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A RAND NOTE

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ORACLE and Requirements Forecasting, Vol. I: Extensions of ORACLE

Z. F. Lansdowne

May 1988

40 Years 1948-1988

RAND

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ORACLE (Oversight of Resources and Capability for Logistics Effectiveness) is a methodology that was developed to abstract aggregate relations from the U.S. Air Force's computation system for spares and repair requirements. Because ORACLE currently works with only a simplified version of this system, this Note investigates ways of extending ORACLE so that it can work with the actual version.

A RAND NOTE

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ORACLE and Requirements Forecasting, Vol. I: Extensions of ORACLE

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PREFACE

ORACLE (Oversight of Resources and Capability for Logistics Effectiveness) is a methodology that was developed to abstract aggregate relations from the U.S. Air Force's Recoverable Item Requirements Computation System, which is also known as D041. Because the current method for computing the ORACLE database is designed to work with only a simplified version of D041, the purpose of Volume 1 of this Note is to investigate how this method can be modified to work with the actual version. This volume presents technical details, and it is intended for readers who may wish to implement or extend this methodology, or to understand its technical limitations. Volume II examines how an ORACLE-like procedure could be used to improve requirements forecasts.

This Note was prepared for the Office of the Assistant Secretary of Defense for Production and Logistics under the auspices of RAND's National Defense Research Institute, the OSD-sponsored Federally Funded Research and Development Center at RAND. The work was performed under the project titled "Effective Modeling."

SUMMARY

ORACLE (Oversight of Resources And Capability for Logistics Effectiveness) is a methodology that assesses the effects of changing certain resource levels on the peacetime materiel readiness and wartime sustainability of U.S. air forces, to improve the estimation and justification of resource requirements. The Air Force Logistics Command (AFLC) uses the D041 system to help manage approximately 150,000 aircraft components. During each quarter, the D041 system is used to estimate how much of each component should be repaired and purchased for about three years into the future. The goal of the ORACLE methodology is to construct an aggregate database having the following features: The database is an additional product of a standard D041 quarterly computation, it is small enough to fit in a portable microcomputer, and it can be readily manipulated by a spreadsheet-like program to mimic in aggregate form the responses of D041 to program changes.

The current ORACLE methodology was designed to work with only a simpiifed version of D041, not the actual version. D041 is now being modified to incorporate the logic that is in the Logistics Management Institute's (LMI) Aircraft Availability Model (AAM). In particular, D041 will incorporate the AAM's approach to common items, indenture relationships, and an availability objective based upon no cannibalization. Because the current methodology does not allow for any of these new features, this Note investigates how these features can be included.

Altogether, five different methods for developing the ORACLE database are described. The first method is based upon differentiating the necessary conditions for optimality and mean pipeline equations, which is an extension of the approach used by the preliminary version of ORACLE. Unfortunately, the indenture relationships make this approach difficult to implement. The second method is an approximate procedure in which the indenture relationships are essentially eliminated. The third method retains the indenture relationships but in a simplified

form. The fourth is based upon making multiple runs of the AAM for the same year. The fifth method uses regression analysis to analyze data obtained for several years. These methods are compared with respect to simplicity, accuracy, and execution time.

The only method for which there is some computational experience is the final one, regression over time. Volume II discusses the performance of that approach in the context of requirements forecasting.

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TABLE

I. INTRODUCTION

ORACLE (Oversight of Resources and Capability for Logistics Effectiveness) is a methodology that was developed to relate dollars expended on recoverable components to the goals set in the Planning, Programming, and Budgeting (PPB) process.[1] This methodology is designed to systematically abstract aggregate relations between dollars and goals from the U.S. Air Force's Recoverable Item Requirements Computation System, which is also referred to as D041. However, the current version of ORACLE is designed to work with only a simplified version of D041, rather than the actual version.

The Program Objective Memorandum (POM) describes the activities and capabilities to be achieved over a five-year period, as well as the time-phased resources required to achieve them. The Air Force Logistics Command (AFLC) uses the D041 computation as the basis for its inputs to the POM. However, the D041 computation makes forecasts for only about three years beyond the asset cutoff date, while the POM years extend to about seven years beyond asset cutoff. In the past, a cost-per-flying-hour rate has been used to forecast POM requirements beyond D041's horizon, but in recent years these forecasts have been substantial underestimations.

This Note will investigate the following two questions:

1. How can the ORACLE estimation methodology be modified so that it can work with the actual version of D041?
2. How can an ORACLE-like procedure be used to improve the accuracy of the POM requirements forecasts?

Volume I will consider the first question, and Vol. II will consider the second.

To develop the ORACLE database for a weapon system (or end item) included within D041, the first step is to identify a set of independent variables Z_m, $m = 1, \ldots, M$ (such as an availability target and total flying hours) and a set of dependent variables D_n, n = 1, ..., N (such as buy requirement, serviceable assets, and base repairs). The second step is to run D041 for a nominal case, which means for a specified set of values for the independent variables for each weapon system. And the final step is to compute the partial derivative of each dependent variable with respect to each independent variable, as evaluated at the nominal case. The ORACLE database for each weapon system consists of the collection of partial derivatives $\partial D_n/\partial Z_m$, where m = 1, ..., M and n = 1, ..., N.

After the ORACLE database has been constructed, the outputs of D041 can be approximated in the following way. For a given weapon system, let Z_m^0 be the value of the mth independent variable for the nominal case, and let D_n^0 be the corresponding value of the nth dependent variable as computed by D041. If a scenario is considered for which the independent variables are Z_m, for m = 1, ..., M, then the corresponding value of the nth dependent variable that would be computed by D041 can be approximated with

$$D_n = D_n^0 + \sum_{m=1}^{M} \partial D_n/\partial Z_m (Z_m - Z_m^0). \qquad (1)$$

D041 is now being modified to incorporate the logic that is currently in the Logistics Management Institute's (LMI) Aircraft Availability Model (AAM).[2] In particular, D041 will incorporate the AAM's approach to common items, indenture relationships, and an availability objective based upon no cannibalization. Because the current method for computing the ORACLE database does not allow for any of these features, one purpose of this Note is to investigate how these new features could be included. Next, each of these features is described separately.

The term "common item" refers to an item that is installed on more than one weapon system. Thus when a common item is backordered, the availabilities of several weapon systems may be degraded. When a spare unit of a common item is procured, however, several weapon systems share the benefit. Based upon the relative amount of flying hours, the approach taken by the AAM is to allocate a portion of a common item's backorders and cost to each associated weapon system.

The indenture relationships indicate which items are subassemblies of other items. A Line Replaceable Unit (LRU) refers to a component that can be removed and replaced on the flight line; a Shop Replaceable Unit (SRU) refers to a subassembly that can be removed and replaced in a repair shop. The AAM treats the effect of LRU backorders differently from that of SRU backorders. Although an LRU backorder affects aircraft availability, an SRU backorder lengthens LRU repair times, thus lowering the probability of an LRU spare being in a serviceable condition.

The objective function used by the AAM is the probability that a random aircraft is not missing any of its reparable LRUs, assuming that there is no cannibalization. In particular it is assumed that the holes in the aircraft, because of a lack of serviceable LRUs, are uniformly spread over all appropriate aircraft. This objective function depends upon the expected numbers of LRU backorders at the base echelon, which in turn depend upon the expected numbers of LRU backorders at the depot echelon and upon the expected numbers of SRU backorders at both the base and depot echelons.

Section II will state the equations in the AAM, as well as the underlying assumptions and approximations. This model has the structure of a constrained nonlinear programming problem, and it is solved using a marginal analysis technique.

