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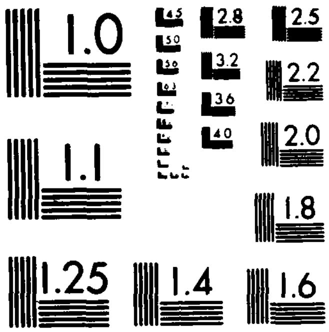

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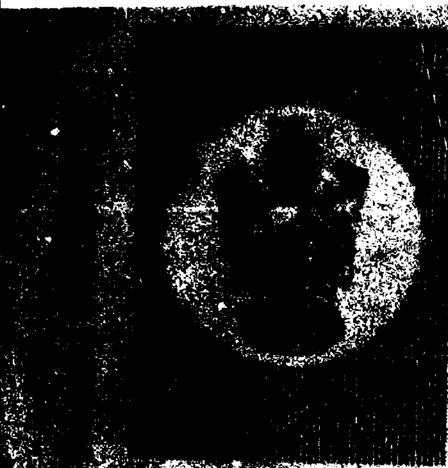

ROYAL SIGNALS AND RADAR ESTABLISHMENT, MALVERN

THE AVOIDANCE OF COLLISIONS FOR NEWTONIAN BODIES WITH HIDDEN VARIABLES

Author: B D Bramson

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October 1987

ROYAL SIGNALS AND RADAR ESTABLISHMENT

Report 87013

Title: The Avoidance of Collisions for Newtonian Bodies with Hidden Variables

Author: B D Bramson

Date: October 1987

SUMMARY

The collision avoidance of a pair of uniformly moving bodies is considered in three dimensions. The relative motion of the bodies yields an expression relating the time to closest approach, the minimum range, the current range and its rate of change, other variables being unobservable. A Boolean relation is then proposed that is satisfied whenever the minimum range and time to closest approach simultaneously fall below given thresholds. The relation is further studied, in particular with regard to the issue of false and premature alarms. An airborne collision avoidance system is a possible application.

CONTENTS

1 INTRODUCTION

This paper concerns the relative motion in three spatial dimensions of a pair of uniformly moving bodies, idealised as point particles. The purpose is to generate relations between observables that will be useful for the prediction and avoidance of collisions. Applications include the design of an airborne collision avoidance system.

The problem would appear to be mathematically trivial save for a crucial engineering constraint. It is given that the apparatus for collision avoidance, carried by at least one of the bodies, is capable of measuring directly only the relative range of the bodies and its rate of change in time. It is assumed that further time derivatives are not computable with sufficient accuracy and that other observables, like relative bearing and velocity, are hidden.

Section 2 discusses the relative motion of the two bodies, expressions being presented for the time, t, to closest approach, henceforth called the warning time, and the minimum range, m. Both m and t depend on the relative velocity, v, and are therefore unobservable; but an expression is found, independent of v, that relates m and t to the current range, r, and its rate of change, $\dot{r}$.

In section 3, a pair of threshold constants are introduced, mu and tau, for m and t respectively. mu defines a minimum safe separation for the two bodies while tau defines a minimum warning time whenever mu exceeds m. Indeed, a Boolean relation, danger, is defined between r, $\dot{r}$, m, t, mu and tau but, being dependent on v, it is unobservable. An expression is further derived for the time for r to reach mu, given appropriate pre-conditions.

In section 4, an observable Boolean relation, alarm, is presented involving r, $\dot{r}$, mu and tau. Its strength lies in its ability to flag danger reliably while its weakness lies in its tendency to generate either false or premature alarms.

Section 5 briefly illustrates the alarm in configuration space.

2 UNIFORM RELATIVE MOTION

At time u, consider a pair of uniformly moving point particles with relative position vector X(u) and (constant) relative velocity vector $\dot{X}$. ($\dot{}$ denotes d/du.) Let their relative range be r(u), being the magnitude of X(u), so that their relative range rate is $\dot{r}(u)$. Further, let their relative speed be v, being the magnitude of $\dot{X}$.

