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Coherent Combination of Signals from
Multiple Receiving Sites of NAVSPASUR

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The Naval Space Surveillance system (NAVSPASUR) is designed to determine orbits for satellites which reflect signals from the NAVSPASUR transmitters. There are several NAVSPASUR receiving sites, each of which has 11 or 12 individual antennas. The relative phase of reflected signals as received at these closely spaced antennas (typically within 200 to 1200 feet of each other) is used to determine the angular location of the satellites. Because of the short baselines between the antennas, the accuracy of an angular position measurement is not high. Typical accuracies are 0.01 degree. If it were possible to combine signals from several NAVSPASUR sites (with typical separations of many hundreds to thousands of kilometers), much higher position accuracies could potentially be determined. It might also be expected that some imaging information could be gained for some larger satellites.

This report discusses the requirements for coherent combination of signals received at different NAVSPASUR sites, and notes the limits set by equipment, propagation effects, and the characteristics of the NAVSPASUR signals themselves. By coherent combination, we mean determining the relative phases of signals received at different sites. We conclude that little useful information is likely to result from such operation, mostly because it is unlikely that accurate, absolute phases can be determined.

Signals and Processing

Consider the form of a NAVSPASUR signal received at a particular antenna:

\[ S_j(t) = V_j(t)e^{i[2\pi(\nu_{LOj} - \nu_{NS})t + \phi_j(t)]} + N_j(t), \]  

where \( S_j(t) \) is the output signal from antenna \( j \) at time \( t \), after conversion by mixing with the local oscillator (LO).

\( V_j(t) \) is the signal amplitude at time \( t \). \( V_j(t) \) varies slowly compared to the period of the NAVSPASUR carrier frequency (216.98 MHz). Variations in \( V_j(t) \) appear to come mostly from spatial structure of both the transmit and receive beam patterns. This term may also include variations caused by the size and shape of the satellite. The variation time scale of \( V_j(t) \) leads to a finite bandwidth for \( S_j(t) \).
$\nu_{NS}$ is the transmit frequency of NAVSPASUR. $\nu_{LO}$ is the net receiver local oscillator frequency.

$\phi(t)$ is the received phase as a function of time, above and beyond carrier phase (which is the $\nu_{NS}$ term). This term includes Doppler shift (which is an offset from $\nu_{NS}$), propagation path effects, and any phase shift of the reflected signal caused by the satellite. $\phi(t)$ varies slowly also, compared to the period of the NAVSPASUR carrier. This term contains the satellite position information.

$N(t)$ is the receiver noise.

The phase difference between signals received at separate locations contains information about the location of a satellite with respect to those sites. The phase difference can be conveniently extracted by correlating (multiplying) signals received at several sites. This correlation process gives an output of

$$R_{jk}(r) = S_j(t_j) \cdot S^*_k(t_k)$$

$$= V_j(t_j)V_k(t_k)e^{2\pi i[\nu_{NS}r + (\nu_{LO}t_j - \nu_{LO}t_k)]}e^{i[\phi_j(t_j) - \phi_k(t_j + r)]} + \text{cross terms involving noise} \tag{2}$$

$S^*_k(t_k)$ means complex conjugate of $S_k(t_k)$, and $r = t_k - t_j$.

If the noise in the receivers is well-behaved statistically (i.e. a Gaussian random process), the noise terms will not bias any results which we discuss below, but they will set limits on how well any quantity can be determined. The effect of noise can be reduced by averaging several determinations of $R_{jk}$.

If all the contributions to the phase terms in (2) are known, the satellite angular location follows immediately from the $\phi_j - \phi_k$ term and knowledge of the antenna positions. (The transmitter location does not matter, it adds the same phase to both $\phi_j$ and $\phi_k$.) Actually, one determination of $\phi_j - \phi_k$ results in a locus of possible satellite positions. This locus is a hyperbolic surface of revolution, which is the surface of constant phase difference between the two antenna sites. Other information may provide additional constraints on the satellite location. For NAVSPASUR, the satellite must lie in (or at least close to) the plane of the transmitter beam. If signals from several receiving sites are correlated, multiple hyperbolas result, and the satellite must be located at a point common to all of them, i.e. their intersection.

