				-				_				_	
VIICLA	91 425 SSIFIE	EFF ON AER D SD-	ECTS CN CHI OPHYSI TR-88-	DF TRA Emical ICS La -40 F0-	NSLATI LAS B H H 4701-9	ONAL ((U) AE IRELS	NID RO ROSPA 81 MAN 86	TATION CE COR R 88 T	AL NON P EL S R-0006	EQUIL EQUID A(206 F/G	D CR)-1)/3	5./ NL	1
		······································											



IN FILE COPY

いたいで

DTIC

APR 0 5 1988

H

4

88

4

121

and Rotational

ew Commical Laser Performance

B: MIDGLS Association Labor With Discourse Connection Tax Association Connection Connection Connection Connection Connection Connection

ะกับกัง เป็นไปที่เหลือข้อข้องไห้ เป็นจะไปที่ เหลือข้อที่สายไปที่สายไม่ที่สายไม่ได้เหลือ<mark>เหลือไปไม่ม</mark>ีเหล<mark>่เป็นส</mark>มันเหลือไป

AD-A191 425

ana: 2 22 € 2

1 March 1968

Depart for SPACE DIVISION ANE FORCE SYNCTIMS COMMAND Los America An Porce Date O Bio: Static Watdows Postal Center Los America CA 9009-2950

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED This report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-85-C-0086-P00016 with the Space Division, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by W. P. Thompson, Jr., Director, Aerophysics Laboratory. Lt Scott W. Levinson/CNW was the Air Force project officer.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

SCOTT W. LEVINSON, Lt., USAF Project Officer SD/CNW

RAYMOND M. CLEONG, Major, USAF Deputy Director, AFSTC West Coast Office AFSTC/WCO OL-AB

