

# Tuning Stability of a Digitally-Tuned, Electrically-Short Monopole Element Mounted on Circular Groundplanes of Different Radii

By

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  - Monopole Antenna
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  - Single-Parameter Tuning
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## PREFACE

The Air Force Airborne SINGARS (AFABS) VHF-FM radio is a frequency-hopped, antijam radio which utilizes an electrically-small antenna enclosed in a blade-shaped radome to minimize aerodynamic drag on airborne platforms. The size of the antenna groundplane is platform dependent but is usually small or comparable to an rf wavelength. The development of an efficient digitally-tuned antenna, which remains stably tuned for various operating conditions and platforms, is of interest for this radio.

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G. Korbani and S. F. McGrady wrote the computer program of the theoretical model and performed the computer runs upon which the data of this paper is based. S. P. Cruze contributed to the initial effort on single-parameter tuning.

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## SECTION 1

### INTRODUCTION

This paper investigates the tuning stability of a digitally-tuned, electrically-short monopole element mounted at the center of different-sized circular groundplanes. In particular, double-parameter and single-parameter tuning is investigated to determine the range of groundplane radii for which the same tuning words can be utilized in maintaining a specified antenna gain over a given frequency band.

Efficient, electrically-small antennas have instantaneous percentage bandwidths which are inherently narrow<sup>(1),(2)</sup>. By loading the antenna or its impedance-matching network with sufficient ohmic loss, large bandwidths can be achieved but at the expense of appreciably reduced efficiency. An alternative method for obtaining potentially much larger efficiency is electronic tuning of the antenna system. The antenna system comprises the antenna and its impedance-matching network.

An electrically-short monopole antenna has an input impedance whose real part (equal to the radiation resistance for zero ohmic loss) is small compared to 50 ohms and whose imaginary part is negative and large compared to 50 ohms. Both the radiation resistance and input reactance are functions of frequency over the desired frequency band of operation (30-150 MHz).

Generally, two parameters in the impedance-matching network must be varied (designated "double-parameter tuning") to provide impedance matching of both the real and imaginary parts of the antenna input impedance. However, single-parameter tuning is

sufficient provided the radiation resistance is proportional to the square of the frequency, large compared to ohmic loss resistances, and small compared to the 50 ohm source impedance<sup>(3)</sup>. These conditions for perfect impedance matching with single-parameter tuning are generally not realizable over large frequency bands as will be shown in this paper.

Impedance matching of the imaginary part of the antenna input impedance may usually be achieved by digital switching of appropriate inductances from a fixed inductance bank in series with the antenna element. Impedance matching of the real part of the antenna input impedance may usually be achieved by feeding the inductances at a variable (digitally-switched) tap point in double-parameter tuning and at a fixed tap point in single-parameter tuning (see figures 1 and 2 of section 2). The conditions for which such circuits are sufficient are given by Eqs. (10) and (15) of section 2.

Consider now the impedance-matching network which provides an impedance match to 50 ohms when the monopole element is mounted on a reference groundplane of specified radius. The set of switching bits, which achieves an impedance match at a given frequency when the monopole element is mounted on the reference groundplane, is denoted the "tuning word" for that frequency. This paper investigates the range of groundplane radii for which the same tuning words can be utilized in maintaining a specified antenna gain on the radio horizon.

The antenna gain on the radio horizon (dBi) is the sum of the antenna directive gain on the horizon (dBi), antenna system mismatch gain (equal to minus the mismatch loss) dB, and the radiation efficiency of the antenna system (dB). Although the directive gain and radiation efficiency are functions of the groundplane size, it is

shown in this paper that the mismatch gain is particularly sensitive to groundplane size for groundplane radii appreciably different from that of the reference groundplane.

The analytical model is described in section 2. Numerical results and conclusions are given in sections 3 and 4, respectively.

SECTION 2  
ANALYTICAL MODEL

2.1 ANTENNA CIRCUIT

Consider a monopole element, of height  $h = 16$  in. and radius  $b = 0.5$  in., which is mounted at the center of a circular groundplane of radius  $a = 0.5$  to 40 ft. in the frequency band 30-150 MHz (see figure 1). The element is in series with an impedance-matching network comprising the inductances  $L_1$  and  $L_2$  of a tapped coil, one end of which is connected to the groundplane. The coil tap point is fed by the center conductor of a low-loss coaxial line whose outer conductor is also connected to the groundplane. The ohmic resistance of the coil and antenna is approximated by a resistance  $R_{\text{ohmic}}$  in series with  $L_1$ . In this paper, the tapped coil is treated as a lumped circuit which does not radiate.

The equivalent circuit of the monopole antenna and its impedance-matching network is shown in figure 2. In double-parameter tuning, both  $L_1$  and  $L_2$  are variable with the radian frequency  $\omega$  (see figure 2a). In single-parameter tuning, only  $L_1$  is variable with frequency (see figure 2b). The monopole antenna has an input impedance  $Z(\omega)$ . The transmission line has a characteristic impedance of 50 ohms which is matched to the output impedance  $Z_g = R_g = 50$  ohms of the source generator. The input impedance  $Z_{\text{IN}}$  to the matching network is a function of the antenna input impedance  $Z(\omega)$ ,  $R_{\text{ohmic}}$ , and the assigned values of  $L_1$  and  $L_2$  at any given frequency.

The central issue is the range of groundplane sizes for which the same tuning "words" can be utilized in maintaining a specified antenna gain over the frequency band. In double-parameter tuning,

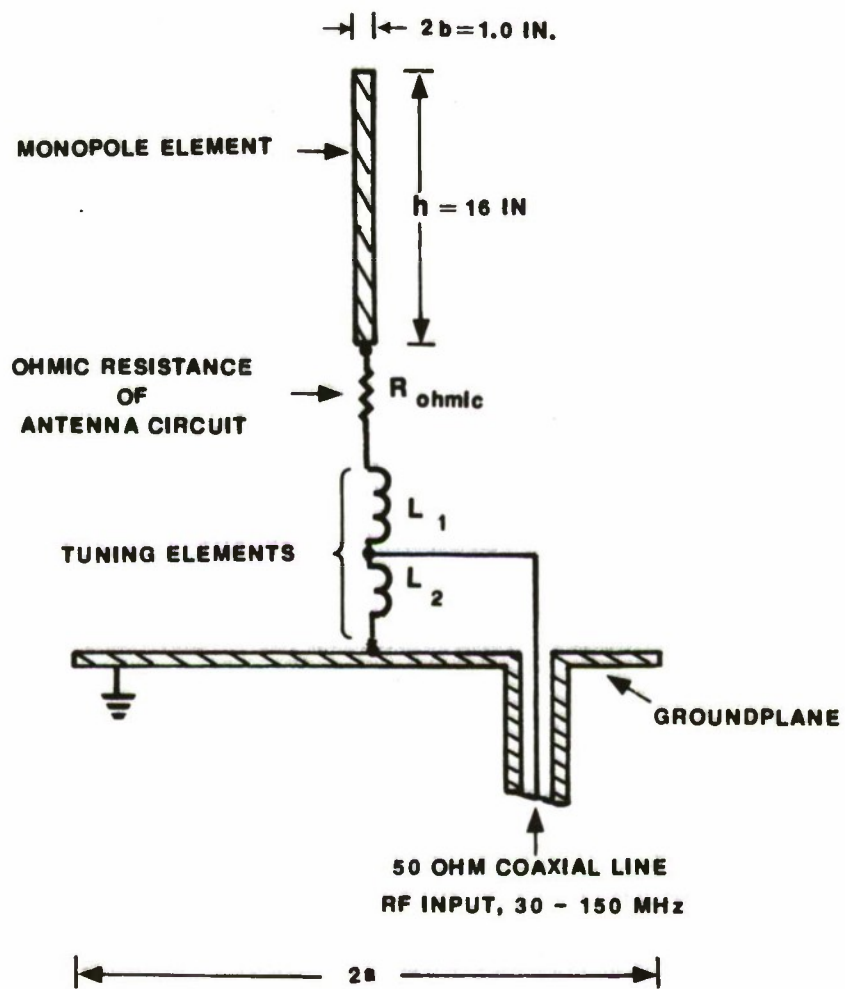


Figure 1. Monopole Antenna with Impedance-Matching Network

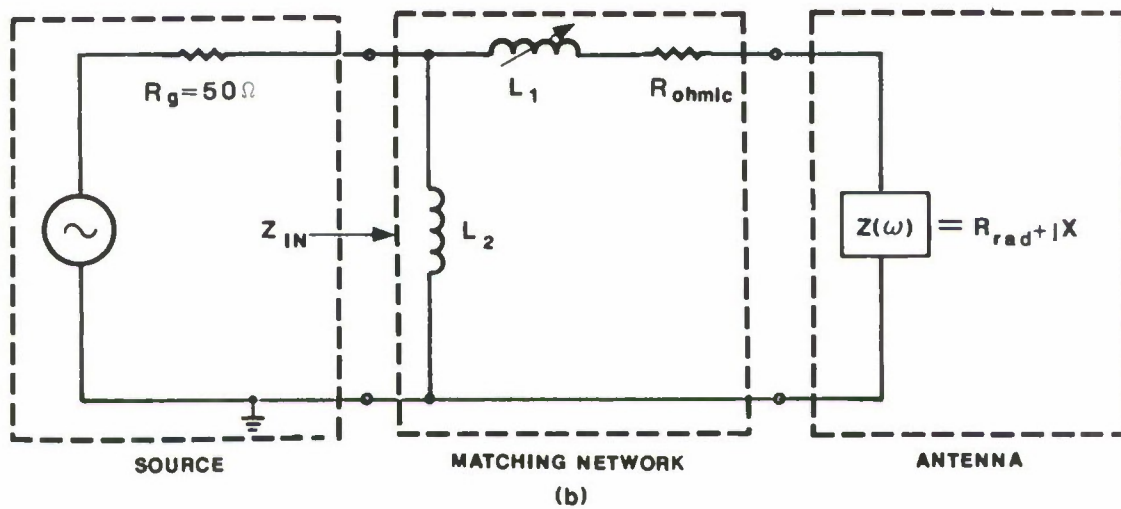
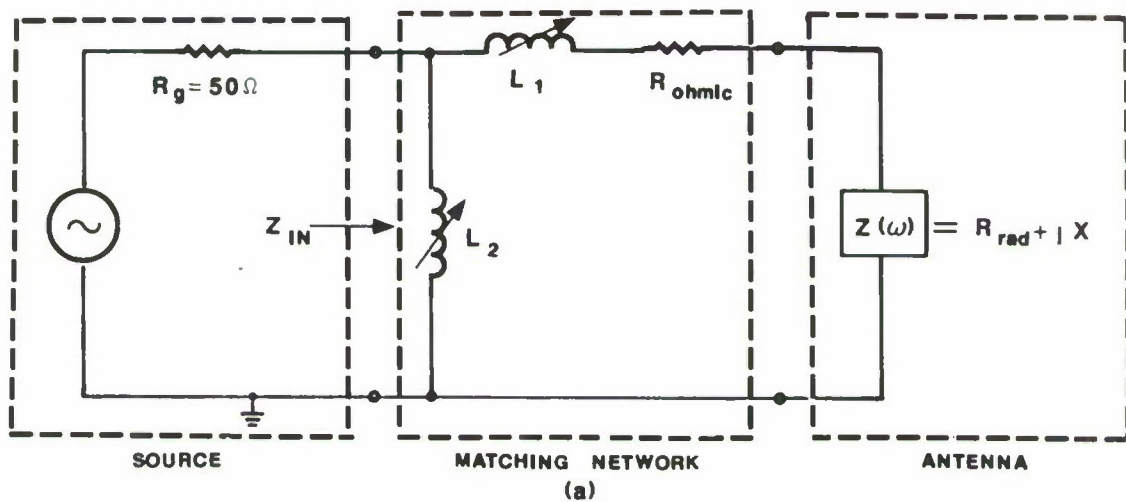


Figure 2. Equivalent Circuit of Monopole Antenna with Impedance - Matching Network  
 (a) Double-Parameter Tuning  
 (b) Single-Parameter Tuning

the tuning "words" (values of the inductances  $L_1$  and  $L_2$  at each frequency) are chosen so that when the monopole element is mounted on a reference groundplane of radius  $a = 4$  ft.,  $Z_{IN} = R_g = 50$  ohms at each of these frequencies. In single-parameter tuning, the tuning words are chosen so that, when the monopole element is mounted on a reference groundplane of radius  $a = 4$  ft.,  $\text{IM}(Z_{IN}) = 0$  at each frequency and  $\text{Re}(Z_{IN}) = R_g = 50$  ohms at 30 MHz.

## 2.2 INPUT IMPEDANCE OF THE MONOPOLE ANTENNA

The input impedance  $Z(\omega)$  of the monopole antenna is given by

$$Z(\omega) = R_{\text{rad}} + jX \quad (1)$$

where  $R_{\text{rad}}$  is the antenna radiation resistance and  $X$  is the antenna reactance.

The input impedance  $Z(\omega)$  of the monopole antenna was determined by the method of moments RICHMD1 program<sup>(4)</sup> for each of the above groundplanes. The radiation resistance varies from 0.3 to 0.6 ohms at 30 MHz and 10 to 30 ohms at 150 MHz (see figure 3). The reactance varies from -500 to -750 ohms at 30 MHz and -20 to -90 ohms at 150 MHz (see figure 3).

## 2.3 DETERMINATION OF THE TUNING WORDS

The input impedance  $Z_{IN}$  to the matching network is given by

$$Z_{IN} = j\omega L_2 // [j\omega L_1 + R_{\text{ohmic}} + Z(\omega)] = \text{Re}(Z_{IN}) + j\text{Im}(Z_{IN}) \quad (2)$$



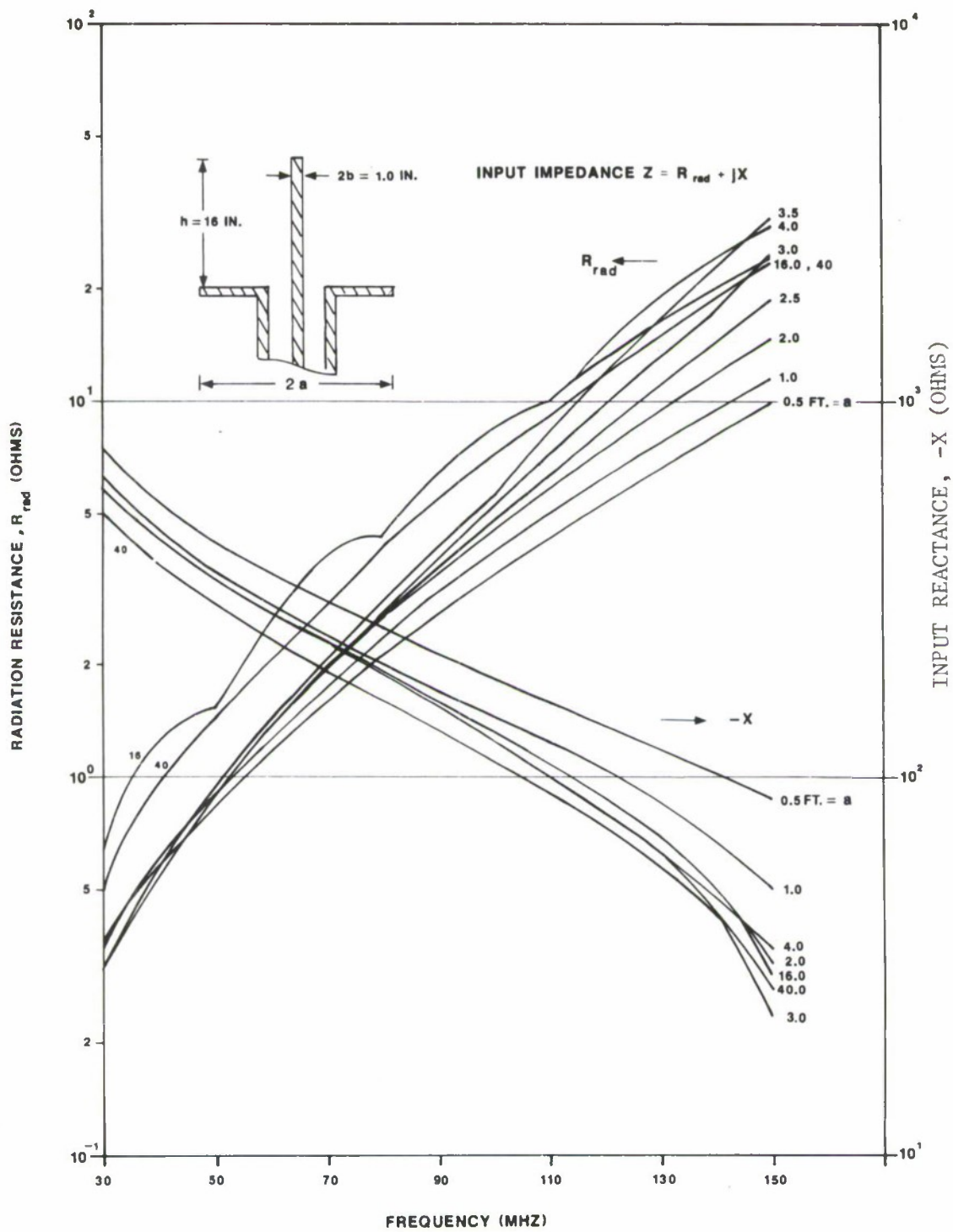


Figure 3. Input Impedance, for Zero Ohmic Loss, of Electrically-Short Monopole Element at the Center of a Circular Groundplane of Radius  $a = 0.5 - 40$  ft.

where

//  $\equiv$  "in parallel with"

$$\text{Re}(Z_{\text{IN}}) = \frac{\omega^2 L_2^2 R}{(\omega L_2 + B)^2 + R^2} \quad (3)$$

$$\text{Im}(Z_{\text{IN}}) = \frac{\omega L_2 [B(\omega L_2 + B) + R^2]}{(\omega L_2 + B)^2 + R^2} \quad (4)$$

$$R \equiv R_{\text{rad}} + R_{\text{ohmic}} \quad (5)$$

$$B \equiv \omega L_1 + X \quad (6)$$

The parameters  $L_1$  and  $L_2$  must be positive real quantities which are determined by requiring, when the monopole element is mounted on a reference groundplane of radius  $a = 4$  ft., that (1)  $\text{Im}(Z_{\text{IN}}) = 0$  at each frequency of interest and (2)  $\text{Re}(Z_{\text{IN}}) = R_g = 50$  ohms at each frequency of interest for double-parameter tuning and at 30 MHz only for single-parameter tuning. Let the subscript "o" denote the reference groundplane. Thus,  $Z_{\text{INo}}$ ,  $L_{1o}$ ,  $L_{2o}$ ,  $B_o$ ,  $R_o$ , and  $X_o$  denote the quantities  $Z_{\text{IN}}$ ,  $L_1$ ,  $L_2$ ,  $B$ ,  $R$ ,  $X$ , respectively, when the monopole element is mounted on the reference groundplane.

