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20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

This report examines the combination of two ellipses associated with fixing the same emitter. Extreme cases are analyzed while intermediate cases are simulated. The effect of combination on the confidence level is examined. The meaning of acceptance or rejection of combining the ellipses is discussed in detail.

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U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

Mixing Combination and the Acceptance Test

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PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

Mixing Combination and the Acceptance Test

I. INTRODUCTION

When all of the appropriate hypotheses apply, combinations of 95% confidence ellipses produce resultant ellipses which are themselves 95% ellipses. In other reports it has been shown that when one or more of the hypotheses is violated (such as using estimated covariance matrices instead of the true ones) the result need not be a 95% ellipse. The question addressed in this report is how applying acceptance tests before combination affects the confidence level of the resultant ellipse.

The discussions in this memo are organized as follows:

- Section II - A model of the way combination and testing mix;
- Section III - A discussion of results and conclusions;
- Section IV - Analysis of limit cases of extreme relative size;
- Section V - Discussion of when shape can and cannot be ignored, with an example; (Orientation is also discussed.)
- Section VI - Simulation results plus a discussion of the meaning of the values computed. Some readers may wish to skip to this section.

In a later report the impact of the number of such combinations performed will be analyzed. Also note that in this report the possibility that one might attempt to combine data from different emitters is not addressed. This type of error could also have a significant impact but these impacts will be addressed in a later report.

II. MODEL OF COMBINATION AND TESTING MIXTURE

For this report it is assumed that

(1) There is truly only one emitter.
(2) There is no initiation of new ellipses to consider.
(3) The acceptance test is performed at the 95% level
(4) If the acceptance test says accept then the incoming ellipse is combined with the ellipse already in the data base. In this case the resultant ellipse will be the combined ellipse.
(5) If the acceptance test says reject then the incoming ellipse will be dropped and the original ellipse will be the resultant ellipse.

Finally note that in this report we only discuss the results of bringing in one incoming ellipse one time.

III. RESULTS AND CONCLUSIONS

It appears that confidence levels are somewhat lower when combination and testing are mixed than they would otherwise be.

Theory says that if combination is done without testing, the resultant should capture the true emitter 95% of the time. From this we inferred that:

If passing the acceptance test implies that
=> the combined ellipse contains the true (95+ε)% of the time

Then failing the acceptance test implies that
=> the combined ellipse contains the true (95-19ε)% of the time

However, failing the acceptance test really implies that
=> the combination is not done
=> the original ellipse (failing combination) is used instead
=> the true is caught X% of the time by the original ellipse

(Note that comparing X% versus (95-19ε)% was MARC's first concept for studying this problem.)

X% depends on the relative size of the ellipses being combined. If R denotes the ratio of the major axes of similarly oriented and shaped ellipses then

	Ratio of New/Old R	percent of times true is caught by old if test fails X%	Source
(limit case)	0	0%	Analysis + simulation
	1	about 70%	Simulation
(limit case)	∞	95%	Analysis + simulation

The impact of shape can only be inferred in a general way at this time. It appears, however, that X% for nonsimilar ellipses could be based on ratio if the ambiguity of that ratio in different directions is tolerable. Unfortunately it is not possible to rotate the ellipse to reduce the range of ratios. For example differently oriented ellipses of the same size and shape produce different results depending on orientation which in general are not the same as the R=1 case above. (See the attached examples for an extreme case.)

The natural quantity to compare with X% above is (95-19ε)%. MARC was not able to do enough simulations to determine ε by this method so our results are based on analysis. (Simulations do suggest ε is small or even 0.) Analysis can show that ε=0 for R=0 and for R=∞. Furthermore, symmetry shows that R and 1/R would produce the same value for ε.

What these results suggest is that if the test rejects combination of a large ellipse and a small ellipse when in fact they do come from the same emitter then the large ellipse probably does not contain the emitter but the confidence in the small ellipse is not impaired. If ellipses are comparable in size then one's confidence in both ellipses is impaired but to a lesser extent. Using this information will be difficult, however as other considerations are likely to come into play in practice.