	REPORT DOCU	MENIATION	PAGE			
a. REPORT SECURITY CLASSIFICATION		16. RESTRICTIVE	MARKINGS			
a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION	AVAILABILIT	OF REPOR	T	
- DECLASSIEICATION / DOWNGRADING SCHEDU	II F	Approved for	public r	elease;		
	/ L L	uistribution	unituite			
PERFORMING ORGANIZATION REPORT NUMBE	R(S)	5 MONITORING	ORGANIZATIO	N REPORT N	UMBER(S)
R-0086A(2060)-1		SD-TR-88-40				
a. NAME OF PERFORMING ORGANIZATION	66. OFFICE SYMBOL	7a. NAME OF M	ONITORING OF	RGANIZATIO	N	
he Aerospace Corporation	(It applicable)	Space Divisi	.on			
Aboratory Operations	L	7b ADDRESS (Cit	ty, State, and	ZIP Code)		
		Los Angeles	Air Force	Station	l	
1 Segundo, CA 90245		Los Angeles,	, CA 90009	-2960		
a. NAME OF FUNDING / SPONSORING	85. OFFICE SYMBOL	9. PROCUREMEN	T INSTRUMEN			MBER
ORGANIZATION	(If applicable)		0086 0000	16		
ADDRESS (City, Change and 710 Control	1	104/01-85-C-	-0000-P000			
k. AUURESS (LITY, STATE, and ZIP LODE)		PROGRAM	PROJECT	TASK		WORK UNIT
		ELEMENT NO.	NO.	NO		ACCESSION NO
n cw Chemical Laser Performance 2. PERSONAL AUTHOR(S) 1. FINE CHARGE INFORMATION INFORMA	OVERED	14. DATE OF REPO	DRT (Year, Mor	nth, Day) 1	5 PAGE	COUNT
Chemical Laser Performance PERSONAL AUTHOR(S) Mirels_Harold Tab. TIME C FROM SUPPLEMENTARY NOTATION	e OVERED	14. DATE OF REPO 1988 Marc	DRT (Year, Mor h 1	nth, Day) 1	5 PAGE 49	COUNT
2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 6. SUPPLEMENTARY NOTATION		14. DATE OF REPC 1988 Marc	DRT (Year, Mor h 1	nth, Day) 1	5 PAGE 49	COUNT
2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 6. SUPPLEMENTARY NOTATION 7. COSATL CODES		14. DATE OF REPO 1988 Marc	DRT (Year, Mor h 1	and identifi	5 PAGE 49	
2. PERSONAL AUTHOR(S) 2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 13b. TIME C FROM 6. SUPPLEMENTARY NOTATION 7 COSATI CODES FIELD GROUP SUB-GROUP	e OVERED TO	14. DATE OF REPC 1988 Marc	DRT (Year, Mor h 1 e if necessary	and identif	5 PAGE 49 y by bloc	COUNT k number)
on cw Chemical Laser Performance 2. PERSONAL AUTHOR(S) 4irels_Harold 3a. TYPE OF REPORT 13b. TIME C FROM	e OVERED TO 18 SUBJECT TERMS Chemical Laset GW Chemical Li	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers (Contin	DRT (Year, Mor h 1 e if necessary uous Wave	and identif	5 PAGE 49 y by bloc 1 Lase	COUNT k number) rs)
2. PERSONAL AUTHOR(S) 2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 13b. TIME C FROM	e OVERED TO 18 SUBJECT TERMS Chemical Laser GW Chemical List and identify by block	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number)	DRT (Year, Mor h 1 e if necessary uous Wave	and identif	5 PAGE 49 y by bloc 1 Lase	COUNT k number) rs)
A previous model used to descri	e OVERED TO 18 SUBJECT TERMS Chemical Lasel GM Chemical Lasel Chemical Chemical Chemical Lasel Chemical Chemical C	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem	DRT (Year, Mor h 1 e if necessary uous Wave hical lase	and identif	5 PAGE 49 y by bloc 1 Lasen mance	COUNT k number) rs) ⁽ is
ABSTRACT (Continue on reverse if necessary A previous model used to descri generalized to include rotation	e OVERED TO TO 18 SUBJECT TERMS Chemical Lasen Chemical Lasen Chemical Lasen Chemical Lasen of Chemical Lasen for Chemical Lasen of Che	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational	DRT (Year, Mor h 1 e if necessary uous Wave hical lase nonequili	and identif Chemica r perform brium.	5 PAGE 49 y by bloc 1 Lasen The rea	COUNT k number) rs) ⁽ is sultant
2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 13b. TIME OF REPORT <td>e OVERED TO TO TO TO TO TO TO TO TO TO</td> <td>(Continue on revers Model, asers; (Contin number) ave (cw) chem ranslational mption that t</td> <td>DRT (Year, Mor h 1 e if necessary uous Wave hical lase nonequili ranslatio l pumping</td> <td>and identify Chemica r perform brium. nal and and co</td> <td>5 PAGE 49 y by bloc 1 Lasen mance The re- rotatio 1] isio</td> <td>COUNT k number) rs) ⁽ is sultant onal nal</td>	e OVERED TO TO TO TO TO TO TO TO TO TO	(Continue on revers Model, asers; (Contin number) ave (cw) chem ranslational mption that t	DRT (Year, Mor h 1 e if necessary uous Wave hical lase nonequili ranslatio l pumping	and identify Chemica r perform brium. nal and and co	5 PAGE 49 y by bloc 1 Lasen mance The re- rotatio 1] isio	COUNT k number) rs) ⁽ is sultant onal nal
ABSTRACT (Continue on reverse if necessary A previous model used to descri generalized to include rotation equations are simplified by the relaxation rates are fast compa deactivation rates. As a consec	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot	DRT (Year, Mor h 1 se if necessary uous Wave hical lase nonequili ranslatio l pumping ational r	and identif and identif Chemica r perform brium. nal and , and co elaxatio	5 PAGE 49 y by bloc 1 Lasen The rea rotation 1 lision n are	COUNT k number) rs) ⁽ is sultant onal nal in equi-
2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 13b. TIME OF REPORT <td>e OVERED TO TO TO TO TO TO TO TO TO TO</td> <td>14. DATE OF REPC 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification</td> <td>DRT (Year, Mor h 1 e if necessary uous Wave fical lase nonequili ranslatio l pumping ational r h is intro</td> <td>and identified and identified Chemica r perform brium. nal and , and co elaxatio duced by</td> <td>5 PAGE 49 y by bloc 1 Lasen mance The re- rotation n are the a</td> <td>COUNT k number) rs) (is sultant onal nal in equi- ssumption</td>	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPC 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification	DRT (Year, Mor h 1 e if necessary uous Wave fical lase nonequili ranslatio l pumping ational r h is intro	and identified and identified Chemica r perform brium. nal and , and co elaxatio duced by	5 PAGE 49 y by bloc 1 Lasen mance The re- rotation n are the a	COUNT k number) rs) (is sultant onal nal in equi- ssumption
ABSTRACT (Continue on reverse if necessary A previous model used to descri generalized to include rotation equations are simplified by the relaxation rates. As a consec librium with stimulated emission $R_r/R_t - 1 (\Delta \nu_r/\Delta \nu_d) << 1$, wher	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Donnler wi	DRT (Year, Mor h 1 se if necessary uous Wave hical lase nonequili ranslatio l pumping ational r h is intro tional to dths STh	and identif and identif Chemica r perform brium. nal and , and co elaxatio duced by transla e result	s PAGE 49 y by bloc l Lasen The re- rotation llision n are the a tional ant sv	COUNT k number) rs) ' is sultant onal nal in equi- ssumption relaxation stem of
ABSTRACT (Continue on reverse if necessary A previous model used to descri generalized to include rotation equations are simplified by the relaxation rates are fast compa deactivation rates. As a consec librium with stimulated emission $ R_r/R_t - 1 (\Delta v_r/\Delta v_d) << 1$, wher relaxations is independent of rot	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPC 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Doppler wi ion. An ampl	DRT (Year, Mor h 1 e if necessary uous Wave fical lase nonequili ranslatio l pumping ational r h is intro tional to dths. >Th ifier sol	and identify and identify Chemica Chemica r perform brium. nal and , and co elaxatio duced by transla e result ution is	s PAGE 49 y by bloc l Lasen The rea rotation llision n are the a tional ant sy prese	COUNT k number) rs) ⁽ is sultant onal nal in equi- ssumption relaxation stem of nted that
ABSTRACT (Continue on reverse if necessary A previous model used to descri generalized to include rotation equations are simplified by the relaxation rates are fast compa deactivation rates. As a consec $ R_r/R_t - 1 (\Delta v_h/\Delta v_d) << 1$, wher rates and $\Delta v_h/\Delta v_d$ is the ratio equations is independent of rot predicts saturation effects in	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Doppler wi ion. An ampl eriments. Fa	DRT (Year, Mor h 1 e if necessary uous Wave hical lase nonequili ranslatio l pumping ational r h is intro tional to dths. >Th ifier sol abry-Perot	nth, Day) and identif Chemica r perform brium. nal and , and co elaxatio duced by transla e result ution is oscilla	5 PAGE 49 y by bloc 1 Lasen The resonance The resonance the a tional ant sy presenance to so	COUNT k number) rs) ' is sultant onal nal in equi- ssumption relaxation stem of nted that lutions are
on cw Chemical Laser Performance 2. PERSONAL AUTHOR(S) Airels, Harold 3a. TYPE OF REPORT 6. SUPPLEMENTARY NOTATION 7. COSATI CODES FIELD 6. SUPPLEMENTARY NOTATION 7. COSATI CODES FIELD 6. SUPPLEMENTARY NOTATION 7. COSATI CODES 7. C	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPO 1988 Marc (Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Doppler wi ion. An ampl eriments. Fa and for a pa fication of	DRT (Year, Mor h 1 e if necessary uous Wave fical lase nonequili ranslatio l pumping ational r h is intro tional to dths. >Th ifier sol abry-Perot artly satu	and identified and identified Chemica Chemica r perform brium. nal and , and co elaxatio duced by transla e result ution is oscilla rated si codes	s PAGE 49 y by bloc l Lasen The rea rotation llision n are the a tional ant sy prese tor so ngle-1 It is	COUNT k number) rs) ⁽ is sultant onal nal in equi- ssumption relaxation stem of nted that lutions are ine laser. concluded
InterviewInterview2. PERSONAL AUTHOR(S)2. PERSONAL AUTHOR(S)3a. TYPE OF REPORT3a. TYPE OF REPORT13b. TIME OF REPORT6. SUPPLEMENTARY NOTATION77COSATI CODESFIELDGROUPSUB-GROUPABSTRACT (Continue on reverse if necessaryA previous model used to descrigeneralized to include rotationequations are simplified by therelaxation rates are fast compadeactivation rates. As a conseclibrium with stimulated emissic $ R_r/R_t - 1 (\Delta v_h / \Delta v_d) << 1$, wherrates and $\Delta v_h / \Delta v_d$ is the ratioequations is independent of rotpredicts saturation effects inalso presented for a multilineThe present results provide a tthat a reasonable first estimation	e OVERED TO TO TO TO TO TO TO TO TO TO	(Continue on revers r Model, asers; (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Doppler wi ion. An ampl eriments. Fa and for a pa fication of r al laser perf	DRT (Year, Mor h 1 e if necessary uous Wave hical lase nonequili ranslatio l pumping ational r h is intro tional to dths. >Th ifier sol bry-Perot artly satu numerical formance c	and identify and identify Chemica r perform brium. nal and , and co elaxation duced by transla e result ution is oscilla rated si codes. an be ob	5 PAGE 49 y by bloc 1 Lasen mance The rea rotational ant sy prese tor so ngle-1 It is otained	COUNT k number) rs) is sultant onal nal in equi- ssumption relaxation stem of nted that lutions are ine laser. concluded by
con cw Chemical Laser Performance 2. PERSONAL AUTHOR(S) Mirels, Harold 3a. TYPE OF REPORT 6. SUPPLEMENTARY NOTATION 7. COSATI CODES FIELD GROUP SUB-GROUP ABSTRACT (Continue on reverse if necessary A previous model used to descri generalized to include rotation equations are simplified by the relaxation rates are fast compa deactivation rates. As a consec librium with stimulated emission $ R_r/R_t - 1 (\Delta v_r / \Delta v_d) << 1$, wher rates and $\Delta v_r / \Delta v_d$ is the ratio equations is independent of rot predicts saturation effects in also presented for a multiline The present results provide a t that a reasonable first estimate equations potational equilibrium	e OVERED TO TO TO TO TO TO TO TO TO TO	14. DATE OF REPO 1988 Marc (Continue on reverse r Model, asers'(Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Doppler wi ion. An ampl eriments. Fa and for a pa fication of r al laser perfinal nonequili	DRT (Year, Mor h 1 e if necessary uous Wave dical lase nonequili ranslatio l pumping ational r is intro tional to dths. >Th ifier sol bry-Perot artly satu numerical formance o	and identified and identified Chemica r performing r p	s PAGE 49 y by bloc l Lasen The rei rotational ant sy prese tor so ngle-1 It is ptained	COUNT k number) rs) is sultant onal nal in equi- ssumption relaxation stem of nted that lutions are ine laser. concluded by
on cw Chemical Laser Performance 2. PERSONAL AUTHOR(S) Mirels_Harold 3a. TYPE OF REPORT 13b. TIME OF REPORT 14b. TO DES 15b. TIME OF REPORT 16b. TIME OF REP	e OVERED TO TO TO TO TO TO TO TO TO TO	(Continue on revers r Model, asers (Contin number) ave (cw) chem ranslational mption that t tion, chemica ional and rot implification ratio of rota to Doppler wi ion. An ampl eriments. Fa and for a pa fication of r al laser perf nal nonequili	DRT (Year, Mor h 1 e if necessary uous Wave hical lase nonequili ranslatio l pumping ational ro is intro tional to dths. Th ifier sol abry-Perot artly satu numerical formance of brium. ECURITY CLAS	and identify Chemica Chemica Chemica r perform brium. nal and , and co elaxatio duced by transla e result ution is oscilla rated si codes. an be ob <u>ke (1) (</u>	s PAGE 49 y by bloc l Lasen The rea rotational ant sy prese tor so ngle-1 It is otained	COUNT k number) rs) ' is sultant onal nal in equi- ssumption relaxation stem of nted that lutions are ine laser. concluded by

