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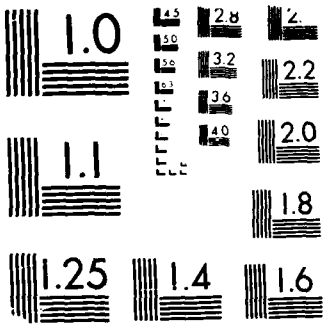

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Analysis of Anomalous Resistivity During the Conduction Phase of the Plasma Erosion Opening Switch

R. KULSRUD,* P. F. OTTINGER, AND J. M. GROSSMANN

Plasma Technology Branch
Plasma Physics Division

JAYCOR, Inc.
Vienna, VA 22180-2270

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During the conduction phase of the plasma erosion opening switch, the current channel width in the body of the plasma away from the electrodes has been observed to be many times wider than the collisionless skin depth. Anomalous collisions have been invoked to explain this descrepancy. Here the problem is analyzed using an electrostatic Vlasov approach and an unstable ion acoustic mode is identified. The derived growth rate is fast enough and the non-linear saturation level is high enough to explain the observed magnetic field penetration.

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ANALYSIS OF ANOMALOUS RESISTIVITY DURING THE CONDUCTION PHASE OF THE PLASMA EROSION OPENING SWITCH

I. INTRODUCTION

The plasma erosion opening switch (PEOS) is a fast opening vacuum switch which has been used to compress the output pulse from conventional pulsed power generators.[1-8] It is also being used in research for developing high power inductive storage pulsed power generators.[9] The PEOS consists of a low density ($n_i \sim 10^{13}$ cm^{-3}) C^{++} plasma injected ($V_D \sim 10$ cm/μs) between the electrodes of a coaxial transmission line as shown schematically in Fig. 1. The plasma is classically collisionless on the timescale of interest (~ 100 ns). However, the plasma has been found to conduct current (~ 1 MA) across a strong (~ 10-50 kG) self-generated magnetic field until a current threshold is reached and then to open (i.e. cease to conduct) quickly (~ 10 ns).[7] The current threshold is determined by the injected plasma properties and the geometry. A theory which describes the mechanisms involved in setting this threshold current and which describes the opening mechanisms is found elsewhere.[10] Here field penetration and current conduction in the body of the plasma during the conduction phase of the PEOS will be addressed. This is important for understanding PEOS scaling.

While the PEOS is conducting, electrons are emitted from the cathode and enter the plasma after being accelerated across a

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cathode sheath. Theoretical analysis shows that the potential drop across the sheath can be large (~ MV).[11] In fact, the floating potential of the plasma can be much larger than the induced voltage across the plasma. These energetic electrons are the current carriers and are also capable of driving instabilities in the plasma. Magnetic field probe data[12] shows that the current channel in a PEOS is many times wider than the collisionless skin depth of the plasma (~ 0.1 cm). In the absence of anomalous resistivity radial current flow can only be accomplished by $\mathbf{E}_z \times \mathbf{B}_\theta$ drift, where E_z results from charge separation. Under these conditions PIC code simulations[13,14], fluid code simulations[15,16,17] and analytic fluid work[11] have all predicted current channel widths that are narrower than observed in experiments. PIC code[18] and fluid code[19] simulations with anomalous collisions modeled by introducing a collision term in the electron equations of motion have compared well with experimental data with an anomalous collision frequency, ν_a, on the order of ω_{ce}.

Because the switch carries a large current, the magnetic field is large and the electron gyration radius is small compared with the anode-cathode separation. The ions are essentially unmagnetized. As stated above, without anomalous collisions due to instabilities electrons must move with $\mathbf{E} \times \mathbf{B}$ velocities, and actually are tied to the magnetic field lines.

Such a motion can be shown to lead to a relation between n_e and B that must be satisfied along electron trajectories. This constraint relating n_e and B can be combined with other plausible assumptions to show that the electrons cannot cross the plasma from the cathode to the anode but must flow essentially on lines

of constant r in contradiction to the direct observation of current flow across the PEOS plasma. This contradiction then provides the *reductio ad absurdum* argument that motivates the belief that there must be a mechanism in the plasma to disturb the pure **E** x **B** motion of the electrons. In this paper it is suggested that the mechanism involves a collection of unstable ion acoustic waves that interacts with the electrons by wave particle interactions to provide such anomalous collisions. The collisions produce the penetration of current and magnetic field into the plasma at an anomalously large rate. They explain the ability of the plasma to carry the large currents across the strong magnetic field as observed.

The *reductio ad absurdum* argument is now presented. Assume that:

(1) There are no electron collisions and that the electrons move with the **E** x **B** velocity. This is the primary assumption whose absurdity will be shown. On the basis of this assumption it can be stated that the magnetic field is frozen into the electron fluid. By a standard proof it can also be shown that the flux in a magnetic tube remains constant in time as the tube moves with the **E** x **B** velocity. That is, for cylindrical geometry

$$\frac{n_e r}{B} = \text{const.} \tag{1a}$$

on the electron trajectory even for a nonsteady state.

(2) The plasma is charge neutral. Usually the density is high enough for this to be a good assumption. On the basis of this, Eq. (1a) becomes

$$\frac{Zn_i r}{B} = \text{const.} \tag{1b}$$

on the electron trajectory, where Z is the ion charge state.

(3) The ions move very little during the switch conduction time. This can be shown to be the case for many short conduction time switches since the J x B force on the ions moves them only a short distance. Also their motion of injection in the original plasma is too slow to move them far. As a consequence, n_i in Eq. (1b) can be taken to be that of the plasma before the power pulse arrives at the switch.

(4) The plasma is in a quasi-steady state so that the electron current lines are coincident with the electron trajectories. From assumption (3) the ion current is negligible, so that the electron trajectories are the total current lines. Now from cylindrical geometry together with Ampere's law one can conclude that B must vary as 1/r along current lines and thus along electron trajectories. Hence, Eq.(1b) becomes

$$Zn_i r^2 = \text{const.} \tag{1c}$$

along electron trajectories, where n_i is the initial ion density.