Section III will present a method for computing the ORACLE database for the AAM based upon differentiating the necessary conditions for optimality and mean pipeline equations, which is an extension of the approach used by the current version of ORACLE. Because only LRUs are considered in the current version, that version is able to compute the ORACLE database by treating only one item at a time. However, one effect of including indenture relationships is to couple the partial derivatives for an LRU with those for all associated SRUs. In that case, computing the ORACLE database requires solving a system of simultaneous linear equations for each LRU in the weapon system.

Because the indenture relationships make the foregoing approach difficult to implement, Sec. IV will discuss four alternative estimation procedures: The first method treats the SRUs as though they were LRUs, which essentially eliminates the indenture relationships; the second retains the indenture relationships but in a simplified form; the third is based upon making multiple runs of the AAM for the same year; and the fourth uses regression analysis to analyze data obtained for several years.

Altogether, Secs. III and IV will present five methods for constructing the ORACLE database. Section V will compare these methods with respect to simplicity, accuracy, and execution time.

II. AIRCRAFT AVAILABILITY MODEL

This section will describe the formulas and assumptions underlying the Aircraft Availability Model (AAM) based upon the most recent documentation for that model.[2] Because many of these formulas were not explicitly stated in this documentation, it was necessary to verify their accuracy by checking with the developers of the AAM. Consequently, this section may provide the most complete mathematical statement of the AAM that is available.

The following simplifying assumptions will be used:

1. There is only one level of indenture, which means that each SRU is a subassembly of an LRU, rather than of another SRU;
2. An SRU can be a subassembly for only one type of LRU.

Although AAM does not require either of these assumptions, they will be used to simplify the presentation given here.

OBJECTIVE FUNCTION

The objective function of AAM will be written as a direct function of the expected numbers of LRU backorders at bases. However, it will be shown later that the expected numbers of LRU backorders at bases depend upon the expected numbers of LRU backorders at the depot echelon and upon the expected numbers of SRU backorders at both the base and depot echelons.

A weapon system refers to a particular mission and design of an aircraft, which may have more than one series. For a given weapon system, the index i will refer to the ith LRU and the index k will refer to the kth series. The index e will refer to the repair echelon: e = 1 represents the base echelon and e = 2 the depot echelon.

Suppose that a particular weapon system has been specified and define

a_{ki0} = quantity per application of LRU i on series k

b_{ki} = fraction of aircraft in series k that includes LRU i

n_k = number of aircraft in series k.

Thus the number of units of LRU i installed on aircraft of series k is

$$t_{ki0} = a_{ki0} b_{ki} n_k .$$

If K is defined to be the number of series in the specified weapon system, then the number of aircraft in that weapon system is

$$N = \sum_{k=1}^{K} n_k .$$

Define

EBO_{i0e} = expected number of backorders for LRU i at echelon e.

Thus EBO_{i01} refers to the expected number of backorders for LRU i at all bases. Because the AAM allows the possibility that LRU i could be installed upon several series of the same weapon systems, or upon different weapon systems, it is necessary to compute the fraction of the total backorders that is associated with a particular series of a particular weapon system, which is done as follows. First define

F_k = number of flying hours for series k of the specified weapon system

IP_i = number of flying hours accumulated by units of LRU i over *all* weapon systems

and then compute the use factor

$$U_{ki} = F_k / IP_i .$$

The AAM considers $U_{ki}EBO_{i01}$ to be the expected number of backorders for LRU i that is associated with series k of the specified weapon system.

Assuming that the backorders are randomly distributed over the available aircraft, then the probability that an aircraft of series k is not missing a unit of LRU i is

$$q_{ki0} = 1 - b_{ki} + b_{ki}(1 - U_{ki}EBO_{i01}/t_{ki0})^{a_{ki0}}.$$

The probability that a randomly chosen aircraft in the specified weapon system is not missing a unit of LRU i is

$$Q_{i0} = \sum_{k=1}^{K} (n_k/N)q_{ki0}.$$

If I is defined to be the number of LRUs in the weapon system and it is assumed that the various LRUs fail independently, then the probability that a randomly chosen aircraft is not missing any of its reparable LRUs is

$$A = \prod_{i=1}^{I} Q_{i0}. \tag{2}$$

This expression is a measure of aircraft availability without cannibalization and is the objective function that will eventually be maximized.

EXPECTED SRU BACKORDERS AT DEPOT

The index pair (i, j) will be used to refer to the jth SRU belonging to the ith LRU. However, if j = 0, then (i, 0) will refer to the ith LRU. Define

λ_{ije} = daily demand rate for SRU (i, j) at echelon e

DC_{ij} = depot condemnation rate for SRU (i, j)

DL_{ij} = depot procurement time for SRU (i, j)

RT_{ije} = average repair time for SRU (i, j) at echelon e

x_{ije} = stock level for SRU (i, j) for each site at echelon e

m_{ije} = mean pipeline for SRU (i, j) for each site at echelon e.

The depot pipeline for an SRU consists of the units being repaired at the depot, plus those on order from a commercial vendor. Thus the mean depot pipeline for SRU (i, j) is

$$m_{ij2} = \lambda_{ij2}[RT_{ij2}(1 - DC_{ij}) + DL_{ij}DC_{ij}], \tag{3}$$

where e = 2 refers to the depot echelon.

Define

$p(y|m_{ije})$ = probability of having y units in the pipeline given that the mean is m_{ije}.

The AAM uses either a Poisson or negative binomial distribution as the probability distribution for each pipeline. Given the stock level x_{ij2}, the expected backorders for SRU (i, j) at the depot are given by

$$EBO_{ij2} = \sum_{y>x_{ij2}} (y - x_{ij2})p(y|m_{ij2}).$$

EXPECTED LRU BACKORDERS AT DEPOT

The depot pipeline for an LRU consists of the units on order, plus those being repaired at the depot, plus those awaiting SRUs at the depot before being repaired. However, AAM makes the approximation that the calculation of LRU depot backorders can be *uncoupled* from that for SRU depot backorders. In particular, the mean depot pipeline for LRU is computed as

$$m_{i02} = \lambda_{i02}[RT_{i02}(1 - DC_{i0}) + DL_{i0}DC_{i0}], \tag{4}$$

which is the same as Eq. (3) except that j = 0.

Later in Eq. (6), the mean SRU backorders at the depot, which should be affecting LRU backorders at the depot, are added to the mean LRU pipeline at the base echelon. In effect, AAM assumes that each additional SRU depot backorder will cause an additional LRU to be awaiting parts at the depot, causing an additional LRU to be backordered at the depot, adding one unit to the mean LRU pipeline at the base echelon. This approximation will overestimate the mean LRU base pipeline for two reasons: A single LRU may be awaiting parts for more than one SRU, and an additional LRU that is awaiting parts may not necessarily cause an additional LRU to be backordered.

Given the stock level x_{i02} and ignoring the SRU contribution, the expected backorders for LRU i at the depot is

$$EBO_{i02} = \sum_{y>x_{i02}} (y - x_{i02})p(y|m_{i02}).$$

EXPECTED SRU BACKORDERS AT BASES

Define

RTS_{ij} = fraction reparable this station (at base level) for SRU (i, j)

OST_{ij} = order and ship time from depot to base for SRU (i, j)

OIM_{ij} = fraction of depot demand for SRU (i, j) originating from bases

NB_{ie} = number of activities that use LRU i at echelon e.

The AAM treats the depot echelon as though it is a single aggregated entity, which corresponds to having $NB_{i2} = 1$. However, multiple bases are allowed, which corresponds to having $NB_{i1} \geq 1$.

