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<p>→ Coherence and chaos in partial differential equations was studied, with particular emphasis on (1) the damped-driven sine Gordon equation and (2) an optically bistable laser cavity. In addition, the propagation of rapidly oscillating nonlinear integrable waves was investigated.</p> <p>The principal results about propagation in an optically bistable ring cavity may be summarized as the identification of the interplay between coherent transverse spatial structures and temporal chaos in the characteristics of the laser beam. → next page</p>			
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Principal mathematical results on the damped-driven sine-Gordon equation include (1) a numerical study of low dimensional chaotic attractors with coherent spatial structures, including dynamical system diagnostics of their time series, and direct numerical measurements establishing that the attractor is well co-ordinatized by a few nonlinear normal modes; (2) complete analytical identification of all homoclinic structures for the integrable sine-Gordon equation; (3) direct numerical detection of homoclinic crossings along the chaotic attractor of the full system.

Principal mathematical results about the propagation of rapidly oscillating integrable waves include (1) the identification and derivation of a Hamiltonian structure for the modulation equations and (2) a study of the process by which singularities are smoothed by dispersion through the injection of additional degrees of freedom into the field.

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FINAL REPORT

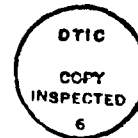
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Final Report -- H. Flaschka  
Statement of the Problems studied

1) The structure of integrable and nonintegrable dynamical systems in the complex domain was investigated. The object is to relate singularities of solutions in the complex domain to the behavior of solutions for real time.

2) The Poisson geometry associated with various dynamical systems was studied. One wants to understand the connection between integrability and the existence of multiple Poisson structures.

3) Work continued on the analysis of the phase-space geometry of a particular dynamical system, the Neumann system. The goal is to understand the symmetries that underlie integrability.

These are all long-term projects. The research is continuing (with students and collaborators) and while the results have been presented in lectures, the final papers are not yet written.

Results

1) a) The Painlevé analysis for Nahm's equations was carried out. This is a rather intricate system arising in the theory of monopoles. The computation relies on representation theory of  $sl(2, \mathbb{C})$ . (To be published)

b) In collaboration with Y. Zeng, the Painlevé analysis for generalized integrable Toda lattices was carried out. The Painlevé data are given Lie-algebraic interpretation. (This work will be completed in China in May 1988.)

c) A student, M. Zou, has used complex domain techniques to prove nonexistence of analytic integrals of perturbed Toda lattices. He has also found various new examples relevant to Painlevé analysis: nonintegrable systems with the "weak Painlevé" property, integrable systems with polynomial Hamiltonian but nonalgebraic first integral, etc. (This work will form part of his Ph.D. thesis, expected by Spring 1989.)

2) a) A student, P. Damianou, has constructed an infinite set of Poisson structures on the Toda lattice phase space. This is a new example, which does not fit the hypotheses used in other work on bi-Hamiltonian structures. (A paper is being written.)

b) In collaboration with T. Ratiu, it was shown that the simultaneous resolution of simple singularities is a momentum map. It turned out that this result was known in representation theory; a

detailed study of the Poisson-geometric implications (not known) is to be part of Damianou's thesis.

3) In collaboration with N. Ercolani, the Neumann system was related to the geometry of Kummer varieties. This was published. Further connections between this geometry and the formalism of Hirota equations are still under investigation.

#### Publications and papers in progress

N. Ercolani, H. Flaschka, "The geometry of the Hill equation and of the Neumann system," Phil. Trans. R. Soc. Lond. A 315, 405-422 (1985)

H. Flaschka, "Remarks on integrable Hamiltonian systems," submitted for publication.

H. Flaschka, Y. Zeng, "Lie-algebraic Kovalevskaya analysis for the Toda lattice," in preparation.

H. Flaschka, "Painleve property of Nahm's equations," in preparation.

Also, Ph.D. theses of Pantelis Damianou and Maorong Zou, in progress.