The argument can now be completed as follows. If Zn_i is a function of r alone and Zn_i does not vary as $1/r^2$, then the electrons must move keeping r a constant. It is more plausible experimentally that Zn_i is independent of r than varying as $1/r^2$,

so that the electrons are constrained from changing radius and prevented from moving from cathode to anode in contradiction with experiment. Even if Zn_i is not a function of r alone, it is difficult to envision a reasonable profile that will allow electrons to cross the plasma in light of the constraint expressed in Eq. (1c).

This argument requires certain comments. First, the argument depends heavily on the cylindrical geometry. Theoretical analysis involving planar geometry will not encounter constraint (1c). Second, assumption (2) concerning charge neutrality is not rigorous. At lower densities it is possible to violate charge neutrality over narrow channels (a few skin depths) through which current can pass. In this case Eq. (1a) must still be satisfied but Eqs. (1b) and (1c) need not be satisfied. This appears to be what is happening in the PIC code simulations of the PEOS. However, for larger densities the channel is very narrow in contrast to the broad channels observed in experiments. Narrow current channels with high current density would enhance the development of the plasma instabilities described in this paper and produce collisions to broaden the channel. These instabilities could be three dimensional (as is the one described herein) and would not occur naturally in two dimensional simulations.

The conclusion is that the electrons cannot carry out a pure **E** x **B** motion. There needs to be an anomalous collision rate, ν_a, that disturbs their motion so that they are governed by

$$-e\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) - m_e \nu_a \mathbf{v} = 0 \quad . \tag{2}$$

For ν_a to make a large enough difference, we see that ν_a must be of order $\omega_{ce} = eB/m_e c$, the electron cyclotron frequency. The necessary size of ν_a has been confirmed by including ν_a in numerical calculations of **E**, **B** and the fluid velocity field, **v**, which showed that ν_a must be $\geq .2\ \omega_{ce}$ to get current patterns similar to those observed. For typical PEOS parameters $\omega_{pe} = (4\pi e^2 n_e/m_e)^{1/2} \sim 2 \times 10^{11}\ s^{-1}$ and $\omega_{ce} \sim 1\text{-}5 \times 10^{11}\ s^{-1}$ at the end of the conduction phase.

II. THE ION ACOUSTIC INSTABILITY (LINEAR)

A possible mechanism to produce anomalous collisions is ion acoustic waves driven unstable by the **E** x **B** velocity of the electrons, $\mathbf{v}_E = c\mathbf{E} \times \mathbf{B}/B^2$, if v_E exceeds several times $c_s = (ZT_e/m_i)^{1/2}$, the ion acoustic speed. Here T_e is the azimuthal electron temperature along the magnetic field lines. Such waves are unstable for all wave lengths down to the Debye length. Since the wave lengths are in general small, a local theory can be applied neglecting any inhomogeniety in the plasma. First the linearized theory of this instability will be considered.

The local **E** field can be eliminated by transforming to a frame moving with the velocity $\mathbf{v}_E$ in the **E** x **B** direction. In this frame the ions have a mean drift velocity $\mathbf{v}_o = -\mathbf{v}_E$. The ions are treated as cold and the plasma is treated as homogeneous. A coordinate system is chosen with x axis in the direction of $\mathbf{v}_E$ and z axis in the direction of **B** as shown in Fig. 2. (Note that the z direction differs from the conventional

one along coaxial geometry.) The electrostatic wave dispersion relation for a wave varying as $\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$ is

$$\varepsilon'(\mathbf{k},\omega') = k^2 - \frac{k^2\omega_{pi}^2}{(\omega' - \mathbf{k}\cdot\mathbf{v}_o)^2} + k_\perp^2 \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(\frac{2e^{-b}I_1(b)}{b}\right)$$

$$+ \frac{\omega_{pe}^2}{v_e^2}\left(1 + \xi_e Z(\xi_e)e^{-b}I_0(b)\right) = 0 , \qquad (3)$$

where $\mathbf{k}_\perp = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}$, $b = k_\perp^2 T_\perp m_e c^2/e^2B^2$, $\xi = \omega'/|k_z|v_e$, $v_e^2 = T_z/m_e$ and $Z(x)$ is the plasma dispersion function. $T_\perp$ and T_z $(= T_e)$ are the electron temperatures perpendicular and parallel to $\mathbf{B}$ respectively. Taking $b \ll 1$ and $\xi \ll 1$, yields

$$\varepsilon'(k,\omega') = 1 - \frac{\omega_{pi}^2}{(\omega' - \mathbf{k}\cdot\mathbf{v}_o)^2} + \frac{\omega_{pe}^2}{k^2v_e^2}\left(1 + i\sqrt{\pi/2}\,\xi\, e^{-\xi^2/2}\right) = 0 . \qquad (4)$$

Dropping the imaginary term, it is found that

$$\omega' = \mathbf{k}\cdot\mathbf{v}_o \pm \frac{\omega_{pi}}{\sqrt{1 + 1/k^2\lambda_D^2}} . \qquad (5a)$$

If $k\lambda_D \ll 1$, then

$$\omega' \simeq \mathbf{k}\cdot\mathbf{v}_o \pm |\mathbf{k}|c_s , \qquad (5b)$$

where $\lambda_D = v_e/\omega_{pe}$ is the Debye length and $c_s = \omega_{pi}\lambda_D = (ZT_z/m_i)^{1/2}$ is the ion acoustic speed. Since $k_z < 0$ and $k_z > 0$ are

equivalent, $k_z > 0$ is chosen for simplicity. Keeping the imaginary term in next order, the growth rate, γ, is found to be

$$\gamma = -\sqrt{\frac{\pi}{8}}\,\frac{(\omega' - \mathbf{k}\cdot\mathbf{v}_o)^3\omega'}{k^2 k_z c_s^2 v_e}\,\exp\left(\frac{-\omega'^2}{2k_z^2 v_e^2}\right) \quad . \tag{6}$$