IV. ANALYSIS OF EXTREME CASES IN SIZE RATIO BETWEEN THE TWO ELLIPSES

A. Background Material

The analysis performed in this section shall use results from MARC's report entitled "Testing and Combination of Confidence Ellipses: A Geometric Analysis." In particular in that report it was established that there is an intersection interpretation of the acceptance test where:

1. If one ellipse contains the center of the other the test accepts.

2. If the ellipses do not intersect then the test rejects.
Note there are unresolved cases left by these two criteria. The limit cases we will study have the property that the probability of being unresolved by these two rules goes to zero in the limit and hence can be ignored.

B. Analysis of Cases

Case 1- The ellipse already in the data base is very much smaller than the incoming ellipse.

Fact 1: If combined then the resultant ellipse would be very similar to the ellipse already in the data base.

Fact 2: The cases of interest are those which occur with some reasonable frequency (say perhaps within the 99.99% ellipse.)

Fact 3: For cases that occur with any reasonable frequency (as defined above) and for a sufficiently extreme difference in size between the existing and incoming ellipses, then the acceptance test accepts in approximately the same cases for which the incoming ellipse contains the emitter.

The justification may be formalized (epsilon-delta type arguments) but may more easily be thought of in the following way. Graphs are provided to clarify asertions found here.

i) In the limit of size ratio the true emitter must be relatively close to the small ellipse in the cases of interest. Technically one must decide how many digits of precision on needs then go out to the ellipse of a correspondingly high confidence level (say 99.9999%) and then assert a sufficiently high percentage cases of interest (depending on # of digits of accuracy needed) are accounted for. Although it means going further out in the limit variable before it occurs the 99.9999% ellipse will eventually be 'small' relative to the larger ellipse.

ii) The test accepts in approximately the same cases for which the incoming ellipse contains the emitter and with the ellipse sizes so different intersection means the incoming ellipse probably will contain either all or none of existing ellipse and surrounding region. (Surrounding region meaning the '99.9999% confidence region' and hence would include the true in cases of interest.)

The conclusion that results from this analysis is if combined the resultant ellipse will contain the true emitter location 95% of the time since that ellipse is little different from the original ellipse in the data base. If test fails, however, then one would either be in

i) the theoretically very rare case where the location estimate of the original ellipse is significantly less accurate than suggested by the confidence ellipse

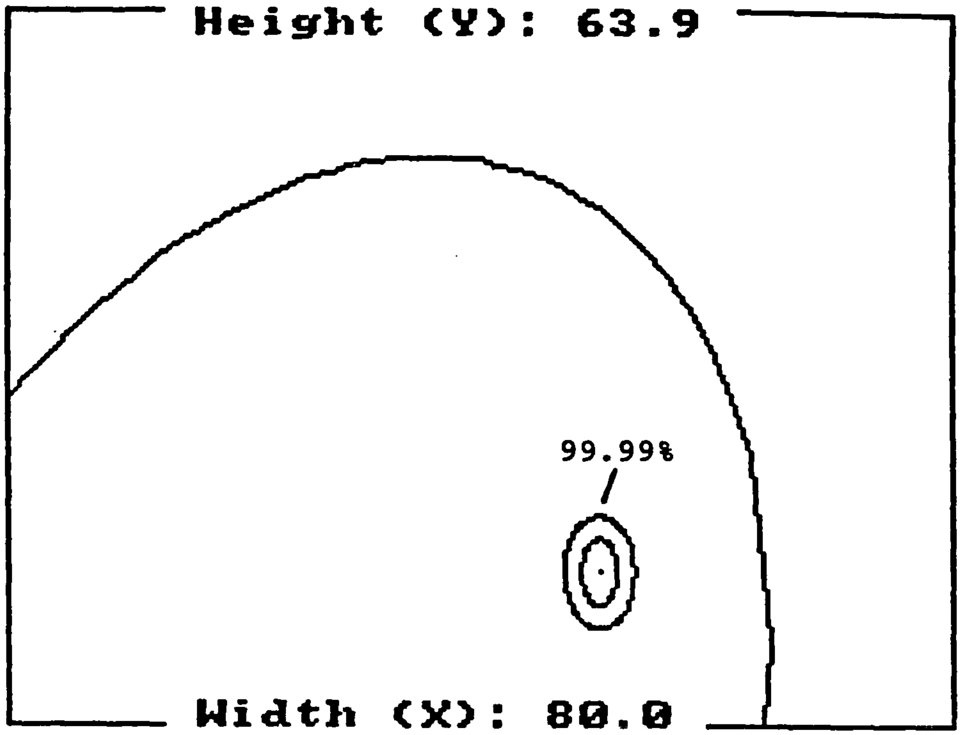
or ii) the case where the incoming ellipse did not contain the true.

