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AFGL-TR-87-0251
ENVIRONMENTAL RESEARCH PAPERS, NO. 985

The 3D-BSW Model Applied To Climatology of Small Areas and Lines

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17 August 1987

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ATMOSPHERIC SCIENCES DIVISION PROJECT 6670
AIR FORCE GEOPHYSICS LABORATORY
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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFGL-TR-87-0251 ERP, No. 985	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Air Force Geophysics Laboratory	6b. OFFICE SYMBOL (If applicable) LYA
7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Hanscom AFB Massachusetts 01731-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO
62101F	6670	09	12

11. TITLE (Include Security Classification)

The 3D-BSW Model Applied to Climatology of Small Areas and Lines

12. PERSONAL AUTHOR(S) Irving I. Gringorten and Albert R. Boehm, Major, USAF

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Scientific. Interim.	FROM ______ TO ______	1987 August 17	102

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	CFLOS; Stochastic process; Cloud-free intervals; Simulation; Sawtooth wave model; Areal coverage.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This is a continuing effort to improve stochastic modeling to simulate weather conditions and events, specifically cloud cover spread over space and time. The Sawtooth Wave simulation model has been applied with encouraging results. In a previous report the model was limited to two dimensions, and to the two-dimensional Boehm Sawtooth Wave (2D-BSW) model. A second report introduces the 3D-BSW model, to the extent that it can be used to simulate synoptic events, such as cloud cover. There has remained the task of developing and describing the three-dimensional model to yield the probabilities of events in one-, two-, and three-dimensional space, also to link spatial events to time changes and their probabilities. As in the previous reports, the determination of such probabilities was accomplished by applying Monte Carlo methods of approximation to the frequency of events. This report includes the concomitant fields of correlation, both spatial and temporal. The probability of fractional cover, in an area or on a line, and the probability of line intervals that are free of specified exceedances, (Contd)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☒ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Irving I. Gringorten	22b. TELEPHONE (Include Area Code) (617) 377-5954 — 22c. OFFICE SYMBOL LYA

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

Unclassified

19. (Contd)

are among the estimates. Algorithms have been compiled to fit the results of the Monte Carlo exercises. *Keywords: CFL [illegible] d-l[illegible] Line [illegible] 14) [illegible]*

Preface

The requirements for simulation of weather phenomena, such as total cloud cover, layered cloud cover, cloud-free line-of-sight, areal extent of temperature, moisture content of the air, or snow depth, have been increasing along with the *questions about the likelihood or possibility of events and combinations of events.*

This report, one of an ongoing series, presents the three dimensional version of the Boehm sawtooth-wave model (3D-BSW), conceived by the co-author. This model has been modified and expanded since the last report.

In this effort we have been assisted greatly by our Branch Chief, Donald D. Grantham, our former associate Charles F. Burger and recent associate Capt. Oliver Muldoon. We also acknowledge the guidance and recommendations of USAF-ETAC, especially of Capt. Dewey Harms, who has made valuable contributions to the subject. We are especially indebted to our secretaries Mrs. Helen M. Connell and Mrs. Carolyn Fadden for their excellent copy of the report, no easy task.

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1. INTRODUCTION

A previous report[1] describes the procedures for the simulation of a spatial set of conditions by use of the three-dimensional Boehm Sawtooth Wave Model (3D-BSW). The present report describes the use of the model for estimating the climatic probability of spatial events by algorithms based on repetitive simulations.

Special attention is given to cloud-cover probabilities. Fields of clouds are simulated stochastically, incorporating areal sizes and lineal separations of the clouds, both in the horizontal and in the vertical. Using these simulations, synthetic climatology is developed to estimate the likelihood of events such as an overcast of clouds covering a floor space of varying dimensions or the probability of a clearing in the clouds. One special problem addressed is the probability of a cloud-free interval along a line of travel.

(Received for publication 7 August 1987)

1. Gringorten, I.I. (1984) A Simulation of Weather in 3D Space, AFGL-TR-84-0267, ADA 155221.

The approach, as in previous reports,[2,3,4] is to develop a model that will depict a synoptic situation that statistically resembles the natural state of the weather. In many repetitive applications of the model the correlation of events at points of known distance apart should resemble the natural-state correlations. Additionally, a multitude of applications of the model should yield frequencies of events that will approximate their climatic probabilities.

In the previous[1] report a set of isopleths of rainfall, in one actual 24-hr period in New England, was shown as an example of the kind of field, with all its variability, that could be simulated by a model. The 3D-BSW model is the basis for a Monte Carlo method to produce climatological probability estimates. To illustrate this purpose, Figure 1 gives such estimates of the probability of 24-hr rainfall in New England, in reasonable agreement with the results of surveying January 24-hr amounts, in 10 years at some 140 stations through New England. As Figure 1 shows, the single-station frequency of no rain is 63 percent; that of rain not exceeding 13 mm (1/2 inch) throughout Massachusetts is 84 percent, and is accurately estimated as such by the model. The 3D-BSW model extends the estimates to varying areal sizes, and shows, for example, that in 1000 km^2 the probability of no rain is 57 percent. Or, the probability of rain, of any amount, somewhere in 1000 km^2 is 43 percent. Also, somewhere in the 1000 km^2 area there is an 8 percent probability of a 13-mm (1/2 inch) rainfall, and so on.

(One exception to the application of the model is on Mount Washington, N.H., where the precipitation is heavier than in the rest of the region.)

As in preceding reports, model development is done with the equivalent normal deviate (END), symbolized as y, of the meteorological variable (X). The transformation, back and forth, between X and y, is accomplished through the cumulative probability, Pr(X) or Pr(y). Symbolically,

$$X \longleftrightarrow Pr(X) = Pr(y) \longleftrightarrow y \quad . \tag{1}$$

Several methods of accomplishing this transformation have been mentioned and illustrated in previous reports. Whatever method is adopted, a one-to-one relation will exist between y and X, or between y and Pr(X).

2. Gringorten, I.I. (1979) Probability models of weather conditions occupying a Line or an area, J. Appl. Meteorol., 18:957-977.

3. Whiton, R.C., Berecek, E.M., and Sladen, J.G. (1981) Cloud Forecast Simulation Model, USAFETAC Scott AFB, IL 62225, USAFETAC/TN-81-004, 126 pp.

4. Burger, C.F., and Gringorten, I.I. (1984) Two-Dimensional Modeling for Lineal and Areal Probabilities of Weather Conditions, AFGL-TR-84-0126, ADA 147970, 58 pp.

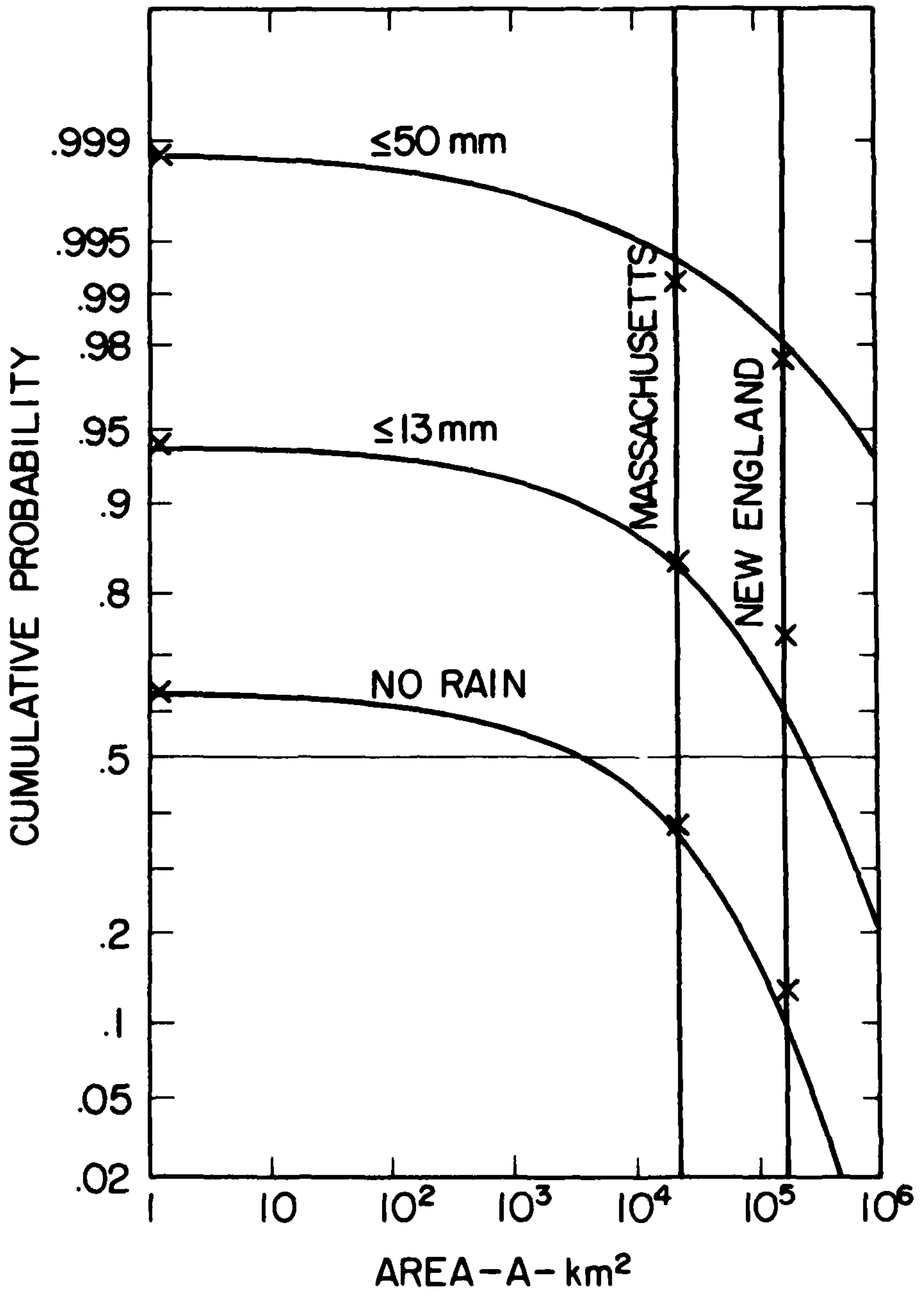

Figure 1. Showing the Frequency, at the X's of 24-hr January Rainfall at a Typical New England Station (on the left-hand ordinate); of the Maximum in Massachusetts, Area 21,350 km^2; and in the Six-State New England Region, Area 169,824 km^2. The solid curves are found by the application of the 3D-BSW model, with scale distance of 6.22 km. They show the probability of no rain, of rain less than 13 mm (1/2 inch) and rain less than 50 mm (2 inches), throughout an area (A)

Throughout this work we assume the weather is stationary stochastic. While there is much variation in the condition (X) from place to place, with level of elevation and in time of day or season, it is assumed that the "true" average, median or mode, and the true probability distribution of values remain unchanged with time or era. Although this premise conflicts with the occasional declaration of changing climate, it is acceptable in a human lifetime of climatic events, including extremes.

The problems and solutions, as presented herein, might also be viewed as a generalization of the often-asked questions about joint frequencies or probabilities of events, such as simultaneous cloud cover at two or more stations. This study deals with the aggregate of events throughout a specified area, answering such questions as the likelihood of consistently high values throughout the area or in a measurable fraction of the area. That is, the 3D-BSW model is extended to estimate the probability of events occurring in an area or given fraction of an area, or along a line.

2. THE 3D-BSW MODEL

With the 3D-BSW model, a value of the END (y) is obtainable at every point in 3D-space. Care must be taken to modify the vertical measure (w') by multiplying it by a factor q (= 50, say) to make the modified vertical measure (w) comparable to the horizontal measure in the (u, v)-plane. Thus:

$$w = q.\, w' \quad . \tag{2}$$

2.1 Development

For the 3D-BSW model each formation of uniform sawtooth waves fills all of three-dimensional space. (The waves do not move.) Each wave formation has a constant wavelength (Λ) that becomes a characteristic parameter of the model. The wavelength may vary from tens of kilometers to hundreds of kilometers.

A parameter, called the scale distance, symbolized as r, is proportional to Λ. In earlier work the scale distance was defined as the distance over which the correlation coefficient is 0.99. In the study of clouds, r was found to be approximately 1 km when Λ was several hundred kilometers. In the 3D-BSW model, we set the ratio of Λ to r at

$$\Lambda = 256\, r \text{ km} \quad . \tag{3}$$

This equivalency relationship permits us to use the scale distance of the earlier Gringorten Model B[2] as the scale distance of the 3D-BSW model (see Section 2.4.1 for amplification).

2.1.1 ELEMENTS NEEDED FOR A SINGLE WAVE FORMATION

Consider one wave formation in the 3D-space, oriented so that the flat surface or plane that contains one constant phase is parallel to the plane surface ABC (Figure 2). Let X(u, v, w) be a point on the plane surface (ABC) with coordinates (u, v, w). Let P be the closest point on this plane to the origin 0 (0, 0, 0). Then the line OP will be perpendicular to the plane, oriented by angles (α, β, γ) between OP and the three axes. If the length of OP is D, then from solid geometry the equation of the plane containing P and X is

$$u.\cos\alpha + v.\cos\beta + w.\cos\gamma = D \ . \tag{4}$$

Let λ be the angular measure from the U-W plane to the vertical plane containing OP. (If the W-axis were the earth's axis, λ would be longitude.) From spherical triangles (Figure 3),

$$\begin{aligned} \cos\alpha &= \sin\gamma \ . \ \cos\lambda \\ \cos\beta &= \sin\gamma \ . \ \sin\lambda \end{aligned} \tag{5}$$

from which

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \ . \tag{6}$$

Or, one of (α, β, γ) is determined when the other two are given.

Let the leading edge of the middle wave of a wave formation miss the origin by a distance H, and define

$$h = H/\Lambda \ . \tag{7}$$

There are three degrees of freedom in the random choice of a wave formation. We choose h by random process, thus using one of the three degrees of freedom:

$$0 \leqq h \leqq 1.0 \ , \tag{8}$$

h being uniform on (0, 1).

The remaining two degrees of freedom must be used to obtain values for (α, β, γ). We have two alternative procedures for doing this. While one is a more direct procedure, the other will save computer time.

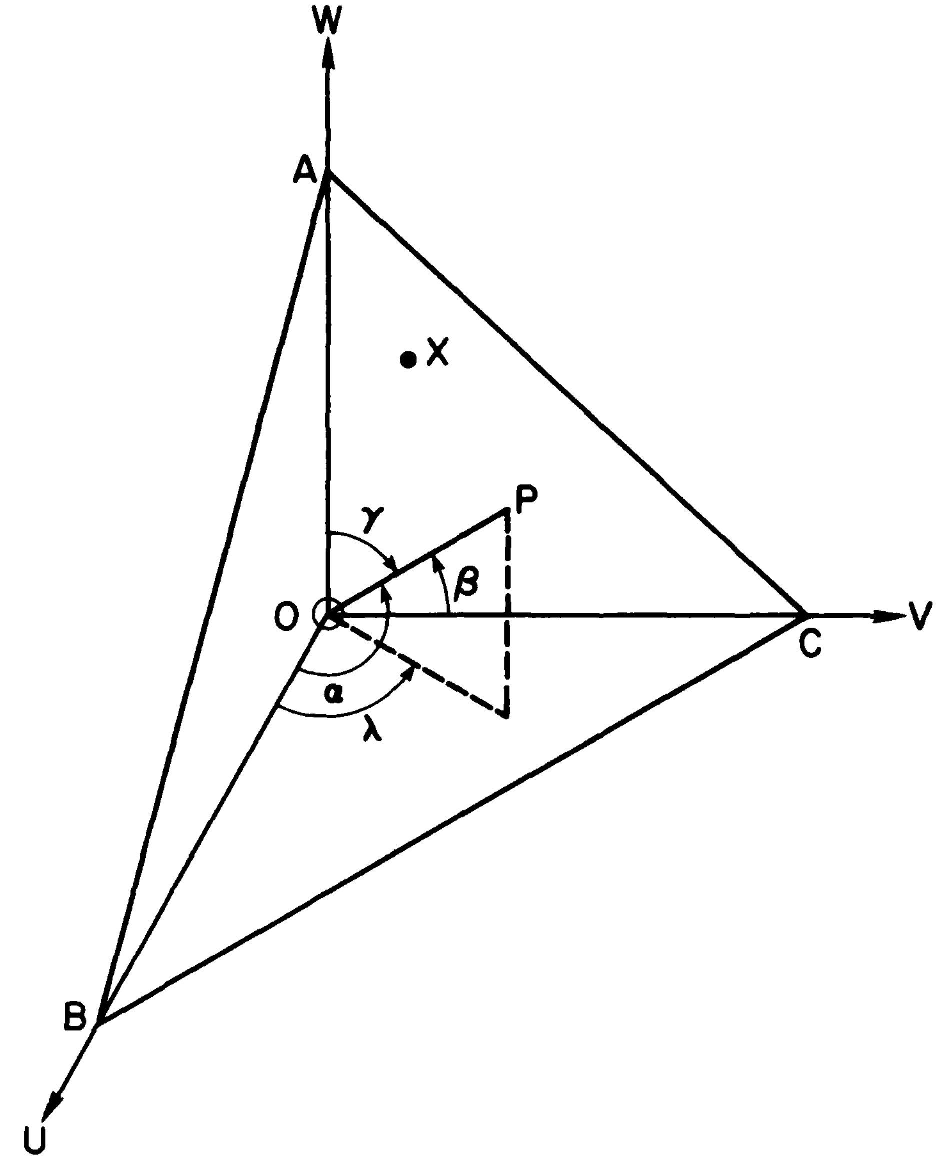

Figure 2. Illustration of a Plane (ABC). The point (X) is considered to be on this plane surface. Point (P) is such that OP is the shortest distance from the origin to the plane, and has angles (α, β, γ) with the U-, V-, W-axes. The projection of OP onto the U-V plane has angle λ with the U-axis

2.1.2 THE LAMBDA METHOD FOR OBTAINING (α, β, γ)

Consider the sphere with radius (OP). (Figure 3 shows one octant.) The shell has a total area of $4\pi D^2$. An element of the area (dA) is given by:

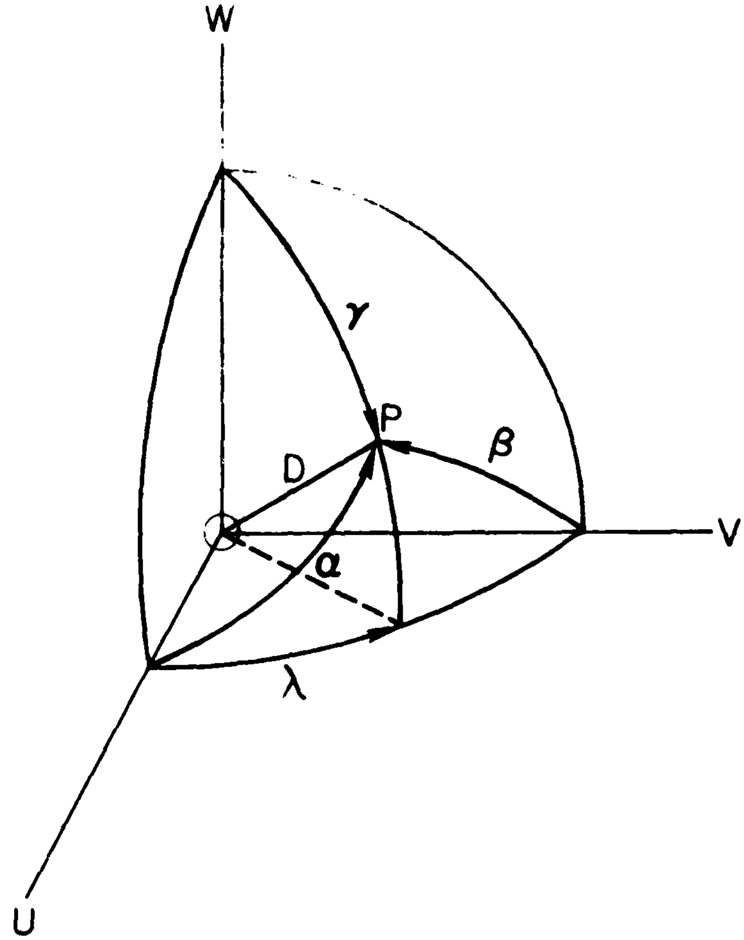

Figure 3. Illustration of One Octant of the Sphere Centered at the origin (0). The line (OP) of length (D) has angular sides (α, β, γ) with the three axes (U, V, W) and lies in the plane that makes angle (λ) with the (U - W) plane

$$
\begin{aligned}
dA &= (D, d\gamma) \, . \, (D . \sin\gamma \, . \, d\lambda) \\
&= D^2 \, . \, d(-\cos\gamma) \, . \, d\lambda \, .
\end{aligned}
\tag{9}
$$

In the random selection of the wave formation, the point P should have the same probability of appearing in one element of area (dA) as in any other on the sphere. Thus we choose two random numbers (R_γ, R_λ), each between 0 and 1, and set

$$\cos\gamma = 2R_\gamma - 1, \qquad \text{to make} \quad -1.0 \leqq \cos\gamma \leqq 1.0 \tag{10}$$

where $\cos\gamma$ is uniform on (-1, +1)

$$\lambda = 2\pi R_\lambda, \quad \text{to make} \quad 0.0 \leqq \lambda \leqq 2\pi .$$

where λ is uniform on (0, 2π)

Equations (5) give $\cos\alpha$ and $\cos\beta$, which, together with $\cos\gamma$ define D [Eq. (4)] for each point (u, v, w) in the 3D space. We will refer to this as Alternative (1).

2.1.3 THE ACCEPTANCE-REJECTION METHOD FOR OBTAINING (α, β, γ)

Choose three random numbers (R_u, R_v, R_w) between 0.0 and 1.0, and set

$$\begin{aligned} r_u &= R_u - 1/2 && \text{to make } -1/2 \leqq r_u \leqq 1/2 , \\ r_v &= R_v - 1/2 && \text{to make } -1/2 \leqq r_v \leqq 1/2 , \\ r_w &= R_w - 1/2 && \text{to make } -1/2 \leqq r_w \leqq 1/2 . \end{aligned} \tag{11}$$

The point with coordinates (r_u, r_v, r_w) will lie within a unit cube. To assure that any one direction of the line from the origin to (r_u, r_v, r_w) will be as likely as any other direction, we impose the restriction:

$$(OP') = \sqrt{r_u^2 + r_v^2 + r_w^2} \leqq 1/2 . \tag{12}$$

If the length (OP') is greater than (1/2), then we reject the three random numbers and try another group of three random numbers. The probability that the set (R_u, R_v, R_w) will be rejected will be $[1 - 4/3\pi(1/2)^3] = 0.476$. On the average, this alternative procedure requires the generation of (3/0.524) = 5.7 random numbers per wave formation for orientation plus an additional random number for phase.

With the first acceptable set of random numbers, we find

$$\begin{aligned} \cos\alpha &= r_u/(OP') , \\ \cos\beta &= r_v/(OP') , \\ \cos\gamma &= r_w/(OP') , \end{aligned} \tag{13}$$

which then define D [Eq. (4)] for each point (u, v, w) in the space. We will refer to this method as Alternative (2).

2.1.4 COMPARISON OF ALTERNATIVES FOR GENERATING WAVES

While Alternative (2) requires, on the average, the generation of 6.7 random numbers, as opposed to three for Alternative (1), it does not require the determination of a sinusoidal function, that consumes more computer time than finding the random numbers. There is another attractive feature in Alternative (2): it can be extended to higher dimensions than the three for space, to include an additional dimension for time or to introduce additional influences on the wave formation. (This will be explored in later reports.) However, care must be taken to eliminate the selection of absolute values for (r_u, r_v, r_w) that will make (OP') too small for computer operation.

2.1.5 WAVE HEIGHT

The wave height (x) of one wave formation, at a point (u, v, w) is given by

$$x = \text{FRA}\ (h + D/\Lambda) \tag{14}$$

where FRA denotes the fractional part of the bracketed quantity. While x(u, v, w) is termed "wave height", it is difficult to visualize in 3D space. For our purpose, it is acceptable as a property of the wave formation at the point (u, v, w), and is defined, mathematically and unequivocally, by Eq. (14). Where $x' = h + D/\Lambda$, x' can be negative. The symbol INT(x') is used for the integer part of x'. If, as in FORTRAN, this is taken to be the integer part with its sign, then the wave height (x) at a point (u, v, w) is given by

$$x = x' - \text{INT}(x') \qquad \text{when } x' \geqq 0\ ,$$

$$= x' - \text{INT}(x') + 1 \qquad \text{when } x' < 0\ .$$

If, as in BASIC, the symbol INT(x') is taken to be the next lesser whole number, then, simply, the wave height is given by

$$x = x' - \text{INT}(x') \text{ for all } x'\ .$$

The wave height (x) varies uniformly from 0 to 1.0 in the 3D-BSW model.

2.1.6 GENERATING MULTIPLE WAVE FORMATIONS

The above procedure produces one wave formation and one set of values for $\cos\alpha$, $\cos\beta$, $\cos\gamma$, and h. Repeated L times, there will be L wave formations. with L wave heights (x_ℓ, ℓ = 1(1)L) at (u, v, w) whose average wave height

$$\bar{x} = \frac{1}{L} \sum_{i=1}^{L} x_i (u, v, w) \tag{15}$$

will have a mean of (1/2) and variance of (1/12L), and will have an approximate normal (Gaussian) distribution. For sufficiently large L, the END (y) at X(u, v, w) is given, practically, by

$$y(u, v, w) = \sqrt{12L} \cdot (\bar{x} - 1/2) \,. \tag{16}$$

The END (y) can thus be calculated for any point (u, v, w) in the 3D space.