The base pipeline for an SRU consists of the units on order from the depot, plus those being repaired at the base. However, the units on order must include the requisitions that have been backordered at the depot, as well as those in transit between the depot and base. Because OIM_{ij} is the fraction of depot backorders that correspond to orders received from the base echelon, the mean pipeline for SRU (i, j) at an average base is computed as

$$m_{ij1} = \{\lambda_{ij1}RTS_{ij}RT_{ij1} + \lambda_{ij1}(1 - RTS_{ij})OST_{ij} + OIM_{ij}EBO_{ij2}\}/NB_{i1}, \tag{5}$$

and the expected backorders at all bases are

$$EBO_{ij1} = \sum_{y>x_{ij1}} NB_{i1}(y - x_{ij1})p(y|m_{ij1}).$$

EXPECTED LRU BACKORDERS AT BASES

The base pipeline for an LRU consists of the units on order, plus those being repaired at the base, plus those awaiting parts at the base before being repaired. The units on order include the requisitions that have been backordered at the depot, plus those in transit between the depot and base.

The AAM makes two key assumptions when computing the mean LRU base pipeline: One LRU at a base is awaiting parts for each backordered SRU at that base, which ignores the possibility that a single LRU could be waiting for several SRUs; and the fraction $(1 - OIM_{ij})$ of the backordered SRUs at the depot will have the same effect as the same number of backordered LRUs at the depot, which enables the calculation of depot SRU backorders to be uncoupled from that for depot LRU backorders. For simplicity, this section has also assumed that each SRU can be associated with only one type of LRU. Under these circumstances, the mean pipeline for LRU i at an average base is

$$m_{i01} = \{\lambda_{i01} RTS_{i0} RT_{i01} + \lambda_{i01}(1 - RTS_{i0}) OST_{i0}$$

$$+ EBO_{i02} + \sum_{j=0}^{J(i)} [EBO_{ij1} + (1 - OIM_{ij}) EBO_{ij2}]\}/NB_{i1}, \quad (6)$$

where J(i) is the number of SRUs associated with LRU i. Thus the expected backorders for LRU i at all bases are

$$EBO_{i01} = \sum_{y>x_{i01}} NB_{i1}(y - x_{i01}) p(y|m_{i01}).$$

SAFETY STOCK CONSTRAINTS

The safety stock is defined to be the difference between the stock level and the mean pipeline; that is, $x_{ije} - m_{ije}$. Although not mentioned in the AAM documentation, the planned D041 implementation of the AAM will restrict the safety stock to be nonnegative. For completeness, this section will allow an upper bound restriction to be present as well. Define

$S(m_{ije})$ = pipeline standard deviation for SRU (i, j) at echelon e as a function of the corresponding mean.

For example, D041 currently assumes that

$$S(m_{ije}) = \alpha^{1/2}(m_{ije})^{(\beta+1)/2},$$

where α and β are constants determined by a statistical study. For the purposes of this section, it is assumed that the stock level x_{ije} has the constraint

$$LK_{ije} \le (x_{ije} - m_{ije})/S(m_{ije}) \le UK_{ije} \quad (7)$$

for i = 1, ..., I, j = 0, ..., J(i), and e = 1, 2, where LK_{ije} and UK_{ije} are specified lower and upper bounds. For example, $LK_{ije} = 0$ corresponds to the planned nonnegativity restriction in D041.

OPTIMIZATION PROBLEM

The final task is to formulate the optimization problem. First define

STK_{ije} = available assets for SRU(i, j) at echelon e.

The available assets STK_{ije} include: serviceable spares on hand and on order, plus carcasses waiting for induction and in repair, less any backorders. Because broken carcasses are included as available assets, this formulation assumes that everything that can be repaired will be repaired. However, AAM does allow another option, the so-called "repair option," in which the magnitude of depot repairs is also a decision variable.

Next define

c_{ij} = replacement cost for SRU(i, j).

Just as an earlier formula prorated backorders for common items among different weapon systems, it is also necessary to prorate costs. Compute

$$V_i = \left\{ \sum_{k=1}^{K} a_{ki0} b_{ki} F_k \right\} \Big/ IP_i,$$

which is the proportion of the total item flying program generated by the specified weapon system. The AAM considers $V_i c_{ij}$ to be the cost actually charged to this weapon system when buying one unit of SRU(i, j).

The optimization problem is to determine the stock levels x_{ije} for $i = 1, \ldots, I$, $j = 0, \ldots, J(i)$, and $e = 1,2$ in order to maximize the objective (2) subject to the safety stock restrictions (7) and the budget constraint

$$\sum_{i=1}^{I} \sum_{j=0}^{J(i)} \sum_{e=1}^{2} V_i c_{ij} [NB_{ie} x_{ije} - STK_{ije}]^+ \le B, \qquad (8)$$

where

$$[y]^+ = \begin{cases} y & \text{for } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

and B is the available procurement budget.

The AAM has the structure of a constrained nonlinear programming problem. The approximations in the foregoing equations were deliberately made so that this problem can be efficiently solved by using a marginal analysis technique. Because the depot SRUs are uncoupled from depot LRUs, it is possible to make tradeoffs among SRUs for a given LRU, before making tradeoffs among LRUs. The LRUs can then be ranked in terms of the increase in the objective function that would occur if the corresponding spare unit, or equivalent investment in SRUs, were added to the inventory.

For a component that is common to several weapon systems, the AAM will in general compute a different stock level for each of these weapon systems. In this circumstance, the AAM will implement the largest of these stock levels, so that the weapon system "wanting" the component the most will drive the decision. For the purpose of computing the ORACLE database, which is the set of partial derivatives for the dependent variables with respect to independent variables belonging to the same weapon system, this interaction between different weapon systems will be ignored. Incorporating this interaction would correspond to adding extra terms to the basic linear model (Eq. (1)), where these extra terms involve the partial derivatives for the dependent variables belonging to one weapon system with respect to independent variables belonging to other weapon systems. Although in the future it may prove desirable to extend ORACLE so that these extra terms are added, such an extension will not be addressed here.

III. NECESSARY CONDITIONS FOR OPTIMALITY

For the purpose of developing the ORACLE database for the AAM, it is possible to use formulas given by Bigelow [3] for computing the derivatives of several dependent variables, once the derivatives for the stock levels, base repairs, and depot repairs are known, and for computing the derivatives for base and depot repairs. The only formulas missing are for computing the partial derivatives of the AAM stock levels with respect to the independent variables. This section will show how to compute these latter derivatives by differentiating the necessary conditions for optimality and mean pipeline equations, which is an extension of the method originally developed by Bigelow.[3,4]

A key approximation made throughout this section is to use the normal distribution as the probability distribution for each pipeline, for the formulas for computing the partial derivatives even to be defined. If discrete distributions, as used in the AAM (the Poisson and negative binomial) were used, then the derivatives would not be defined. It is well known that if the mean pipeline size is large, then the normal distribution does provide a good approximation.[5] But if the mean is small, the normal approximation introduces some error. However, this approximation is not needed for the last two approaches considered in Sec. IV.

When working with discrete probability laws, Bigelow [4] has suggested an alternative approach that approximates a derivative by using differences. The advantage of such an approach is that the normal approximation is not needed. The disadvantages are that the resulting equations are more cumbersome and may not assure more accuracy than simply using the normal approximation. Consequently, the normal approximation will be used in what follows.

