

$$Q_h(\text{comp}) = \sum_{i=1}^n \sum_{j=1}^k [ER_{ij}(\text{comp})]^2 \times h_j(\text{comp}) \quad [2.20a]$$

Notice however that this quantity is based on equating errors from members of our comparison group only. Haebara defines an analogous quantity that expresses the contribution of our base group examinees to the equating criterion. This quantity is defined as:

$$Q_h(\text{base}) = \sum_{i=1}^n \sum_{j=1}^k [ER_{ij}(\text{base})]^2 \times h_j(\text{base}) \quad [2.20b]$$

where $h(\text{base})$ is the relative frequency distribution of our base group examinees and:

$$\begin{aligned} ER_{ij}(\text{base}) &= P_i(Q_j(\text{base}); a_i(\text{base}), b_i(\text{base}), c_i(\text{base})) \\ &- P_i([Q_j(\text{base}) - B]/A; a_i(\text{comp}), b_i(\text{comp}), c_i(\text{comp})) \end{aligned} \quad [2.21]$$

Notice that if we allow the quantity:

$$Q^*(\text{base}) = Q(\text{comp}) \times A + B \quad [2.22a]$$

to represent the transformation of our comparison group ability parameter to the base group metric, then solving [2.22a] for $Q(\text{comp})$, we obtain:

$$Q^*(\text{comp}) = [Q(\text{base}) - B] / A \quad [2.22b]$$

which represents the transformation of our base group ability parameters to the comparison group metric, as implied by [2.21]. Then Haebara suggests:

$$Q^* = Q_h(\text{comp}) + Q_h(\text{base}) \quad [2.23]$$

as the final form of the criterion function. Our task has now been reduced to one of finding the values of our equating constants A and B that minimize Q^2 . As in the case of the technique suggested by Stocking and Lord (1982) there is no closed form solution for A and B. Thus, to find the value of A and B that minimize [2.23] we must employ an iterative numerical procedure.

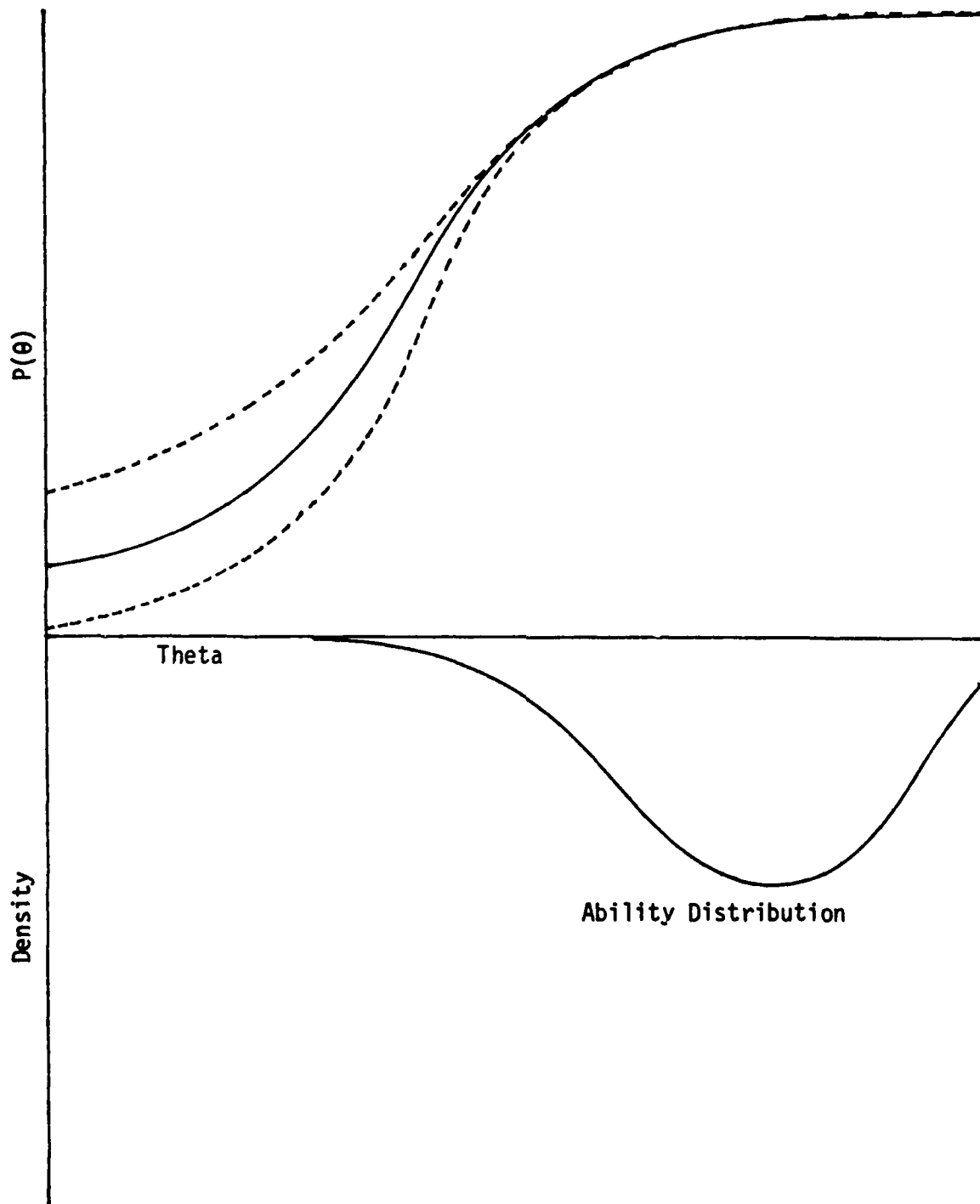
One criticism with Haebara's technique deals with how the squared differences between corresponding ICC's are weighted. To understand fully the rationale behind this criticism, a slight diversion might be helpful.

Figure 5 displays an arbitrary item characteristic curve (solid line) and a hypothetical distribution of examinees (represented by the u-shaped curve along the base line). If we used a sample from this distribution of examinees to estimate the shape of the ICC in Figure 5, we could very likely end up with an estimated curve represented by either of the dashed lines in Figure 5. Notice that where we have the greatest number of examinees, the agreement between the true ICC (solid line) and our estimates is very close. Where we have very few, or no examinees, the discrepancy between our estimated and true ICC may be very large. In general, we can place a great deal of confidence in the shape of a particular segment of an ICC, if in the region of that segment we have a substantial number of examinees. Conversely, if along a particular segment of an ICC we have very few examinees, we should place very little confidence in its estimated shape.

We may generalize this argument to our present situation, where we have two estimates for every ICC, each estimated from a potentially

Figure 5

Relation of True ICC and Hypothetical Estimates of the ICC
with Distribution of Ability

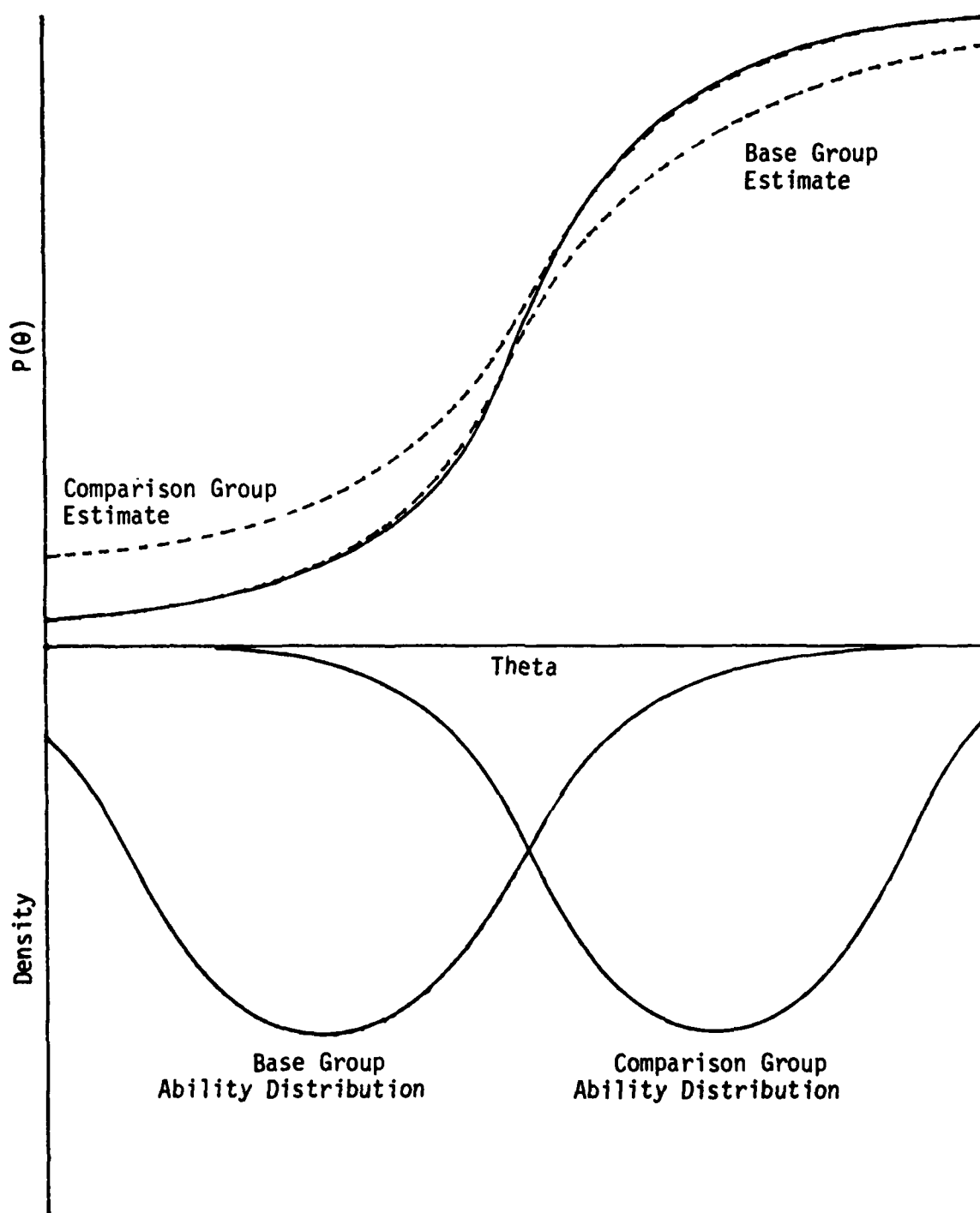


different distribution of ability. A hypothetical example is given in Figure 6. In this illustration we have two distributions of ability as indicated by the two u-shaped curves along the base line. From each sample obtained from these hypothetical distributions, we derive an estimate for our true ICC. (The true ICC is indicated by the solid line, and each of the estimates, from each sample, by the dashed lines.) Notice, as we might expect, that the base group estimate is very close to the true ICC along the range where there are a large number of base group examinees. Similarly, the estimated comparison group ICC is very close to the true ICC along the range where there are a large number of comparison group examinees. Also, as we might have anticipated, the discrepancy between the true and estimated ICC can be very large in areas where there are very few or no examinees, from the corresponding group, from which to derive the estimate.

The weighting scheme suggested by Haebara (1980) weights those segments of the estimated ICC differences in accordance with the relative frequency of examinees falling in the region. Returning to Figure 6, the weight function for the base group $h(\text{base})$ would weight the squared differences between one curve that was estimated very well along this range (the base group estimate) and one curve that was estimated very poorly along this range (the comparison group estimate). An analogous point can be made concerning the weight function for the comparison group, $h(\text{comp})$. The point to be stressed here is that a substantial portion of the weighted squared differences between corresponding ICC's may be due to error of estimation when a weighting scheme such as the one suggested by Haebara is employed.

Figure 6

Relation of True ICC and Two Independent Estimates of the ICC
With Two Different Distributions of Ability



Segall and Levine (1983) suggest a weighting scheme that attempts to eliminate the above criticism. Their approach is outlined in the following section.

Segall and Levine Method:

The quantity to be minimized by the technique suggested by Segall and Levine (1983) is Q_f , define as:

$$Q_f = \sum_{i=1}^n \sum_{j=1}^k [ER_{ij}]^2 \times f_j^* \quad [2.24]$$

where:

$$ER_{ij} = P_i(\theta_j(\text{comp}); a_i(\text{comp}), b_i(\text{comp}), c_i(\text{comp})) \\ - P_i(\theta_j(\text{comp}) \times A + B; a_i(\text{base}), b_i(\text{base}), c_i(\text{base})) \quad [2.25]$$

f_j^* represents a new weight function which is obtained in the following manner. First the relative frequency distribution of $\theta(\text{comp})$ is constructed by dividing the range of $\theta(\text{comp})$ into k small intervals with the midpoints $\theta_j(\text{comp})$. Next these relative frequencies are transformed to relative proportions to produce $f_j(\text{comp})$. Then, $f_j(\text{comp})$ is transformed to the scale of our base group (using our equating constants A and B) and $f_j(\text{base})$ is computed using these transformed cut-points on our distribution of base group examinees. Our complete weight function is then computed as:

$$f_j^* = \sqrt{[f_j(\text{comp}) \times A + B] \times [f_j(\text{base})]} \quad [2.26]$$

Returning to Figure 6, notice that our new weight function will be largest over the range where the overlap of the two estimated distributions of ability is the greatest. The weight function will be

smallest, or zero, where we have little or no overlap of our two estimated distributions of ability. This weighting scheme places the heaviest emphasis on that portion of the squared difference between corresponding ICC's that are relatively well estimated in both groups. That portion of the squared difference between corresponding ICC's that is computed from an ICC segment which is poorly estimated for either group, receives a small or zero weight.

As in the case of the two previous methods there is no closed form expression for the minimization of Q_f in eq [2.24], so an iterative numerical procedure must be used.

The method described in the following section utilizes a somewhat different approach to control the influence of parameter sampling error on the estimation of our equating constants. This technique, suggested by Segall(1982), employs estimated covariance matrices for our item parameters in the framework of a maximum likelihood estimation procedure.

Estimation of Equating Constants

Using Vectors of Item Parameter Differences

In this section a method is introduced by adopting maximum likelihood estimation concepts to the problem of estimating the equating constants. A heuristic discussion will be used to review the reasoning that led to the method. Of course, the heuristic argument is not

essential. First consider one item and the two vectors of item parameter estimates

$$\alpha_i(\text{base}) = \begin{bmatrix} a_i(\text{base}) \\ b_i(\text{base}) \\ c_i(\text{base}) \end{bmatrix}$$

$$\alpha_i(\text{comp}) = \begin{bmatrix} a_i(\text{comp}) \\ b_i(\text{comp}) \\ c_i(\text{comp}) \end{bmatrix}$$

Since these vectors are maximum likelihood estimates from large samples, they will be approximately multivariate normal. Since they are estimated from different samples they will be independent, and any linear combination of them will be multivariate normal.

Let A_0 and B_0 denote the "true" equating constants, i.e. the unique pair of constants that transforms the ability scale of the comparison group to the ability scale of the base group after the two scales have been specified. For each A and B the vector

$$v_i = v_i[A, B] = \begin{bmatrix} a_i(\text{base}) \\ b_i(\text{base}) \\ c_i(\text{base}) \end{bmatrix} - \begin{bmatrix} a_i(\text{comp}) / A \\ b_i(\text{comp}) \times A + B \\ c_i(\text{comp}) \end{bmatrix} \quad [2.27]$$

must be multivariate normal because it is a linear combination of multivariate normal vectors.

The covariance matrix of v_i can be easily determined from the covariance matrices of $\alpha_i(\text{base})$ and $\alpha_i(\text{comp})$. Maximum likelihood

estimation theory can be used to estimate the covariance matrices of each of the component vectors. For each possible value of the equating constants v_i will be multivariate normal with an approximated covariance matrix $C(A,B)$. (In fact, $C(A,B)$ is independent of B , but that fact is not needed here.)

If only the expectation of the random vector v_i were known, its multivariate normal density could be specified. If A and B are equal to A_0 and B_0 respectively, and the maximum likelihood estimates are based on large enough samples to be considered unbiased, the expectation $E[\alpha_i(\text{base})]$ will equal the linearly transformed $E[\alpha_i(\text{comp})]$, and v_i will have expectation equal to zero.

To summarize, for each A and B the hypothesis $A=A_0$ and $B=B_0$ implies that v_i is multivariate normal with zero expectation and specified covariance matrix. The hypothesis implies a specific formula for the multivariate density of v_i . If the estimates for different items were independent, then the joint distribution of all the v_i would also be multivariate normal with density

$$L[v_1(A,B), v_2(A,B), \dots, v_n(A,B) | A=A_0 \text{ and } B=B_0] = \prod_{i=1}^n L[v_i(A,B) | A=A_0 \text{ and } B=B_0] \quad [2.28]$$

Unfortunately the estimates for different items are not completely independent when item and person parameters are estimated simultaneously. Minor dependencies are expected and observed when the same group of subjects are used in the estimation of each item. The method introduced in this section ignores these interdependencies and

treats formula [2.28] as the joint conditional density of the v_i . An estimate of A_0 and B_0 is obtained by maximizing this joint density.