For satellites at distances large compared to the separation between antenna sites, the hyperbolas are nearly tangential to each other, and the satellite position may be constrained to a plane (if only great circle stations are used) or a line (if out of plane stations are used). For low altitude satellites, much better two-dimensional (for great circle sites) or three-dimensional (for out of plane stations) locations will result.
Equipment

The preceding discussion assumes that the instrument (including its surroundings such as the atmosphere and ionosphere) is perfectly calibrated, so that the only unknown contributions to the phase terms come from satellite location and structure effects. In general, this will not be the case. Here we discuss these effects.

Clock Synchronization - In order for an interferometer to work, the signals from the separate antennas must be aligned before correlation to within the coherence time of the signals. The coherence time is roughly 1/B, where B is the bandwidth of the signals. For NAVSPASUR, B is very narrow, so that relative timing accuracy need not be very good in order to correlate the signals. This is not a blessing, however. While it makes relative timing easy, it also means that there is not much useful information to be gained from the correlation in terms of relative timing accuracy. Hence, there is no useful group delay observable.

Since the NAVSPASUR signal is basically monochromatic, it is very nearly a pure sine wave. This quasi-sinusoidal behavior is readily apparent in the processing done by Andrews, who finds that a simple model accounts for most of the signal phase characteristics on time scales of fractions of a second; and in signals sampled with high time resolution, in which the sine wave is seen directly (Andrews and Beda, private communication). Thus, any sample of a NAVSPASUR signal looks almost like any other sample. In other words, the signal will correlate with itself over a large range of time offsets. The correlated output does give the relative phase of the signals, as noted earlier [equation (2)]. However, without additional information, this phase is useless for NAVSPASUR applications because it can be ambiguous by many cycles.

There is some effective bandwidth to the NAVSPASUR signals because of variations in the amplitude of the reflected signal caused by the transmitter antenna pattern and variations in the aspect of the satellite. These time domain variations are observed to have time scales less than 0.1 second. Hence, they result in bandwidths of about 10 Hz in the frequency domain. The signals can be aligned in time to an accuracy of about 0.1 of the reciprocal modulation bandwidth, or about 0.01 second, by looking for the maximum of \( R_{jk} \) as a function of time offset \( \tau \). This accuracy corresponds to some 2 million cycles of phase uncertainty. In terms of group delay measurements of relative arrival times, this accuracy would result in position errors of \( 0.01c \) (\( c = \text{speed of light, 300,000 km per second} \)), or 3000 km. This hardly aids orbit determination!

If each NAVSPASUR site had a GPS timing receiver, they could establish relative time to about 10 nanoseconds (Klepczynski, W. J., 1984, Proceedings of the Sixteenth Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, pp. 385-402). This is equivalent to only 2 cycles of the NAVSPASUR carrier. In the absence of other phase errors, this would allow useful phase information to be obtained about the position of satellites. The cycle ambiguity would be much reduced, to a few cycles. Depending on the orbit and the spacing between antennas, in many cases it would be
possible to unambiguously fix the location of the satellite, or at least the line of position it must occupy. Observations of satellites with known positions could be used to calibrate the actual timing effects.

**ANTENNA LOCATION** – In order to exploit the full precision of phase measurements, the location of the receiving antennas must be known to high accuracy. A location error of size $d$ will cause phase errors of order $\frac{d}{\lambda}$. The exact size of the error depends on the relation between $d$ and the direction to the satellite. The impact on determination of satellite position similarly depends on the site location error.