In other words 5% of the time one would have incoming ellipses not containing the true. These ellipses might go back into the data base for possible classification as a new entity.

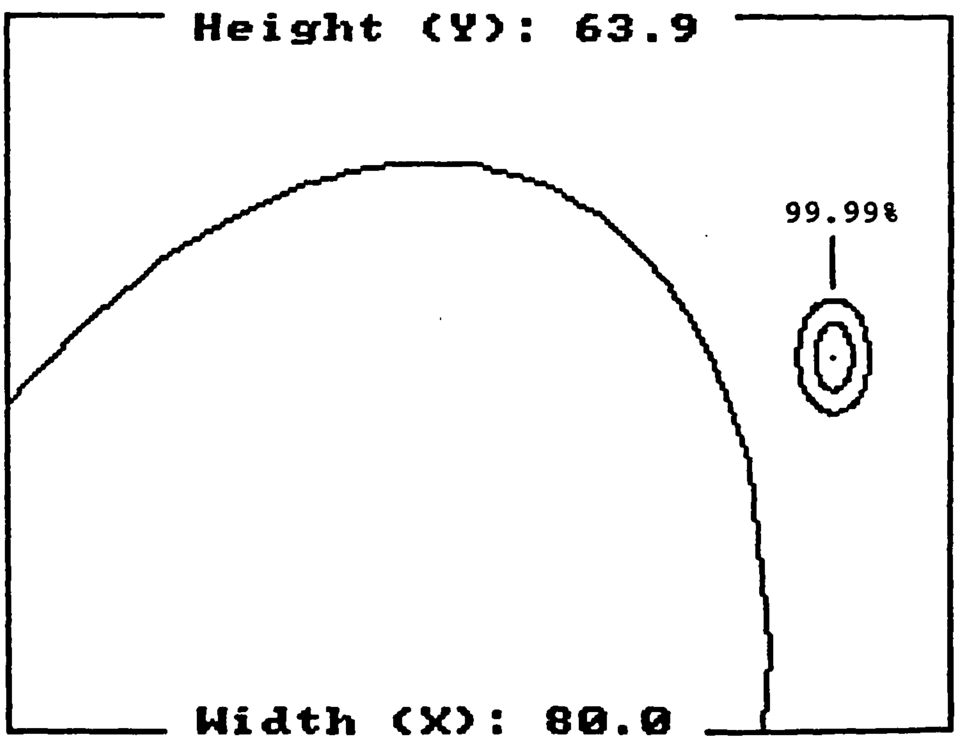
These conclusions are also supported by the pattern of simulations in attachments I and II.

Case 2-The incoming ellipse is much smaller than the ellipse already in the data base.