For  $\text{Im}(Z_{\text{INo}}) = 0$  [Requirement (1)], Eq. (4) reduces to

$$B_o (\omega L_{2o} + B_o) + R_o^2 = B_o^2 + \omega L_{2o} B_o + R_o^2 = 0 \quad (7)$$

Solving for  $B_o$  and using Eq. (6) to find  $L_{1o}$ ,

$$B_o = \frac{-\omega L_{2o} \pm [(\omega L_{2o})^2 - 4 R_o^2]^{1/2}}{2} \quad (8)$$

$$L_{1o} = \frac{-X_o}{\omega} - \frac{L_{2o}}{2} \pm \left[ \frac{L_{2o}^2}{2} - \frac{R_o^2}{\omega^2} \right]^{\frac{1}{2}} \quad (9)$$

Since  $L_{1o}$  must be a real quantity,

$$R_o \leq \omega L_{2o}/2, \quad \text{Im}(Z_{INo}) = 0 \quad (10)$$

If  $R_o > \omega L_{2o}/2$ , then the circuit of figure 2 cannot satisfy the condition  $\text{Im}(Z_{INo}) = 0$ . For such a case, let

$$L_{1o} = -\frac{X_o}{\omega} - \frac{L_{2o}}{2}, \quad R_o > \omega L_{2o}/2 \quad (11)$$

If condition (10) is satisfied, then the choice of root in Eq. (9) is determined by selecting the root which yields  $L_{1o} \geq 0$  and which yields the smaller modulus of the voltage reflection coefficient  $\rho_o$  at the input to the impedance-matching network where  $\rho_o$  is given by

$$\rho_o = \frac{Z_{INo} - 1}{Z_{INo} + 1} \quad (12)$$

For  $\text{Re}(Z_{INo}) = R_g = 50$  ohms at each frequency of interest in double-parameter tuning and at 30 MHz in single-parameter tuning [Requirement (2)], and for  $B_o$  given by Eq. (8), Eq. (3) reduces at those frequencies to

$$R_g = \frac{\omega^2 L_{2o}^2 R_o}{\omega L_{2o} + B_o + R_o} = \frac{2R_o}{1 \pm \left[ 1 - \frac{R_o^2}{(\omega L_{2o}/2)^2} \right]^{\frac{1}{2}}} \quad (13)$$

where the + and - signs correspond to the positive and negative radicals in Eq. (9). Solving Eq. (13) for  $L_{20}$ ,

$$L_{20} = \frac{1}{\omega} \left[ \frac{R_g R_o}{1 - \frac{R_o}{R_g}} \right]^{\frac{1}{2}} = \begin{cases} L_{20}(\omega_A), \text{ single parameter tuning} \\ L_{20}(\omega), \text{ double-parameter tuning} \end{cases} \quad (14)$$

where  $\omega_A/2\pi = 30$  MHz.

Eq. (14) is independent of which root is taken in Eq. (13). In double-parameter tuning,  $L_{20}$  is determined by Eq. (14) at each frequency of interest. In single-parameter tuning  $L_{20}$  is determined by Eq. (14) only at the radian frequency  $\omega = \omega_A$ .

For  $L_{20}$  to be a real quantity in Eq. (14),

$$R_o \leq R_g, \quad L_{20} \text{ real} \quad (15)$$

If  $R_o > R_g$ , then the circuit of figure 2 cannot satisfy the condition  $\text{Re}[Z_{INo}] = R_g$  for any value of  $L_{20}$ .

Root selection of  $L_{10}$  in Eq. (9) can be simplified by use of Eq. (13) for double-parameter tuning but not for single-parameter tuning. For double-parameter tuning, Eq. (13) imposes restrictions on which root in Eq. (9) to select in determining  $L_{10}$ . If  $R_g/2R_o \geq 1$ , Eq. (13) is satisfied only by the negative radical. If  $R_g/2R_o < 1$ , Eq. (13) is satisfied only by the positive radical. For single-parameter tuning, Eq. (13) cannot be utilized for root selection of  $L_{10}$  (except when  $\omega = \omega_A$ ) because Eq. (13) is not applicable to single-parameter tuning except at the frequency  $\omega_A/2\pi = 30$  MHz.

The algorithms for determining  $L_{1o}$  and  $L_{2o}$ , may be summarized as follows:

Double-parameter tuning:

$$L_{2o} = L_{2o}(\omega) = \begin{cases} \frac{1}{\omega} \left[ \frac{R_g R_o(\omega)}{1 - \frac{R_o(\omega)}{50}} \right]^{\frac{1}{2}}, & 0 \leq R_o(\omega) \leq R_g \text{ ohms} \\ \text{not realizable, } R_o(\omega) > R_g \text{ ohms} \end{cases} \quad (16)$$

$$L_{1o} = L_{1o}(\omega) = \begin{cases} \left[ -\frac{X_o(\omega)}{\omega} - \frac{L_{2o}}{2} - \left[ \left( \frac{L_{2o}}{2} \right)^2 - \frac{R_o^2(\omega)}{\omega^2} \right]^{\frac{1}{2}} \right], & 0 \leq R_o(\omega) \leq R_g/2 \text{ ohms} \\ \left[ -\frac{X_o(\omega)}{\omega} - \frac{L_{2o}}{2} + \left[ \left( \frac{L_{2o}}{2} \right)^2 - \frac{R_o^2(\omega)}{\omega^2} \right]^{\frac{1}{2}} \right], & R_g/2 < R_o(\omega) < R_g \text{ ohms} \end{cases} \quad (17)$$

Single-parameter tuning:

$$L_{2o} = L_{2o}(\omega_A) = \begin{cases} \frac{1}{\omega_A} \left[ \frac{R_g R_o(\omega_A)}{1 - \frac{R_o(\omega_A)}{R_g}} \right]^{\frac{1}{2}}, & 0 \leq R_o(\omega_A) \leq R_g \text{ ohms} \\ \text{not realizable, } R_o(\omega_A) > R_g \text{ ohms} \end{cases} \quad (18)$$

$$L_{10} = L_{10}(\omega) = \begin{cases} -\frac{X_o(\omega)}{\omega} - \frac{L_{20}}{2} + \left[ \left( \frac{L_{20}}{2} \right)^2 - \frac{R_o^2(\omega)}{\omega^2} \right]^{\frac{1}{2}}, & R_o(\omega) \leq \omega L_{20}/2. \\ -\frac{X_o(\omega)}{\omega} - \frac{L_{20}}{2}, & R_o(\omega) > \omega L_{20}/2 \end{cases} \quad (19)$$

Choose root which yields  $L_{10} \geq 0$  and which yields the smaller  $|\rho_o|$  when  $\rho_o$  is given by Eq. (12).

When the monopole element is mounted on the reference ground-plane, and  $R_o(\omega) \leq R_g$  ohms, then it is always possible with double-parameter tuning to provide over the operating band a perfect match of the antenna input impedance  $Z(\omega)$  to the generator source impedance  $R_g$ . It is generally not possible with single-parameter tuning.

However, there is a case for which impedance matching over the operating band can be obtained with single-parameter tuning. Consider the case in which the radiation resistance is proportional to the square of the frequency, large compared to ohmic loss resistances, and small compared to the source impedance. Stated algebraically, these conditions are:

$$R_{ohmic} \ll R_{rad} = K \omega^2 \ll R_g \quad (20)$$

where  $K$  is a constant. For Conditions (20), the double-parameter tuning Eqs. (16) and (17) reduce to

$$L_{20} \approx \frac{1}{\omega} [R_g R_o(\omega)]^{1/2} \approx \frac{1}{\omega} (R_g R_{rad})^{1/2} = (R_g K)^{1/2} \quad (21)$$

$$L_{10} \approx -\frac{X_o(\omega)}{\omega} - \frac{L_{20}}{2} - \left[ \left( \frac{L_{20}}{2} \right)^2 - K\omega^2 \right]^{1/2} \quad (22)$$

Since  $L_{20}$  in Eq. (21) is approximately independent of frequency, an approximate match over the operating band can be obtained with single-parameter tuning if Conditions (20) are satisfied over the operating band.

At the high end of the band, the condition  $R_{\text{rad}} \ll R_g$  is not satisfied for  $R_g = 50$  ohms (see figure 3). Furthermore, the condition  $R_{\text{rad}} = K\omega^2$  is not physically realizable over large operating bands (see figure 4). In figure 4, the radiation resistance of figure 3 is replotted on log log paper whose abscissa corresponds to decreasing values of frequency. It should be noted that the plots are not straight lines with a slope of two as would be the case if the radiation resistance were proportional to the square of the frequency. However, for electrically-short, electrically-thin, low-loss monopole elements mounted on groundplanes which are very much larger than a wavelength, the radiation resistance is approximately proportional to the square of the frequency. For such a case, the constant  $K$  is given by<sup>(5)</sup>

$$K = K_\epsilon \gg 1 = \frac{\eta h^2}{12\pi c^2} \approx 10 h^2/c^2 ; h \ll \lambda, a \gg \lambda, b \ll \lambda \quad (23)$$

$$\epsilon \equiv 2\pi a/\lambda$$

$$\eta = \text{wave impedance of free space} = 376.73 \text{ ohms} \approx 120\pi \text{ ohms}$$

$$c = \text{wave velocity in free space} = 2.9979 \times 10^8 \text{ m/s}$$

$$h = \text{element length}$$

$$b = \text{element width}$$

$$a = \text{groundplane radius}$$

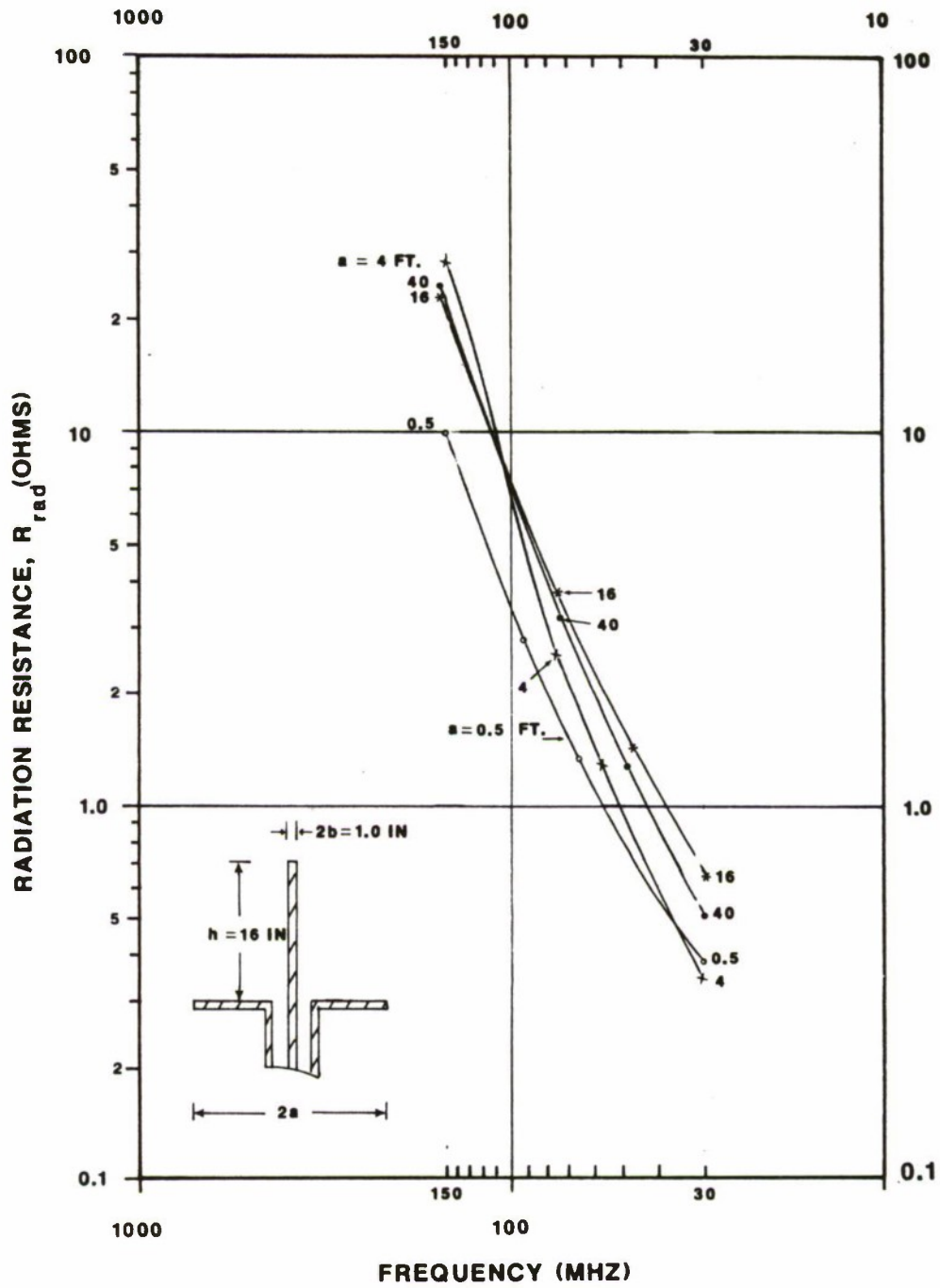


Figure 4. Frequency Dependence of Radiation Resistance



For the same monopole elements, the condition  $R_{\text{rad}} = K\omega^2$  is also physically realizable over small operating bands for groundplane radii  $\epsilon = 2\pi a/\lambda \approx 0, 3.6, 5.3, 7.0, 8.5, 10.0, 11.8, \dots$  wavenumbers for which the derivative of radiation resistance with respect to groundplane radius (in wavenumbers) is zero<sup>(6)</sup>. The proportionality constant  $K_\epsilon$  for each of the above groundplane radii is given by<sup>(7)</sup>

$$\begin{aligned} K_\epsilon / K_{\epsilon \gg \gg 1} &= d_{\epsilon \gg \gg 1} / d_\epsilon = 0.750 / d_\epsilon, \\ \epsilon &\approx 0, 3.6, 5.3, 7.0, 8.5, 10.0, 11.8, \dots \end{aligned} \tag{24}$$

where  $d_\epsilon$  is the numeric directive gain on the horizon for an electrically-short monopole element mounted on a groundplane of radius  $\epsilon$  (in wavenumbers) and where  $K_\epsilon \gg \gg 1$  is given by Eq. (23). For  $\epsilon=0$ ,  $K_\epsilon / K_{\epsilon \gg \gg 1} = 0.5$  since  $d_0 = 1.5$  (see ref. 5).

In summary, single-parameter tuning cannot provide a perfect impedance match over a 5:1 operating band (30-150 MHz) on a groundplane whose radius is either electrically-small or comparable to a wavelength. When the monopole element is mounted on the reference groundplane, double-parameter tuning can provide a perfect impedance match over the operating band whereas in single-parameter tuning the tuning words can be chosen to provide a perfect impedance match only at one frequency (30 MHz) and to provide zero input reactance at other frequencies.