To calculate the warning time, t(u), and the minimum range, m, the equation of relative motion is used. This determines the relative position vector at time u from that at time 0 according to

$$X(u) = X(0) + \dot{X}\, u \quad , \qquad (2.1)$$

whence the behaviour of the relative range is given by

$$r^2(u) = r^2(0) + 2r(0)\,\dot{r}(0)\,u + v^2\,u^2 \ . \qquad (2.2)$$

Henceforth assume that v /= 0. This allows us to choose the origin of time to correspond to the instant of closest approach. This being so,

$r(0) = m$, $\dot{r}(0) = 0$ and $t = -u$. Thus, dropping the argument (u),

$$r^2 = m^2 + v^2 u^2 \ . \tag{2.3}$$

Differentiating with respect to u and using $t = -u$ provides an expression for the warning time,

$$t = - r \dot{r} / v^2 \ , \tag{2.4}$$

while the minimum range, m, is given by

$$m^2 = r^2 (1 - \dot{r}^2 / v^2) \ . \tag{2.5}$$

Note from (2.4) that

$$(r > 0 \text{ AND } \dot{r} < 0) = t > 0 \ . \tag{2.6}$$

For $v = 0$, m and t are both undefined. Even for $v /= 0$, neither m nor t are known since v is a hidden variable. Nevertheless, they possess a relation that is independent of v and this follows from equations (2.3) and (2.4):

$$m^2 = r^2 + r \dot{r} t \ . \tag{2.7}$$

When the bodies are converging in range, $\dot{r} < 0$, it follows that

$$m^2 + r |\dot{r}| t = r^2 \text{ AND } t > 0 \tag{2.8}$$

which, for given r and $\dot{r}$ (< 0), is illustrated in figure 1.

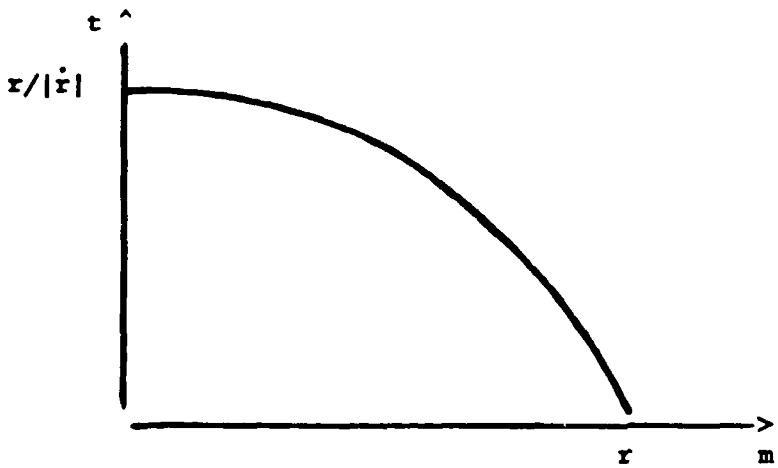

Figure 1: Knowing r and $\dot{r}$ (< 0) yields a relation between the warning time t and the minimum range m.

3 THE IMPOSITION OF THRESHOLDS

Let us now impose a minimum safe separation on the two bodies, call it mu (> 0). We require that r > mu for all time. Further, if m <= mu < r and $\dot{r} < 0$, we shall wish to be alerted should the warning time be less than a certain threshold, tau (> 0), this being considered dangerous. Within this context it is vital to have a rigorous definition of danger. So, consider the time dependent Boolean expression

$$\text{danger} = (r \le \mu \text{ OR } \dot{r} < 0 \text{ AND } m \le \mu \text{ AND } t \le \tau) \; . \qquad (3.1)$$

(AND binds tighter than OR.)

Sadly, unless $r \le \mu$, danger is unobservable being dependent on v and an attempt to resolve this is made in the following section.

To conclude this section, we calculate the time, t_μ, for r to reach the critical value, μ, given that $m \le \mu$.

Using equation (2.3) we see that if $m \le \mu$, r reaches μ when

$$u^2 = (\mu^2 - m^2) / v^2 \; . \qquad (3.2)$$

The two solutions for $m < \mu$ correspond to events before and after closest approach. It follows that the time to reach threshold before the instant of closest approach is given by

$$t_\mu = t - (\mathrm{sqrt}(\mu^2 - m^2)) / v \; . \qquad (3.3)$$

The relative motion of the two bodies is illustrated in figure 2.