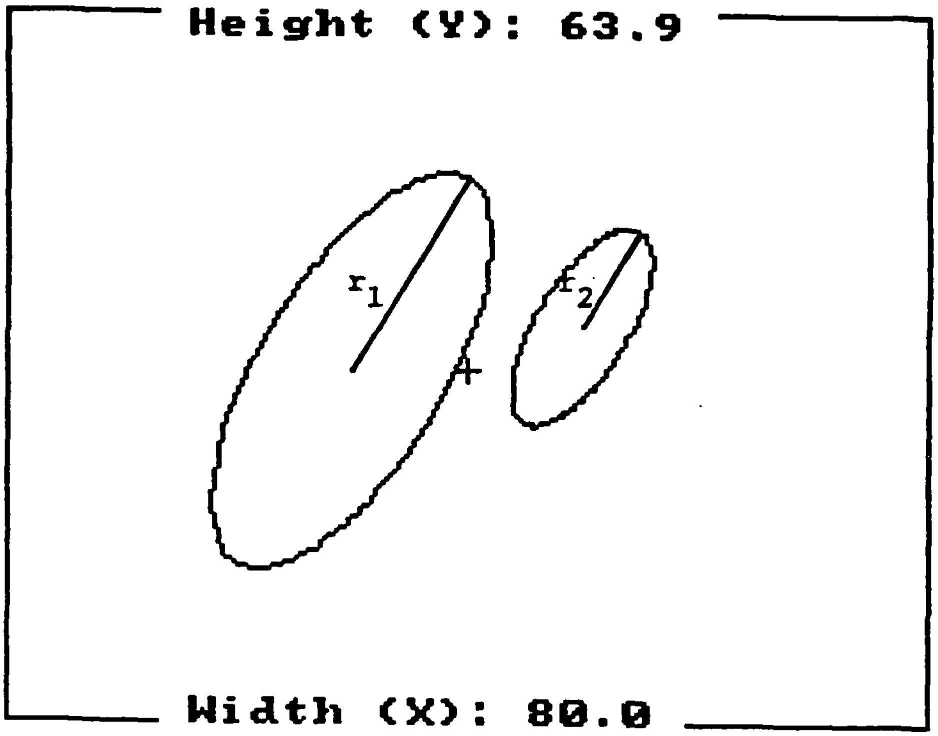
The analysis of this case is the same as the previous case with roles



"ACCEPT"

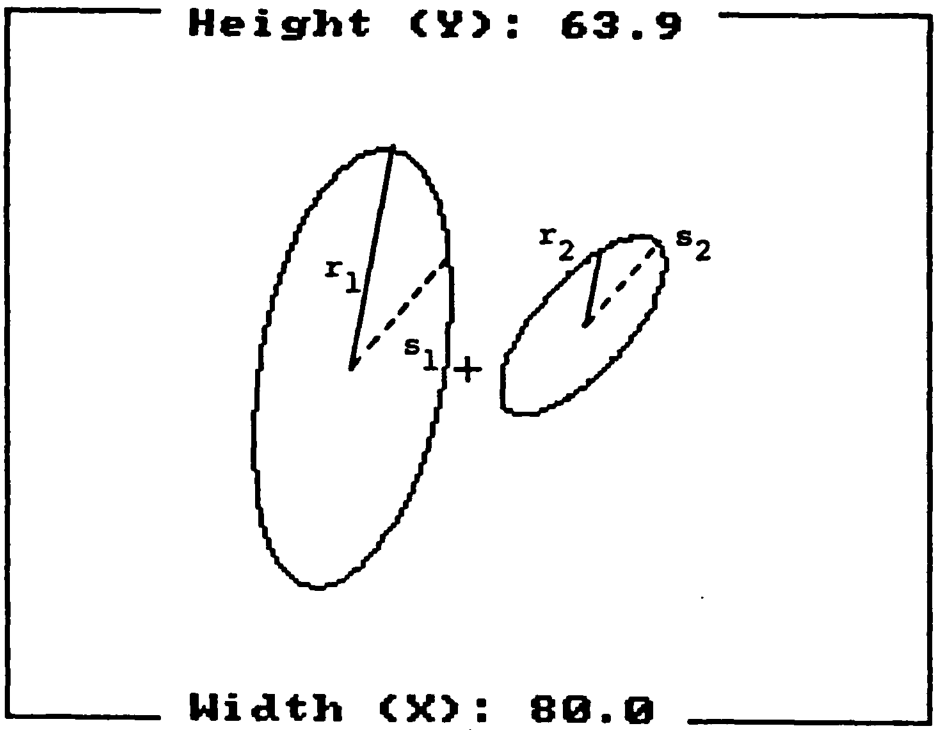


"REJECT"



Ratio of size

$R=r_1/r_2$



Ratio of size

$R=r_1/r_2$?