STATEMENT OF CONDITIONS

Because the natural logarithm $\ell(\bullet)$ is strictly monotonically increasing, the objective function A can be replaced with $\ell(A)$, since a solution maximizes A if and only if it also maximizes $\ell(A)$. This replacement will be made in order to convert the product expression in the objective function (2) into a summation. A Lagrange multiplier ν will be used to include the budget constraint (8) into the modified objective function. Thus the problem of maximizing $\ell(A)$ becomes: determine the stock levels x_{ije} to maximize

$$L = \ell(A) + \nu \sum_{i=1}^{I} \sum_{j=0}^{J(i)} \sum_{e=1}^{2} V_i c_{ij} [NB_{ije} x_{ije} - STK_{ije}]^{+} \tag{9}$$

subject to the constraints

$$LK_{ije} \le (x_{ije} - m_{ije})/S(m_{ije}) \le UK_{ije}. \tag{10}$$

For the value x_{ije} to be optimal, this stock level must satisfy one of the following four conditions necessary for optimality:

$$x_{ije} = STK_{ije}/NB_{ie} \tag{11}$$

$$x_{ije} = S(m_{ije})LK_{ije} + m_{ije} \tag{12}$$

$$x_{ije} = S(m_{ije})UK_{ije} + m_{ije} \tag{13}$$

$$\partial\ell(A)/\partial x_{ije} + \nu NB_{ie} V_i c_{ij} = 0. \tag{14}$$

Constraint (11) is a lower bound because of existing assets, (12) is a lower bound on safety stock, (13) is an upper bound on safety stock, and (14) must be satisfied if none of the preceding bounds are active.

It is convenient to define the normalized stock level

$$k_{ije} = (x_{ije} - m_{ije})/S(m_{ije}).$$

When using the normal approximation for the pipeline distribution, it can be shown [6, p. 446] that the expected backorders can be computed as

$$EBO_{ije} = NB_{ie}S(m_{ije})[\phi(k_{ije}) - k_{ije}\Phi(k_{ije})],$$

where the normal density is given by

$$\phi(r) = (1/2\pi)^{1/2} \exp(-r^2/2)$$

and the right tail of the distribution function by

$$\Phi(k) = \int_k^\infty \phi(r)dr.$$

In the derivations that follow, two key results are used repeatedly: the derivative of $\Phi(r)$ is $-\phi(r)$, and the derivative of $\phi(r)$ is $-r\phi(r)$.

Equation (14) will be written in detail for each possible case. First consider the partial derivative of the Lagrangian Eq. (9) with respect to the LRU stock level at a base. According to the chain rule for derivatives,

$$\partial\ell(A)/\partial x_{i01} = (1/Q_{i0})(\partial Q_{i0}/\partial EBO_{i01})(\partial EBO_{i01}/\partial x_{i01}), \tag{15}$$

where

$$\partial Q_{i0}/\partial EBO_{i01} = \sum_{k=1}^{K} (n_k/N)\partial q_{ki0}/\partial EBO_{i01}$$

$$\partial q_{ki0}/\partial EBO_{i01} =$$

$$-(a_{ki0}b_{ki}U_{ki}/t_{ki0})(1 - U_{ki}EBO_{i01}/t_{ki0})^{a_{ki0}-1}$$

and

$$\partial EBO_{i01}/\partial x_{i01} = -NB_{i1}\Phi(k_{i01}).$$

Consequently Eq. (14) becomes

$$-(1/Q_{i0})\partial Q_{i0}/\partial EBO_{i01}\Phi(k_{i01}) + \nu V_i c_{i0} = 0, \tag{16}$$

and $\partial Q_{i0}/\partial EBO_{i01}$ is given above.

Next consider the partial derivative of the Lagrangian Eq. (9) with respect to the LRU stock level at the depot. According to the chain rule,

$$\partial \ell(A)/\partial x_{i02} =$$

$$(1/Q_{i0})(\partial Q_{i0}/\partial EBO_{i01})(\partial EBO_{i01}/\partial m_{i01})(\partial m_{i01}/\partial x_{i02}),$$

where

$$\partial EBO_{i01}/\partial m_{i01} = NB_{i1}[\Phi(k_{i01}) + \phi(k_{i01})S'(m_{i01})]$$

$$\partial m_{i01}/\partial x_{i02} = -\Phi(k_{i02})/NB_{i1}$$

and $S'(\bullet)$ is the derivative of $S(\bullet)$. Because $NB_{i2} = 1$, Eq. (14) becomes

$$-(1/Q_{i0})\partial Q_{i0}/\partial EBO_{i01}[\Phi(k_{i01})$$

$$+ \phi(k_{i01})S'(m_{i01})]\Phi(k_{i02}) + \nu V_i c_{i0} = 0. \tag{17}$$

Next consider the partial derivative of the Lagrangian Eq. (9) with respect to the SRU stock level at a base. According to the chain rule,

$$\partial \ell(A)/\partial x_{ij1} =$$

$$(1/Q_{i0})(\partial Q_{i0}/\partial EBO_{i01})(\partial EBO_{i01}/\partial m_{i01})(\partial m_{i01}/\partial x_{ij1}),$$

where

$$\partial m_{i01}/\partial x_{ij1} = -\Phi(k_{ij1})$$

and the other derivatives were given earlier. Thus Eq. (14) becomes

$$-(1/Q_{i0})\partial Q_{i0}/\partial EBO_{i01}[\Phi(k_{i01})$$

$$+ \phi(k_{i01})S'(m_{i01})]\Phi(k_{ij1}) + \nu V_i c_{ij} = 0. \tag{18}$$

And, finally, consider the partial derivative of the Lagrangian Eq. (9) with respect to the SRU stock level at the depot. According to the chain rule,

$$\partial \ell(A)/\partial x_{ij2} =$$

$$(1/Q_{i0})(\partial Q_{i0}/\partial EBO_{i01})(\partial EBO_{i01}/\partial m_{i01})(\partial m_{i01}/\partial x_{ij2}),$$

where

$$\partial m_{i01}/\partial x_{ij2} = [\partial EBO_{ij1}/\partial x_{ij2} + (1 - OIM_{ij})\partial EBO_{ij2}/\partial x_{ij2}]/NB_{i1}$$

$$\partial EBO_{ij1}/\partial x_{ij2} = (\partial EBO_{ij1}/\partial m_{ij1})(\partial m_{ij1}/\partial x_{ij2})$$

$$\partial EBO_{ij1}/\partial m_{ij1} = NB_{i1}[\Phi(k_{ij1}) + \phi(k_{ij1})S'(m_{ij1})]$$

$$\partial m_{ij1}/\partial x_{ij2} = -\Phi(k_{ij2})OIM_{ij}/NB_{i1}$$

$$\partial EBO_{ij2}/\partial x_{ij2} = -\Phi(k_{ij2}).$$

Consequently Eq. (14) becomes

$$-(1/Q_{i0})\partial Q_{i0}/\partial EBO_{i01}[\Phi(k_{i01}) + \phi(k_{i01})S'(m_{i01})]\{(1 - OIM_{ij})\Phi(k_{ij2})$$

$$+ [\Phi(k_{ij1}) + \phi(k_{ij1})S'(m_{ij1})]\Phi(k_{ij2})OIM_{ij}\} + \nu V_i c_{ij}\nu = 0. \tag{19}$$

DIFFERENTIATION OF CONDITIONS

Define the column vectors $X_i = (x_{ije})$ and $M_i = (m_{ije})$. Let Z represent an arbitrary independent variable. Suppose that the AAM has been solved for a nominal case, corresponding to the value Z^0, thereby determining the nominal values X_i^0 and M_i^0 for the two vectors. For the purpose of developing ORACLE's database, it is desirable to compute the partial derivatives $\partial X_i/\partial Z$ and $\partial M_i/\partial Z$, as evaluated at the nominal values. This subsection will show how these derivatives can be computed by differentiating the necessary conditions for optimality and mean pipeline equations.