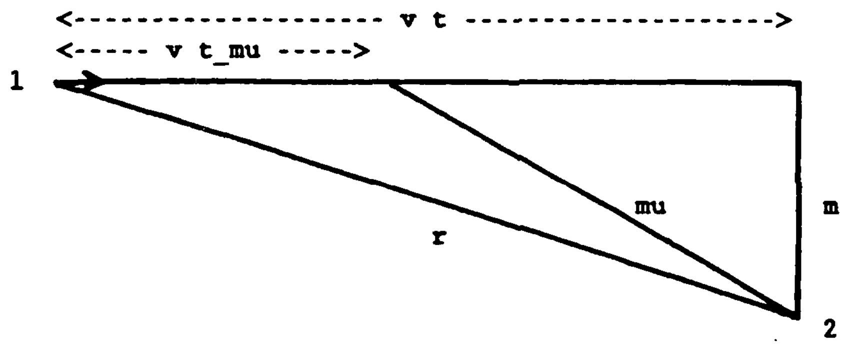

Figure 2: The motion of 1 relative to 2 when $\dot{r} < 0$ AND $m \le \mu < r$.

4 AN OBSERVABLE ALARM

The Boolean relation, danger, defined in equation (3.1) is unobservable; but in section 2 we derived a relation (2.7) between m and t in terms only of observable quantities. Constraining m and t as in (3.1) imposes constraints on r^2 from (2.7). Accordingly, define the Boolean expression, alarm, by

$$\text{alarm} = (r \le \mu \text{ OR } \dot{r} < 0 \text{ AND } r^2 \le \mu^2 - r\,\dot{r}\,\tau) \; . \qquad (4.1)$$

Using the relations (3.1), (4.1) and (2.8), it follows that

$$\text{danger} \Longrightarrow \text{alarm} \; . \qquad (4.2)$$

Unlike danger, alarm is observable and the question therefore arises as to whether it may be used in place of danger. To examine this possibility, we use equations (2.3), (2.4) and (2.6) together with the fact that $u = -t$, to rewrite (4.1) as

$$\begin{aligned}&\text{alarm} = \\ &(m^2 + v^2\, t^2 <= mu^2 \text{ OR} \\ &\ t > 0 \text{ AND } m^2 + v^2\, t(t - tau) <= mu^2)\ .\end{aligned} \tag{4.3}$$

To examine this further, take the conjunction with

$$t <= 0 \text{ OR } t > 0 \tag{4.4}$$

and use the property that

$$\begin{aligned}&(t > 0 \text{ AND } m^2 + v^2\, t^2 <= mu^2\ ) \implies \\ &(t > 0 \text{ AND } m^2 + v^2\, t(t - tau) <= mu^2)\ .\end{aligned} \tag{4.5}$$

This yields the expression

$$\begin{aligned}&\text{alarm} = \\ &(t <= 0 \text{ AND } m^2 + v^2\, t^2 <= mu^2 \text{ OR} \\ &\ t > 0 \text{ AND } m^2 + v^2\, (t - tau/2)^2 <= mu^2 + v^2\, tau^2/4)\ ;\end{aligned} \tag{4.6}$$

which defines the union of the interiors of quadrants of two ellipses in the (t, m) plane, with centres (0, 0) and (tau/2, 0) respectively, and is illustrated in figure 3.