$R=s_1/s_2$?

reversed. The significance of this reversal of roles is that the acceptance test will pass as long as the original ellipse contains the true emitter location. If original ellipse doesn't contain the true emitter location then it will fail to combine. The larger the original ellipse the longer this problem would be expected to persist.

V. SHAPE AND ORIENTATION

The extreme case analysis of the impact of size was based in the geometry of testing and combination. All the arguments would still apply if the shapes were slightly different. Shape only matters to the extent that it makes the ratio of sizes ambiguous. Orientation also affects the ambiguity of the ratio of sizes as can be seen in example X. In fact this is an extreme, but still important case. The ellipses shown are of equal size but because of orientation the ratio of sizes is ambigous. The results shown for this case are typical.

In cases where the major axes of the two ellipses point in the same direction and in cases where ellipses are nearly round, ratio techniques may be used. Note that two ellipses that have equal axes radii but different compass orientations are different shapes for the purposes of this report. Also note that differences in size outweigh the impact of shape.

VI. SIMULATIONS

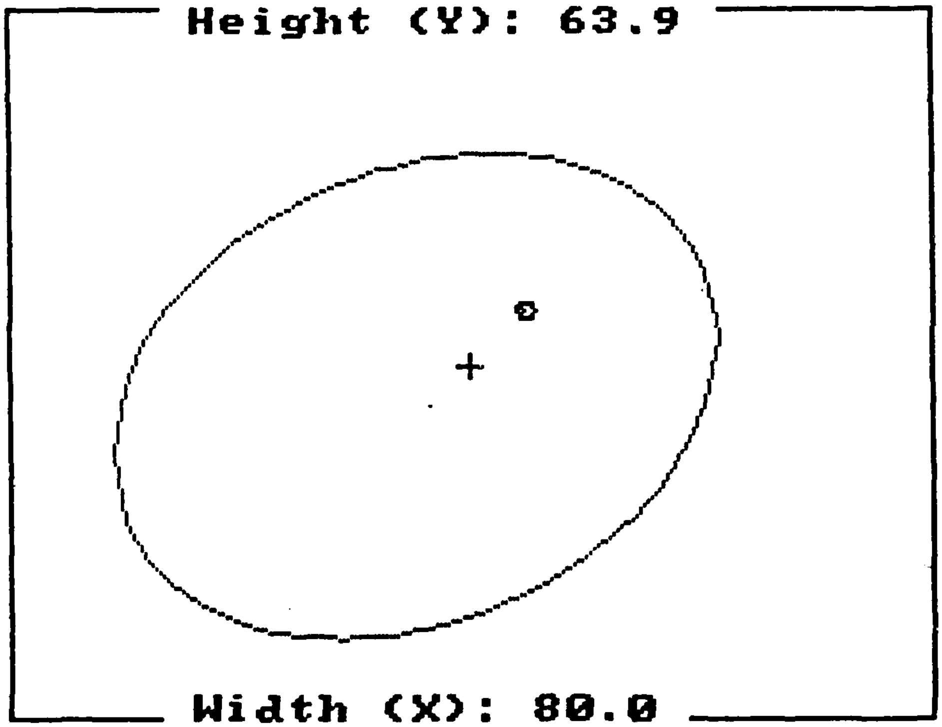
In reading the simulation output reports:

Accepts are the cases where the acceptance test accepts and the combination of ellipses is performed. The number catching the true in this case is the number of combined ellipses catching the true.

Rejects are the cases where the acceptance test accepts and the combination of ellipses is <u>not</u> performed. The number catching the true in this case is the number of times the ellipse preexisting in the database catches the true. (The other ellipse referred to as the 'incoming' ellipse has no impact on the score.)