This admittedly heuristic argument led to the formulation of a new technique which in fact performs quite well. The vectors of item parameters for corresponding items are treated as if they were independent multivariate normal vectors with covariance matrices specified by inverting estimated information matrices. A and B are estimated by maximizing the joint density under the hypothesis that $A=A_0$ and $B=B_0$. It will be seen that inspite of the correlations between the parameters of different items, that the method performs quite well. Some details on the implementation of the method follow.

For the present let us develop our criterion on a basis of one item only. The generalization to an n -item test is straight forward and will be discussed later in this section. For the moment, our task is to make explicit a probability function for the v_i where the estimated vectors

$$\begin{bmatrix} a_i(\text{base}) \\ b_i(\text{base}) \\ c_i(\text{base}) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a_i(\text{comp}) / A \\ b_i(\text{comp}) \times A + B \\ c_i(\text{comp}) \end{bmatrix}$$

are independently sampled from a multivariate normal population with known covariance matrix. That is, we seek a formula for

$$\text{Prob}\{v_i(A,B) | A=A_0, B=B_0\} \quad [2.29a]$$

where A_0 and B_0 are the true equating constants. The more compact and suggestive notation

$$L(v_i | A=A_0, B=B_0) \quad [2.29b]$$

will be used to denote this multivariate probability density. Because

the item parameters are maximum likelihood estimates of essentially the same parameters in each group, we can regard the \mathbf{v}_i as normally distributed with expectation equal to zero. That is, because the expectation of each vector of item parameters is their true value, and because their true values are the same for corresponding items i , then the expectation of their difference is a zero vector.

Next, we would like to examine the sampling variance of our estimated item parameters. For the moment, let us concern ourselves with a single vector of parameter estimates. Maximum likelihood theory specifies that the variance - covariance for these estimates may be obtained from the inverse of the information matrix (\mathbf{I}). When the ability parameters are known, formulas for the elements of the information matrix are given in Lord (1980, p.191). Thus the sampling variance-covariance (\mathbf{C}_i), for item i , of our item parameter estimates may be obtained from:

$$\mathbf{C} = \mathbf{I}^{-1} = \begin{bmatrix} I_{aa} & I_{ab} & I_{ac} \\ I_{ba} & I_{bb} & I_{bc} \\ I_{ca} & I_{cb} & I_{cc} \end{bmatrix}^{-1} \quad [2.30]$$

We will have two covariance matrices for each item. One covariance matrix from our base group parameter estimates and the other from our comparison group parameter estimates. We may represent these as:

$$\mathbf{C}(\text{base}) = \mathbf{I}^{-1}(\text{base}) = \begin{bmatrix} C_{aa}(\text{base}) & C_{ab}(\text{base}) & C_{ac}(\text{base}) \\ C_{ba}(\text{base}) & C_{bb}(\text{base}) & C_{bc}(\text{base}) \\ C_{ca}(\text{base}) & C_{cb}(\text{base}) & C_{cc}(\text{base}) \end{bmatrix} \quad [2.31a]$$

$$C(\text{comp})^{-1} = I(\text{comp})^{-1} = \begin{bmatrix} Caa(\text{comp})/A^2 & Cab(\text{comp}) & Cac(\text{comp})/A \\ Cba(\text{comp}) & Cbb(\text{comp}) \times A^2 & Cbc(\text{comp}) \times A \\ Cca(\text{comp})/A & Ccb(\text{comp}) \times A & Ccc(\text{comp}) \end{bmatrix} \quad [2.31b]$$

Notice that certain values of the elements of the covariance matrix for the comparison group, in eq [2.31b], depend on the value of our equating constant A. This is because we are transforming the metric of the comparison group to that of the base group, and we would expect this transformation to have some effect on our variance - covariance elements which were computed in our original comparison group metric.

Next, because our two estimates of parameters for an item are independent (each coming from a different group), the sampling variance of our vectors of parameter differences (v_i) is equal to:

$$C_{di} = C_i(\text{base}) + C_i^*(\text{comp}) \quad [2.32]$$

Finally to specify the criterion based on eq [2.29] we can examine the surface of a tri-variate normal density function, with $N[0, C_{di}]$:

$$f_i(v_i|A,B) = (2\pi)^{-3/2} |C_{di}|^{-1/2} \exp(-cs/2) \quad [2.33]$$

where:

$$cs = x_i' C_{di}^{-1} x_i \quad [2.34]$$

and:

$$x_i = v_i - E(v_i) = v_i - 0 = v_i \quad [2.35]$$

Thus [2.33] gives us a criterion related to the likelihood of obtaining a single vector of parameter differences for given values of A and B.

To obtain the analogous quantity that incorporates information supplied by all the items in our test, we formulate the following objective function,

$$L(v_1, v_2, v_3, \dots, v_n | A=A_0 \text{ \& } B=B_0) = \prod_{i=1}^n f(v_i | A, B) \quad [2.36]$$

To find the values of A and B that maximize [2.36] an iterative numerical procedure is employed.

The next two chapters present a comprehensive comparison of the techniques outlined in this chapter. The relative abilities of these techniques to estimate accurately the linear transformation necessary to develop a common metric are assessed. This assessment involves, both simulated and real data, covering a variety of conditions.

CHAPTER 3
ASSESSMENT OF EQUATING TECHNIQUES
USING SIMULATED DATA

This chapter presents a study whose objective is to assess the ability of each technique described in the previous chapter to accomplish its intended goal: to transform two sets of parameters to a common metric. To accomplish this assessment, two different approaches were taken.

The approach described in this chapter involves the use of simulated responses based on a simulated test and several different distributions of examinees. The use of simulated data provides greater control and knowledge concerning the true relation between the two sets of independently estimated parameters. Because the true relation between the two sets of parameters is known with simulated data, firm statements can be made concerning the ability of each technique to recover this relation. The main problem, however, with simulated data, is that it is based on a model whose assumptions are almost certainly violated to varying degrees by real people answering real items.

As an answer to this criticism, Chapter 4 presents an approach that examines the relative ability of the equating techniques to recover the proper transformation using "real" data. "Real" is used here in the sense that the data are actual responses to items on an actual test. The obvious drawback of this approach is that the true relation between the two sets of estimated parameters is not known. This presents a special problem for evaluating the accuracy of the estimated equating

constants. This problem, along with one solution, is presented in Chapter 4.

Experimental Design using Simulated Data

Unfortunately, there are an infinite number of simulated tests and simulated distributions of examinees that could be selected for inclusion in this study. Because it is feasible to examine only a few different simulated tests and distributions of examinees, we should select these carefully. First, it would be desirable to structure the test as closely as possible to the type of test that we would find in practice. Similarly, the distributions of ability should also be modeled after the types of distributions commonly observed. Modeling our simulated test parameters and item responses as closely as possible to actual tests makes generalization to "real" data easier. The design presented below attempts to specify values of these parameters that are similar to those found in many applied testing situations.

The Test

Table 3 lists the item parameters for a 60 item test used to generate the dichotomous item responses. The a-parameters of this test were specified by sampling numbers from a uniform distribution in the interval $[.3, 1.4]$. The b values were sampled from a uniform distribution in the interval $[-3., +3.]$, and the c-parameters were drawn from a uniform distribution in the interval $[\cdot 11, \cdot 33]$. These parameters identify the items included in tests of three different

Table 3

Item Parameters for Simulated Test
Used to Create Simulated Response Vectors

ITEM	<u>a</u>	<u>b</u>	<u>c</u>
1	.7948	-2.8090	.2373
2	.3031	2.6931	.2594
3	1.0263	-.8141	.1978
4	.8035	-2.4044	.1300
5	.5906	-1.3875	.2613
6	.8063	1.8960	.1477
7	1.2265	-.6636	.2661
8	.3778	1.6944	.2494
9	1.2698	1.8150	.2885
10	.8935	.9261	.1688
11	.7382	.8374	.3083
12	1.3302	-1.8851	.2026
13	1.2925	2.9963	.1886
14	1.3972	1.4620	.2623
15	1.0194	1.2102	.1791
16	1.3878	.3561	.1377
17	1.0037	2.2912	.1355
18	.7901	.1613	.1786
19	.7742	-1.3758	.3221
20	1.0598	1.4167	.2454
21	.9478	2.8291	.3085
22	.9324	-1.5024	.1467
23	1.2279	2.8816	.3175
24	1.2978	-1.3201	.2998
25	.9555	.8139	.2150
26	1.0750	-.7296	.2105
27	.9491	1.6064	.3189
28	.6723	-1.0248	.3271
29	.4617	2.0528	.2669
30	.5510	1.7964	.3047
31	1.1574	.8547	.2619
32	.9271	.2455	.3261
33	.5457	-1.4978	.2188
34	1.3730	2.3566	.2960
35	.9273	.9723	.2323

(Table 3 continued)

ITEM	<u>a</u>	<u>b</u>	<u>c</u>
36	1.3273	2.6651	.2454
37	.5960	2.5812	.2540
38	1.3299	2.8950	.1447
39	1.2710	1.4454	.2847
40	.8222	-.6592	.2898
41	.6442	-2.9478	.2106
42	.4047	-2.5030	.2071
43	.5023	-1.5370	.2074
44	.7106	-1.7613	.2441
45	.7164	2.4118	.2937
46	.9033	-1.2684	.2822
47	.9312	-.1995	.2227
48	1.0233	1.3889	.2237
49	1.1059	2.8210	.2981
50	.6400	.8782	.1829
51	.8730	-.5741	.3145
52	.4710	-.7691	.2760
53	1.2326	-2.0874	.2392
54	1.3953	-1.5240	.1539
55	.7350	.5290	.1776
56	1.2183	-.6919	.2190
57	.4101	-.1285	.1582
58	1.2872	1.0834	.1707
59	1.2091	-1.5559	.3179
60	1.2616	-1.9413	.1828

lengths. In one set of conditions, Conditions 1 through 5 (see Table 4), parameters for all 60 items were used to generate dichotomous responses for a test of length 60. For Condition 6, the first 30 items parameters listed in Table 3 were used to generate responses for a test of length 30. And finally, in Condition 7, the first 15 items listed in Table 3 were used to generate responses for a test of length 15.

Ability Distributions

Several pairs of base group - comparison group ability distributions were examined. These pairs constitute the basic conditions of the simulation portion of this study.

In each condition, the mean and SD of the distribution of ability for the base group remained unchanged. These distributions were normal, with mean equal to zero, and standard deviation equal to one. In each condition, base group ability parameters were sampled from a normal $[0,1]$ distribution. Because these sampled values were to be treated as true parameters, they were transformed to have zero mean and unit variance. (This transformation is similar to a z-score transformation.)

Several different comparison group ability distributions were specified, one for each condition listed in Table 4. Each of these distributions were generated by sampling values from a normal distribution. For the Conditions 1 through 7, the comparison and base group distributions differed by varying amounts. Table 4 lists the means and standard deviations for the ability distributions used in each condition. Notice that the mean and standard deviation are constant across conditions for the base group (with mean equal to zero and SD

Table 4

Summary of Simulated Conditions

Condition	Number of Subjects	Number of Items	Base Group		Comp Group		Scale Transformation	
			Mean	SD	Mean	SD	A	B
1	1000	60	0.0	1.0	-0.5	.80	.80	-0.5
2	500	60	0.0	1.0	-0.5	.80	.80	-0.5
3	250	60	0.0	1.0	-0.5	.80	.80	-0.5
4	500	60	0.0	1.0	0.0	.80	.80	0.0
5	500	60	0.0	1.0	-1.0	.80	.80	-1.0
6	1000	30	0.0	1.0	-0.5	.80	.80	-0.5
7	1000	15	0.0	1.0	-0.5	.80	.80	-0.5

equal to one). On the other hand, the mean and standard deviation for the comparison group did not necessarily remain constant from one condition to the next.

Samples of several different sizes were generated: 250, 500, and 1000 simulated examinees. In each condition the number of comparison group examinees equaled the number of base group examinees.

Generation of Item Responses

Data, or item responses for these simulated examinees were generated in accordance with Birnbaum's (1968) three parameter logistic model (see Table 1). Note that once the item parameters are specified (Table 3) the probability of a correct response to an item is solely a function of examinee ability. As examinee ability increases, so does the probability of a correct response. The probability of a correct response can be used to generate an observable dichotomous right-wrong response by comparing it to a uniformly distributed random number between 0 and 1. A response was coded as correct when its associated probability was greater than the random number, and incorrect when it was less.

Using this procedure 14 data sets of dichotomous responses were generated, two data sets per condition (Table 4). Each data set contained the equivalent of N examinees answering an n item test. The item parameters from Table 3 along with person parameters sampled from a normal distribution were used in the above procedure to generate the data sets.

Estimation of Item and Person Parameters

The LOGIST computer program (Wood, Wingersky & Lord, 1976) was used to estimate all person and item parameters for each data set described above. Briefly:

Given the responses of a group of examinees to a set of items, the computer program, LOGIST, has been developed to estimate the item characteristic curve parameters for each item and the ability of each examinee in terms of Birnbaum's three parameter logistic model. The parameters are estimated by a method analogous to the maximum likelihood method described by Lord (1968) with the likelihood function modified to handle omits. (Wood, & Lord, 1976, p.1)

Of special interest, for our purpose is the method LOGIST uses to specify the origin and unit of the measurement scale. Remember that these are arbitrary, and both person and item parameters are unidentifiable until these have been specified. LOGIST selects the unit and origin in such a manner that the final theta estimates ($\hat{\theta}$) have a mean of zero and a standard deviation of one, for all estimated thetas inside the range $-\text{THLIM}$ to $+\text{THLIM}$. THLIM is either specified by the user or the default of 3.0 is used.

The method LOGIST uses to select the unit and origin was an important consideration in the selection of criteria used to judge the relative ability of the equating techniques to transform all parameters to a common metric. In a following section, several such criteria are discussed.

Estimation of Equating Constants

Each of the seven approaches described in Chapter 2 was used to

estimate the A and B equating constants for each condition listed in Table 4. For each of the seven conditions in Table 4 there was an associated pair of base group - comparison group estimated parameter sets from LOGIST. These base and comparison group parameters were input into each of the seven equating techniques. Notice that within each condition, each equating technique used as data the same estimated base and comparison group parameters. To obtain the estimated equating constants, computer programs were written for each of the seven techniques. These programs are listed in the APPENDIX.

Criteria for Recovery of the Equating Constants

The least complicated criterion for determining how closely each technique recovered the true transformation is to compare the estimated equating constants with the true values of these constants. The true value of these constants, for each condition, are listed in the last two columns of Table 4. Notice that the true values of the equating constants listed in the last two columns of table 4 are identical to the mean and SD of the comparison group distributions, listed in the previous two columns of Table 4. This relation was anticipated on the basis of the method used by LOGIST to specify the unit and origin of the theta metric. Remember that to generate a normal distribution with mean equal to B and a SD equal to A, one can take the values from a normal [0,1] distribution and apply the following linear transformation:

$$\theta_j^* = \theta_j \times A + B \quad [3.1]$$

We have the identical situation with our LOGIST estimated person parameters. Remember that LOGIST fixes the mean of the estimated thetas to zero and their standard deviation to one. Thus the parameters for the base group will automatically be set to their original metric. The estimated thetas for the comparison group, however, will also have a mean of zero and a standard deviation of one. Then, it follows that the A and B components of the linear transformation used to set the estimated comparison group parameters to their original metric will correspond exactly to the true SD and mean, respectively, of the comparison group distribution for that condition. These values are listed in Table 4.

To judge the accuracy of each of the seven approaches, a comparison of the estimated equating constants with the true values can be made. The technique whose estimates come the closest to the true A and B can be ranked highest; the technique whose estimates come next closest, can be ranked second, etc.

One drawback with this approach is exemplified by the situation in which the estimated A constant of one equating technique is closest to the true A, while the estimated B constant of another equating technique is closest to the true B. Thus, in addition to examining the size of the difference between our true and estimated constants for each technique, it would be desirable to have an additional criterion that incorporated the effect of errors in both, the A and the B constants simultaneously.

One such criterion might involve the root mean squared error (RMSE) difference between the true thetas and the estimated comparison group

thetas, after equating. That is, from each of the seven equating techniques, under each condition, we obtain values of our A and B equating constants. From eq [1.2d], we can use these values to transform the estimated thetas of our comparison group. Accordingly one criterion would be:

$$RMSE_{tc} = \sqrt{\frac{1}{N} \sum_a^N [\theta_{ac}(\text{comp}) - (\hat{A}_{tc} \times \hat{\theta}_{ac}(\text{comp}) + \hat{B}_{tc})]^2} \quad [3.2]$$

where the subscript t represents equating technique; c represents Condition; and N equals the number of subjects in the comparison group. The transformed thetas in the above RMSE belong to the comparison group only. If the estimated equating constants are close to their true values, we would expect the RMSE to be relatively small. On the other hand, we would expect a relatively large value of the RMSE for poorly estimated equating constants.