If $|\mathbf{v}_o| >> c_s$, the sign of ω' depends on the sign of $\mathbf{k}\cdot\mathbf{v}_o$. It is seen that $\gamma > 0$ for the plus sign in Eq. (5) when $\mathbf{k}\cdot\mathbf{v}_o < 0$ and for the minus sign when $\mathbf{k}\cdot\mathbf{v}_o > 0$. Thus for unstable waves

$$\gamma = \sqrt{\frac{\pi}{8}}\,kc_s\xi\,\exp(-\xi^2/2) \quad , \tag{7a}$$

where now

$$\xi = \frac{|\mathbf{k}\cdot\mathbf{v}_o| - kc_s}{k_z v_e} < 1 \quad . \tag{7b}$$

In the lab frame the unstable wave always propagates in that direction along $\mathbf{k}$ making an angle less than 90° with the $\mathbf{E} \times \mathbf{B}$ velocity $\mathbf{v}_E$ (i.e. $-\mathbf{v}_o$). Since waves with $\mathbf{k}$ and minus $\mathbf{k}$ are equivalent, we discuss only those with $\mathbf{k}\cdot\mathbf{v}_o < 0$, $\omega' - \mathbf{k}\cdot\mathbf{v}_o = kc_s$.

In general $|\mathbf{v}_o| >> c_s$ so that the kc_s term is negligible unless $\cos\phi << c_s/|\mathbf{v}_o|$, where ϕ is the angle which $\mathbf{k}$ makes with $\mathbf{v}_E$. Neglecting the c_s term, the condition $\xi = k_x v_o/k_z v_e < 1$ becomes $\cos\phi/\cos\theta < |\mathbf{v}_e|/|\mathbf{v}_o|$, where θ is the angle between $\mathbf{k}$ and $\mathbf{B}$. The relation $\xi = 1$ determines a plane through the vector $\mathbf{E}$ and making an angle $\tan^{-1}(|\mathbf{v}_e|/|\mathbf{v}_o|)$ with the $\mathbf{E},\mathbf{B}$ plane. The maximum growth rate $\gamma_m \approx (\pi/8)^{1/2}kc_s$ is near this plane. If the condition $k\lambda_D << 1$ is relaxed, the maximum growth is found to be of order $0.15\omega_{pi}$, corresponding to a typical growth time of

2.7 ns for $n_i = 10^{13}$ cm^{-3} and C^{++}. Although γ is considerably smaller than the anomalous collision frequency ν_a that was discussed in the introduction, it will be shown that the nonlinear saturation of the wave is relatively weak, so that the waves can produce an anomalous collision rate which is faster than their growth rate.

III. THE WAVE PARTICLE INTERACTION

The interaction of these ion acoustic waves will now be considered and it will be demonstrated that this interaction leads to an effective collision frequency. In the laboratory frame

$$\omega = \omega' + \mathbf{k}\cdot\mathbf{v}_E = \omega' - \mathbf{k}\cdot\mathbf{v}_o = kc_s \quad , \tag{8}$$

so that the wave appears as an ordinary ion-acoustic wave. However, in the **E** x **B** frame it is a negative energy wave driven unstable by resonant interaction with electrons moving in the negative direction parallel to **B**. In the laboratory frame it extracts energy by removing energy from the **E** x **B** motion. This removal is the origin of the anomalous collision term in Eq. (2). In addition, the electrons are heated in the parallel direction.

To see this, consider the energy, W'_w, and the momentum, $\mathbf{P}'_w$, of a wave of amplitude $\tilde{E}$ in the moving **E** x **B** frame:

$$W'_w = \omega' \frac{\partial \varepsilon'}{\partial \omega'} \frac{\tilde{E}^2}{8\pi} \quad ,$$

$$= \frac{2\omega_{pi}^2 \omega'}{(\omega' - \mathbf{k}\cdot\mathbf{v}_o)^3} \frac{\tilde{E}^2}{8\pi} = \frac{2\omega_{pi}^2 (\mathbf{k}\cdot\mathbf{v}_o)}{k^3 c_s^3} \frac{\tilde{E}^2}{8\pi} < 0 \quad , \tag{9}$$

$$\mathbf{P}'_w = \frac{\mathbf{k}}{\omega'} W'_w = \frac{2\omega_{pi}^2}{k^3 c_s^3} \frac{\tilde{E}^2}{8\pi} \mathbf{k} \quad . \tag{10}$$

Thus, in the moving frame the unstable waves are negative energy waves, propagating in the negative **k** direction, with momentum parallel to **k**. In the lab frame

$$W_w = W'_w + \mathbf{v}_E \cdot \mathbf{P}'_w = W'_w - \mathbf{v}_o \cdot \mathbf{P}'_w \quad ,$$

$$= \frac{2\omega_{pi}^2 \omega}{k^3 c_s^3} \frac{\tilde{E}^2}{8\pi} \quad , \tag{11}$$

$$\mathbf{P}_w = \mathbf{P}'_w \tag{12}$$

Thus, in the lab frame the waves are positive energy waves, propagating in the plus **k** direction with momentum in the positive **k** direction.