#### 2.4 ANTENNA GAIN

It is of interest to determine the antenna gain on the radio horizon (elevation angle  $\theta=\pi/2$  rad) as a function of groundplane

radius for the same tuning words which are selected when the monopole element is mounted on the reference groundplane. The antenna gain,  $G(\pi/2)$  (dBi), on the radio horizon is given by

$$G(\pi/2) = D(\pi/2) + M + \eta \text{ (dBi)} \quad (25)$$

where

$D(\pi/2)$  = directive gain on the horizon of the monopole antenna (dBi)

$\eta$  = radiation efficiency of the antenna circuit (dB)

$M$  = mismatch gain (= - mismatch loss) of the antenna circuit (dB)

The directive gain  $D(\pi/2)$  is determined in section 3.1 by the method of moments RICHMD2 program<sup>(4)</sup>.

The radiation efficiency  $\eta$  is given by

$$\eta = 10 \log_{10} \left( \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{ohmic}}} \right) \text{ (dB)} \quad (26)$$

where

$R_{\text{ohmic}}$  = series ohmic resistance of antenna system (ohms)

$R_{\text{rad}}$  = radiation resistance of the monopole antenna (ohms)

The mismatch gain M is given by

$$M = 10 \log_{10}(1 - |\rho|^2) \text{ (dB)} \quad (27)$$

where

$\rho$  = voltage reflection coefficient at the input to  
the impedance-matching network

The voltage reflection coefficient  $\rho$  is given by

$$\rho = \frac{\left(\frac{Z_{IN}}{Z_g}\right) - 1}{\left(\frac{Z_{IN}}{Z_g}\right) + 1} \quad (28)$$

where  $Z_{IN}$  is given by Eq. (2) and  $Z_g = R_g = 50$  ohms is the output impedance of the source generator.

Although the mismatch gain M is determined by specifying the voltage reflection coefficient  $\rho$ , it may also be determined by specifying the equivalent voltage standing wave ratio (VSWR) which would be measured along a transmission line between the source generator and the impedance-matching network. The VSWR,  $\rho$ , and M are related by

$$VSWR = \frac{1 + |\rho|}{1 - |\rho|} \quad (29)$$

$$M = 10 \log_{10} \left[ \frac{4 \text{ VSWR}}{(\text{VSWR} + 1)^2} \right] \text{ (dB)} \quad (30)$$

Numerical values of  $D(\pi/2)$ ,  $\eta$ , M, VSWR, and  $G(\pi/2)$  are given in section 3.

The analytical model does not include the effect of any impedance pad which might be placed between the source generator and the impedance-matching network. A two-port impedance pad between the source generator and impedance-matching network does not improve the mismatch gain of the antenna but does help protect the source generator from being damaged by power reflected from the antenna. The impedance pad reduces the amount of power reflected back to the source generator but also reduces the antenna gain by an amount equal to the insertion loss  $L$  (dB) of the pad (for a pad whose input and output impedances are equal to the source impedance when the opposite port is terminated in the source impedance). For example, a pad of 3 dB insertion loss will ensure a VSWR  $\leq 3.0:1$  between the generator and the pad for any load impedance at the output of the pad. However, the antenna gain will be reduced by 3 dB since the voltage reflection coefficient at the input to the impedance-matching network will be unchanged by the presence of the pad and the power delivered to the antenna will be reduced by 3 dB.

## SECTION 3

### NUMERICAL RESULTS

Numerical results for the antenna gain,  $G(\pi/2)$ , on the radio horizon (elevation angle  $\theta = \pi/2$  rad) are given in this section for the antenna circuit of figure 1 and for groundplane radii  $a = 0.5 - 40$  ft. For all groundplane radii, the parameters  $L_1$  and  $L_2$  are those selected for impedance matching when the monopole element is mounted on a groundplane of radius  $a = 4$  ft. Single- and double-parameter tuning of  $L_1$  and  $L_2$  are discussed in section 2.3.

The antenna gain,  $G(\pi/2)$ , is given by Eq. (25). The antenna gain is a function of the directive gain on the radio horizon, radiation efficiency of the antenna circuit, and the impedance mismatch gain at the input to the impedance-matching network.

#### 3.1 DIRECTIVE GAIN ON THE HORIZON

The directive gain of the monopole antenna was determined by the method of moments RICHMD2 program<sup>(4)</sup> for each of the above groundplanes. The directive gain,  $D(\pi/2)$ , on the radio horizon (elevation angle  $\theta = \pi/2$  rad) varies from 1.76 dBi to -0.95 dBi at 30 MHz and 1.81 dBi to -1.95 dBi at 150 MHz (see figure 5). For a given groundplane size, the elevation pattern for the 16 in. length element is within approximately 0.4 dB of that for a quarter-wave element. An electrically-short thin element has a directive gain on the radio horizon which is less than that of a quarter-wave element by 0.12 dB for very small groundplanes and by 0.39 dB for very large groundplanes<sup>(5)</sup>.

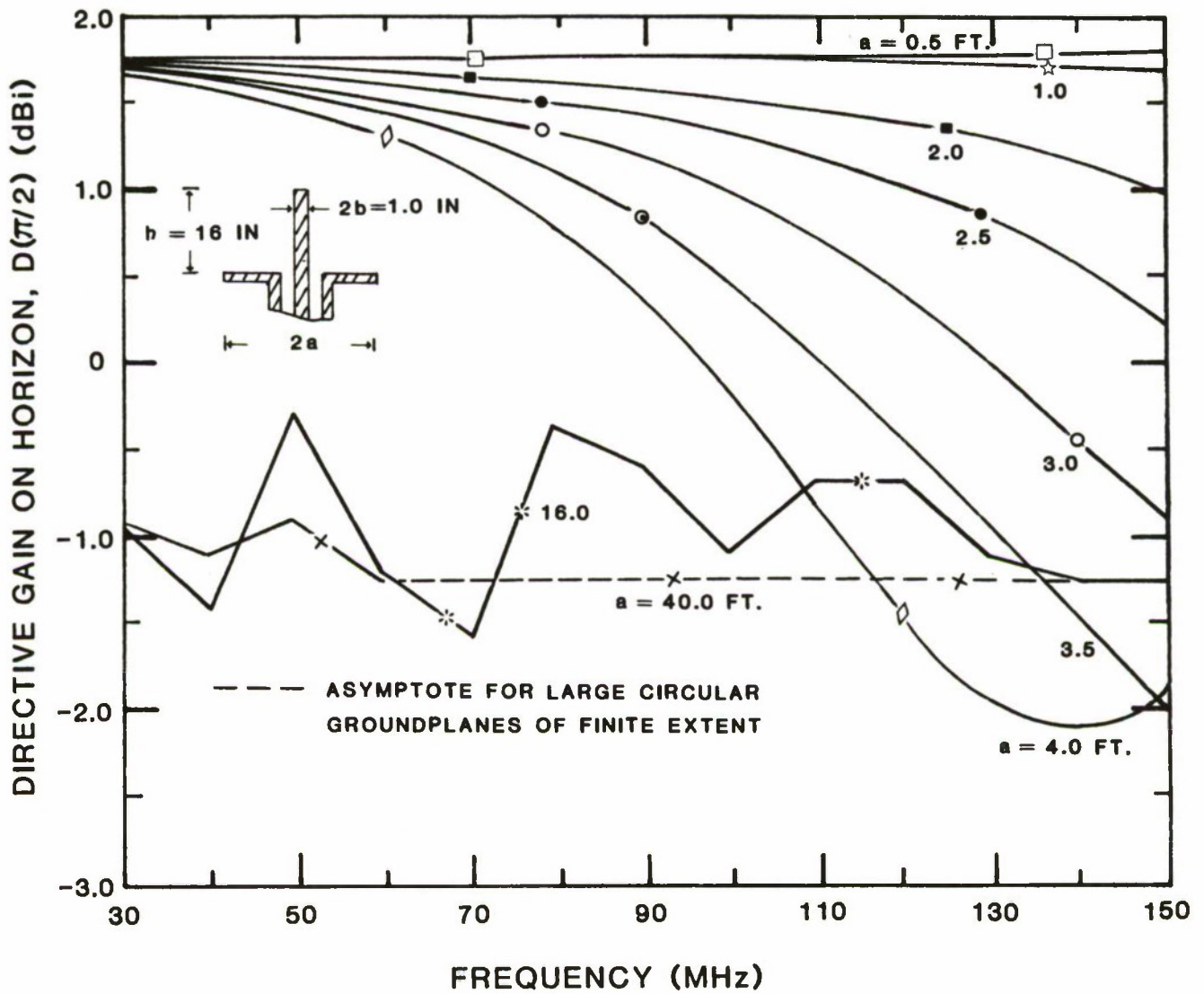


Figure 5. Directive Gain on Horizon of Electrically-Short Monopole Element at the Center of a Circular Groundplane of Radius  $a = 0.5 - 40$  ft.

### 3.2 RADIATION EFFICIENCY

The radiation efficiency  $\eta$  of the antenna circuit was determined by substituting into Eq. (26) the values of radiation resistance  $R_{\text{rad}}$  plotted in figure 3.

Numerical results are given for  $R_{\text{ohmic}} = 0, 1.7, \text{ and } 7.8$  ohms. The ohmic resistances  $R_{\text{ohmic}} = 0, 1.7, \text{ and } 7.8$  ohms yield radiation efficiencies of 0, -8, and -14 dB, respectively, at 30 MHz when the monopole element is mounted on a groundplane of radius  $a = 4.0$  ft. The radiation efficiency improves significantly (by several dB) with increasing frequency but is only weakly dependent (varies by approximately 2 dB) upon groundplane radius (see figure 6).

### 3.3 MISMATCH GAIN

The mismatched gain  $M$  was determined by evaluating Eqs. (27) and (28) for the single- and double-parameter tuning conditions discussed in section 2.3

The mismatch gain  $M$  at 30 MHz as a function of groundplane radius is plotted in figure 7. The range of groundplane radii, for which the mismatch loss (= - mismatch gain in dB) does not exceed a specified level, increases with increasing ohmic resistance. The range of groundplane radii, for which the mismatch loss is within 3 dB, is approximately 4 to 16 ft., 3 to 16 ft., and 2 to 16 ft. for  $R_{\text{ohmic}} = 0, 1.7, \text{ and } 7.8$  ohms, respectively. With very small and very large groundplane radii, the mismatch loss can exceed 90 dB for  $R_{\text{ohmic}} = 0$  ohms.

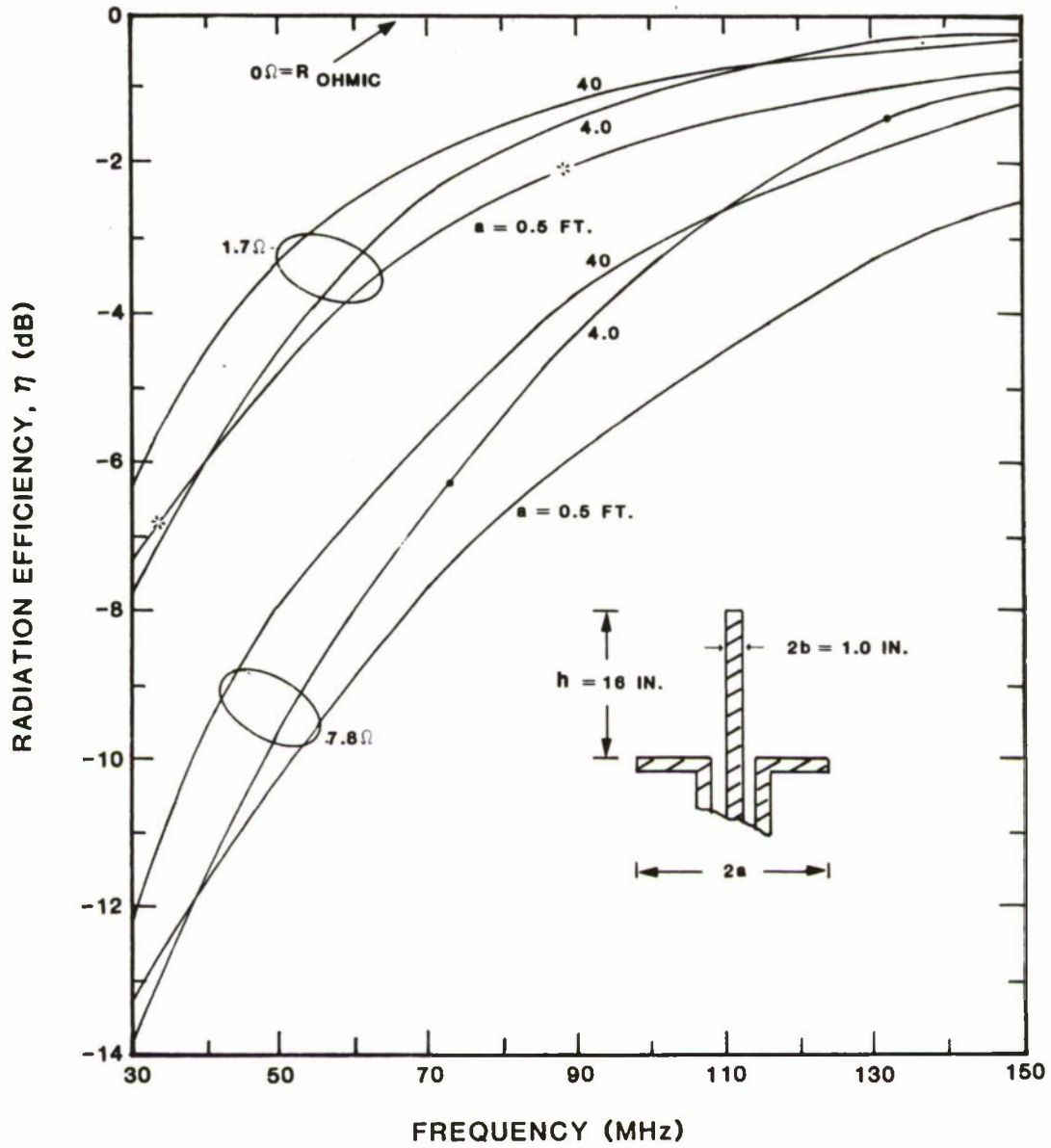


Figure 6. Radiation Efficiency of Monopole Antenna for System Ohmic Resistances of 0, 1.7, and 7.8 Ohms



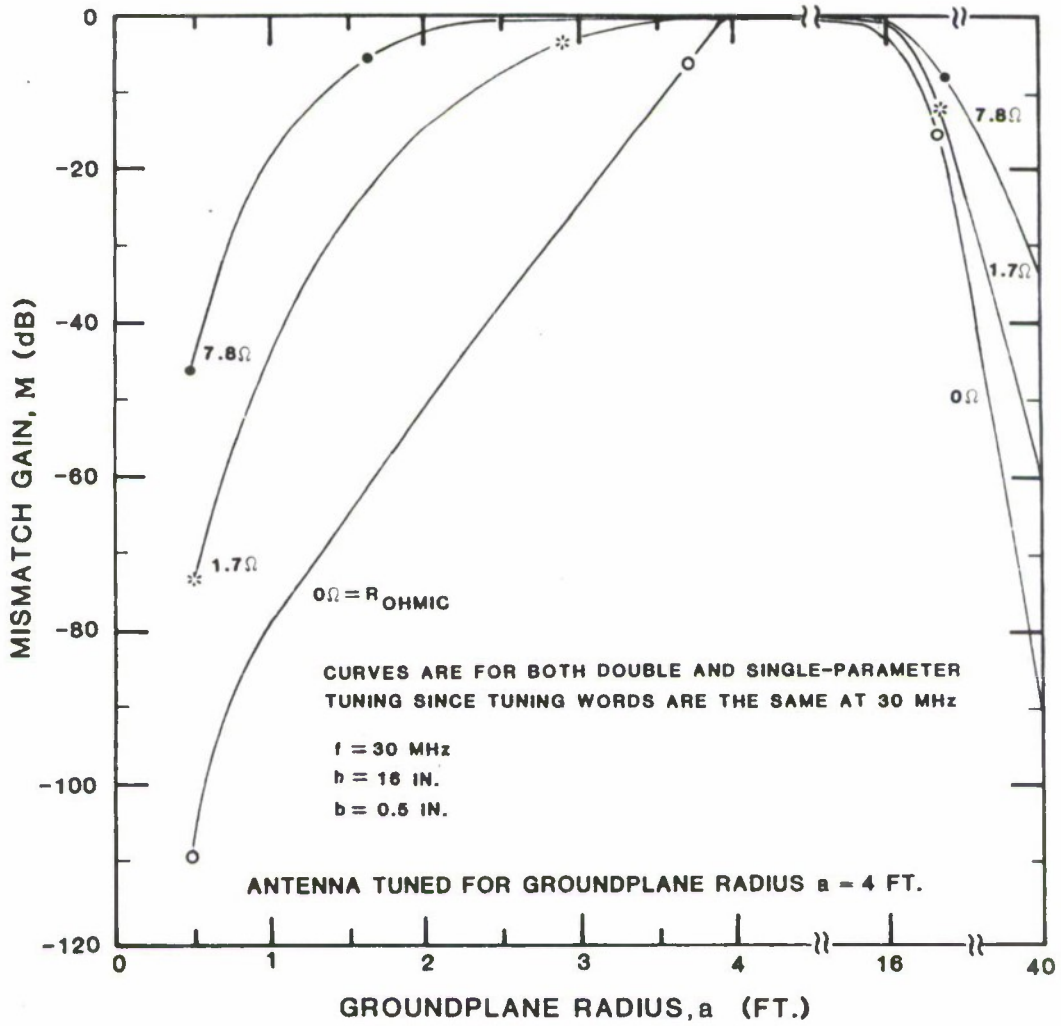


Figure 7. Mismatch Gain at 30 MHz

The mismatch gain and equivalent VSWR are plotted as a function of frequency in figures A1-A6 of appendix A for single- and double-parameter tuning and for  $R_{\text{ohmic}} = 0, 1.7, \text{ and } 7.8$  ohms with groundplane radius as a parameter. The results of figures A1-A6 are summarized here:

1. For a groundplane radius  $a = 4$  ft., the  $\text{VSWR} = 1$  at all frequencies with double-parameter tuning. For the same groundplane radius with single-parameter tuning, the  $\text{VSWR} = 1$  at 30 MHz and may be as large as 7.3 at 100 MHz (see figure A5).
2. With  $R_{\text{ohmic}} = 0$  ohms and double-parameter tuning, the  $\text{VSWR} \leq 6$  (corresponding to a mismatch loss of less than 3 dB) over the frequency band for groundplane radii  $3 \leq a \leq 16$  ft. (see figure A2). For the same conditions but with single-parameter tuning, the range of groundplane radii is reduced to  $4 \leq a \leq 16$  ft. (see figure A1).
3. With  $R_{\text{ohmic}} = 1.7$  ohms and double-parameter tuning, the  $\text{VSWR} \leq 6$  for groundplane radii  $2.5 \leq a \leq 16$  ft. (see figure A4). For the same conditions but with single-parameter tuning, the range of groundplane radii is approximately the same as with double-parameter tuning (see figure A3).
4. With  $R_{\text{ohmic}} = 7.8$  ohms and double-parameter tuning, the  $\text{VSWR} \leq 6$  for groundplane radii  $2.0 \leq a \leq 16$  ft. (see figure A6). For the same conditions but with single-parameter tuning, there is no groundplane radius for which the  $\text{VSWR} \leq 6$  over the whole frequency band (see figure A5). In figure A5 it should be noted that the mismatch loss is much larger at mid-band frequencies than at other frequencies. The reason is that, for the conditions of figure A5, the

$\text{Re}(Z_{\text{IN}})$  is much larger than 50 ohms at midband frequencies because the sum of the radiation resistance plus the ohmic resistance is not small compared to 50 ohms.