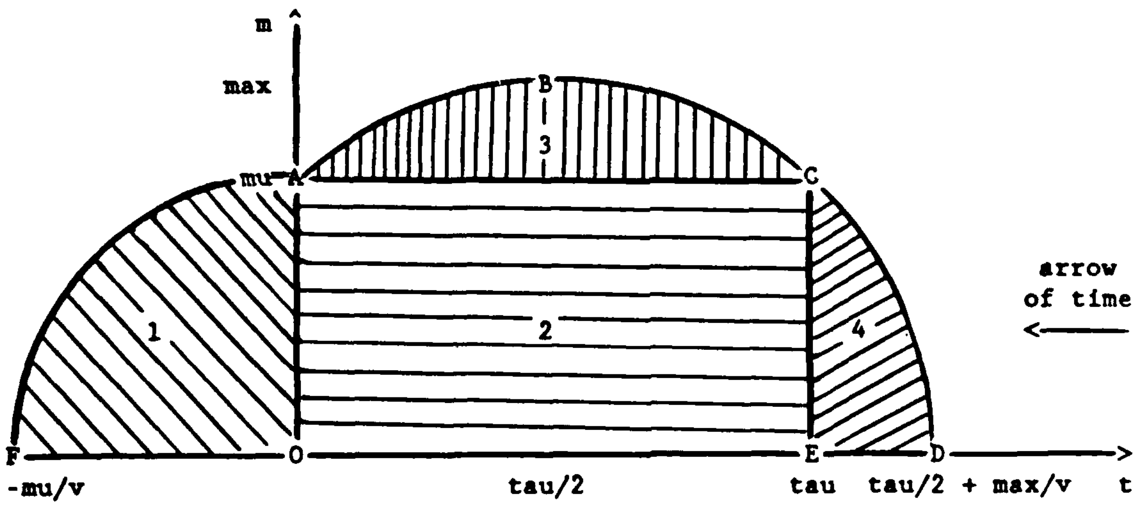

Figure 3: The shaded regions comprise alarm. The arrow of time is the direction of increasing u.

Note that the alarm is raised only if m is bounded above by a certain velocity dependent expression:

$$\text{alarm} \implies m <= max\ , \tag{4.7}$$

with $$max = sqrt(mu^2 + v^2\, tau^2/4)\ . \tag{4.8}$$

Further, the boundary of alarm includes the points:

	A	(0, mu)	,
	B	(tau/2, max)	,
	C	(tau, mu)	,
	D	(tau/2 + max/v, 0)	
and	F	(-mu/v, 0)	.

(4.9)

In fact alarm comprises the disjunction of four Boolean expressions corresponding to the four distinct regions in figure 3. These may be identified formally by forming the conjunction of (4.3) with the expression

(m <= mu OR m > mu) AND (4.10)
(t <= 0 OR 0 < t <= tau OR t > tau) ,

yielding

alarm = (4.11)
1 (t <= 0 AND $m^2 + v^2 t^2$ <= mu^2 OR
2 0 < t <= tau AND m <= mu OR
3 0 < t < tau AND m > mu AND $m^2 + v^2$ t(t - tau) <= mu^2 OR
4 t > tau AND m < mu AND $m^2 + v^2$ t(t - tau) <= mu^2) .

Region 1 is bounded by the lines OA and OF and the ellipse segment AF while region 2 is bounded by the rectangle OACE. The union of regions 1 and 2 correspond to danger. From (4.2) we expect the alarm to be raised here. The remainder of alarm comprises: region 3, a false alarm, bounded by the ellipse segment ABC and the line AC; and region 4, a premature alarm, bounded by the ellipse segment CD and by the lines CE and ED.

We now examine the behaviour of alarm for the two cases, m <= mu and m > mu. In each case consider the motion of a point in the (t, m) plane, parallel to the t axis and in the direction of t decreasing, representing the relative motion of the two bodies.

CASE 1 m <= mu. The alarm comes on when the warning time satisfies

t = tau/2 + 1/v sqrt($max^2 - m^2$) >= tau , (4.12)

corresponding to a point on the ellipse segment CD, and goes off when

t = -1/v sqrt($mu^2 - m^2$) , (4.13)

corresponding to a point on the ellipse segment AF.

When m = mu the motion follows CA, the alarm coming on when t = tau and going off when t = 0.

When m < mu a premature warning is given when the motion encounters the segment CD. Indeed, the alarm comes on for a value of t, given by equation (4.12), that may be arbitrarily large for v small enough, approximating to mu/v for a slow, collision course (along DEOF). However, the time t_mu for r to reach its threshold mu is bounded above by tau. To see this, use equations (3.3) and (4.3) to show that, on the segment CD,

t_mu = t - sqrt(t(t - tau)) AND t >= tau . (4.13)

t_mu decreases monotonically from tau as t increases from tau and is bounded below by tau/2 (see appendix). The expression (4.13) includes the case when m = mu (t_mu = t = tau) and is illustrated in figure 4.

After region 4 the motion proceeds into region 2 and thence, following the instant of closest approach, into region 1 where the bodies while diverging in range are still within a distance mu apart.