For each combination of indices (i, j, e), the previous subsection showed that there is a necessary condition for optimality in the form

$$F_{ije}(Z^0, X_i^0, M_i^0, \nu^0) = 0, \tag{20}$$

where F_{ije} is either (11), (12), (13), or (14), and Eq. (14) is either (16), (17), (18), or (19). Equations (18)-(19) couple the variables for an SRU with those for its associated LRU.

Also for each combination of indices (i, j, e), Sec. II showed that there is a mean pipeline equation in the form

$$G_{ije}(Z^0, X_i^0, M_i^0) = 0, \tag{21}$$

where G_{ije} is either (3), (4), (5), or (6), and the expected backorders in these expressions are evaluated with (15). Equation (6) couples the variables for an LRU with those for all associated SRUs.

Differentiating (20) implies

$$\sum_{s=0}^{J(i)} \sum_{t=1}^{2} [(\partial F_{ije}/\partial x_{ist})(\partial x_{ist}/\partial Z) + (\partial F_{ije}/\partial m_{ist})(\partial m_{ist}/\partial Z)]$$
$$+ \partial F_{ije}/\partial Z + \partial F_{ije}/\partial \nu \; \partial \nu/\partial Z = 0, \tag{22}$$

and differentiating (21) implies

$$\sum_{s=0}^{J(i)} \sum_{t=1}^{2} [(\partial G_{ije}/\partial x_{ist})(\partial x_{ist}/\partial Z) + (\partial G_{ije}/\partial m_{ist})(\partial m_{ist}/\partial Z)] + \partial G_{ije}/\partial Z = 0. \quad (23)$$

Because (6) and (18)-(19) couple the variables for an LRU with those for its associated SRUs, differentiating these equations will couple the partial derivatives for an LRU with those for its associated SRUs. It therefore follows that the partial derivatives for a single SRU cannot be determined separately from those for other SRUs associated with the same LRU, or from the partial derivatives for the LRU itself. In other words, (22)-(23) represent a system of simultaneous linear equations.

If we consider only LRU i and assume for the moment that $\partial\nu/\partial Z$ is known, then there are $2[J(i)+1]$ unknown variables $\partial x_{ije}/\partial Z$ plus $2[J(i)+1]$ unknown variables $\partial m_{ije}/\partial Z$, or $4[J(i)+1]$ unknown variables altogether. Because (22)-(23) represent a system of $4[J(i)+1]$ simultaneous linear equations, there are sufficient equations to compute a unique situation.

Because F_{ije} is a linear function of the Lagrange multiplier ν, (22) will be independent of the nominal value ν^0 for this multiplier. Thus it is not necessary to know this nominal value to compute the partial derivatives of the stock levels and pipelines. This latter observation is important, because the AAM does not use a Lagrange multiplier solution procedure, so the nominal value for this multiplier may not be known.

Define the column vector Y_i to consist of both X_i and M_i. It is convenient to rewrite (22)-(23) as

$$H_i \partial Y_i/\partial Z + E_i + R_i \partial\nu/\partial Z = 0, \quad (24)$$

where each element of the matrix H_i is either $\partial F_{ije}/\partial x_{ist}$, $\partial F_{ije}/\partial m_{ist}$, $\partial G_{ije}/\partial x_{ist}$, or $\partial G_{ije}/\partial m_{ist}$; each element of the matrix E_i is either $\partial F_{ije}/\partial Z$ or $\partial G_{ije}/\partial Z$; and each element of the matrix R_i is either

$\partial F_{ije}/\partial\nu$ or zero. The solution to this system of simultaneous linear equations can be obtained by first inverting the matrix H_i and then computing

$$\partial Y_i/\partial Z = H_i^{-1}[-E_i - R_i\partial\nu/\partial Z]. \tag{25}$$

The appropriate value for the partial derivative $\partial\nu/\partial Z$ can be determined in the following way. Define the row vector C_i such that the product $C_i\partial Y_i/\partial Z$ is equal to

$$\sum_{j=0}^{J(i)} \sum_{e=1}^{2} NB_{ie}V_i c_{ij}\partial x_{ije}/\partial Z. \tag{26}$$

If the total budget B remains fixed, then the budget constraint (8) implies that

$$\sum_{i=1}^{I} C_i\partial Y_i/\partial Z = 0.$$

After substituting (25) into (26), we find that

$$\sum_{i=1}^{I} C_i\{H_i^{-1}[-E_i - R_i\partial\nu/\partial Z]\} = 0,$$

implying that

$$\partial\nu/\partial Z = -\sum_{i=1}^{I} C_iH_i^{-1}E_i \Big/ \sum_{i=1}^{I} C_iH_i^{-1}R_i. \tag{27}$$

By substituting (27) into (25), the desired partial derivatives of the stock levels can be obtained.

In summary, the algorithm for computing the partial derivatives of the stock levels with respect to an arbitrary independent variable has the following steps:

1. First solve the AAM for the nominal case, thereby determining the nominal vector values X_i^0 and M_i^0 for $i = 1, \ldots, I$;

2. By differentiating the necessary conditions for optimality, differentiating the mean pipeline equations, and then evaluating these relations at the nominal values, construct the matrices H_i, E_i, and R_i for $i = 1, \ldots, I$;

3. Invert each matrix H_i, which is equivalent to solving a system of linear equations with $4[J(i)+1]$ unknowns for $i = 1, \ldots, I$;

4. By using (27), compute the partial derivative of the Lagrange multiplier with respect to the independent variable;

5. And by using (25), compute the partial derivatives for the stock levels with respect to the independent variable.

The main computational work in this algorithm seems to be inverting the matrix H_i for each LRU i, which is step 3. However, this matrix has a special structure that may be exploited. In particular, this matrix may be written as

$$H_i = \begin{bmatrix} H11 & H12 & 0 & 0 \\ H21 & H22 & H23 & H24 \\ H31 & H32 & H33 & H34 \\ 0 & 0 & 0 & H44 \end{bmatrix}$$

where column 1 corresponds to base stock levels, column 2 to base pipelines, column 3 to depot stock levels, and column 4 to depot pipelines, and the rows are similarly defined. Those portions of the matrix that have nonzero elements are indicated by the symbol HIJ, where I and J can be 1, 2, 3, or 4. Because H44 is the only nonzero submatrix on the fourth row, it follows that the overall matrix H_i can be inverted in two stages: First invert H44, and then invert the matrix consisting of the submatrices HIJ for I and J between 1 and 3. However, the inversion of H44 is trivial because it is a diagonal matrix. Thus the amount of work required to invert H_i can be reduced to solving a system of simultaneous linear equations with only 3[J(i) + 1] unknowns.

It is possible to exploit the structure of H_i even further by developing an inversion algorithm that takes advantage of the zeroes on the first row. However, such an algorithm would be more complex than the one just described because H11 is not diagonal and H12 is not zero.

For simplicity, Sec. II gave the equations for the AAM for the case in which there is only one level of indenture and in which an SRU can be associated with only one type of LRU. However, the new version of D041 will allow several levels of indenture, allow an SRU to be associated with several types of LRUs, plus use a product relationship (rather than a summation) for the SRU expected backorders in (6). Because Eqs. (20) and (21) are sufficiently general to cover the latter changes, the foregoing algorithm is also applicable to the new version of D041.