#### Scientific personnel

N. M. Ercolani, Assoc. Prof., Dept. of Mathematics,  
University of Arizona;

Y. B. Zeng, Assoc. Prof., Dept. of Mathematics,  
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Statement of the Problem Studied.

Coherence and chaos in partial differential equations was studied, with particular emphasis on (1) the damped-driven sine Gordon equation and (2) an optically bistable laser cavity. In addition, the propagation of rapidly oscillating nonlinear integrable waves was investigated.

The principal results about propagation in an optically bistable ring cavity may be summarized as the identification of the interplay between coherent transverse spatial structures and temporal chaos in the characteristics of the laser beam. We were the first to study transverse effects in an optically bistable laser cavity with the natural nonlinear evolution equation of the system. Solitary wave profiles were shown to be the fixed points of an infinite dimensional map which describes the system; these solitary waves form the basis of a projection method which reduced the infinite dimensional map to a two dimensional one, from which the physical characteristics of the fixed points were deduced with extreme accuracy; two competing instabilities (propagational through the nonlinear cavity versus feedback) were identified and analyzed; a chaotic response was identified and its features connected to these instabilities. The above study was carried out in one transverse dimension. At the end of the grant period, a two dimensional study was initiated which is still in progress.

References [1, 3, 5, 13, 17; Ph.D. thesis, H. Adachiara].

Principal mathematical results on the damped-driven sine-Gordon equation include (1) a numerical study of low dimensional chaotic attractors with coherent spatial structures, including dynamical system diagnostics of their time series, and direct numerical measurements establishing that the attractor is well co-ordinatized by a few nonlinear normal modes; (2) complete analytical identification of all homoclinic structures for the integrable sine-Gordon equation; (3) direct numerical detection of homoclinic crossings along the chaotic attractor of the full system. This study is a first in two respects: the first complete mathematical classification of all homoclinic structures for an integrable pde and the first direct numerical correlation of these objects with the chaotic attractors of the perturbed pde. At present we are using the mathematical expressions for these homoclinic structures in analytical (as opposed to

numerical) studies. References [2, 4, 5, 6, 8, 9, 10, 11, 12, 16, 18, 19, 22].

Principal mathematical results about the propagation of rapidly oscillating integrable waves which were obtained during the duration of this grant include (1) the identification and derivation of a Hamiltonian structure for the modulation equations and (2) a study of the process by which singularities are smoothed by dispersion through the injection of additional degrees of freedom into the field. Current work is in progress with N. Ercolani and D. Levermore. [References 7, 14, 15, 18, 20, 21, Ph.D. thesis, Jin Shan].



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18. "Oscillations and Instabilities in Near-Integrable PDE's", (with N. Ercolani and M.G. Forest), *Lect. Appl. Math* 23, 3 (1986).
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## PAPERS FROM THE PERIOD 1984-1987 FOR ARMY GRANT

The main contributions during this period were:

- i. A series of papers developing a method for obtaining macroscopic equations for describing the dynamics of patterns. # 1, 7, 12, 13, 19, 20, 21
  - ii. A series of papers on nonlinear optics. # 2, 4, 5, 8, 9, 18, 22, 23, 24
  - iii. Some new ideas on Turbulent transport. # 6
- 
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22. "A New Class of Instabilities in Passive Optical Cavities" (with J. V. Moloney and A. C. Newell), *Phys. Rev. Lett.* 54, 7 (1985).
23. "Convection of Microstructure and Related Problems" (with G. Papanicolaou and O. Pironneau), *SIAM J. Appl. Math* 45, 5 (1985).
24. "The Origin and Saturation of Modulational Instabilities" (with N. Ercolani and M. G. Forest), *Physica* 18D, 472 (1986).
25. "Focusing Singularity of the Cubic Schroedinger Equation," *Phys. Rev. A* 34, 2, 1200 (1986).
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28. "Oscillations and Instabilities in Near Integrable PDE's" (with N. Ercolani and M. G. Forest), *Lect. Appl. Math.* 23 3 (1986).
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31. "Phase-Pulling and Space-Time Complexity in an AC Driven Damped One-Dimensional Sine-Gordon System" (with A. Mazor and A. Bishop), *Phys. Lett. A* (1987).
32. "Homoclinic Orbits for the Periodic Sine Gordon Equation" (with N. Ercolani and M. G. Forest), to appear in *Physica D*.
33. "A Quasi-Periodic Route to Chaos in a Near-Integrable PDE: Homoclinic Crossings" (with A. Bishop and E. Overman), to appear in *Phys. Lett. A*.
34. "Solitary Waves as Fixed Points of Infinite-Dimensional Maps for an Optical Bistable Ring Cavity: Analysis" (with H. Adachihara, J. Moloney, and A. C. Newell), to appear in *J. Math. Phys.*
35. "Hamiltonian Structure for the Modulation Equations of a Sine-Gordon Wavetrain" (with N. Ercolani, M. G. Forest, and R. Montgomery), to appear in *Duke Math. J.* (1988).

36. "On the Weak Limit of Rapidly Oscillating Waves" (with L. Chierchia and N. Ercolani), to appear in *Duke Math. J.* (1988).
37. "Geometry of the Modulational Instability. Part 1: Local Results; Part 2. Global Results" (with N. Ercolani and M. G. Forest), submitted to *Comm. Pure Appl. Math.*

#### CONFERENCE PROCEEDINGS

1. "Soliton Perturbation Theory" (with A. C. Scott), in Nonlinear Evolution Equations Soluble by the Spectral Transform, ed. F. Calogero, *Research Notes in Mathematics* (Pitman, London, 1978).
2. "A Multisoliton Perturbation Theory" (with A. C. Scott), in Solitons in Action, eds. K. Konngren and A. C. Scott (Academic Press, NY, 1978).
3. "Concrete Periodic Inverse Spectral Transform" (with W. E. Ferguson and H. Flaschka), in Proc. Kyoto Conf. on Theory of Nonlinear Waves, Inst. Math. Sciences, Kyoto University, January 1978.
4. "A Physical Description of the Spectral Transform," in Nonlinear Electromagnetics, ed. G. Uslenghi (Academic Press, 1980).
5. "Overview of Soliton Mathematics," in Proc. Conf. on One-Dimensional Physics, Fribourg, eds. J. Bernasconi and T. Schneider (1981).
6. "Nonlinear Evolution Equations with Rapidly Oscillating Initial Data" (with G. Papanicoloau and O. Pironneau), in Macroscopic Properties of Disordered Media, eds. R. Burridge, S. Childress, and G. Papanicoloau, *Lecture Notes in Physics* 154 (Springer-Verlag, 1981).
7. "Convection of Microstructure" (with G. Papanicoloau and O. Pironneau), in Computing Methods in Applied Sciences and Engineering, V, eds. R. Glowinski and J. L. Lions (North-Holland, 1982).
8. "Solitary Waves with Dispersion and Dissipation," in Proc. Conf. on Structure and Dynamics of Proteins, ed. E. Clementi (1982).
9. "An Infinite Dimensional Map from Optical Bistability Whose Regular and Chaotic Attractors Contain Solitary Waves" (with J. V. Moloney and A. C. Newell), in Chaos in Nonlinear Dynamical Systems, ed. J. Chandra (SIAM, Philadelphia, 1984).
10. "Oscillations and Instabilities in Near-Integrable PDE's" (with N. Ercolani and M. G. Forest) *Lect. Appl. Math.* 23, 3 (1986).
11. "Modulational Instabilities of Periodic Sine Gordon Waves: A Geometric Analysis" (with N. Ercolani and M. G. Forest), *Lect. Appl. Math.* 23, 47 (1986).
12. "On the Construction of Modulating, Multiphase Wave Trains." Proceedings of Conference on Nonlinear Oscillations, University of Minnesota, 1985, eds. D. Kinderlehrer and M. Slemrod (1986).
13. "Weak Limits of Nonlinear Conservation Laws with Oscillating Data," in Proceedings of Conference on Nonlinear Oscillations (with G. Papanicoloau and L. Tartar), University of Minnesota, 1985, eds. D. Kinderlehrer and M. Slemrod (1986).
14. "A Quasiperiodic Route to Chaos in a Near Integrable PDE" (with A. Bishop, M. G. Forest, and E. A. Overman), Proceedings of Los Alamos Conference on Spatial Coherence and Temporal Chaos, 1987, *Physica* 23D, 293-328 (1986).