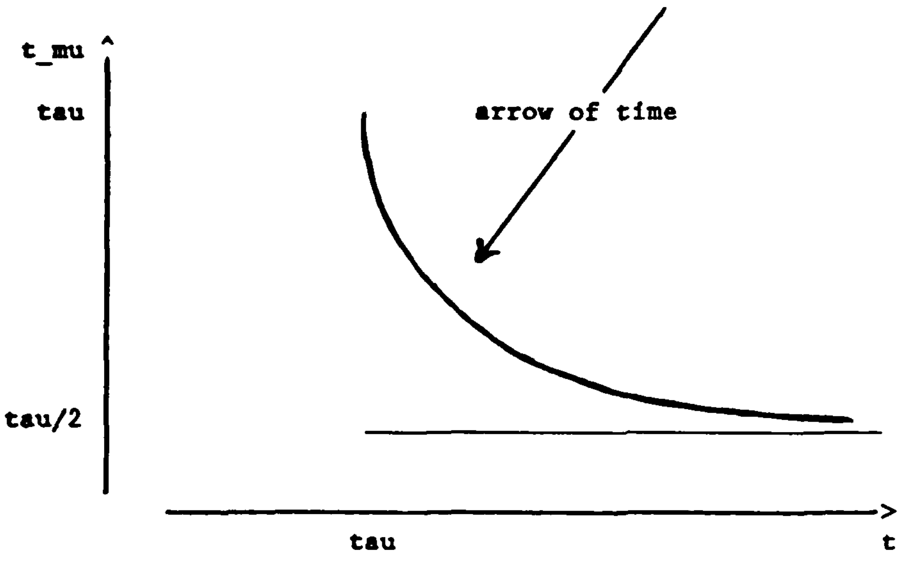

Figure 4: When m <= mu, the alarm is raised as soon as t_mu reaches a value that satisfies tau/2 < t_mu <= tau.

CASE 2 m > mu. This appears to be a false alarm. The alarm is raised if m <= max, defined in (4.8). It comes on when

$$t = tau/2 + 1/v \; sqrt(max^2 - m^2) \; , \qquad (4.14)$$

which satisfies tau/2 <= t < tau, and goes off when

$$t = tau/2 - 1/v \; sqrt(max^2 - m^2) \; , \qquad (4.15)$$

satisfying 0 < t < tau/2.

For v small compared with mu/tau, the upper bound for m, max, achieved at B, lies close to mu while for large v it approximates to v tau/2.

To conclude this section a brief comparison will be made between alarm and one version of the modified tau criterion [1]. Versions differ on their behaviour following the instant of closest approach, the one chosen here being given by

$$\begin{aligned} &modified\_tau = &(4.16)\\ &(r <= mu \text{ OR } \dot{r} < 0 \text{ AND } r <= mu - \dot{r} \; tau) \; . \end{aligned}$$

THEOREM alarm ==> modified_tau . (4.17)

In other words, modified_tau is raised whenever alarm is raised. This follows swiftly from the definition (4.1) on noting that

$$r > mu \Longrightarrow mu^2 - r \, \dot{r} \; tau < r \; (mu - \dot{r} \; tau) \; . \qquad (4.18)$$

5 CONFIGURATION SPACE PICTURE

The analysis in the previous section concerned relations between the minimum range, m, the warning time, t, and the time, t_mu, for the range threshold to be reached, when the alarm is raised. Following the spirit of work by Ford [1, 2], it is also convenient to illustrate the boundary of alarm in configuration space.

Using X and $\dot{X}$ to denote the relative position and velocity vectors at time u, as in section 2, and defining max by (4.8), the expression (4.1) for alarm may be rewritten

$$\text{alarm} = (|X| \le \text{mu OR} \quad X.\dot{X} < 0 \text{ AND } |X + \text{tau}/2\ \dot{X}| \le \text{max}) \ ; \qquad (5.1)$$

which represents the union of the interiors of a pair of spheres whose radii are mu and sqrt($\text{mu}^2 + v^2\ \text{tau}^2/4$) and whose centres lie in the direction of relative motion, a distance v tau/2 apart. This is illustrated in figure 5 from which many of the properties previously derived receive a ready interpretation.

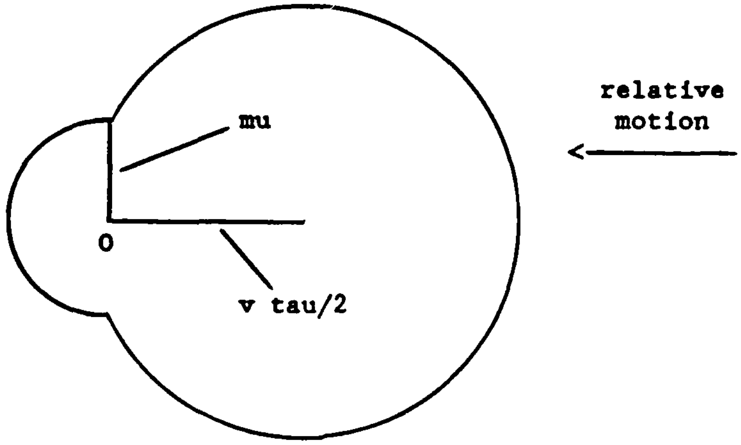

Figure 5: Alarm comprises the union of the interiors of a pair of spheres in configuration space.

