7057-70

U.S. Army Intelligence Center and School Software Analysis and Management System

Comparison Of Fix Methodologies

Technical Memorandum No. 25

MARC

Mathematical Analysis Research Corporation

24 January 1987

National Aeronautics and
Space Administration

JPL

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

JPL D-4622
ALGO_PUB_0095

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER ALGO_PUB_0095	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Technical Memo 25, "Comparison of Fix Methodologies"		5. TYPE OF REPORT & PERIOD COVERED FINAL 6. PERFORMING ORG. REPORT NUMBER D-4622
7. AUTHOR(s) Mathematical Analysis Research Corp. (MARC)		8. CONTRACT OR GRANT NUMBER(s) NAS7-918
9. PERFORMING ORGANIZATION NAME AND ADDRESS Jet Propulsion Laboratory, ATTN: 171-209 California Institute of Technology 4800 Oak Grove, Pasadena, CA 91109		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS RE 182 AMEND #187
11. CONTROLLING OFFICE NAME AND ADDRESS Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		12. REPORT DATE 24 Jan 87 13. NUMBER OF PAGES 7
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Commander, USAICS ATTN: ATSI-CD-SF Ft. Huachuca, AZ 85613-7000		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE NONE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Dissemination

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Prepared by Jet Propulsion Laboratory for the US Army Intelligence Center and School's Combat Developer's Support Facility.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fix Estimation, Minimization of Squared Angular Error, Sine Variation of Squared Angular Error, Weighted Perpendicular Fix, Perpendicular Fix, Taylor Series, Bias, Variance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report four methods of fix estimation are compared. These are: Minimization of squared angular error, Sine variation of minimization of squared angular error, Weighted perpendicular fixing, and Perpendicular fixing. These methods are compared via Taylor series thru the second derivative. By this method the effects of both variance and bias can be compared. This memo is an outgrowth of Tech Memo 20, "Two Dimensional Uncorrelated Bias in Fix Algorithms", DTIC #AD-A189473.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

7057-70

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

Comparison Of Fix Methodologies

Technical Memorandum No. 25

24 January 1987

Author:

MARC
Mathematical Analysis Research Corporation

Approval:

James W. Gillis, Subgroup Leader
Algorithm Analysis Subgroup

Edward J. Records, Supervisor
USAMS Task

Concur:

A. F. Ellman, Manager
Ground Data Systems Section

Fred Vote, Manager
Advanced Tactical Systems

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

JPL D-4622

PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

This specific work was performed in accordance with the FY-87 statement of work (SOW #2).

EXECUTIVE SUMMARY

This Technical Memorandum was prepared to summarize the results of work preformed under both the FY-86 and FY-87 Statements of Work and was funded by the FY-86 funds.

The purpose of this Technical Memorandum is to compare the principal fix estimation methods found in Army Tactical Systems with respect to statistically biased location estimates.

Comparison of Fix Methodologies

INTRODUCTION

In this memo Perpendicular fixing, 2-dimensional Weighted Perpendicular fixing, Minimization of Squared Angular Error, and the Sine variation of minimization of square angular error used by FFIX will be compared. A number of accuracy comparison criteria and assumptions will be used. (Speed of computation will not be addressed. With respect to these criteria the following ranking from best to worst will be determined:

Method	Problem
1) (a) Minimization of Squared Angular Error	2nd Order Bias.
(b) --- Tie --- Sine Variation of 1a above.	2nd Order Bias.
2) Weighted Perpendicular	2nd Order Bias that does not go to zero with increasing sample size.
3) Perpendicular	As above plus a larger first order variance than the other methods

A way of looking at the ranking is that these properties depend on the coefficients of a Taylor Series and other methods share the good properties of the Minimization of Squared Angular Error method as long as their Taylor Series matches to enough terms. Based on matching Taylor Series coefficients around zero angular observed error alone, one would come up with the ranking

Method	Taylor Series Match
1) Minimization of Squared Angular Error	***********
2) Sine Variation of 1)	Second derivative term match at least
3) Weighted Perpendicular	First derivative term match
4) Perpendicular	Constant term match only unless the distance from the sensors to the emitter are equal - in that case there is a first derivative match

The bad properties related to the Perpendicular method are that it has a larger uncertainty (measured by variance) than the other methods and is the most likely to have detectable bias. The variance problem is probably the more serious of the two. Furthermore, Weighted Perpendicular will have a reasonable likelihood of significant bias in many of the same situations as the Perpendicular method.

THE CRITERIA

Typically when one is looking for a 'good' statistical estimator one looks for a minimum variance unbiased estimator.

The first derivative in the Taylor Series expanded about zero errors is used to check for being minimum variance of the four methods under consideration at least to within order sigma squared. This is done by evaluating and truncating the Taylor Series for the fix about zero measured errors. Although truncating the Taylor Series after the first derivative terms may seem restrictive, using more than first derivatives one does not get elliptical Error Ellipses, or the applicability of Chi-square or F-Statistics.

All location estimation methods that yield the true when there is no error in the observed angles and which have Taylor Series about the true must be unbiased to the first order term in the Taylor Series. The second derivative term in the Taylor Series will be used to check for being nearly asymptotically unbiased. Totally unbiased is impossible if the theory is to reduce to intersection of LOBs in the sample size 2 case. Furthermore computing beyond the second derivative is reasonably unimportant. Recall that the entire Error Ellipse theory begins to break down if expansion to the first derivative about the true does not give a reasonable approximation. Finally if one does not compute at least the second derivative then one does not find bias at all assuming that the angular error likelihood function is symmetric about zero. Furthermore, bias is the most serious problem to look for in any estimator so it is a reasonable candidate for a property at the second derivative level.

ADDITIONAL ANALYSIS CRITERIA POSSIBILITIES DISCUSSED

The most obvious additional criteria is to require the second order Taylor Series term to yield no bias. The property of 2 LOB's combining to yield the intersection would have to be abandoned in this case. The problem with these type corrections is that they require knowledge of the true angular standard deviation. (This is obvious in the two LOB case.) Since perfect knowledge of the standard deviation is probably not available this does not appear to be feasible. For reasons of dimensionality the fact that this objective can not be achieved in the 2 LOB case may not rule out the possibility that it could be achieved for 3 or more LOBs. Basically it would require developing an analysis that included both the location estimate statistic and the angular error estimate statistic. This sounds like a difficult problem, but MARC has not really begun to work on it. JPL appears to be more interested in other work at this time.

A complete analysis of the expansion of the Taylor Series to the second derivative term could be examined. This would involve squares of normals. MARC does intend to pursue this analysis from time to time as it useful in analysis of existing algorithms. (As opposed to the bias problem which is really construction of a new Fixing technique.) The important question concerning the "full blown analysis" is how well does the approximation theory work out in the 'tails.' 'Tails' translates to error ellipses at high confidence levels in our 2-dimensional problem.

Another direction being pursued is a generalization of uniformly minimum variance unbiased estimator. It is possible to compute a Cramer-Rao type bound. The concept of sufficient statistics will also be examined. This study will be done in connection with a Senior Thesis at Claremont McKenna College. The proof of the validity of the generalization of the Cramer-Rao bound technique has already been finished at MARC.