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AD-A196 504

REPORT DOCUMENTATION PAGE (When Data Entered)

1. REPORT NUMBER: ALGO PUB 0120	2. GOVT ACCESSION NO.	READ INSTRUCTIONS BEFORE COMPLETING FORM 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Technical Memo 31, "Calculation the CEP"		5. TYPE OF REPORT & PERIOD COVERED FINAL 6. PERFORMING ORG. REPORT NUMBER D-4710
7. AUTHOR(s) Mathematical Analysis Research Corp. (MARC)		8. CONTRACT OR GRANT NUMBER(s) NAS7-918
9. PERFORMING ORGANIZATION NAME AND ADDRESS Jet Propulsion Laboratory, ATTN: 171-209 California Institute of Technology 4800 Oak Grove, Pasadena, CA 91109		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS RE 182 AMEND #187
11. CONTROLLING OFFICE NAME AND ADDRESS Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		12. REPORT DATE 25 Aug 87 13. NUMBER OF PAGES 8
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE NONE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Dissemination

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Prepared by Jet Propulsion Laboratory for the US Army Intelligence Center and School's Combat Developer's Support Facility.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fix Estimation, Error Ellipse, EEP, CEP, Analysis of Extreme Case, Confidence Level

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report compares the probability contained in the CEP associated with an EEP to that of the EEP at a given confidence level. The levels examined are 50% and 95%. The CEP is found to be both more conservative and less conservative than the associated EEP, depending on the eccentricity of the ellipse. The formulas used are derived in the appendix.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

357-102

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
SOFTWARE ANALYSIS AND MANAGEMENT SYSTEM

CALCULATING THE CEP

TECHNICAL MEMORANDUM NO. 31

MARC

Mathematical Analysis Research Corporation

25 August 1987

National Aeronautics and
Space Administration

JPL

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

JPL D-4710
ALGO_PUB_0120

7057-102

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PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

This specific work was performed in accordance with the FY-87 statement of work (SOW #2).

Calculating the CEP

The calculation of the Circular Error Probable (CEP) in some systems is based on the lengths of the major and minor axes of the Elliptical Error Probable (EEP). The CEP (see Figure 1) is centered about the estimated location of the emitter with the following radius:

$$\text{Radius} = .75*\text{SQRT}[(\text{EEP Major Axis})^2 + (\text{EEP Minor Axis})^2]$$

One measure for determining the accuracy of the CEP calculations is to examine the following two extreme cases:

1) <u>Major axis = Minor axis</u> (in length).
   This means that the uncertainty in the estimated location is equal in all directions. In this case,
   a) The CEP and EEP should have the same size and shape.
   b) The CEP and EEP do have the same shape (circular).
   c) The CEP and EEP do not have the same size. The CEP will contain 12.5% more area (6% further out in all directions) than the EEP. See Figure 2.
   d) The 50% CEP calculated in the above manner will actually contain 54% probability of containing the emitter and the 95% CEP will actually contain 97% probability.

2) <u>Major axis significantly longer than the minor axis.</u>
   In this case, the CEP's radius is approximately 3/4 the length of the longer EEP axis.
   a) The CEP and EEP have completely different shape. The CEP is circular. The EEP is long and thin.
   b) Approximately 14.4% of the area within the EEP will lie outside of the CEP. See Figure 3.
   c) The amount of probability within the CEP will depend on the 'confidence level' of the EEP.
   Two cases of interest are:
      i) EEP with 50% 'confidence level' -- the CEP will contain 62% probability of containing the emitter.
      ii) EEP with 95% 'confidence level' -- the CEP will contain 93% 'confidence'.