1 4 1 4 4 4 YO AVA ALA ALA

PERSON SUCCESSION DECEMBER OF

2222220 2222224

5552233

04060606060406060606060606060	<u>,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,</u>	<u>,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,</u>	and the second of the second of the

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

18. SUBJECT TERMS (Continued)

Inhomogeneous Broadening Effects Lasers Multiline Performance Rotational Nonequilibrium Translational Nonequilibrium

INCLASSIELED

SECURITY CLASSIFICATION OF THIS PAGE

CONTENTS

I.	INTRODUCTION	7
11.	FORMULATION	9
	A. Flow Model	9
	B. Distribution Functions	9
	C. Laser Equations 1	.2
111.	EQUILIBRIUM CASES 1	.7
IV.	NONEQUILIBRIUM CASES 1	.9
	A. Simplified Equations 1	9
	B. Amplifier 2	:6
	C. Oscillator	15
v.	DISCUSSION 4	1
VI.	CONCLUDING REMARKS 4	3
APPE	NDIX A: PARTIAL LIST OF SYMBOLS 4	15
APPE	NDIX B: PARAMETER EVALUATION 4	7
REFE	RENCES	;3



1

Accession For	·
NTIS GRA&I	
DTIC TAB	
Juanno unced	
Justilicati r	·
	· · · · · · · · · · · · · · · · · · ·
8v	
Stote (Platfou)	,
Av=11=1111++	0/0 8
fyster og	⊡ ć ∕⊂⊥
at Spr -	1
	1
	r

10.223

122222000 - Constant

LUNDY XXXXX DUDDY XXXXX XXXXX

FIGURES

. . . .