If, for convenience, L is made equal to 12, then

$$y(u, v, w) = \sum_{i=1}^{12} x_i (u, v, w) - 6 \,. \tag{17}$$

For a single simulated field, a minimum of 36 random numbers using Alternative (1), or by Alternative (2) an expected maximum of 80 random numbers, are needed to generate all values of the END (y) at all points in the three-dimensional field.

2.2 Simulation or Depiction

To obtain a picture in a horizontal surface, within, say a square 200 km to the side, we keep w constant (conveniently zero), and compute y(u, v, w = 0) for as many points (u, v, w = 0) in the square as we wish. Likewise we can generate a vertical cross-section by keeping u constant, or v constant, or by keeping a linear combination of (u, v) constant. For an overall procedure to produce a field of values, see Appendix A.

Samples of simulated cloud structure are given in the previous report[1] (October 1984), one in a horizontal plane representing a cloud cover, and four other examples of clouds in a vertical cross-section.

2.3 Probabilities - Description of the Monte Carlo Method

The above-described procedure produces END-values in all three dimensions, or in the two dimensions of a horizontal surface or vertical cross-section, or along a line of travel or line of sight. Suppose a field of END-values (y) is generated in a horizontal square of area A. Among the many values of y in the square there will be a maximum, y_{max}, also a value that will be the (9/10)ths maximum, also (8/10)ths maximum, and so on. Taking any single value, y_0 (say = 0.4), as a threshold, there will be a certain probability that it will be the maximum in the area and will not be exceeded in the entire square. There will be a higher probability that it will not be exceeded in all except (1/10)th of the area, still higher in all except (2/10)ths of the area, and so on. Lastly there is the probability that is 1.0 minus the probability that it will be exceeded everywhere, when it will be the minimum in the square. The areal climatology of a region should consist of information on such probabilities. The same kind of climatology could be obtained for events or conditions along a line.

At this stage we must admit to an unfulfilled wish. An analytical solution is desirable to determine the probability distribution of the maximum in a space, similar to the solution for the maximum in an Ornstein-Uhlenbeck temporal process.[5,6] Not having such an analytical approach, we have resorted to approximate solutions by the Monte Carlo method.

2.3.1 MULTIPLE FIELDS OF ENDs

In an area (A) or on a line (L) a finite number of END-values can be found by the above-described procedures. For a single such field, a maximum value of y, the above percentiles and minimum value can be determined. The greater the number of y-values that are calculated, the closer will be the maximum to the true maximum of the area or the line. Now, let us generate many such fields or "snapshot" pictures (M = 25,000, say) of area (A) or line (L). There will be M trial values of the maximum in A or L. These values collectively constitute a frequency distribution of the maximum, and may be regarded as a good approximation to the true probability distribution of the maximum. The same will be true for all percentiles and for the minimum.

---

5. Keilson, J., and Ross, H.F. (1975) Passage time distributions for Gaussian Markov (Ornstein-Uhlenbeck) statistical process, Selected Tables in Mathematical Statistics, 3:233-237.

6. Gringorten, I.I. (1982) The Keilson-Ross Procedure for Estimating Climatic Probabilities of Duration of Weather Conditions, AFGL-TR-82-0116, ADA 119860.

2.3.2 TYPES OF PROBABILITIES DEFINED

For clarification in the following development, the symbol P_o' is used for probability of exceedance of the value (X) at a single point. The symbol P_o, as used herein, is the cumulative probability, and is the probability that X will not be exceeded at a single point. That is,

$$P_o = (1 - P_o') \ . \tag{18}$$

It is assumed that P_o is the same for any point in the area or the line. The climatic probability of the weather condition defines P_o, which, in turn, defines its corresponding END (y_o). As mentioned in Section 1 there is a one-to-one correspondence between the END (y) and the meteorological variable (x). A threshold (X) such that $x \leq X$ has its corresponding value, y_o, such that $y \leq y_o$.

The single-point END (y_o) will be negative when $P_o < 1/2$, positive when $P_o > 1/2$. (For sky cover P_o is the cloud-free probability in a single direction.) The symbol PA will be used for the probability of a fraction or less coverage of an area. An example is the probability of 3/10 or less cloud cover in 1000 km^2. Coverage can also apply to the threshold of a continuous variable such as temperature. An example is the probability that 0.25 of the area has temperatures below freezing.

The symbol PL will be used for the probability of a fraction or less coverage of a line segment, for example, the probability that along the route between New York City and Washington, D.C., rain would occur 0.33 or less of the distance. Coverage is without regard to whether the rain occurred in one unbroken interval or many small intervals as long as their sum is 0.33 or less of the total distance.

The symbol PI will be used for the probability that at least one interval somewhere along a line segment has weather continuously below the threshold (y_o). For example, suppose a helicopter flying through patchy fog can visually navigate if no fog patch is greater than 1.5 km. What is the probability that there are one or more fog patches greater than 1.5 km anywhere along a 200-km route? A weather condition can be a specific kind of weather (for example, thunderstorm) or all values exceeding (above or below) a specific threshold (for example, pressures below 992 mb). Note that, when the interval is as long as the total line segment, then PI is the same as PL for zero coverage.

A line interval (s') may be visualized as a window sliding along a longer line of travel (T); at any position of the window, within s' there will be a maximum value; the lowest of these maximum values is the threshold condition whose probability (PI) we wish to find.

2.4 Areal Coverage (PA)

The goal of this section is to find a satisfactory approximation for

$$PA(X, A, F, r) = PA(y_o, A, F, r)$$

which is the probability that a weather condition (x) will not exceed the threshold (X) (or the END (y) will not exceed y_o), except in (F/10)ths or less of the area (A), given the scale parameter (r). That is, 1-PA (y_o, A, F, r) is the probability that y_o is the (F/10) highest value of y in the area (A).

If F = 0, then the threshold weather condition is exceeded nowhere in the area. For example, what is the probability of no clouds anywhere in New Jersey, that is, the state is completely clear?

If F = 10, then the threshold weather condition is exceeded everywhere in the area. For example, what is the probability the relative humidity is above 80 percent everywhere in Iowa?

If $0 < F < 10$, then PA is the probability that the threshold weather condition is not exceeded except in a fraction of the area less than or equal to (F/10).

2.4.1 STANDARDIZED UNITS

At this stage of the development, we need to standardize our units. A square, of area A (km^2), has side s' where

$$s' = \sqrt{A}\ \text{km} \quad . \tag{19}$$

The standardized distance (s) is

$$s = s'/r \tag{20}$$

where r is the scale distance. Or

$$r = s'/s\ \text{km} \quad .$$

The scale distance (r) had been defined, in earlier work, as the distance over which the correlation coefficient is 0.99. In the study of clouds, (r) was approximately 1 km when the wavelength was found to be several hundred kilometers. Arbitrarily we have chosen, for standardization, the ratio of Λ to r at 256:1. That is, $\Lambda = 256$ r km in the 3D-BSW model, making the two parameters (Λ and r) alternatives to each other.

In previous work with New England 24-hr precipitation, we found the scale distance equal to 5 to 10 km (6.22 km in the example of Figure 1). For radar echoes on a PPI-scope the scale distance ranged from 1 to 2 km in summer, 2-1/2 to 4-1/2 km in winter. The following sections will present typical results for sky-cover data.

For convenience, the term (z) is introduced to serve as a dimensionless substitute for A or s', such that

$$z = \ell n(s'/r)/\ell n\ 2 = \ell n(\sqrt{A}/r)/\ell n\ 2 \tag{21}$$

or

$$s = 2^z \text{ where } s = s'/r \tag{22}$$

or

$$A = (r.2^z)^2 \quad .$$

With the parameter A or r given, there will be a one-to-one correspondence between z and A. Symbolically

$$PA(y_o, A, F, r) = PA(y_o, z, F) \; .$$

2.4.2 GENERATING MAPS OF END-VALUES

A computer program was compiled to provide the Monte Carlo estimates of probabilities: $PA(y_o, z, F)$ for $y_o = -4.0(0.5)4.0$, $z = 0(1)8$, and $F = 0(1)5$. Corresponding to the nine choices for z, nine square areas were selected for the study, that had standardized sides:

$$s = 2^z = 1,\ 2,\ \ldots\ 256$$

for which

$$A = 1,\ 4,\ \ldots.\ (256)^2 \text{ for } r = 1) \; .$$

By the method described on the previous page, and as described in the previous report, a field of y-values was generated to simulate each map of a total of 25,000 maps. (In applying Eq. (14) the parameter was set at Λ = 256 km.) Each square was gridded to yield $(21)^2 = 441$ grid points at which the y-values were found, which were then ordered by size. The largest value was assigned to the distribution for y(max). The (n_F)th largest value from the top, where

$$n_F = \text{TRUNC} \{441F/10\} + 1, \quad F = 1(1)5 \tag{23}$$

was assigned to the distribution of the (F/10)th highest value of y in the area (A).

The (F/10)th lowest y-value should be comparable to the (F/10)th highest with the sign of y reversed. Another y-value, therefore, was obtained by selecting the (n_F')th value of y where

$$n_F' = 441 - \text{TRUNC} \{441F/10\}, \quad F = 4(-1)0 \tag{24}$$

and, after changing the sign of y, assigning it to the distribution of the (F/10)th highest of y in the area (A).

The six probability distributions of PA for the six values of F = 0(1)5 were approximated by the frequency distributions of the y-values in the 25,000 maps. The results, in nine columns for the nine z's, in order of increasing END (y_o), are shown in Tables 1(0) to 1(5), each table for one value of F.

These values were plotted in six charts [Figures 4(0(to 4(5)], and curves were drawn, with smoothing at the extremes. Each chart, one for each value of F = 0(1)5, has probability along the ordinate, z along the abscissa. Each curve on each chart is drawn for one value of y_o [= -3.5(0.5)4.0]. Each chart shows the cumulative probability of y_o that will be exceeded in only (F/10)ths of the area.

Table 1(0). Estimates of the Probability Distribution of PA (y_o, z, F = 0) Where y_o is the Maximum in the Area $A = (r.\ 2^z)^2$, z = 0(1)8, r is the Given Scale Distance (km). The sample size was 25,000 maps

y_o	Z 0	1	2	3	4	5	6	7	8
-3.5	0.00009	0.00015	0.00011	0.00001	-	-	-	-	-
-3.0	0.00097	0.00101	0.00081	0.00059	0.00027	-	-	-	-
-2.5	0.00489	0.00481	0.00401	0.00251	0.00123	0.00013	-	-	-
-2.0	0.0216	0.0198	0.0162	0.0106	0.00525	0.0075	-	-	-
-1.5	0.0611	0.0581	0.0486	0.0376	0.0212	0.00623	0.00033	-	-
-1.0	0.151	0.143	0.127	0.102	0.0643	0.0263	0.00289	-	-
-0.5	0.298	0.285	0.260	0.220	0.155	0.0798	0.0171	0.00009	-
0	0.483	0.471	0.443	0.390	0.306	0.190	0.0637	0.00223	-
0.5	0.674	0.663	0.637	0.590	0.503	0.368	0.183	0.0224	0.00001
1.0	0.828	0.819	0.799	0.760	0.693	0.571	0.374	0.110	0.00157
1.5	0.9298	0.9234	0.9132	0.890	0.848	0.765	0.605	0.313	0.0332
2.0	0.9749	0.9718	0.9677	0.9591	0.9387	0.897	0.802	0.590	0.227
2.5	0.99377	0.99275	0.99059	0.9880	0.9814	0.9666	0.9260	0.827	0.598
3.0	0.99871	0.99833	0.99771	0.99697	0.99511	0.99095	0.9794	0.9489	0.868
3.5	0.99975	0.99965	0.99965	0.99941	0.99905	0.99799	0.99581	0.9888	0.9728
4.0	0.99993	0.99995	0.99995	0.99987	0.99977	0.99965	0.99929	0.99853	0.99627
4.5	-	-	-	0.99995	0.99995	0.99995	0.99991	0.99983	0.99967

Table 1(1). The Probability Distribution of PA (y_o, z, F = 1), Where y_o is the (1/10)th Maximum of the Area (A) Given by $\sqrt{A} = r.\ 2^z$. The sample size was 25,000 maps

y_o	Z 0	1	2	3	4	5	6	7	8
-3.5	0.00015	0.00015	0.00017	0.00001	-	-	-	-	-
-3.0	0.00097	0.00115	0.00103	0.00069	0.00053	0.00009	-	-	-
-2.5	0.00501	0.00519	0.00475	0.00329	0.00221	0.00029	0.00003	-	-
-2.0	0.0222	0.0209	0.0185	0.0142	0.00929	0.00383	0.00039	-	-
-1.5	0.0628	0.0612	0.0549	0.0460	0.0328	0.0166	0.00331	-	-
-1.0	0.154	0.149	0.138	0.123	0.0924	0.0569	0.0190	0.00067	-
-0.5	0.302	0.294	0.277	0.251	0.206	0.144	0.0697	0.00787	-
0	0.491	0.482	0.465	0.430	0.376	0.297	0.193	0.0616	0.00035
0.5	0.680	0.673	0.655	0.626	0.578	0.499	0.389	0.223	0.0239
1.0	0.832	0.826	0.814	0.792	0.757	0.699	0.616	0.499	0.288
1.5	0.9319	0.9267	0.9204	0.9080	0.889	0.855	0.814	0.771	0.768
2.0	0.9758	0.9737	0.9712	0.9666	0.9597	0.9462	0.9334	0.9300	0.9680
2.5	0.99431	0.99341	0.99241	0.99057	0.9879	0.9844	0.9816	0.9872	0.99827
3.0	0.99877	0.99857	0.99835	0.99769	0.99713	0.99609	0.99603	0.99817	0.99997
3.5	0.99975	0.99967	0.99969	0.99965	0.99929	0.99941	0.99945	0.99985	-
4.0	0.99993	0.99995	0.99995	0.99995	0.99991	0.99989	0.99997	0.99995	-
4.5	-	-	-	-	0.99995	-	-	-	-

Table 1(2). The Probability Distribution of PA (y_o, z, F = 2), where y_o is the (2/10)ths Maximum of the Area (A) Given by $\sqrt{A} = 2^z r$

	Z								
y_o	0	1	2	3	4	5	6	7	8
-3.5	0.00019	0.00015	0.00017	0.00009	0.00001	-	-	-	-
-3.0	0.00107	0.00115	0.00111	0.00087	0.00077	0.00005	-	-	-
-2.5	0.00511	0.00537	0.00489	0.00385	0.00275	0.00091	0.00005	-	-
-2.0	0.0224	0.0218	0.0196	0.0168	0.0121	0.00643	0.00103	-	-
-1.5	0.0634	0.0630	0.0575	0.0518	0.0408	0.0246	0.00743	0.00035	-
-1.0	0.155	0.152	0.143	0.135	0.1101	0.0772	0.0372	0.00387	-
-0.5	0.305	0.299	0.287	0.267	0.234	0.184	0.118	0.00318	0.00007
0	0.493	0.487	0.476	0.451	0.413	0.356	0.279	0.150	0.00861
0.5	0.683	0.678	0.665	0.645	0.612	0.563	0.500	0.395	0.183
1.0	0.834	0.830	0.821	0.806	0.784	0.755	0.720	0.689	0.703
1.5	0.9330	0.9285	0.9243	0.9157	0.9051	0.888	0.882	0.891	0.9656
2.0	0.9761	0.9748	0.9726	0.9704	0.9664	0.9617	0.9620	0.9774	0.99865
2.5	0.99445	0.99383	0.99287	0.99189	0.99027	0.9896	0.99203	0.99683	-
3.0	0.99879	0.99863	0.99845	0.99821	0.99769	0.99743	0.99825	0.99973	-
3.5	0.99979	0.99975	0.99973	0.99977	0.99955	0.99965	0.99995	0.99995	-
4.0	0.99993	0.99995	-	-	0.99993	-	-	-	-
4.5	-	-	-	-	-	-	-	-	-

Table 1(3). The Probability Distribution of PA (y_o, z, F = 3), Where y_o is the (3/10)ths Maximum of the Area (A) Given by $\sqrt{A} = 2^z \cdot r$

	Z								
y_o	0	1	2	3	4	5	6	7	8
-3.5	0.00019	0.00015	0.00021	0.00009	0.00005	-	-	-	-
-3.0	0.00111	0.00121	0.00113	0.00093	0.00089	0.00023	-	-	-
-2.5	0.00527	0.00553	0.00517	0.00443	0.00335	0.00153	0.00019	-	-
-2.0	0.0227	0.0223	0.0206	0.0186	0.0150	0.00919	0.00235	0.00003	-
-1.5	0.0641	0.0634	0.0600	0.0569	0.0490	0.0329	0.0144	0.00107	-
-1.0	0.158	0.156	0.149	0.143	0.126	0.0984	0.0589	0.0114	-
-0.5	0.307	0.304	0.295	0.281	0.258	0.221	0.165	0.0734	0.00101
0	0.495	0.492	0.484	0.468	0.444	0.407	0.355	0.258	0.0600
0.5	0.685	0.682	0.673	0.662	0.642	0.615	0.585	0.546	0.493
1.0	0.836	0.833	0.828	0.817	0.806	0.794	0.789	0.810	0.9225
1.5	0.9333	0.9307	0.9276	0.9230	0.9168	0.9122	0.9200	0.9500	0.99665
2.0	0.9763	0.9754	0.9739	0.9739	0.9720	0.9723	0.9783	0.99285	0.99991
2.5	0.99455	0.99409	0.99317	0.99281	0.99213	0.99245	0.99575	0.99907	-
3.0	0.99879	0.99865	0.99853	0.99839	0.99813	0.99837	0.99923	0.99995	-
3.5	0.99979	0.99975	0.99975	0.99977	0.99971	0.99979	0.99995	-	-
4.0	0.99993	0.99995	-	-	0.99995	-	-	-	-
4.5	-	-	-	-	-	-	-	-	-

Table 1(4). The Probability Distribution of PA (y_o, z, F = 4), Where y_o is the (4/10)ths Maximum of the Area (A)

y_o	z: 0	1	2	3	4	5	6	7	8
-3.5	0.00019	0.00017	0.00021	0.00009	0.00013	0.00003	-	-	-
-3.0	0.00111	0.00123	0.00117	0.00107	0.00097	0.00029	-	-	-
-2.5	0.00533	0.00563	0.00553	0.00491	0.00417	0.00257	0.00049	-	-
-2.0	0.0229	0.0228	0.0218	0.0204	0.0180	0.0123	0.00437	0.00025	-
-1.5	0.0645	0.0655	0.0632	0.0618	0.0559	0.0440	0.0238	0.00307	-
-1.0	0.159	0.159	0.155	0.152	0.142	0.120	0.0848	0.0278	0.00003
-0.5	0.309	0.308	0.303	0.295	0.281	0.260	0.222	0.136	0.0118
0	0.498	0.496	0.492	0.483	0.472	0.455	0.429	0.377	0.225
0.5	0.687	0.686	0.682	0.675	0.671	0.660	0.656	0.673	0.781
1.0	0.838	0.836	0.834	0.829	0.825	0.824	0.843	0.888	0.9853
1.5	0.9340	0.9319	0.9311	0.9285	0.9259	0.9308	0.9444	0.9791	0.99995
2.0	0.9765	0.9761	0.9759	0.9760	0.9759	0.9788	0.9865	0.99769	-
2.5	0.99455	0.99417	0.99353	0.99359	0.99393	0.99477	0.99787	0.99975	-
3.0	0.99879	0.99867	0.99857	0.99849	0.99859	0.99891	0.99975	-	-
3.5	0.99979	0.99977	0.99975	0.99977	0.99971	0.99989	-	-	-
4.0	0.99993	0.99995	-	-	0.99995	-	-	-	-
4.5	-	-	-	-	-	-	-	-	-

Table 1(5). The Probability Distribution of PA (y_o, z, F = 5), Where y_o is the (5/10)ths Maximum of the Area (A) Corresponding to Z = 0(1)8, Λ = 256

y_o	Z 0	1	2	3	4	5	6	7	8
-3.5	0.00019	0.00017	0.00023	0.00011	0.00021	0.00009	-	-	-
-3.0	0.00119	0.00131	0.00121	0.00141	0.00113	0.00053	0.00007	-	-
-2.5	0.00539	0.00571	0.00613	0.00569	0.00487	0.00421	0.00115	0.00001	-
-2.0	0.0231	0.0234	0.0227	0.0225	0.0208	0.0162	0.00801	0.00085	-
-1.5	0.0654	0.0671	0.0656	0.0666	0.0642	0.0548	0.0378	0.00845	-
-1.0	0.160	0.162	0.160	0.161	0.157	0.146	0.117	0.0591	0.00147
-0.5	0.311	0.311	0.310	0.309	0.305	0.298	0.280	0.222	0.0606
0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.5	0.689	0.689	0.690	0.691	0.695	0.702	0.720	0.778	0.9394
1.0	0.840	0.838	0.840	0.839	0.843	0.854	0.883	0.9409	0.99851
1.5	0.9346	0.9329	0.9343	0.9333	0.9358	0.9452	0.9622	0.99153	-
2.0	0.9769	0.9766	0.9773	0.9774	0.9791	0.9838	0.99197	0.99913	-
2.5	0.99459	0.99427	0.99385	0.99429	0.99511	0.99577	0.99883	0.99997	-
3.0	0.99879	0.99867	0.99877	0.99857	0.99885	0.99945	0.99991	-	-
3.5	0.99979	0.99981	0.99975	0.99987	0.99977	0.99991	-	-	-
4.0	0.99993	-	-	-	-	-	-	-	-
4.5	-	-	-	-	-	-	-	-	-

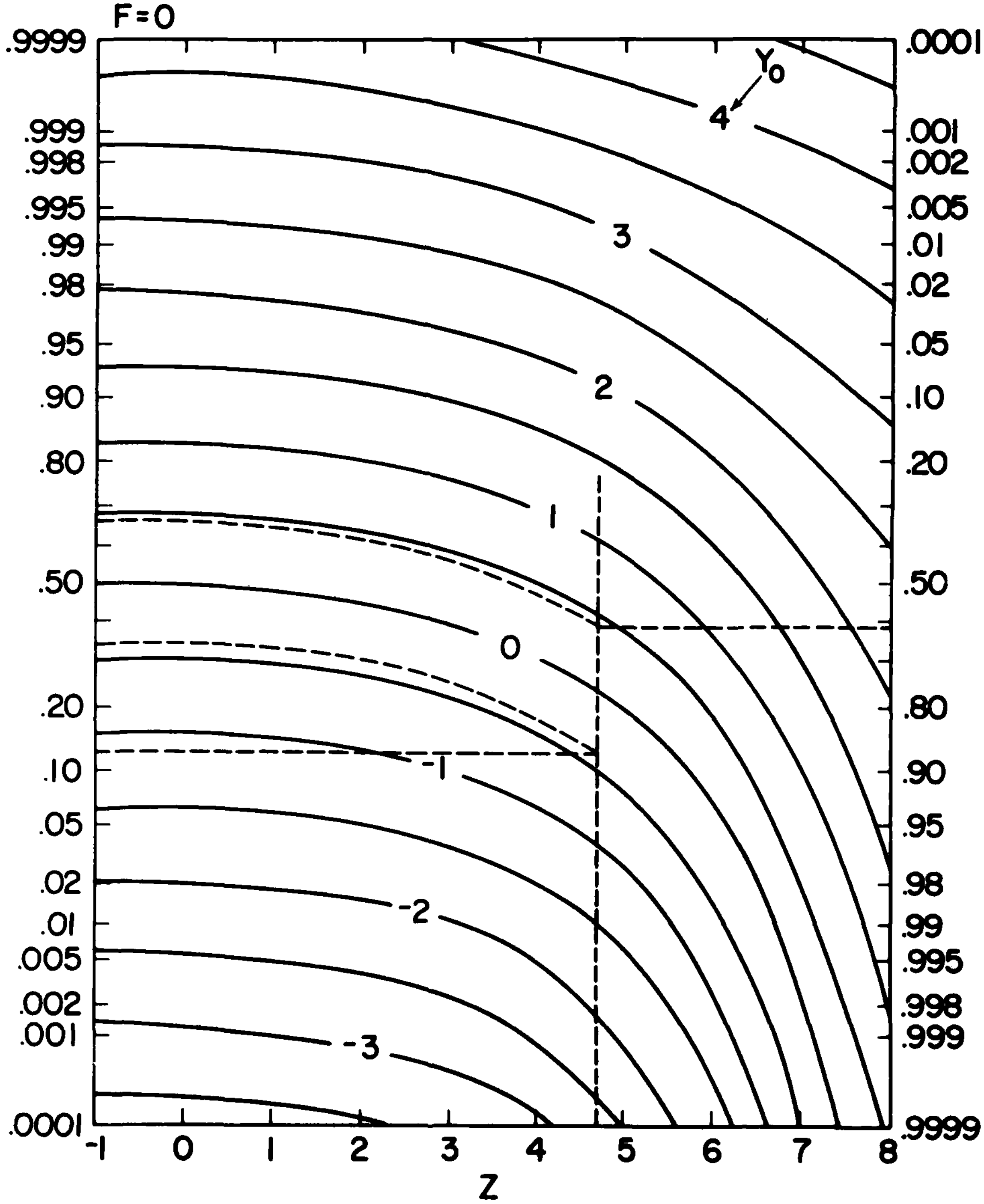

Figure 4(0). For the Graphical Solution of P (y_o, z, F), the Probability That an END (y) Will Exceed (y_o) Only in (F/10) or Less of the Area (A) When Scale Distance, r= 1. (This chart is for F = 0.) The horizontal scale is for the standardized measure (z) of the area [Eq. (21)]