#### 3.4 ANTENNA GAIN ON THE HORIZON

The antenna gain on the radio horizon,  $G(\pi/2)$ , at 30 MHz is plotted in figure 8 as a function of groundplane radius. Although antenna gain is plotted as a continuous curve, please note the discontinuities in the abscissa and that the only data points for groundplane radii  $a \geq 4$  ft. are for  $a = 4, 16,$  and  $40$  ft. The same comment applies to figures B1-B4 of appendix B. At 30 MHz, the antenna horizon gain as a function of groundplane radii is a maximum for groundplane radii approximately equal to that of the reference groundplane with gains of 2, -6, and -12 dBi for  $R_{\text{ohmic}} = 0, 1.7,$  and  $7.8$  ohms, respectively. The range of groundplane radii  $a$ , for which the horizon gain is within 3 dB of the maximum horizon gain, is  $4 \leq a \leq 16$  ft.,  $3 \leq a \leq 16$  ft., and  $2 \leq a \leq 16$  ft. for  $R_{\text{ohmic}} = 0, 1.7,$  and  $7.8$  ohms, respectively. These results are independent of whether double- or single-parameter tuning is utilized because at 30 MHz the tuning words are identical for double- and single-parameter tuning.

The antenna gain on the radio horizon at 60, 90, 120, and 150 MHz is plotted in figures B1-B4 of appendix B. At these frequencies, the maximum antenna gain on the horizon as a function of groundplane radius generally decreases with increasing values of  $R_{\text{ohmic}}$ , whereas the range of groundplane radii for which the gain is within 3 dB of the maximum gain generally increases with increasing value of  $R_{\text{ohmic}}$ . At frequencies other than 30 MHz, the maximum antenna gain on the horizon as a function of groundplane radius is

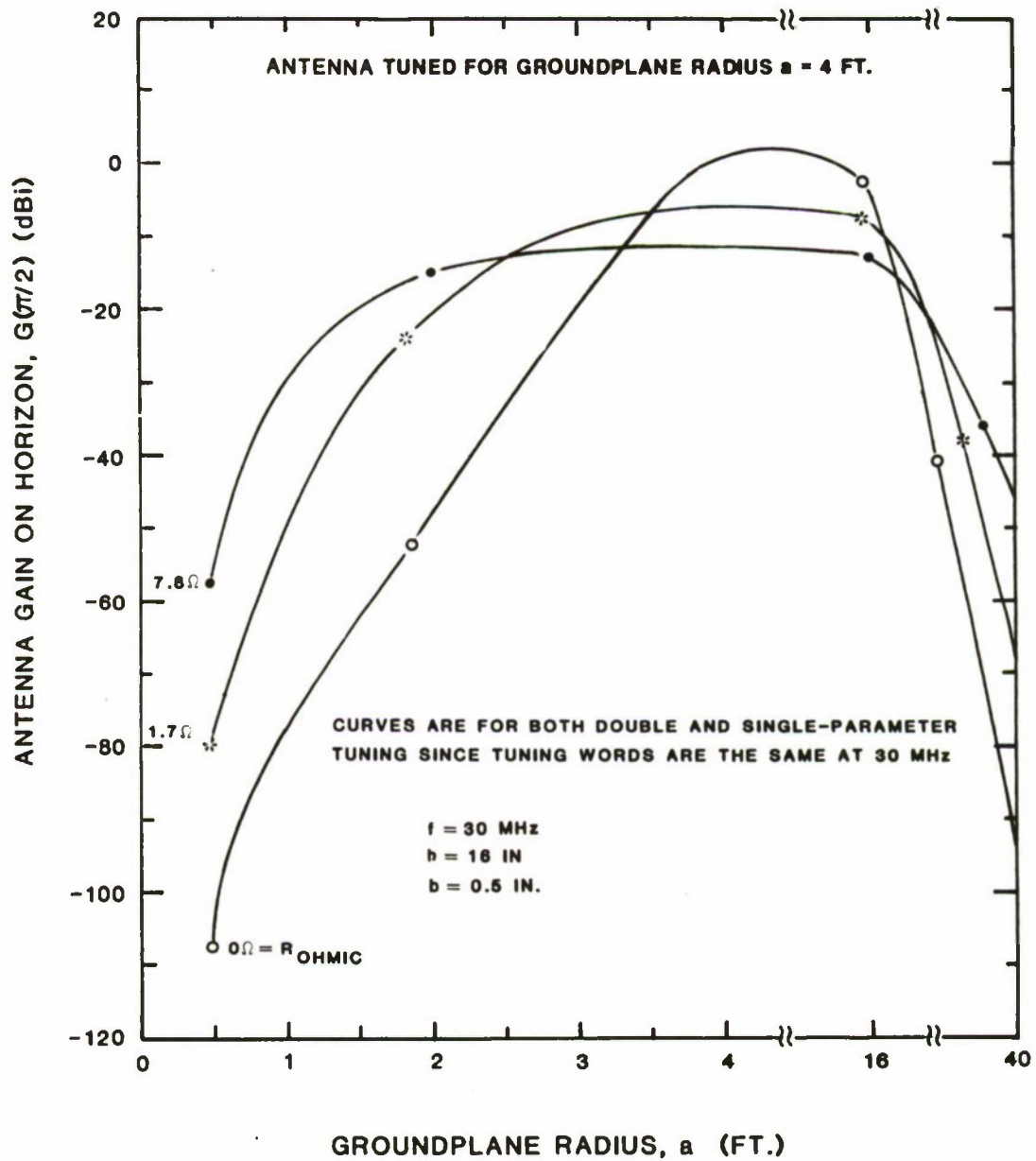


Figure 8. Antenna Gain on Horizon at 30 MHz

generally greater (0.1 - 10 dB) with double-parameter tuning than with single-parameter tuning. However, the range of groundplane radii for which the gain is within 3 dB of the maximum gain is generally not significantly increased with double-parameter tuning except at the high end of the frequency band for  $R_{\text{ohmic}} = 0$  ohms.

The antenna gain on the radio horizon for the monopole element mounted on the reference groundplane of radius  $a = 4$  ft. is plotted as a function of frequency in figure 9. The discontinuity in antenna gain for single-parameter tuning at 110 MHz for  $R_{\text{ohmic}} = 7.8$  ohms is because of a change in the algorithm for  $L_{10}$  when  $R_0(\omega) > \omega L_{20}/2$  (see Eq. 19). The antenna gain at any frequency over the frequency band is larger with double-parameter tuning than with single-parameter tuning. For zero ohmic loss,  $G(\pi/2)$  over the entire frequency band is greater than -2 dBi and -7 dBi with double-parameter and single-parameter tuning, respectively. Unfortunately, as was mentioned earlier, the tuning stability is not significantly better with double-parameter tuning than with single-parameter tuning.

The antenna gains on the radio horizon for groundplane radii  $a = 0.5 - 40$  ft. are plotted as a function of frequency in figures C1-C8. The antenna gain on the horizon as a function of frequency is a minimum at 30 MHz for any given groundplane except the reference groundplane. The antenna gain is 40 to 100 dB less at 30 MHz than at 150 MHz for groundplane radii much less or greater than that of the reference groundplane. At 30 MHz, the antenna gains for  $a = 0.5$  ft. and 40 ft. are -107 dBi and -94 dBi, respectively, with  $R_{\text{ohmic}} = 0$  ohms.

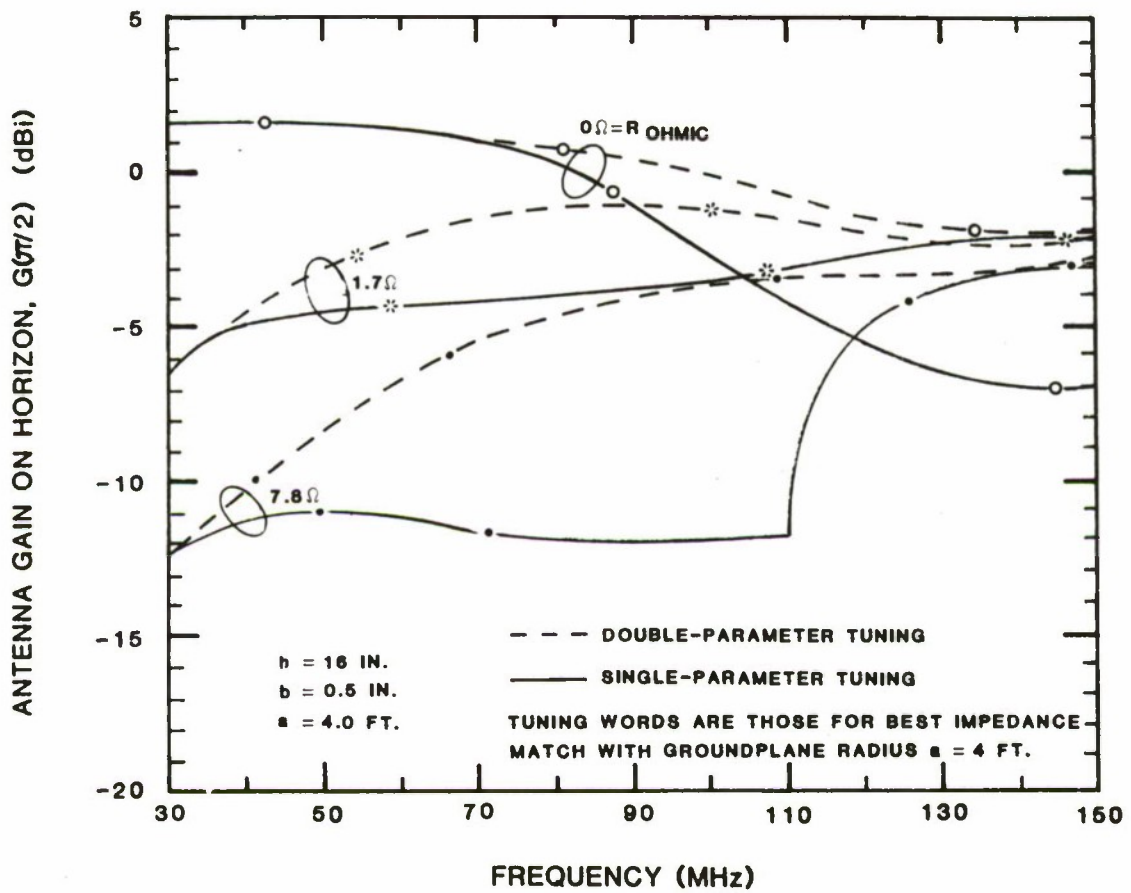


Figure 9. Antenna Gain on Horizon,  $a = 4.0$  ft.

SECTION 4  
CONCLUSIONS

Double-parameter and single-parameter tuning of an electrically-short monopole antenna is investigated in this paper to determine the range of groundplane radii for which the same tuning words can be utilized in maintaining a specified antenna gain over the 30-150 MHz frequency band.

In double-parameter tuning, the tuning words are chosen by varying two inductances of the impedance-matching network to provide a perfect impedance match at each frequency of interest when the antenna is mounted on a reference groundplane of 4 ft. radius. In single-parameter tuning, the tuning words are chosen by varying one inductance to provide a perfect impedance match at 30 MHz and zero input reactance at other frequencies when the antenna is mounted on a groundplane of 4 ft. radius.

Edge diffraction by the circular groundplane significantly alters the antenna input impedance so that it is not possible to utilize the same tuning words to provide a good impedance match for groundplane radii appreciably different from that of the reference groundplane and, in the case of single-parameter tuning, even for the reference groundplane.

The maximum antenna gain on the horizon as a function of groundplane radius is generally greater with double-parameter tuning than with single-parameter tuning. However, the range of groundplane radii for which the gain is within 3 dB of the maximum gain is generally not significantly increased with double-parameter tuning.

The maximum antenna gain on the horizon as a function of groundplane radius generally decreases with increasing ohmic resistance  $R_{\text{ohmic}}$  of the antenna circuit whereas the range of groundplane radii for which the gain is within 3 dB of the maximum gain generally increases with increasing values of  $R_{\text{ohmic}}$ .

The antenna gain on the horizon as a function of frequency is a minimum at 30 MHz for any given groundplane except the reference groundplane. For example, the antenna gain is 40 to 100 dB less at 30 MHz than at 150 MHz for groundplane radii much less or greater than that of the reference groundplane.

At 30 MHz the antenna gain on the horizon as a function of groundplane radius is a maximum for groundplane radii approximately equal to that of the reference groundplane with gains of 2, -6, and -12 dBi for  $R_{\text{ohmic}} = 0, 1.7, \text{ and } 7.8$  ohms, respectively. At 30 MHz, the range of groundplane radii  $a$ , for which the horizon gain is within 3 dB of the maximum horizon gain, is  $4 \leq a \leq 16$  ft.,  $3 \leq a \leq 16$  ft., and  $2 \leq a \leq 16$  ft., for  $R_{\text{ohmic}} = 0, 1.7, \text{ and } 7.8$  ohms, respectively.

It is concluded that it may not be possible to use the same tuning words for all groundplanes of interest without a substantial loss in antenna gain because of mismatch loss at some frequencies, particularly at the low end of the band. Therefore, it may be necessary to have different tuning words for different aircraft platforms.

Different tuning words for different aircraft platforms may be implemented by utilizing more than one antenna model or by sensing in real time the impedance mismatch and then modifying the tuning word at a given frequency to minimize the impedance mismatch.



The latter method is preferable because it would:

- 1) eliminate the difficult logistics problem of having to field several antenna models;
- 2) solve the problem of tuning instabilities arising from environmental changes in humidity, temperature, and stores in addition to that of platform size; and
- 3) improve the antenna radiation efficiency since it would not be necessary to load the antenna circuit with antenna loss in order to provide tuning stability.

An alternative to the implementation of different tuning words for different platforms might be to utilize a different antenna element, such as a dipole, whose input impedance might not be as sensitive to groundplane size as that of a monopole. However, even if such an element would prove to have better tuning stability with varying platform size and to have the desired gain pattern characteristics, it is not clear how such an alternative would solve the problem of tuning instabilities arising from changes in humidity, temperature, and stores without having to load the antenna circuit with antenna loss in order to provide tuning stability.

Consequently, implementation of different tuning words, by sensing in real time the impedance mismatch and then modifying the tuning word at a given frequency to minimize the impedance mismatch, is a preferable design objective. Such an objective has been realized at HF frequencies and an rf power level of 400W with sensing and tuning times of approximately 20  $\mu$ s and 1s, respectively, by a circuit comprising a directional coupler, digital processor, and electromechanical switches. The substitution of P-I-N diode

switches for the electromechanical switches might prove to be a feasible technique for achieving such an objective at VHF frequencies with an rf power level of 10W and a tuning time of less than 1 ms provided that the intermodulation products generated by the use of such switches are not excessive for the intended application.