1.	Continuous Wave Chemical Laser (a) Flow Field and Fabry- Perot Resonator and (b) Flame Sheet Model of Reaction Zone	10
2.	Variation of Inversion Number Density with Streamwise Distance for Case of Uniformly Illuminated Amplifier with Laminar Mixing	27
3.	Line Shape for Laser Medium with Single Transition at $X_j = 0.0$ or 0.4 and a Homogeneous Width $\Delta v_h / \Delta v_d = 0.024$; (a) Case $X_j = 0.0$; (b) Case $X_j = 0.4$	29,3 0
4.	Variation of Gain with Frequency at Streamwise Station $\zeta = 0.305$ in a Uniformly Illuminated Amplifier with a Single Transition at $X_j = 0.4$; (a) Case $\Delta v_h / \Delta v_d = 0.024$; (b) Case $\Delta v_h / \Delta v_d = 0.048$	31,32
5.	Effect of Input Intensity on Line Center ($v = v$) Gain at Fixed Streamwise Station for Uniformly Illuminated Amplifier	34
6.	Performance of Saturated Multiline Fabry-Perot Oscillator in the Limit $R_r = R_t >> 1$, $\Delta v_h \ll \Delta \lambda_c$, $Y_{J,j} \ll 1$, $G_c \neq 0$, and $S = O(1)$	38
7.	Performance of Single Line Fabry-Perot Oscillator in Limit $R_r = R_t >> 1$ and $Y_{i,i} << 1$	40

TABLES

8"+" 8"+" #L."+8."+8. "+1. "+8. "+1." *** 4+

I.	Numerical Values of Parameters for cw HF Laser	20-23
11.	Maxima for Amplifier with Uniform Incident Radiation and Laminar Diffusion	28
111.	Homogeneous Width and Gas Kinetic Collision Rate Data for HF + M_i + HF + M_i	48
IV.	Vibrational Deactivation Rates for Reaction HF $(v + 1) + - 1$	
	$M_i \overset{K_{cd}}{\rightarrow} HF(v) + M_i$	5 0
V.	Maximum Value of \overline{f}_{J} and Corresponding Value of J for fixed θ_{R}	51

 ~ 1

Continuous wave (cw) chemical lasers generally operate at pressures of the order of 1 Torr in order to achieve good efficiency. At this pressure level, the gain medium is inhomogeneously broadened. In addition, the lasing process tends to modify the gain medium so that the lasing particles are neither in translational nor rotational equilibrium. Nonetheless, early analytic models of cw chemical laser performance assumed that the lasing medium was in translational and rotational equilibrium (e.g., Refs. 1 and 2). In these models, reasonable estimates were provided for net output power, but the spectral content was not predicted. The latter requires consideration of finite translational and rotational relaxation rates.

Subsequently, analytic models and numerical codes were developed that included either rotational nonequilibrium $^{3-5}$ or translational nonequilibrium. $^{6-8}$ A recent review of rotational nonequilibrium rate processes and models is given in Ref. 9.

The combined effect of translational and rotational nonequilibrium has received less attention. An analytic model for a low pressure CO_2 laser is presented in Ref. 10, whereas a model for a generic molecular laser with applications to CO_2 and cw chemical lasers is presented in Ref. 11. A numerical code that treats both translational and rotational nonequilibrium has been developed by D. Bullock and co-workers;¹² limited numerical results have been published.¹³

The present study generalizes a previous simple $model^{6-8}$ in order to include rotational as well as translational nonequilibrium effects. The object is to provide analytic expressions for cw chemical laser amplifier and oscillator performance as well as to delineate the parameters that characterize nonequilibrium effects. The appropriate equations are first deduced. Equilibrium and nonequilibrium solutions are then obtained. Symbols are defined in Appendix A.

II. FORMULATION

Equations are deduced that define effects of translational and rotational nonequilibrium on cw chemical laser performance.

A. Flow model

A cw chemical laser is illustrated in Fig. la. The present simplified mixing model is illustrated in Fig. lb. The reactants are assumed to be premixed but do not react until a flame sheet, $y_f(x)$, is reached. The flame sheet shape is specified, a priori, from diffusion theory. The streamwise station where the flame sheet reaches the channel center line is denoted x_D and characterizes the diffusion rate. The width per semichannel and the number of semichannels are denoted w and n_{sc} , respectively. Laser radiation is in the $\pm y$ direction.

B. Distribution functions

Let $n_v(J,v)dv$ denote the number of particles (moles/cm³) with vibrational energy level v and rotational energy quantum number J that are resonant with radiation of frequency v. The following notation is introduced

$$\int_{-\infty}^{\infty} n_{v}(J,v) dv \equiv n_{v}(J)$$
 (1a)

$$\sum_{J}^{n} n_{v}(J) \equiv n_{v}$$
(1b)

$$\sum_{\mathbf{v}} n_{\mathbf{v}} \equiv n_{\mathbf{T}}$$
(1c)

For radiation in the $\pm y$ direction, the resonant frequency v is related to particle thermal velocity v_v by the Doppler relation

$$\frac{v}{v_0} - 1 = \pm \frac{v_y}{c}$$
(2)

where v is the resonant frequency for particles at rest. We neglect the dependence of $v_{\rm c}$ on v,J.

For particles with a Maxwellian thermal velocity distribution

$$\overline{p} = \frac{n_v(J,v)}{n_v(J)} = \frac{\left[(4\epsilon n_2)/\pi\right]^{1/2}}{\Delta v_d} \exp\left[-(4\epsilon n_2)\left(\frac{v-v_0}{\Delta v_d}\right)^2\right]$$
(3)



X T P T S T A T K T S

Fig. 1. Continuous Wave Chemical Laser (a) Flow Field and Fabry-Perot Resonator and (b) Flame Sheet Model of Reaction Zone^{1,6-8}

$$\bar{p}_{o} = [(4\ell n 2)/\pi]^{1/2} / \Delta v_{d}$$
(4)

We define $p_v = \bar{p}/\bar{p}_o$ and note

$$\int_{\infty}^{\infty} p_{v} dv = \frac{1}{p_{o}}$$
(5)