The NAVSPASUR wavelength is about 1.4 meters. To limit phase errors caused by uncertainties in the location of the antennas, it would be desirable to know the positions of their phase centers to a few centimeters. This level of accuracy is available from Very Long Baseline Interferometer (VLBI) or laser ranging geodetic measurements (VLBI: Clark, T. A. *et al.*, 1985, *IEEE Trans. Geoscience & Remote Sensing*, GE-23, pp. 438-449; laser ranging: Degnan, J. J., 1985, *IEEE Trans. Geoscience & Remote Sensing*, GE-23, pp. 398-413). However, for antennas like the NAVSPASUR phased arrays, the phase center is likely to depend on the elevation and azimuth to a satellite. Careful calibration of this effect would be required, for many directions.

**LOCAL OSCILLATOR** – Any unknown phase offset in the local oscillator (LO) at a receiving site adds to the observed phase as an error. Transmission delays in the receiving antennas or receivers will cause additional phase offsets. These terms can be calibrated by observations of an object whose position is known. They can then be removed from other observations. However, these terms will change with time. How long they are stable sets constraints on calibration intervals.

Antenna and receiver phase errors are likely to change slowly with time. Current NAVSPASUR techniques can largely correct for their effects. Thus, drifts in the phase of the LO will probably set the most serious equipment limits for coherent operation. Since we are considering coherent combination of signals from widely separated sites, there will be no direct link between them. The local oscillators must be generated independently at each site, yet remain in phase with each other.

Atomic frequency standards have the requisite stability. There are three types of these standards readily available today: Cesium (Cs) beam, Rubidium (Rb) vapor, and Hydrogen (H) maser. Of these, masers offer the best stability. The Cs and Rb standards would be useful if the array can be calibrated at intervals on the order of one to a few hours.

The best laboratory Cesium standards have fractional frequency stability $(\frac{\Delta f}{f})$ of order a few parts in $10^{-14}$ for time scales of order $10^4$ - $10^5$ seconds (Figure 9.14 in *Interferometry and Synthesis in Radio Astronomy* by Thompson, Moran, and Swenson (1986, John Wiley & Sons, New York). These standards would keep NAVSPASUR in phase (phase error less than about 0.1 turn) for many (5 to 10) hours. Commercial Cs standards are factors of 5 to 10 less stable (HP 5061A specifications in Hewlett-Packard catalog; Thompson *et al.* (*op.

The best rubidium standards are almost as good as the laboratory cesium standards for time scales up to a few times $10^4$ seconds (HP 5065A specifications from Hewlett-Packard; and R. Beard, private communication). These Rb standards would keep NAVSPASUR in phase for up to 5 hours.

Hydrogen masers provide much better frequency stability than any Cs or Rb standards, with $\delta f$ of less than $10^{-15}$ for times scales of $10^3$ to several times $10^5$ seconds. This stability means that H masers used as LO references would provide several days of coherent operation of NAVSPASUR before recalibration was required. We thus deem H masers highly desirable if such operation is to be implemented. Periodic calibration, by observations of satellites with well-known orbits, would be required for NAVSPASUR operation, regardless of type of frequency standard. It is unknown to us if there are an adequate number of satellites that could be used for this calibration. In the case of masers, calibration would probably need to be performed no more often than daily.

The size of the ionospheric perturbations, discussed next, makes most of the above discussion academic. However, our intent here is to note all aspects of coherent combination of NAVSPASUR signals, and independently identify all requirements.

**Propagation Errors**

**Neutral Atmosphere** – At the zenith, the earth's atmosphere is equivalent to about two meters of propagation path length because the index of refraction of air is not unity. At lower elevations, this path length increases, by about a factor of $1/\sin(\text{elevation})$. By measuring temperature and pressure, it is possible to model this delay to much better than one percent (Davis, J. L. et al., 1985, *Radio Science*, 20, pp. 1593-1607). Except at the lowest elevations (< 10 degrees), the error in the path delay model will be so small that it would not compromise coherent NAVSPASUR operation.