## LIST OF REFERENCES

1. Chu, L. J., "Physical Limitations of Omni-Directional Antennas," Journal of Applied Physics, Vol. 19, December 1948, pp. 1163-1175.
2. Harrington, R. F., Time-Harmonic Electromagnetic Fields, NY: McGraw-Hill, 1961, Chapter 6, problem 6-31, p. 316.
3. Webster, R. E., "A Single-Control Tuning Circuit for Electrically Small Antennas," IRE Transactions on Antennas and Propagation, Vol. AP-3, January 1955, pp. 12-15.
4. Weiner, M. M., S. P. Cruze, C. C. Li, and W. J. Wilson, Monopole Elements on Circular Ground Planes, Norwood, MA: Artech House, 1987, Part 1, Section 4.2; Part 2, Appendix B2.
5. Weiner, op. cit., table 4.
6. Weiner, op. cit., figures 9 and 11.
7. Weiner, op. cit., Eq. (2.3-2).

APPENDIX A

PLOTS OF MISMATCH GAIN AND EQUIVALENT  
VSWR AS A FUNCTION OF FREQUENCY

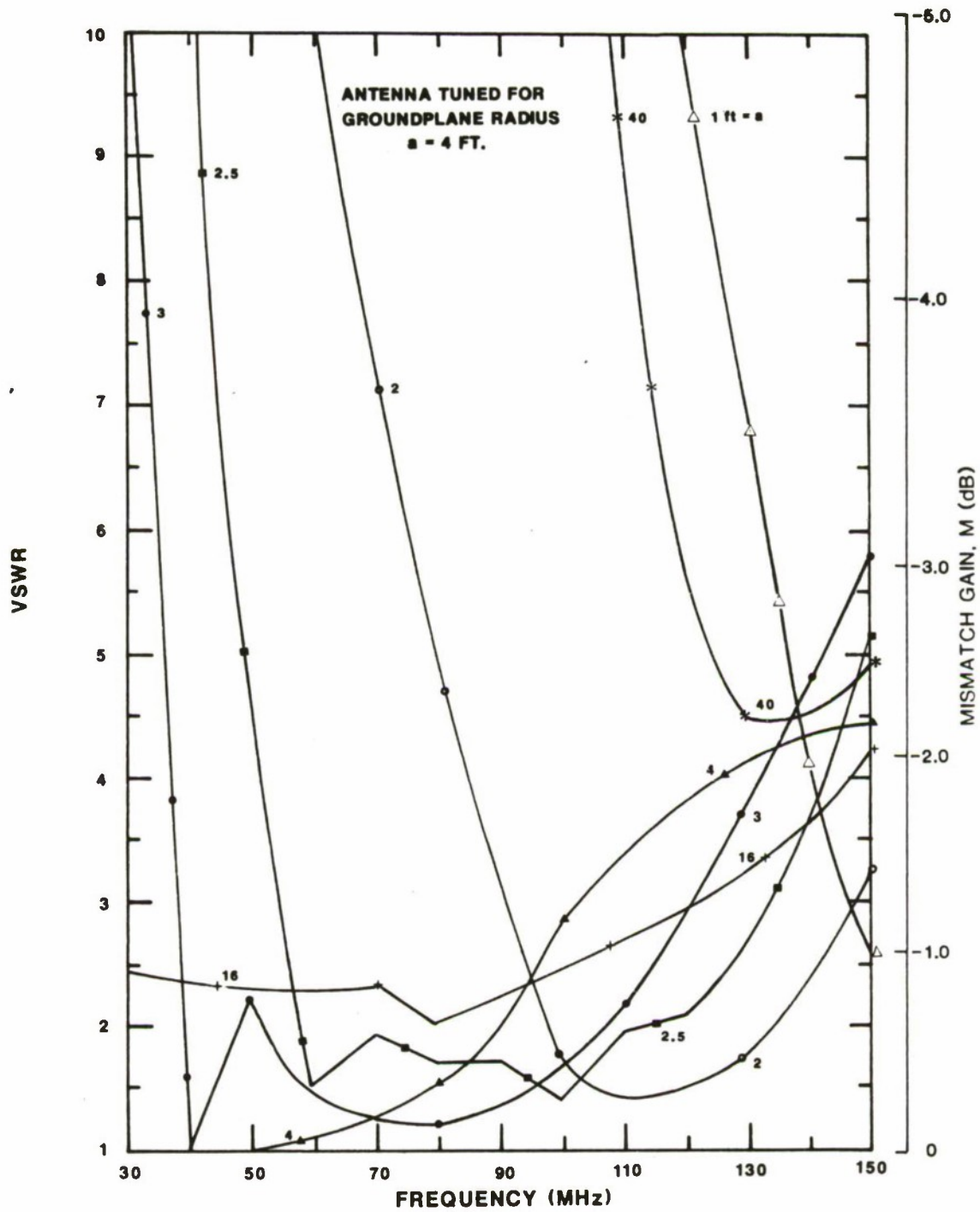


Figure A1. Mismatch Gain, Equivalent VSWR  
(Single-Parameter Tuning,  $R_{ohmic} = 0$  ohms)

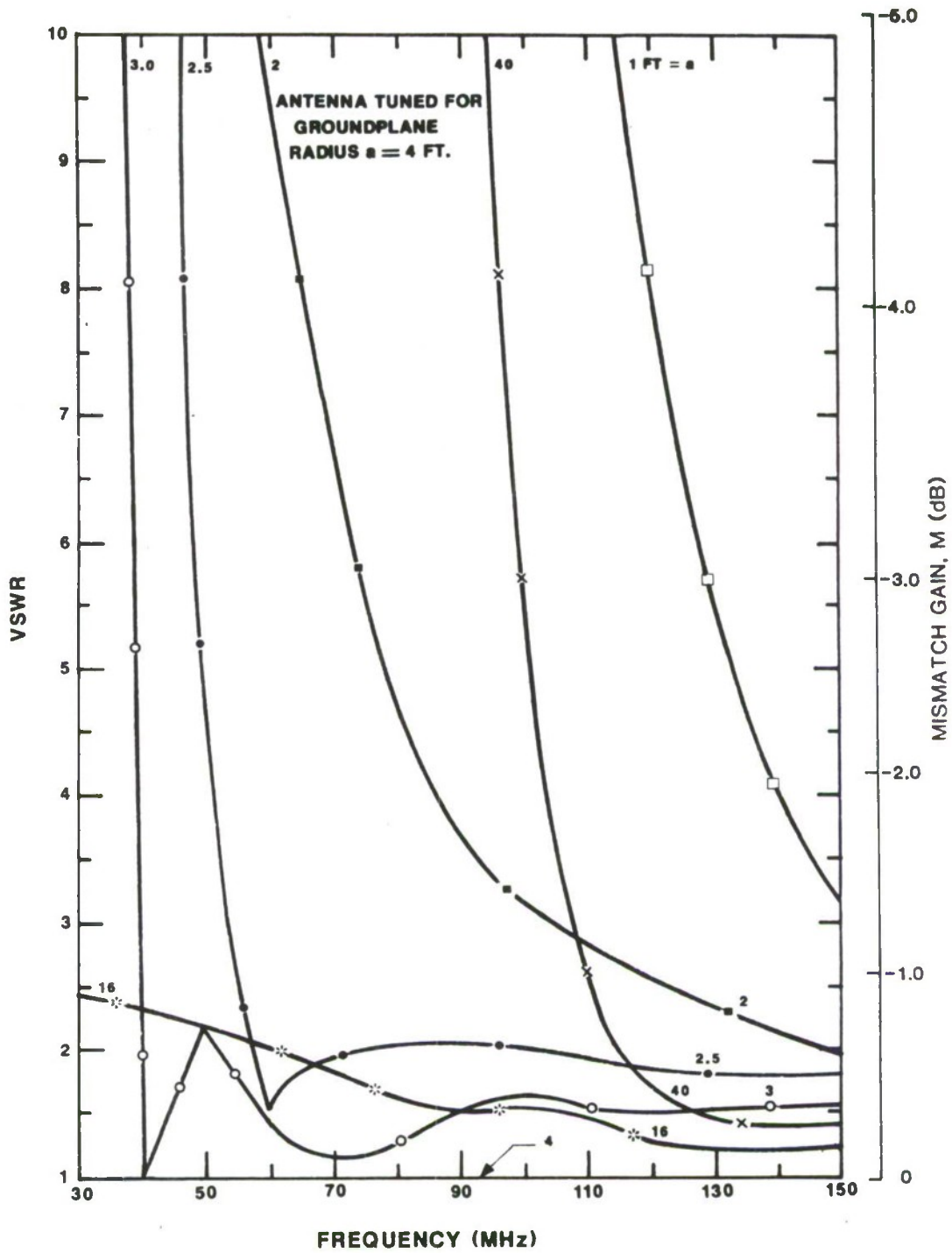


Figure A2. Mismatch Gain, Equivalent VSWR  
(Double-Parameter Tuning,  $R_{ohmic} = 0$  ohms)

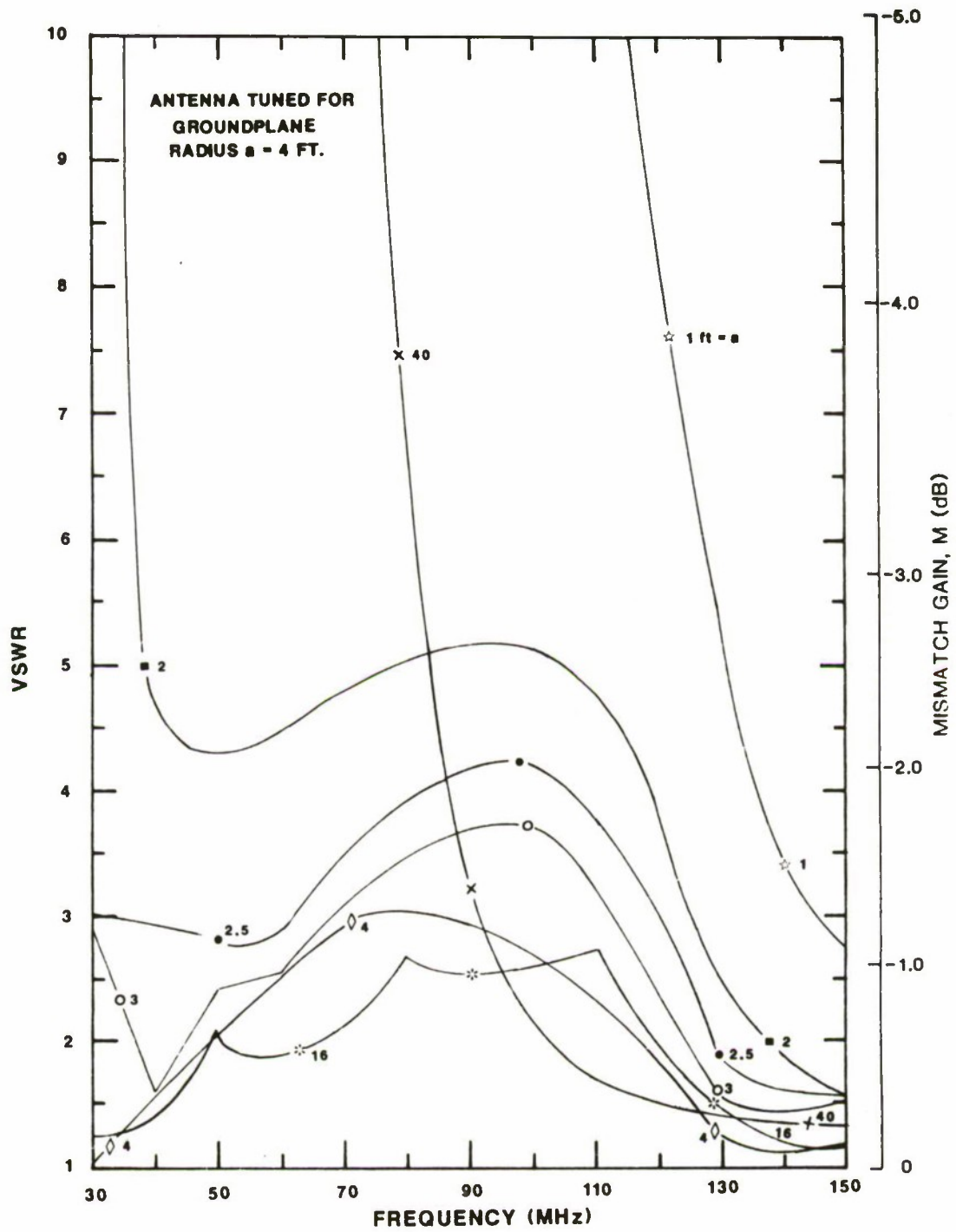


Figure A3. Mismatch Gain, Equivalent VSWR  
(Single-Parameter Tuning,  $R_{ohmic} = 1.7$  ohms)

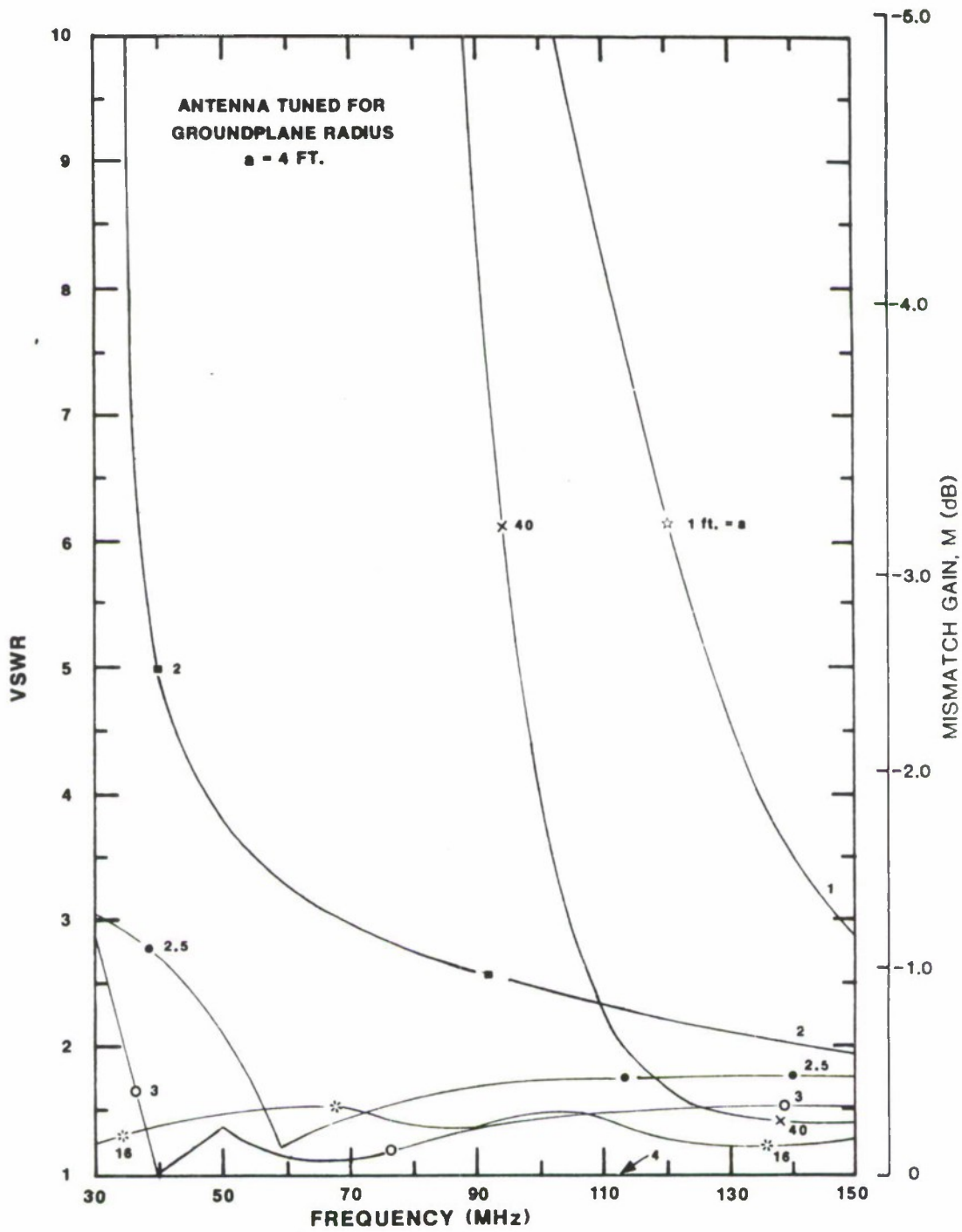


Figure A4. Mismatch Gain, Equivalent VSWR  
(Double-Parameter Tuning,  $R_{ohmic} = 1.7$  ohms)