Consider now the interchange of momentum and energy of a wave with the electrons and the ions in **E** x **B** frame. Now $k_z > 0$ and the wave propagation velocity along z is $\mathbf{k}\cdot\mathbf{v}_o/k_z < 0$. This wave interacts with electrons moving at this same velocity in the z direction in the moving frame. The electrons are accelerated by this wave, gaining momentum and energy at rates

$$\frac{dW_e}{dt} = -2\gamma W'_w = \sqrt{2\pi} \frac{\omega_{pi}^2}{k^2 c_s^2} \frac{(\mathbf{k}\cdot\mathbf{v}_o)^2}{k_z v_e} \frac{\tilde{E}^2}{8\pi} \quad , \tag{13}$$

$$\frac{d\mathbf{P}_e}{dt} = -2\gamma \mathbf{P}_w = -\sqrt{2\pi}\ \frac{\mathbf{k}\omega_{pi}^2 \left|\mathbf{k}\cdot\mathbf{v}_o\right|}{k^2 c_s^2 k_z v_e}\ \frac{\tilde{E}^2}{8\pi} \quad . \tag{14}$$

The resonant electrons gain a positive amount of energy in amplifying the wave. They gain momentum in the negative **k** direction. (These resonant electrons move in this direction.) The ions are unaffected except through their nonresonant interaction with the wave.

In the lab frame the electrons lose energy to the wave. This arises by their **E** x **B** velocity being slowed down, giving up energy both to the wave and also to the parallel part of the electron energy. There is consequently a force on the electrons. The part of this force perpendicular to **B** is

$$\mathbf{F}_\perp = -\sqrt{2\pi}\ \frac{\mathbf{k}_\perp \omega_{pi}^2}{k^2 c_s^2}\ \frac{\left|\mathbf{k}\cdot\mathbf{v}_o\right|}{k_z v_e}\ \frac{\tilde{E}^2}{8\pi} \quad . \tag{15}$$

In general, the waves are symmetrical with respect to the $\mathbf{v}_E$,**B** plane so that any component of $\mathbf{F}_\perp$ perpendicular to $\mathbf{v}_E$ cancels out. The component of $\mathbf{F}_\perp$ along $\mathbf{v}_E$ is in the opposite direction to the **E** x **B** motion. Denoting this component by $-\nu_a n_e m_e \mathbf{v}_E$, and setting $\tilde{\mathbf{E}} = -i\mathbf{k}\tilde{\phi}$ yields

$$\nu_a = \sqrt{\frac{\pi}{2}}\ \frac{k_x^2 v_e}{k_z}\ \left(\frac{e\tilde{\phi}}{T_z}\right)^2 \quad . \tag{16}$$

The heating in the parallel direction, Eq. (13) can be reduced to the form

$$\frac{dW_e}{dt} = \frac{dW_e'}{dt} = -\mathbf{F}_\perp\cdot\mathbf{v}_E = \nu_a n_e m_e v_E^2 \quad , \tag{17}$$

where Eq. (15) has been employed. This corresponds to the frictional heating of the electrons by the anomalous collisions with the ions.

The wave-particle interaction in the laboratory frame is illustrated in Fig. 3. The wavevector **k** is chosen so that $k_z > 0$. The ion acoustic wave propagates along **k** with velocity c_s. The condition for an electron with guiding center velocity $\mathbf{u} = \mathbf{v}_E + v_z\hat{\mathbf{z}}$ to be in resonance with the wave is

$$\mathbf{k}_\perp \cdot \mathbf{v}_E + k_z v_z = \omega = kc_s \ . \tag{18}$$

Neglecting c_s compared to v_E, we have

$$v_z = \frac{-\mathbf{k}_\perp \cdot \mathbf{v}_E}{k_z} < 0 \quad , \tag{19}$$

so that the resonant electron moves in the negative z direction. The vector **u** makes an angle of nearly 90° with **k** (if **k** is in the $\mathbf{v}_E$,**B** plane).

Because of the slope of the electron distribution function in the parallel direction, the resonant electrons extract negative momentum from the wave (see Fig. 3). Since the momentum of the wave is in the positive z direction this means that the wave must amplify, the energy coming from the $\mathbf{v}_E$ motion of the electrons. (Figure 3 has been drawn for **k** in the $\mathbf{v}_E$,**B** plane but **k** can be in a general direction and amplification will occur provided that $\mathbf{k}\cdot\mathbf{v}_E > 0$.)

From Eq. (16) it can be seen that if $k_x \sim k_z \sim 1/\lambda_D$ and $e\tilde{\Phi}/T_z \simeq 1$, then $\nu_a \simeq \omega_{pe}$, a value typically of order of the

electron cyclotron frequency as required by the discussion in the introductory section. The magnitude of $e\tilde{\Phi}/T_z$ is estimated in the next section.

IV. NONLINEAR THEORY

In this section the nonlinear amplitudes of the unstable ion acoustic waves are estimated in order to determine the actual magnitude of ν_a from Eq. (16). Because of the negative energy character of the waves they do not saturate by quasilinear saturation. Instead the resonant particles are accelerated and heated in the direction parallel to B as the waves are amplified. When the waves reach a certain amplitude, they are saturated by induced scattering off of ions. The equation for the evolution of the wave amplitudes is given by Kadomtsev[20] for the limit $T_i << T_e = T_z$ and can be written as

$$\frac{\partial I_{\mathbf{k}}}{\partial t} - \gamma_{\mathbf{k}} I_{\mathbf{k}} = \frac{\omega_{pi}^2 T_i I_{\mathbf{k}}}{n_i m_i^2 k^2} \int \frac{\left(\mathbf{k}\cdot\mathbf{k}'\right)^2}{\omega_{\mathbf{k}'}^3} \left[\mathbf{k} \times \mathbf{k}'\right]^2 \delta'(\omega'') I_{\mathbf{k}'} d^3\mathbf{k}', \tag{20}$$

where $I_{\mathbf{k}} d^3\mathbf{k}$ is the mean contribution to $\langle\tilde{\Phi}^2\rangle$ from waves with $\mathbf{k}$ in $d^3\mathbf{k}$, $\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$, $\omega'' = \omega_{\mathbf{k}} - \omega_{\mathbf{k}'}$, $\omega_{\mathbf{k}}$ is the frequency of the wave with wave vector $\mathbf{k}$ and δ' is the derivative of the delta function. The right hand side represents the rate of transfer of intensity from the $\mathbf{k}'$ mode to the $\mathbf{k}$ mode by induced scattering. In a steady state, when modes are saturated, energy should be scattered predominantly from unstable modes to damped modes, although there should be some scatter among the unstable modes as well as among the damped modes. The unstable modes are separated

from the damped modes by the plane in **k** space through the origin and perpendicular to $\mathbf{v}_E$.

