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A UNIQUENESS PROOF FOR SONAR MOTION ESTIMATION

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To establish the groundwork for estimating circular motion from sonar measurements, necessary and sufficient conditions for the existence and uniqueness of a motion parameter solution are derived. The measurements, range, range rate, and bearing, for a point on a moving object, are assumed to be matched [1], ie associated with measurements of the same point at a later time. These six measurements; $R_1, R_2, \dot{R}_1, \dot{R}_2, B_1$, and B_2; subscripted to denote time, are used to estimate five parameters of planar motion: initial position, x_1, y_1; speed, V; initial heading, H_1; and heading rate, ω. The range and bearing measurements are transformed to initial position and (equivalent) straight line travel distance, D, and direction, ϕ, using

$$x_1 = R_1 \cos B_1 \quad y_1 = R_1 \sin B_1$$

$$x_2 = R_2 \cos B_2 \quad y_2 = R_2 \sin B_2$$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \phi = \tan^{-1}[\frac{y_2 - y_1}{x_2 - x_1}].$$

Substituting $p = \omega T/2$, $\psi_1 = \phi - B_1$, and $\psi_2 = \phi - B_2$ leaves three measurement equations,

$$D = V\ T \sin p\ /p \tag{1}$$

$$\dot{R}_1 = V \cos(\psi_1 - p) \tag{2}$$

$$\dot{R}_2 = V \cos(\psi_2 + p) \tag{3}$$

in two unknowns, V and p. Once a solution for p is found the initial heading is given by $H_1 = \phi - p$. Assume $V > 0$ and $-\pi < p < \pi$ to limit the number of solutions. The variable V can be eliminated by combining equations (1) and (2) to give

$$\frac{\sin p}{p} = \frac{D}{T\dot{R}_1} \cos(\psi_1 - p) \tag{4}$$

with p the only nonmeasured quanity. Dividing by $\dot{R}_1$ is for convenience and can be shown to not alter results when $\dot{R}_1 = 0$. In (4) the ratio D/T is assumed positive as is the left hand side of (4) for the range of p considered. In the example shown in Figure 1 a solution of (4) exists if the amplitude of the sinusoid is negative or positive and large enough for a tangential intersection with the sinc function. Using the equations for tangential intersection gives the condition for existence of a solution of (4) (ie consistency of $\dot{R}_1$) as

$$\dot{R}_1 \le \frac{D}{T A_0(\psi_1)} \qquad -\frac{\pi}{2} \le \psi_1 \le \frac{\pi}{2} \tag{5a}$$

$$\dot{R}_1 \ge \frac{-D}{T A_0(\psi_1)} \qquad -\pi \le \psi_1 \le -\frac{\pi}{2},\ \frac{\pi}{2} \le \psi_1 \le \pi \tag{5b}$$

where the values for $A_0(\psi_1)$, which is symmetric about $\psi_1 = 0$ and $\psi_1 = \pi$, are tabulated in Table 1.

If (5) is met with equality, equation (4) has a unique solution, and the solution for the motion parameters is unique if

$$\dot{R}_2 = \frac{D\,p}{T \sin(p)} \cos(\psi_2 + p). \tag{6}$$

Otherwise no solution for the motion parameters exists.

If (5) is met with inequality, equation (4) has two solutions. The consistency of (3) is more easily assessed when it is combined with (2) by removing V to give

$$\dot{R}_2 \cos(p - \psi_1) = \dot{R}_1 \cos(p + \psi_2). \tag{7}$$

Equation (7) can have two solutions for p that differ by π. The sinusoid of equation (2) has opposite signs at these two solutions and the same sign at both solutions of (4). Therefore equations (4) and (7) have at most one solution in common. For consistency, equation (6) must be true for one of the solutions of (4).

Equation (7) can also be true for all p leading to an ambiguity if either

$$\psi_2 = -\psi_1 + 2\pi n \qquad \dot{R}_2 = \dot{R}_1 \ \rightarrow\ \phi = \frac{B_1 + B_2}{2} + \pi n \tag{8}$$

or

$$\psi_2 = -\psi_1 + \pi + 2\pi n \quad \dot{R}_2 = -\dot{R}_1 \ \rightarrow\ \phi = \frac{B_1 + B_2 + \pi}{2} + \pi n \tag{9}$$

for any integer n. Eq. (8), physically interpreted, states the straight line travel is along the line of sight. ($B_1 = B_2$). Equation (9) is satisfied when the observer lies on the perpendicular bisector of the straight line travel. It is also satisfied if the starting and ending position, (x_1, y_1) and (x_2, y_2), are on opposite sides of the observer. These geometries cause two-way ambiguities in the motion parameter solutions unless (5) is met with equality.

REFERENCES

1. Tsai, R.Y., and Huang, T.S., Estimating three-dimensional motion parameters of a rigid planar patch. III: finite point correspondences and three-view problem, *IEEE Trans ASSP* vol 32, pp 213-219, Apr 1984.

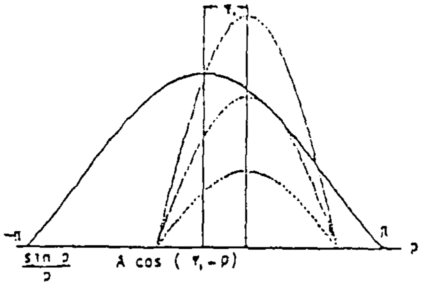

Figure 1. Graph of equation (4) showing 0, 1, and 2 solution cases as a function of A.

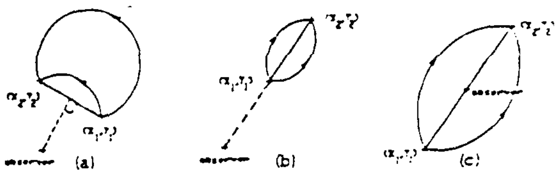

Figure 2. Ambiguous trajectories associated with (a) observer on perpendicular bisector, (b) observer outside and colinear and (c) observer inside and colinear.

TABLE 1

ψ_1 ($\div \pi/30$)	$A_o(\psi_1)$	ψ_1 ($\div \pi/30$)	$A_o(\psi_1)$
0	1	8	0.82808
1	0.99726	9	0.79330
2	0.98905	10	0.73442
3	0.97541	11	0.67822
4	0.95638	12	0.61707
5	0.93206	13	0.54839
6	0.90251	14	0.46743
7	0.86783	15	0.31831