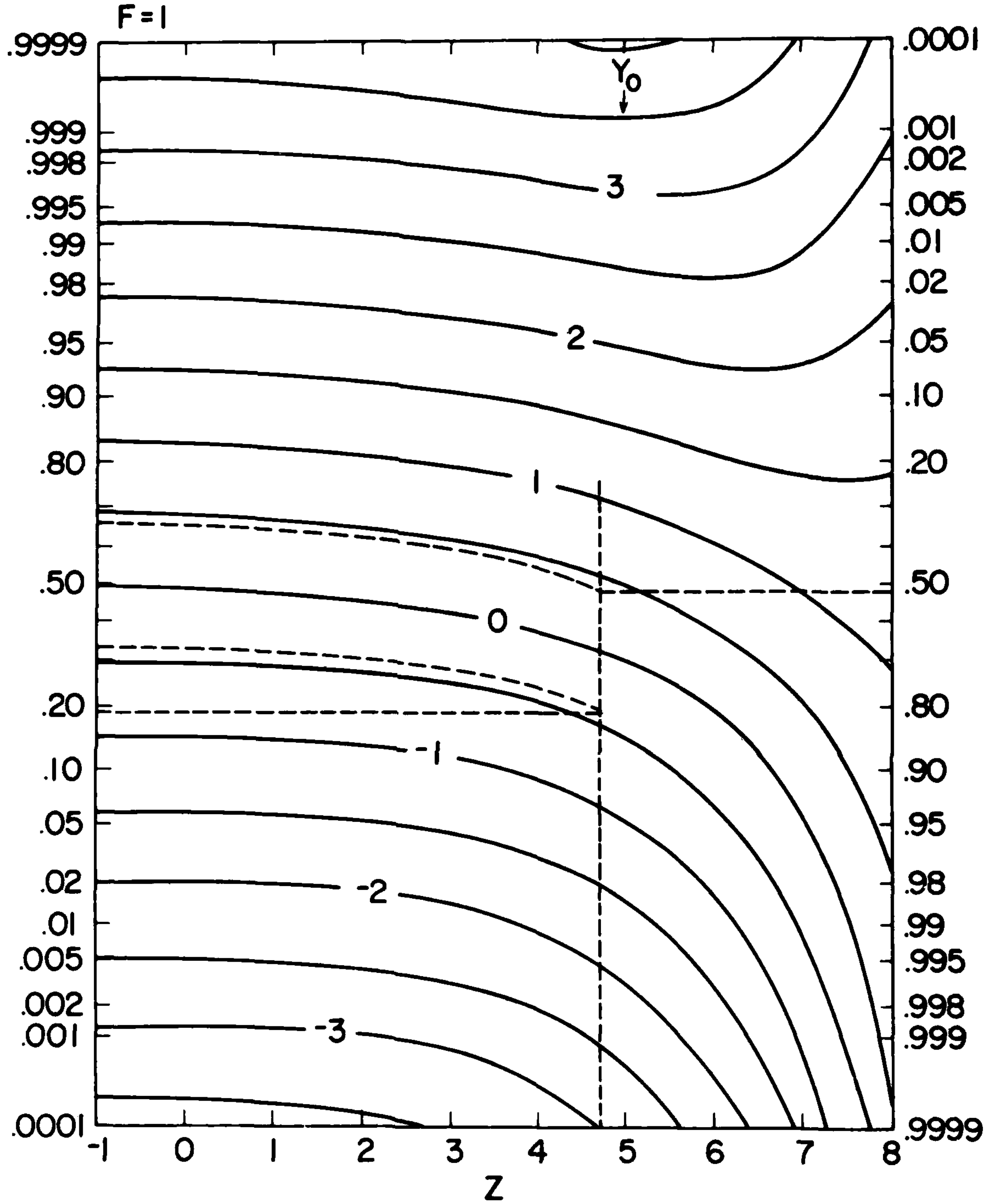

Figure 4(1). For the Graphical Solution of P (y_0, z, F), the Probability That an END (y) Will Exceed (y_0) Only in (F/10) or Less of the Area (A) When Scale Distance, r = 1. (This chart is for F = 1.) The horizontal scale is for the standardized measure (z) of the area [Eq. (21)]

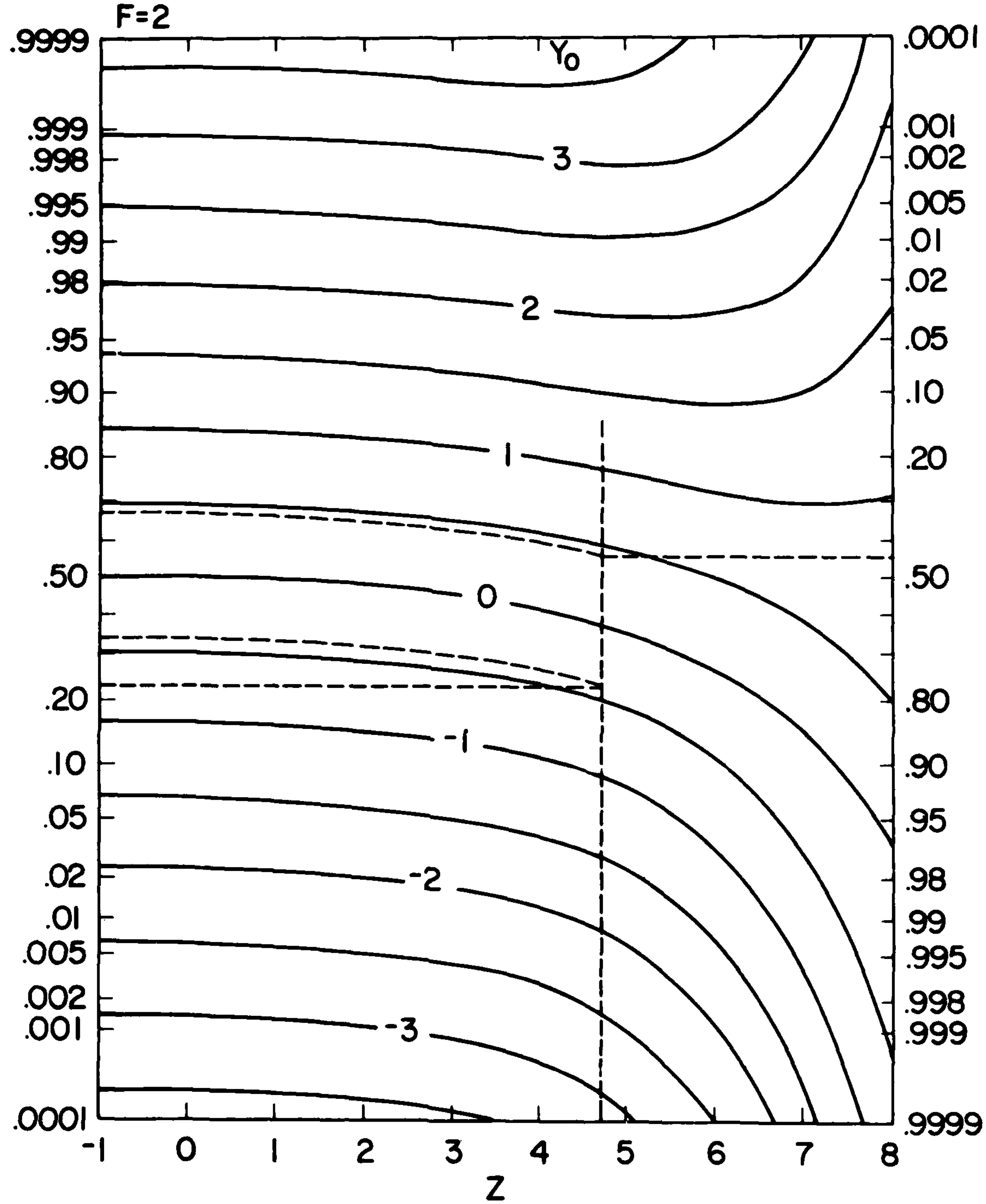

Figure 4(2). For the Graphical Solution of P (y_o, z, F), the Probability That an END (y) Will Exceed (y_o) Only in (F/10) or Less of the Area (A) When Scale Distance, r = 1. (This chart is for F = 2.) The horizontal scale is for the standardized measure (z) of the area [Eq. (21)]

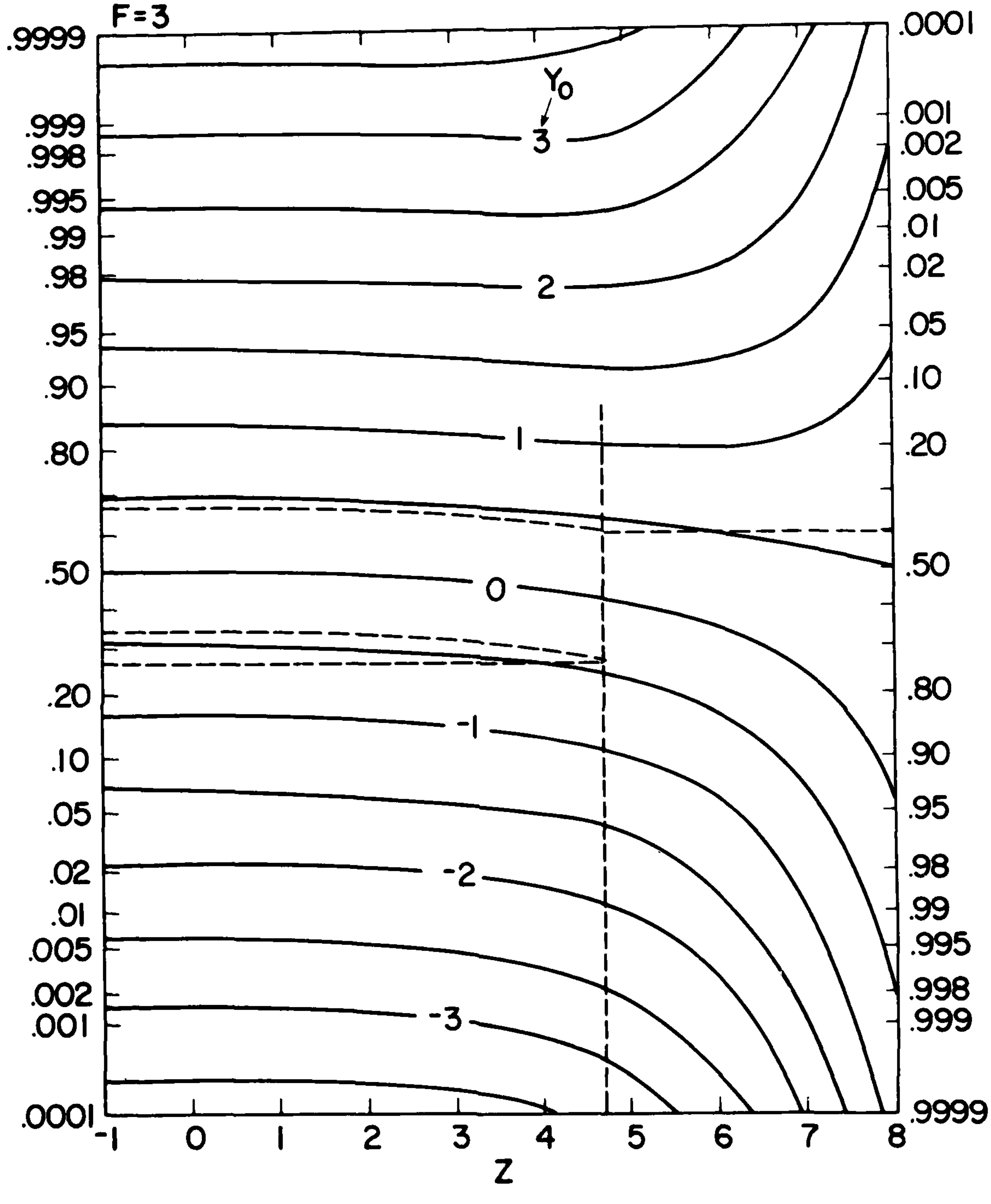

Figure 4(3). For the Graphical Solution of P (y_0, z, F), the Probability That an END (y) Will Exceed (y_0) Only in (F/10) or Less of the Area (A) When Scale Distance, r = 1. (This chart is for F = 3.) The horizontal scale is for the standardized measure (z) of the area [Eq. (21)]

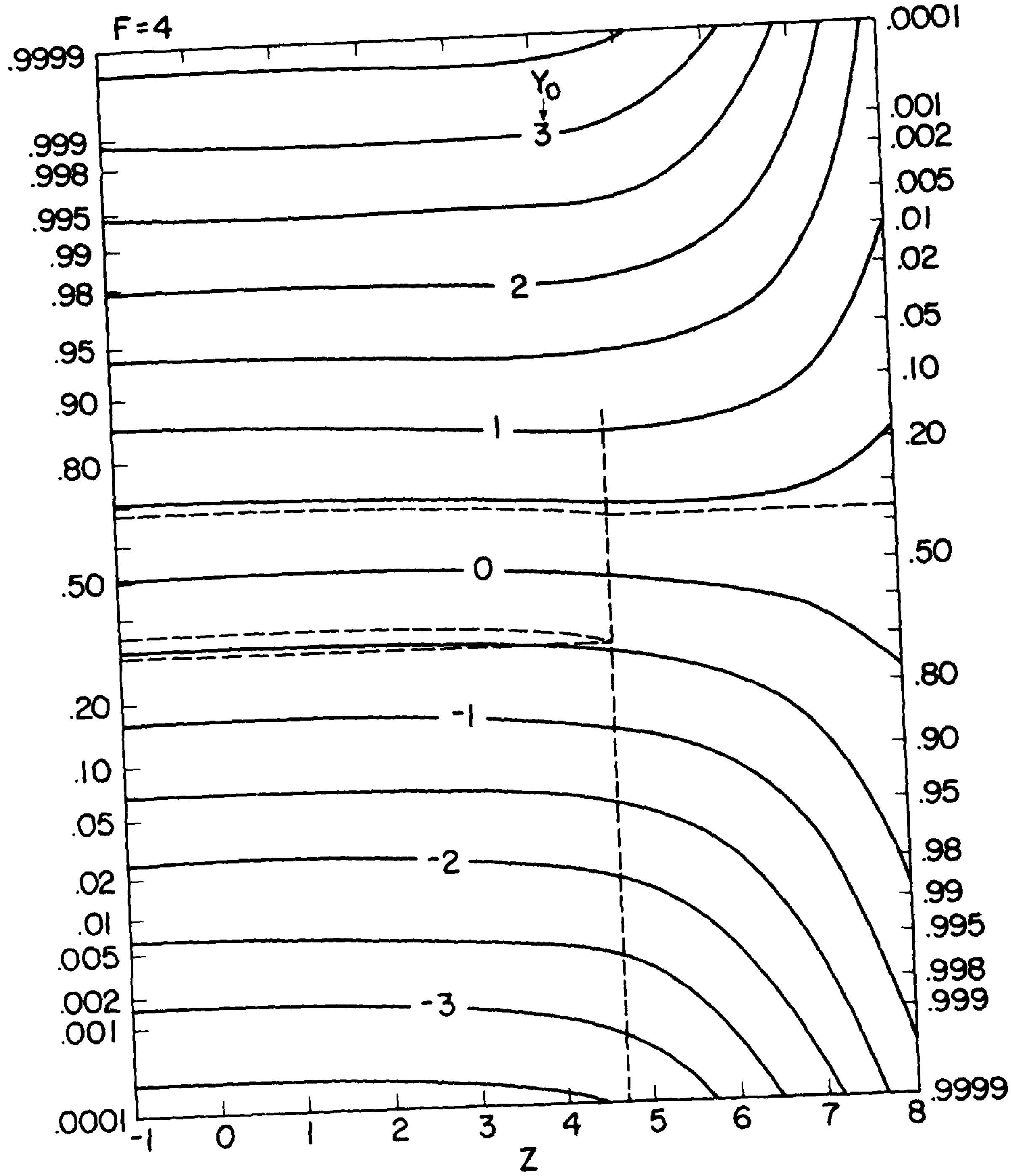

Figure 4(4). For the Graphical Solution of P (y_0, z, F), the Probability That an END (y) Will Exceed (y_0) Only in (F/10) or Less of the Area (A) When Scale Distance, r = 1. (This chart is for F = 4.) The horizontal scale is for the standardized measure (z) of the area [Eq. (21)]

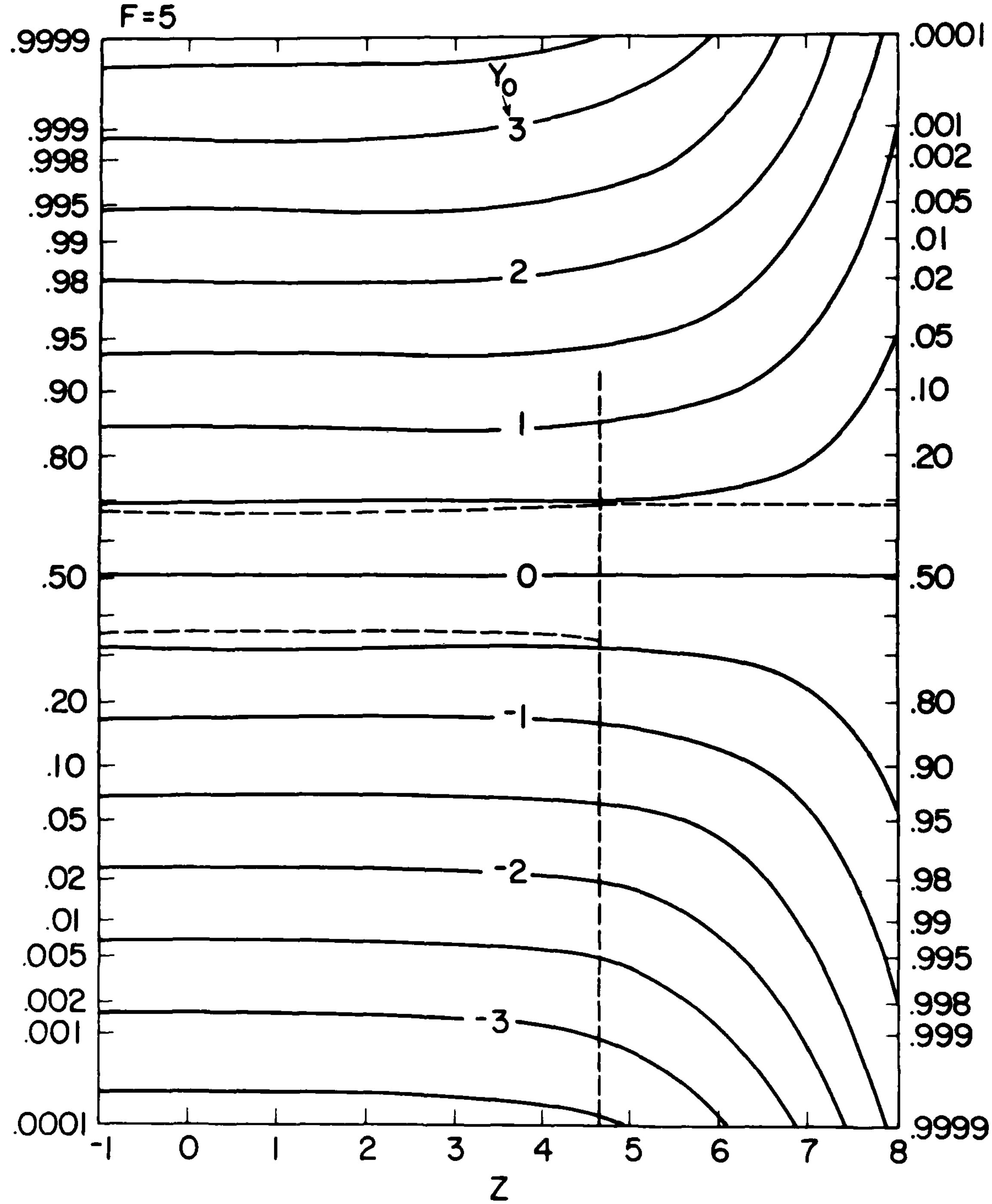

Figure 4(5). For the Graphical Solution of P (y_0, z, F), the Probability That an END (y) Will Exceed (y_0) Only in (F/10) or Less of the Area (A) When Scale Distance, r = 1. (This chart is for F = 5.) The horizontal scale is for the standardized measure (z) of the area [Eq. (21)]

2.4.3 CLOUD-COVER DISTRIBUTION BY GRAPHICAL SOLUTION

Begin with:	mean cloud cover:	P'_o
	scale distance:	r km
	area of floor space:	A km^2.

(For the sky dome, as the surface observer sees it, the floor space is assumed to have an area of A = 2424 km^2 equivalent to a circular area with a 15-nm radius.)

Find

$$z = \ln(\sqrt{A}/r)/\ln 2 .$$

Find the cumulative probability

$$P_o = 1 - P'_o \tag{25}$$

which is also the single-point probability of no cloud. Find its END, y_o. On each chart, for each F = 0(1)5, the intersection of the y_o-curve with the ordinate line through z, interpolated as necessary, gives the reading of the probability, PA (y_o, z, F).

For F = 6(1)10 we take advantage of mirror-image symmetry. Enter Figure 4(4) down to Figure 4(0) respectively, and find the intersection of the curve of ($-y_o$) with the ordinate line through z. Read the probability PA (y_o, z, F) on the right-hand scale.

Or, for F = 6(1)10 we can approach the problem as follows: look for the solution for PA ($-y_o$, A, (10-F), r) on the (10-F)-chart at the intersection of the ($-y_o$)-curve with the z-ordinate. Then,

$$PA(y_o, A, F, r) = 1 - PA(-y_o, A, (10-F), r) . \tag{26}$$

<u>Example 1</u>: Suppose we wish to find the probability that the single-point lower 10 percent of visibility is to be exceeded in no more than (2/10)ths of a region of area 250 km^2. The scale parameter is given (say, r = 2 km). Then

$$z = \ln(\sqrt{A}/r)/\ln 2 = 2.98 ,$$

$$P_o = 0.1, \text{ for which } y_o\ -1.28,$$

and

$$F = 2 .$$

Entering Figure 4(2), (the dashed curves and lines are for the next example) and following the curve for $y_o = -1.28$ to the intersection with $z = 2.98$, we obtain, on the left-hand scale, the probability

$$PA(y_o, A, F, r) = 0.082 .$$

That is, the answer is 8.2 percent, only slightly lower than the basic single-point probability. If this visibility were not to be exceeded anywhere in the 250 km^2 area the answer [from Figure 4(0)] would have been 6 percent.

Example 2: For Hanscom AFB, MA the RUSSWO[13] gives a mean sky cover of 0.66 in January, noontime, assume for this example, that the scale distance $r = 1.9$ km then,

$$P'_o = 0.66, \; P_o = 0.34, \; y_o = -0.41 \; .$$

For the sky cover, the area A is taken to be 2424 km^2 which, together with $r = 1.9$ km, gives

$$z = \ell n(\sqrt{2424}/1.9)/\ell n2 = 4.70 . \tag{27}$$

By finding the curve $y_o = -.041$ in each Figure 4(0) to Figure 4(5) successively, and following the curve until $z = 4.70$, the probabilities of all clear, 1/10 cover, ---, (5/10)ths cover were found, as shown in the column for estimates by graph (Table 2). By entering Figures 4(4) down to Figure 4(0) successively at $y_o = 0.41$ and following each curve until $z = 4.70$, the probabilities of 6/10 to full overcast were found on the righthand scale, as shown in the rest of Table 2. The RUSSWO figures are shown for comparison. The broken lines for this example have been plotted in Figures 4(0) to Figure 4(5).

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* RUSSWO - Revised Uniform Summaries of Surface Weather Observations.

Table 2. Frequency Distribution of Sky Cover at Hanscom AFB, MA, January, Noontime, as Given in the RUSSWO, and as Estimated by the 3D-BSW Model. The mean sky cover is $P_o = 0.66$, scale distance, $r = 1.9$ km. The sky-dome area is assumed to be $A = 2424$ km^2, giving $z = 4.70$

			3D-BSW Estimates		
Sky Cover	F	RUSSWO	By Graph		By Computer (Appendix B)
		Freq.		Freq.	Freq.
Clear	0	0.155	4(0)	0.140	0.127
1/10	1	0.198	4(1)	0.190	0.191
≦ 2/10	2	0.247	4(2)	0.230	0.230
≦ 3/10	3	0.288	4(3)	0.262	0.266
≦ 4/10	4	0.329	4(4)	0.297	0.296
≦ 5/10	5	0.360	4(5)	0.325	0.330
≦ 6/10	6	0.387	4(4)	0.375	0.366
≦ 7/10	7	0.431	4(3)	0.400	0.401
≦ 8/10	8	0.491	4(2)	0.440	0.446
≦ 9/10	9	0.537	4(1)	0.51	0.499
≦ 10/10	10		4(0)	0.61	0.605
Overcast		0.462		0.39	0.395

2.4.4 CLOUD COVER DISTRIBUTION BY ALGORITHM AND COMPUTER SOLUTION

Algorithms, previously published[4] with the 2D-BSW model, were found to approximate the answers of the 3D-BSW model with rmse = 0.005, bias -0.003. If limited to $z \leqq 7$, the bias is as low as -0.001. This high degree of accuracy permits us to use the previous algorithms (Appendix B) for the probability of areal coverage, PA (y_o, A, F, r).