Making use of the result that $\omega_k = kc_s$ and from the delta function factor which shows that $k \simeq k'$, we may reduce Eq. (20) to the form

$$\frac{\partial \Phi_k}{\partial t} - \gamma_k \Phi_k = - \frac{kc_s T_i \Phi_k}{ZT_e} \int_{|\mathbf{k}'|=|\mathbf{k}|} d\Omega_{\mathbf{k}'} \mu''^2 (1 - \mu''^2) \frac{\partial}{\partial k'} \left(\Phi_{k'}\right) \quad , \tag{21}$$

where $\mu'' = \mathbf{k}\cdot\mathbf{k}'/kk'$ and where we have introduced

$$\Phi_{\mathbf{k}} = \frac{e^2}{T_e^2} k^3 I_{\mathbf{k}} \quad . \tag{22}$$

In terms of $\Phi_{\mathbf{k}}$, which is dimensionless, the expression for ν_a may be written as

$$\nu_a = \sqrt{\frac{\pi}{2}}\, v_e \int \frac{k_x^2 \Phi_{\mathbf{k}}}{|k_z| k^3} d^3\mathbf{k} \quad . \tag{23}$$

Equation (21) is an integro-differential equation for Φ_k, which must be solved numerically. However, its interpretation is clear. Waves with positive k_x are unstable with $\gamma_k > 0$, so that their amplitude grows due to the γ_k term in Eq. (21). Similiarly, $\gamma_k < 0$ for the damped modes with $k_x < 0$, and their amplitude decreases due to the γ_k term. The nonlinear scattering term on the righthand side of Eq. (21) transfers energy between all the modes with $k' \simeq k$. In equilibrium the transfer of energy from unstable to stable modes must balance the growth of the

unstable modes, and the damping of the stable modes. The sign of the right hand side must be such as to give this balance. The transfer of energy among the unstable modes can be regarded as a rearrangement of the energy of the unstable modes, but does not contribute to the equilibrium balance.

To get an idea as to the magnitude of the energy in the unstable modes, consider the assumption that $\Phi_{\mathbf{k}}$ is independent of angle for the unstable modes ($k_x < 0$) and a function of k alone, say $\Phi^U(k)$. Similiarly, assume that $\Phi_{\mathbf{k}}$ is a different function of k alone, say $\Phi^S(k)$, for the stable modes. This assumption for Φ_k will not solve the full Eq. (21) of course, but it can be chosen to make two moments of this equation vanish. We average Eq. (21) for this choice for $\Phi_{\mathbf{k}}$ over all unstable directions $k_x > 0$, and again over all stable directions $k_x < 0$, to obtain two differential equations for Φ^U and Φ^S. [This corresponds to the Eddington approximation in the theory of radiative transfer. It can be generalized by expanding Φ in a series of Legendre functions and taking Legendre moments of Eq. (21).]

These assumptions can be written as

$$\Phi_{\mathbf{k}} = \Phi^U(k) \qquad k_x > 0 \quad , \tag{24a}$$

$$\Phi_{\mathbf{k}} = \Phi^S(k) \qquad k_x < 0 \quad . \tag{24b}$$

In taking the angular averages we must neglect the coupling of two unstable modes, or two damped modes, since physically this coupling does not lead to a change in either the stable or unstable energy. This fact is not represented by our approximation.

Substituting Eqs. (24a) and (24b) into Eq. (21) and integrating first over the hemisphere $k_x > 0$ with the angular integral over k' restricted to $k_x' < 0$ yields

$$\frac{\partial \Phi^U}{\partial t} = \langle \gamma_{\mathbf{k}} \rangle_U \, \Phi^U - \frac{4\pi k^2 c_s T_i}{15 \, Z \, T_e} \, \Phi^U \, \frac{\partial \Phi^S}{\partial k} \quad . \tag{25a}$$

Carrying out the two integrals over the opposite hemispheres yields

$$\frac{\partial \Phi^S}{\partial t} = \langle \gamma_{\mathbf{k}} \rangle_S \, \Phi^S - \frac{4\pi k^2 c_s T_i}{15 \, Z \, T_e} \, \Phi^S \, \frac{\partial \Phi^U}{\partial k} \quad . \tag{25b}$$

Here $\langle \gamma_{\mathbf{k}} \rangle_U$ and $\langle \gamma_{\mathbf{k}} \rangle_S$ are the growth rates given by Eq. (7) angularly averaged over the unstable and stable hemispheres. Their averages are given by

$$\langle \gamma_{\mathbf{k}} \rangle_U = \sqrt{\frac{\pi}{8}} \, \langle \xi \exp(-\xi^2/2) \rangle_U k c_s = \frac{\alpha}{\sqrt{8\pi}} \exp(\alpha^2/2) \, E_1(\alpha^2/2) \, k c_s \quad , \tag{26a}$$

$$= 0.184 \, f(\alpha) \, k c_s \quad ,$$

$$\langle \gamma_{\mathbf{k}} \rangle_S = - \, 0.184 \, f(\alpha) \, k c_s \quad , \tag{26b}$$

where $\alpha \equiv v_E/v_e$, $E_1(x)$ is the exponential integral[21] and $f(\alpha)$ is normalized to unity at $\alpha = 1$. For small α, $f(\alpha) \sim 2.17\alpha \ln(1/\alpha)$, while for large α, $f(\alpha) \sim 2.17/\alpha$.