15. "On the Construction of a Modulating Multiphase Wavetrain for a Perturbed KdV Equation," Proceedings of Conference on Nonlinear Oscillations and Compensated Compactness, University of Minnesota (1987).
16. "Weak Limits of Semi-Linear Hyperbolic Systems with Oscillating Data," Proceedings of Conference on Nonlinear Oscillations and Compensated Compactness, University of Minnesota (1987).

#### OTHER PUBLICATIONS

1. "Theory and Applications of Solitons" (ed. by H. Flaschka), Proc. of NSF Conf. on Solitons, Special Volume of the Rocky Mountain Math. Journal (1978).
2. "Inverse Problems" (ed. D. McLaughlin), Proc. of AMS-SIAM Conf., 1983.

#### INVITED LECTURES

- 1981 Fifth International Symposium on Computing Methods in Applied Sciences and Engineering, Paris.
- 1982 International Conference on Soliton Perturbation Theory, Nice.
- 1982 International Conference on Structure and Dynamics of Proteins, La Jolla.
- 1983 Workshop on Coherence and Chaos, Los Alamos.
- 1983 International Conference on Nonlinear Biophysics, Loma Linda.
- 1983 Analysis Colloquium, Duke University.
- 1984 Western States Mathematical Physics Meeting, Cal. Tech.
- 1984 Lectured at Bucharest Institute for Physics, Romania.
- 1984 Conference on Raman Scattering, Los Alamos.
- 1984 Conference on Applied Solitons, Los Alamos.
- 1984 Applied Mathematics Colloquium, Stanford University.
- 1984 Conference on Nonlinear Transport, Los Alamos.
- 1984 AMS-SIAM Conference on Nonlinear Evolution Eq<sup>s</sup>, Santa Fe.
- 1984 Mathematics Colloquium, Penn State University.
- 1984 Conference on Turbulent Flows, Nice.
- 1984 Inaugural Conference of Nonlinear Research, Berkeley.
- 1985 Conference on Oscillation Theory, Mathematics Institute, Minnesota.
- 1986 Seminar, College of France.
- 1986 Lecture on Dynamical Systems at INRIA Workshop, Paris.
- 1986 PDE Seminar, Ecole Normale Supérieure, Paris.
- 1986 Mathematics Seminar, Université de Paris, XIII.
- 1986 Seminar, Observatoire de Nice.
- 1986 Physics Seminar, University of Montpellier.
- 1986 Mathematics Seminar, Heriot-Watt University, Edinburgh.
- 1986 Aerospace and Mechanical Engineering Seminar, University of Southern California.
- 1986 Lecture at Inaugural MIDIT Workshop, Lyngby, Denmark.
- 1987 Cornell University
- 1987 Mathematical Sciences Research Institute, Berkeley, California.
- 1987 Montpellier, France
- 1987 Oberwolfach, West Germany
- 1987 Cetraro, Italy
- 1987 Joint Summer Research Conferences in the Mathematical Sciences, University of Colorado
- 1987 Second Howard University Symposium on Nonlinear Semigroups, Partial Differential Equations, and Attractors.