Example 2 (Contd.): Table 2 shows, in the last column, the results of solution by computer (following the technique described in Appendix B). The values are close to the values by graphical solution. (The graphical values might be read slightly differently by another user of the graphs.)

The difference in the estimates (by graphs or by algorithms) from the RUSSWO frequencies can be party explained by the fact that the RUSSWO categories include up to 5 percent additional cover, except for the overcast that includes breaks of 5 percent. The model, on the other hand, gives probability of strictly clear, and strictly overcast without breaks. For the same reason each partial cover should appear slightly less, in cumulative frequency, than in the RUSSWO. This effect is clearly illustrated in Figure 5. The latter shows

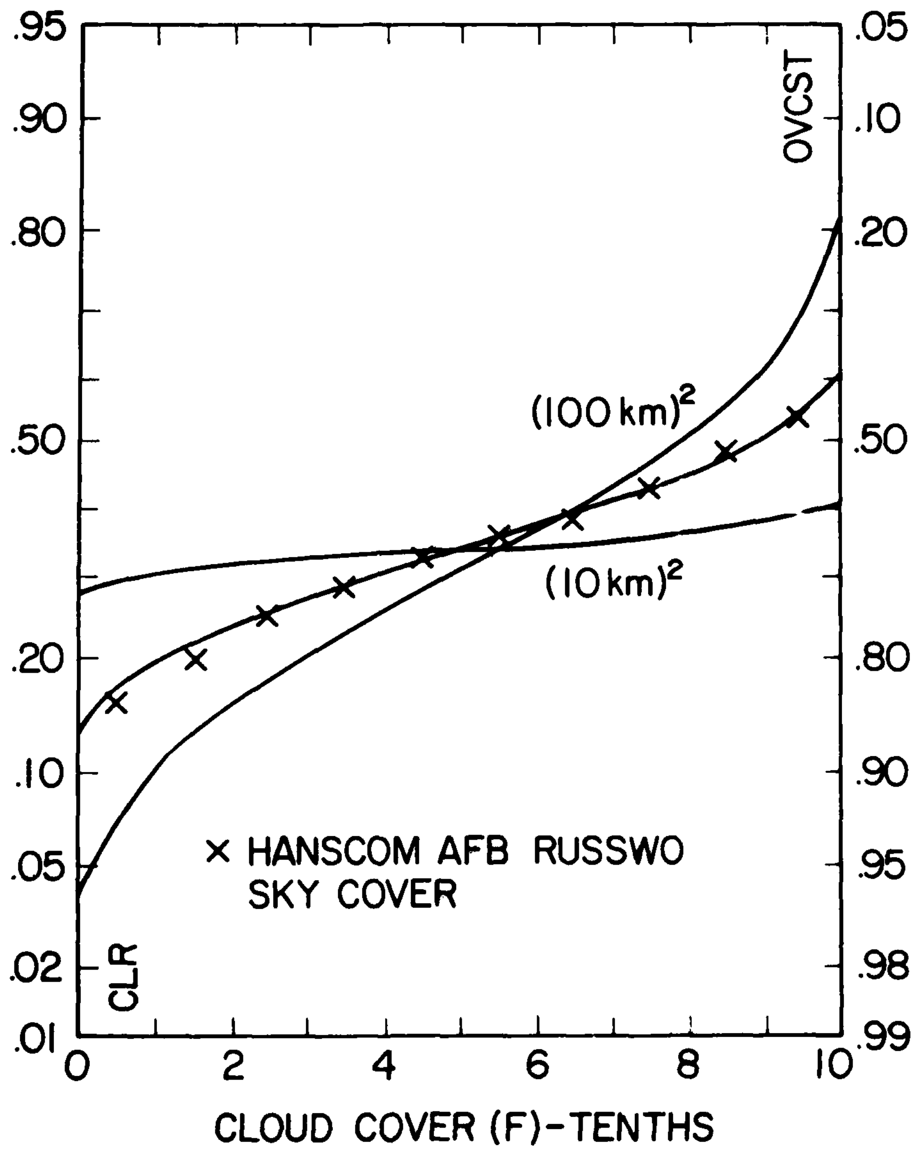

Figure 5. A Plot of the Cumulative Probability, PR($\leq$ F), vs Cloud Cover ($0 \leq F \leq 10$) in Tenths. The X's are plotted values from the RUSSWO for Hanscom AFB, January, noontime, with mean 0.66. With scale distance determined at r = 1.9 km, the 3D-BSW model provided the three curves for areas (10 km)^2, 2424 km^2 (sky cover) and (100 km)^2

the plot of the RUSSWO sky-cover frequencies (at the X's), the 3D-BSW model probability curve for the sky cover of area 2424 km^2, and, in addition, the model curves for cloud cover over areas of 100 km^2 and 10^4 km^2. The additional two curves are estimates without benefit of verification.

2.5 Line Coverage (PL)

The probability distributions of conditions along a line, or fraction of a line, were obtained, together with algorithms, for solution by the previous 2D-BSW model. This exercise is not repeated for the 3D-BSW model, since a more important goal takes precedence, as described in the following section. For an algorithm solution of PL(y_o, s', F, r), (the probability that y_o will be exceeded in no more than (F/10)ths of the line of length (s')) based on results with the 2D-BSW model see Appendix C.

2.6 Line Intervals (PI)

The goal of this section is to estimate the probability, PI(X, s', T, r), that a condition (X) is the minimized maximum in a linear interval, s'(km), somewhere along a longer line of travel of length T(km). The given parameter is either the scale distance r(km) or the wavelength Λ(km) where $\Lambda = 256\,r$, in the 3D-BSW model.

In terms of s', T, and r, the following standardized terms are obtained:

$$s = s'/r$$

$$z = \ln s/\ln 2 \tag{28}$$

$$\omega = \ln (T/s')/\ln 2 .$$

With r = 1 or s' = s, the Monte Carlo exercise, previously described, was used to produce 25,000 maps (250,000 would have been better but prohibitive), each with 24 wave formations that produced a field of END- or y-values throughout the 3D space. For this exercise the y-values were determined along a line, arbitrarily chosen to be the V-axis (Figure 2). The line interval (s) was made equal to 2^z for z -1(1)8. The overall line of travel (T) was made equal to $2^{z+\omega}$, $\omega = 0(1)(10\text{-}z)$, so that the length of travel (T) varied from a minimum equal to one-half of the single-unit interval to a maximum of 1024 units of distance. The y-values were generated along the line, equally spaced at small intervals:

$$\delta s = s/N . \tag{29}$$

The number N was arbitrarily set at 20, so that y-values were determined at 21 points along the length of (s).

The interval (s) was moved along the line (T) to find the lowest maximum in that interval. This was done 25,000 times, yielding a frequency distribution of y (minmax) for each combination of s and T, or z and ω.

Tables 3(-1) to 3(8) show the results for the frequencies that approximate the probability distribution, $PI(y_o, z, \omega)$, applicable to s' and T when the scale distance (r) is given, thus:

$$\begin{aligned} s' &= r \cdot 2^{z} && \text{for } z = -1(1)8 \\ T &= r \cdot 2^{z+\omega} && \text{for } \omega = 0(1)(10-z) \end{aligned} \tag{30}$$

2.6.1 BY GRAPHICAL SOLUTION

Figures 6(-1) to 6(8) show the same results, except that the curves have been smoothed as deemed desirable, especially for the very high and very low probabilities. Each figure is for a single value of z. In practice there would need to be a system of interpolation between the values in the tables or figures.

2.6.2 BY ALGORITHM AND COMPUTER SOLUTION

While the figures could be used directly, it is much more desirable, in fact imperative, to compose algorithms for the estimates of probabilities. This was done, with results as contained in Appendix D. The basic equation is of the form: $y = A(z, \omega) + B(z, \omega) \cdot y_o + C(z, \omega) \cdot y_o^2$. After the y_o - curves were drawn of P vs ω, for each Z = -1(1)8 they were extrapolated beyond (10-Z) to (12-Z), by eye, to give a useful extension of the model's application. The additional values of A (i, j), B(i, j), C (i, j) are included in the tables (Appendix D). Example: Suppose that a cloud-free interval of 1 km is sought in a flight track of length of 50 km, when the sky cover (f) is observed to be (9/10)ths. The Stanford Research Institute (SRI)[7] formula for a cloud-free line of sight (CFLOS) overhead gives

$$P_n = 1 - f(1 + 3f)/4 = 0.168 .$$

Accepting this as the single-point probability (P_o), then $y_o = -0.96$.

Suppose, further, that the parameter value is r = 0.109 km, or Λ = 27.6 km (see below). Using Appendix D, with this value of r, together with s' = 1 km, T = 50 km, for which we obtain z = 3.21, ω = 5.64, the solution to the algorithms gives

$$PI(s' = 1 \text{ km}, T = 50 \text{ km}) = 0.84 .$$

That is, there is an 84 percent probability of having a 1-km window of observation through the clouds on a 50-km track, when the mean cloud cover is (9/10)ths.

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7. Allen, J.H., and Malick, J.D. (1983) The frequency of cloud-free viewing intervals, Twenty-first Aerospace Science Meeting, 10-13 January 1983, Reno, NV., Copyright AIAA Inc.

Table 3(-1). Estimate of the Probability Distribution, $Pr(y_o, z = -1, \omega)$, of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z - 1/2$) Over a Total Line of Travel ($T = 2^{\omega + z} - 1/2$ to 1024 units)

P_o	y_o	ω											
		0	1	2	3	4	5	6	7	8	9	10	11
0.000032	-4.0								0.00013	0.00022	0.00040	0.00078	0.0016
0.000233	-3.5	0.000235	0.000245	0.00025	0.00029	0.00033	0.00045	0.00065	0.00100	0.0017	0.0030	0.0062	0.0130
0.00135	-3.0	0.00145	0.00146	0.00148	0.00150	0.0018	0.0025	0.0039	0.0062	0.011	0.020	0.037	0.071
0.00621	-2.5	0.00622	0.0063	0.0065	0.0070	0.0080	0.015	0.0160	0.0245	0.042	0.078	0.140	0.250
0.02275	-2.0	0.023	0.024	0.0245	0.027	0.030	0.040	0.055	0.083	0.138	0.238	0.400	0.600
0.06681	-1.5	0.0670	0.0675	0.0679	0.073	0.085	0.100	0.138	0.200	0.322	0.500	0.722	0.890
0.1587	-1.0	0.160	0.161	0.163	0.173	0.194	0.228	0.290	0.408	0.570	0.772	0.923	0.9855
0.3085	-0.5	0.310	0.312	0.315	0.330	0.360	0.402	0.480	0.612	0.767	0.912	0.982	0.9950
0.5000	0	0.510	0.518	0.520	0.535	0.555	0.600	0.670	0.780	0.900	0.977	0.9982	-
0.6915	0.5	0.695	0.698	0.700	0.712	0.735	0.775	0.831	0.906	0.972	0.9974	-	-
0.8413	1.0	0.842	0.843	0.845	0.855	0.870	0.896	0.933	0.971	0.9952	0.99984	-	-
0.93319	1.5	0.934	0.935	0.938	0.9405	0.950	0.962	0.979	0.9943	0.99952	-	-	-
0.97725	2.0	0.978	0.9785	0.979	0.980	0.9831	0.9900	0.9956	0.99915	-	-	-	-
0.9938	2.5	0.9939	0.9940	0.9941	0.9946	0.9959	0.9979	0.99935	-	-	-	-	-
0.99865	3.0	0.99868	0.99869	0.99872	0.9989	0.99930	0.99969	-	-	-	-	-	-
0.999767	3.5	0.99977	0.99978	0.99980	0.99984	-	-	-	-	-	-	-	-

Table 3(0). Estimates of the Probability Distribution, PI (y_o, z= 0, ω), of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z = 1$) Over a Total Line of Travel (T= 1 to 1024 units)

y_o	ω										
	0	1	2	3	4	5	6	7	8	9	10
-4.0	0.00010	0.00010	0.00010	0.00022	0.00025	0.00026	0.00046	0.00065	0.00102	0.00142	0.00230
-3.5	0.00030	0.00030	0.00038	0.00038	0.00054	0.00078	0.00174	0.00218	0.00418	0.00662	0.0131
-3.0	0.00145	0.00150	0.00154	0.00202	0.00238	0.00398	0.00698	0.0109	0.0200	0.0363	0.0654
-2.5	0.00562	0.00590	0.00642	0.00798	0.01046	0.01586	0.0241	0.0395	0.0738	0.138	0.242
-2.0	0.02350	0.02414	0.02590	0.02974	0.03758	0.05222	0.0768	0.124	0.218	0.374	0.590
-1.5	0.06494	0.06630	0.07085	0.0810	0.09990	0.13166	0.188	0.287	0.461	0.682	0.881
-1.0	0.15666	0.16298	0.17378	0.19290	0.22318	0.28002	0.368	0.514	0.733	0.9101	0.9869
-0.5	0.31206	0.31918	0.33206	0.35622	0.39986	0.47254	0.582	0.737	0.9091	0.9875	0.99974
0	0.50058	0.50802	0.52182	0.54554	0.59313	0.67030	0.775	0.895	0.9813	0.99882	0.99998
0.5	0.68846	0.69438	0.70646	0.72862	0.77190	0.83054	0.9022	0.9689	0.99770	0.99998	-
1.0	0.83550	0.83958	0.84906	0.86562	0.89338	0.92862	0.9669	0.99378	0.99994	-	-
1.5	0.93398	0.93578	0.94114	0.95022	0.95974	0.9764	0.99222	0.99942	0.99998	-	-
2.0	0.97614	0.97758	0.97966	0.98242	0.98774	0.99434	0.99866	0.99998	-	-	-
2.5	0.99414	0.99486	0.99530	0.99634	0.99774	0.99922	0.99990	-	-	-	-
3.0	0.99910	0.99910	0.99926	0.99934	0.99974	0.99982	0.99998	-	-	-	-
3.5	0.99990	0.99990	0.99990	0.99990	0.99998	0.99998	-	-	-	-	-
4.0	0.99998	0.99998	0.99998	0.99998	-	-	-	-	-	-	-

Table 3(1). Estimates of the Probability Distribution, $PI(y_o, z = 1, \omega)$, of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z = 2$) Over a Total Line of Travel ($T = 2^{z+\omega} = 2$ to 1024 units)

y_o	ω									
	0	1	2	3	4	5	6	7	8	9
-4.0	-	-	-	-	-	0.00018	0.00026	0.00066	0.00114	0.00234
-3.5	0.00010	0.00014	0.00034	0.00046	0.00066	0.00134	0.00202	0.00338	0.00614	0.0125
-3.0	0.00138	0.00138	0.00182	0.00226	0.00350	0.00622	0.0100	0.0192	0.0345	0.0645
-2.5	0.00570	0.00630	0.00862	0.0110	0.0153	0.0221	0.0380	0.0703	0.130	0.237
-2.0	0.0235	0.0251	0.0286	0.0362	0.0489	0.0719	0.115	0.207	0.358	0.571
-1.5	0.0645	0.0675	0.0772	0.0954	0.126	0.183	0.277	0.449	0.666	0.871
-1.0	0.156	0.166	0.186	0.218	0.272	0.360	0.504	0.719	0.9003	0.9866
-0.5	0.306	0.319	0.344	0.385	0.459	0.568	0.725	0.9031	0.9855	0.99978
0	0.496	0.509	0.534	0.582	0.658	0.767	0.888	0.9790	0.99914	0.99998
0.5	0.685	0.697	0.719	0.761	0.821	0.897	0.9661	0.99742	0.99998	-
1.0	0.833	0.843	0.860	0.889	0.9233	0.9648	0.99394	0.99994	-	-
1.5	0.9287	0.9333	0.9440	0.9562	0.9738	0.99110	0.99916	0.99998	-	-
2.0	0.9752	0.9773	0.9802	0.9860	0.99346	0.99790	0.99994	-	-	-
2.5	0.99366	0.99482	0.99598	0.99770	0.99914	0.99970	0.99998	-	-	-
3.0	0.99842	0.99878	0.99914	0.99960	0.99974	0.99998	-	-	-	-
3.5	0.99970	0.99970	0.99970	0.99974	0.99994	-	-	-	-	-
4.0	0.99998	0.99998	0.99998	0.99998	0.99998	-	-	-	-	-

Table 3(2). Estimates of the Probability Distribution, $PI(y_o, z = 2, \omega)$, of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z = 4$) Over a Total Line of Travel ($T = 2^{z+\omega} = 4$ to 1024 units)

y_o	ω								
	0	1	2	3	4	5	6	7	8
-4.0	-	-	-	-	0.00014	0.00018	0.00050	0.00090	0.00170
-3.5	-	0.00018	0.00026	0.00054	0.00098	0.00150	0.00254	0.00514	0.00986
-3.0	0.00114	0.00142	0.00202	0.00278	0.00494	0.00846	0.0166	0.0305	0.0573
-2.5	0.00530	0.00738	0.00926	0.0132	0.0197	0.0345	0.0641	0.119	0.217
-2.0	0.0211	0.0238	0.0300	0.0424	0.0649	0.107	0.193	0.337	0.545
-1.5	0.0609	0.0672	0.0855	0.115	0.170	0.259	0.424	0.642	0.857
-1.0	0.149	0.167	0.199	0.252	0.336	0.478	0.695	0.886	0.9835
-0.5	0.292	0.318	0.361	0.433	0.544	0.705	0.892	0.9832	0.9950
0	0.479	0.504	0.554	0.634	0.750	0.878	0.9753	0.99846	0.99998
0.5	0.672	0.695	0.742	0.807	0.885	0.9615	0.99694	0.99994	-
1.0	0.826	0.844	0.877	0.9153	0.9601	0.99302	0.99998	0.99998	-
1.5	0.9243	0.9339	0.9504	0.9706	0.9892	0.99898	-	-	-
2.0	0.9744	0.9784	0.9835	0.99246	0.99758	0.99982	-	-	-
2.5	0.99286	0.99474	0.99650	0.99890	0.99970	0.99998	-	-	-
3.0	0.99838	0.99890	0.99918	0.99970	0.99998	-	-	-	-
3.5	0.99970	0.99970	0.99974	0.99994	-	-	-	-	-
4.0	0.99998	0.99998	0.99998	0.99998	-	-	-	-	-

Table 3(3). Estimates of the Probability Distribution, PI (y_o, z = 3, ω), of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z = 8$) Over a Total Line of Travel ($T = 2^{z+\omega} = 8$ to 1024 units)

y_o	ω							
	0	1	2	3	4	5	6	7
-4.0	-	-	-	0.00006	0.00006	0.00014	0.00014	0.00050
-3.5	0.00002	0.00002	0.00054	0.00078	0.00086	0.00154	0.00354	0.00646
-3.0	0.00134	0.00190	0.00306	0.00462	0.00710	0.0125	0.0221	0.0423
-2.5	0.00414	0.00622	0.00922	0.0152	0.0267	0.0508	0.0963	0.178
-2.0	0.0184	0.0240	0.0340	0.0533	0.0902	0.162	0.289	0.484
-1.5	0.0535	0.0671	0.0945	0.141	0.224	0.374	0.585	0.823
-1.0	0.131	0.165	0.210	0.291	0.433	0.645	0.855	0.9772
-0.5	0.264	0.309	0.383	0.494	0.665	0.861	0.9740	0.99906
0	0.451	0.498	0.582	0.703	0.848	0.9672	0.99782	0.99998
0.5	0.643	0.689	0.765	0.857	0.9463	0.99530	0.99982	-
1.0	0.802	0.836	0.887	0.9443	0.9889	0.99966	0.99998	-
1.5	0.9132	0.9313	0.9591	0.9833	0.99778	0.99998	-	-
2.0	0.9707	0.9774	0.9895	0.99734	0.99990	-	-	-
2.5	0.99150	0.99378	0.99790	0.99970	0.99998	-	-	-
3.0	0.99798	0.99870	0.99970	0.99982	-	-	-	-
3.5	0.99982	0.99982	0.99998	0.99998	-	-	-	-
4.0	0.99998	0.99998	-	-	-	-	-	-

Table 3(4). Estimates of the Probability Distribution, PI (y_o, z = 4, ω), of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z = 16$) Over a Total Line of Travel ($T = 2^{z+\omega} = 16$ to 1024 units)

y_o	ω						
	0	1	2	3	4	5	6
-4.0	-	-	0.00006	0.00006	0.00014	0.00014	0.00042
-3.5	-	0.00010	0.00018	0.00026	0.00066	0.00154	0.00342
-3.0	0.00094	0.00186	0.00250	0.00454	0.00746	0.0128	0.0247
-2.5	0.00298	0.00530	0.00878	0.0168	0.0327	0.0611	0.114
-2.0	0.0135	0.0213	0.0355	0.0624	0.118	0.212	0.377
-1.5	0.0409	0.0605	0.0977	0.162	0.289	0.485	0.731
-1.0	0.105	0.149	0.224	0.351	0.553	0.782	0.9507
-0.5	0.222	0.292	0.403	0.576	0.798	0.9502	0.99718
0	0.402	0.481	0.611	0.783	0.9416	0.99474	0.99998
0.5	0.601	0.678	0.797	0.9165	0.9899	0.99966	-
1.0	0.773	0.831	0.9133	0.9790	0.99906	0.99998	-
1.5	0.897	0.9286	0.9712	0.99586	0.99998	-	-
2.0	0.9633	0.9780	0.99434	0.99982	-	-	-
2.5	0.9897	0.99422	0.99914	0.99998	-	-	-
3.0	0.99754	0.99902	0.99978	-	-	-	-
3.5	0.99978	0.99994	0.99998	-	-	-	-
4.0	0.99982	0.99998	-	-	-	-	-

Tables 3(5) and 3(6). Estimates of the Probability Distribution, PI (y_o, z = 5 or 6, ω), of the Minimized Maximum (y_o) in a Linear Interval ($S = 2^z$ = 32 or 64) Over a Total Line of Travel ($T = 2^{z+\omega}$ = 32 or 64 to 1024 units)

y_o	ω(z = 5)						ω(z = 6)				
	0	1	2	3	4	5	0	1	2	3	4
-4.0	-	-	-	-	-	-	0.00006	-	-	-	-
03.5	-	0.00010	0.00018	0.00054	0.00078	0.00134	0.00006	-	-	-	-
-3.0	0.00018	0.00058	0.00138	0.00202	0.00378	0.00794	0.00006	-	0.00006	0.00006	0.00014
-2.5	0.00142	0.00286	0.00678	0.0138	0.0255	0.0493	0.00030	0.00094	0.00162	0.00278	0.00686
-2.0	0.00762	0.0148	0.0289	0.0558	0.108	0.201	0.00138	0.00502	0.0122	0.0249	0.0506
-1.5	0.0235	0.0451	0.0867	0.166	0.312	0.521	0.00762	0.0243	0.0526	0.110	0.216
-1.0	0.0700	0.124	0.215	0.381	0.611	0.851	0.0313	0.0767	0.167	0.328	0.558
-0.5	0.166	0.261	0.415	0.645	0.870	0.9836	0.0976	0.202	0.389	0.643	0.880
0	0.328	0.446	0.639	0.862	0.9761	0.99954	0.228	0.389	0.647	0.886	0.99042
0.5	0.526	0.649	0.829	0.9653	0.99866	0.99998	0.417	0.605	0.855	0.9834	0.99970
1.0	0.719	0.818	0.9419	0.99494	0.99990	-	0.632	0.798	0.9635	0.99866	0.99998
1.5	0.862	0.9221	0.9851	0.99974	0.99998	-	0.810	0.9175	0.99398	0.99994	-
2.0	0.9475	0.9738	0.99650	0.99998	-	-	0.9231	0.9717	0.99950	0.99998	-
2.5	0.9855	0.99486	0.99982	-	-	-	0.9774	0.99330	0.99998	-	-
3.0	0.99658	0.99902	0.99994	-	-	-	0.99482	0.99942	-	-	-
3.5	0.99970	0.99998	0.99998	-	-	-	0.99974	0.99998	-	-	-
4.0	0.99982	-	-	-	-	-	0.99990	-	-	-	-

Tables 3(7) and 3(8). Estimates of the Probability Distribution, $PI(y_0;\ z = 7 \text{ or } 8,\ \omega)$, of the Minimized Maximum (y_0) in a Linear Interval $(S = 2^z = 128 \text{ or } 256)$ Over a Total Line of Travel $(T = 2^{z+\omega} = 128 \text{ or } 256 \text{ to } 1024 \text{ units})$

y_0	$\omega(z = 7)$				$\omega(z = 8)$		
	0	1	2	3	0	1	2
-4.0	0.00006	-	-	-	-	-	-
-3.5	0.00006	-	-	-	-	-	-
-3.0	0.00006	-	-	-	-	-	-
-2.5	0.00006	-	0.00002	0.00018	-	-	-
-2.0	0.00038	0.00102	0.00174	0.00306	-	-	0.00006
-1.5	0.00098	0.00482	0.0123	0.0260	-	-	0.00010
-1.0	0.00694	0.0256	0.0691	0.145	0.00046	0.00186	0.00366
-0.5	0.0327	0.103	0.248	0.452	0.00374	0.0146	0.0399
0	0.116	0.276	0.552	0.824	0.0249	0.0901	0.211
0.5	0.271	0.523	0.838	0.9810	0.109	0.297	0.581
1.0	0.492	0.755	0.9687	0.99958	0.305	0.610	0.896
1.5	0.720	0.9050	0.99686	0.99998	0.580	0.856	0.99074
2.0	0.880	0.9707	0.99978	-	0.815	0.9648	0.999[illegible]2
2.5	0.9625	0.99318	0.99998	-	0.9410	0.99222	0.99998
3.0	0.99162	0.99886	-	-	0.9858	0.99830	-
3.5	0.99934	0.99982	-	-	0.99822	0.99978	-
4.0	0.99982	0.99994	-	-	0.99950	0.99998	-