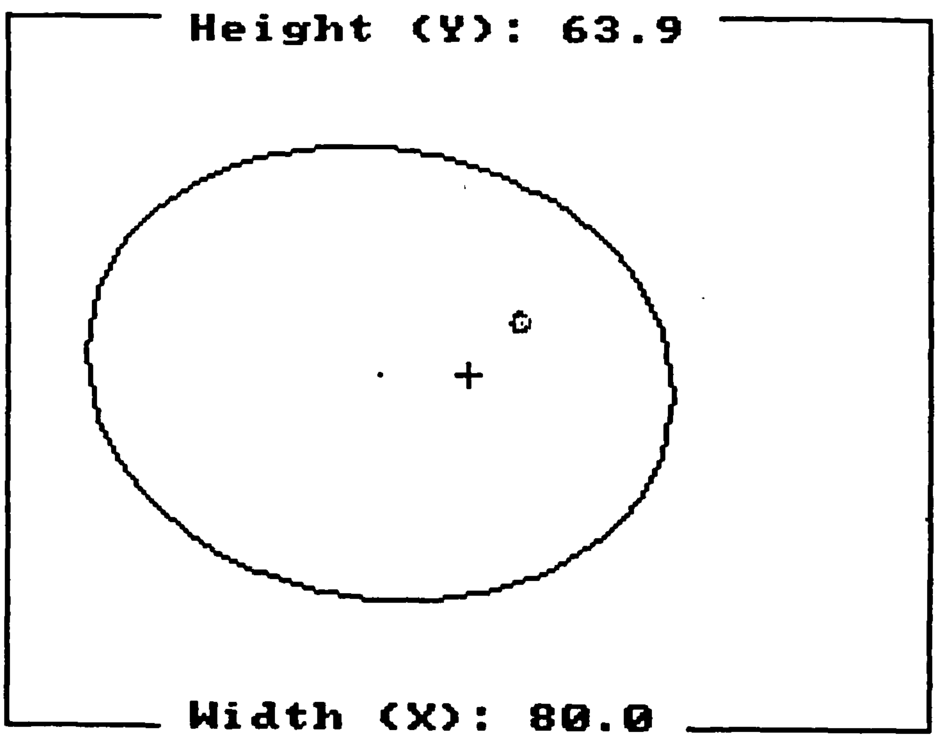
Case Description: Incoming Ellipse is Much Bigger Than The Data Base Ellipse

Acceptance Test Results	Total Number Of Occurences	Number Catching The True	Percentage Catching The True
Accepts	9491	9004	0.9487
Rejects	509	480	0.9430
Totals	10000	9484	0.9484



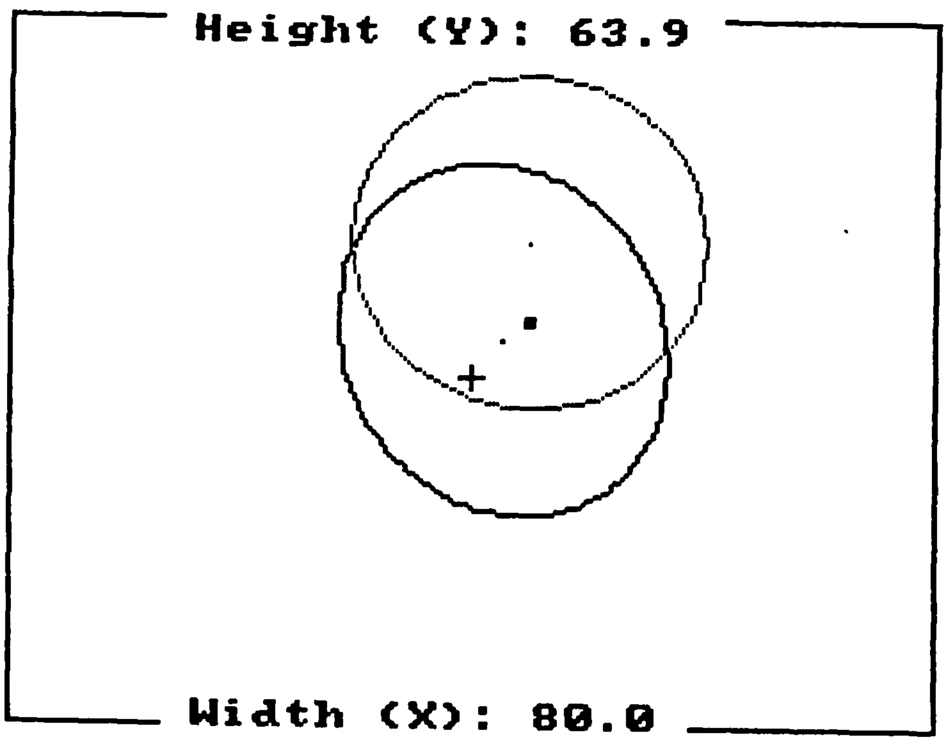
Case Description: Incoming Ellipse is Much Smaller Than The Data Base Ellipse