Assuming a steady state in Eqs. (25) yields

$$\beta = k \, \frac{d\Phi^S}{dk} \quad , \tag{27a}$$

$$-\beta = k \frac{d\Phi^U}{dk} \quad , \tag{27b}$$

where $\beta = 0.22(ZT_e/T_i)f(v_E/v_e)$. The solutions of these equations are

$$\Phi^S = \beta \ln(k/k_{min}) \quad , \tag{28a}$$

$$\Phi^U = \beta \ln(k_D/k) \quad . \tag{28b}$$

Boundary conditions have been chosen so that Φ^S is zero at some minimum value of k denoted by k_{min}. That k_{min} is not zero is a reflection of the fact that for small k the growth rate is too slow to build up the waves instantaneously to the steady state value. (k_{min} should decrease in time as 1/t.) Also the condition that $\Phi^U = 0$ at $k = k_D = 1/\lambda_D$ has been employed since at shorter wavelengths, the waves are damped by ion Landau damping. However, a finite amplitude for the unstable waves can be sustained by nonlinear coupling.

One may now substitute the expressions for Φ_k, Eqs. (24), into Eq. (23) to evaluate ν_a. There results

$$\nu_a = \sqrt{\frac{\pi}{2}}\, 2\pi v_e \left\langle \frac{k_x^2}{|k_z|k} \right\rangle \int_{k_{min}}^{k_D} \left(\Phi^U + \Phi^S \right) dk \quad , \tag{29}$$

where $\langle k_x^2/|k_z|k\rangle$ represents the angular average of $k_x^2/(|k_z|k)$. This average is logarithmically divergent for small k_z. However, because the growth rate, given in Eq. (7), is exponentially small

for small k_z, we may reasonably cut this average off, say at $k_z = 0.1$. This yields $\langle k_x^2/|k_z|k\rangle = 0.9$. Substituting Eqs. (28a) and (28b) into Eq. (29), we find

$$\nu_a = 1.56 \frac{ZT_e}{T_i} f\left(\frac{v_E}{v_e}\right) \ln\left(\frac{k_D}{k_{min}}\right) \omega_{pe} \quad . \tag{30}$$

As noted above k_{min} decreases with time, so ν_a should slowly increase with time.

Although the procedure employed to solve Eq. (21) is admittedly very approximate, it does bring out the physics. The resulting value for the anomalous collision frequency, ν_a, is correspondingly approximate, but it should be correct in order of magnitude. Equation (30) indicates that the ion acoustic mode driven unstable by the large **E x B** drift can reach an amplitude that is sufficiently large to *produce* a ν_a as large as the electron plasma frequency, ω_{pe}, even though the growth rate of the mode is of order ω_{pi}. The reason that this is possible is that the mode coupling that saturates the waves is so weak that the waves grow to a very large amplitude independent of its linear growth rate. Further, because the waves propagate at the ion acoustic speed, they can achieve this large amplitude without leaving the plasma.

It should be noted that it was assumed that the mode is strongly unstable, so that $v_E \gg c_s$ and so that the c_s term can be neglected in Eq. (7b). If ν_a is too large, v_E will be reduced, and c_s will increase through heating of the electron temperature parallel to **B**. Eventually the unstable angular

region in the $k_x > 0$ hemisphere will be reduced and ν_a will be correspondingly reduced.

Finally, it should be noted that the equations employed in the derivation of the saturated mode amplitude and of ν_a are based on the small amplitude limit $e\tilde{\phi}/T_e << 1$. If $e\tilde{\phi}/T_e \sim 1$, many more nonlinear effects must be included. Hence, the coefficient of ω_{pe} is not accurate. At most it can be said that $e\tilde{\phi} \sim T_e$ and that $\nu_a \sim \omega_{pe}$.

V. CONCLUSIONS

It has been shown that an anomalously large electron-ion collision frequency is needed to allow the flow of current across the large magnetic field in the PEOS. From general considerations this collision frequency is shown to be of order of a few tenths of the electron cyclotron frequency.

It is proposed here that one mechanism for producing this anomalous collision frequency is a large amplitude ion acoustic waves. These waves, when propagating in a direction which makes an angle of less than 90 degrees with respect to the **E x B** direction, are driven unstable by the large **E x B** velocity of the electrons through the ions. The growth rates of these waves is comparable with the ion plasma frequency. In most cases this frequency is very large compared with the time scales for switch operation, so that there is time for many exponentiations of the wave amplitude. Since the waves propagate at the ion acoustic speed, they do not propagate very far during their growth times

so that they are absolutely rather than convectively unstable.

Examination of the quasilinear wave-particle interactions of these waves has shown that they remove momentum and energy from the directed motion of the electrons perpendicular to the magnetic field at a rate that can be represented by the anomalous collision frequency ν_a. The resulting frictional energy goes to heat the thermal motion of the electrons parallel to the magnetic field. The thermal motion perpendicular to the field is unchanged because of the adiabatic invariance of the magnetic moment of the electrons. When the modes damp, they transfer the momentum to the ions.

To determine the magnitude of ν_a it is necessary to determine the saturated amplitude of the waves. There is no saturation of the waves by quasilinear wave-particle interactions because the energy of the waves is effectively negative. The only quasilinear saturation effect is to heat the parallel motion of the electrons so as to raise the acoustic speed c_s. This speed is usually much smaller than the driving **E x B** velocity v_E. The saturation mechanism is consequently the much weaker one of nonlinear scattering of the unstable ion waves into stably propagating directions. The nonlinear integrodifferential equation which determines the saturation was approximated by a simple Eddington type assumption, in order to derive an approximate value for the saturated amplitudes.

The result is that because the saturation process is weak, the ion acoustic waves grow to large amplitudes ($e\tilde{\phi}/T_e \sim 1$), and the resulting anomalous collision frequency is of order of the

electron plasma frequency. In most cases of PEOS operation this is of the same order of magnitude as the electron cyclotron frequency and is large enough to explain the cross field current.