We now consider the case of rotational equilibrium and neglect the effect of vibrational energy level on the characteristic rotational energy temperature T_R . The fraction of particles in rotational energy level J is denoted \overline{f}_J and is found from (Appendix B)

$$\bar{f}_{J} \equiv \frac{n_{v}(J)}{n_{v}} = \frac{(2J+1)\exp[-J(J+1)\theta_{R}]}{\sum (2J+1)\exp[-J(J+1)\theta_{R}]}$$
(6)

where the summation is from J = 0 to $J = \infty$. Let \overline{f}_r denote a reference value of \overline{f}_J and introduce $f_J = f_J / \overline{f}_r$. It follows that

 $\sum_{J} \overline{f}_{J} = 1 ; \quad \sum_{J} f_{J} = 1/\overline{f}_{r}$ (7)

Convenient values for \overline{f}_r are noted in Appendix B. Finally, we observe that the gain at frequency v for laser transition v,J can be expressed

$$g_{v,J}(v) = \overline{\sigma}_{v,J} \int_{-\infty}^{\infty} \left\{ \frac{2J_{\ell} + 1}{2J_{u} + 1} \left[n_{v}(J,v') \right]_{u} - \left[n_{v}(J,v') \right]_{\ell} \right\} L(v - v')dv' \quad (8a)$$

where subscript v,J refers to lower laser level values, subscripts u and ℓ denote upper and lower laser level values, respectively, and

$$L(v - v') = \left[1 + 4\left(\frac{v - v'}{\Delta v_{h}}\right)^{2}\right]^{-1}$$
(8b)

where Δv_h is the homogeneous line width. The evaluation of Δv_h and the cross section $\overline{\sigma}_{v,J}$ is discussed in Appendix B. Numerical estimates are provided in Table I.

C. Laser equations

In order to simplify the mathematical development, we assume a "Q" type laser transition

$$v + 1, J + v, J$$
 (9a)

rather than the "P" type laser transition

$$v + 1, J - 1 + v, J$$
 (9b)

TRACTED REPORT IN

222

appropriate for cw chemical lasers. This approximation is consistent with other simplifying assumptions used in the present model. In addition, we consider a two-vibrational energy level model and denote the upper and lower levels by subscripts 2 and 1, respectively.

The variation of upper level number density with streamwise distance, in the present model, is found from

$$\frac{u}{y_{f}} \frac{d[n_{2}(J,v)y_{f}]}{dx} = u \frac{\bar{p} \bar{f}_{J}}{y_{f}} \frac{d(n_{T}y_{f})}{dx} - k_{cd}n_{2}(J,v) + k_{tr}[\bar{p}n_{2}(J) - n_{2}(J,v)] + k_{rr}\bar{p}[\bar{f}_{J}n_{2} - n_{2}(J)]$$
(10a)

 $-\frac{\bar{\sigma}_{v,J}}{\varepsilon_{v,J}} [n_2(J,v) - n_1(J,v)] \sum_{j} L(v - v_{J,j}) \overline{I}_{J,j}$

where $\varepsilon_{v,J}$ is the energy per mole of photons. The terms on the right-hand side of the equation represent the effects of chemical pumping, collisional deactivation, translational cross relaxation (assumed to be proportional to departure from translational equilibrium), rotational cross relaxation (assumed to be proportional to departure from rotational equilibrium), and stimulated emission and absorption. For the term involving k_{tr} , it is assumed that within a given rotational level J, the creation of $n_2(J,v)$ is proportional to the departure from translational equilibrium. For the term involving k_{rr} , it is assumed that rotational relaxation into level J results in particles with a Maxwellian velocity distribution. Equation (10a) provides the proper limits $n_2(J,v) + \bar{p}n_2(J)$ and $n_2(J) + \bar{f}_J n_2$ as $k_{tr} + \infty$ and $k_{rr} + \infty$, respectively. It is assumed that the chemical reaction creates only upper level particles and that these are in translational and rotational equilibrium. The notation $\overline{I}_{J,j}$ and $v_{J,j}$ refers to the jth resonator mode for the v + l, J + v,J transition. The corresponding expression for the lower laser level is

$$\frac{u}{v_{f}} \frac{d[n_{1}(J,v)y_{f}]}{dx} = k_{cd} \bar{p} \bar{f}_{J}n_{2} + k_{tr}[\bar{p}n_{1}(J) - n_{1}(J,v)]$$

$$+ k_{rr}\bar{p}[\bar{f}_{J}n_{1} - n_{1}(J)]$$

$$+ \frac{\bar{\sigma}_{v,J}}{\varepsilon_{v,J}} [n_{2}(J,v) - n_{1}(J,v)] \sum_{j} L(v - v_{J,j})\bar{I}_{J,j}$$
(10b)

The first term on the right-hand side follows from the assumption that collisional deactivation of n_2 results in an equilibrium distribution of $n_1(J,v)$. The following nondimensional quantities are introduced

$$\zeta = \frac{k_{cd}x}{u} \qquad \qquad N_{2J} = \frac{n_2(J)y_f}{n_r \bar{f}_r w} \qquad (11a)$$

$$N_{2Jv} = \frac{n_2^{(J,v)y_f}}{n_r \bar{f}_r \bar{p}_0 w} \qquad N_2 = \frac{n_2^{y_f}}{n_r w}$$
(11b)

$$N_{J\nu}^{\pm} = N_{2J\nu} \pm N_{1J\nu}$$
 $N_{J}^{\pm} = N_{2J} \pm N_{1J}$ (11c)

$$N^{\pm} = N_2 \pm N_1 \qquad I_{J,j} = \overline{I}_{J,j} / \overline{I}_{S,L} \qquad (11d)$$

$$R_{t} = k_{tr}/k_{cd} \qquad R_{r} = k_{rr}/k_{cd} \qquad (11e)$$

$$G_{J}(v') = \frac{g_{J}(v')y_{f}}{n_{r}\bar{f}_{r}\bar{p}_{o}\Delta v_{h}\bar{\sigma}_{v,J}w} \qquad G_{J,j} = G_{J}(v_{j})$$
(11f)

$$= \int_{-\infty}^{\infty} N_{Jv} L(v - v') \frac{dv}{\Delta v_h} \qquad S = \pi \bar{p}_o \bar{f}_r \Delta v_h R_t \qquad (11g)$$

where n_r is a reference value of n_2 and

$$\bar{I}_{S,L} = \varepsilon_{v,J} k_{tr} / (2\bar{\sigma}_{v,J})$$
(12)

is an intensity that characterizes line shape distortion resulting from saturation.