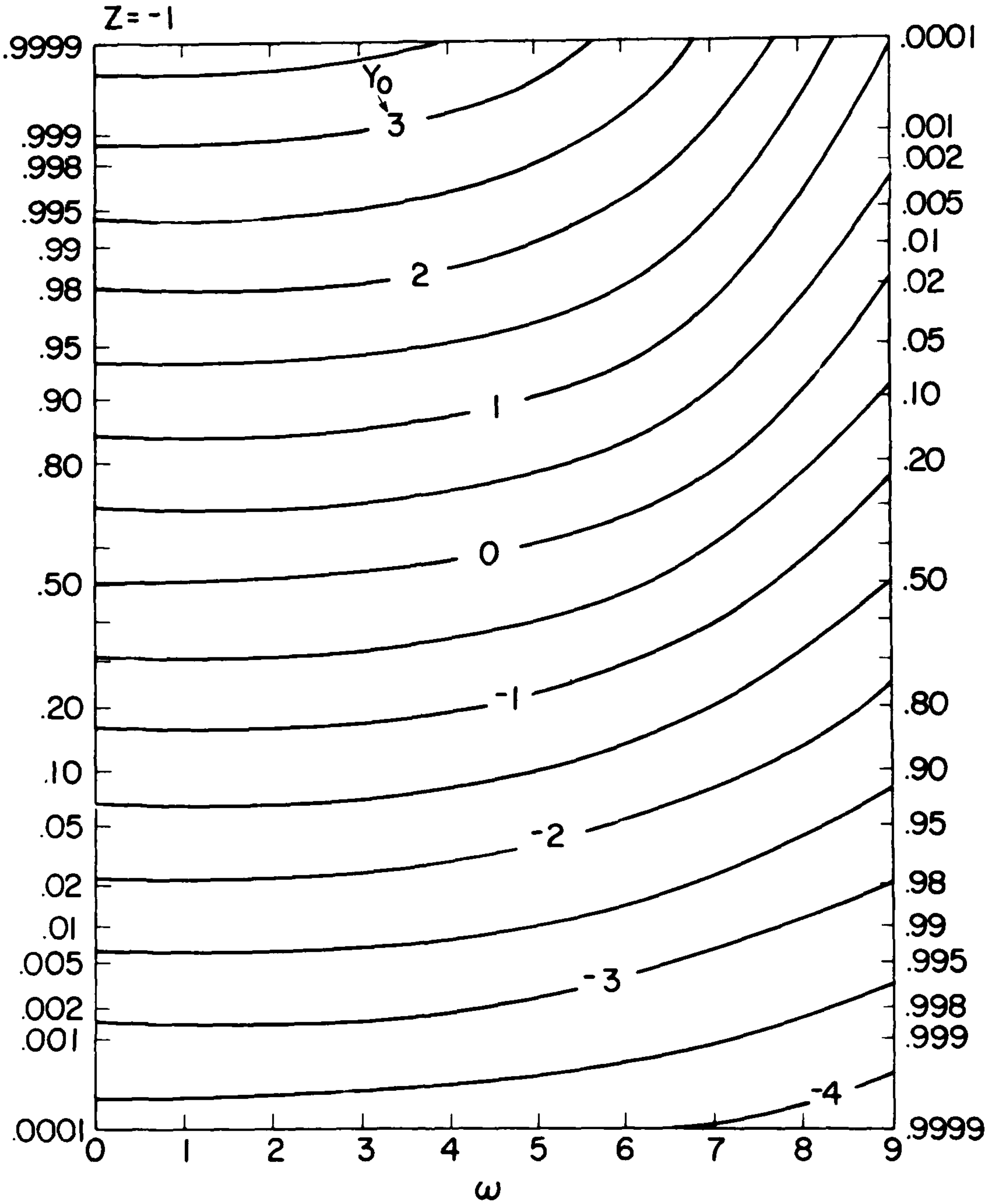

Figure 6(-1). The Graphical Solution of $P(y_0, z, \omega)$, the Probability That (y_0) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r (km).
This chart is for $z = -1$: T ranges from 1/2 to 1024 standardized units

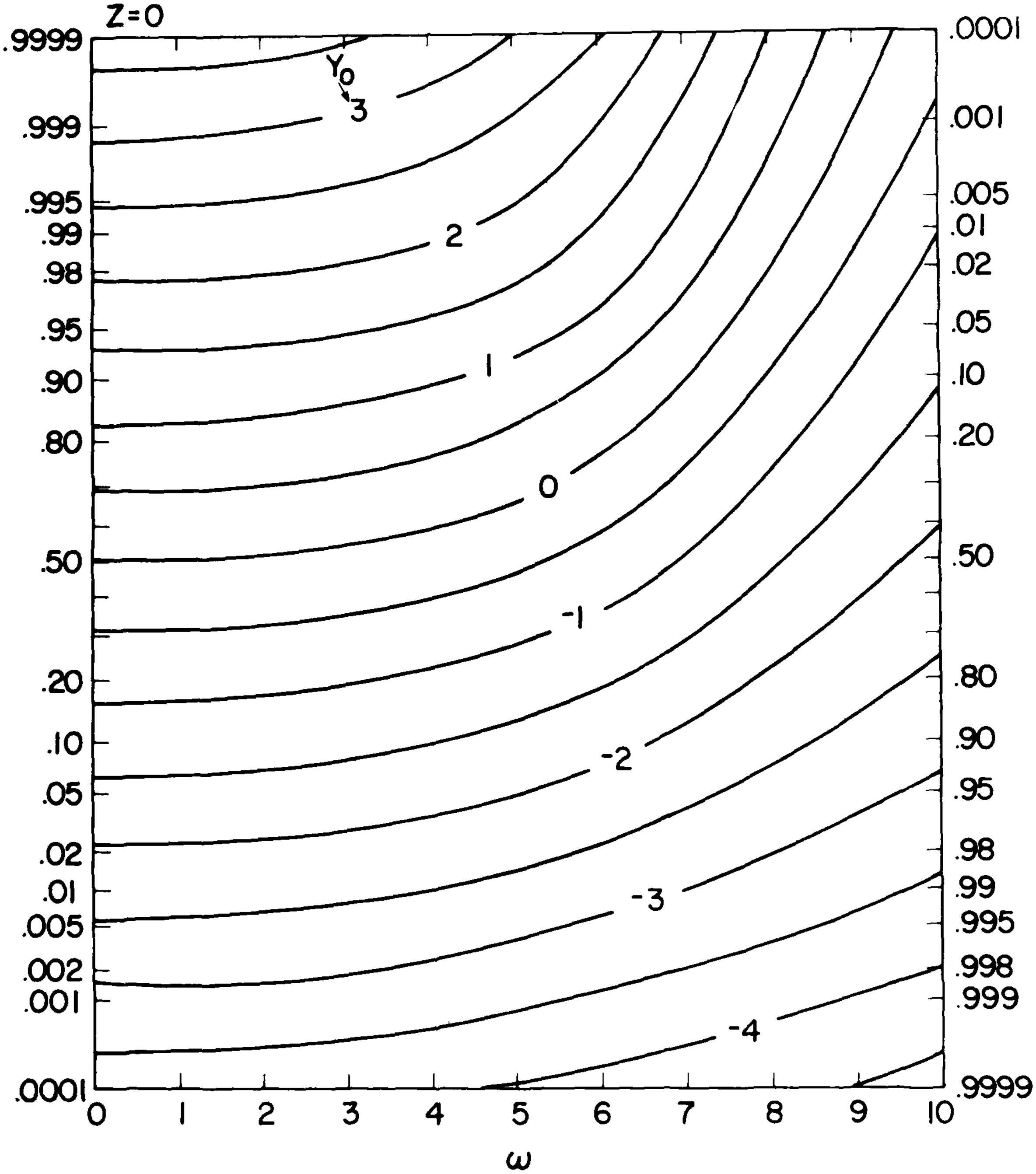

Figure 6(0). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r(km).
This chart is for z = 0: T ranges from 1 to 1024 units

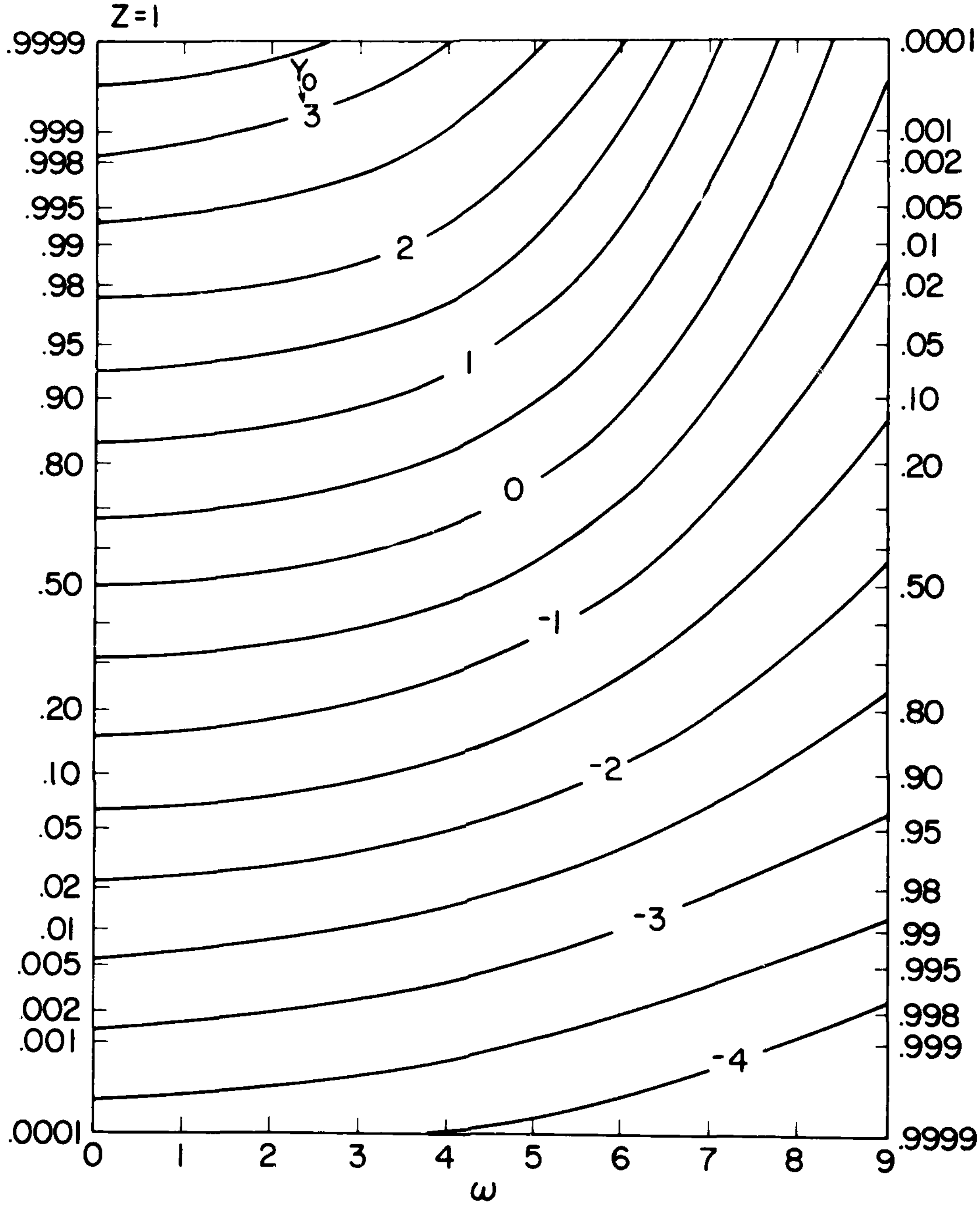

Figure 6(1). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r(km).
This chart is for z = 1: T ranges from 2 to 1024 units

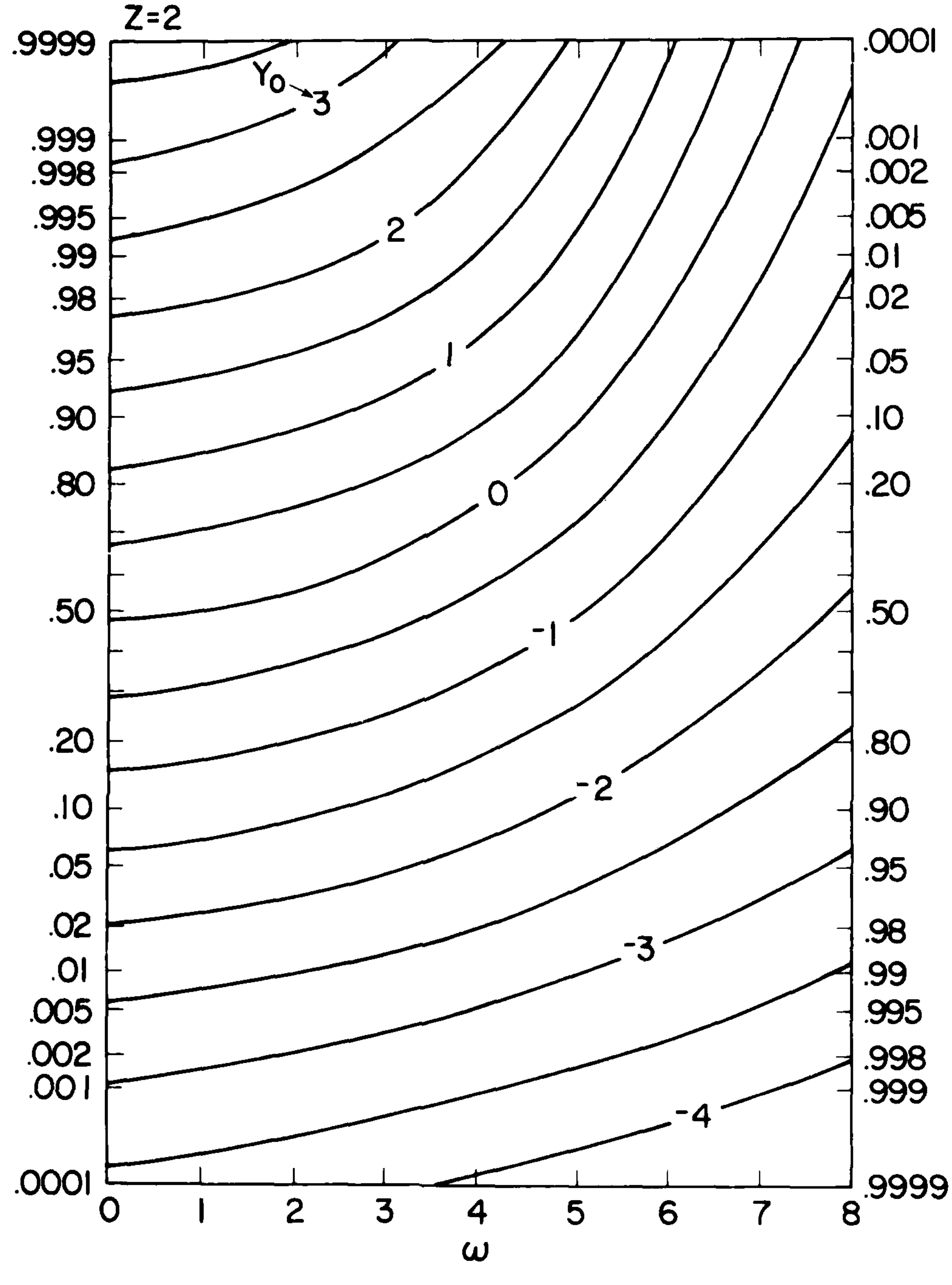

Figure 6(2). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r(km).
This chart is for z = 2: T ranges from 4 to 1024 units

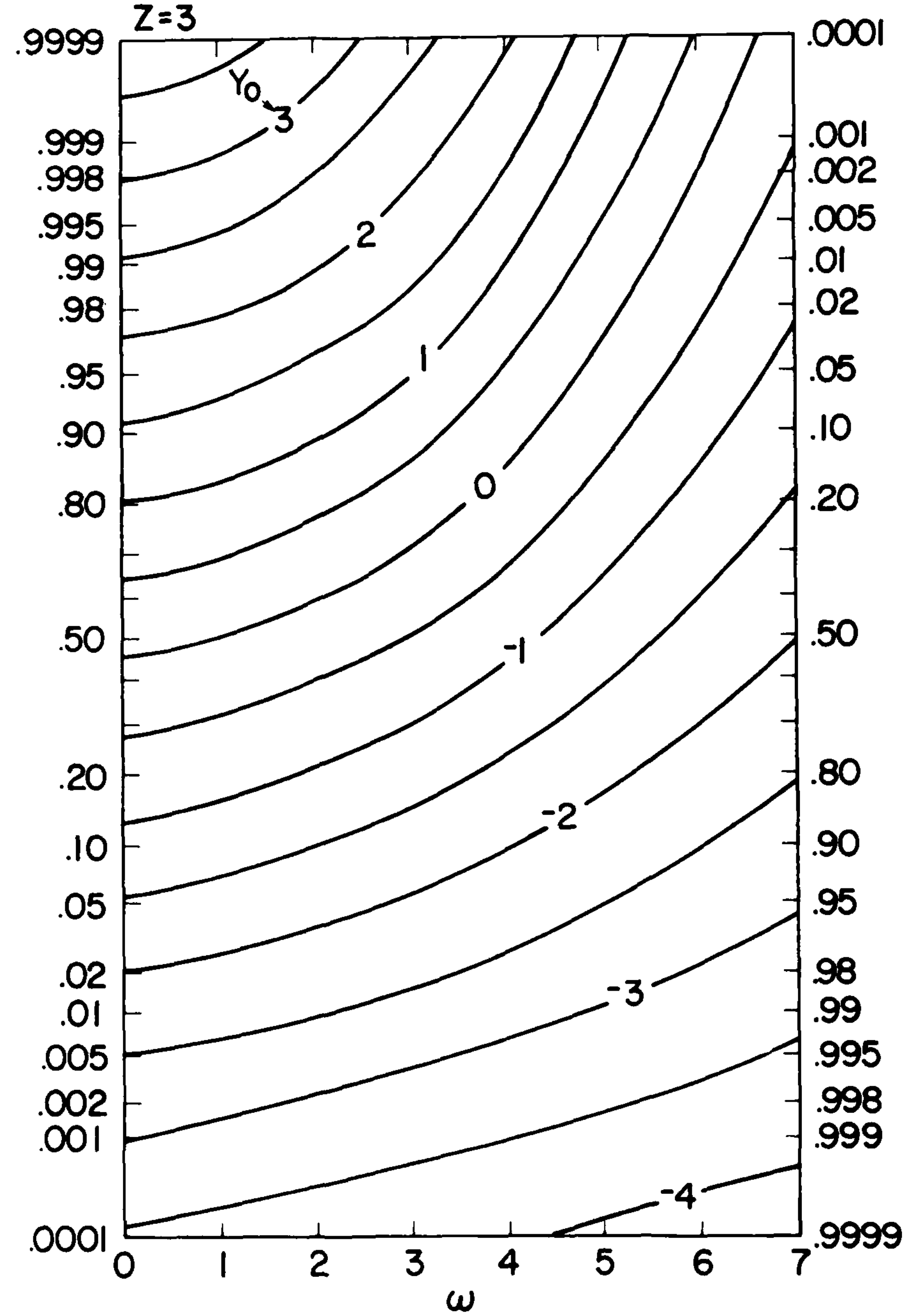

Figure 6(3). The Graphical Solution of P (y_o, z, ω), the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r(km).
This chart is for z = 3: T ranges from 8 to 1024 units

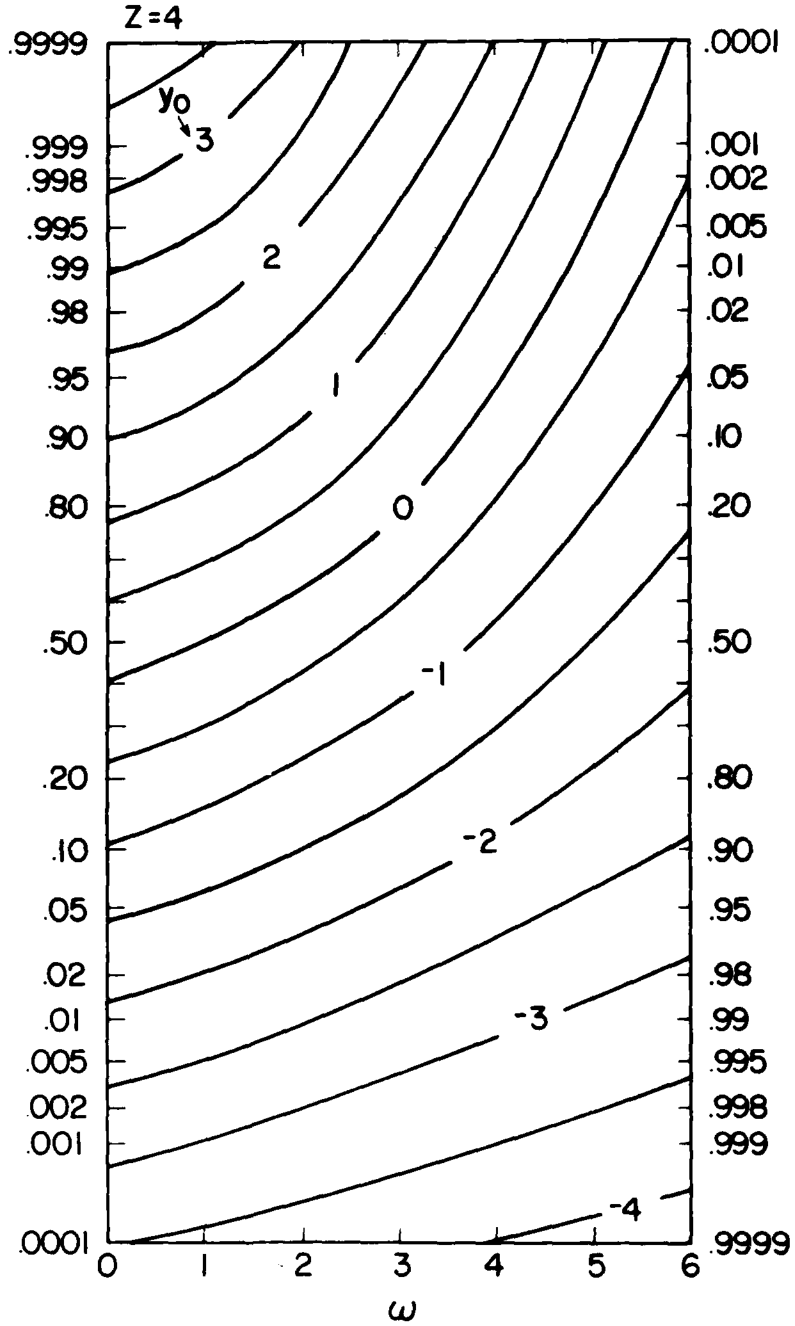

Figure 6(4). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r(km).
This chart is for z = 4

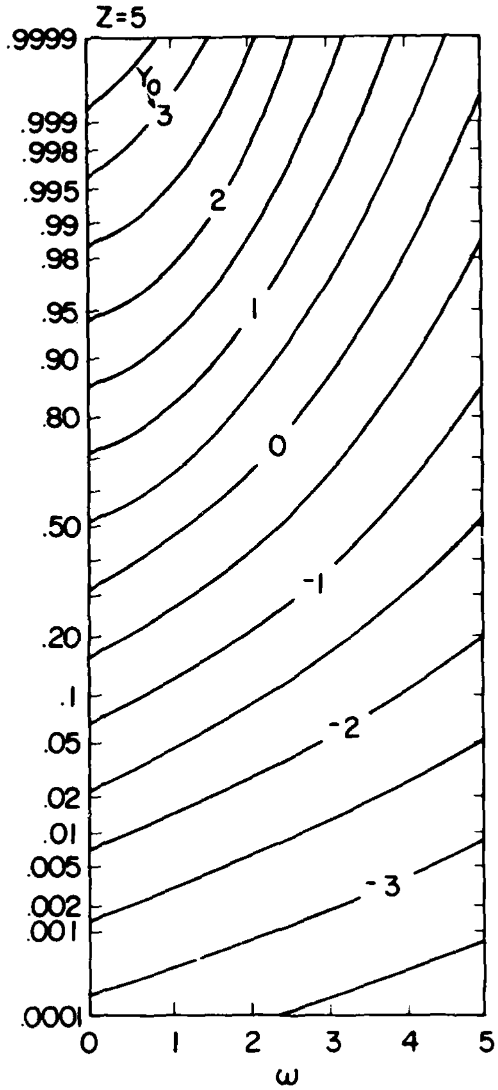

Figure 6(5). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r (km). This chart is for z = 5