Notice that the size of the RMSE described by eq [3.2] is influenced by two factors. First, the size of each RMSE is influenced by error in the estimation of the comparison group thetas. For poorly estimated thetas we would expect a relatively large RMSE. For good estimates of these thetas we would expect to see a small RMSE. Second, the RMSE in eq [3.2] is also influenced by the error in the estimated equating constants. This is the portion of the RMSE that is of primary interest for our purposes. One possible improvement over the criterion given by [3.2] would be an index that was influenced only by the errors in the estimated equating constants. That is, it would be desirable to have an

index that was influenced only by errors due to equating and not by errors of estimate in the comparison group thetas. This desire motivates the following specification for a criterion that can be used to judge the accuracy of the equating constant estimates in each condition:

$$RMSE_{tc} = \sqrt{\frac{1}{N} \sum_a \left(\left\{ \hat{A}_{tc} [(\theta_{ac} - B_c) / A_c] + \hat{B}_{tc} \right\} - \theta_{ac} \right)^2} \quad [3.3]$$

Notice that the transformation in the inner brackets represents one that will give the true comparison group thetas a mean of zero and SD of one (just as they would have if LOGIST had estimated these parameters without error). This quantity is then transformed to the base group metric using the estimated constants. Finally the squared difference between the transformed theta and its true value is obtained and summed across examinees, to obtain the final RMSE. Notice that this index uses only true theta values, and thus avoids the problem associated with sensitivity to errors in person parameter estimation. The index given in eq [3.3] was computed for each technique in each of the seven conditions. Values of this index are listed in Tables 6 through 12.

Results

Tables 6 through 12 summarize the results of the simulated portion of this study. Each table lists the results of one condition given in Table 4. For each technique the estimated equating constants are given, along with the RMSE given by eq [3.3].

Table 13 list the average of the RMSE values across all seven

conditions for each technique. These averages were computed by taking the RMSE values listed in Tables 6 through 12, and averaging the RMSE values across the seven conditions, for each technique.

Table 5
Key for Tables 6 through 13

TECHNIQUE	Description
ALL b's	b-parameter equating (using all b_i 's)
SELECTED b's	b-parameter equating (using well estimated b_i 's)
WEIGHTED b's	b-parameter equating (using weighted b_i 's)
TRUE SCORE	True score equating (Stooping & Lord)
ICC (H)	ICC equating (Haebara)
ICC (S/L)	ICC equating (Segall & Levine)
MLE	MLE equating based on vectors of item parameter differences

Table 6
(Condition 1)

A = .80		B = -.50	
NUMBER OF SUBJECTS			
BASE GROUP: 1000		COMPARISON GROUP: 1000	
NUMBER OF ITEMS: 60			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	.3679	-.5940	.4419832
SELECTED b's	.7456	-.6102	.1228830
WEIGHTED b's	.7797	-.5707	.0735640
TRUE SCORE	.7491	-.5191	.0543682
ICC (H)	.7695	-.5238	.0387308
ICC (S/L)	.7557	-.5257	.0512494
MLE	.7758	-.5025	.0242760

Table 7
(Condition 2)

A = .80 B = -.50			
NUMBER OF SUBJECTS			
BASE GROUP:	500	COMPARISON GROUP:	500
NUMBER OF ITEMS: 60			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	.1984	-.1578	.6916400
SELECTED b's	.8415	-.4719	.0500991
WEIGHTED b's	.7937	-.4246	.0756928
TRUE SCORE	.7249	-.4560	.0869498
ICC (H)	.7700	-.4637	.0470628
ICC (S/L)	.7236	-.4733	.0808992
MLE	.7708	-.4533	.0550536

Table 8
(Condition 3)

A = .80 B = -.50			
NUMBER OF SUBJECTS			
BASE GROUP:	250	COMPARISON GROUP:	250
NUMBER OF ITEMS: 60			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	.9640	-1.5273	1.0402315
SELECTED b's	.8006	-.4035	.0964754
WEIGHTED b's	.9011	-.4419	.1164287
TRUE SCORE	.7738	-.4799	.0329480
ICC (H)	.8038	-.4359	.0642359
ICC (S/L)	.7254	-.4655	.0820452
MLE	.7816	-.4146	.0873095

Table 9
(Condition 4)

A = .80 B = 0.00			
NUMBER OF SUBJECTS			
BASE GROUP:	500	COMPARISON GROUP:	500
NUMBER OF ITEMS: 60			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	2.2061	-.5012	1.4914301
SELECTED b's	.8067	-.0186	.0197350
WEIGHTED b's	.8509	-.0159	.0532301
TRUE SCORE	.7797	.0095	.0224022
ICC (H)	.8101	.0234	.0254557
ICC (S/L)	.7777	.0092	.0241489
MLE	.8196	.0526	.0561538

Table 10
(Condition 5)

A = .80 B = -1.00			
NUMBER OF SUBJECTS			
BASE GROUP:	500	COMPARISON GROUP:	500
NUMBER OF ITEMS: 60			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	.3859	-2.5905	1.6433882
SELECTED b's	.8670	-.8234	.1888696
WEIGHTED b's	.8725	-.8123	.2012114
TRUE SCORE	.7645	-.8925	.1132213
ICC (H)	.8104	-.8636	.1367862
ICC (S/L)	.7612	-.8802	.1259345
MLE	.8050	-.8475	.1525574

Table 11
(Condition 6)

A = .80		B = -.50	
NUMBER OF SUBJECTS			
BASE GROUP: 1000		COMPARISON GROUP: 1000	
NUMBER OF ITEMS: 30			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	.2734	-.0522	.6910274
SELECTED b's	.7294	-.4557	.0832805
WEIGHTED b's	.8112	-.4340	.0669083
TRUE SCORE	.6850	-.4549	.1234248
ICC (H)	.7520	-.4466	.0718140
ICC (S/L)	.7390	-.4478	.0802616
MLE	.7470	-.4416	.0787914

Table 12
(Condition 7)

A = .80		B = -.50	
NUMBER OF SUBJECTS			
BASE GROUP: 997		COMPARISON GROUP: 1000	
NUMBER OF ITEMS: 15			
TECHNIQUE	A	B	RMSE-COMP
ALL b's	.5796	-.6473	.2650133
SELECTED b's	1.0096	-.4619	.2129480
WEIGHTED b's	.8863	-.2355	.2781787
TRUE SCORE	.8776	-.5151	.0790203
ICC (H)	.9005	-.3897	.1491785
ICC (S/L)	.8600	-.3953	.1206400
MLE	.9383	-.3644	.1936142

Table 13

Root Mean Squared Errors
Averaged Over all Seven Conditions
For Each Equating Technique

TECHNIQUE	MEAN-RMSE
ALL b's	.8950
SELECTED b's	.1106
WEIGHTED b's	.1236
TRUE SCORE	.0732
ICC (H)	.0762
ICC (S/L)	.0807
MLE	.0925

CHAPTER 4
ASSESSMENT OF EQUATING TECHNIQUES
USING REAL DATA

This section examines the relative ability of the equating techniques to estimate the transformation using "real" data. Real in this context means the data are responses to items made by real people on an actual test, as opposed to simulated data described in Chapter 3.

The major emphasis of this portion of the study is to examine the effect that naturally occurring violations to the assumptions of IRT have on the equating procedures. We suspect that such assumptions as local independence, fit of the three parameter logistic model, and unidimensionality are violated to some degree by real people answering real items. By examining the performance of the equating techniques using real data, we may gain some insight into the effect these naturally occurring violations have on the performance of the equating techniques.

Study I

Data

Data for these analyses were obtained from the Anchor Test Study (Bianchini & Loret, 1974) equating study files. Item response data from the Word Knowledge (50 items) and the Reading Comprehension sections (45

items) of form F of the Metropolitan Achievement Tests (Durost, Bixler, Wrightstone, Prescott, & Balow, 1970) were obtained. The Reading Comprehension and Word Knowledge sections were combined and treated as a single 95 item test in the analyses described below. The subjects consisted of 2000 5th and 6th grade, white and black examinees.

Assignment of Subjects into Base and Comparison Groups

The 2000 subjects were randomly assigned to the base and comparison groups. This assignment resulted in 1000 examinees in each group.

Note that this random assignment of subjects results in expected values of the equating constants of $A=1$ and $B=0$. This is because random assignment of subjects results in expected distributions of ability that are equivalent across groups. If the distributions are equivalent, then their first two moments are also identical. Remember that LOGIST sets the unit of the scale equal to the SD of the estimated thetas and the origin of the scale equal to the mean of the estimated thetas. If the expected values of the mean and SD are equivalent for the base and comparison groups then the linear transformation that places the comparison group scale on the metric of the base group is simply:

$$\theta^* = A \times \theta + B$$

where $A=1$ and $B=0$.

Estimation of Item and Person Parameters

The LOGIST computer program (Wood, Wingersky & Lord, 1976) was used to estimate all person and item parameters for each group independently.

Results

Each of the seven approaches described in Chapter 2 were used to estimate the A and B equating constants. The base and comparison group parameter estimates from LOGIST were used as input for each of the seven techniques. Table 14 lists the estimated equating constants from each of the seven techniques.

Notice that all the techniques did extremely well at estimating the scale transformation. The first observation to be made is one concerning the effect of naturally occurring violations of the IRT model. With these data, none of the techniques appeared to be adversely effected by violations to the model. This is evidenced by the close agreement of all the estimated equating constants to their expected values.

The observation that all the techniques did well is not surprising. In this condition, there were a relatively large number of subjects in each group, each subject answering a relatively long (95 item) test. Perhaps most importantly for the three b-parameter equating techniques was the heterogeneity of the samples used to estimate these item parameters. As a result of selecting two samples with wide range of abilities (and in part due to the test's suitability to these samples), all the b-parameters had small sampling errors. Thus the transformation based on these 95 well estimated b-values was very close to the expected transformation.

Because all the techniques did so well in estimating the transformation, it may be interesting to assess the performance of these techniques under less ideal conditions than the one discussed above. The study outlined in the following section examines the performance of the

Table 14

MAT

<hr/>		
A = 1.00 B = 0.00		
<hr/>		
<hr/>		
NUMBER OF SUBJECTS		
BASE GROUP: 1000 COMPARISON GROUP: 1000		
<hr/>		
<hr/>		
NUMBER OF ITEMS: 95		
<hr/>		
<hr/>		
TECHNIQUE	A	B
ALL b's	1.0042	.0011
SELECTED b's	1.0042	.0011
WEIGHTED b's	1.0079	-.0160
TRUE SCORE	1.0154	-.0154
ICC (H)	1.0151	-.0136
ICC (S/L)	1.0148	-.0131
MLE	1.0157	-.0144
<hr/>		

See key on p.62 for identification of techniques.

equating techniques when smaller samples and fewer items are used.

Study II

Data

Data for these analyses were again obtained from the Anchor Test Study (Bianchini, et al.) equating study files. In this study, only item response data from the Reading Comprehension section (45 items) of form F of the Metropolitan Achievement Tests (Durost, et al.) were used. A new sample of 1000 5th and 6th grade, white and black examinees was used. Note that the examinees used in these analyses were exclusive of those used in Study I.

Assignment of Subjects into Base and Comparison Groups

The 1000 subjects were randomly assigned to the base and comparison groups. This assignment resulted in 500 examinees in each group. Notice that this random assignment of subjects again resulted in expected values of the equating constants of $A=1$ and $B=0$.

Estimation of Item and Person Parameter

The LOGIST computer program (Wood, et al.) was used to estimate all item and person parameters for each group independently.

Results

Each of the seven approaches described in Chapter 2 were used to

estimate the A and B equating constants. The base and comparison group parameter estimates from LOGIST were used as input for each of the seven equating techniques. Table 15 lists the estimated equating constants from each of the seven techniques.

Notice, from Table 15, that although the estimated transformations are not as close to the expected values as were those from Study I, the estimates from the present study all appear in fairly close agreement. Again, the relatively good performance of the three b-parameter techniques is most likely a result of the heterogeneous samples used to estimate these values.

The implications of the finding of these results along with those of the simulation portion of this study are discussed in detail in the following Chapter.

Table 15

MAT

<hr/>		
A = 1.00 B = 0.00		
<hr/>		
<hr/>		
NUMBER OF SUBJECTS		
BASE GROUP:	500	COMPARISON GROUP: 500
<hr/>		
<hr/>		
NUMBER OF ITEMS: 45		
<hr/>		
<hr/>		
TECHNIQUE	A	B
ALL b's	.9227	-.0058
SELECTED b's	.9227	-.0058
WEIGHTED b's	.9186	.0029
TRUE SCORE	.8902	.0410
ICC (H)	.9077	.0265
ICC (S/L)	.9071	.0265
MLE	.9071	.0297
<hr/>		

See key on p.62 for identification of techniques.

CHAPTER 5
DISCUSSION AND IMPLICATIONS
FOR THE SELECTION OF A TECHNIQUE TO
TRANSFORM PARAMETERS TO A COMMON METRIC
IN ITEM RESPONSE THEORY

The results of Chapters 3 and 4 suggest that a relatively cautious approach should be taken by researchers when choosing a technique for placing two sets of independently estimated parameters on a common metric. The findings of the simulation section of this study (Chapter 3) did not indicate that one technique possesses a uniform advantage over other techniques across the various conditions examined. The results suggest that the choice of equating technique by the researcher may be influenced by such factors as sample size, test length, and differences between the two distributions of ability. The findings of the analyses involving real data (Chapter 4) suggest that under some circumstances, the transformation of scale estimated by even the simplest techniques may be satisfactory.

Before any specific recommendations regarding the appropriate choice of equating technique are made, the results of Chapters 3 and 4 will be reviewed and discussed in detail. The recommendations made later in this chapter will be based on these observations and insights.

Discussion of Simulation Results

Comment on Experimental Design

One major limitation of the design of the simulation study involves the effects of sampling error of the equating constants on the RMSE criterion. If the analyses for a particular condition were replicated, it is possible that a different rank ordering of the techniques would be observed.

Each estimated transformation involves the estimation of two parameters, the A and the B equating constants. For a given technique, under a specified condition, each of these parameters possesses a specific standard error of estimate. The smaller the standard error of estimate, the closer we would expect the estimated transformation to be to the true transformation over numerous replications of the experiment (assuming all the techniques are unbiased). If the standard error of estimate for a technique were relatively large, we would expect to observe a wide range of estimated scale transformations over numerous replications, some close to the true transformation and others very far from the true transformation. Notice for techniques with relatively large standard errors of estimate, it is difficult to predict how close the estimated transformation will be to the true transformation on any one replication of the experiment. Predictions are usually made in terms of expected differences over many replications of the experiment. Techniques which possess smaller standard errors of estimate are desired over techniques which possess larger standard errors of measurement. Over many replications of the experiment the technique with the smaller

sampling variance would produce estimates that are closer to the true values than those produced by the technique with the larger sampling variance.

The current simulation design can be thought of as an experiment which assesses the performance of a technique using a single observation in each of seven conditions. We would like to identify the techniques with the smallest sampling errors. For a given observation, we would expect the estimates from a technique with a small sampling variance to be close to the true values (but we may occasionally observe values that are far from the true values). Notice also that it is possible for a technique with a large error of estimate to produce values close to the true values. Thus by observing a single observation it is difficult to make accurate inferences about the standard errors of estimate for the equating techniques.

One solution to the above problem would be to perform numerous replications of the experiment under each of the seven conditions. The empirical distributions of the parameter estimates could then be used to make inferences about their true sampling distributions. If a large number of replications were performed, reliable and consistent differences in the standard errors of estimate may be detected. Unfortunately the cost of replicating all the analyses in each condition is prohibitive with the resources currently available.

To gain some insight into making inferences about the performance of the techniques using a single observation, the analyses for Condition 2 were replicated an additional four times for a total of five replications. Each replication involved the sampling of new samples of

ability parameters, the generation of dichotomous responses, estimation of item and person parameters, and finally the estimation of the equating constants. The analyses are summarized in Tables 16 and 17. Table 16 displays the RMSE values and their rank orders for each technique for each of the five replications. The RMSE values are in parentheses, with the rank order of the RMSE across techniques for a given replication displayed directly to the right. Table 17 displays the mean, median, and range of RMSE values for each of the seven techniques for the same replications.

Notice (from Table 16), as anticipated the rank ordering of techniques does not remain constant across replications of the experiment. As discussed earlier this result is most likely due to sampling error of the equating constants. Table 17 gives some insight into the effect of sampling error on the RMSE criterion for each technique. Notice that the range of RMSE values for the three b parameter techniques is relatively large compared to the remaining techniques.

The results of Tables 16 and 17 indicate that small differences in RMSE values between two techniques should not be interpreted as evidence for differential performance. It is likely that small differences may be due to sampling error. Only large discrepancies in RMSE values should be treated as significant differences in performance.