Substitution of normalized variables into the sum of Eqs. (10a) and (10b) and subsequent integration with respect to v and summation with respect to J indicates

$$\frac{dN'_{J\nu}}{d\zeta} = p_{\nu}f_{J}\frac{dN_{T}}{d\zeta} + p_{\nu}f_{J}N_{2} - N_{2J\nu} + R_{t}(p_{\nu}N_{J}^{+} - N_{J\nu}^{+})$$

$$+ R_{r}p_{\nu}(f_{J}N^{+} - N_{J}^{+})$$
(13a)

$$\frac{dN_{J}^{+}}{d\zeta} = f_{J} \frac{dN_{T}}{d\zeta} + (f_{J}N_{2} - N_{2J}) + R_{r}(f_{J}N^{+} - N_{J}^{+})$$
(13b)

$$N^{+} = N_{T}$$
(13c)

The difference between Eqs. (10a) and (10b) indicates

$$\frac{dN_{J_{\nu}}}{d\zeta} = p_{\nu}f_{J}\frac{dN_{T}}{d\zeta} - (p_{\nu}f_{J}N_{2} + N_{2J\nu}) + R_{t}(p_{\nu}N_{J} - N_{J\nu})$$
(14a)

$$+ R_{r} P_{v} (f_{J} N^{-} - N_{J}^{-}) - R_{t} N_{Jv} \sum_{j} L(v - v_{J,j}) I_{J,j}$$

$$\frac{dN_{J}^{-}}{d\zeta} = f_{J} \frac{dN_{T}}{d\zeta} - (f_{J} N_{2} + N_{2J}) + R_{r} (f_{J} N^{-} - N_{J}^{-})$$
(14b)

$$\frac{-P_{0}^{\Delta v}h^{R}t \sum_{j}^{G}J_{j}j^{I}J_{j}j}{d\zeta} = \frac{dN_{T}}{d\zeta} - (N_{T} + N^{-}) - \frac{S}{\pi}\sum_{j}^{G}G_{J,j}J_{j}J_{j}j$$
(14c)

Equations 13 and 14 can be solved for $N_{J\nu}^{\pm}$, N_{J}^{\pm} , and N^{\pm} , if the chemical pumping rate $dN_T/d\zeta$ is specified and if $G_{J,j}$ (oscillator) or $I_{J,j}$ (amplifier) is specified. For laminar mixing

$$N_{\rm T} = (z/z_{\rm D})^{1/2}$$
(15)

where $\zeta_D = k_{cd} x_D/u$ is the normalized diffusion distance. For a Fabry-Perot resonator where each mirror has a reflectivity R_m , $g_{J,j}y_f n_{sc} = -2nR_m$, and

$$\frac{\overline{\sigma}_{v,J}}{\overline{\sigma}_{r}}G_{J,j} = \frac{-\ln R_{m}}{n_{sc}n_{r}\overline{f}_{r}\overline{p}_{o}\Delta v_{h}\overline{\sigma}_{w}} = \text{constant}$$
(16)

The output power per semichannel, released up to station x, is denoted \overline{P} and is found from

$$P = \frac{P}{n_r w u \varepsilon_r} = \frac{S}{2\pi} \sum_{J j} (\varepsilon_{v,J} / \varepsilon_r) \int_{0}^{\zeta} G_{J,j} I_{J,j} d\zeta$$
(17a)

If we assume $\varepsilon_{v,J} = \varepsilon_r$, Eq. (14c) indicates

$$2P = N_{T} - N^{-} - \int_{0}^{\zeta} (N_{T} + N^{-}) d\zeta$$
 (17b)

The solution of these equations is discussed in the following sections.

III. EQUILIBRIUM CASES

The assumption of translational and rotational equilibrium corresponds to taking the limits $R_t \neq \infty$ and $R_r \neq \infty$, respectively, in Eqs. (13) and (14). The results are

$$\frac{N_{J_{v}}^{+}}{P_{v}f_{J}} = \frac{N_{J}^{+}}{f_{J}} = N^{+} = N_{T}$$
(18a)

$$\frac{\overline{N}_{J_{U}}}{\overline{p_{v}f_{J}}} = \frac{\overline{N}_{J}}{\overline{f}_{J}} = \overline{N}$$
(18b)

as expected and

$$\frac{dN^{-}}{d\zeta} = \frac{dN_{T}}{d\zeta} - (N_{T} + N^{-}) - \frac{S}{\pi} \sum_{J j} G_{J,j} I_{J,j}$$
(19a)

Also, from Eqs. (18b) and (11f), in the limit $\Delta v_h \ll \Delta v_d$,

$$\frac{2}{\pi} \frac{G_{J,j}}{f_J^N} = \exp\left[-4\ell n 2\left(\frac{\nu_j - \nu_o}{\Delta \nu_d}\right)^2\right] \left[1 + 0\left(\frac{\Delta \nu_h}{\Delta \nu_d}\right)\right]$$
(19b)

In the case of an oscillator, threshold gain is specified $(G_{J,j} = G_c)$, and Eqs. (19a) and (19b) are solved for $I_{J,j}$. In the present two-vibrational level model, one lasing transition J,j occurs, namely the transition with highest gain. (In a multivibrational level model, there is one lasing transition for each upper vibrational level.) Amplifier solutions are obtained by specifying $I_{J,j}$ in Eq. (19a). Equations (19a) and (19b) correspond to the equilibrium model presented in Ref. 1.

IV. NONEQUILIBRIUM CASES

We now consider effects of translational and rotational nonequilibrium. The quantities R_r and R_t are large in cw chemical lasers (Table I and Ref. 9). Therefore, we consider the limit

$$R_r \gg 1, R_t \gg 1$$
 (20)

A similar approximation was introduced in Ref. 11. Simplified laser equations are deduced. Amplifier and oscillator solutions are then obtained.