Actually developing this algorithm would be quite time consuming. Such development must include program design, coding, and validation. The main difficulties would be the complex matrix manipulations, including construction of the matrices, inversion, and multiplication.

IV. ALTERNATIVE ORACLE ESTIMATION METHODS

The indenture relationships make the approach of the last section difficult to implement, because these relationships result in a set of simultaneous linear equations that must be solved for each LRU in the weapon system. Consequently, this section will discuss four simpler alternative methods. The first method treats the SRUs as though they were LRUs, eliminating the indenture relationships. The second method retains the indenture relationships but in a simplified form. Either of these approximate methods would be used only to compute the partial derivatives, as the nominal values would still be obtained by solving the AAM without any additional approximation. The third method is based upon making multiple runs of the AAM for the same year. The fourth uses regression analysis to analyze data obtained for several years.

ELIMINATION OF INDENTURE RELATIONSHIPS

For the purpose of computing partial derivatives, first consider the approach of treating the SRUs as though they were LRUs. In other words, all items (indentured or not) are treated as though they were installed directly on the aircraft.

Bigelow has also recommended this approximation scheme but suggested that the data be modified in the following ways: The time spent working on an LRU to isolate the failed SRUs should be added to the present SRU repair times; the price of an LRU should be reduced by the purchase price of a full complement of SRUs; and the SRUs installed on LRUs should be counted as part of the available SRU assets.[3] Bigelow's suggested modifications have several disadvantages: Additional data are required that may not be available; the residual price of the parent LRU may become small or even negative; the resulting necessary conditions for optimality for the LRUs are different from those given in Sec. III and thus have different solutions; and in the absence of extensive cannibalization, it is unrealistic to consider the installed SRU components as being assets that are available for repairing other LRUs. Consequently, the approach considered here is to treat the SRUs as though they were LRUs, but without making any of the foregoing data modifications, which in turn avoids all of the foregoing disadvantages.

To derive the formulas for this approximation, it is necessary to generalize the definitions given in Sec. II. Define

a_{kij} = quantity per application of SRU (i,j) on series k.

Thus the number of units of SRU (i,j) installed on aircraft of series k is

$$t_{kij} = a_{kij} b_{ki} n_k,$$

where b_{ki} and n_k were defined in Sec. II. The probability that an aircraft of series k is not missing a unit of SRU (i, j) is

$$q_{kij} = 1 - b_{ki} + b_{ki}(1 - U_{ki} EBO_{ij1}/t_{kij})^{a_{kij}},$$

where U_{ki} was defined in Sec. II. The probability that a randomly chosen aircraft is not waiting for a spare unit of SRU (i, j) is

$$Q_{ij} = \sum_{k=1}^{K} (n_k/N) q_{kij},$$

where N was defined in Sec. II. And, finally, the probability that a randomly chosen aircraft is not missing any of its reparable components is

$$A = \prod_{i=1}^{I} \prod_{j=0}^{J(i)} Q_{ij},$$

which replaces Eq. (2) as the objective function.

The next step is to modify the mean pipeline equations that were given in Sec. II. For $i = 1, \ldots, I$ and $j = 0, \ldots, J(i)$, the mean base pipeline is

$$m_{ij1} = [\lambda_{ij1} RTS_{ij} RT_{ij1} + \lambda_{ij1}(1 - RTS_{ij}) OST_{ij} + EBO_{ij2}]/ND_{i1},$$

and the mean depot pipeline is

$$m_{ij2} = \lambda_{ij2}[RT_{ij2}(1 - DC_{ij}) + DL_{ij} DC_{ij}],$$

where the other parameters were defined in Sec. II.

Equations (11)-(14) in Sec. III still represent the necessary conditions for optimality, except that Eq. (14) can now be simplified. For $i = 1, \ldots, I$ and $j = 0, \ldots, J(i)$, the partial derivative of the Lagrangian (9) with respect to the stock level at a base is

$$-(1/Q_{ij}) \partial Q_{ij}/\partial EBO_{ij1} \Phi(k_{ij1}) + \nu V_i c_{ij} = 0,$$

and the partial derivative with respect to the stock level at the depot is

$$-(1/Q_{ij}) \partial Q_{ij}/\partial EBO_{ij1} [\Phi(k_{ij1}) + \phi(k_{ij1}) S'(m_{ij1})] \Phi(k_{ij2}) + \nu V_i c_{ij} = 0,$$

where

$$\partial Q_{ij}/\partial EBO_{ij1} = \sum_{k=1}^{K} (n_k/N) \partial q_{kij}/\partial EBO_{ij1}$$

and

$$\partial q_{kij}/\partial EBO_{ij1} = -(a_{kij} b_{kij} U_{ki}/t_{kij})(1 - U_{ki} EBO_{ij1}/t_{kij})^{a_{kij}-1}.$$

Let Z be an arbitrary independent variable. The approach of Sec. III can be used to compute the partial derivatives of the stock levels and mean pipelines with respect to Z, namely $\partial x_{ij1}/\partial Z$, $\partial x_{ij2}/\partial Z$, $\partial m_{ij1}/\partial Z$, and $\partial m_{ij2}/\partial Z$. This approach requires that a four-by-four system of linear equations be solved for each item (i, j), where these equations are obtained by differentiating the foregoing necessary conditions for optimality and mean pipeline equations. However, these equations imply that $\partial m_{ij2}/\partial Z$ can be evaluated independently of the other variables, and that $\partial m_{ij1}/\partial Z$ can be expressed as a linear function of $\partial x_{ij2}/\partial Z$. Thus through substitution, the four-by-four set of equations for each item can be reduced to only a two-by-two set, leaving only the variables $\partial x_{ij1}/\partial Z$ and $\partial x_{ij2}/\partial Z$ to be determined. In other words, treating SRUs as LRUs can reduce the main computational effort of the Sec. III algorithm to solving only a two-by-two system of equations for each item, which basically is the same approach that was described by Bigelow for minimizing expected backorders.[4]

Of course, treating the SRUs as LRUs is an approximation. Its disadvantage is that there will be some inaccuracy, while its advantage is that the resulting algorithm will be much easier to program and faster to run compared with the more exact approach considered in Sec. III.

SIMPLIFICATION OF INDENTURE RELATIONSHIPS

The second approach toward simplifying the Sec. III algorithm is to retain the indenture relationships but in a simplified form. In particular, the following approximations are made for the purpose of computing the partial derivatives:

1. The normal distribution is used as the probability distribution for each pipeline;
2. There is a single level of indenture;
3. After setting $OIM_{ij} = 0$ for each SRU (i, j), the mean base pipelines for SRUs and LRUs are given by Eqs. (5)-(6);
4. In the base case, conditions (11)-(13) are not active for LRU depot stock levels.

Approximation 1 was also made in Sec. III for the formulas for computing the partial derivatives even to be defined.

Although approximation 2 was also used in Secs. II and III, the AAM does allow several indenture levels and the algorithm considered in Sec. III can be extended to allow several indenture levels. But, and this point must be emphasized, having a single indenture level is required by the approach considered here.

The AAM already makes the approximation that the calculation of depot LRU backorders can be uncoupled from that for depot SRU backorders. But in an analogous way, approximation 3 also uncouples the calculation of base SRU backorders from that for depot SRU backorders with the effect that each additional SRU depot backorder causes an additional LRU depot backorder or an additional SRU base backorder, in either case adding one unit to the mean LRU base pipeline. Although this approximation will overestimate the mean LRU base pipeline, it may be acceptable for the purpose of computing the partial derivatives of the stock levels.