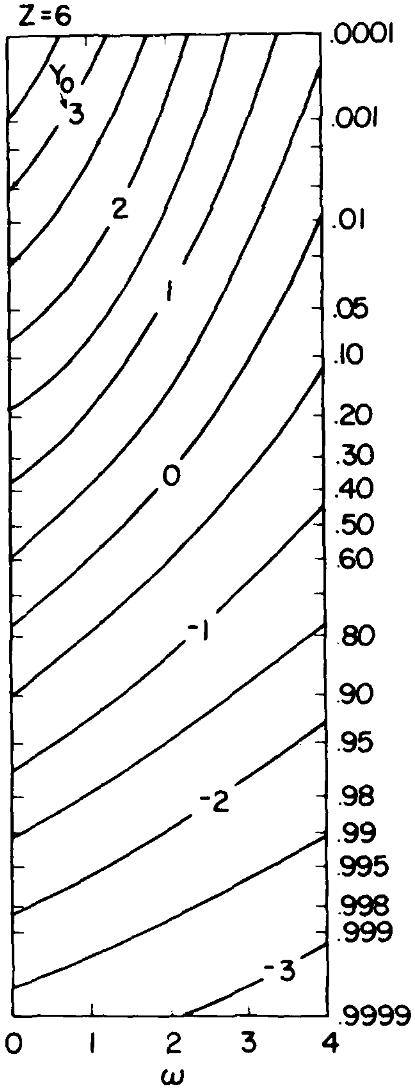

Figure 6(6). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r (km). This chart is for z = 6

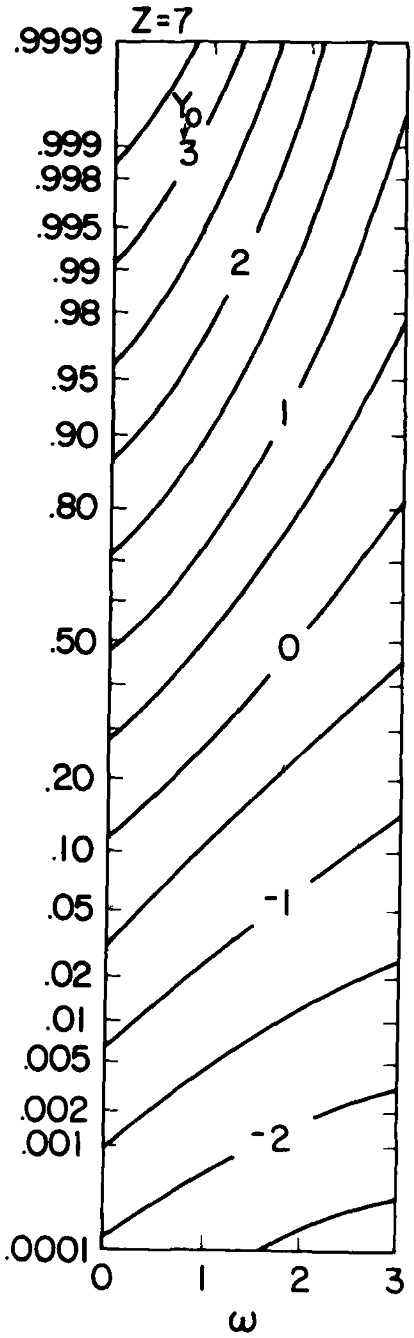

Figure 6(7). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r (km).
This chart is for z = 7

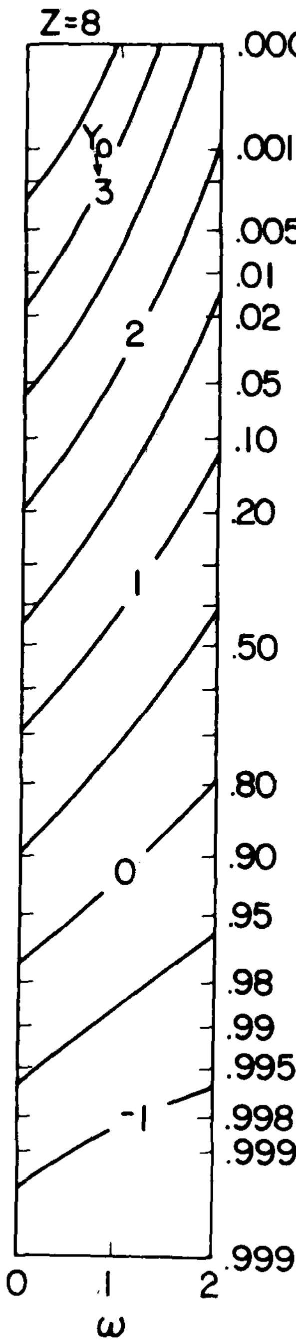

Figure 6(8). The Graphical Solution of $P(y_o, z, \omega)$, the Probability That (y_o) is the Minimized Maximum of y in a Line Interval s $(= 2^z)$ Throughout a Total Length of Travel $(T = 2^{z+\omega})$, Which Become Real Measures When Multiplied by the Scale Distance, r (km).
This chart is for z = 8

3. THE SRI ALTERNATIVE FOR LINE INTERVAL

Allen and Malick[7] of the Stanford Research Institute (SRI) proposed a model in which the mean length of a cloud-free interval (μ km) and the mean length of a cloudy interval (λ km) both depend on the probability of the cloud-free line-of-sight (PCFLOS). If P denotes PCFLOS and Q = 1 - P, then the SRI formula provides

$$\left.\begin{aligned} \mu(P) &= (14P + 1.7) \\ \lambda(P) &= \frac{Q}{P}(14P + 1.7) \end{aligned}\right\} \quad \text{for } P \leqq 1/2 \tag{31}$$

and

$$\left.\begin{aligned} \mu(P) &= \frac{P}{Q}(14Q + 1.7) \\ \lambda(P) &= (14Q + 1.7)\,. \end{aligned}\right\} \quad \text{for } P > 1/2$$

For a given length (s') within the track length (T), the SRI formula becomes

$$P'_s = 1 - \left\{1 - \exp(-s'/\mu)\right\}^{T/(\mu+\lambda)} \text{ for } T > (\mu+\lambda) \tag{32}$$

or

$$P'_s = P.\exp(-s'/\mu) + Q\left\{1 - \exp(-(T-s')/\lambda)\right\}.\exp(-s'/\mu) \text{ for } T < (\mu+\lambda)\,.$$

Malick, Allen and Zakanycz,[8] with satellite data on cloud-free and cloudy intervals, in the month of June, 1978, were able to plot a frequency of clear runs, for lengths between 1 and 11 km, cloud cover such that single point cloud-free probability is 0.27. We read the frequencies of run lengths, as shown (Table 4), and found the best estimates of the scale distance, for each run length in the 3D-BSW model, by using the model in successive trial-and-error steps. With the resulting geometric mean value of the scale distance (r = 0.109 km), the 3D-BSW model probabilities were estimated, to compare them with the plotted frequencies, as shown (Table 4).

8. Malick, J.D., Allen, J.H., and Zakanycz, S. (1979) Calibrated analytical modeling of cloud-free intervals. Proceedings of Conference SPIE, Vol. 195, Atmospheric Effects on Radiative Transfer (1979).

Table 4. Readings, From an SRI Diagram,[8] of the Frequency of Cloud-free Run Length, When the Single-point Probability of No Cloud is 0.27. Included also are estimates of corresponding scale distance in the 3D-BSW model, and the latter's probability estimates

Run Length (km)	Frequency (SRI)	Scale Distance r (km)	3D-BSW Probability Estimate r = 0.109 km
1	0.27	0.50	0.22
2	0.25	0.60	0.18
3	0.14	0.10	0.15
4	0.12	0.11	0.12
5	0.06	0.07	0.10
6	0.06	0.08	0.086
7	0.025	0.06	0.075
8	0.025	0.06	0.051
9	0.025	0.07	0.051
10	0.02	0.07	0.043
11	0.015	0.07	0.036
	Geometric mean = 0.109		

The greatest departure of estimate is for 1-km and 2-km run lengths. Actually, since the single-point probability is taken to be 0.27, the 1-km cloud-free run should be significantly less, but was not recorded that way, since the pixels in the satellite pictures, in 1-km squares, were treated as the smallest elements.

3.1 Example of SRI Alternative

If, as in the example of Section 2.6.2 with PCFLOS = 0.168, we want to find the probability of a 1-km clear interval in track of 50 km, the SRI formula gives

$\mu = 4.05$ km

$\lambda = 20.10$ km

$PI(s' = 1 \text{ km}, T = 50 \text{ km}) = 0.96$.

That is, the SRI formula estimates 96 percent probability of a 1-km interval in 50 km, compared with the 84 percent probability of the 3D-BSW model.

3.2 Comparing the SRI and 3D-BSW Procedures

First, the number of parameters used to characterize a cloud cover differs between SRI and 3D-BSW. In the SRI formula the amount of cloud cover is essentially the only parameter. In the 3D-BSW model there is the additional parameter of scale distance, which is found to vary with time of day and time of year, as well as geographical location and altitude above the ground. Eventually scale distance may be found to vary with cloud types, although this has not yet been studied. The data used by SRI are best fitted by the 3D-BSW model when the scale distance is chosen to be 0.109 km, or wavelength 28 km. In previous work on clouds, rainfall and radar echoes, we have found scale distances of an order of magnitude greater. This has prompted a further comparison of results of the SRI model with the 3D-BSW model (Table 5). If the nature of the clouds, especially in horizontal persistence, as represented by the scale distance (r), should differ from the clouds sampled by SRI, then the estimated likelihood of seeing through breaks in the clouds could differ dramatically.

Second, the algorithmic solution for the SRI model is simpler than for the 3D-BSW model. However, once the software program is assembled for the latter procedure, it should make little difference to the computer.

Third, in deriving their equations, the SRI authors assumed independence of cloud cover between successive non-overlapping segments of the track. In the 3D-BSW model, on the other hand, a field of correlation is deliberately constructed into the model, with the consequent effect of reducing the likelihood of a clear run when the mean cloud cover is large (for example, 9/10). This effect is noticeable in Table 5.

Table 5. The Estimates of Probability of a Clear Interval of Length s' (km) Over a Track of 50 km, When the Sky Cover is 0.9, for Which CFLOS Overhead Probability is 0.168 (scale distance (r) is varied from 0.05 km to 5 km)

Interval Length s' (km)	SRI Model	3D-BSW Model			
		r = 0.05 km	r = 0.11 km	r = 1 km	r = 5 km
0.5	0.988	0.977	0.883	0.35	0.32
1	0.96	0.93	0.83	0.34	0.27
5	0.51	0.26	0.45	0.32	0.21
10	0.17	0.016	0.16	0.25	0.21
50 km	negligible	negligible	negligible	0.05	0.13

4. PARAMETER DETERMINATION

The 3D-BSW model depends basically on two parameters. In this sense it resembles most statistical models, in which the most likely parameters are mean and standard deviation. In the 3D-BSW model, the mean is used but standard deviation is replaced by the scale distance (r) or its alternate the wavelength (Λ).

While the mean or single-point probability is relatively easy to obtain, the scale distance usually takes considerable effort that must make the most of climatological records. Accordingly, an in-depth description of how these parameters are obtained with the 3D-BSW model, is postponed to a later report.

A description of parameter determinations can be found using the 2D-BSW model.[4] Gringorten's[9] report on cloud distributions in the vertical finds scale distance at Bedford, MA sky cover, ranging from 0.5 km at noon in summer to 10 km at midnight in winter. World-wide, Burger's[10] atlas of sky cover presents similar values, which show considerable variation with geography. An earlier paper showed that scale distance is of the order of 1 to 5 km for radar echoes, 4 to 10 km for 24-hr rainfall in New England.

9. Gringorten, I.I. (1982) Climatic Probabilities of the Vertical Distribution of Cloud Cover, AFGL-TR-82-0078, ADA 118753.

10. Burger, C.F. (1985) World Atlas of Total Sky Cover, AFGL-TR-85-0198, ADA 170474.

5. CORRELATIONS

As described above, the BSW model was developed to provide useful simulations of the weather, particularly clouds and to provide the probabilities of events or conditions in 3D-space. The field of correlation coefficients (cc) associated with the model, while not a goal in itself, is an important indicator of the validity of the model, and may be used as a tool in achieving the primary goals.

While studies of the spatial field of correlation have not been as numerous as studies of time-lapse correlation, they have not been neglected. Buell[11] found patterns in the cc's of horizontal winds aloft, as functions of distance between stations. Bertoni and Lund[12] published some revealing diagrams of winter cc's, in the horizontal, of pressure, temperature and air density from the surface to altitudes as high as 16 km. For temperature they found horizontal cc's to decrease to zero at distances of roughly 1,000 nm, then become negative to nearly -0.2, then back to zero at roughly 2,200 nm (Figure 7).

5.1 Correlation in the 3D-BSW Model

The cc of the END's at two points is to be found as a function of their separation (s). The vertical scale is multiplied (see above) by a factor q (= 50, say) to make the vertical measure cause a decay in correlation more rapid than the horizontal measures.

5.2 Derivation of the Analytical Expression for the cc

The END (y), at any point (u, v, w) in a three-dimensional space, is a standardized average of a set of N heights (x_i, i = 1, N) of N wave formations, thus:

$$y = (\sum x_i - N.\bar{x})/(\sigma_x . \sqrt{N}) \tag{33}$$

where $\bar{x}$ is the true mean of all x_i, σ_x is the true standard deviation of x_i. In the 3D-BSW model, the x's have a rectangular distribution of values between 0 and 1, hence

$$\bar{x} = 1/2$$
$$\sigma_x = 1/\sqrt{12} \tag{34}$$

11. Buell, C.E. (1962) Two-Point Variability of Wind. Final Report, AF19(604)-7282, AFCRL-62-889, Kaman Nuclear, Colorado Springs, Colorado.

12. Bertoni, E.A., and Lund, I.A. (1964) Winter Space Correlations of Pressure, Temperature and Density to 16 km. Environment Res. Papers (No. 75), AFCRL-64-1020, ADA 611002, 29 pp.

to make

$$y = \left(\sum x_i - N/2\right)/\sqrt{N/12}\,. \qquad (35)$$

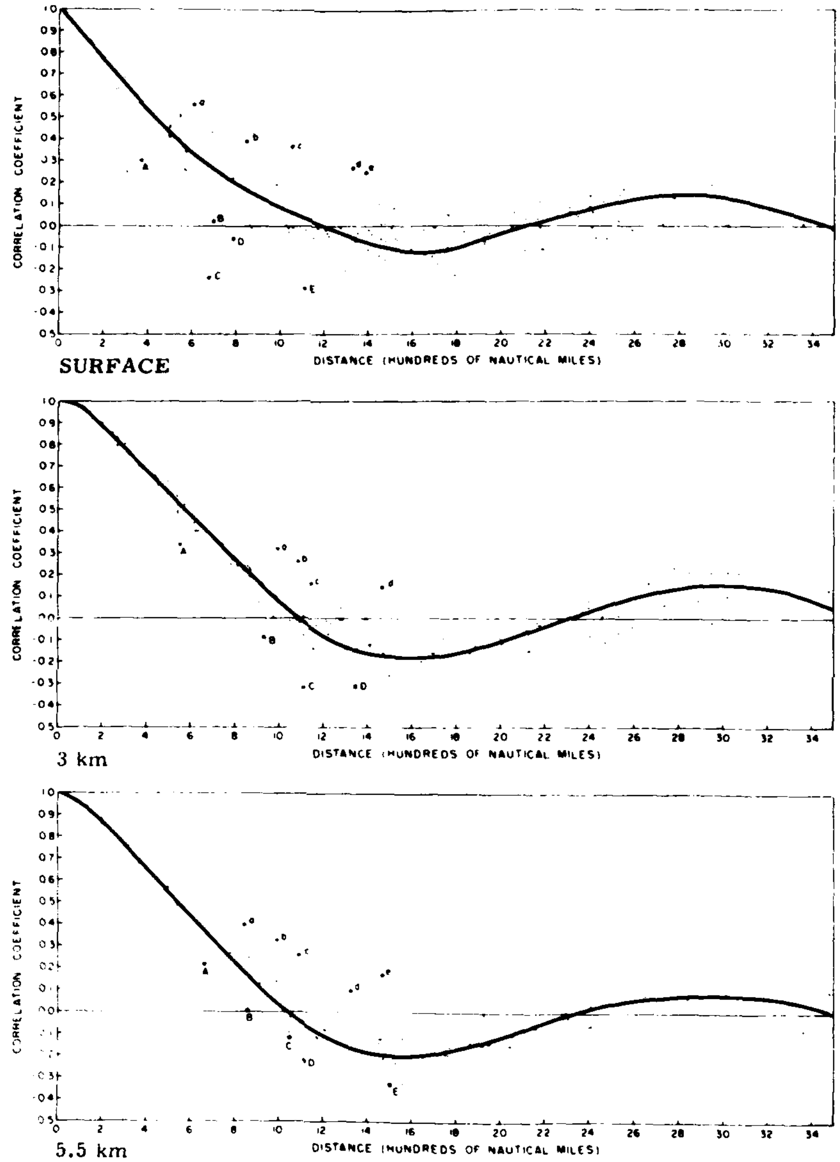

Figure 7. Horizontal Decay of Temperature Correlation as a Function of Distance (nm) Between Stations[12]

In any one wave formation the cc between x_1 and x_2, that are separated by the distance s, is given by

$$\rho_x(s) = \frac{E(x_1 x_2) - \bar{x}^2}{\sigma_x^2} . \tag{36}$$

In the set of N wave formations, the cc between y_1 and y_2, at any two points separated by distance s, is given by

$$\rho_y(s) = E(y_1 y_2) . \tag{37}$$

A brief manipulation of these equations will reveal that

$$\rho_y(s) = \rho_x(s) . \tag{38}$$

Hence, to find $\rho_y(s)$ it is sufficient to find $\rho_x(s)$, now simply denoted as $\rho(s)$.

It is convenient, at this stage, to measure all distances in units of the wavelength (Λ). That is, in each wave formation the crests will be one unit of distance apart ($\Lambda = 1$ temporarily).

Without loss of generality we can limit our attention to one wave formation and think of the wave crests as thin flat sheets of plywood separated by one unit of distance, all sheets parallel to the V-W plane. In Figure 8 the U, W-axes are in the flat surface of the paper, the V-axis and the plywood sheets rise out of the paper.

We place the first point (C) along the U-axis at distance x_1 from the leading edge of the middle wave, and restrain x_1 to vary uniformly from zero to 1.0. In the 2D-BSW model the wave height at C was its phase (x_1). In the 3D-BSW model the locus of the second point (P) at distance (s) from the point C is a spherical surface of radius (s), represented only by the circle in Figure 8. The height (x_2) of the second point at P is given by

$$x_2 = x_1 + s . \cos\theta + \delta \tag{39}$$

where θ is the angle between the line CP and the U-axis, and where

δ = -1, -2, --- if P is in the first, second, --- right-hand wave,

= 0 if P is in the middle wave (as shown in Figure 8),

= 1, 2, --- if P is in the first, second, --- left-hand wave.

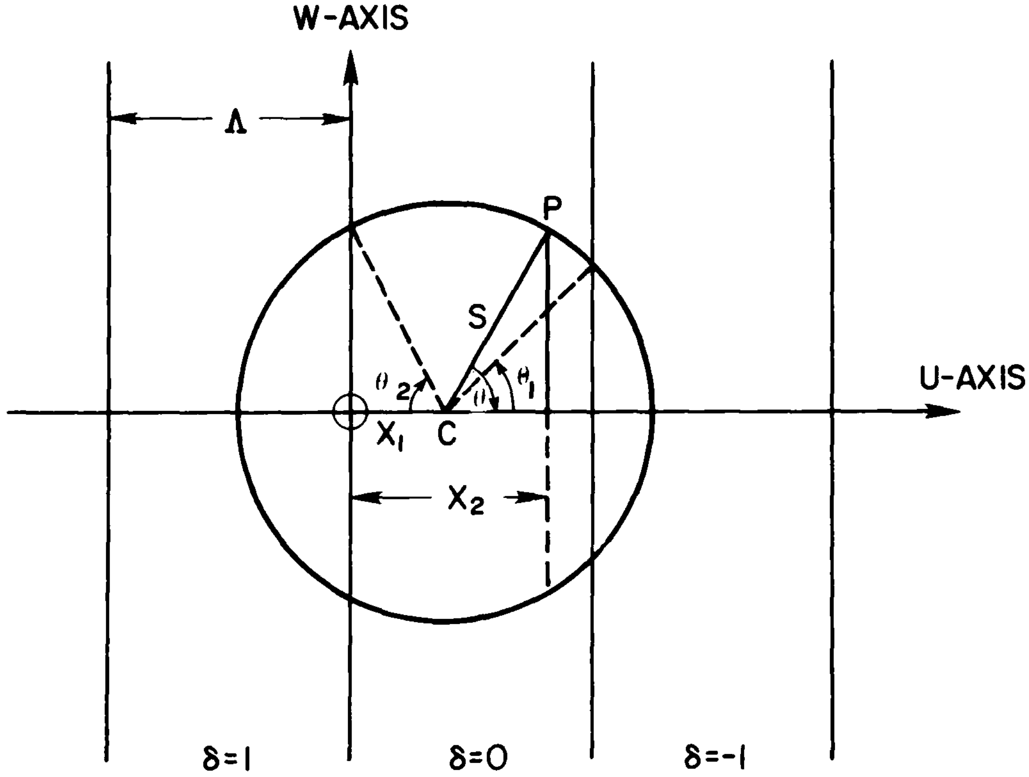

Figure 8. A Cross-section of a Wave Formation in the U-W Plane to Illustrate the Relation Between Two Points (C, P) Separated by Distance (s). The wave phase resembles a flat sheet extending into the V-dimension (into and out of the paper)

The covariance of (x_1, x_2) is obtained by finding the average of $(x_1 x_2)$ over the sphere of radius s, whose surface area is $4\pi s^2$. The element of area at P is given by

$$\delta A = (s.\delta\theta).(s.\sin\theta.\delta\zeta) \tag{40}$$

where ζ is an angular measure around the small circle of radius $s.\sin\theta$, centered on the U-axis. For integration we must have

$$0 \leqq \theta \leqq \pi$$

$$0 \leqq \zeta \leqq 2\pi .$$

Hence

$$E(x_1 x_2) = \frac{1}{4\pi s^2} \int_{x=0}^{1} \int_{\theta=0}^{\pi} \int_{\zeta=0}^{2\pi} x(x + s\cos\theta + \delta) \,.\, s^2 \sin\theta \,.\, dx \,.\, d\theta \,.\, d\zeta \,. \tag{41}$$

Integrating with respect to ζ, and with some terms vanishing,

$$2E(x_1 x_2) = \frac{2}{3} + \int_{x=0}^{1} x\,dx \int_{\theta=0}^{\pi} \delta \,.\, \sin\theta \; d\theta \,. \tag{42}$$

When $0 \geqq s \leqq 1$, δ is restricted to $\delta = -1, 0, 1$. Hence

$$2E(x_1 x_2) = \frac{2}{3} + \int_{x=0}^{s} x\,dx \int_{\theta=\pi-\theta_2}^{\pi} \sin\theta \,.\, d\theta + \int_{x=1-s}^{1} x\,dx \int_{\theta=0}^{\theta 1} (-1)\sin\theta\, d\theta$$

where

$$\cos\theta_1 = (1 - x)/s \tag{43}$$

$$\cos(\pi - \theta_2) = x/s$$

which simplifies to

$$12.\, E(x_1 x_2) = 4 - 3s + 2s^2$$

whence, by Eqs. (34) to (36)

$$\rho(s) = 1 - 3s + 2s^2 \qquad \text{for } 0 \geqq s \leqq 1 \,. \tag{44}$$

When $1 < s \leqq 2$, then δ can have two or more values. Or

$$\delta = -2, -1, 0, 1 \text{ or } 2 \,.$$

The solution for $E(x_1 x_2)$ is much longer and involved, but equally valid. It gives, finally

$$\rho(s) = (1 - 3s + 2s^2) - 6\,(s - 1)^2/s \qquad \text{for } 1 < s \leqq 2 \,. \tag{45}$$

Figure 9 is a plot of $\rho(s)$ vs s, where s is in units of the wavelength. The cc becomes zero at one-half wavelength, becomes negative with minimum -0.12, then zero again at 1, 1.5 and 2.0 wavelengths. As s increases, the value of $\rho(s)$ will oscillate from positive to negative, and will become vanishingly small.

When $\Lambda = 300$ km, the cc very nearly equals 0.99 over the unit distance of 1 km. For other Λ we make cc a function of distance, (s' km), thus:

$$\rho(s') = 1 - 3s + 2s^2 \text{ where } s = s'/\Lambda \quad \text{for } 0 \leq s' \leq \Lambda \,. \tag{46}$$

The value of Λ that will minimize the root mean square difference between the cc's of the previous Model B and the 3D-BSW Model, for distances up to the distance at which Model B yields cc equal to zero, is close to 260 km. We choose to set $\Lambda_o = 2^8 = 256$ km. The cc will drop to zero at $s' = 2^7 = 128$ km; the cc at $s' = 1$ km will be 0.9883---.

5.3 Correlation in the Vertical

The climatic records (WBAN10 and TDF14) give information on the amount of cover (in tenths) at one to four levels, plus the height of the cloud bases. But how can we derive cc's from this information? The Air Weather Service[13] nephanalysis model 3-D Neph, gives 3-hourly information on cloud presence at some 15 levels in the atmosphere, from which tables of the frequency distribution of cloud cover, in layers, have been compiled. The record might be explored to see if cc's also could be computed or estimated. We are unaware of any published results, and for the present, we consider cc in the vertical to be unknown. (In the earlier sections of this report, the value of q = 50 is only a guess.)

The application of the 3-D model to produce clouds in three dimensions should bypass the cc. The approach to the distribution of cloud cover in the vertical is to treat a cloud layer, of thickness (h), as though it were an entity with one mean cloud cover (P_o) and scale distance (r) for that layer. The mean cloud cover, in layers, is obtainable from the 3-D Neph data. It is also obtainable from the TDF14 information on cloud cover from the ground to height (H) and by using the parameter G (h, H), previously described.[1]

For the present, the scale distance of a layer that has specific lower and upper levels is taken to be the same scale distance as that determined for the whole layer from the ground up to the top of the cloud layer.