6 CONCLUDING REMARKS

This paper has discussed the collision avoidance of two Newtonian bodies in uniform motion when only the relative range and its rate of change are observable. In section 3 the concept of danger was defined in terms of given threshold constants, mu and tau, by

$$\text{danger} = (\text{range} \le \text{mu OR} \quad \text{range rate} < 0 \text{ AND minimum range} \le \text{mu} \quad \text{AND warning time} \le \text{tau}) \ . \qquad (6.1)$$

However, danger is in general unobservable, the minimum range, m, and warning time, t, depending on the relative velocity, v. Hence, an observable Boolean relation, alarm, was introduced in section 4:

$$\text{alarm} = (r \le \text{mu OR} \quad \dot{r} < 0 \text{ AND } r^2 \le \text{mu}^2 - r\,\dot{r}\ \text{tau}) \ ; \qquad (6.2)$$

and this is raised whenever there is danger:

$$\text{danger} \Longrightarrow \text{alarm} \ . \qquad (6.3)$$

The question then arose as to the use of alarm to indicate danger. In particular, if the two bodies are converging in range ($\dot{r} < 0$) what are the implications of alarm? The conclusions, illustrated in figures 3 and 4, amount to two Boolean cases in addition to danger. One of these involves the time t_mu for r to reach its threshold mu.

To summarise, for m <= mu the alarm comes on with t >= tau as soon as t_mu becomes small enough, its value lying between tau/2 and tau and depending on v; and this will happen before danger is reached. Whether this is regarded as a premature alarm depends on whether a warning that r is approaching mu is considered useful. After the instant of closest approach the alarm goes off as soon as r exceeds mu.

Secondly, an alarm is raised for mu < m <= sqrt(mu^2 + v^2 tau^2/4) as soon as t becomes small enough, its value, dependent on v, lying between tau/2 and tau. The expression bounding m here behaves like mu for v small compared with mu/tau and like v tau/2 for large v. Finally, the alarm goes off for some value of t lying between 0 and tau/2. Whether this case is regarded as a false alarm depends on whether a level of protection that increases with v is considered useful.

With regard to its use for a collision avoidance system, Ford [2] has compared the behaviour of alarm with that of other criteria. In relation to this it was proved, at the end of section 4, that

$$\text{alarm} \Longrightarrow \text{modified\_tau} \,, \qquad (6.4)$$

where
$$\text{modified\_tau} = (r \le \text{mu OR } \dot{r} < 0 \text{ AND } r \le \text{mu} - \dot{r}\,\text{tau}) \,. \qquad (6.5)$$

Thus modified_tau is raised whenever alarm is raised. However, the converse is not true: modified_tau is raised more frequently than alarm.

ACKNOWLEDGEMENT

I am indebted to Dr R L Ford for stimulating my interest in this problem and for many helpful conversations.

REFERENCES

1 R L Ford: The protected volume of airspace generated by an airborne collision avoidance system; J Nav 39 pp 139-158, 1986.

2 R L Ford: A new range test criterion for ACAS; ICAO SICASP Working Group 2, Working Paper, July 1987.

APPENDIX

Consider the behaviour of the real-valued partial function

$$f(t) = \text{IF } t \ge \text{tau THEN } t - \text{sqrt}(t(t - \text{tau})) \,, \qquad (A1)$$

with tau (> 0) regarded as fixed. To examine f, put

$$f(t) = g(x) \quad \text{with} \quad t = \text{tau} \cosh^2 x \,, \text{ for } x \ge 0 \,. \qquad (A2)$$

As x increases from 0, so t increases from tau. Standard properties of hyperbolic functions may then be used to show that

$$g = \text{tau}/2\ (1 + \exp(-2x)) \,, \qquad (A3)$$

which decreases monotonically from tau, tending to tau/2.

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