Approximation 4 requires that the LRU depot stock levels for the base case exceed the lower bounds because of existing assets and safety stocks, while being less than the upper bounds because of safety stock. However, these bounds are allowed to be active for LRU base stocks, SRU depot stocks, and SRU stocks.

Equations (11)-(14) still represent the necessary conditions for optimality, except that approximation 3 enables (14) to be simplified in one case. The partial derivative of the Lagrangian (9) with respect to the SRU stock level at the depot is now

$$-(1/Q_{i0})\partial Q_{i0}/\partial EBO_{i01}[\Phi(k_{i01}) + \phi(k_{i01})S'(m_{i01})]\Phi(k_{ij2}) + \nu V_i c_{ij} = 0, \qquad (28)$$

whereas the partial derivatives for the other cases are still given by Eqs. (16)-(18).

For each LRU i, approximation 4 implies that Eq. (17) is the necessary condition for optimality corresponding to the depot stock level, which means that

$$\Phi(k_{i02}) = \nu V_i c_{i0} Q_{i0}/\{\partial Q_{i0}/\partial EBO_{i01}[\Phi(k_{i01}) + \phi(k_{i01})S'(m_{i01})]\}. \qquad (29)$$

Suppose that the necessary conditions (11)-(13) are not active for the stock level of SRU (i, j) at a base. Then Eq. (18) must be one of the necessary conditions for optimality, implying that

$$\Phi(k_{ij1}) = \nu V_i c_{ij} Q_{i0}/\{\partial Q_{i0}/\partial EBO_{i01}[\Phi(k_{i01}) + \phi(k_{i01})S'(m_{i01})]\}.$$

Substituting Eq. (29) implies that

$$\Phi(k_{ij1}) = (c_{ij}/c_{i0})\Phi(k_{i02}).$$

After differentiating this expression with respect to an arbitrary independent variable Z, we have

$$\phi(k_{ij1})\partial k_{ij1}/\partial Z = (c_{ij}/c_{i0})\phi(k_{i02})\partial k_{i02}/\partial Z,$$

which expresses $\partial k_{ij1}/\partial Z$ as a linear function of $\partial k_{i02}/\partial Z$.

Next suppose that condition (11) is active for the stock level of SRU(i,j) at a base, implying that $\partial x_{ij1}/\partial Z = 0$. Because the normalized stock level is defined as $k_{ij1} = (x_{ij1} - m_{ij1})/S(m_{ij1})$, it must be true that

$$\partial k_{ij1}/\partial Z = -\partial m_{ij1}/\partial Z[1 + k_{ij1}S'(m_{ij1})]/S(m_{ij1}). \tag{30}$$

Because approximation 3 implies that $\partial m_{ij1}/\partial Z$ can be evaluated directly as a known function of Z and the base case solution, Eq. (30) implies that $\partial k_{ij1}/\partial Z$ can also be evaluated directly.

Finally, suppose that either condition (12) or (13) is active for SRU (i, j) at a base. In this case, the normalized stock level k_{ij1} is equal to a constant, implying that the partial derivative $\partial k_{ij1}/\partial Z$ equals zero.

In summary, depending upon which of the conditions (11)-(14) is active, $\partial k_{ij1}/\partial Z$ can be expressed as a linear function of $\partial k_{i02}/\partial Z$, as a known function of Z, or as zero. However, the last two options are special cases of the first option, namely that $\partial k_{ij1}/\partial Z$ can be expressed as a linear function of $\partial k_{i02}/\partial Z$. By employing the same argument, it can be seen that the partial derivative $\partial k_{ij2}/\partial Z$ can also be expressed as a linear function of $\partial k_{i02}/\partial Z$.

It follows from differentiating (15) that

$$\partial EBO_{ij1}/\partial Z = NB_{ij1}\{-S(m_{ij1})\Phi(k_{ij1})\partial k_{ij1}/\partial Z$$

$$+ S'(m_{ij1})\partial m_{ij1}/\partial Z[\phi(k_{ij1}) - k_{ij1}\Phi(\kappa_{ij1})]\}. \tag{31}$$

As already noted, $\partial m_{ij1}/\partial Z$ can be evaluated directly, and $\partial k_{ij1}/\partial Z$ can be expressed as a linear function of $\partial k_{i02}/\partial Z$. Thus (31) implies that $\partial EBO_{ij1}/\partial Z$ can be expressed as a linear function of $\partial k_{i02}/\partial Z$. Using the same argument, both $\partial EBO_{i02}/\partial Z$ and $\partial EBO_{ij2}/\partial Z$ can be expressed as a linear function of $\partial k_{i02}/\partial Z$. And, finally, (6) implies that $\partial m_{i01}/\partial Z$ can also be expressed as a linear function of $\partial k_{i02}/\partial Z$.

Thus, for each LRU i, only two partial derivatives need to be determined, namely $\partial k_{i01}/\partial Z$ and $\partial k_{i02}/\partial Z$. Once $\partial k_{i02}/\partial z$ is determined, then $\partial k_{ij1}/\partial Z$ and $\partial k_{ij2}/\partial Z$ can be evaluated using the previously described linear relationships. Because $\partial m_{i02}/\partial Z$, $\partial m_{ij1}/\partial Z$, and $\partial m_{ij2}/\partial Z$ can all be evaluated directly, $\partial m_{i01}/\partial Z$ can be computed by differentiating (6). After the foregoing derivatives have been determined, then the partial derivatives of the stock levels can be evaluated with

$$\partial x_{ije}/\partial Z = S(m_{ije})\partial k_{ije}/\partial Z + [1 + k_{ije}S'(m_{ije})]\partial m_{ije}/\partial Z,$$

for $i = 1, \ldots, I$, $j = 0, \ldots, J(i)$, and $e = 1, 2$.

The column vectors $X_i = (x_{ije})$, $M_i = (m_{ije})$, and $Y_i = (X_i, M_i)$ were defined in Sec. III. Also define $K_i = (k_{i01}, k_{i02})$. As indicated by the preceding discussion, approximations 1-4 enable (24) to be simplified as

$$H_i'\partial K_i/\partial Z + E_i' + R_i'\partial\nu/\partial Z = 0$$

$$\partial Y_i/\partial Z = T_i'\partial K_i/\partial Z + P_i',$$

where the matrices H_i', E_i', R_i', T_i', and P_i' are all functions of the nominal case values, and H_i' is a two-by-two matrix. The solution to this system of linear equations can be obtained by first inverting H_i' and then computing

$$\partial Y_i/\partial Z = -T_i'\{(H_i')^{-1}[E_i' + R_i'\partial\nu/\partial Z]\} + P_i'. \tag{32}$$

The appropriate value for the partial derivative $\partial\nu/\partial Z$ can be determined by substituting (32) into (26).

Consequently, approximations 1-4 enable the computational effort to be greatly reduced, because the main effort is solving only a two-by-two system of linear equations for each LRU in the weapon system. The earlier approximation method, which treated the SRUs as LRUs, required that a two-by-two system of linear equations be solved for each item in the weapon system. However, both of these approximation schemes should be contrasted with the more exact algorithm considered in Sec. III, which required that a system of linear equations having 3[J(i)+1] variables be solved for each LRU i in the weapon system.

MULTIPLE RUNS

Another approach to constructing the ORACLE database is based upon making multiple runs of the AAM. This approach is much simpler and easier to implement than the algorithms considered earlier.