Acceptance Test Results	Total Number Of Occurences	Number Catching The True	Percentage Catching The True
Accepts	9504	8996	0.9465
Rejects	496	16	0.0323
Totals	10000	9012	0.9012



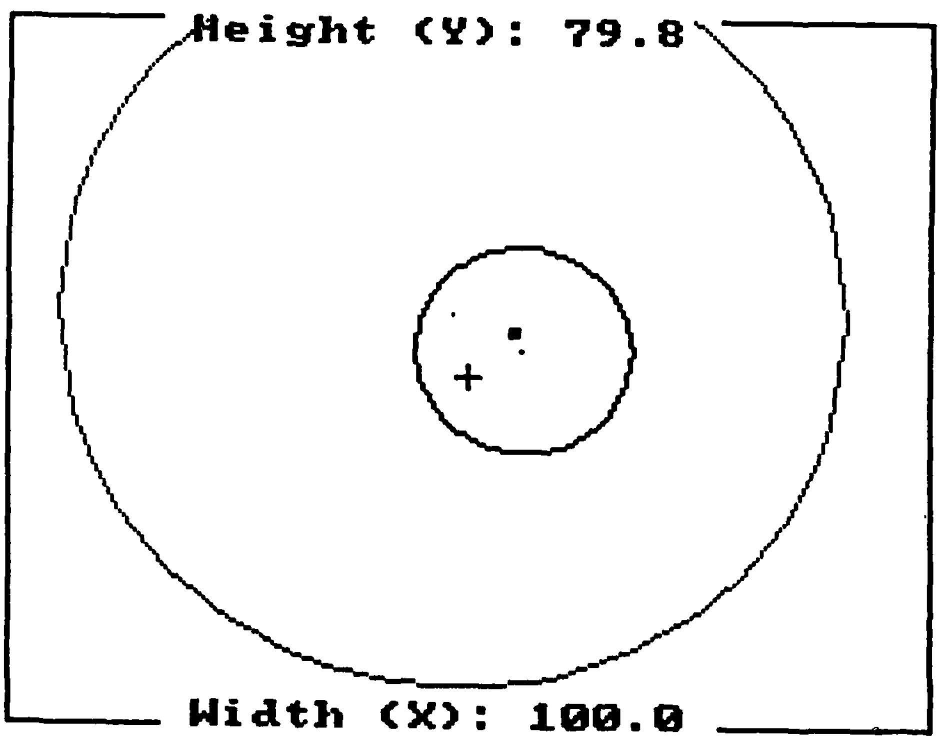
Case Description: Approximately Equal Ellipse Sizes

Acceptance Test Results	Total Number Of Occurences	Number Catching The True	Percentage Catching The True
Accepts	9511	9031	0.9495
Rejects	489	323	0.6605
Totals	10000	9012	0.9012



Case Description: Incoming Ellipse is 5 Times Bigger Than The Base Ellipse

Acceptance Test Results	Total Number Of Occurences	Number Catching The True	Percentage Catching The True
Accepts	9495	9014	0.9493
Rejects	505	482	0.9545
Totals	10000	9496	0.9496



Case Description: Incoming and Base Ellipse Are Orthogonal and 'Thin'

Acceptance Test Results	Total Number Of Occurences	Number Catching The True	Percentage Catching The True
Accepts	9497	8995	0.9471
Rejects	503	261	0.5189
Totals	10000	9256	0.9256