13. Fye, F.K., Major USAF, (1978) The AFGWC Automated Cloud Analysis Model, AFGWC Technical Memorandum 78-002, ADA 057176, Hq AFGWC, Offutt AFB, Nebraska.

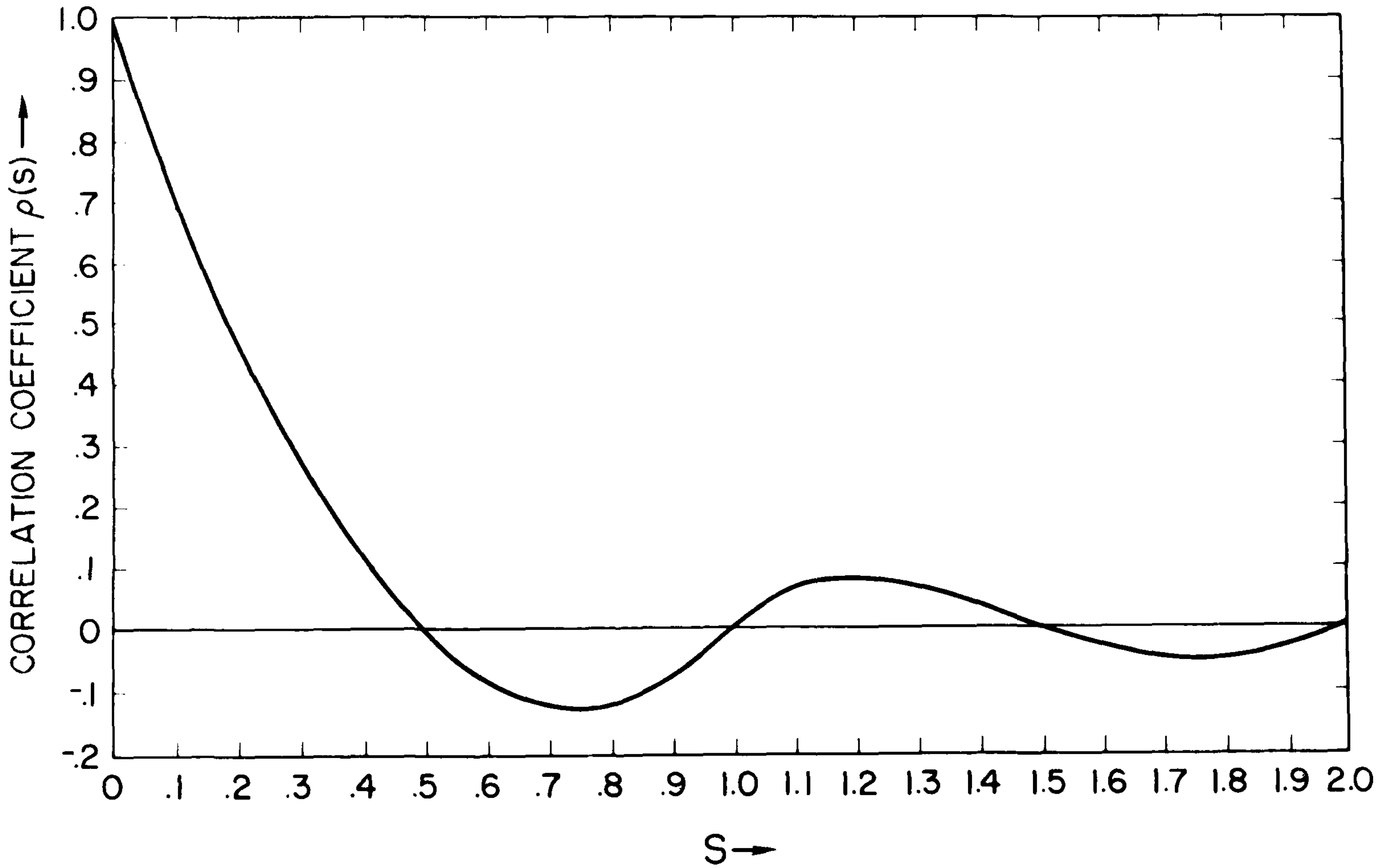

Figure 9. The Relation of Correlation Coefficient $\rho(s)$ to Distance (s) in the 3D-BSW Model. In this diagram the unit of length for s is the wavelength

5.4 Probability of Joint Events at Two or more Stations

The main thrust of this report is aimed at spatial events within a known space, rather than specific points within the space. However, since the 3D-BSW model provides the correlation coefficient between points of known distance apart, it is possible to obtain the probability of joint events, through application of the multivariate normal distribution.[14]

The joint probability of two points exceeding a threshold is given by the integration of the bivariate normal distribution. Of the many ways to calculate this integral, we prefer to use Eq. (31) of Owen[15] along with Algorithm No. 15 of Yamauti.[16]

For the joint probability of three or more points, algorithms become increasingly complex. We find it simpler to cumulate the results using a multitude of simulations.

6. ALTERNATIVES TO THE 3D-BSW MODEL

The 3D-BSW model is basic to the methodology of this report. It succeeds the previous Gringorten model B and the 2D-BSW model, and is subject to change or improvement in the future. A few alternatives have been considered, including multi-dimensional models to include characteristics other than the spatial, such as time. Instead of sawtooth waves, one model has wedge-shaped waves and another sinusoidal waves.

6.1 The 4D-BSW Alternative

If simulation is to be realized by the 3D-BSW model in space, coupled with a process in time, the procedure will call for the stochastic production of a three-dimensional field of END-values, $y_t(u, v, w)$, at each point (u, v, w) for an initial time (t). Then another field of END-values must be generated stochastically, and linked to the initial field, to give time-related END-values. For every additional time another field would be generated, with its attendant random numbers. For a 20-year run with a 1-min time step, this would require the random generation of over 10 million fields.

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To avoid the dilemma just described, the 3D-BSW model has been advanced to the 4D-BSW model, in which the fourth dimension is devoted to time. The representation of time, however, is just one of many possibilities for a 4D-model. As of this writing, the simulation, by the 4D-BSW model, of a field of values in one, two, three or four dimensions has been tried successfully. The decay of correlation with distance for one wavelength (Λ) is given by

$$\rho(s) = 1 - 8s/\pi + 3s^2/2 \quad \text{for } s = s'/\Lambda \leqq 1 . \tag{47}$$

The application of the climatic probabilities has been explored. The areal coverage probabilities have been found to be similar to the 3D coverage. Much more, however, needs to be done.

6.2 Sinusoidal-Wave Model

The sinusoidal-wave model has its attractions and may yet be developed intensively. The primary drawback is the time required to produce synoptic maps. Wave formations are produced stochastically, as in the 3D-BSW model, but, instead of Eq. (14) for the wave height (x) at a point (u, v, w), the equation becomes

$$x = \sin \{ 2\pi(h + D/\Lambda) \} . \tag{48}$$

It differs, also, from the 3D-BSW model in the shape of the curve of correlation vs distance (Figure 10). For s near zero, the 3D-BSW correlation decreases nearly exponentially, while the sine correlation has an exponential-squared decay.

Instead of Eqs. (44) or (45), the formula for cc vs distance becomes

$$\rho(s) = \frac{\sin(2\pi s)}{2\pi s} \quad \text{for all } s \geqq 0 . \tag{49}$$

While the exponential-type decay of cc serves us well for meso-scale phenomena, such as cloud and precipitation patterns, the exponential-squared type, which Figure 10 suggests, may be more appropriate for macro-scale phenomena. There was a hint of this preference when the earlier Gringorten Models A and B were studied, in particular for upper-air temperatures.

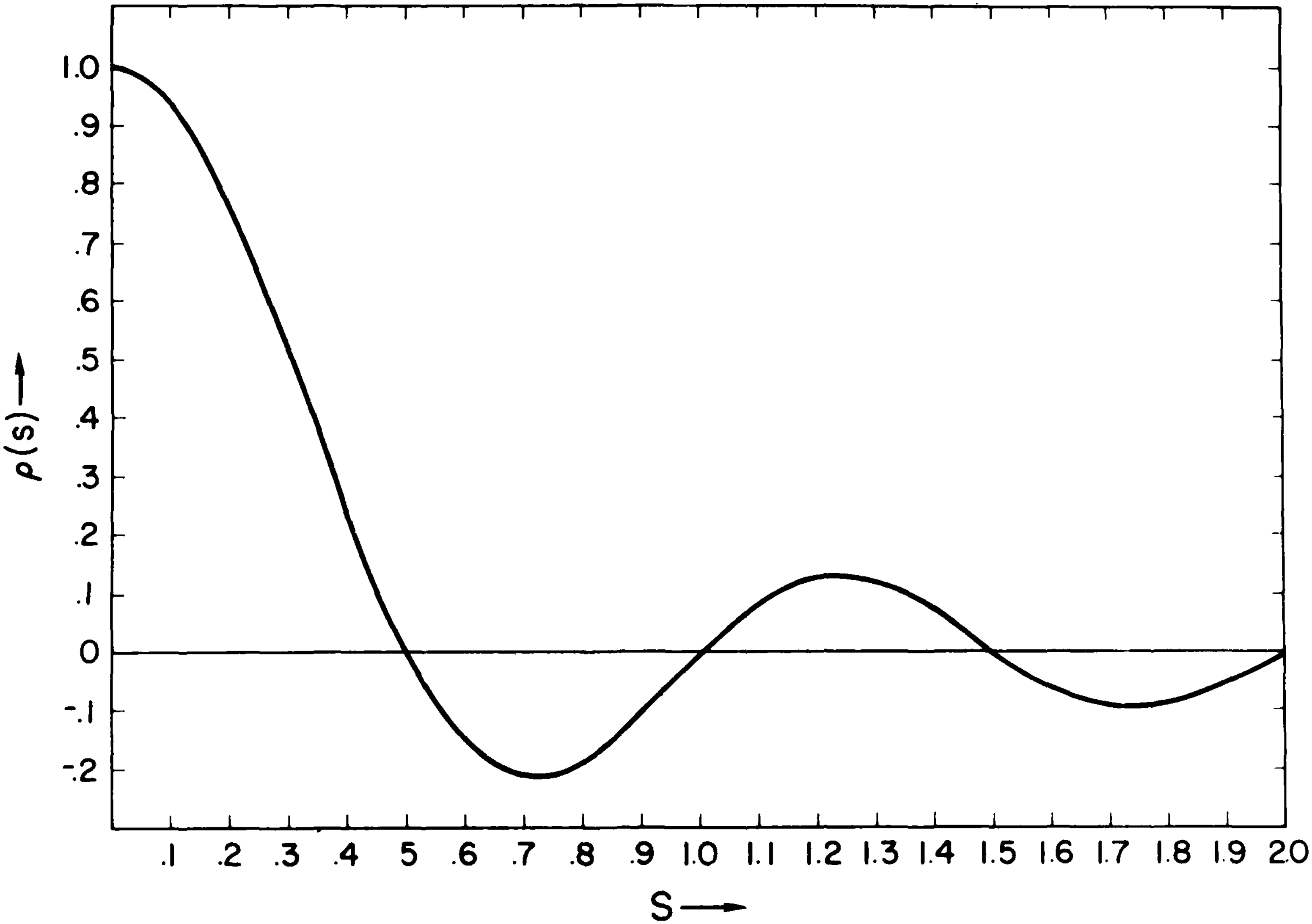

Figure 10. The Relation of Correlation Coefficient $\rho(s)$ to Distance (s) Between Two Stations, in the 3D-Sinusoidal Model

7. SUMMARY AND CONCLUSIONS

The 3D-BSW model was developed to estimate probabilities of events (X) in one-, two- or three-dimensional space. It requires a parameter (Λ) such that the correlation coefficient of the END's (y) of X at two stations will reduce to zero at a separation of (Λ/2) km between the stations.

Symbolically the algorithms have been written to give:

(a) PA (X, A, F, r), the probability that a threshold condition (X) will be exceeded only in a fraction (F/10) or less of the area (A),

(b) PL (X, s', F, r), the probability that X will be exceeded only over (F/10)'s or less of the line of length (s'),

(c) PI (X, s', T, r), the probability that there is a distance (s') out of an overall line of travel (T), over which the condition (X) is not exceeded (that is, the minimized maximum).

The 3D-BSW model has special application to cloud cover. If the threshold is considered to be the no-cloud condition, then the algorithms provide the probabilities of a clear condition in the fraction (F) of an area (A) or line of length (s'), along with the probability of a cloud-free interval of length (s') in a total line of travel (T).

Methods for the determination of the parameter (Λ) or (r) are still to be published. The data sources are primarily the climatological summaries of hourly data.

As might be expected, the fields generated by the model are characterized by correlation, varying with the distance of separation between points. A vertical distance, with respect to correlation, is comparable to a much greater distance in the horizontal. This relationship needs to be studied further.

Paradoxically the determination of the joint probability of a threshold condition at three or more points is more difficult than finding such probability for a whole area.

The 3D-BSW model may be improved, or replaced, eventually. So far, it has emerged as the optimum tool for simulating synoptic images and climatic frequencies of conditions or events in space.

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Appendix A

Simulating a 1- or 2-Dimensional Field of END's

Purpose: To produce stochastically a one- or two-dimensional field or portrayal of END-values of a weather element using the 3D-BSW model. A one-dimensional field is produced by keeping two components (u, v or w) constant, a two-dimensional field is produced by keeping one component constant.

Comment: Data are not required. Instead a software program, to generate random numbers, each from a rectangular distribution of numbers from 0 to 1, is used. Each map or configuration will require the generation of several dozen random numbers.

Step 0. Choose Λ: a wavelength, the parameter of the 3D-BSW model.

Choose R_o: an initial random number or seed. Note: It has been found helpful to record this number, since it may be desired to intensively explore a small part of the synoptic field in a supplementary investigation.

Choose L: the number of wave formations per map.

Choose (u_o, v_o, w_o) the starting point.

Choose M_u: the number of intervals to the side in the u-direction

Choose M_v: the number of intervals in the v-direction.

Choose M_w: the number of levels in the w-direction.

Choose $\delta s'$ (km): the interval in the horizontal.

Choose $\delta w'$ (km): the vertical distance between levels.

Choose q: the quotient to "expand" the vertical scale.

Note: The following description of the method is strongly influenced by the BASIC language.

Step 1. For ℓ = 1 to L (for generating the ℓth wave formation)

Find R_h, a random number between 0.0 and 1.0

Set $h_\ell = R_h$

Step 1a. To find $\cos \alpha_\ell$, $\cos \beta_\ell$, $\cos \gamma_\ell$.

Find: R_u, R_v, R_w, three random numbers between 0.0 and 1.0

Find: $r_u = R_u - 1/2$,

$r_v = R_v - 1/2$,

$r_w = R_w - 1/2$.

Find $(OP)^2 = r_u^2 + r_v^2 + r_w^2$.

If $(OP)^2 > 1/4$, then reject the three random numbers, and repeat Step 1a.

Else, when $(OP)^2 \leqq 1/4$, find $\cos \alpha_\ell = r_u/(OP)$

$\cos \beta_\ell = r_v/(OP)$

$\cos \gamma_\ell = r_w/(OP)$.

Continue Step 1, as long as $\ell < L$.

Step 2. For m_w = 0 to M_w (for the m_wth level)

Set $w = (w_o + m_w \cdot \delta w') \cdot q$.

Step 3. For m_v = 0 to M_v (for the m_vth point in the V-direction).

Set $v = v_o + m_v . \delta s'$.

Step 4. For m_u = 0 to M_u (for the m_uth point in the U-direction).

Set $u = u_o + m_u \cdot \delta s'$.

Initialize $\Sigma x_\ell (u, v, w) = 0$.

Step 5. For ℓ = 1 to L (for the ℓth wave formation).

Find $D = u. \cos \alpha_\ell + v \cdot \cos \beta_\ell + w \cdot \cos \gamma_\ell$.

Find $T = h_\ell + D/\Lambda + I$ (where I is a sufficiently large integer, to make T positive; if BASIC is used, I isn't necessary).

Find $x_\ell (u, v, w)$ = FRA (T) = T - INT(T).

Note: Section 2.1.5 describes how to find the wave height (x), tailored to both FORTRAN and BASIC.

Add $x_\ell(u, v, w)$ to $\sum_{\ell=1}^{L} x_\ell(u, v, w)$.

Continue Step 5 as long as $\ell \leq L$.

Find $y(u, v, w) = \sqrt{(12L)} \cdot [\frac{1}{L} \sum_{\ell=1}^{L} x_\ell(u, v, w) - 1/2]$.

Continue Step 4, as long as $m_u \leq M_u$.

Continue Step 3, as long as $m_v \leq M_v$.

Continue Step 2, as long as $m_w \leq M_w$.

(This exercise provides END-values at all specified grid points, in space. Since this appendix was written, a modification of the 3D-BSW model has enabled us to use one of its three dimensions to closely resemble the O-U process for time, which will be described in a later report.)

Appendix B

Estimating the Probability That a Threshold Value Will be Exceeded in Only (F/10)ths of a Square Area (A)

<u>Purpose:</u> To estimate PA (y_o, A, F, r), the probability that a threshold value of X, or its END (y_o), will not be exceeded, except in (F/10)ths or less of the square area (A); scale distance is (r).

<u>Algorithm:</u>

Step 0: Begin with: threshold (y_o) or the single-point cumulative probability (P_o) of X, plus scale distance (r km), plus area of floor space (A km^2).

Step 1. Compute entries y_o (if necessary) and z.
Find $z = (\ln \sqrt{A}/r)/\ln 2$.

If given P_o, find y_o. The recommended formula is the NBS approximation:

$$y_o = k\,[t - (a_o + a_1 \cdot t)/(1 + b_1 t + b_2 t^2)] \tag{B1}$$

where

$a_0 = 2.30753$

$a_1 = 0.27061$

$b_1 = 0.99229$

$b_2 = 0.04481$

and where

$k = -1, \quad t = \sqrt{(\ell n\ 1/p^2)}$ for $p = P_o \leqq 1/2$

$k = 1, \quad t = \sqrt{(\ell n\ 1/(1-p)^2)}$ for $p = P_o > 1/2$.

Step 2. To solve for the probability of the fractional cover (F/10):
Note: In this algorithmic solution, F need no longer be an integer.
If $0 \leqq F \leqq 5$, substitute $F' = F$, $y_o' = y_o$.
If $5 < F \leqq 10$, substitute $F' = 10 - F$, $y_o' = -y_o$.
The END, $y(F')$, is obtained by interpolation between $y\{INT(F')\}$ and $y\{INT(F')+1\}$. Both of these latter values are obtained by

$$y(F') = \alpha + \beta + \gamma + \delta \qquad \text{(B2)}$$

where α, β, γ, δ are solved by the equations in Table B1, supported supported by Tables B2 and B3.

The interpolation procedure is accomplished by

$$y(F') = y\{INT(F')\} + \{F' - INT(F')\}\ \{y[INT(F')+1] - y[INT(F')]\}. \qquad \text{(B3)}$$

Find the probability, P corresponding to the End, $y(F')$.
The recommended formula is the NBS approximation

$$P = \ell + m\ [2(1 + c_1\chi + c_2\chi^2 + c_3\chi^3 + c_4\chi^4)^4]^{-1} \qquad \text{(B4)}$$

where

$c_1 = 0.196\ 854$

$c_2 = 0.115\ 194$

$c_3 = 0.000\ 344$

$c_4 = 0.019\ 527$

$\chi = |y(F')|$

$\ell = 0$, $m = 1$ for $y(F') \leqq 0$

$\ell = 1$, $m = -1$ for $y(F') > 0$.

For $0 \leqq F \leqq 5$, set $PA(y_o, A, F, r) = P$

For $5 < F \leqq 10$, set $PA(y_o, A, F, r) = 1 - P$.

Table B1. Expressions for Each Term of Eq. (B2)

Term	Expression	Conditions
α	$y_o + (0.006e^{0.8z} - 0.003)(F - 5)$	$-1 \leq z \leq 1.0$
β	$(0.9981 + 0.0011e^{0.89z})y_o + (a_n + b_n e^{c_n z})(F + n - 6) - \sum_{i=0}^{n-1} (a_i + b_i e^{c_i z})$ $n = 5 - INT(F)$	$1.0 < z \leq 7$
γ	$(0.0017e^{0.95z} - 0.3129)(0.75 - 0.3y_o)(1 - 0.25F)$	$5.5 \leq z \leq 7$ and $0 \leq F \leq 4$ and $y_o \geq 2.5$
δ	$\sum_{j=1}^{m} (d(F)_j + f(F)_j e^{g(F)_j y_o})R_j + y'(z = 7)$ $R_j = z - 7$ (when $m = 1$) $= 0.5$ ($m = 2$ and $j = 1$) $= z - 7.5$ ($m = 2$ and $j = 2$) $m = INT(z - 5.5)$ $\left.\begin{array}{l} d(F)_j = -d(F)_j \\ f(F)_j = -f(F)_j \\ g(F)_j = g(F)_j \end{array}\right\}$ ($F = 5$ and $y_o < 0$)	$7.0 < z \leq 8.0$

Note: y_o should be read as y_o', F as F'

Table B2. Constants for the Expression (β) in Table B1

n	a_n	b_n	c_n
0	0.0000	0.0000	0.00
1	-0.0071	0.0086	0.53
2	-0.0205	0.0097	0.52
3	0.0229	0.0045	0.64
4	-0.0517	0.0224	0.45
5	0.0260	0.0087	0.71

Table B3. Constants for the Expression (δ) in Table B1

	m = 1			m = 2		
F	$d(F)_m$	$f(F)_m$	$g(F)_m$	$d(F)_m$	$f(F)_m$	$g(F)_m$
0	0.1420	-2.4090	-0.44	0.1908	-3.8406	-0.56
1	1.0686	-2.2854	-0.46	1.8710	-4.2412	-0.55
2	1.9440	-2.7842	-0.41	3.0540	-4.4752	-0.39
3	2.2560	-2.7356	-0.40	3.3960	-4.5228	-0.50
4	2.0328	-2.2910	-0.41	2.0590	-2.5702	-0.75
5	1.2356	-1.2300	-1.30	1.6124	-1.5434	-1.51