**Ionosphere** – The ionosphere causes propagation delays which are a function of frequency. At the NAVSPASUR frequency, these delays are so large and unpredictable, that they will set the limits on how well this technique will work. In fact, they probably preclude useful phase connected operation.

For typical electron densities ($10^{10}$ to $10^{12}$ cm$^{-3}$ at altitudes of 100 to 500 kilometers), propagation through the ionosphere will cause added phase delays of many 10's of cycles along any signal path. However, for propagation paths which are almost parallel, the differential variations will be small. This is why the current system works: at each NAVSPASUR site, the signal paths to the individual elements are nearly identical, so total propagation path lengths can be ignored, and phases received at the antennas have essentially the same relative phases that they would have had if there were no ionosphere.
Thus, these phases correctly determine the angle of arrival of signals. As the propagation paths become more widely separated, the ionospheric phase delays become less correlated and more random.

For coherent combination of signals received at widely separated sites, some type of correction for ionospheric effects would be necessary. Since ionospheric propagation effects have a frequency-squared behavior, a two frequency system could provide the necessary corrections. However, the choice of frequencies would be a compromise between wide frequency coverage (for high accuracy) and narrow coverage (to be sure that there are no lobe ambiguities - i.e. choosing the wrong cycle of phase difference). A frequency agile system would be most desirable, so that the second frequency could be chosen based on prevailing ionospheric conditions. This does not seem feasible in the current NAVSPASUR system, so we do not pursue it further here.

Ionosondes (RF soundings of the ionosphere) could be used for calibration, but they would have to be taken in the direction of the satellite(s) being observed, which is not necessarily known in real time. Any feasibly sized antenna for the ionosondes would not have adequate resolution to isolate the line of site to the satellite, and ionosondes can not fully probe the ionosphere. (i.e. One cannot probe above the peak electron concentration.)

Calibration of the ionosphere from observation of satellites above it is feasible, but only for the particular set of directions to those satellites. Other directions must be estimated from models for the distribution of the ionosphere electrons. NAVSPASUR observations of satellites at lower altitudes (< 500 or 1000 km) would be complicated by the fact that they are in the ionosphere, so total electron content (TEC) measurements would require additional interpolation for height. Something proportional to TEC is routinely determined by observations of geostationary satellites, but only in certain directions, and a model for the earth's magnetic field is needed to interpret the results, thereby adding one more layer to the problem.

Satellites of the Global Positioning System (GPS) transmit at two frequencies. This allows ionospheric path delays to be determined, but only for the instantaneous directions to the satellites. Observations of these satellites could map TEC, but with crude time and angular resolutions. When the full GPS constellation is operational, there will typically be 5 or 6 satellites visible at any one time from any one site. Again, models for the distribution of electrons in height and angle would be needed to interpolate GPS observations to NAVSPASUR look angles.

The best models for total electron content are unlikely to give values more accurate than 10% (Andrews, M. A., in Workshop on Current Ideas in Longwave Propagation, proceedings of meeting held at NRL in April, 1986). Since the total differential ionosphere between NAVSPASUR sites is likely to cause tens to hundreds of turns of phase, this accuracy is not sufficient to even identify the correct phase lobe.

It thus appears to us that full coherent operation of NAVSPASUR is not likely. The phase uncertainties caused by the ionosphere, even after best efforts at calibration, are so
large that one would not know which phase cycle applied to a certain satellite.

More information about actual observing conditions with a coherent NAVSPASUR can be gained from observations with similar systems. We are aware of two similar systems. There have been several successful radio astronomical very long baseline interferometry experiments carried out at 327 MHz. These experiments use independent LO references (Rb standards and H masers) at widely separated sites. The goal of these experiments is to study radio source structure. Only relative phases are necessary for these studies. However, these experiments provide information about such matters as short term phase stability of rubidium standards and H maser oscillators, as well as the phase behavior of the ionosphere. There is little discussion about experimental effects in the literature from these experiments. Briggs (1983, *Astronomical Journal*, 88, pp. 239-242) has found that coherence times range from a few minutes to many tens of minutes on a baseline of several thousand kilometers at 430 MHz. However, it appears that turbulence in the solar corona is responsible for some of the shorter coherence times that Briggs finds.