The reader is cautioned against directly applying the results displayed in Tables 16 and 17 to the interpretation of results in the remaining six conditions. First, these results are based on a small number of replications and are also subject to the effects of sampling

Table 16
 RMSE Values and Rank Orders
 for Condition 2
 for Five Replications of the Analysis

TECHNIQUE	Replication				
	1	2	3	4	5
ALL b's	(.692)7	(.390)7	(.715)7	(.713)7	(.864)7
SELECTED b's	(.050)2	(.163)6	(.125)5	(.167)5	(.070)4
WEIGHTED b's	(.076)4	(.096)4	(.198)6	(.223)6	(.118)6
TRUE SCORE	(.087)6	(.075)1	(.065)1	(.082)3	(.054)2
ICC (H)	(.047)1	(.093)3	(.123)4	(.079)2	(.069)3
ICC (S/L)	(.081)5	(.090)2	(.089)2	(.076)1	(.034)1
MLE	(.055)3	(.109)5	(.115)3	(.108)4	(.076)5

Table 17
Summary of Five Replications
of Condition 2

Technique	Mean	Median	Min-----Max	Range
ALL b's	.6748	.7133	.3904--.8643	.4738
SELECTED b's	.1151	.1255	.0501--.1673	.1172
WEIGHTED b's	.1421	.1178	.0757--.2229	.1472
TRUE SCORE	.0723	.0746	.0536--.0869	.0334
ICC (H)	.0821	.0789	.0471--.1228	.0757
ICC (S/L)	.0738	.0809	.0336--.0899	.0563
MLE	.0925	.1077	.0551--.1147	.0597

error. Second, the sampling error of a given technique might be expected to vary from condition to condition. Standard errors of estimate in general would be expected to increase as sample sizes and test length decrease, and as overlap of the two ability distributions decreases. The main point to be stressed is that differences in RMSE values should not be translated into literal differences in performance, but rather should be interpreted in the context of sampling error of the equating constants.

Summary of Simulation Results

Table 18 summarizes the results of the simulation study presented in Chapter 3 (Tables 6 through 12). In parentheses are the RMSE values for each technique, for each condition, copied from the last column of Tables 6 through 12. To the left of each RMSE value is the rank order (from smallest to largest) of the seven RMSE values for a given equating technique, rank ordered across conditions. Directly to the right of each RMSE value is the rank order (also from smallest to largest) of the RMSE values for a given condition, rank ordered across techniques.

Below, the rank ordering of RMSE values for a given equating technique are examined. Each technique is examined in turn. Some theoretical predictions and considerations from Chapter 2 are integrated with the empirical findings of Table 18.

Table 19 lists three sets of comparisons that are of special interest: Test Length (Conditions 1,6,7), Sample Size (Conditions 1,2,3), and Ability Distribution Overlap (Conditions 4,2,5). As can be

Table 18

Root Mean Squared Error Values
and Rank Orders for
Simulation Conditions

TECHNIQUE	Condition						
	1	2	3	4	5	6	7
ALL b's	2(.44)7	4(.69)7	5(1.04)7	6(1.49)7	7(1.64)7	3(.69)7	1(.27)6
SELECTED b's	5(.123)6	2(.050)2	4(.096)5	1(.020)1	6(.189)5	3(.083)5	7(.213)5
WEIGHTED b's	3(.074)5	4(.076)4	5(.116)6	1(.053)5	6(.201)6	2(.067)1	7(.278)7
TRUE SCORE	3(.054)4	5(.087)6	2(.033)1	1(.022)2	6(.113)1	7(.123)6	4(.079)1
ICC (H)	2(.039)2	3(.047)1	4(.064)2	1(.025)4	6(.137)3	5(.072)2	7(.149)3
ICC (S/L)	2(.051)3	4(.081)5	5(.082)3	1(.024)3	7(.126)2	3(.080)4	6(.121)2
MLE	1(.024)1	2(.055)3	5(.087)4	3(.056)6	6(.153)4	4(.079)3	7(.194)4
Summary of Conditions							
N	1000	500	250	500	500	1000	1000
n	60	60	60	60	60	30	15
Comp Mean	-.5	-.5	-.5	0	-1	-.5	-.5

Table 19
Comparisons

Effect	Conditions		
Test Length	1(60 Items)	6(30 Items)	7(15 Items)
Sample Size	1(1000 Subs)	2(500 Subs)	3(250 Subs)
Distribution	4($x=0$)	2($x=-0.5$)	5($x=-1.0$)
Small \longrightarrow (Expected RMSE) \longrightarrow Large			

seen from Table 4, within each set of comparisons listed in Table 19, only one of the three factors (Sample size, Test Length or Distribution Overlap) varies, while the values of the other two factors remain constant. By examination of these sets of conditions, some insight into the effect of each of these factors on each technique may be obtained. Notice however that sampling error of the equating constants may in large part influence the observed ordering of the RMSE values for each of these contrasts. If for a particular technique the expected ordering of RMSE values is not observed, we should not conclude that the factor of interest has no effect, but rather should interpret the ordering of RMSE values in the context of sampling error.

1. b-parameter Equating (using all the b_i 's). This technique finds the transformation of the comparison group scale that equates the first two moments of the two distributions of estimated b-parameters.

The sample size comparison (Conditions 1, 2 and 3) shows the expected ordering. This rank ordering of RMSE values suggests that as sample size decreases (from 1000 to 500 to 250) our RMSE values increase (from .44 to .69 to 1.04). This finding may have been anticipated on theoretical grounds, for as sample size decreases the standard error of our b-parameters increases. This increase in the standard error of the b values from conditions 1, 2 to 3 appears likely to have produced the observed ordering of RMSE values for those conditions. The other two comparisons of interest, Test Length (Conditions 1, 6 and 7) and overlap of ability distributions (Conditions 4, 2 and 5) did not display the expected rank orderings.

Perhaps the most important observation to be made concerning the RMSE values for the first technique is to note their large values relative to the other values in the table. None of the transformations estimated by the simple b-parameter technique appears close to the true transformation. Thus, for a test with item parameters similar to those specified in Table 3 and under conditions similar to those examined, the simple b-parameter technique which incorporates all the values to estimate the transformation appears unsatisfactory.

2. b-parameter Equating (using well estimated b_i 's). The objective of this technique was to obtain a smaller set of better estimated b_i 's from which to compute the transformation of scale. This was accomplished by excluding items with extreme estimated difficulty values or small estimated discrimination values.

Table 18 reveals a substantial reduction across each of the seven conditions, of the RMSE values for the present technique. That is, by removing the items with large standard errors from the items used to estimate the sample moments, a substantial improvement of the estimated transformation is observed.

Of the three comparisons that examine the effects of Test Length, Sample Size, and Ability Distribution Overlap, only the latter (Ability Distribution Overlap) displays the anticipated rank order of RMSE values. Notice however that the test length comparison is partially confounded by the exclusion of some items in each condition (due to large standard errors). Table 20 lists the actual number of items selected under each condition used to compute the transformation of

Table 20
Number of Items Selected by
Technique 2 for
Estimation of Sample Moments

Condition	Number Selected	Total no. of Items
1	42	60
2	43	60
3	42	60
4	45	60
5	40	60
6	17	30
7	10	15

scale. These are the number items determined to possess relatively small standard errors by examination of the estimated a and b parameters. Rather than basing our transformation on the full 60, 30 and 15 items for Conditions 1, 6 and 7; the transformation is actually based on 42, 17 and 10 items respectively.

Although this b-parameter technique did very well in Conditions 2 and 4, it is important to note that even after eliminating items with large standard errors, that there is no guarantee that the transformation based on the remaining items is adequate under any of the conditions examined.

3. b-parameter Equating (using weighted b_i's). This technique controls for the effects of poorly estimated b_i's by the use of weights that are inversely proportional to the estimated standard errors of the estimated item difficulties.

With respect to the first technique, there is a substantial reduction across most conditions, of the RMSE values. However, it is surprising to see that this technique did not display a large improvement over the second technique for most of the Conditions in Table 18. Additional evidence for this lack of large improvement is displayed in Table 16. Notice that for the replications of Condition 2, this technique displayed larger RMSE values than the restricted b technique in four of the five replications.

One hypothesis consistent with the above results deals with the estimator of the standard error of the b parameters (see Chapter 2 for a review). The formulas used for estimating the standard error are

asymptotic in nature, and converge to the true values as N (sample size) tends towards infinity. For relatively small samples (say 500 or less) the estimated standard error may be a poor approximation to the true standard error. By examining Table 18, we can observe that the two conditions in which the present technique possesses smaller RMSE values than the second technique (Conditions 1 and 6) are conditions which possess samples of 1000. Although this reversal of ordering for these two conditions may be attributed to sampling error, the results are consistent with the hypothesis that sample sizes of 500 or less are too small to produce accurate estimates of the standard errors of the difficulty parameters.

Both the Sample Size and Ability Distribution Overlap comparisons displayed the expected ordering of RMSE values. The Test Length comparison did not display the anticipated ordering.

4. True Score Equating (Stocking & Lord). This technique finds the linear transformation necessary to develop a common metric by using estimated true scores.

For five of the seven conditions, the RMSE values for the True score technique are smaller than the corresponding RMSE values of the three b parameter techniques. Thus, over most of the conditions examined, the scale transformation estimated by the True score technique appeared closer to the true transformation than those estimated by any of the three b-parameter techniques. Additional evidence for the superior performance of the True score technique is displayed in Table 16. The True score technique displayed smaller RMSE values than all three

b-parameter techniques in four of the five replications of Condition 2.

Of the three comparisons that examine the effects of Test Length, Sample Size, and Ability Distribution Overlap, only the latter (Ability Distribution Overlap) displayed the anticipated rank ordering of RMSE values.

5. ICC equating (Haebara) and

6. ICC equating (Segall & Levine)

Both these techniques estimate the scale transformation by examining the sum (across items) of the weighted sum of squared differences between corresponding ICC's. These two techniques differ with respect to the way that the squared differences for each ICC are weighted. The weighting scheme suggested by Haebara (1980) weights those segments of the estimated ICC differences in accordance with the relative frequency of examinees falling in the region. The weighting scheme suggested by Segall & Levine (1983), however, is formed from the product of the two empirical pdf's from each group. This weight function is largest over the range where the overlap of the two estimated distributions of ability is the greatest, and zero where there is no overlap. The goal here, remember was to place the heaviest emphasis on that portion of the squared difference between corresponding ICC's that is relatively well estimated in both groups.

Both sum of squares techniques out performed the three b-parameter techniques in almost all the conditions examined (see Table 18). The two sums of squares techniques produced RMSE values that were smaller than those produced by the True score method in 3 out of the seven

conditions examined. From Table 18 we can observe that Haebara's method produced relatively smaller RMSE values than the Segall & Levine method in all but two of the seven conditions examined. From Table 16 however a different ordering of RMSE values is displayed for the two techniques. The Segall & Levine method displays smaller RMSE values than Haebara's method in four of the five replications of Condition 2. This reversal of RMSE ordering for these two techniques is most likely due to sampling error. Although there may be real differences in performance between the two methods, these differences, if they exist, appear too small to be detected by the present design.

A closer examination of some intermediate results suggest one alteration to the Segall & Levine method that may improve its performance. The weight function suggested by Segall & Levine is formed by taking the square root of the product of the two empirical pdf's (after transforming the comparison group distribution to the base group metric). This weighting scheme resulted in a weight function that contains a relatively large number of zero elements. As a result the weighted sums of squared difference between corresponding ICC's was formed from a relatively small number of points because of the large number of zero elements contained in the weight function. The sum of squared differences estimated from a small number of points probably resulted in a less accurate estimate of the sums of squares criterion than if the squared differences had been evaluated at a larger number of points. One improvement to this method would involve recomputing the weight function in such a manner that it examined only the range of distribution overlap, thus avoiding the problem associated with zero

elements.

In Chapter 2 it was predicted that when the two distributions of ability were roughly equal, the performance of the two sum of squares techniques should produce similar results. This prediction is confirmed by examining the RMSE values of Condition 4 for the two techniques.

7. Equating Based on Vectors of Item Parameter Differences. This technique finds the scale transformation that maximizes an approximation to the the likelihood of observing the vectors of item parameter differences. In its present form, this technique relies on several approximate properties of maximum likelihood estimators: (1) maximum likelihood estimators are approximately normally distributed with mean equal to the true parameter value; and (2) the asymptotic variance covariance matrix for these estimates may be obtained from the inverse of an approximated information matrix. The variance covariance estimates are used to define the objective function that is used to estimate the equating constants.

Over most of the conditions and replications (Tables 16 and 18) examined the RMSE values for the MLE technique appear larger than the corresponding values for the True score and sums of squares techniques. One possibility is that this observed ordering is due to sampling error, as discussed earlier. These results are also consistent with the hypothesis that the observed performance may be explained by the heavy reliance of this technique on the asymptotic properties of maximum likelihood estimators. Some support for this explanation is achieved by examination of Condition 1, (where there are 1000 examinees and 60

items). In this condition, with a relatively large number of subjects, the MLE procedure displayed smaller RMSE values than in other conditions with smaller samples and shorter tests. It also performed relatively well compared to the other methods (although this result may not be reliable). It may be that sample sizes of 500 and test lengths of 30 are insufficient to yield good estimates of covariances matrices and normally distributed estimates. Further analyses however would be needed to confirm this hypothesis.

Of the three comparisons that examine the effects of Test Length, Sample Size, and Ability Distribution Overlap, only the latter (Ability Distribution Overlap) failed to display the anticipated rank ordering of RMSE values.

Discussion of Real Data Results

As discussed in Chapter 4, Study I appears to indicate that none of the techniques were adversely effected by violations to the IRT model. This is evidenced by the close agreement of all the estimated equating constants to their expected values. Similar results were obtained from Study II. Although the estimated transformations were not as close to the expected values as were those from Study I, the estimated equating constants from Study II all appear in fairly close agreement.

One of the most surprising and important findings of Chapter 4 was the excellent performance of the simple b parameter equating technique. Notice that these results contradict the findings of the simulation

results (Chapter 3), which showed this technique produced poor estimates under all the conditions examined. It may be possible, however, to reconcile these findings by considering differences in the heterogeneity of the samples used to estimate the item parameters, as well as the relative difficulty of the items with respect to these samples.

Remember that b parameter values and the ability parameters " θ " are measured on the same scale. If the b value for a particular item possesses a value close to many of the true ability parameters, the estimate of that b value will possess a small standard error. If on the other hand, the b value for a particular item possesses an extreme value, far from most ability parameters, the estimate of that b value will possess a relatively large standard error.

One hypothesis for the discrepant findings between the simulation and real data studies is that there exists a different relation between the difficulty parameters and ability distributions of the two analyses. It may be that the test used to generate the simulated item responses contained many more items with extreme b values than did the real test examined in Chapter 4. These extreme b values, for the simulation analyses, resulted in poor estimates of the scale transformation. This hypothesis is examined in further detail in the following section.

The appropriateness of the simple b parameter technique is an especially important issue. It is probably the most used technique for transforming parameters to a common metric and thus deserves special attention. One of the most widely used estimation programs LOGIST (Wood, et al.) allows the metric of the theta scale to be specified by standardizing on the estimated b values. That is, there is an option

which sets the unit and origin of the theta scale to values that result in the estimated item difficulties possessing a mean of zero and a standard deviation of one. Notice that if two sets of parameters are estimated independently, this option would automatically equate the first two moments of the distribution of estimated item difficulties. This procedure is identical to the simple b parameter technique discussed in this paper. Notice that a simple rule that examined the estimated parameter values from LOGIST to judge the appropriateness of the simple b parameter technique would be extremely useful. One such technique is developed in the following section. As a basis this rule uses the results of the simulation and real data analyses of Chapters 3 and 4.

Recommendations for the Selection of Appropriate Techniques for Transforming Parameters to a Common Metric

The first issue to be addressed in this section is the specification of a simple rule for governing the use of the simple b parameter technique. The discrepant findings of Chapters 3 and 4 will be reviewed in detail and used as a basis for developing this criterion.

The second goal of this section is the specification of general guidelines concerning the use of all the equating techniques with respect to certain test and sample characteristics. These guidelines will also incorporate the results of Chapters 3 and 4, and are discussed

in the latter portion of this section.

Guidelines for Use of the Simple b-Parameter Technique

The results of Chapter 3 indicate that the simple b parameter technique performed poorly under all seven conditions examined. The results of Chapter 4, the real data analyses, indicated satisfactory performance by the equating procedure. The most likely explanation for the difference in performance can be traced to the differences in the distributions of the difficulty parameters, relative to the ability distributions. That is, for the simulation (Chapter 3) conditions, the b values were specified in a manner that resulted in a large number of extreme values (relative to the distribution of ability examined). These extreme values, of course, possess large sampling errors, which in turn result in a poor estimated scale transformation. The MAT, on the other hand, appears to have the heaviest concentration of b values in the region with the heaviest concentration of thetas. This b parameter - ability distribution relationship produces well estimated difficulty values, which in turn result in a well estimated scale transformation.