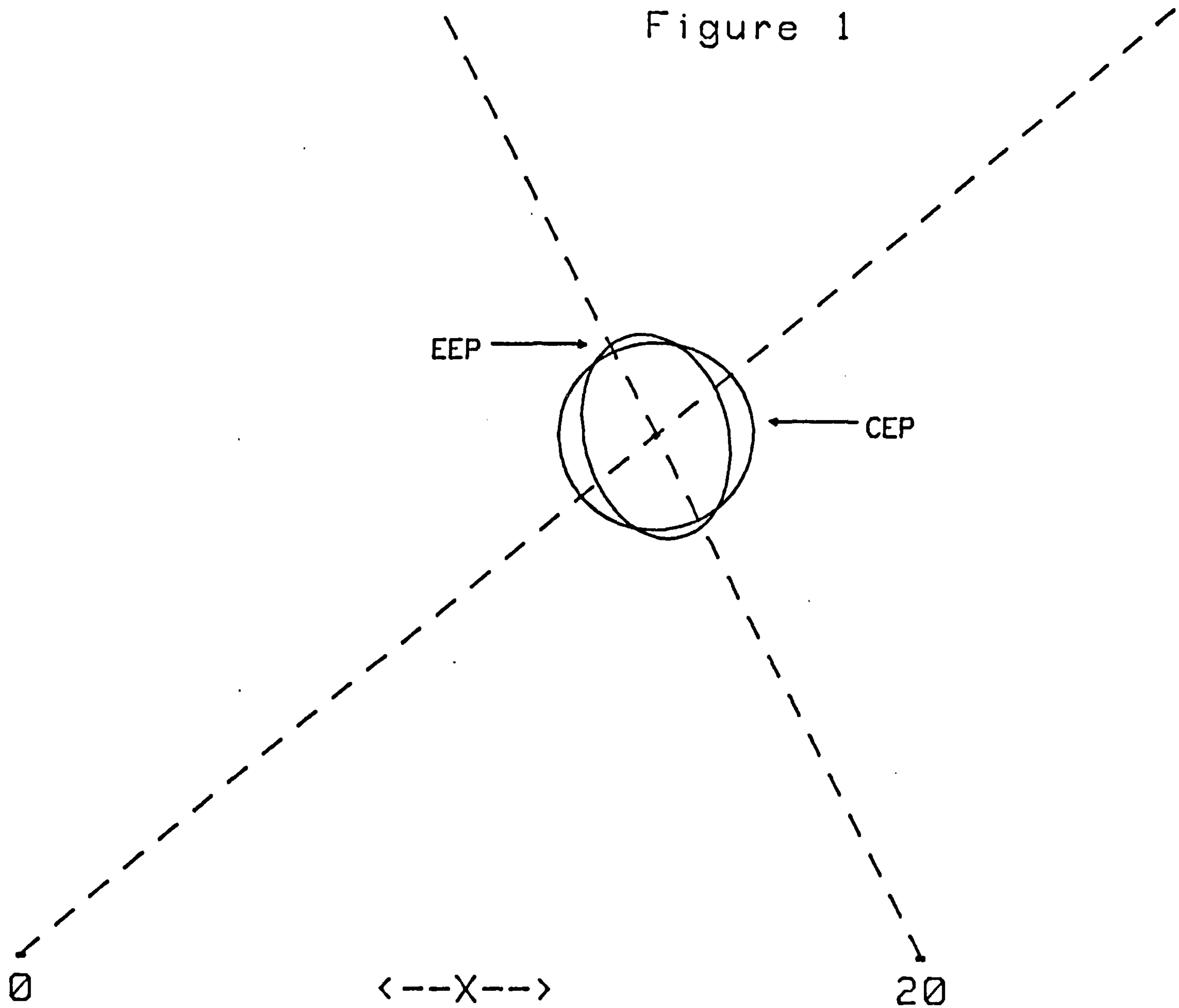
Thus, in the first case, the CEP contains slightly more area than the EEP, while in the second case, the CEP misses some of the area of the EEP. Yet, although the above two cases demonstrate the two extremes for the geometric shape of the EEP, they are not necessarily extreme in terms of their probability of occurring. For instance, given similar standard deviations and a symmetric sensor layout, it is quite likely for an EEP with circular shape to develop. The probability of case 2 (the skewed EEP) occurring is more unlikely, however; either the angular standard deviations must vary significantly among sensors or the sensor layout must be markedly skewed in one direction.

Changing 'confidence levels' of the EEP alters the amount of probability that the CEP will pick up from outside of the EEP. For instance, for a circular shaped EEP with a 50% confidence level, the slightly larger CEP will have approximately 54% confidence associated with it. Similarly, a 95% confidence ellipse (circular shaped) will have a corresponding CEP with 97% confidence (for the derivation of these numbers, see the Math Appendix).