The Very Large Array in New Mexico, operated for the U.S. National Science Foundation, has been outfitted to operate at 327 MHz also, on baselines up to about 30 km. This system endeavors to provide full phase accountability, and it could provide information about small scale ionospheric irregularities, as well as give a feel for the variation of ionospheric effects for different directions of observation.

### Imaging

An interferometer array can be used to provide images of certain objects. Basically, the interferometer samples the coherence field of the object under observation. If enough samples of that field are obtained, one can determine what the object looks like.

In radio astronomical applications of interferometric imaging, the object being observed has complete spatial incoherence. (By this, we mean that there is no fixed phase relationship between the emission from any one part of the object and any other part.) In this case, there is a simple Fourier transform relation between the coherence function samples and the brightness distribution of the object (Fomalont, E. B. and Wright, M. C. H., 1974, in *Galactic and Extra-Galactic Radio Astronomy*, chapter 10, Verschuur, G. L. and Kellermann, K. I. eds., Springer-Verlag, New York).

In the case of NAVSPASUR signals reflected from a satellite, the scattered radiation will be completely coherent. *i.e.* there exists a fixed phase relationship between samples of the scattered radiation for any two locations in space. For a monochromatic signal and no relative motion between the transmitter, receiver, and satellite, this is readily apparent. For moving objects, this is still the case from instant to instant (perhaps with appropriate corrections for Doppler shift). The slight bandwidth of the reflected signals for NAVSPASUR means that this case holds for time intervals shorter than about 0.01 second or spatial sizes smaller than about 3000 km. We consider only the fully coherent scattering case.
In this case, we may consider that the target satellite reflects a complex field (amplitude and phase) into space. We sample this field at the several NAVSPASUR sites. These samples form a hologram of the target. With enough of them, we might hope to get an image of the target by various means of analyzing the data.

We want samples of the true amplitude and phase at each site. Cross-correlation determines only the relative amplitudes and phases for two sites. Thus, data are combined in the correlation process and some information may be lost (depending on redundancy of baselines), although calibration may be simplified, since only relative phase calibration is needed. However, as noted earlier, such calibration is very difficult for NAVSPASUR, especially when the ionosphere is considered.

Closure phases (the sum of phases around a loop of three or more antennas) are very useful for radio astronomical imaging. Equipment and propagation phases drop out of the closure sum, leaving only combinations of the true coherence field phases. With appropriate analysis, this is enough information to make good maps (Pearson, T. J., and Readhead, A. C. S., 1984, *Annual Review of Astronomy and Astrophysics*, 24, pp. 97-130). Unfortunately, for the case of coherent objects, all closure phases are zero. Hence, we must have accurately calibrated phases on all baselines in order to use phases for imaging.

The amplitudes measured at various stations are potentially useful for imaging, since relative gains are much easier to determine than relative phases. The signal amplitudes represent directly the amplitudes of the coherence pattern. No correlation is needed to recover these amplitudes. In general, amplitudes by themselves are not very useful for image determination. However, if there is *a priori* knowledge about the size or shape of a particular satellite, the amplitude information could help refine that knowledge.

Conclusion

We conclude that coherent combination of signals from the several sites of the U.S. NAVSPASUR is feasible, *i.e.* determination of relative, but essentially uncalibrated, phases. This combination is not likely to give useful results, however. The phase errors introduced by the ionosphere effectively prevent meaningful determination of true position or structure phases. Very accurate timing and *a priori* phase calibration of the instrument would be necessary, even if the ionosphere were not such a problem.