To add credibility to the above explanation, Figures 7 through 12 display the relation between the distribution of difficulty parameters and the distribution of ability, for the MAT and simulation studies. In each figure, the S-shaped curve (extending from the lower left hand corner to the upper right hand corner) represents the cumulative distribution function of the ability parameters. Each b value is represented by a pair of horizontal and vertical lines, superimposed on the same figure. Thus each b parameter for an item is represented by

one vertical and one horizontal line. The vertical line, terminating on the theta axis, represents the value of the parameter. The horizontal line for the same item, terminating on the cdf axis, indicates the proportion of thetas falling at or below the corresponding b value.

Figures 7 and 8 display the relation between the estimated b values and ability parameters for the MAT (Chapter 4, Study I). From either of the base or comparison group calibrations, we observe a heavy concentration of b values in the range -1.7 to $+1.7$ (vertical lines). From examination of the horizontal lines for those same items, we observe very few items that fall in the extreme tails of the cdf. Thus, almost all the b values fall in a range surrounded by a substantial number of thetas.

Figures 9 and 10 display the relation between the estimated b values and ability parameters for the shorter 45 item version of the MAT (Chapter 4, Study II). Again from either the base or comparison group calibrations, we observe a heavy concentration of b values in the range -1.7 to $+1.7$. As before, we observe practically no items falling in the extreme tails of the cdf. Thus, here also the b values fall in a range at or near a substantial number of examinees.

Figures 11 and 12 however, suggest an entirely different relation. These figures display the relation between the true b parameter values used to generate the simulated responses (Chapter 3) and the true ability parameters for 1000 subjects (for Condition 1). Remember that these b values were sampled from a uniform distribution in the range -3 to $+3$. The uniformity of these values is evidenced by the roughly even scatter of the vertical lines. From examination of the horizontal

Figure 7

Relation of Distribution of Estimated Difficulty Parameters
with Cumulative Distribution of Estimated Person Parameters

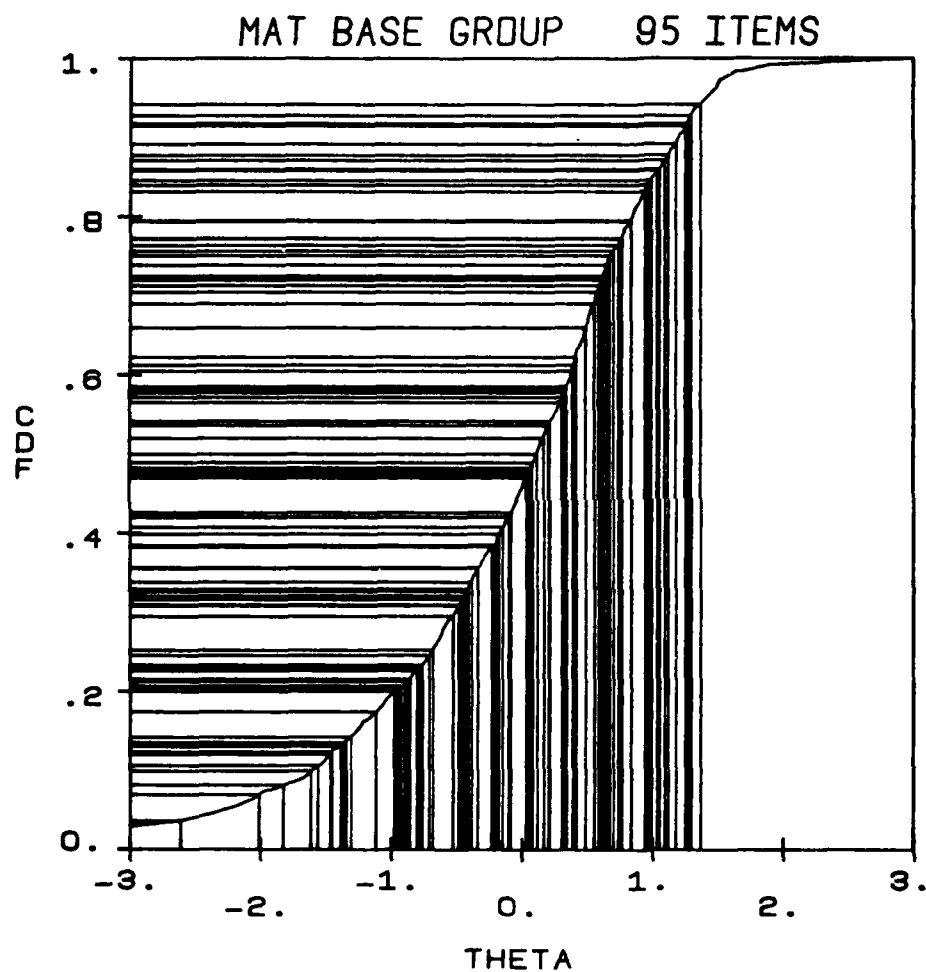


Figure 8

Relation of Distribution of Estimated Difficulty Parameters
with Cumulative Distribution of Estimated Person Parameters

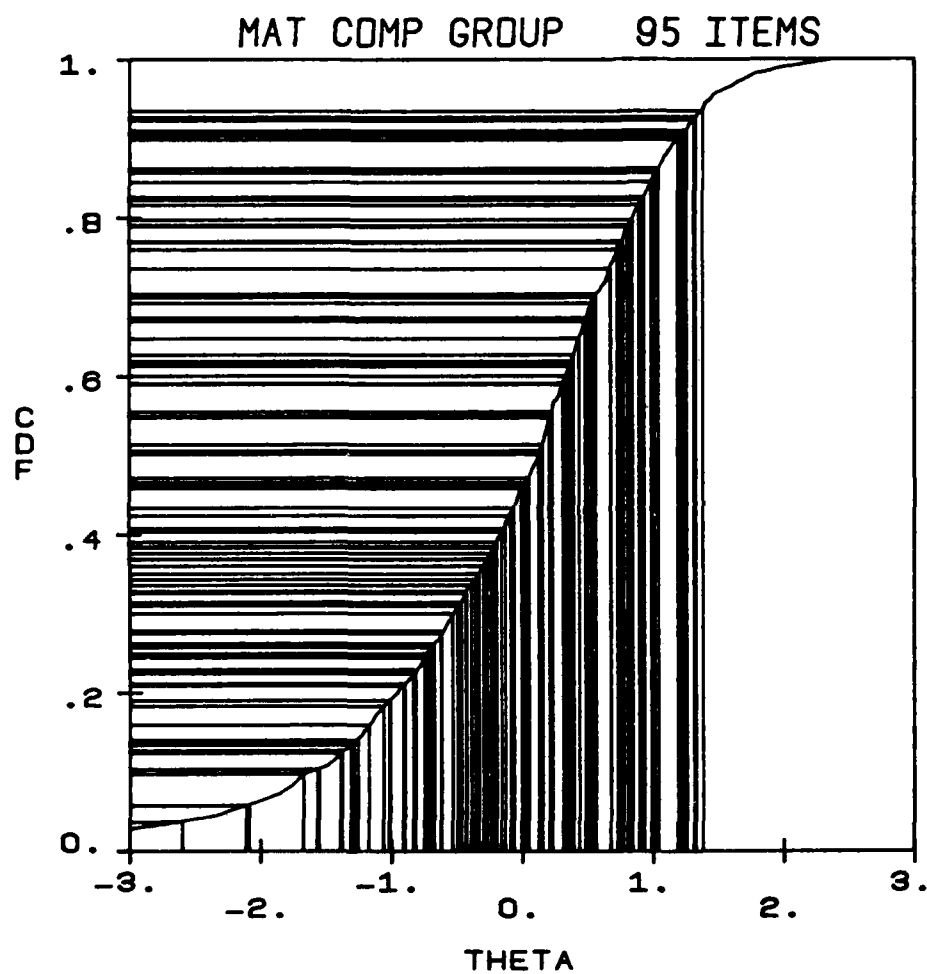


Figure 9

Relation of Distribution of Estimated Difficulty Parameters
with Cumulative Distribution of Estimated Person Parameters

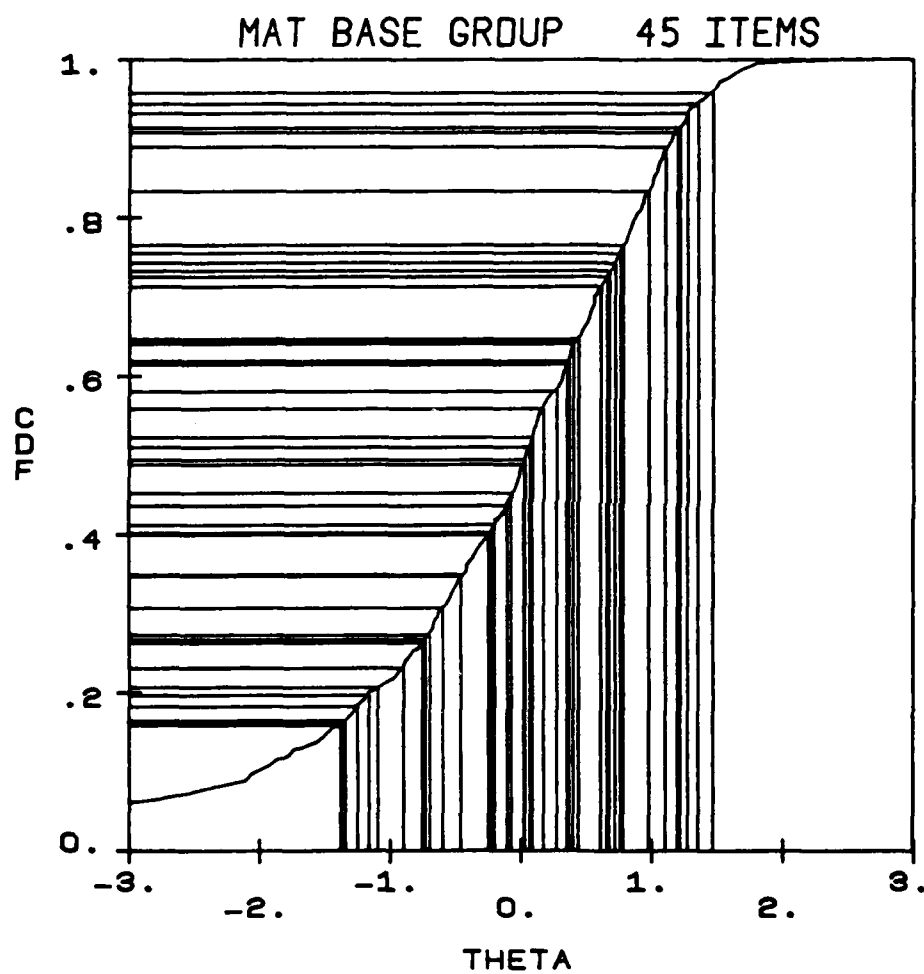


Figure 10

Relation of Distribution of Estimated Difficulty Parameters
with Cumulative Distribution of Estimated Person Parameters

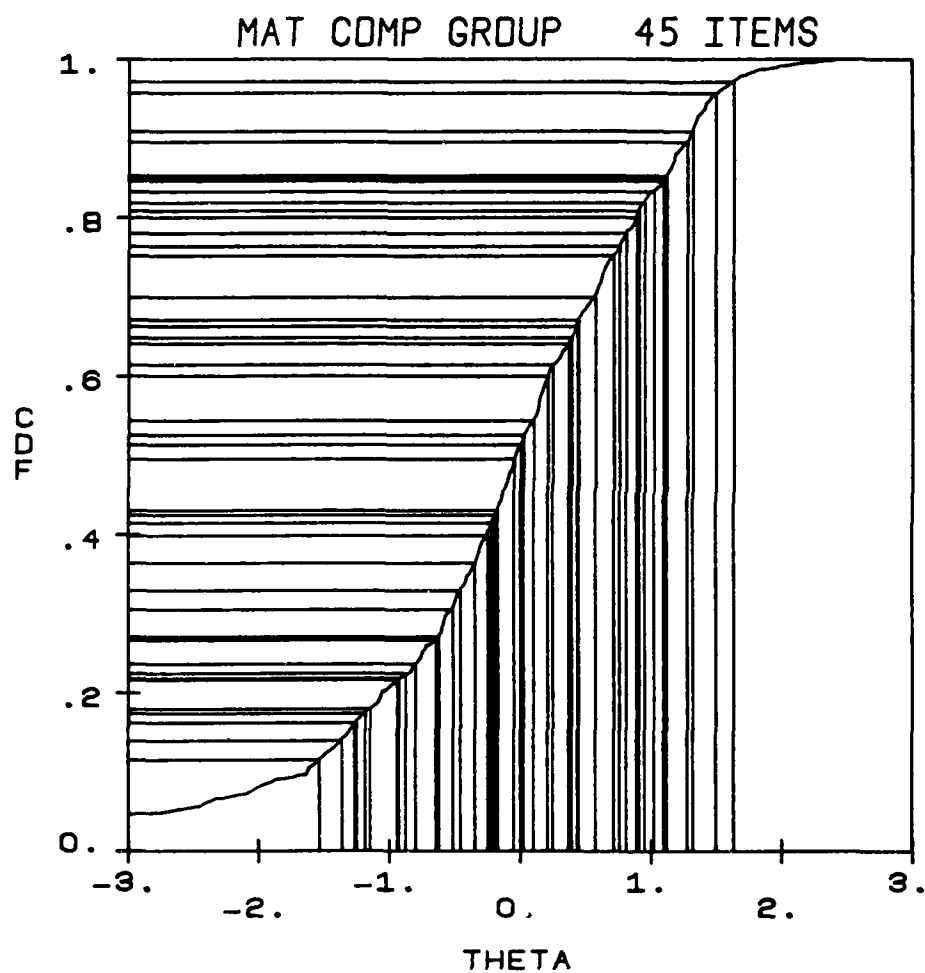


Figure 11

Relation of Distribution of True Difficulty Parameters used
in the Simulation Analyses with the
Cumulative Distribution of True Person Parameters (from Condition 1)

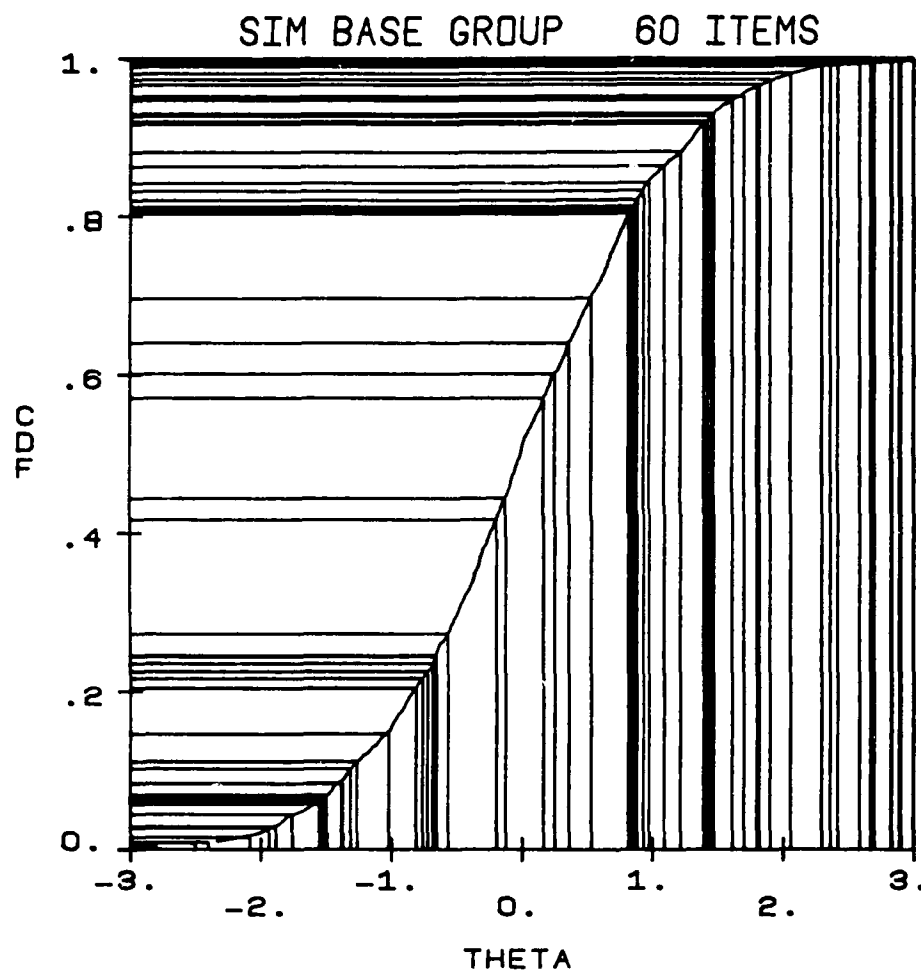
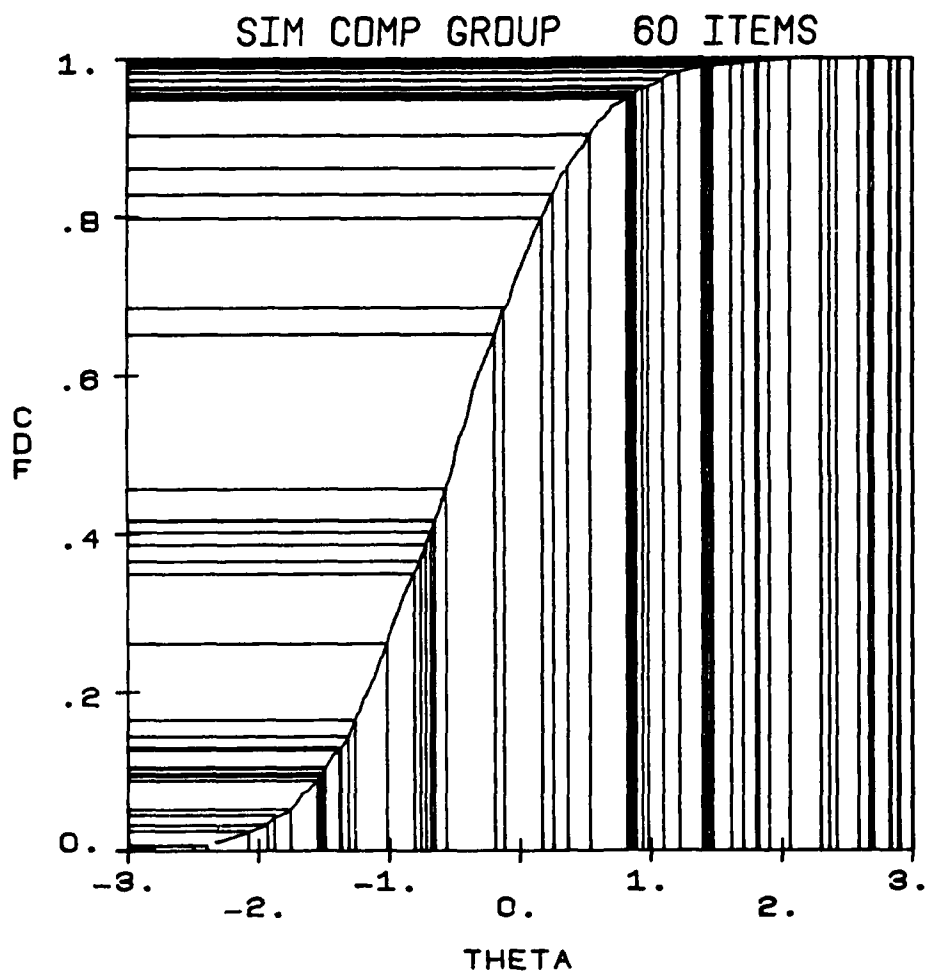


