AD-A191 567 RESEARCH IN ENUMERATION AND GRAPH THEORY WITH APPLICATIONS TO COMPUTER SCIENCE(U) CALIFORNIA UNIV SAN DIEGO LA JOLLA DEPT OF MATHEMATICS E A BENDER 1/1

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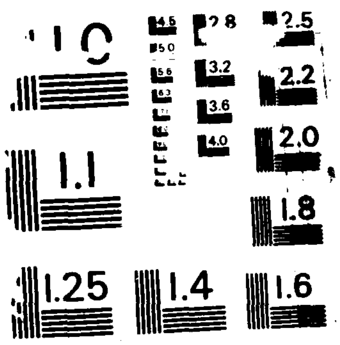
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Final Report
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Research in Enumeration and Graph Theory
with Applications to Computer Science

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Section 1: Overview

In addition to being at UCSD, I attended a month long working meeting in Vancouver, B.C., and visited the University of Georgia and the University of Auckland, N.Z. At the meeting I learned a considerable amount and began various lines of research, some leading nowhere and others that reached fruition in Georgia.

During the contract, almost half of my effort has been directed toward studying maps on surfaces other than the sphere, an area in which very little progress had been made. I was very pleasantly suprised by what I and my colleagues were able to accomplish. The concluded research is described in [1–4]. Canfield and I are currently pursuing additional research.

Other completed research was on convex polyhedra, reported in [5] and [6], and the size of PLA's, the conclusion of work begun earlier and reported in [7].

At present, research is underway on three problems in graph theory. The current state is described in the next section. One of these, [C], involves the use of a PC purchased under the grant.

The visit to the University of Georgia lasted for two quarters. It involved some teaching, paid for by UGA, and some research, paid for partially by UGA but primarily by ONR. (The UGA sponsored research is not reported here.) During this time, [1], [2], [4] and [6] were completed, [A] was started, [B] was worked on and issue of the feasibility of [C] was raised.

The visit to the University of Auckland lasted about two months. During this time [3] and [5] were completed and some unfruitful lines of research were pursued. For part of this visit, Brendan McKay was also visiting and [B] was begun.

At UCSD, [7] was finished and serious study of [C] was initiated.

Section 2: Completed Research

1. (with E.R. Canfield and R.W. Robinson) The enumeration of maps on the torus and projective plane. *Canad. Math. Bull.*, to appear.
   The enumeration of rooted maps by number of edges on the torus and projective plane is studied. Explicit expressions for the generating functions are obtained.

Asymptotic results, recurrence relations and numerical tables are also presented. (Some of this material is preparatory for [2].)

2. (with E.R. Canfield) The asymptotic number of rooted maps on a surface. *J. Combin. Theory Ser. A* **43** (1986), 244–257.
We asymptotically enumerate two classes of maps by edges on a fixed but arbitrary surface: rooted maps and rooted smooth maps. This is done by obtained a complicated set of recursive equations that expression generating functions for maps with distinguished faces on a given surface in terms of similar generating functions on surfaces of lower genus. Asymptotics are obtained directly from the recursions without solving them.

3. (with N.C. Wormald) The asymptotic number of rooted nonseparable maps on a surface. *J. Combin. Theory Ser. A*, to appear.
We obtain asymptotics for the number of rooted nonseparable maps on a fixed but arbitrary surface. This is done by first enumerating a more restrictive class of maps, called nonsingular, by using the results in [2]. Then it is shown that almost all nonseparable maps are nonsingular.

4. (with E.R. Canfield and R.W. Robinson) The asymptotic number of tree–rooted maps on a surface. *J. Combin. Theory Ser. A*, to appear.
Tree rooted maps were studied by Walsh and Lehman on orientable surfaces. Asymptotic formulas for tree–rooted maps and smooth tree–rooted maps can be obtained from their results. We also obtain results on nonorientable surfaces using the methods in [2].

5. (with N.C. Wormald) The number of rooted convex polyhedra. *Canad. Math. Bull.*, to appear. (partially supported)
Using Lagrange inversion, we obtain a previously overlooked single summation formula with decreasing terms for the number of rooted convex polyhedra. This leads to the pretty asymptotic formula for the number of convex polyhedra with $i+1$ vertices and $j+1$ faces:
$$\frac{1}{4 \times 3^5 ij(i+j)} \binom{2i}{j+3} \binom{2j}{i+3}.$$

6. (with E.R. Canfield) Face sizes of 3–polytopes. *J. Combin. Theory Ser. B*, to appear.
We study the root face valency of random convex polyhedra as the number of vertices gets large.

7. (with J.T. Butler) On the size of PLA's required to realize binary and multiple–

valued functions. *IEEE Trans. Comp.*, submitted. (partially supported)
We study the average number of product terms required to realize binary and multiple–valued functions as a function of the number of nonzero output values. This improves on previously known bounds. Furthermore, we study the variance, which had previously been intractable. This allows us to obtain estimates of the probability that a function with relatively few nonzero outputs will be realizable with a given PLA.

Section 3: Research in Progress

[A] (with E.R. Canfield and R.W. Robinson) We are studying labeled planar graphs (a map is embedded, a planar graph *can be* embedded). The embedding of a graph is unique if and only if it is 3–connected. This fact and [6] together provide a way to go from 3–connected maps to 3–connected labeled planar graphs. Methods developed by Walsh (and others) allow one to then go to 2–connected and to connected planar maps. Unfortunately, this is complicated by the presence of parametrizations. We are able to obtain exact numbers and are working on asymptotics. (This was tabled after I left Georgia.)

[B] (with E.R. Canfield and B.D. McKay) Assume that the probability of that a random labeled, n–vertex, q–edge graph is connected approached the form $A(x)B(x)^n$ where $x = q/n$. McKay and I obtained an equation for $B(x)$ and began attempts at proving that the nth root of the probability approached $B(x)$. While McKay obtained strong numerical evidence for the conjecture, Canfield and I obtained an equation for $A(x)$ and worked on proving the conjecture. We are continuing our efforts while Canfield is visiting UCSD. The proof is nearly complete. It involves a double induction on n and q and utilizes Wright's results for small $q - n$. So far we have written up nearly fifty pages.

[C] (with E.R. Canfield) Although the results in [2] allow one, in principle, to determine the generating function for maps on any given surface, the amount of calculation involved is tremendous. As stated, the calculations involve intermediate equations with $g+2$ variables, where g is the genus of the surface; however, we can show that the final result must have a relatively simple form. An attempt to use a symbolic language on a supercomputer bogged down due to lack of memory. Further study has shown ways to reduce the amount of calculations considerably. It should be possible to obtain some generating functions if one tailors a symbolic language to the problem. Canfield and I plan to return to this shortly. I have already done some programming for it using a PC.

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