It is thus concluded that the unstable ion acoustic waves are strong enough to produce the collisions necessary to explain the cross field currents and the magnetic field penetration into the PEOS plasma.

VI. ACKNOWLEDGEMENTS

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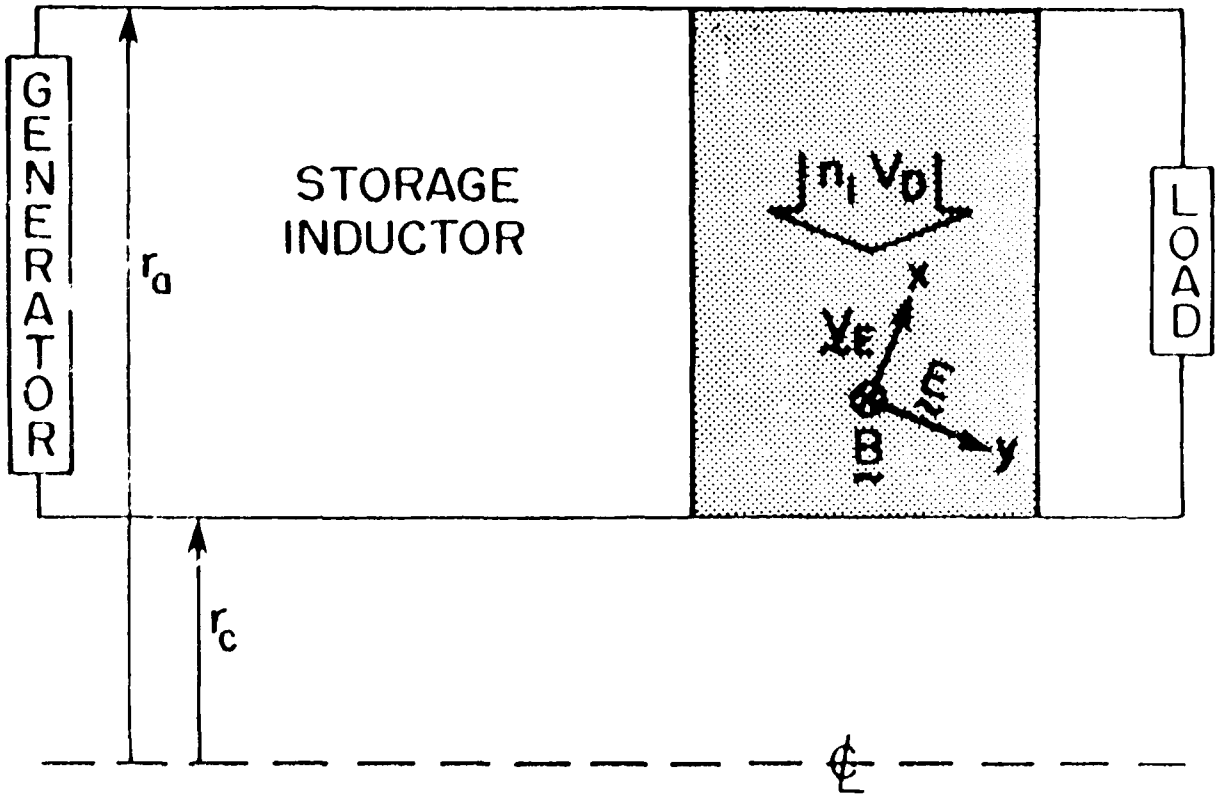

Fig. 1 - The geometry of the PEOS. The directions of the electric field and the resulting E x B drift at a point are indicated. The coordinate system employed in Secs. II-V is also displayed.