Figure 12

Relation of Distribution of True Difficulty Parameters used
in the Simulation Analyses with the
Cumulative Distribution of True Person Parameters (from Condition 1)



lines, for these same items, we observe a very sparse concentration of values near the center of the cdf, and very heavy concentrations near the extremes of the odf.

Figures 11 and 12 display very dramatically the differences in the distribution of difficulty values between the real and simulation portions of this study. In the simulation portion of the study, there were many b values concentrated at the extreme tails of the odf. These b values fall in a range surrounded by few or no examinees, thus possessing large sampling errors. These large sampling errors resulted in poor estimated transformations from the simple b parameter equating technique.

It is interesting to note that had the b values for the simulation parameters been sampled from a normal, rather than uniform distribution, results similar to those found in Figures 7 through 10 for the MAT, would probably have been observed.

Table 21 summarizes some key information found in Figures 7 through 12. For each of these figures, the number of b values falling in the extremes of the odf were tabulated. Here, an extreme b value is one in which fewer than 5% of the ability parameters possess more extreme values. Thus, any b values falling in the cdf regions 0 to .05, and .95 to 1., were considered extreme as listed in Table 21. These are the b values likely to possess the largest standard errors.

As can be observed from Table 21, the percent of simulation b values falling in the extreme cdf range (40% and 58%) is much higher than any of the MAT calibrations (ranging from 1% to 4%). This explains very satisfactorily the differences in performance of the simple b parameter

Table 21
Frequency and Percent of b Values
with Extreme Associated cdf Values

Test	Test Length	Sample Size	Frequency		Percent	
			Base Comp		Base Comp	
MAT	95	1000	1	1	1%	1%
MAT	45	500	1	2	2%	4%
Simulation Condition 1	60	1000	24	35	40%	58%

equating technique in the simulation and real data analyses. In the simulation analyses, a large proportion of the b values possessed large standard errors, while in the real data analyses only 1 to 4 percent at most, possessed relatively large standard errors.

Notice that Table 21 suggests a relatively useful criterion. When 4% or less of the b values fall in the extreme cdf range (as defined above) for each group, and none of these b values possess a corresponding cdf value of 0 or 1, the simple b parameter technique appears to produce satisfactory results. Thus a criterion of 4 to 5 percent (with none of the items having cdf values of 0 or 1), may be very useful in determining the adequacy of the simple b parameter equating technique. Notice however, that a cut of 4% may represent a relatively conservative estimate of the percent of items allowed to possess large values relative to the distribution of ability. Further study may show that slightly larger percentages are admissible.

Notice, also that one nice feature of the above guideline, is that it is metric free, and can be used on any set of parameter estimates, no matter how the unit and origin were specified. Thus, it would be possible, for example, to estimate simultaneously the item parameters and standardize on the estimated b parameters for the two groups independently. Then, the above procedure could be used on each set of estimated parameters to check the adequacy of this standardization, after the standardization had already been performed.

Recommendations for Appropriate Use of the Seven Equating Techniques

As mentioned at the beginning of this chapter, a relatively cautious

approach should be taken by the researcher when choosing a technique for placing two sets of independently estimated parameters on a common metric. Under the best of circumstances all the techniques appear to perform well and transforming the parameters can be performed using one of the simple b parameter techniques. Under the worst of circumstances, the more complicated sums of squares and MLE techniques offer a clear advantage over the simple b parameter techniques, and their use is encouraged.

The first step, should be one of determining how well the simple b parameter technique is suited to the test and ability distribution at hand. The analysis presented in the previous section may provide very useful insights into the suitability of the simple b parameter technique. Remember, for each group separately, the estimated ability parameters are sorted, and the proportion of thetas at or below each b value is computed. If there is a large number of the b values with extreme proportions, for either group, the researcher should consider using one of the other techniques. If there are a small number of b values in each group with extreme values, all the techniques would be expected to perform relatively well. Thus, in the latter instance, choice of equating method is not critical.

When a relatively large number of b values with extreme proportions have been encountered, choice of equating technique may be further influenced by such factors as sample size and test length. Again for relatively large samples (1000 or more in each group) and relatively long tests (60 or more items) the True Score, Sums of Squares, and MLE procedures would be expected to produce satisfactory results.

For shorter tests and smaller samples, the True Score or either of the ICC techniques would be likely to produce the most favorable results. Because of the heavy reliance of the MLE technique on certain asymptotic properties, its use is not recommended with small samples and short tests.

Recommendations for Further Study

The results and insights gained from the current research raise a number of issues deserving further investigation. Several of these areas are described below.

Asymptotic Sampling Variance of Item Parameters

The results of portions of the simulation analyses raise several questions concerning the relation among the asymptotic properties of maximum likelihood estimates of item parameters, the estimators of the standard error of these parameters, and sample size. The results concerning the performance of the weighted b parameter technique appear consistent with the hypothesis that samples of 1000 may be needed before the estimates of the standard error of the b parameters are close approximations to the true values. The results concerning the performance of the MLE equating technique also appear consistent with the hypothesis that samples of 1000 may be needed before the asymptotic properties of the item parameters, and their variance covariance estimates are realized. Further investigation into the relation between

sample size and the assumed asymptotic properties of the item parameters for the logistic model would be relevant to many other areas of IRT as well as the current research.

Guidelines for Simple b-Parameter Technique

Remember from the previous section that 4% was suggested as the maximum proportion of items in a test possessing extreme cdf values for the acceptable use of the simple b parameter technique. Remember that this criterion was selected on a basis of the results from the actual MAT analyses, where the b parameter technique performed relatively well. As indicated earlier, a criterion of 4% of the test items may represent a relatively conservative criterion. A systematic study into the effects of varying the number of items in a test with extreme b values may indicate whether in fact a criterion of 4% is too conservative.

Improvements to Equating Techniques

Results from the analyses produced several insights for further modifications to several of the techniques.

True Score Equating Technique (Stocking & Lord). The criterion minimized by this technique only involves minimizing the sums of squared differences between true scores for one of the two groups of examinees. An increase in power may perhaps be achieved by adding another term to the loss function which reflects the analogous term for members of the other group of examinees, thus incorporating transformed and untransformed true score estimates from both samples.

Sums of Squares Equating Technique (Segall & Levine). The weight

function used in calculating the criterion contains zero's over a substantial range of theta, producing a weighted sums of squares estimate for each item that was formed on the basis of a very small number of points. One improvement to this technique would involve a modification to the weighting scheme that evaluated the ICC using a larger number of points in the same range of ability distribution overlap. This would produce a more accurate estimate of the weighted sums of squares criterion, which in turn may improve the performance of the technique.

APPENDIX

Fortran Computer Programs Used to Estimate
Scale Transformation for All Seven Equating Techniques

EQUATE - Subroutine

This routine estimates the linear transformation necessary to place two independently estimated sets of parameters on a common metric. Seven approaches are available. These are described in Chapter 2. This routine makes several calls to IMSL (Version 9) subroutines. All other subroutines are listed below. Program was written for use on CDC system using Fortran Extended Version 4.

EQUATE(PARB,PARC,IROW,NITEMS,THETAB,THETAC,NSUBSB,
NSUBSC,NPAR,IPT,A,B,IOUT,TIME,IERROR)

- PARB:** Matrix of NITEMS rows by 3 columns containing the estimated item parameters for the base group. Row 1 contains the item parameters for item 1, Row 2 contains the item parameters of item 2, etc. Column 1 contains the a-parameters, column 2 contains the b-parameters and column 3 contains the c-parameters. If using the 2 parameter model all values in column 3 should be set to zero.
- PARC:** Matrix of NITEMS rows by 3 columns containing the estimated item parameters for the comparison group. Format is same as PARB.
- IROW:** Row dimension of PARB and PARC exactly as specified in the calling program.
- NITEMS:** Number of items in PARB and PARC.
- THETAB:** Vector of length NSUBSB containing ability parameter estimates for base group examinees. A value of 999 is treated as missing.
- THETAC:** Vector of length NSUBSC containing ability parameter estimates for comparison group examinees. A value of 999 is treated as missing.
- NSUBSB** Number of subjects in base group (including 999's).

- NSUBSC** Number of subjects in comparison group (including 999's).
- NPAR:** Number of item parameters in model. If using the 2 parameter model, NPAR=2, otherwise, NPAR=3.
- IOPT:** Vector of length seven. To obtain equating constant estimates for technique k, set IOPT(k)=1, where:
- IOPT(1) = b-parameter equating (using all b's)
 - IOPT(2) = b-parameter equating (using well estimated b's)
 - IOPT(3) = b-parameter equating (using weighted b's)
 - IOPT(4) = True score equating (Stocking & Lord)
 - IOPT(5) = ICC equating (Haebara)
 - IOPT(6) = ICC equating (Segall & Levine)
 - IOPT(7) = MLE equating based on parameter differences
- A:** Vector of length seven containing the A constant estimates for techniques specified in IOPT. The estimate for technique k are in A(k).
- B:** Vector of length seven containing the B constant estimates for techniques specified in IOPT. The estimate for technique k are in B(k).
- IOUT:** Tape number specified in program statement to which error messages will be written.
- TIME:** Vector of length seven containing the number of CPU seconds used to estimated the equating constants. The amount of time used by technique k is in TIME(k).
- IERROR:** Vector of length seven containing error message codes. A value of zero for the kth element signifies that satisfactory estimates were obtained by technique k.

Subroutine Listings:

```

SUBROUTINE EQUATE(PARB,PARC,IROW,NITEMS,THETB,THETC,
  NB,NC,NPAR,IOPT,A,B,IOUT,TIME,ERROR)
C
  REAL PARB(IROW,3),PARC(IROW,3),THETB(NB),
  THETC(NC),A(7),B(7),THETAB(2000),THETAC(2000),
  TIME(7)
  INTEGER IOPT(7),ERROR(7)
C
C   INITIALIZE CONSTANTS TO ZERO

```

```

DO 11 L = 1,7
  A(L) = 0.
  B(L) = 0.
11  CONTINUE
C
C   CHECK FOR MISSING THETAS IN BASE GROUP
  K = 1
DO 17 J = 1,NB
  IF(THETB(J) .EQ. 999.) GOTO 17
  THETAB(K) = THETB(J)
  K = K + 1
17  CONTINUE
  NSUBSB = K - 1
C
C   CHECK FOR MISSING THETAS IN COMP GROUP
  K = 1
DO 19 J = 1,NC
  IF (THETC(J) .EQ. 999.) GOTO 19
  THETAC(K) = THETC(J)
  K = K + 1
19  CONTINUE
  NSUBSC = K - 1
C
C   SELECT SPECIFIED EQUATING TECHNIQUES:
  T1 = SECOND(CP)
  IF (IOPT(1) .EQ. 1)
    * CALL BEQUAT(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
    ERROR(1) = IER
    T2 = SECOND(CP)
    TIME(1) = T2 - T1
    T1 = T2
C
  IF (IOPT(2) .EQ. 1)
    * CALL BEQUAR(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
    ERROR(2) = IER
    T2 = SECOND(CP)
    TIME(2) = T2 - T1
    T1 = T2
C
  IF (IOPT(3) .EQ. 1)
    * CALL BEQUAC(PARB,PARC,IROW,NITEMS,THETAB,THETAC,NSUBSB,
    * NSUBSC,NPAR,A,B,IOUT,IER)
    ERROR(3) = IER
    T2 = SECOND(CP)
    TIME(3) = T2 - T1
    T1 = T2
C
  IF (IOPT(4) .EQ. 1)
    * CALL TESLRD(PARB,PARC,IROW,NITEMS,THETAB,NSUBSB,A,B,IOUT,IER)
    ERROR(4) = IER
    T2 = SECOND(CP)
    TIME(4) = T2 - T1

```

```

      T1 = T2
C
      IF (IOPT(5) .EQ. 1)
      * CALL HAEBAR(PARB,PARC,IROW,NITEMS,THETAB,THETAC,NSUBSB,
      * NSUBSC,A,B,IOUT,IER)
      ERROR(5) = IER
      T2 = SECOND(CP)
      TIME(5) = T2 - T1
      T1 = T2
C
      IF (IOPT(6) .EQ. 1)
      * CALL SLSUMS(PARB,PARC,IROW,NITEMS,THETAB,THETAC,NSUBSB,
      * NSUBSC,A,B,IOUT,IER)
      ERROR(6) = IER
      T2 = SECOND(CP)
      TIME(6) = T2 - T1
      T1 = T2
C
      IF (IOPT(7) .EQ. 1)
      * CALL EQUMLB(PARB,PARC,IROW,NITEMS,THETAB,THETAC,
      * NSUBSB,NSUBSC,NPAR,A,B,IOUT,IER)
      ERROR(7) = IER
      T2 = SECOND(CP)
      TIME(7) = T2 - T1
C
      RETURN
      END
C
C ***** UNRESTRICTED DIFFICULTY EQUATING *****
C
      SUBROUTINE BEQUAT(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
C
      REAL PARB(IROW,3),PARC(IROW,3),A(7),B(7)
C
      INITIALIZE VALUES
      BTOT = 0.
      CTOT = 0.
      BTOT2 = 0.
      CTOT2 = 0.
      IER = 0
C
      COMPUTE MEANS AND SDS
      DO 30 I = 1,NITEMS
      BTOT = BTOT + PARB(I,2)
      BTOT2 = BTOT2 + PARB(I,2)**2.
      CTOT = CTOT + PARC(I,2)
      CTOT2 = CTOT2 + PARC(I,2)**2.
30    CONTINUE
C
      SDB = (SQRT(FLOAT(NITEMS)*BTOT2 - BTOT**2.))/FLOAT(NITEMS)
      SDC = (SQRT(FLOAT(NITEMS)*CTOT2 - CTOT**2.))/FLOAT(NITEMS)
      BMEAN = BTOT / FLOAT(NITEMS)