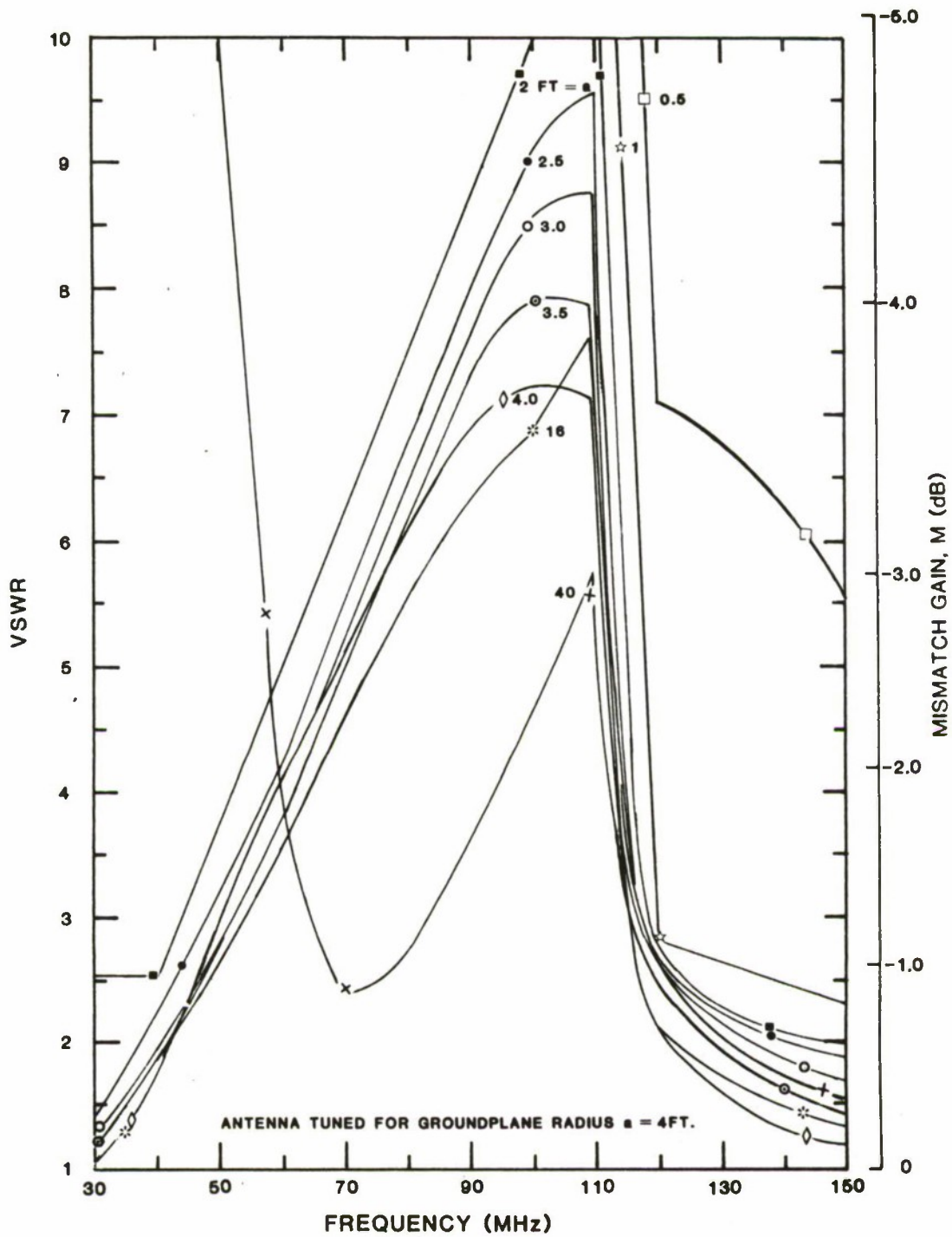


Figure A5. Mismatch Gain, Equivalent VSWR  
 (Single-Parameter Tuning,  $R_{ohmic} = 7.8$  ohms)

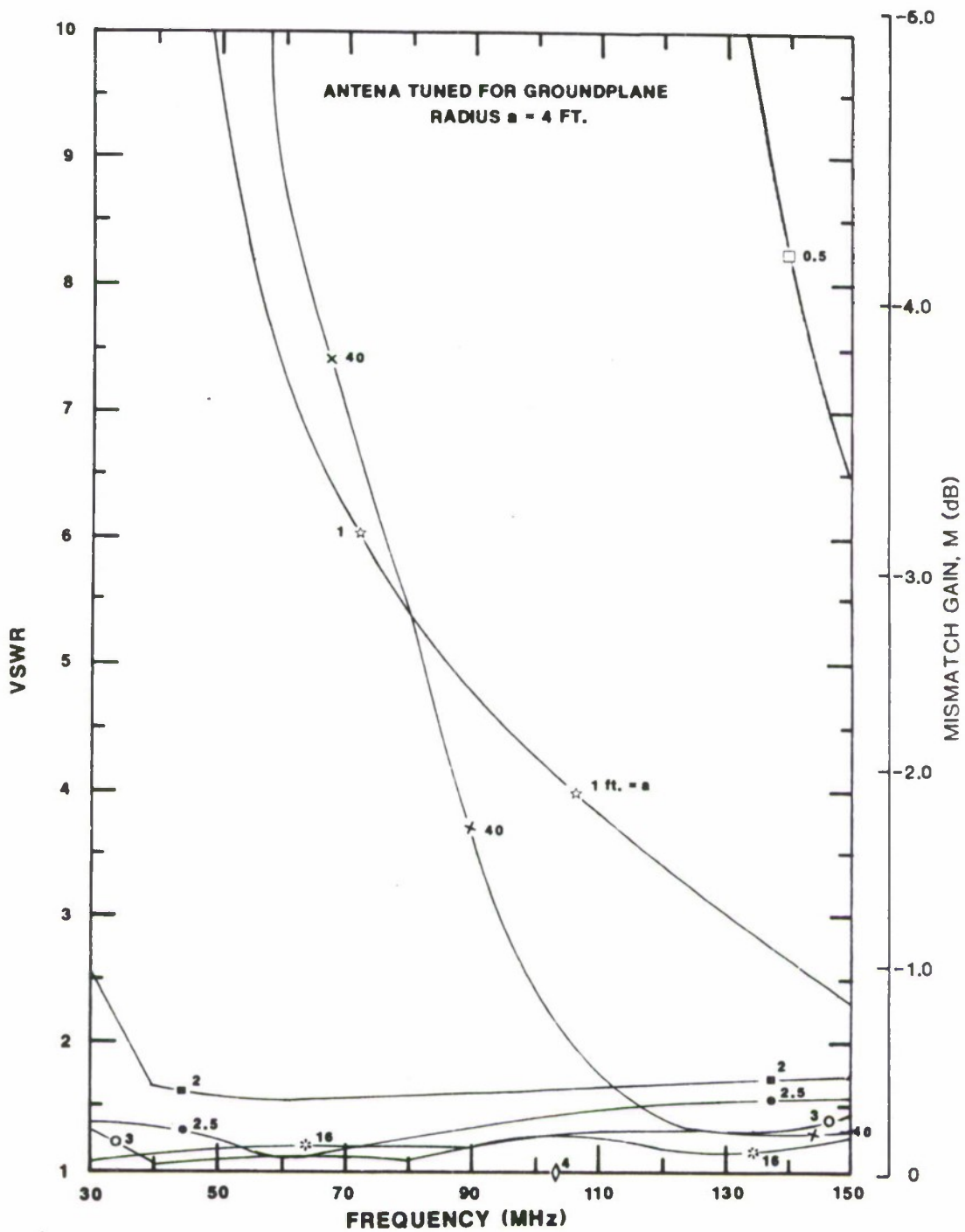


Figure A6. Mismatch Gain, Equivalent VSWR  
(Double-Parameter Tuning,  $R_{ohmic} = 7.8$  ohms)