The following BASIC program (filename B:PYOAFR.BAS) estimates PA (y_o, A, F, r). It requires the inputs:

scale distance, entered as R (km),
area of floor space, entered as A (km^2),
fraction of area, entered as F,
the cumulative probability of the threshold (X) entered as P_o,

and yields the output (P), the probability of the event.

```
10 REM B:PYOAFR.BAS ON GRINGORTEN DISK NO. 4. 18 DEC 1986
20 REM TO FIND THE PROBABILITY THAT X. OR ITS END (Y0) WHOSE CLIMATIC FREQUENCY
IS P0. WILL BE EXCEEDED IN (F/10)THS OR LESS OF AREA A: PARAMETER IS SCALE
DISTANCE (R)
30 REM P0 HEREIN WILL BE CUMULATIVE PROBABILITY
40 REM THE FOLLOWING ARE CONSTANTS FOR THE ALGORITHMS
50 A0 = 2.30753
60 A1 = .27061
70 B1 = .99229
80 B2 = .04481
90 C1 = .196854
100 C2= .115194
110 C3 = .000344
120 C4 = .019527
121 OPTION BASE0
130 DIM AN(6), BN(6), CN(6)
140 DIM DFM(2.6)
150 DIM FFM(2,6)
160 DIM GFM(2,6)
170 DIM RD(2,2)
180 FOR I = 0 TO 5
190 READ AN(I), BN(I), CN(I)
200 NEXT I
210 DATA 0,0,0,-.0071, .0086, .53, -.0205, .0097, .52
220 DATA .0229, .0045, .64, -.0517, .0224, .45, .0260, .0087, .71
230 FOR  M = 1 TO 2
240 FOR J = 0 TO 5
250 READ DFM(M,J), FFM(M,J), GFM(M.J)
260 NEXT J
270 NEXT M
280 DATA .1420. -2.4090, -.44. 1.0686. -2.2854, -.46
290 DATA 1.9440. -2.7842, -.41. 2.2560, -2.7356, -.40
300 DATA 2.0328. -2.2910, -.41, 1.2356. -1.2300, -1.30
310 DATA .1908, -3.8406, -.56, 1.8710, -4.2412, -.55
320 DATA 3.0540. -4.4752. -.39, 3.3960. -4.5228. -.50
330 DATA 2.0690. -2.5702, -.75, 1.6124, -1.5434. -1.51
340 REM STEP 0     FOR THE INPUTS
350 INPUT "SCALE DISTANCE =";R
360 INPUT "AREA=";A
370 INPUT "FRACTION =";F
380 INPUT "P0=";P0
390 REM STEP 1     TO TRANSFORM THE VARIABLES INTO ALGORITHM ENTRIES
400 REM TO CALCULATE Z
410 TEMP = SQR(A)/R
420 Z = LOG(TEMP)/LOG(2)
430 REM TO CALCULATE Y0 FROM P0
440 IF P0>.5 THEN 480
450 K = -1
460 T = SQR(LOG(1/P0^2))
470 GOTO 500
480 K=1
490 T = SQR(LOG(1/(1-P0)^2))
500 Y0 = K*(T - (A0 + A1*T)/(1+B1*T + B2*T^2))
510 REM TO SUBSTITUTE Y0P=-Y0 AND FP=10-F WHEN F>.5
520 IF F>5 THEN 560
530 FP = F
540 Y0P = Y0
550 GOTO 590
560 FP = 10-F
570 Y0P = -Y0
```

```
580 REM STEP 2        TO CALCULATE THE END (Y) OF THE DESIRED PROBABILITY
590 IF Z>1 THEN 650
600 REM FOR THE CASES WHEN Z<1
610 ALPHA = YOP + (.006*EXP(.8*Z)-.003)*(FP-5)
620 YP = ALPHA
630 GOTO 1410
640 REM FOR THE CASES WHEN Z=>1
650 IF Z>7 THEN 900
660 REM FOR THE CASES WHEN 1=<Z=<7: FIRST WE FIND BETA
670 FIRST = (.9981 + .0011*EXP(.89*Z))*YOP
680 N= 5 - INT(FP)
700 TEMP = EXP(CN(N)*Z)
710 SECOND = (AN(N)+BN(N)*TEMP)*(FP+N-6)
720 SUM = 0
730 IF N = 0 THEN 790
740 NMIN1 = N - 1
750 FOR I = 0 TO NMIN1
770 SUM = AN(I) + BN(I)*EXP(CN(I)*Z) + SUM
780 NEXT I
790 BETA = FIRST + SECOND - SUM
800 REM TO FIND THE GAMMA TERM
810 IF Z =<5.5 THEN 880
820 IF FP >4 THEN 880
830 IF YOP<2.5 THEN 880
840 REM FOR THE CASE WHEM 5<Z<7, 0<F<4, YOP>2.5
850 GAMMA = (.0017*EXP(.95*Z)-.3129)*(.75-.3*YOP)*(1-.25*FP)
860 YP = BETA + GAMMA
870 GOTO 1410
880 YP = BETA
890 GOTO 1410
900 REM FOR THE CASE WHEN Z>7
910 REM WE FIRST FIND Y(Z=7)
920 FIRST = (.9981+.0011*EXP(6.23))*YOP
930 N = 5 - INT(FP)
950 TEMP = EXP(CN(N)*7)
960 SECOND= (AN(N) + BN(N)*TEMP)*(FP+N-6)
970 SUM = 0
980 IF N=0 THEN 1040
990 NMIN1 = N-1
1000 FOR I = 0 TO NMIN1
1020 SUM= SUM + AN(I) + BN(I)*EXP(CN(I)*7)
1030 NEXT I
1040 BETA = FIRST +SECOND - SUM
1050 IF FP>4 THEN 1100
1060 IF YOP<2.5 THEN 1100
1070 GAMMA = (.0017*EXP(6.65)- .3129)*(.75-.3*YOP)*(1-.25*FP)
1080 YP7 = BETA + GAMMA
1090 GOTO 1110
1100 YP7 = BETA
1110 REM FOR THE ADDITONAL TERMS WHEN Z>7
1120 IF FP=5 THEN FP=4.999999
1130 REM FP SHOULD BE JUST LOW ENOUGH TO AVOID ROUNDOFF TO 5 BY THE COMPUTER
1140 M = INT(Z-5.5)
1150 REM M IS EITHER 1 OR 2
1160 RD(1.1) = Z-7
1170 RD(2.1) = .5
1180 RD(2.2) = Z-7.5
1190 REM BECAUSE F MAY NOT BE A WHOLE NUMBER THE FOLLOWING PROCEDURE IS USED
1200 FLOWER = INT(FP)
1210 FUPPER = FLOWER + 1
```

```
1220 FOR FI = FLOWER TO FUPPER
1240 REM FOR THE CORRECTIVE TERM WHEN Z>7
1250 TERM = YP7
1260 IF M=2 THEN 1310
1270 REM THE CASE WHEN M=1
1280 TEMP = EXP(GFM(1,FI)*Y0P)
1290 TERM = TERM + (DFM(1,FI)+FFM(1,FI)*TEMP)*RD(1,1)
1300 GOTO 1360
1310 REM THE CASE WHEN M = 2
1320 FOR J = 1 TO 2
1330 TEMP = EXP(GFM(J,FI)*Y0P)
1340 TERM = TERM + (DFM(J,FI)+FFM(J,FI)*TEMP)*RD(2,J)
1350 NEXT J
1360 IF FI = FUPPER THEN 1390
1370 YFIRST = TERM
1380 NEXT FI
1390 YSECOND = TERM
1400 YP = YFIRST + (FP-FLOWER)*(YSECOND - YFIRST)
1410 REM CONTINUE
1420 IF F>5 THEN Y = -YP
1430 IF F=<5 THEN Y = YP
1440 REM  STEP 3      TO CALCULATE THE DESIRED PROBABILITY
1450 IF Y>0 THEN 1500
1460 L = 0
1470 M=1
1480 X=-Y
1490 GOTO 1530
1500 L = 1
1510 M = -1
1520 X =Y
1530 DEN = (1+C1*X+C2*X^2+C3*X^3+C4*X^4)^4
1540 P = L + M/(2*DEN)
1550 PRINT "FOR FRACTIONAL COVER F=";F;", CLIMATIC PROBABILITY P0=";P0;"IN ARE
A=";A,"     P=";P
1560 GOTO 380
1570 END
```

Appendix C

Estimating the Probability That a Threshold Value (y_o) Will be Exceeded in Only (F/10)ths of a Line Length (s')

Purpose: To estimate PL (y_o, s', F, r), the probability that the threshold of X, or its END (y_o), will not be exceeded, except in (F/10)ths or less of the line of length (s'); scale distance is given (r).

Algorithm:

Step 0. Begin with:
Single-point cumulative probability (P_o) or its END (y_o), scale distance (r km) or wavelength (Λ = 256 r km), line length (s' km).

Step 1. Compute the entries: y_o (if necessary), and z.
Find $z = \ln(s'/r)/\ln 2$. (C1)
Find y_o; the recommended procedure is in Appendix B.

Step 2. To solve for the probability of the fractional cover (F/10)ths of the length (s'): Note: F is not necessarily an integer.
For $0 \leqq F \leqq 5$, substitute $F' = F$, $y_o' = y_o$,
For $5 < F \leqq 10$, substitute $F' = 10-F$, $y_o' = -y_o$.
Find η_1, η_2.
where

$$\eta_i = A_n + B_n y_o' + C_n y_o'^2 \quad \text{(C2)}$$

where

$$A_n = a_n + b_n \cdot \exp(c_n \cdot z) \tag{C3}$$

$$B_n = d_n + f_n \cdot \exp(g_n \cdot z)$$

$$C_n = h_n + q_n \cdot \exp(r_n \cdot z) \text{ for } z > 6$$

$$= 0 \qquad \text{for } z \leqq 6$$

where

$$n = \mathrm{INT}(F') \text{ for } i = 1$$

$$= \mathrm{INT}(F') + 1 \text{ for } i = 2.$$

and where a_n, b_n, ----, r_n are given in Table C1 for n = 0(1)5.
Then find $y(F') = \eta_1 + (\eta_2 - \eta_1) \cdot \{F' - \mathrm{INT}(F')\}$.
Find P corresponding to $y(F')$.

The recommended formula is the same as in Appendix B.

For $0 \leqq F \leqq 5$, set $PL(y_o, s', F, r) = P$

For $5 < F \leqq 10$, set $PL(y_o, s', F, r) = 1 - P$.

Table C1. Constants for Eq. (C3)

n	a_n	b_n	c_n	d_n	f_n	g_n	h_n	q_n	r_n
0	0.0300	-0.0366	0.51	1.0049	7.44×10^{-4}	0.82	8.06×10^{-3}	-1.44×10^{-5}	1.02
1	0.0346	-0.0366	0.47	1.0034	5.03×10^{-4}	0.87	1.89×10^{-3}	-3.77×10^{-7}	1.46
2	0.0272	-0.0283	0.46	1.0054	2.77×10^{-4}	0.95	2.69×10^{-3}	-2.48×10^{-8}	1.81
3	0.0207	-0.0213	0.44	1.0058	2.74×10^{-4}	0.95	-8.76×10^{-4}	-2.34×10^{-12}	2.93
4	0.0096	-0.0103	0.44	1.0021	3.72×10^{-4}	0.91	-2.04×10^{-3}	-7.21×10^{-17}	4.17
5	0.0000	0.0000	0.00	1.0022	3.53×10^{-4}	0.92	0.0000	0.0000	0.00

The following BASIC program (filename B:PYOSPER.BAS) estimates PL(Y_0, s', F, r). It requires these inputs:

scale distance, entered as R (km),

line length, entered as SP (km),

fraction of the line, entered as F (tenths),

the cumulative probability for the threshold value (X), entered as PO,

and yields the output (P), the probability of the event.

```
10 REM B:PYOSPFR.BAS ON GRINGORTEN DISK NO.4. 18 DEC 1986
15 REM
20 REM THE PROBABILITY THAT X. OR ITS END (Y0) WHOSE CLIMATIC FREQUENCY IS P0.
 WILL BE EXCEEDED IN (F/10)THS OR LESS OF A LINE (SP KM): PARAMETER IS SCALE
 DISTANCE (R)
30 REM P0 HEREIN WILL BE CUMULATIVE PROBABILITY
40 REM THE FOLLOWING ARE CONSTANTS FOR THE ALGORITHMS
50 A0 = 2.30753
60 A1 = .27061
70 B1 = .99229
80 B2 = .04481
90 C1 = .196854
100 C2 = .115194
110 C3 = .000344
120 C4 = .019527
130 DIM AN(6).BN(6).CN(6)
131 DIM DN(6). FSN(6), GN(6)
132 DIM HN(6).QN(6).RN(6)
180 FOR I = 0 TO 5
190 READ AN(I),BN(I).CN(I)
191 READ DN(I).FSN(I),GN(I)
192 READ HN(I),QN(I).RN(I)
200 NEXT I
210 DATA .03,-.0366,.51.1.0049,7.44E-4,.82.8.06E-3.-1.44E-5.1.02
220 DATA .0346.-.0366,.47.1.0034.5.08E-4,.87.1.89E-3,-3.77E-7.1.46
230 DATA .0272,-.0283,.46,1.0054,2.77E-4,.95.2.69E-3,-2.48E-8.1.81
240 DATA .0207.-.0213,.44,1.0058.2.74E-4,.95,-9.76E-4,-2.34E-12.2.93
250 DATA .0096,-.0103,.44,1.0021.3.72E-4..91.-2.04E-3.-7.21E-17.4.17
260 DATA .0000..0000. .00,1.0022.3.53E-4..92,.0000,.0000,.00
340 REM STEP 0    FOR THE INPUTS
350 INPUT "SCALE DISTANCE=";R
360 INPUT "LINE LENGTH=":SP
370 INPUT "FRACTION OF LINE IN TENTHS=":F
380 INPUT; "P0=";P0
390 REM STEP 1    TO TRANSFORM THE VARIABLE INTO ALGORITHM ENTRIES
400 REM      TO CALCULATE Z
410 TEMP = SP/R
420 Z = LOG(TEMP)/LOG(2)
430 REM     TO CALCULATE Y0 FROM P0
440 IF P0>.5 THEN 480
450 K = -1
460 T = SQR(LOG(1/P0^2))
470 GOTO 500
480 K = 1
490 T = SQR(LOG(1/(1-P0)^2))
500 Y0 = K*(T-(A0+A1*T)/(1+B1*T+B2*T^2))
510 REM     TO SUBSTITUTE Y0P = -Y0 AND FP = 10-F WHEN F>.5
520 IF F>5 THEN 560
530 FP = F
540 Y0P = Y0
550 GOTO 580
560 FP = 10-F
570 Y0P = -Y0
580 REM STEP 2   TO CALCULATE THE END (Y) OF THE DESCRED PROBABILITY
589 REM WHEN FP=5 IT SHOULD BE LESSENED BY A TRIFLE. TO AVOID A ROUNDOFF OF ERRO
R IN BASIC
590 IF FP = 5 THEN FP = 4.999999
600 FLOWER = INT(FP)
610 FUPPER = FLOWER + 1
620 FOR FI = FLOWER TO FUPPER
```

```
630 I = FI
640 A = AN(I) + BN(I)*EXP(CN(I)*Z)
650 B = DN(I) +FSN(I)*EXP(GN(I)*Z)
660 IF Z=<6 THEN 700
665 REM WHEN Z>6
670 C = HN(I) + QN(I)*EXP(RN(I)*Z)
690 GOTO 710
700 C=0
710 IF FI = FLOWER THEN K=1
720 IF FI = FUPPER THEN K = 2
730 ETA(K) = A + B*YOP + C*YOP^2
740 NEXT FI
745 YP = ETA(1) + (ETA(2) - ETA(1))*(FP-FLOWER)
750 IF F>5 THEN Y = -YP
760 IF F=<5 THEN Y = YP
770 REM STEP 3    TO CALCULATE THE DESIRED PROBABILITY
780 IF Y>0 THEN 830
790 L = 0
800 M = 1
810 X = -Y
820 GOTO 870
830 REM WHEN Y >0
840 L=1
850 M=-1
860 X=Y
870 DEN = (1+C1*X+C2*X^2+C3*X^3+C4*X^4)^4
880 P = L + M/(2*DEN)
890 PRINT  "  THEN  P=";P
900 GOTO 380
1000 END
```

Appendix D

Estimating the Probability That a Threshold Value (y_0) Will Not be Exceeded Over Any Line of Length (s') Within a Longer Line of Travel (T)

Purpose: To estimate $PI(y_o, s', T, r)$, the probability that (y_o) is the lowest maximum END in a linear interval (s' km) somewhere along a line of travel of length (T km); scale-distance parameter is r km.

Algorithm:

Step 0. Begin with

Single-point cumulative probability (P_o) of a threshold condition (X), with corresponding END (y_o);

Scale distance (r km);

Line interval (s' km), and;

Overall line of travel (T km).

Step 1. Compute entries: y_o, z; ω, as needed.

Find $z = \ln(s'/r)/\ln 2$;

$\omega = \ln(T/s')/\ln 2$.

(Note: Neither z nor ω need be an integer.)

Find y_o corresponding to P_o. The recommended formula is the same as in Appendix A.

Step 2. To solve for the probability:

$$PI(y_o, z, \omega) = P(y)$$

find $y(y_o, z, \omega)$ by interpolation:

$$y(y_o, z, \omega) = y(y_o, z_\ell, \omega_\ell).(1-FRA(z)).(1-FRA(\omega))$$

$$+\ y(y_o, z_\ell, \omega_u).(1-FRA(z)).(FRA(\omega))$$

$$+\ y(y_o, z_u, \omega_\ell).(FRA(z)).(1-FRA(\omega))$$

$$+\ y(y_o, z_u, \omega_u).(FRA(z)).(FRA(\omega))$$

where

$$z_\ell = INT(z)$$

$$z_u = INT(z) + 1$$

$$\omega_\ell = INT(\omega)$$

$$\omega_u = INT(\omega) + 1$$

$$FRA(z) = z - INT(z)$$

$$FRA(\omega) = \omega - INT(\omega)$$

and where

$$y(y_o, z_i, \omega_j) = A(i, j) + B(i, j)y_o + C(i, j)y_o^2$$

where A (i, j), B(i, j), C (i, j) are given in Tables D1, D2, D3 respectively.

Lastly, find P corresponding to $y(y_o; z, \omega)$. The recommended formula is the same as in Appendix A.

Set $PI(y_o, s', T, r) = P$

Table D1. A (i, j) for Given z_i, ω_j

z_i/ω_j	0	1	2	3	4	5	6	7	8	9	10	11
-1	0.01	0.02	0.025	0.06	0.13	0.25	0.46	0.82	1.35	2.09	2.95	3.62
0	-0.01	0.01	0.05	0.12	0.23	0.43	0.75	1.26	2.09	3.09	4.50	
1	-0.02	0.02	0.09	0.20	0.40	0.72	1.22	2.02	3.14	4.79		
2	-0.05	0.02	0.14	0.34	0.66	1.18	1.97	2.95	4.52			
3	-0.125	-0.01	0.20	0.51	1.03	1.84	2.84	4.26				
4	-0.25	-0.05	0.29	0.81	1.55	2.52	3.93					
5	-0.45	-0.14	0.37	1.11	2.01	3.30						
6	-0.76	-0.31	0.39	1.23	2.29							
7	-1.23	-0.62	0.13	0.94								
8	-1.91	-1.28	-0.78									

Table D2. B (i, j) for Given z_i, ω_j

z_i/ω_j	0	1	2	3	4	5	6	7	8	9	10	11
-1	1.00	1.00	1.00	1.00	1.01	1.03	1.06	1.12	1.24	1.44	1.65	1.68
0	1.00	1.00	1.00	1.01	1.02	1.06	1.11	1.26	1.54	1.85	2.44	
1	0.99	1.00	1.00	1.02	1.06	1.12	1.24	1.50	1.96	2.72		
2	1.00	1.01	1.01	1.06	1.10	1.20	1.51	1.84	2.55			
3	1.00	0.99	1.04	1.11	1.23	1.50	1.83	2.39				
4	1.01	1.02	1.10	1.23	1.42	1.80	2.42					
5	1.03	1.07	1.20	1.44	1.77	2.39						
6	1.10	1.15	1.39	1.71	2.23							
7	1.21	1.33	1.63	2.20								
8	1.45	1.65	2.00									

Table D3. C (i, j) for Given z_i, ω_j

z_i/ω_j	0	1	2	3	4	5	6	7	8	9	10	11
-1	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.02	0.03	0.00
0	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.05	0.08	0.15	
1	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.04	0.10	0.21		
2	0.00	0.00	0.00	0.01	0.01	0.00	0.05	0.08	0.17			
3	0.00	0.01	0.01	0.02	0.02	0.04	0.07	0.13				
4	0.01	0.01	0.01	0.02	0.03	0.07	0.15					
5	0.01	0.01	0.02	0.03	0.07	0.16						
6	0.00	0.00	0.03	0.05	0.14							
7	0.00	0.00	0.05	0.18								
8	-0.03	-0.08	0.04									

The following BASIC program (filename B:PYOSPTR.BAS) estimates PI (y_o, s', T, r). It requires these inputs:

(a) the single-point probability of exceeding the threshold (X), entered as POP. It is changed by a step in the program to cumulative probability (PO),

(b) scale distance, entered as R (km),

(c) the overall length of the line of travel, entered as T (km),

(d) the length of the window, entered as SP (km),

and yields the output (p), the probability of the event.

```
10 REM B:PYOSPTR.BAS ON GRINGORTEN DISK NO.5. 10 MAR 1987
11 PRINT
20 REM BASIC PROGRAM FOR APPENDIX D OF REPORT ON 3D-BSW MODEL
30 REM TO FIND THE PROBABILITY OF THE LOWEST MAXIMUM (YO) IN AN INTERVAL (S')
   WITHIN A LARGER INTERVAL(T)
40 DIM A(10.14)
50 DIM B(10.14)
60 DIM C(10,14)
70 FOR I = 0 TO 9
80 Z = I - 1
85 TWMINZ = 12-Z
90 FOR J = 0 TO TWMINZ
100 READ A(I,J)
110 NEXT J
120 NEXT I
121 DATA .01,.02,.025,.06,.13,.25,.46,.82,1.35,2.09,2.95,3.62,5.32,7.29
122 DATA -.01,.01,.05,.12,.23,.43,.75,1.26,2.09.3.09,4.5,6.26,6.87
123 DATA -.02,.02,.09,.2,.4,.72,1.22,2.02,3.14,4.79,6.62,8.6
124 DATA -.05,.02,.14,.34,.66,1.18,1.97,2.95,4.52,6.33,8.65
125 DATA -.125,-.01,.2,.51,1.03,1.84,2.84,4.26,6.3,7.74
126 DATA -.25,-.05,.29,.81,1.55,2.52,3.93,5.82,8.18
127 DATA -.45,-.14,.37,1.11,2.01,3.3,5.03,7.91
128 DATA -.76,-.31,.39,1.23,2.29,3.45,5.35
129 DATA -1.23,-.62,.13,.94,1.9,3.16
130 DATA -1.91.-1.28.-.78,-.097,.54
131 FOR I = 0 TO 9
140 Z = I-1
145 TWMINZ = 12 - Z
150 FOR J = 0 TO TWMINZ
160 READ B(I.J)
170 NEXT J
180 NEXT I
190 DATA 1,1,1,1,1.01,1.03,1.06,1.12,1.24,1.44.1.65,1.68,2.51,3.3
191 DATA 1,1,1,1.01,1.02,1.06,1.11,1.26,1.54,1.85,2.44,3.13,3.04
192 DATA .99.1,1,1.02,1.06,1.12,1.24,1.5,1.96,2.72,3.41,4.14
193 DATA 1,1.01,1.01,1.06,1.10,1.2,1.51,1.84,2.55,3.34,4.35
194 DATA 1,.99,1.04,1.11,1.23,1.5,1.83,2.39,3.31,3.61
195 DATA 1.01,1.02,1.1,1.23,1.42,1.8,2.42.3.31,4.38
196 DATA 1.03,1.07,1.2,1.44,1.77,2.39,3.27,4.97
197 DATA 1.1,1.15,1.39,1.71,2.23,2.71,3.91
198 DATA 1.21,1.33,1.63,2.2,2.87,3.99
199 DATA 1.45,1.65,2,2.52.3.6
200 FOR I = 0 TO 9
210 Z = I-1
220 TWMINZ = 12-Z
230 FOR J = 0 TO TWMINZ
240 READ C(I,J)
250 NEXT J
260 NEXT I
270 DATA 0,0,0,0,0,.01,.01,0,.01..02,.03,0,.12,.22
271 DATA 0,0,0,0,0,.01,.01,.02,.05,.08,.15,.23,.19
272 DATA 0,0,0,.01,.01,.01,.02,.04,.1,.21,.27,.35
273 DATA 0,0,0,.01,.01.0,.05,.08,.17,.27,.39
274 DATA 0,.01,.01,.02,.02,.04,.07,.13,.25,.25
275 DATA .01..01,.01,.02,.03,.07,.15,.27,.4
276 DATA .01,.01,.02,.03,.07,.16,.29,.57
277 DATA 0,0,.03,.05,.14,.21,.45
278 DATA 0,0,.05,.18,.32,.6
279 DATA -.03,-.08,.04,.14,.75
300 INPUT "PROBABILITY OF CLOUD COVER": POP
```

```
301 REM PROBABILITY OF CFLOS IS P0 WHERE
310 P0 = 1 - POP
320 REM TO FIND Y0, THE END OF P0
330 GOSUB 700
340 INPUT "SCALE DISTANCE=R":R
350 INPUT "OVERALL DISTANCE=T" :T
359 LIMIT = 256*R: IF T<LIMIT THEN LIMIT = T
360 PRINT "LIMIT WINDOW (SP) TO"; (T/2^13); "<SP<":LIMIT
361 INPUT:"WHEN SP=";SP
370 S = SP/R
380 Z= LOG(S)/LOG(2)
381 IF Z>-1 THEN 384
382 REM WHEN Z =<-1
383 Z=-1
384 REM WHEN Z>-1
385 IF Z<8 THEN 390
386 REM WHEN Z>8
387 PRINT "   NO ANSWER: ALGORITHM WAS NOT EXTENDED FAR ENOUGH.
388 GOTO 361
390 OMEGA = LOG(T/SP)/LOG(2)
400 ZL = INT(Z)
410 ZU = INT(Z) +1
420 WL = INT(OMEGA)
430 WU = WL + 1
435 REM FOR THE FIRST TERM (Y1)
440  ZP= ZL
450 WP = WL
460 GOSUB 810
461 FRAZ = Z - INT(Z)
462 FRAW = OMEGA - WL
470 Y1 = YP*(1-FRAZ)*(1-FRAW)
480 REM FOR THE SECOND TERM (Y2)
490 ZP = ZL
500 WP = WU
510 GOSUB 810
520 Y2 = YP*(1-FRAZ) *(FRAW)
530 REM FOR THE THIRD TERM (Y3)
540 ZP = ZU
550 WP = WL
560 GOSUB 810
570 Y3 = YP*(FRAZ)*(1-FRAW)
600 REM FOR THE 4TH TERM (Y4)
610 ZP = ZU
620 WP = WU
630 GOSUB 810
640 Y4 = YP*(FRAZ)*(FRAW)
650 Y = Y1+Y2+Y3+Y4
660 GOSUB 860
670 PRINT "   THE PROBABILITY =";P
675 GOTO 361
700 REM THE SUBROUTINE TO FIND Y0 GIVEN P0
701 A0 = 2.30753
702 A1 = .27061
703 B1 = .99229
704 B2 = .04481
710 IF P0>.5 THEN 760
720 REM WHEN P0 =<.5
730 TP = SQR(LOG(1/P0^2))
740 K =-1
750 GOTO 790
```

```
760 REM WHEN P0>.5
770 TP = SQR(LOG(1/(1-P0)^2))
780 K =  1
790 Y0 = K*(TP-(A0+A1*TP)/(1+B1*TP+B2*TP^2))
800 RETURN
810 REM SUBROUTINE TO FIND THE END (Y) FOR GIVEN Z, OMEGA AND Y0
820 I = ZP+1
830 J= WP
840 YP = A(I,J) + B(I,J)*Y0 + C(I,J)*Y0^2
850 RETURN
860 REM SUBROUTINE TO FIND THE PROBABILITY P CORRESPONDING TO Y
870 C1 = .196854
880 C2 = .115194
890 C3 = .000344
900 C4 = .019527
910 X = ABS(Y)
915 DEN =(1 + C1*X+C2*X^2 + C3*X^3 + C4*X^4)^4
920 IF Y<0 THEN 980
930 REM WHEN Y=>0
950 L = 1
960 M =-1
970 GOTO 1010
980 REM WHEN Y<0
990 L = 0
1000 M =  1
1010 P = L + M*(.5/DEN)
1015 RETURN
```