A. Simplified equations

1. Limit $R_r >> 1$, $R_t >> 1$

Recall that number density variables in Eqs. (13) and (14) have been normalized to be of order 1. If terms of order R_r^{-1} and R_t^{-1} are neglected, Eqs. (13a)-(13c) indicate

$$\frac{N_{J\nu}^{+}}{f_{J}p_{\nu}} = \frac{N_{J}^{+}}{f_{J}} = N^{+} = N_{T}$$
(21)

Thus, N_{Jv}^+ and N_J^+ retain translational and rotational equilibrium distributions in the present approximation. Similarly, Eqs. (14a)-(14c) become

$$\frac{N_{Jv}}{P_{v}f_{J}N^{-}} = \frac{1 + (\frac{R_{r}}{R_{t}} - 1)(1 - \frac{N_{J}}{f_{J}N^{-}})}{1 + \sum_{j} L(v - v_{J,j})I_{J,j}}$$
(22a)

$$\frac{N_{J}}{f_{T}N} = 1 - 2 \left(\frac{\ell n2}{\pi}\right)^{1/2} \frac{\Delta v_{h}}{\Delta v_{d}} - \frac{R_{t}}{R_{r}} \sum_{j} \frac{G_{J,j}I_{J,j}}{f_{T}N}$$
(22b)

$$\frac{dN^{-}}{d\zeta} = \frac{dN_{T}}{d\zeta} - (N_{T} + N^{-}) - \frac{S}{\pi} \sum_{J} \sum_{i} G_{J,j} I_{J,j}$$
(22c)

Convection, chemical pumping, and collisional deactivation terms no longer appear in Eqs. (22a) and (22b). In the present limit, the rate of increase of $N_{J\nu}$ (or N_{J}) resulting from cross relaxation is just equal to the rate of loss of $N_{J\nu}$ (or N_{J}) resulting from stimulated emission. Thus the cross relaxation and stimulated emission processes are in equilibrium, and other rate processes

	v		211.	113.	115.	141.	051.1	671.5	5. 369	3.615	11.504	53.290	51.648	36.147	
			29.696	41.996	51.434	161.65	2.970	4.200	5.143	666.5	167.	.420	.514	765.	
aser.			ζ θ Ι.	167.	1.466	1.137	761.	167.	4411	1.137	761.	167.	1.066	1.1.1	
cw HF 1 2 6 _m	- 12 - 12 - 12 - 12 - 12 - 12 - 12 - 12		411.	420.	126.	RI0.	141.	157.	.210	281.	1.634	2.569	2,098	1.417	
0.39:0.0 07 × 10 ⁻	6-01 - 0-0 1/sec		. m.	4 14	.115.	514.	. 307	4 14	215.	. 114	101.	74 F.	215.	.614	
λ = 2.7			1.720	1.H'R	4.327	1.641	1.720	1.454	4. 321	1.64]	1.720	З. Я5М	1.12/	1.643	
.J ≡ 6,	k ₁ r × 10 ⁷ 1/sec		511.	ны.	490.	150.	1.146	118.	244.	176.	11.462	8.105	4.414	117.5	
v = 0,	k _{ed} = 10 ⁻¹ -1 1/sec		1.44.	017.	£51.	.157	449.4	101.5	1.524	125.1	h6. 6'X	20017	15.244	115.21	
(a)		B = 1,0	.1265+04	295F+02	,194F+02	. 1201+02	.1275+114	. 3326+02	.21ME+02	, 340E+02	20+3281°	+ 12AF+03	, R44E+02	100F_403	
	3	R = 1.5	20+3182°	10+3681.	00+35 l6*	10+3651*	555£402	10+3716 *	10+3507*	10+3542.	, 253E, 403	, 247E+02	1636402	. 184F +02	
	-}		580°	140.	870.	120.	H.527	£41.4	2.842	7.132	<i>нъг</i> , 692	424, 346	284.231	11.113	
	× 10		1904.	. 406)	006.	1.200	1004	(N)4	006.	1.00	004.		9610	007.1	
	p 10 4 a		261.	<i>2</i> 14.	.132	271.	1.316	1. 116	1.316	1. 115	13.150	13.150	13.160	13.160	

الالكيكيكينات

PASSASSAS

(111) (111)

TABLF I. Numerical values of parameters for CW HF laser. HF: H_2 : H_e : $0_2 = 0.12$:0.47:0.39:0.02

(b) v = 0, J = 9, $\lambda = 2.823 \times 10^{-6}$ m

x z	100*	.073	.164	. 165	4 LU.	121.	₩ ۲۰	424.1	71	1.275	14. 181	14.488
ر.ا . ا. د.ا . ا. دست / مام اه	11.23	44.949	145.72	66.466	1.121	4.7095	452.5	h.h4]	211.	4.70	474.	
- - -	700"	501°	÷11;•	141.		. 194	. 11 .			<u></u> .	. 11.	(65.
к. - сел. 		5,111	1	×10.	1.41.				1.41	t		ואן.
	· •. ·	.17	I	***."		<u>.</u>			· ·		<u>:</u>	, H4
-	1. T	*', * *	, .', ·	1 1	· · ·	н, н.,		11		л, н.,		1, 6.1
	хц,	Ĭ.		2 5 ,1	441.1			ł., ·	. 41. 11	- -	4.4.X	11.2.11
		• • • • •	. 1 . 1	• • • •		· · · · ·	6 . 1	۰.۰۱	81.11.114	1.00	11,141	157.51
	1		1-+16/17	1	2000 1002 1	2000 (ed. 1.	14-04-01	144.441.	2014 49,27 *	. 1644 + OK	1 · + 1 ¹ 1 *	114 111
	··· • 1.11 ·	,	li • milh"	10+3204.	A11+4581 .	ું છે કે છે કે છે.	, 101e+17	15,6,4,01	. 13 36 4/15	{ int don't "	70+3685*	. 3646 +112
n Ser Ser	6.0	2110	100	8[0]	4016-7	1.6.1	2.2.17	1.477	240.619	965.410	243.540	142.655
- - - ×	14.4	· 64.44			. we		(ниђ.	1.70	144.	UN14.	ш.р.	0071
	317		. I 4.		1.115	1. 115	1. 814	1.415	14,140	13.1411	13.160	13.150