APPENDIX B

PLOTS OF ANTENNA GAIN ON THE HORIZON AS  
A FUNCTION OF GROUNDPLANE RADIUS

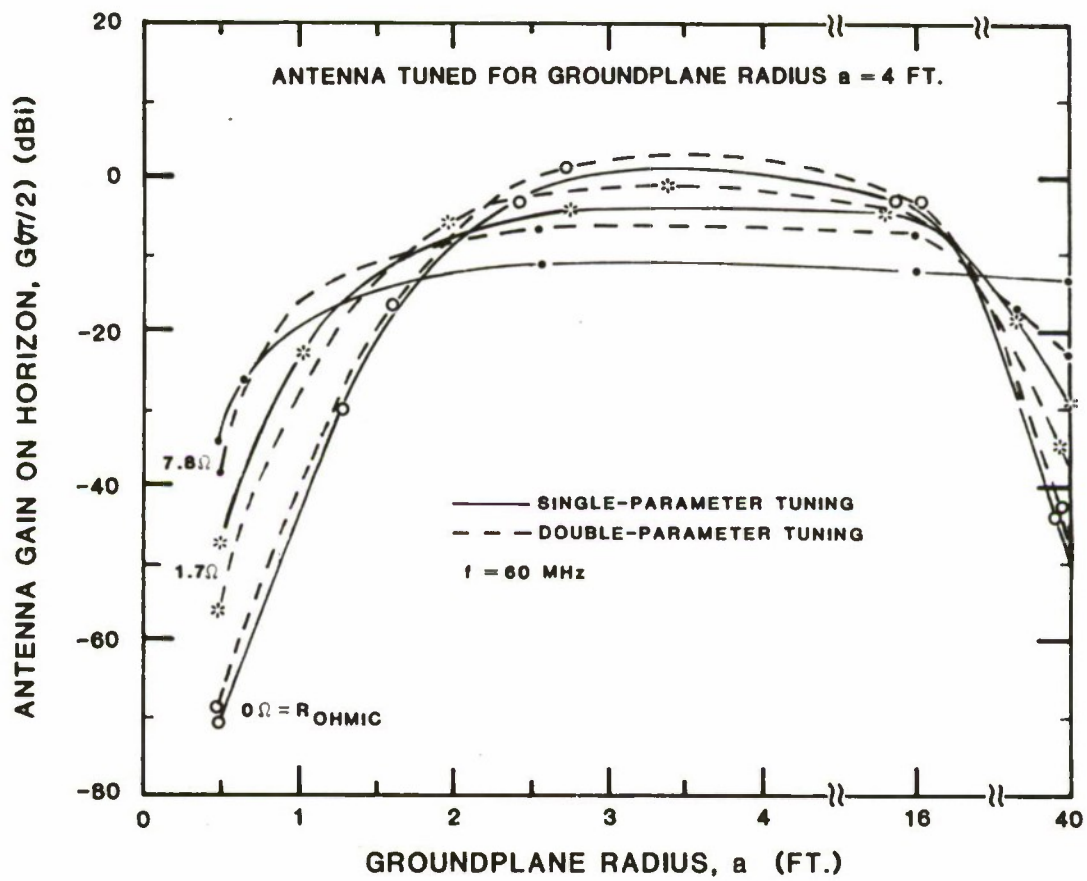


Figure B1. Antenna Gain on Horizon at 60 MHz

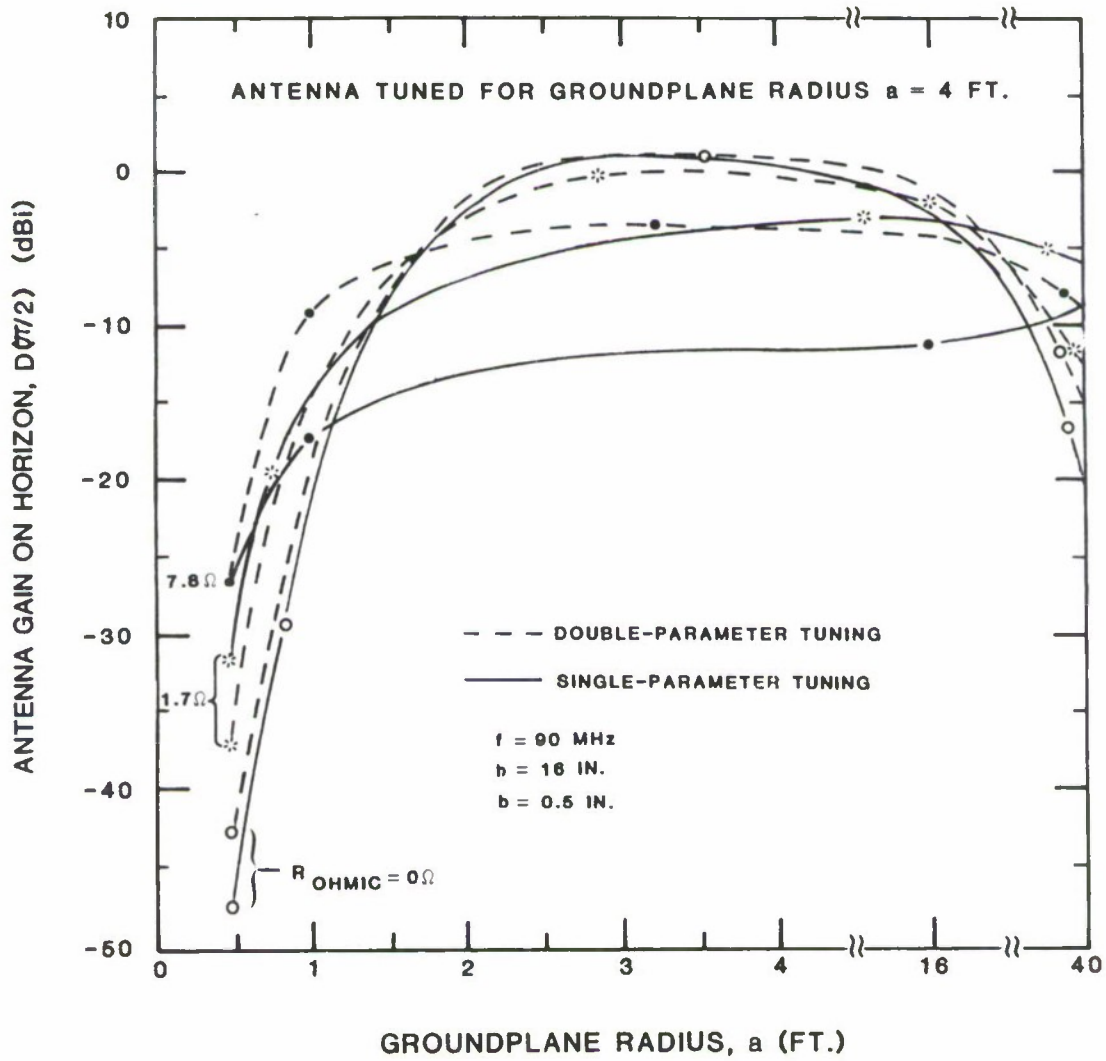


Figure B2. Antenna Gain on Horizon at 90 MHz

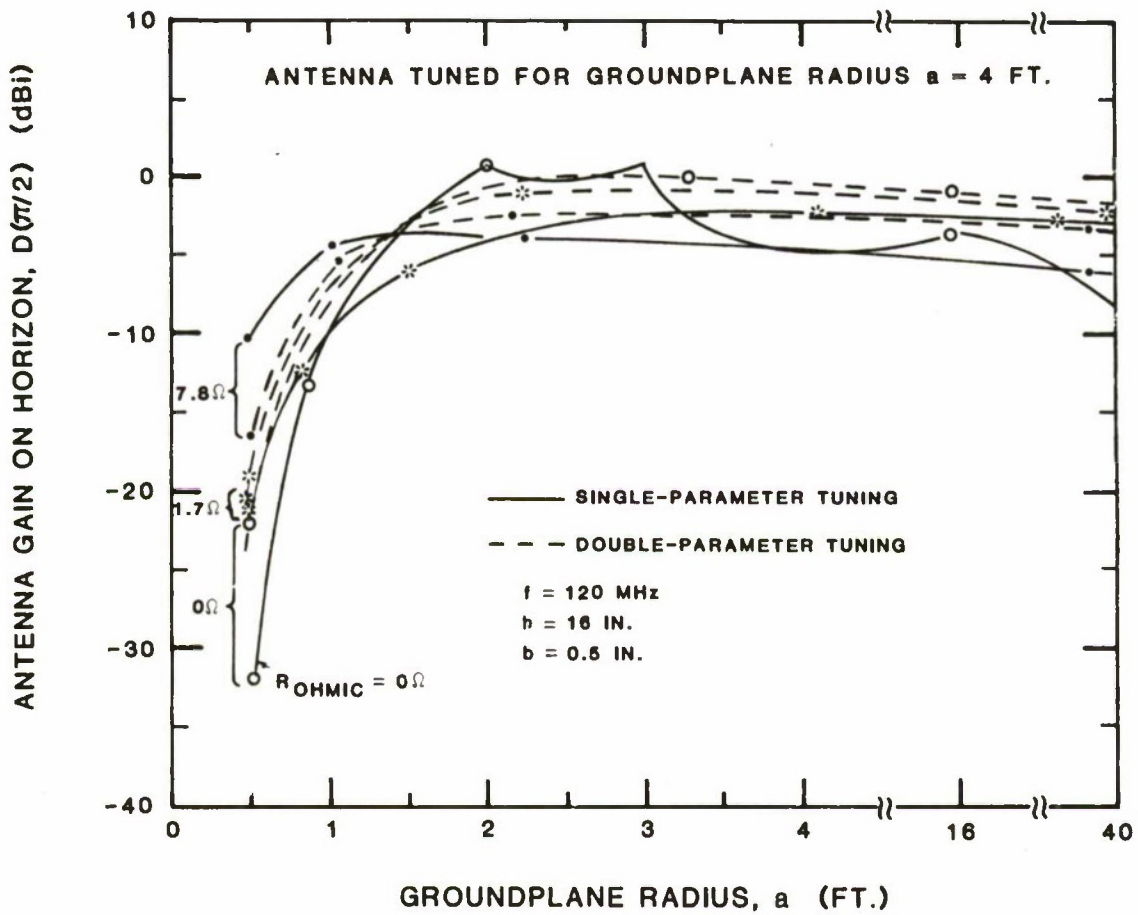


Figure B3. Antenna Gain on Horizon at 120 MHz

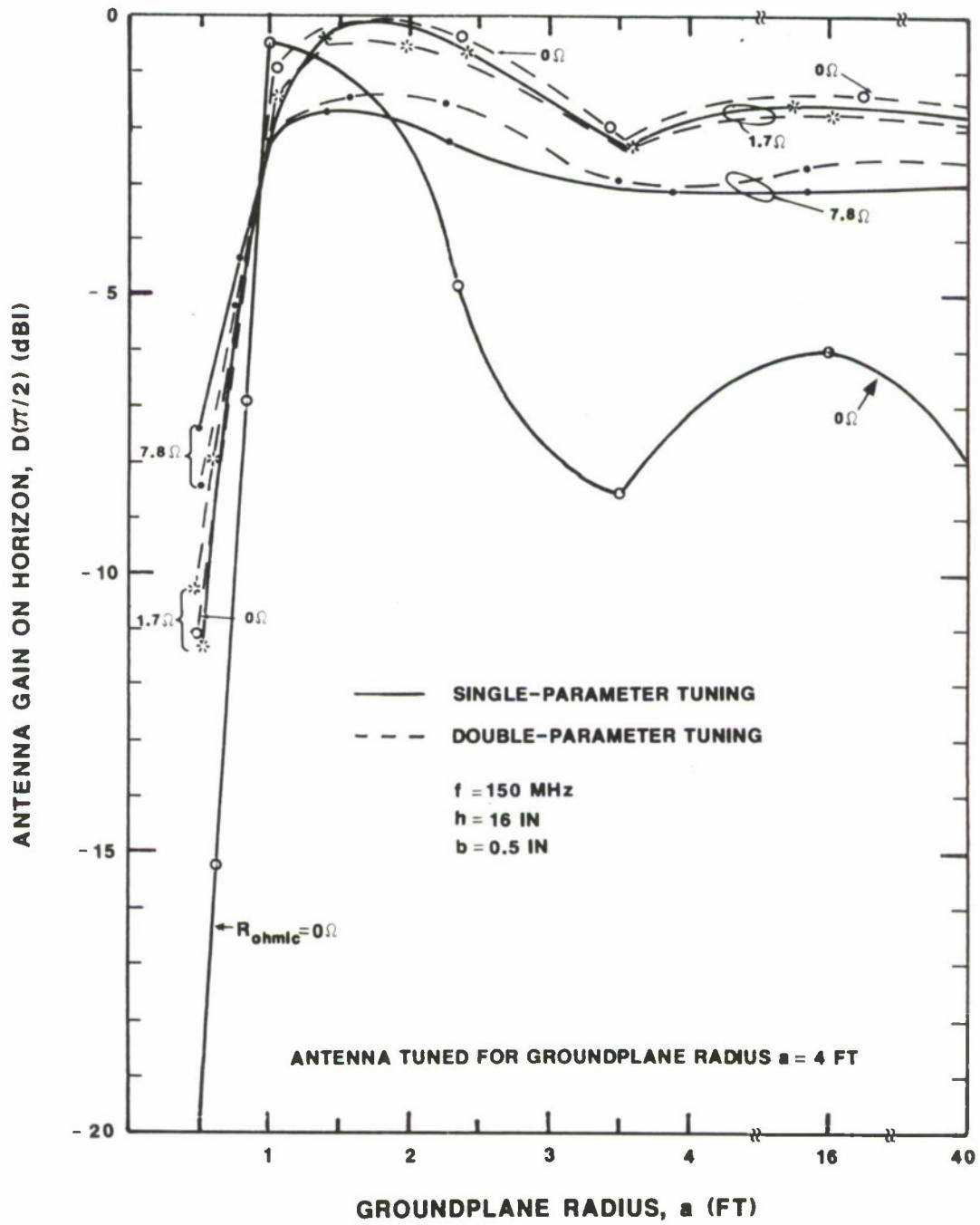


Figure B4. Antenna Gain on Horizon at 150 MHz

APPENDIX C

PLOTS OF ANTENNA GAIN ON THE HORIZON  
AS A FUNCTION OF FREQUENCY



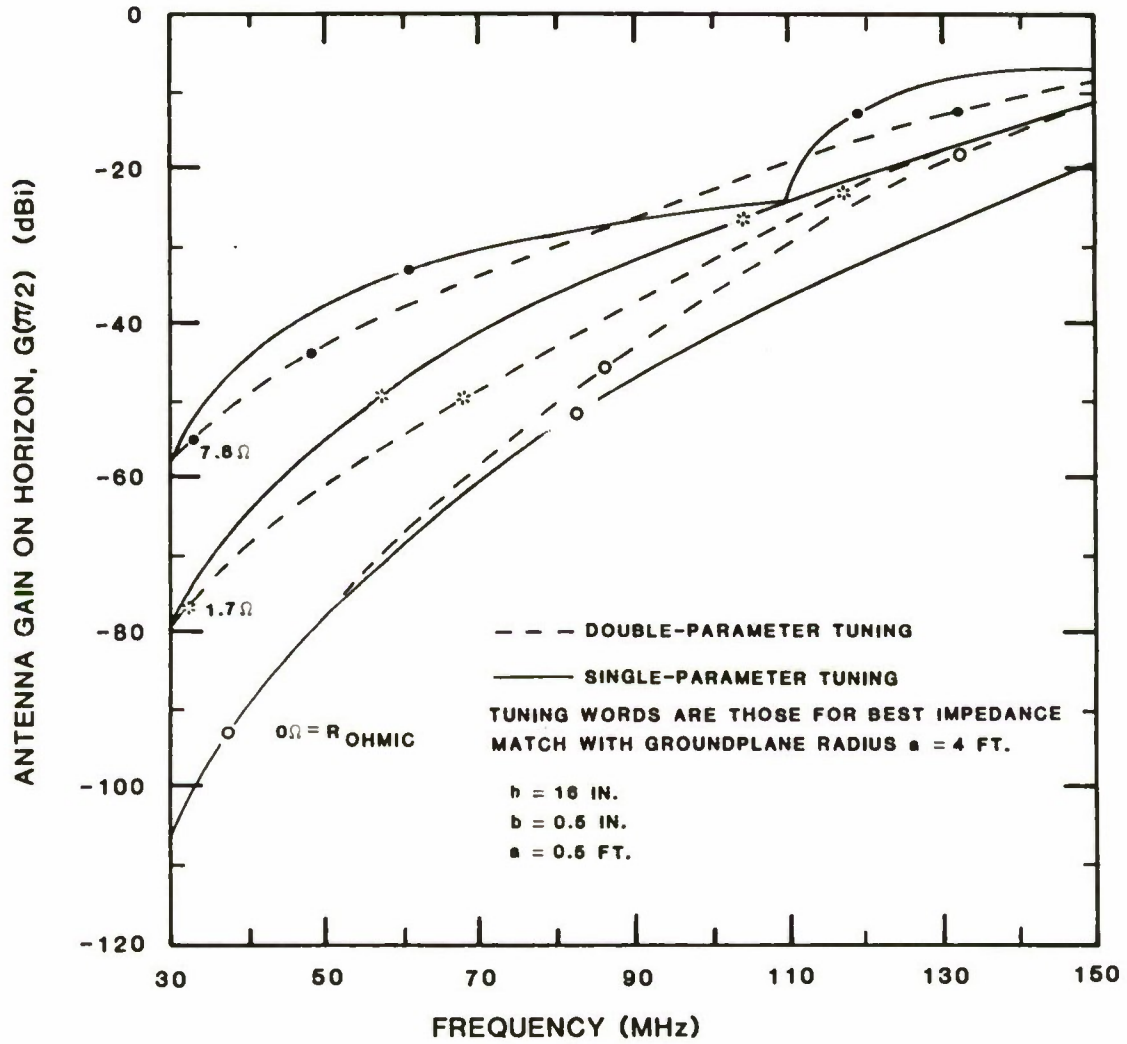


Figure C1. Antenna Gain on Horizon,  $a = 0.5$  ft.

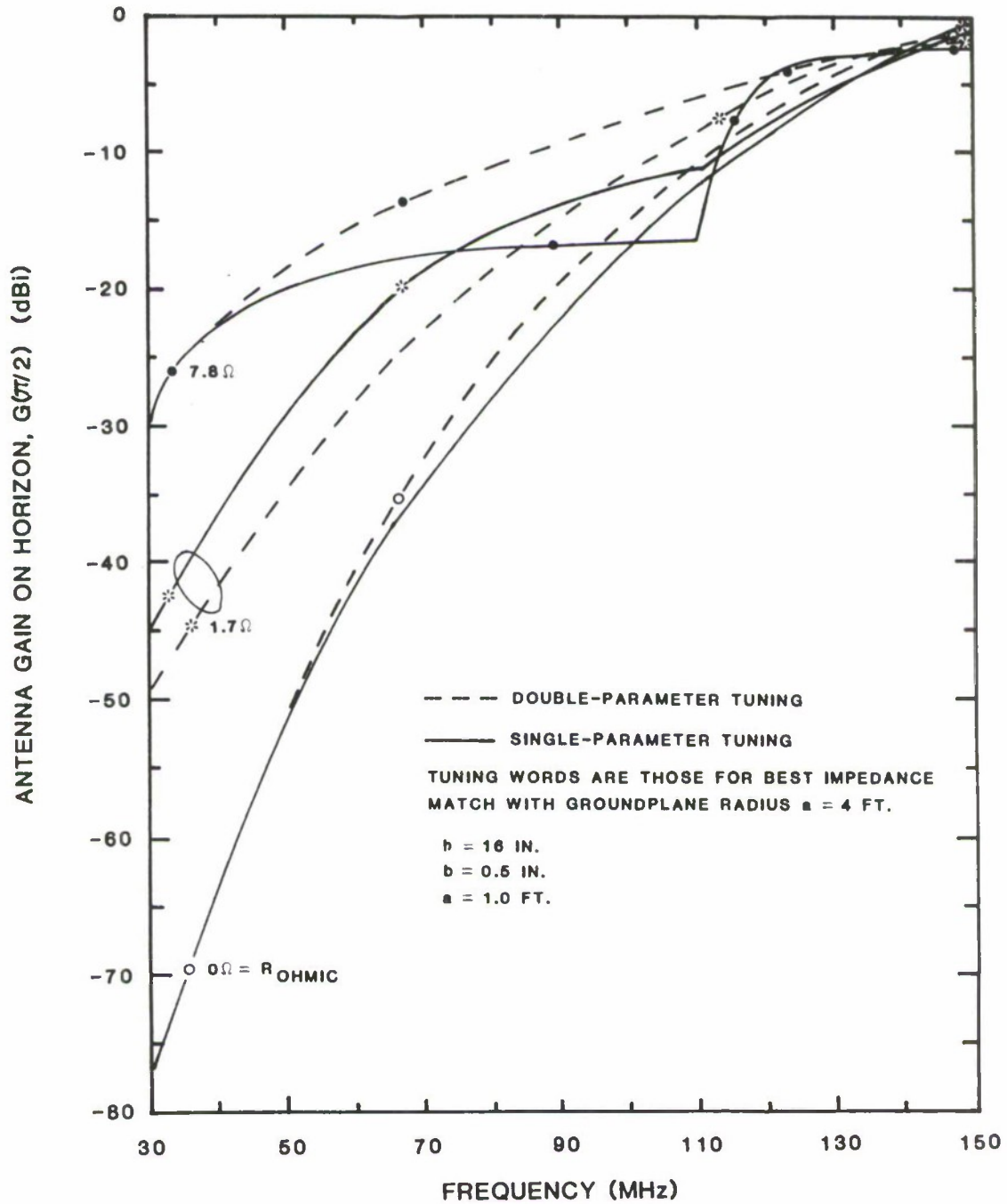


Figure C2. Antenna Gain on Horizon,  $a = 1.0$  ft.

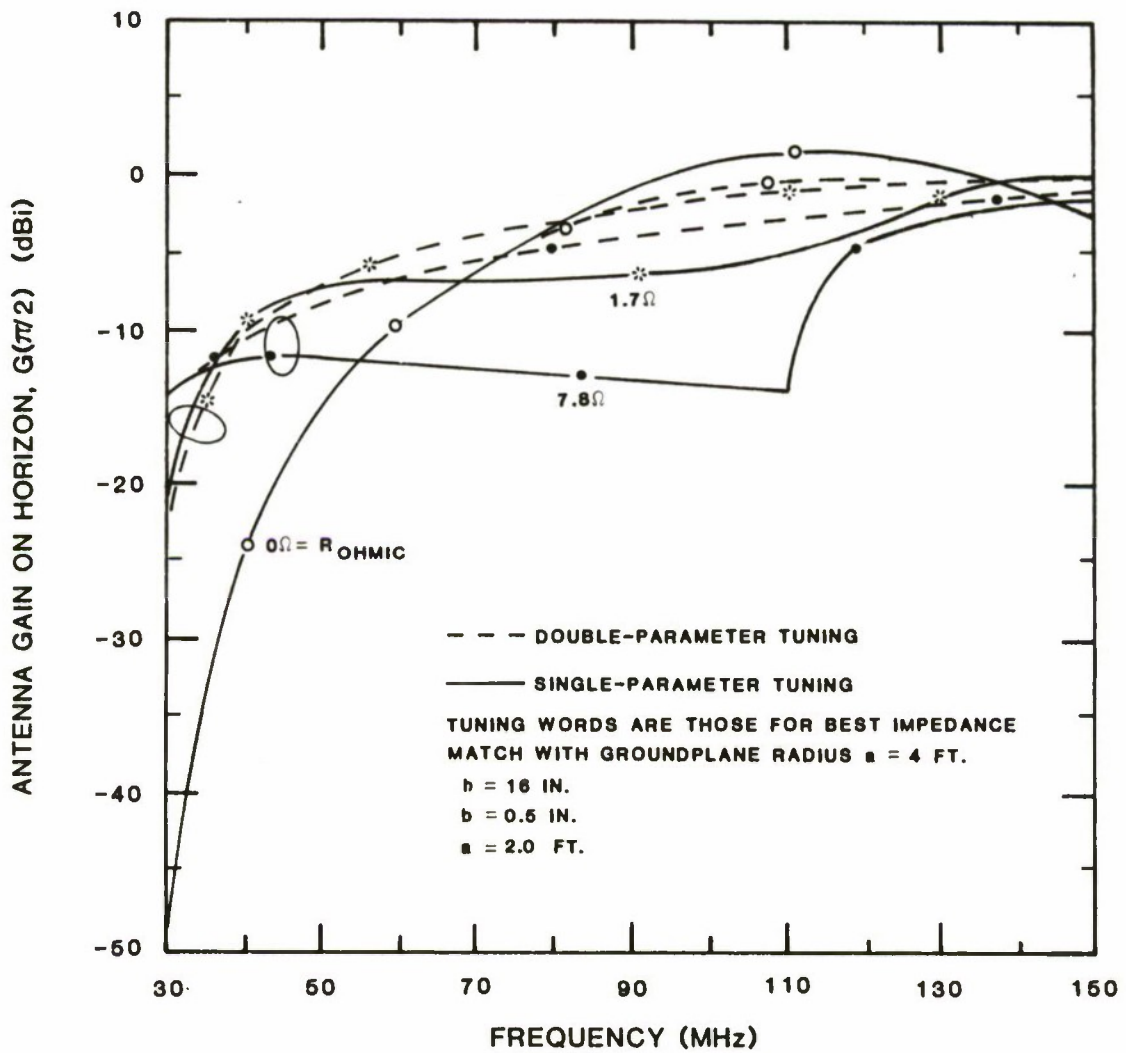


Figure C3. Antenna Gain on Horizon,  $a = 2.0$  ft.