Differences in confidence levels have a more interesting impact on the skewed case, however. Since the CEP may lose as much as 14.4% of the area within the ellipse along the EEP's longer axis, it will also lose some amount of the probability associated with the EEP. But, the CEP will also gain area lying outside of the ellipse (along the EEP's shorter axis) and thus it will gain some probability not associated with the EEP (see Figure 3). The specific amount of probability that the CEP gains outside of the ellipse depends on the confidence level associated with the EEP. For instance, if the skewed EEP has a 50% confidence level, then the amount of probability that the CEP gains outside of the EEP is <u>greater</u> than the probability that the CEP loses from not catching all the probability within the EEP, thus resulting in a CEP with a 62% confidence level. Conversely, if the skewed EEP has a 95% confidence level, then the CEP loses more probability overall than it gains, thus resulting in a CEP with a 93% confidence level.

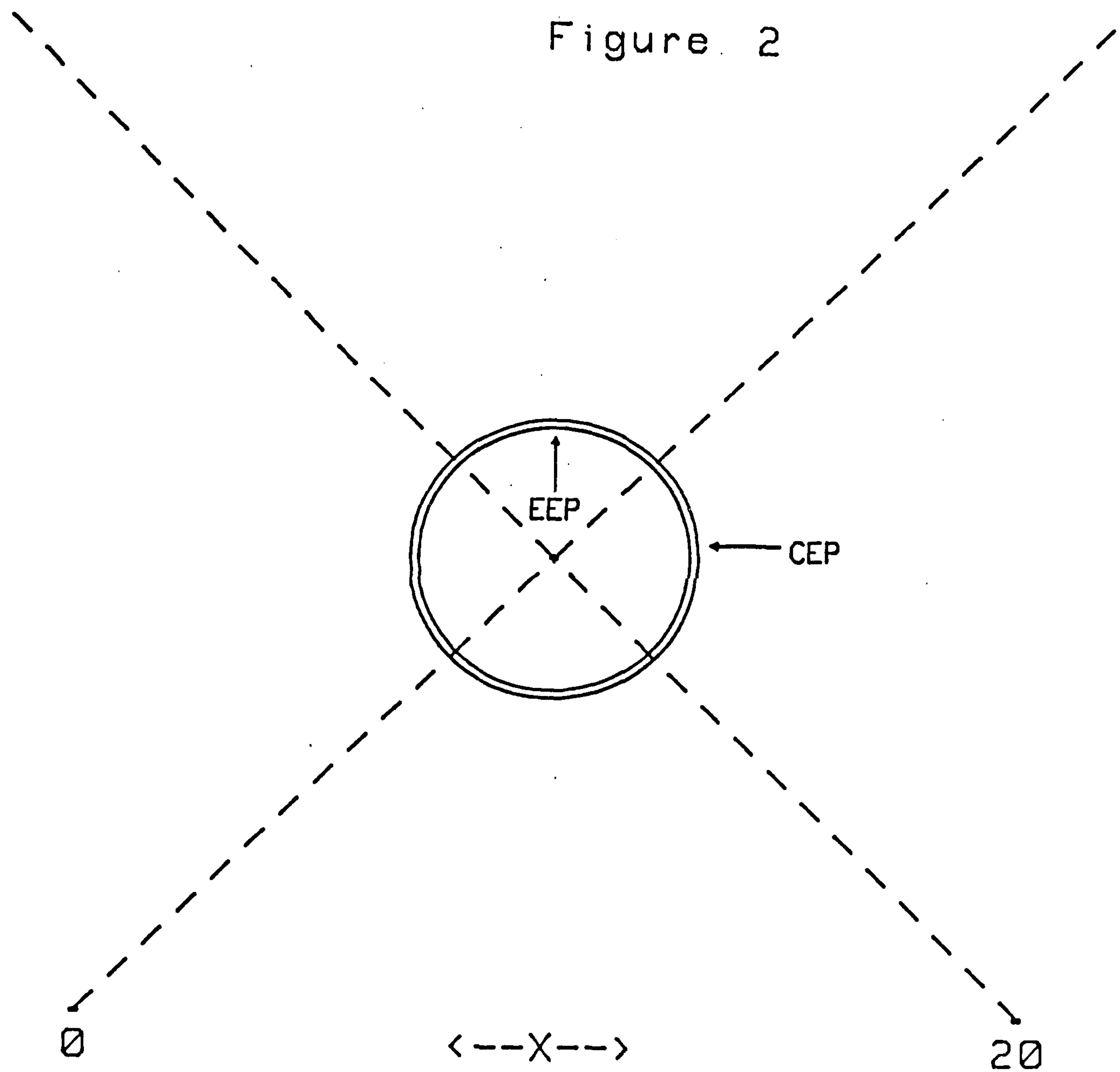
So, the CEP is more conservative at both confidence levels in the circular EEP case. But in the skewed EEP case, the CEP is more conservative at the 50% confidence level but less conservative at the 95% confidence level.

