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Dear Sir,

I enclose the fourth Periodic Report and Invoice for Contract No. DAJA-86-C0040.

Yours faithfully,

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NONLINEAR WAVE AND DIFFUSION EQUATIONS

Principal Investigator: Dr. J.B. McLeod

4th Periodic Report

November 1987 - May 1988

Contract No. DAJA-86-C-0040

University of Oxford.

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Research Projects

Research during the past six months has comprised investigations into a variety of problems, the most relevant of which are contained in the following list.

Wedge Entry

This is the problem of the entry of a wedge into water, and models the impact of seas on a ship's hull. Dr. McLeod, in conjunction with Professor E.G. Fraenkel, whose visit to Oxford was supported in part by the Grant, made an analysis of the integral equation which governs this model, and much qualitative information about the nature of possible solutions was obtained. This should lead to a result about the existence of solutions, a question which has been unresolved for some 50 years.

One-dimensional Solidification Models

a) Solidification with kinetic undercooling

The classical Stefan model for supercooled solidification can be stabilised at high solidification rates by the mechanism of kinetic undercooling, but this model has received little mathematical attention except for the weak formulation of Visintin [1]. However, in the one-dimensional, one-phase case, it has now been possible to prove the existence and uniqueness of a classical solution which tends to the wellknown solution of the Stefan problem as the kinetic undercooling tends to zero, as long as the solution of the latter problem exists [2].

b) Analogues with the oxygen-consumption problem

The formal time-derivative of the one-phase supercooled Stefan problem (whose solution can blow up in finite time) is the well-posed oxygen consumption problem and this suggests a mathematical regularisation of the Stefan problem based on integrating the solution of the oxygen consumption problem. This regularisation and its possible physical implications are listed in [3].

Continuous Casting

An informal 2-day meeting was arranged in collaboration with Professor D. Bland, Cranfield Institute of Technology, in December 1987. As a result of this meeting and the mathematical Study Group in Heriot Watt University, March 1988, work is in progress on

(i) the numerical investigation of the continuous dependence of the solution of the classical continuous casting model on the mould heat transfer coefficient;

(ii) flux consumption during continuous casting [4].

Explicit Solutions in Two Dimensions

Based on earlier work by Ham, some explicit similarity solutions for two-dimensional alloy solidification have been discovered which may be useful for comparison with numerical experiments [5]. Also conformal mapping techniques have been used to solve and unify several related free boundary problems [6].

Geometric Model for Dendritic Growth

The famous equation $\epsilon\dddot{\theta} + \dot{\theta} = \cos\theta$, $\theta(\pm\infty) = \pm\pi/2$ has been studied numerically and analytically, using a high-precision algorithm to deal with cases when ϵ is small (positive or negative). Some preliminary results which confirm non-existence for $\epsilon \neq 0$ are described in [7], [8].

Off-axis Foci in Lens Design

As a preliminary to obtaining rigorous results concerning the design of axially symmetric lenses with arbitrarily prescribed foci, a formal argument has been prescribed which suggests that when the foci are near to or on the axis of symmetry, the lens must be close to one satisfying the Abbé-Sine or Herschel conditions respectively [9].

References

[1] Visintin, A., IMA J. App. Math., 1985, 35, 233-256.

[2] Xie, W., The Stefan problem with a kinetic condition at the free boundary, Submitted to SIAM J. Math. Anal. 1988.

[3] Fasano, A., Howison, S.D., Ockendon, J.R., Primicerio, M., Some remarks on the regularisation of the supercooled one-phase Stefan problem, Submitted to Quart. App. Math. 1988.

[4] Bland, D., IMA J. App. Math., 1984, 32, 89-112.

[5] Howison, S.D., Similarity solution to the Stefan problem and the binary alloy problem, IMA J. App. Math., 1988, to appear.

[6] Howison, S.D. and King, J.R., Explicit solutions to six free boundary problems, IMA J. App. Math., 1988, to appear.

[7] Hammersley, J.D., 1988 preprint.

[8] C.J. Amick and J.B. McLeod, 1988 preprint.

[9] Van-Brunt, B., The Herschel and Abbé-Sine conditions as limiting cases in lens design, accepted for IMACS conference, Paris, July 1988.