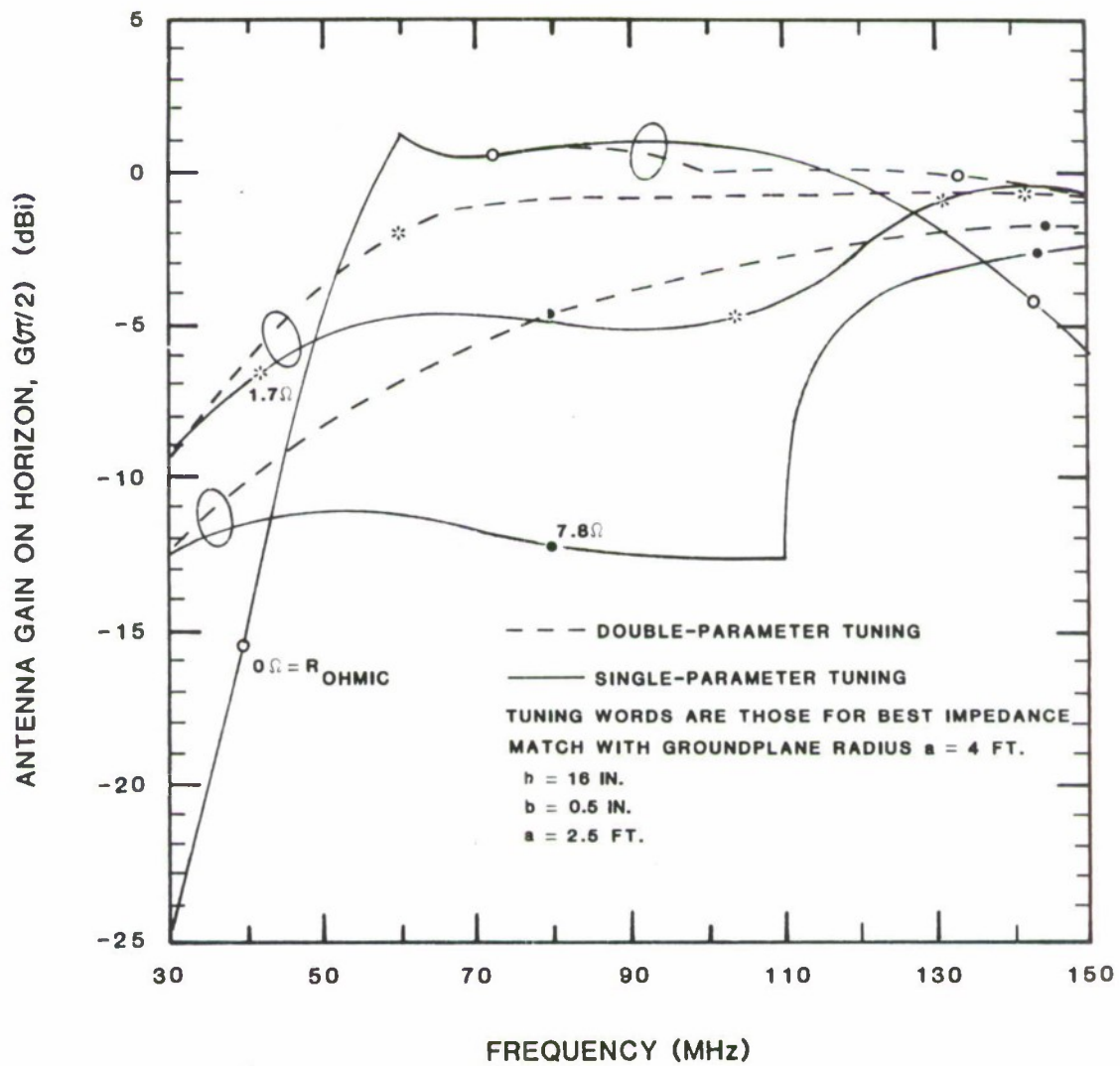


Figure C4. Antenna Gain on Horizon,  $a = 2.5$  ft.

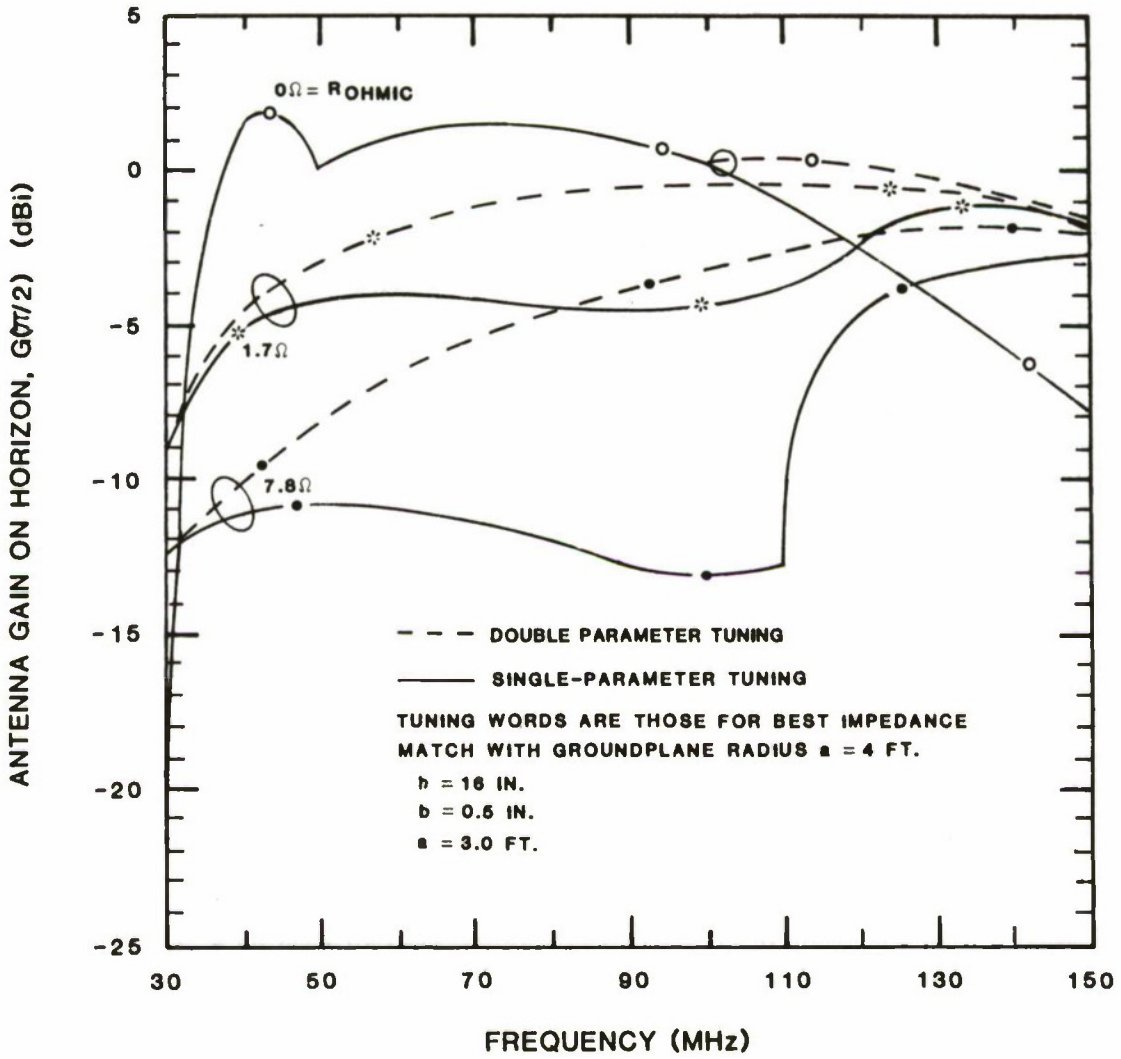


Figure C5. Antenna Gain on Horizon,  $a = 3.0$  ft.

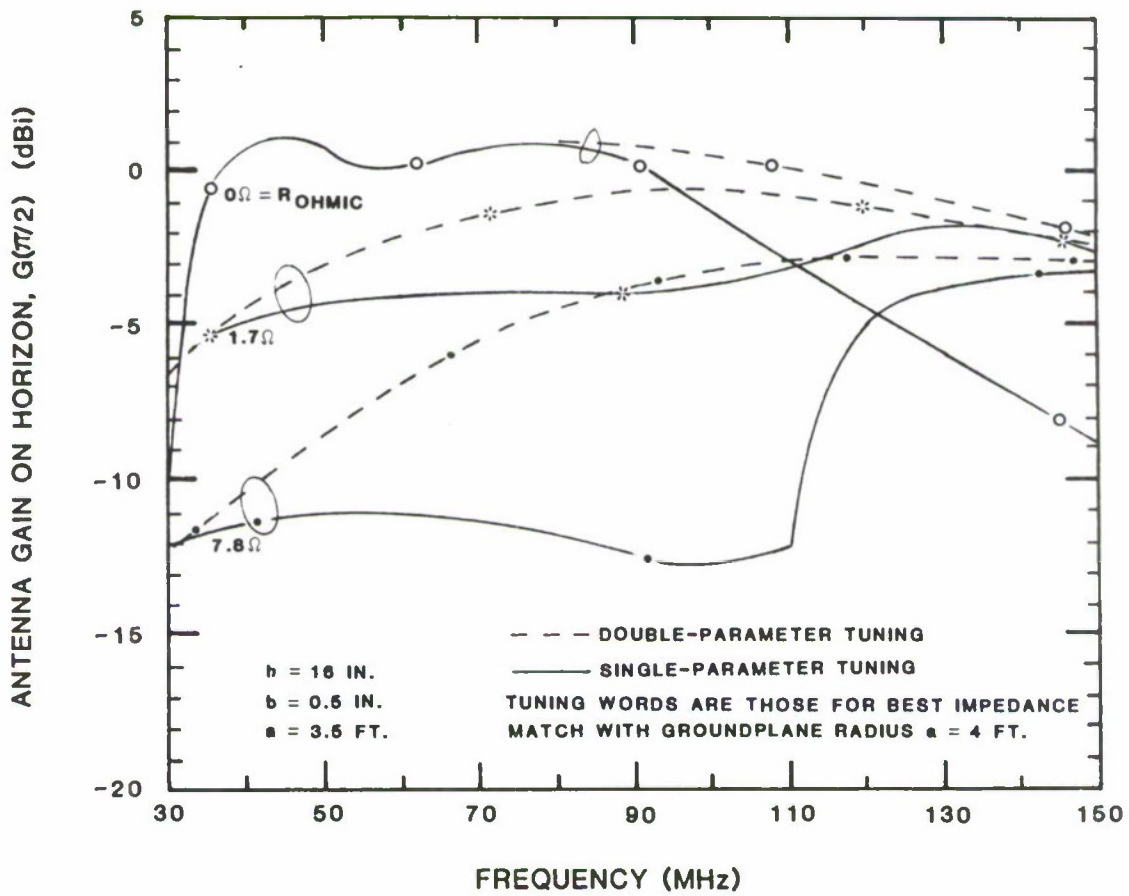


Figure C6. Antenna Gain on Horizon,  $a = 3.5$  ft.

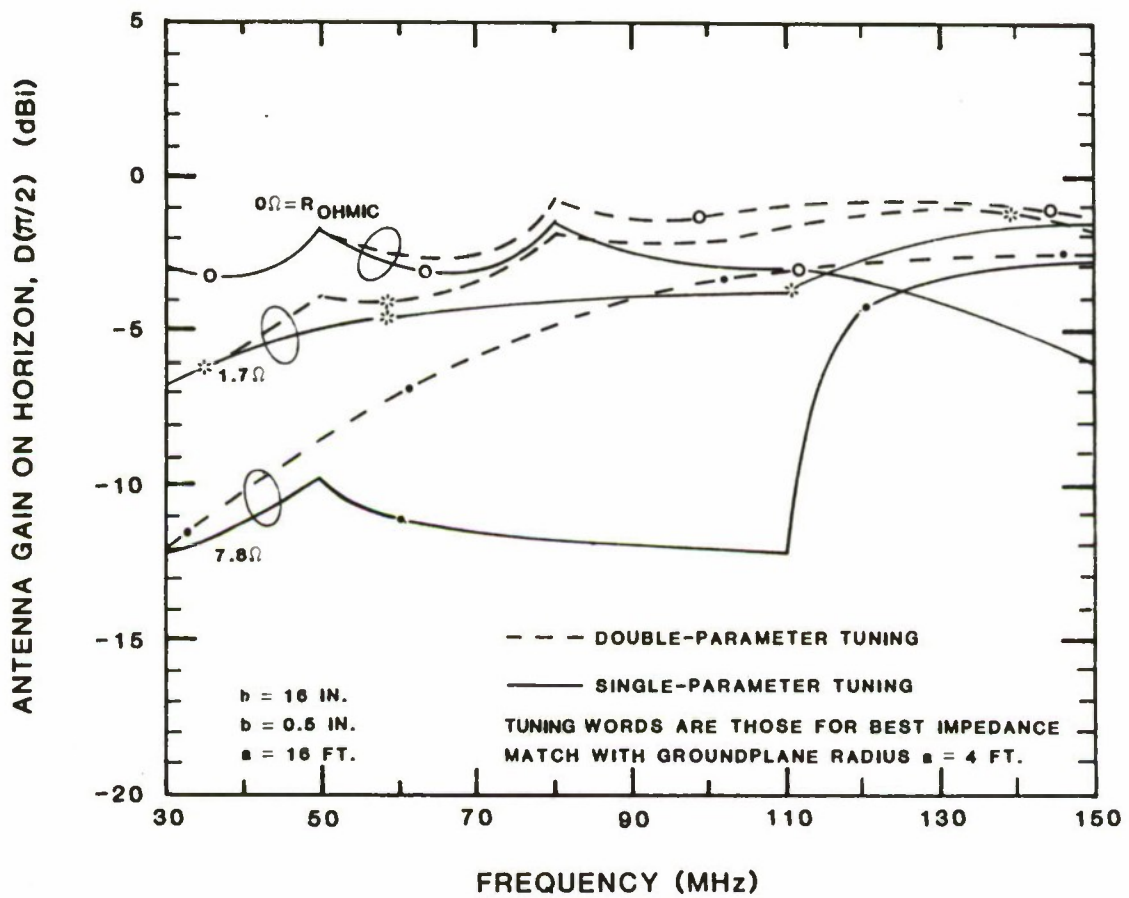


Figure C7. Antenna Gain on Horizon,  $a = 16$  ft.

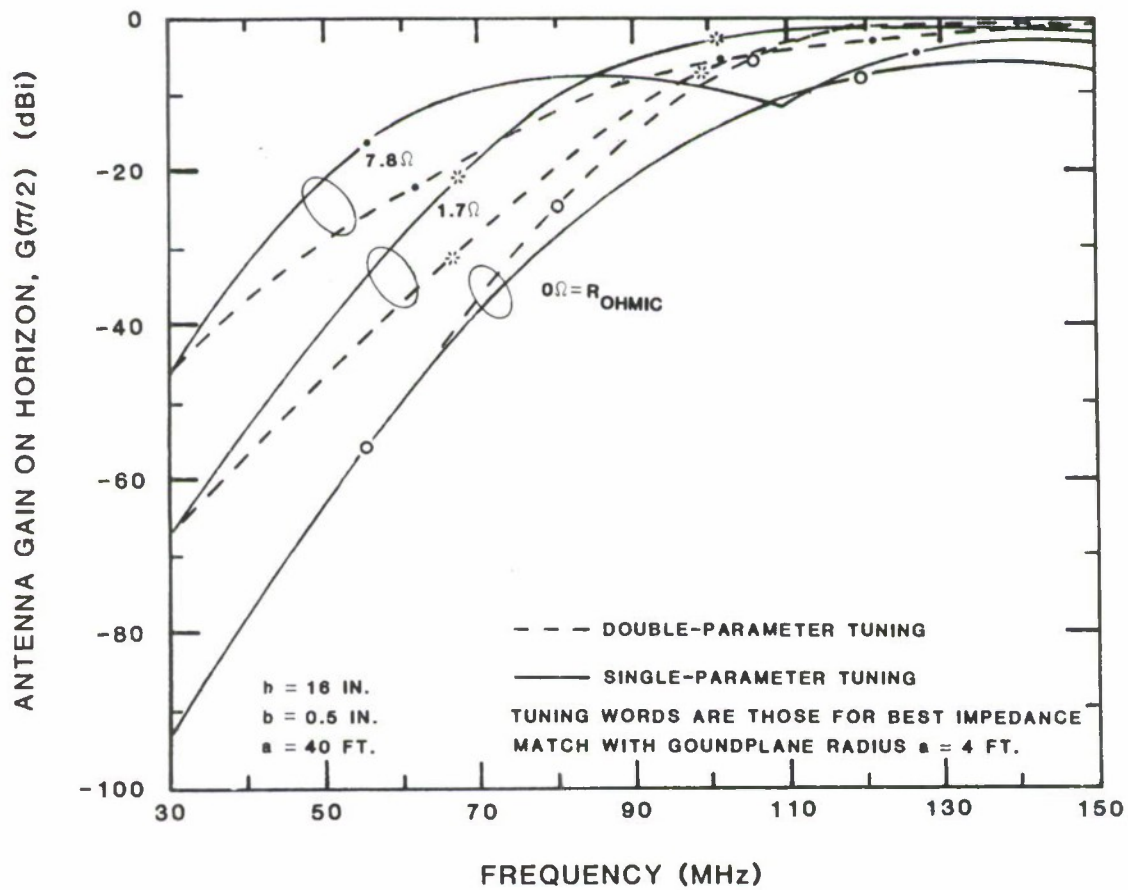


Figure C8. Antenna Gain on Horizon,  $a = 40$  ft.