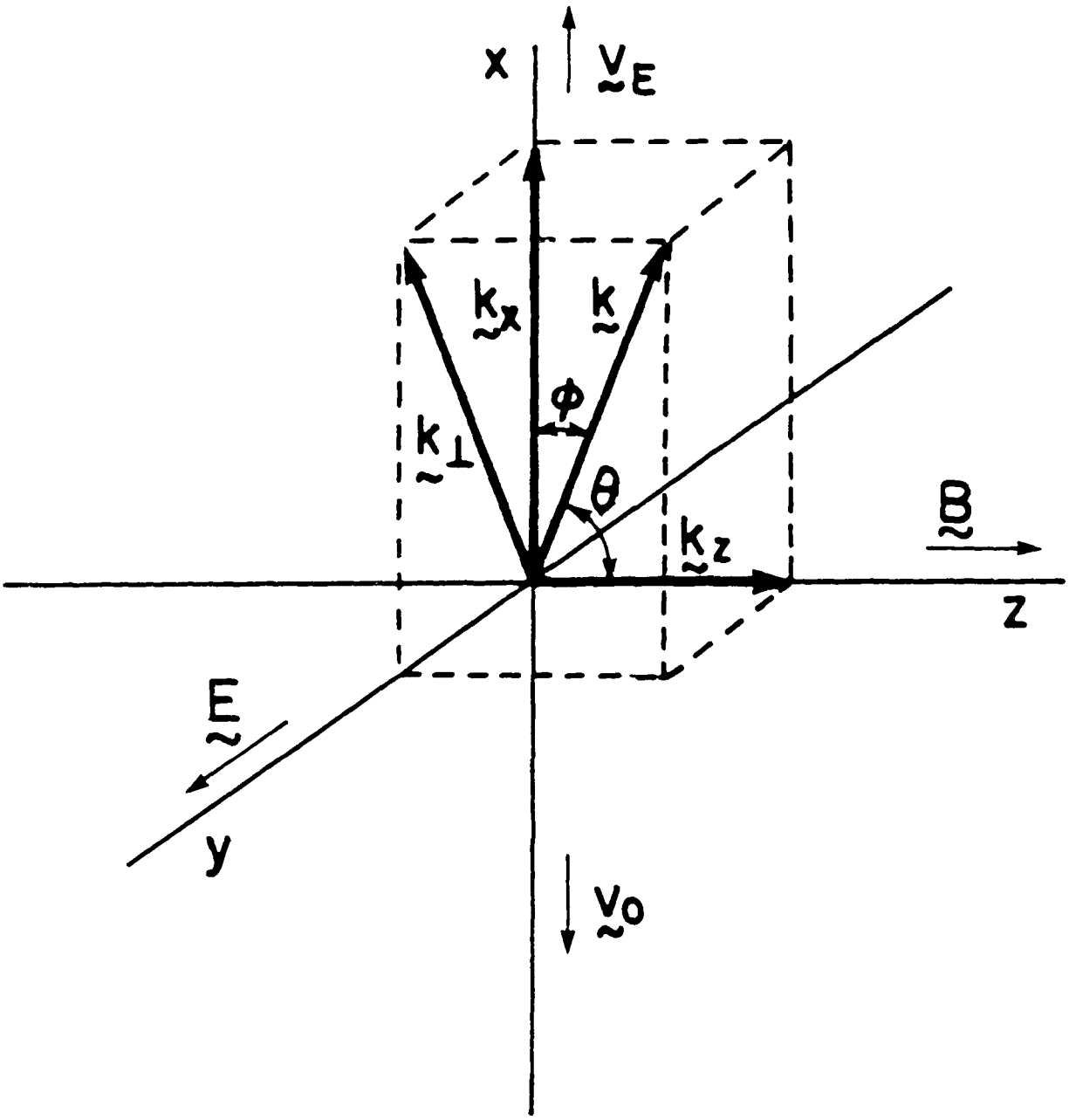

Fig. 2 - The orientation of the coordinate system relative to E, B and v_E is shown. Also for a general wavevector **k**, the various components are given. **k** is chosen in the unstable direction.

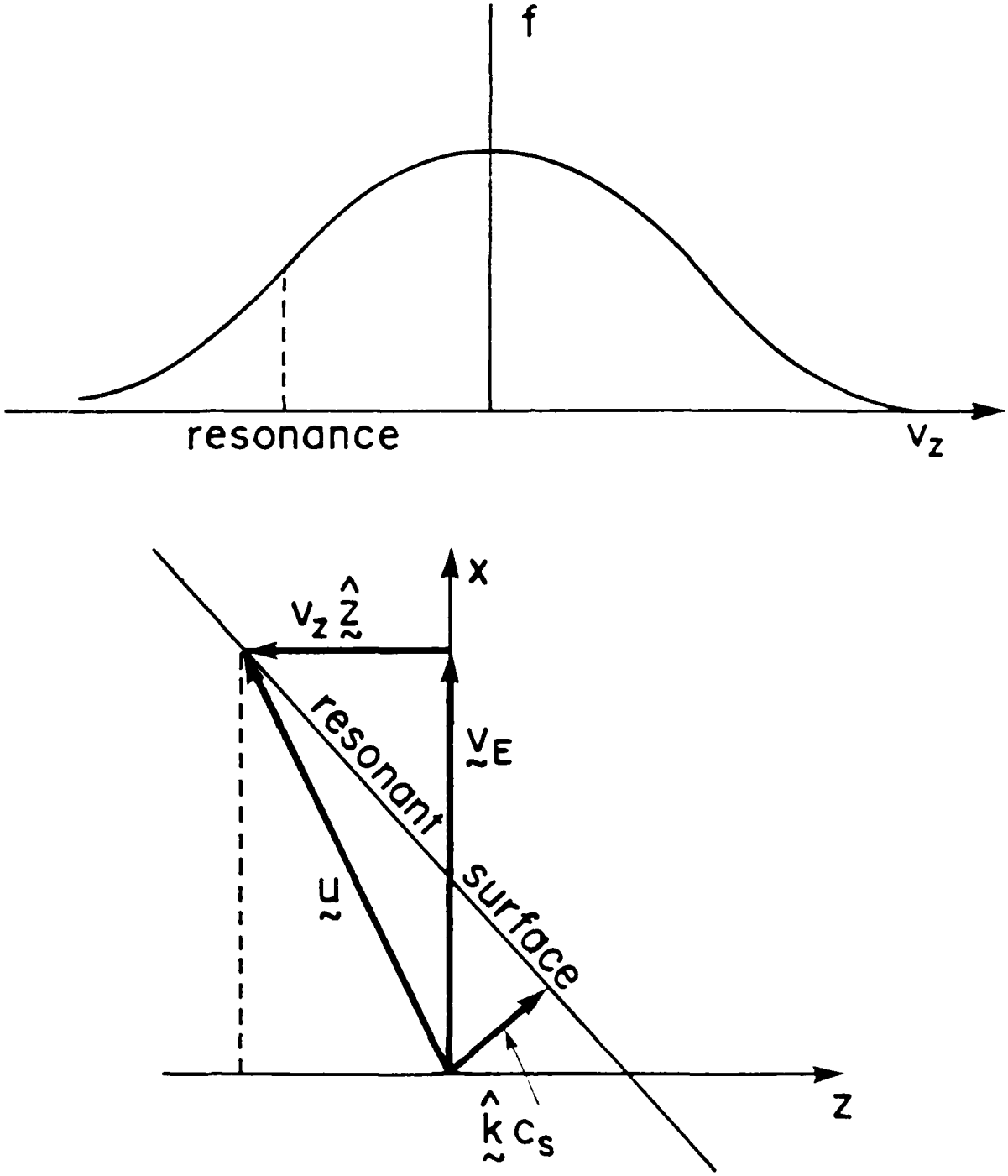

Fig. 3 - The resonant electrons have a parallel velocity in the opposite direction to the wave propagation along **B**. How this happens is shown in the lower figure. Because $\mathbf{v}_E$ is large, $\mathbf{v}_E + v_z\hat{z}$ lies on the constant phase surface of the wave even though $v_z < 0$.

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