```

```

C      CMEAN = CTOT / FLOAT(NITEMS)
C
C      COMPUTE EQUATING CONSTANTS:
C      A(1) = SDB/SDC
C      B(1) = BMEAN - A(1) * CMEAN
C
C      RETURN
C      END
C
C ***** RESTRICTED DIFFICULTY EQUATING *****
C
C      SUBROUTINE BEQUAR(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
C
C      REAL PARB(IROW,3),PARC(IROW,3),A(7),B(7)
C
C      INITIALIZE VALUES
C      BTOT = 0.
C      CTOT = 0.
C      BTOT2 = 0.
C      CTOT2 = 0.
C      XNITEM = NITEMS
C      IER = 0
C
C      COMPUTE MEANS AND SDS
C      DO 30 I = 1,NITEMS
C          IF (PARB(I,1) .LT. 0.15 .OR. ABS(PARB(I,2)) .GT. 3.0
C      * .OR. PARC(I,1) .LT. 0.15 .OR. ABS(PARC(I,2)) .GT. 3.0)
C      * XNITEM = XNITEM - 1
C          IF (PARB(I,1) .LT. 0.15 .OR. ABS(PARB(I,2)) .GT. 3.0
C      * .OR. PARC(I,1) .LT. 0.15 .OR. ABS(PARC(I,2)) .GT. 3.0)
C      * GOTO 30
C          BTOT = BTOT + PARB(I,2)
C          BTOT2 = BTOT2 + PARB(I,2)**2.
C          CTOT = CTOT + PARC(I,2)
C          CTOT2 = CTOT2 + PARC(I,2)**2.
30      CONTINUE
C
C      SDB = (SQRT(XNITEM*BTOT2 - BTOT**2.))/XNITEM
C      SDC = (SQRT(XNITEM*CTOT2 - CTOT**2.))/XNITEM
C      BMEAN = BTOT / XNITEM
C      CMEAN = CTOT / XNITEM
C
C      COMPUTE EQUATING CONSTANTS:
C      A(2) = SDB/SDC
C      B(2) = BMEAN - A(2) * CMEAN
C
C      RETURN
C      END
C
C ***** WEIGHTED DIFFICULTY EQUATING *****
C
C      SUBROUTINE BEQUAC(PARB,PARC,IROW,NITEMS,THETAB,THETAC,

```

```

      * NSUBSB,NSUBSC,NPAR,A,B,IOUT,IER)
C
      REAL PARB(IROW,3),PARC(IROW,3),THETAB(NSUBSB),
      * THETAC(NSUBSC),A(7),B(7),W(200),COVB(3,3),COVC(3,3)
C
C      INITIALIZE ERROR VAR
      IER = 0
C
C      COMPUTE COVARIANCES FOR EACH ITEM FROM EACH GROUP
      DO 101 I = 1,NITEMS
C
C      COMPUTE COV FOR BASE GROUP
      APAR = PARB(I,1)
      BPAR = PARB(I,2)
      CPAR = PARB(I,3)
      CALL COV3PL(THETAB,NSUBSB,APAR,BPAR,CPAR,NPAR,3,COVB,
      * IOUT,IER)
C
C      COMPUTE COV MATRIX FOR COMP GROUP
      APAR = PARC(I,1)
      BPAR = PARC(I,2)
      CPAR = PARC(I,3)
      CALL COV3PL(THETAC,NSUBSC,APAR,BPAR,CPAR,NPAR,3,COVC,
      * IOUT,IER)
C
C      EXTRACT LARGER OF THE TWO VARIANCE ESTIMATES
      W(I) = 1./COVB(2,2)
      IF (COVC(2,2) .GT. COVB(2,2)) W(I) = 1./COVC(2,2)
101  CONTINUE
C
C      COMPUTE WEIGHTED MEANS
      CONST = 0.
      BMEAN = 0.
      CMEAN = 0.
      SDB = 0.
      SDC = 0.
      DO 121 I = 1,NITEMS
          BMEAN = BMEAN + PARB(I,2) * W(I)
          CMEAN = CMEAN + PARC(I,2) * W(I)
          CONST = CONST + W(I)
121  CONTINUE
      BMEAN = BMEAN / CONST
      CMEAN = CMEAN / CONST
C
C      COMPUTE WEIGHTED SD
      DO 178 I = 1,NITEMS
          SDB = SDB + ((PARB(I,2)-BMEAN)**2.) * W(I)
          SDC = SDC + ((PARC(I,2)-CMEAN)**2.) * W(I)
178  CONTINUE
      SDB = SQRT(SDB/CONST)
      SDC = SQRT(SDC/CONST)
C

```

```

C   COMPUTE EQUATING CONSTANTS
A(3) = SDB / SDC
B(3) = BMEAN - A(3) * CMEAN
C
C   RETURN
C   END
C
C ***** STOCKING AND LORD *****
C
C   SUBROUTINE TESLRD(PARB,PARC,IROW,NITEMS,THETAB,NSUBSB,A,B,
C   * IOUT,IER)
C
C   EXTERNAL FUNLRD
C   REAL PARB(IROW,3),PARC(IROW,3),THETAB(NSUBSB),
C   * A(7),B(7),X(2),W(21),SPARC(200,3),PAR(1),F(2),
C   * STHETA(2000),ETAB(2000)
C   INTEGER SNITEM,SNSUBS
C
C   COMMON SPARC,STHETA,SNITEM,SNSUBS,ETAB
C
C   INITIALIZE ERROR VAR
C   IER = 0
C
C   TRANSFER STUFF INTO COMMON ARRAYS
C   DO 105 I = 1,NITEMS
C       DO 101 J = 1,3
C           SPARC(I,J) = PARC(I,J)
101      CONTINUE
105      CONTINUE
C
C   DO 109 J = 1,NSUBSB
C       STHETA(J) = THETAB(J)
109      CONTINUE
C   SNITEM = NITEMS
C   SNSUBS = NSUBSB
C
C   INITIALIZE VALUES OF EQUATING CONSTANTS
C   IF (A(2) .EQ. 0.0 .AND. B(2) .EQ. 0.0)
C   * CALL BEQUAR(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
C   X(1) = A(2)
C   X(2) = B(2)
C   CALL UGETIO(3,0,IOUT)
C
C   COMPUTE ETA FROM BASE GROUP CALIBRATION
C   DO 18 J = 1,NSUBSB
C       THET = THETAB(J)
C       ETAB(J) = 0.
C       DO 29 I = 1,NITEMS
C           APAR = PARB(I,1)
C           BPAR = PARB(I,2)
C           CPAR = PARB(I,3)
C           PROB = P3PL(APAR,BPAR,CPAR,THET)

```

```

                ETAB(J) = ETAB(J) + PROB
29          CONTINUE
18          CONTINUE
C
C      PERFORM MINIMIZATION
        NSIG = 3
        MAXFN = 200
        N = 2
        CALL ZSPOW (FUNLRD,NSIG,N,MAXFN,PAR,X,FNORM,W,IER)
        A(4) = X(1)
        B(4) = X(2)
C
        RETURN
        END
C
        SUBROUTINE FUNLRD(X,F,N,PAR)
C
        REAL X(2),SPARC(200,3),STHETA(2000),ETAB(2000),
        * F(2),PAR(1)
        INTEGER SNSUBS,SNITEM
C
        COMMON SPARC,STHETA,SNITEM,SNSUBS,ETAB
C
        INITIALIZE VALUES
        A = X(1)
        B = X(2)
        DIRA = 0.
        DIRB = 0.
        D = 1.702
C
        FOR EACH SUBJECT
        DO 500 J = 1,SNSUBS
            THET = STHETA(J)
C
            COMPUTE DERIVATIVES OF ETA (FOR COMPARISON GROUP CALIBRATION)
            ETAC = 0.
            DETAA = 0.
            DETAB = 0.
            DO 39 I = 1,SNITEM
                APAR = SPARC(I,1)
                BPAR = SPARC(I,2)
                CPAR = SPARC(I,3)
C
                PARTIAL DERIVATIVE OF P WITH RESPECT TO A
                DPROBA=(EXP(((BPAR*A-THET+B)*APAR*D)/A)*(THET-B)*(CPAR-1.)*
+ APAR*D)/((EXP(((BPAR*A-THET+B)*APAR*D)/A)+1+)**2*A**2)
C
                PARTIAL DERIVATIVE OF P WITH RESPECT TO B
                DPROBB=(EXP(((BPAR*A-THET+B)*APAR*D)/A)*(CPAR-1.)*APAR*D)/((EXP
+ (((BPAR*A-THET+B)*APAR*D)/A)+1+)**2*A)
C
                DETAA = DETAA + DPROBA

```

```

      DETAB = DETAB + DPROBB
C
      APAR = SPARC(I,1) / A
      BPAR = A * SPARC(I,2) + B
      CPAR = SPARC(I,3)
      PROB = P3PL(APAR,BPAR,CPAR,THET)
      ETAC = ETAC + PROB
39  CONTINUE
C
      DIRA = DIRA + (ETAB(J)-ETAC) * DETAA
      DIRB = DIRB + (ETAB(J)-ETAC) * DETAB
500 CONTINUE
C
      F(1) = (-2. / FLOAT(SNSUBS)) * DIRA
      F(2) = (-2. / FLOAT(SNSUBS)) * DIRB
C
      RETURN
      END
C
C ***** HAEBARA SUMS OF SQUARES *****
C
      SUBROUTINE HAEBAR(PARB,PARC,IROW,NITEMS,THETAB,THETAC,NSUBSB,
      * NSUBSC,A,B,IOUT,IER)
C
      EXTERNAL HFUNCT
      REAL PARB(IROW,3),PARC(IROW,3),THETAB(NSUBSB),
      * THETAC(NSUBSC),CUTP(21),MIDP(20),HBASE(20),HCOMP(20),
      * SPARB(200,3),SPARC(200,3),X(2),H(3),G(2),W(6),A(7),B(7)
      INTEGER SNITEM
C
      COMMON HBASE,HCOMP,NINT,SPARB,SPARC,SNITEM,MIDP
C
      INITIALIZE VARIABLES
      NCUT = 21
      NINT = NCUT - 1
      XLOWC = -3.
      XHIC = 3.
      SNITEM = NITEMS
      IER = 0
C
      INITIALIZE CUT POINTS
      CALL ESPNT(CUTP,NCUT,XLOWC,XHIC)
C
      INITIALIZE MIDPOINTS
      HDELT = (CUTP(2) - CUTP(1)) / 2.
      XLOWM = XLOWC + HDELT
      XHIM = XHIC - HDELT
      CALL ESPNT(MIDP,NINT,XLOWM,XHIM)
C
      COMPUTE PROPORTIONS FOR BASE GROUP DISTRIBUTION
      CALL PEDIS(CUTP,NCUT,NINT,THETAB,NSUBSB,HBASE)
C

```

```

C      COMPUTE PROPORTIONS FOR COMPARISON GROUP DISTRIBUTION
      CALL PEDIS(CUTP,NCUT,NINT,THETAC,NSUBSC,HCOMP)
C
C      TRANSFER ITEM PARAMETERS INTO COMMON ARRAYS
      DO 29 I = 1,NITEMS
        DO 27 J = 1,3
          SPARB(I,J) = PARB(I,J)
          SPARC(I,J) = PARC(I,J)
        27 CONTINUE
      29 CONTINUE
C
C      INITIALIZE STARTING VALUES OF EQUATING CONSTANTS
      IF (A(2) .EQ. 0.0 .AND. B(2) .EQ. 0.0)
        * CALL BEQUAR(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
        X(1) = A(2)
        X(2) = B(2)
C
C      PERFORM MINIMIZATION
      CALL UGETIO(3,0,IOUT)
      N = 2
      NSIG = 3
      MAXFN = 500
      IIOPT = 2
C
      CALL ZXMIN(HFUNCT,N,NSIG,MAXFN,IIOPT,X,H,G,F,W,IER)
      A(5) = X(1)
      B(5) = X(2)
C
      RETURN
      END
C
      SUBROUTINE HFUNCT(N,X,F)
C
      INTEGER SNITEM
      REAL HBASE(20),HCOMP(20),SPARB(200,3),SPARC(200,3),MIDP(20),
        * X(2)
C
      COMMON HBASE,HCOMP,NINT,SPARB,SPARC,SNITEM,MIDP
C
C      INITIALIZE PARAMETERS
      A = X(1)
      B = X(2)
      SSC = 0.
      SSB = 0.
C
C      ACCUMULATE SUMS OF SQUARES
      DO 18 I = 1,SNITEM
        ABASE = SPARB(I,1)
        BBASE = SPARB(I,2)
        CBASE = SPARB(I,3)
        ACOM = SPARC(I,1)
        BCOM = SPARC(I,2)

```

```

      CCOM = SPARC(I,3)
C
      DO 14 J = 1,NINT
C
      FOR COMPARISON GROUP
      TC = MIDP(J)
      TB = A * MIDP(J) + B
      SS = (P3PL(ACOM,BCOM,CCOM,TC) -
*         P3PL(ABASE,BBASE,CBASE,TB)) ** 2.
      SSC = SSC + (SS * HCOMP(J))
C
      FOR BASE GROUP
      TC = (MIDP(J) - B) / A
      TB = MIDP(J)
      SS = (P3PL(ABASE,BBASE,CBASE,TB) -
*         P3PL(ACOM,BCOM,CCOM,TC)) ** 2.
      SSB = SSB + (SS * HBASE(J))
14      CONTINUE
18      CONTINUE
C
      F = SSC + SSB
C
      RETURN
      END
C
      SUBROUTINE PEDIS(CUTP,NCUT,NINT,THETA,NSUBS,H)
C
      REAL CUTP(NCUT),H(NINT),THETA(NSUBS)
C
      INITIALIZE TO ZERO
      DO 10 L = 1,NINT
        H(L) = 0.
10      CONTINUE
C
      UPDATE FREQUENCIES
      DO 400 J = 1,NSUBS
        DO 200 K = 1,NINT
          KP1 = K + 1
          IF (THETA(J) .LE. CUTP(KP1) .AND. THETA(J) .GT. CUTP(K))
*             H(K) = H(K) + 1.
200      CONTINUE
400      CONTINUE
C
      TRANSFORM RELATIVE FREQUENCIES TO RELATIVE PROPORTIONS
      DO 500 L = 1,NINT
        H(L) = H(L) / FLOAT(NSUBS)
500      CONTINUE
C
      RETURN
      END
C ***** WEIGHTED SUMS OF SQUARES *****

```

```

C      SUBROUTINE SLSUMS (PARB,PARC,IROW,NITEMS,THETAB,THETAC,NSUBSB,
C      * NSUBSC,A,B,IOUT,IER)
C
C      EXTERNAL SLFUN
C      REAL PARB(IROW,3),PARC(IROW,3),THETAB(NSUBSB),
C      * THETAC(NSUBSC),CUTPC(21),MIDPC(20),HCOMP(20),SPARB(200,3),
C      * X(2),W(21),A(7),B(7),F(2),SPARC(200,3),PAR(1),
C      * CUTPB(21),HBASE(20),WEIGHT(20)
C
C      INTEGER SNITEM
C
C      COMMON MIDPC,SNITEM,SPARB,SPARC,WEIGHT
C
C      INITIALIZE VARIABLES
C      NCUT = 21
C      NINT = NCUT - 1
C      XLOWC = -3.
C      XHIC = 3.
C      SNITEM = NITEMS
C      CRIT = .001
C      ISTAGE = 0
C      IER = 0
C
C      INITIALIZE COMPARISON GROUP CUT-POINTS
C      CALL ESPNT(CUTPC,NCUT,XLOWC,XHIC)
C
C      INITIALIZE COMPARISON GROUP MIDPOINTS
C      HDELT = (CUTPC(2) - CUTPC(1)) / 2.
C      XLOWM = XLOWC + HDELT
C      XHIM = XHIC - HDELT
C      CALL ESPNT(MIDPC,NINT,XLOWM,XHIM)
C
C      COMPUTE PROPORTIONS FOR COMPARISON GROUP DISTRIBUTION
C      CALL PEDIS(CUTPC,NCUT,NINT,THETAC,NSUBSC,HCOMP)
C
C      TRANSFER ITEM PARAMETERS INTO COMMON ARRAYS
C      DO 29 I = 1,NITEMS
C          DO 27 J = 1,3
C              SPARB(I,J) = PARB(I,J)
C              SPARC(I,J) = PARC(I,J)
C          27 CONTINUE
C      29 CONTINUE
C
C      INITIALIZE STARTING VALUES OF EQUATING CONSTANTS
C      IF (A(2) .EQ. 0.0 .AND. B(2) .EQ. 0.0)
C      * CALL BEQUAR(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
C      X(1) = A(2)
C      X(2) = B(2)
C
C      FOR EACH STAGE
C      88 ISTAGE = ISTAGE + 1

```

```

      AL = X(1)
      BL = X(2)
C
C      COMPUTE BASE GROUP CUT-POINTS
DO 17 J = 1,NCUT
      CUTPB(J) = (CUTPC(J) * X(1)) + X(2)
17  CONTINUE
C
C      COMPUTE PROPORTIONS FOR BASE GROUP DISTRIBUTION
CALL PEDIS(CUTPB,NCUT,NINT,THETAB,NSUBSB,HBASE)
C
C      COMPUTE WEIGHT FUNCTION
DO 52 J = 1,NINT
      WEIGHT(J) = SQRT(HBASE(J) * HCOMP(J))
52  CONTINUE
C
C      PERFORM MINIMIZATION
CALL UGETIO(3,0,IOUT)
C      CALL UERSET(0,LEVOLD)
N = 2
NSIG = 3
MAXFN = 200
CALL ZSPOW(SLFUN,NSIG,N,MAXFN,PAR,X,FNORM,W,IER)
C
C      CHECK FOR CONVERGENCE
D1 = ABS(AL-X(1))
D2 = ABS(BL-X(2))
IF (ISTAGE .GE. 10) GOTO 603
IF (D1 .GT. CRIT .OR. D2 .GT. CRIT) GOTO 88
C
C      FINISH
603 CONTINUE
A(6) = X(1)
B(6) = X(2)
C
      RETURN
      END
C
      SUBROUTINE SLFUN(X,F,N,PAR)
C
      REAL SPARB(200,3),PAR(1),F(2),
* MIDPC(20),X(2),
* WEIGHT(20),SPARC(200,3)
C
      INTEGER SNITEM
C
      COMMON MIDPC,SNITEM,SPARB,SPARC,WEIGHT
C
      INITIALIZE VALUES
A = X(1)
B = X(2)
C