Figure 1



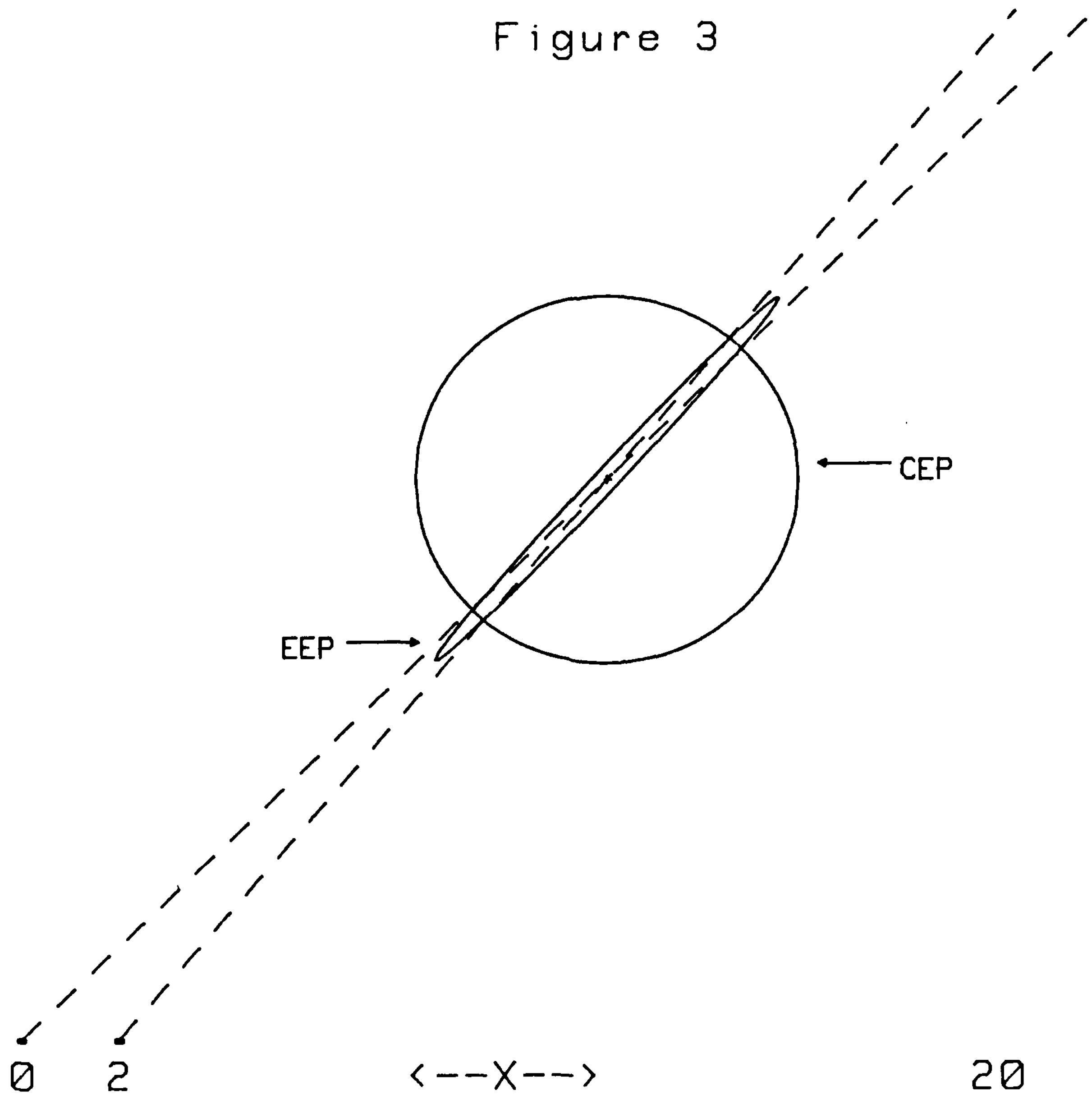
A typical EEP with derived CEP

Figure 2



Case 1: CEPs derived from circular EEPs are 6% larger in all directions.
For a 50% confidence ellipse (like the one shown), the corresponding CEP has approximately 54% confidence associated with it.

Figure 3



Case 2: CEPs derived from skewed EEPs can be as much as 25% shorter along the EEP's longer axis. Thus, the CEP loses some probability by missing the tails of the ellipse but it also gains some from outside of the ellipse.

MATH APPENDIX

Definitions:

p_{eep} = confidence level of the EEP
p_{cep} = confidence level of the CEP
$C(p)$ = chi-squared cutoff at 100p% with 2 degrees of freedom
$\quad = -2\ln(1-p)$
e_a = largest eigenvalue of the inverse covariance matrix
e_b = smallest eigenvalue of the inverse covariance matrix
a = length of EEP's major axis = $\mathrm{SQRT}(C(p_{eep})/e_a)$
b = length of EEP's minor axis = $\mathrm{SQRT}(C(p_{eep})/e_b)$
r = radius of the CEP = $.75*\mathrm{SQRT}([a^2 + b^2])$
N(x) = Standardized cumulative normal distribution

Case 1: Circular EEP

For a circular ellipse, a = b, and r = $.75*\mathrm{SQRT}(a^2+a^2) = 1.06a$

To find p_{cep}, first find the confidence level associated with an ellipse with axes of length r.

$$\mathrm{SQRT}(C(p_{cep})/e_r) = r = 1.06a = 1.06[\mathrm{SQRT}(C(p_{eep})/e_a)]$$