For a given weapon system, Sec. I defined Z_m to be the mth independent variable and D_n to be the nth dependent variable, for $m = 1, \ldots, M$ and $n = 1, \ldots, N$. Let Z_m^0 be the value of the mth independent variable for the nominal case, and let D_n^0 be the corresponding value of the nth dependent variable as computed by the AAM. The ORACLE database consists of the collection of partial derivatives $\partial D_n/\partial Z_m$, as evaluated at the nominal case.

In this alternative method of constructing the ORACLE database for a given weapon system, the first step is to specify a nonzero perturbation ε_m for the mth independent variable, $m = 1, \ldots, M$. The second step is to make M additional runs with the AAM, where the value of the kth independent variable for the mth additional run is

$$Z_k^m = \begin{cases} Z_m^0 & \text{for } k \neq m \\ Z_m^0 + \varepsilon_m & k = m. \end{cases}$$

In other words, for the mth run, the mth independent variable is the only independent variable that is changed from the nominal case. Let D_n^m be the corresponding value of the nth dependent variable for the mth additional run, as computed by the AAM. The final step is to approximate the desired partial derivatives with

$$\partial D_n/\partial Z_m = (D_n^m - D_n^0)/\varepsilon_m \tag{33}$$

for m = 1, ..., M and n = 1, ..., N.

By definition

$$\partial D_n/\partial Z_m = \lim_{\varepsilon_m \to 0} (D_n^m - D_n^0)/\varepsilon_m$$

if this derivative is defined. However, this derivative may not be defined, because of AAM's use of discrete probability distributions to represent the pipelines. If this derivative is defined, then Eq. (33) will approximate that derivative, provided that ε_m is chosen to be suitably small. But if this derivative is not defined, then Eq. (33) could still be used, in which case Eq. (1) could be interpreted as approximating a nonlinear curve with a secant rather than a tangent.

The above scheme requires M additional runs, one for each independent variable. However, it may be desirable to make 2M additional runs to compute two estimates of the derivative for each independent variable, one based upon a positive perturbation and another based upon a negative perturbation. The average of these two estimates could then be used in the linear formula (1). Alternatively, both estimates could be used to define a piecewise linear representation, which is likely to be a more accurate model than the linear formula (1) currently utilized for ORACLE.

REGRESSION OVER TIME

Another method for constructing the ORACLE database utilizes data obtained for several years. This method is also fairly simple and easy to implement.

Let Z_m^t be the value of the mth independent variable in year t, and let D_n^t be the corresponding value of the nth dependent variable as computed by the AAM. A linear formula is used to relate the dependent variables to the independent variables:

$$D_n^t = a_n^0 + \sum_{m=1}^{M} a_n^m Z_m^t. \qquad (34)$$

It follows that a_n^m can be interpreted as being the partial derivative of the dependent variable D_n^t with respect to the independent variable Z_n^t for any year t.

The suggested approach is to obtain actual historical values for the relevant dependent and independent variables and then to compute the coefficients a_n^m by using linear regression analysis. However, such an approach has several difficulties. First, the D041 requirements model has evolved over time, and the AAM has not yet been used during any past year. Consequently, it would be necessary to recompute the dependent variables for past years with the AAM to obtain partial derivatives that would be appropriate for the AAM.

Second, the magnitude of a dependent variable in one year may be partly dependent on decisions made in earlier years. For example, the buy requirement for any year is partly a function of the percentage of the previous year's requirement that was funded. Because some dependent variables are partly functions of past history and not just of the independent variables, there may be a significant residual error in a regression approach.

And third, the values of some independent variables may be correlated. For example, the age of a weapon system and the number of flying hours per year may both be monotonically increasing functions over time. This phenomenon is called multicollinearity or intercorrelation, and it may produce significant errors when using a regression approach.

V. CONCLUSIONS

Five methods have been presented for developing the ORACLE database for a given weapons system: first, differentiation of the necessary conditions for optimality and mean pipeline equations (DIFF); second, elimination of indenture relationships (ELIM); third, simplification of indenture relationships (SIMP); fourth, multiple runs of the AAM (MULT); and fifth, regression over time (REGR). Table 1 ranks these five methods with respect to simplicity, accuracy, and execution time.

Because MULT requires only that multiple runs be made of an existing model, it is judged in Table 1 as being the simplest method to implement. REGR is also a fairly simple method, but it requires some additional work: To obtain partial derivatives appropriate for the AAM, it is necessary to recompute the dependent variables for past years with the AAM and then estimate a regression formula. However, neither of these approaches requires much in the way of new software development. The other approaches, which are based upon differentiating necessary conditions for optimality, require major efforts in software development, as well as changes in software each time that the AAM is changed.

Assuming that Eq. (1) is used to estimate the changes in dependent variables as a function of arbitrary changes in independent variables, which method of computing the partial derivatives would yield the most accurate results? One way of comparing these five methods is to think of DIFF as using a tangent to represent a nonlinear curve, ELIM and SIMP as using approximate tangents, MULT as using a secant, and REGR as using an approximate tangent. If the curve is convex or concave, then the tangent would be the better representation for a small change in an independent variable, whereas the secant would be better for a change in an independent variable that was larger than the perturbation used to define the secant. The relative accuracy of these methods depends upon the changes chosen for the independent variables. However, to compute their tangents, DIFF, ELIM, and SIMP used the normal distribution to

Table 1

RANKING OF METHODS FOR DEVELOPING THE ORACLE DATABASE
(1 = best, 5 = worst)

Method	Simplicity	Accuracy	Execution Time
DIFF	5	3	3
ELIM	3	5	2
SIMP	4	4	1
MULT	1	2	4
REGR	2	1	5

approximate the probability distribution for each pipeline. According to the Central Limit Theorem (4), this approximation would be good for items with fairly high failure rates but not for items with fairly low ones. It is not clear how accurate these methods would be, even for small changes in the independent variables. Because MULT and REGR do not need to use the normal approximation, the latter approaches appear to have a slight advantage with respect to overall accuracy.

If the goal of ORACLE is to predict values of dependent variables during a future year, then REGR has an additional advantage with respect to accuracy: Historical trends can be estimated by using the year as one of the independent variables, and Eq. (34) can be used to project these trends into the future. Because REGR is the only method that uses data from several earlier years, it is the only method that can capture historical trends. If sufficient historical data are available, REGR may be the most accurate model for forecasting.

To compute the partial derivatives of the dependent variables with respect to a single independent variable, DIFF requires that a system of simultaneous linear equations having $3(J(i) + 1)$ variables be solved for each LRU i in the weapon system; ELIM requires that a system of linear equations having two variables be solved for each item in the weapon system; SIMP requires that a system of linear equations having two variables be solved for each LRU in the weapon system; MULT requires that one additional run of the AAM be made, which means solving a nonlinear programming problem; and REGR requires that one additional run of the AAM be made for each past year of data to recompute the dependent variables, followed by estimating one regression formula.

The number of unknown variables is the same for the first four approaches, namely four times the number of items in the weapon system (corresponding to the stock level and pipeline at each echelon for each item). However, solving systems of linear equations should be faster than solving a nonlinear programming problem having the same number of unknown variables. Thus, in Table 1, SIMP is judged as having the smallest execution time, followed by ELIM, DIFF, MULT, and REGR in that order.

The foregoing remarks regarding the five methods are summarized in Table 1 with respect to the three indices of simplicity, accuracy, and execution time. A method is said to dominate some other method if its values for all three indices are smaller than those for the other method. As indicated in Table 1, none of the five methods dominates any other method.

The only method for which there is some computational experience is the final one, REGR. Volume II of this report discusses the performance of that approach in the context of requirements forecasting.

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