```

```

C      COMPUTE PARTIAL DERIVATIVES
      F(1) = PDA(SPARB,SPARC,SNITEM,MIDPC,WEIGHT,A,B)
      F(2) = PDB(SPARB,SPARC,SNITEM,MIDPC,WEIGHT,A,B)
C
      RETURN
      END
C
      REAL FUNCTION PDA(PARB,PARC,NITEMS,MIDT,WEIGHT,A,B)
C
      REAL PARB(200,3),PARC(200,3),MIDT(20),WEIGHT(20)
      PARTIAL DERIVATIVE WITH RESPECT TO A
C
      INITIALIZE VALUES
      PDA = 0.0
      D = 1.702
C
      DO 300 I = 1,NITEMS
        ABI = PARB(I,1)
        BBI = PARB(I,2)
        CBI = PARB(I,3)
        ACI = PARC(I,1)
        BCI = PARC(I,2)
        CCI = PARC(I,3)
        DO 250 J = 1,20
          TJ = MIDT(J)
          TTJ = TJ * A + B
          WJ = WEIGHT(J)
          IF (WJ .EQ. 0.0) GOTO 250
C
          PARTIAL DERIVATIVE OF PB WITH RESPECT TO A
          ANS = (-EXP(BBI*ABI*D+TJ*ABI*D*A+ABI*D*B)*(CBI-1.)*TJ*ABI*D)
          + /(EXP(BBI*ABI*D)+EXP(TJ*ABI*D*A+ABI*D*B))**2
C
          COMPUTE PROBS
          PB = P3PL(ABI,BBI,CBI,TTJ)
          PC = P3PL(ACI,BCI,CCI,TJ)
C
          COMPUTE DIRIVITIVE OF TERM
          TERM = 2. * WJ * ANS * (PB - PC)
C
          PDA = PDA + TERM
        250      CONTINUE
      300      CONTINUE
C
      RETURN
      END
C
      REAL FUNCTION PDB(PARB,PARC,NITEMS,MIDT,WEIGHT,A,B)
C
      REAL PARB(200,3),PARC(200,3),MIDT(20),WEIGHT(20)
C
      INITIALIZE VALUES

```

```

PDB = 0.0
D = 1.702

C
DO 300 I = 1,NITEMS
  ABI = PARB(I,1)
  BBI = PARB(I,2)
  CBI = PARB(I,3)
  ACI = PARC(I,1)
  BCI = PARC(I,2)
  CCI = PARC(I,3)
  DO 250 J = 1,20
    TJ = MIDT(J)
    TTJ = TJ * A + B
    WJ = WEIGHT(J)
    IF (WJ .EQ. 0.0) GOTO 250

C
C   PARTIAL DERIVATIVE OF PB WITH RESPECT TO B
ANS=(-EXP(BBI*ABI*D+TJ*ABI*D*A+ABI*D*B)*(CBI-1.)*ABI*D)/(EXP
+ (BBI*ABI*D)+EXP(TJ*ABI*D*A+ABI*D*B))**2

C
C   COMPUTE PROBS
PB = P3PL(ABI,BBI,CBI,TTJ)
PC = P3PL(ACI,BCI,CCI,TJ)

C
C   COMPUTE DIRIVITIVE OF TERM
TERM = 2. * WJ * ANS * (PB - PC)

C
PDB = PDB + TERM
250   CONTINUE
300   CONTINUE

C
RETURN
END

C
C ***** MLE EQUATING *****
C
C   SUBROUTINE EQUMLE(PARB,PARC,IROW,NITEMS,THETAB,THETAC,
* NSUBSB,NSUBSC,NPAR,A,B,IOUT,IER)
C
C   REAL PARB(IROW,3),PARC(IROW,3),THETAB(NSUBSB),THETAC(NSUBSC),
* COVB(3,3,200),COV1(3,3),COV2(3,3),COVC(3,3,200),
* X(2),
* H(3),G(2),W(6),A(7),B(7),SPARB(200,3),SPARC(200,3)
INTEGER SNITEM,SNPAR

C
EXTERNAL MLEFUN
COMMON COVB,COVC,SNITEM,SPARB,SPARC,SNPAR

C
C   INITIALIZE ERROR VAR
IER = 0

C
C   INITIALIZE STARTING VALUES OF EQUATING CONSTANTS

```

```

      IF (A(2) .EQ. 0.0 .AND. B(2) .EQ. 0.0)
      * CALL BEQUAR(PARB,PARC,IROW,NITEMS,A,B,IOUT,IER)
      X(1) = A(2)
      X(2) = B(2)
      CALL UGETIO(3,0,5)

C
C   TRANSFER STUFF TO COMMON ARRAYS
      DO 441 I = 1,NITEMS
        DO 411 J = 1,3
          SPARB(I,J) = PARB(I,J)
          SPARC(I,J) = PARC(I,J)
411      CONTINUE
441      CONTINUE
      SNITEM = NITEMS
      SNPAR = NPAR

C
C   COMPUTE COVARIANCE MATRICES
      DO 99 ITM = 1,NITEMS
        AB = PARB(ITM,1)
        BB = PARB(ITM,2)
        CB = PARB(ITM,3)
        AC = PARC(ITM,1)
        BC = PARC(ITM,2)
        CC = PARC(ITM,3)
        CALL COV3PL(THETAB,NSUBSB,AB,BB,CB,NPAR,3,COV1,IOUT,IER)
        CALL COV3PL(THETAC,NSUBSC,AC,BC,CC,NPAR,3,COV2,IOUT,IER)
        DO 71 I = 1,NPAR
          DO 69 J = 1,NPAR
            COVB(I,J,ITM) = COV1(I,J)
            COVC(I,J,ITM) = COV2(I,J)
69          CONTINUE
71        CONTINUE
99      CONTINUE

C
C   PERFORM MINIMIZATION
      NSIG = 3
      N = 2
      MAXFN = 500
      IOPT = 2
      CALL ZXMIN(MLEFUN,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)

C
      A(7) = X(1)
      B(7) = X(2)

C
      RETURN
      END

C
      SUBROUTINE MLEFUN(N,X,F)
      REAL COVB(3,3,200),COVC(3,3,200),SPARB(200,3),
      * SPARC(200,3),COV(3,3),WK(3),SCOV(3,3),
      * V(3),X(2)
      INTEGER SNITEM,SNPAR

```

```

COMMON COVB,COVC,SNITEM,SPARB,SPARC,SNPAR

C
C   INITIALIZE VALUES
A = X(1)
B = X(2)
F = 1.
KK = 0

C
DO 500 ITM = 1,SNITEM

C
C   TRANSFORM COVARIANCE MATRIX FOR COMPARISON GROUP
DO 9 I = 1,SNPAR
  DO 7 J = 1,SNPAR
    COV(I,J) = COVC(I,J,ITM)
  7 CONTINUE
  9 CONTINUE
  COV(1,1) = COV(1,1) / (A**2.)
  COV(2,2) = COV(2,2) * (A**2.)
  IF (SNPAR .EQ. 2) GOTO 39
  COV(1,3) = COV(1,3) / A
  COV(3,1) = COV(1,3)
  COV(2,3) = COV(2,3) * A
  COV(3,2) = COV(2,3)
39 CONTINUE

C
C   COMPUTE SUM OF COVARIANCE MATRICES
DO 45 I = 1,SNPAR
  DO 43 J = 1,SNPAR
    SCOV(I,J) = COV(I,J) + COVB(I,J,ITM)
  43 CONTINUE
  45 CONTINUE

C
C   TRANSFORM ITEM PARAMETERS FOR COMPARISON GROUP
APAR = SPARC(ITM,1) / A
BPAR = (SPARC(ITM,2) * A) + B

C
C   COMPUTE VECTOR OF PARAMETER DIFFERENCES
V(1) = SPARB(ITM,1) - APAR
V(2) = SPARB(ITM,2) - BPAR
V(3) = SPARB(ITM,3) - SPARC(ITM,3)

C
C   COMPUTE MULTIVARIATE DENSITY VALUE (PROB)
CALL NORDEN(SCOV,V,SNPAR,PROB,CS)

C
C   INCREMENT CURRENT FUNCTION VALUE
F = F * PROB

C
C   CHECK AND CORRECT FOR UNDERFLOW
70 IF(ABS(F) .GT. 1.) GOTO 100
KK = KK + 1
F = F * 1024.
GOTO 70

```

```

100 CONTINUE
C
500 CONTINUE
C
C   COMPUTE LOG OF FUNCTION VALUE AND MULTIPLY BY -1
F = - ALOG(F) + 10. * FLOAT(KK) * ALOG(2.)
C
RETURN
END
C
SUBROUTINE NORDEN(COV,CEN,P,DEN,CS)
INTEGER P
REAL COV(3,3),CEN(3),WK(3),INVCOV(3,3),B(3),TEMP(3,3),
* TEMP2(3)
C
C   INVERT COVARIANCE MATRIX
DO 11 I = 1,P
    DO 10 J = 1,P
        TEMP(I,J) = COV(I,J)
10    CONTINUE
11    CONTINUE
CALL LINV1F(TEMP,P,3,INVCOV,0,WK,IER)
C
C   COMPUTE CHI-SQUARE
DO 79 I = 1,P
    TEMP2(I) = 0.
    DO 77 J = 1,P
        TEMP2(I) = TEMP2(I) + CEN(J) * INVCOV(J,I)
77    CONTINUE
79    CONTINUE
CS = 0.
DO 48 K = 1,P
    CS = CS + TEMP2(K) * CEN(K)
48    CONTINUE
C
C   COMPUTE DETERMINANT OF COVARIANCE MATRIX
D1 = 5.
CALL LINV3F(COV,B,4,P,3,D1,D2,WK,IER)
DET = D1 * (2.**D2)
C
C   COMPUTE P-VARIATE NORMAL DENSITY
PIE = 3.1415927
DEN = ((2.*PIE)**(-P/2.)) * (DET**(-.5)) * EXP(-CS/2.)
C
RETURN
END
C
C ***** UTILITY SUBROUTINES *****
C
P3PL - Function

```

This routine computes probabilities according to the 3-parameter logistic model for given values of a,b,c and theta.

REAL FUNCTION P3PL(A,B,C,THETA)

A: a-parameter

B: b-parameter

C: c-parameter

THETA: Person parameter

Function listing:

```
REAL FUNCTION P3PL(A,B,C,THETA)
XNUM = 1. - C
DENOM = 1. + EXP(-1.702 * A * (THETA - B))
P3PL = C + (XNUM/DENOM)
RETURN
END
```

ESPNT - SUBROUTINE

THIS SUBROUTINE CREATES A VECTOR OF EQUALLY SPACED POINTS

ESPNT(X,NPONTs,XMIN,XMAX)

X: OUTPUT VECTOR OF LENGTH NPONTs WHICH CONTAINS THE EQUALLY SPACED POINTS.

NPONTs: NUMBER OF POINTS WHICH X WILL CONTAIN.
NPONTs SHOULD BE GREATER THAN OR EQUAL TO 2.

XMIN: MINIMUM VALUE OF X VECTOR. WILL BE PLACED
IN FIRST ELEMENT OF VECTOR X.

XMAX: MAXIMUM VALUE OF X VECTOR. WILL BE PLACED
IN LAST ELEMENT OF VECTOR X.

Subroutine listing:

```

      SUBROUTINE ESPNT(X,NPONTs,XMIN,XMAX)
      REAL X(NPONTs)
C
C
      X(1) = XMIN
      XPONTs = NPONTs
      XNINT = XPONTs - 1.
      XINCRE = (XMAX-XMIN)/XNINT
      DO 10 I = 2,NPONTs
        X(I) = X(I-1) + XINCRE
10    CONTINUE
      RETURN
      END

```

COV3PL - SUBROUTINE

THIS ROUTINE COMPUTES THE ITEM COVARIANCE MATRIX FOR THE TWO OR THREE PARAMETER LOGISTIC MODELS. FORMULAS ARE TAKEN FROM LORD (1980) P.191.

COV3PL (THETAS,NSUBS,A,B,C,NPAR,IR,COV,IOUT,IER)

THETAS: VECTOR OF LENGTH NSUBS CONTAINING THE THETA PARAMETERS FOR THOSE ANSWERING THE ITEM. IF A SUBJECT DID NOT ANSWER THE ITEM, THE THETA FOR THAT PERSON SHOULD BE SET TO 999.

NSUBS: LENGTH OF VECTOR THETAS, INCLUDING 999'S. NUMBER OF SUBJECTS.

A: A-PARAMETER FOR THE THREE PARAMETER LOGISTIC MODEL.

B: B-PARAMETER FOR THE THREE PARAMETER LOGISTIC MODEL.

C: C-PARAMETER FOR THE THREE PARAMETER LOGISTIC MODEL. IF THE TWO PARAMETER MODEL IS DESIRED, C SHOULD BE SET EQUAL TO 0.

NPAR: NUMBER OF ESTIMATED PARAMETERS IN MODEL. NPAR=2 FOR THE TWO PARAMETER MODEL, OR WHEN C IS KNOWN, NPAR=3 FOR THE THREE PARAMETER MODEL.

IR: ROW DIMENSION OF MATRIX COV EXACTLY AS SPECIFIED IN THE

CALLING PROGRAM.

COV: OUTPUT, NPAR BY NPAR MATRIX CONTAINING THE COVARIANCE MATRIX.

IOUT: TAPE NUMBER OF OUTPUT FILE.

IER: IF IER > 0, AN ERROR OCCURED IN AN IMSL SUBROUTINE AND RESULTS IN COV ARE NOT THE ACTUAL ESTIMATES.

Subroutine listing:

```

SUBROUTINE COV3PL(THETAS,NSUBS,A,B,C,NPAR,IR,COV,IOUT,IER)
REAL THETAS(NSUBS),COV(IR,3),
* INF(3,3),WK(3),TCOV(3,3)
CALL UGETIO(3,0,IOUT)
C
C   INITIALIZE VALUES TO ZERO
DO 30 J = 1,NPAR
  DO 25 I = 1,NPAR
    INF(I,J) = 0.0
  25 CONTINUE
  30 CONTINUE
C
C   COMPUTE ADDITIVE TERMS IN COVARIANCE MATRIX
DO 100 J = 1,NSUBS
  THET = THETAS(J)
  IF (THET .EQ. 999.0) GOTO 100
  PROB = P3PL(A,B,C,THET)
  QDP = (1.-PROB)/PROB
C
  INF(1,1) = INF(1,1) + (((THET-B)**2.) * ((PROB-C)**2.) *
  * QDP)
  INF(2,2) = INF(2,2) + (((PROB-C)**2.) * QDP)
  INF(2,1) = INF(2,1) + ((THET-B) * ((PROB-C)**2.) * QDP)
C
  IF (NPAR .EQ. 2) GOTO 100
  INF(3,1) = INF(3,1) + ((THET-B) * (PROB-C) * QDP)
  INF(3,2) = INF(3,2) + ((PROB-C) * QDP)
  INF(3,3) = INF(3,3) + QDP
  100 CONTINUE
C
C   MULTIPLY TERMS BY RESPECTIVE CONSTANTS
D = 1.702
CTERM = 1./((1.-C)**2.)
INF(1,1) = (D**2.) * CTERM * INF(1,1)
INF(2,1) = (D**2.) * A * CTERM * INF(2,1) * (-1.)
INF(1,2) = INF(2,1)
INF(2,2) = (D**2.) * (A**2.) * CTERM * INF(2,2)
C

```

```
IF (NPAR .EQ. 2) GOTO 200
INF(3,1) = D * CTERM * INF(3,1)
INF(1,3) = INF(3,1)
INF(3,2) = (-1.) * D * A * CTERM * INF(3,2)
INF(2,3) = INF(3,2)
INF(3,3) = CTERM * INF(3,3)
200 CONTINUE
C
C   FIND INVERSE OF INFORMATION MATRIX
IER = 0
CALL LINV1F(INF,NPAR,3,TCOV,0,WK,IER)
C
C   COPY RESULTS TO MATRIX: COV
DO 305 J = 1,NPAR
  DO 300 I = 1,NPAR
    COV(I,J) = TCOV(I,J)
  300 CONTINUE
  305 CONTINUE
C
  RETURN
  END
```

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