For a circle, $e_a = e_b = e_r$ and hence

$$\begin{aligned} & \mathrm{SQRT}(C(p_{cep})) = 1.06\mathrm{SQRT}(C(p_{eep})) \\ \Rightarrow\ & C(p_{cep}) = (9/8)*C(p_{eep}) \qquad \text{(recall } 1.06=\mathrm{SQRT}(9/8)) \\ \Rightarrow\ & -2\ln(1-p_{cep}) = (9/8)*(-2)\ln(1-p_{eep}) \\ \Rightarrow\ & (1-p_{cep}) = (1-p_{eep})\hat{}(9/8) \\ \Rightarrow\ & p_{cep} = 1-(1-p_{eep})\hat{}(9/8) \end{aligned}$$

Plugging $p_{eep} = .50$ into the above formula, $p_{cep} = .54$. Similarly, $p_{eep} = .95$ implies $p_{cep} = .97$.

Case 2: Extremely Skewed EEP

Perform a transformation so that the ellipse becomes circular where the limit case is more intuitive. The circle nearly becomes parallel lines, each located at 3/4 of the way along the major axis on both sides of the ellipse. Finding the probability between these parallel lines is in effect a 1-dimensional problem with cutoffs at 3/4 the way along the EEP's major axis.

$$\begin{aligned} p_{cep} &\doteq 2*N(.75*a/[\mathrm{SQRT}(e_a)])-1 \\ &= 2*N(.75*[\mathrm{SQRT}(C(p_{eep}))])-1 \\ &= 2*N(.75*[\mathrm{SQRT}(-2\ln(1-p_{eep}))])-1 \end{aligned}$$

Setting $p_{eep} = .50$ implies that $p_{cep} = .62$
Similarly, $p_{eep} = .95$ implies that $p_{cep} = .93$

Explanation:

1) 2*N(*)-1 is area between two tails (at * and 1-* for *>0)
2) $\mathrm{SQRT}(e_a)$ is one standard deviation in the direction of the major axis of the ellipse.