TABLF. I. Numerical values of parameters for cw HF laser. HF: H_2 : H_e : $0_2 = 0.12$:0.47:0.39:0.02

(c) v = 1, J = 5, $\lambda = 2.795 \times 10^{-6}$ m

	-							-	Ŧ			-	=	
m V <th>, <u>1</u>], 4</th> <th></th> <th></th> <th></th> <th>-</th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th>· · ·</th> <th>c</th>	, <u>1</u>], 4				-							· · ·	c	
No. 1.1 No. 1.1 <t< th=""><th>E</th><th>2</th><th></th><th>3</th><th>F</th><th>- -</th><th>1 1</th><th></th><th>1 and</th><th>1 see</th><th></th><th>cm²/mote</th><th></th></t<>	E	2		3	F	- -	1 1		1 and	1 see		cm ² /mote		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$				н. 1.7	- - -									
				10 1 to 10		X *0.*1			····. ·	40.	5 H .	41.154	1.00.	
(11) (900) (01) (501) (100) (101) (101) (101) (101) (101) (101) (101) (11) (12) (101) (101) (101) (101) (101) (101) (101) (101) (11) (11) (11) (11) (11) (11) (11) (11) (11) (11) (11) (11) (11) (101) (11)	. 11.	11114	P1-1-	1	· · · • (• (• · •	48. 1	1	14.4.	•11.*	4.	h:::	N5.2N4	120.	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	ан .	1116	1107	2000 HT RS *	1.00 AND 1.	124.	NN1-"	664.			. 510	142.921	570.	
1,116 1,910 1,870 1,014 1,116 1,910 1,910 1,914 1,915 1,914 1,915 1,914 1,915 1,914 1,915 1,914 1,915 1,914 <th< td=""><td>A.F.</td><td>1.700</td><td>1.111.</td><td>čov this."</td><td>51-4 16777</td><td>1 110.</td><td>15.01</td><td>1.45.</td><td>125.</td><td>810.</td><td>1417</td><td>122.308</td><td>80.0</td></th<>	A.F.	1.700	1.111.	čov this."	51-4 16777	1 110.	15.01	1.45.	125.	810.	1417	122.308	80.0	
1,116 1,910 <th< td=""><td>1.46</td><td>, true</td><td>1.454.1</td><td>vir.+ 48 ris *</td><td>2004-00-02</td><td>141</td><td>1.146</td><td>1 **</td><td>чн. .</td><td>141.</td><td>710.</td><td>4.115</td><td>{ 10*</td></th<>	1.46	, true	1.454.1	vir.+ 48 ris *	2004-00-02	141	1.146	1 **	чн. .	141.	710.	4.115	{ 10 *	
1, 116 1, 900 1, 341 500 + 1, 710 + 1, 710 + 1, 710 + 1, 710 1, 600 1, 345 1, 346 1, 341 <th 1,<="" td=""><td>1.114</td><td>. NOW.</td><td>1.95</td><td>100 481 2</td><td></td><td>12,460</td><td>118.</td><td>14.27</td><td>• et.</td><td>157</td><td>672.</td><td>H.44R</td><td>.271</td></th>	<td>1.114</td> <td>. NOW.</td> <td>1.95</td> <td>100 481 2</td> <td></td> <td>12,460</td> <td>118.</td> <td>14.27</td> <td>• et.</td> <td>157</td> <td>672.</td> <td>H.44R</td> <td>.271</td>	1.114	. NOW.	1.95	100 481 2		12,460	118.	14.27	• et.	157	672.	H.44R	.271
1,116 1,200 .061 .600 0.12,314 .187 .570 12.234 .187 .187 .187 .181 .183 11,160 .100 187,011 .011000 .012 .101 .612 .131 .183 11,160 .100 187,011 .011000 .012 .612 .612 .131 .131 11,160 .600 192,011 .011000 .012000 .012 .612 .131 11,160 .600 192,011 .110000 .012100 .0120 .012 .612 .131 11,160 .600 128,021 .110000 .01210 .0120 .012 .612 .131 11,160 .600 128,021 .110000 .01210 .0120 .0120 .0129 .0120 .1050 .0120 .1050 .0120 .0120 .0120 .0150 .0160 .1100 .0120 .0120 .0120 .0150 .0100 .1253 .0160 .0120 .0123 .0129 .0120 .1.223 .0129 11,160 .100<	1.1.1	City to	1.2.4	1.446.	turalat.	675 ° 0	1.111	664.	565.	etc.	. 510	10,542	446	
11.160 JOU 185,051 JOU 185,051 JOU 185,051 JOU 185,051 JOU 181,052 JOU JOU </td <td>1.114</td> <td>1.00</td> <td>441</td> <td>(1)+ (cost)*</td> <td>1. .</td> <td>4,8,0</td> <td>1.5.</td> <td>1 85.</td> <td>115.</td> <td>.187</td> <td>007.</td> <td>182.21</td> <td>181.</td>	1.114	1.00	441	(1)+ (cost)*	1. .	4,8,0	1.5.	1 85.	115.	.187	007.	182.21	181.	
11.160 600 19.201 36.000 19.201 36.00 2.700 2.700 2.710 13.160 2000 2.803 2.1000 02.710 2.404 2.710 2.710 13.160 2.900 2.804 1.000 02.710 1.000 4.660 13.160 96.200 1.1000 08.264 5.710 1.223 3.829	11,150	1001	140.281	20143 + 014	2000 18 10 2	128,404	11.46.	1 41.	., 141	1. 4.14	210	14.	1817	
14,160 .900 124,367 .120000.1141000 .05124 4.444 .4450 4.4560 4.4560 14,160 1.000 46,260 14,160 14,000 14,000 14,000 14,000 14,000 14,000 14,000 14,000 14,000 14,000 14,000 14,000	11,160	10 4 .	1.5.101	16.4.4.41		461.81	501°H	.6.11	404.	1°, 364	P21.	. 445	2.710	
13.160 1.200 96,260 111101 1100 100 36,260 3.201 284 266 3.201 1.223 3.829	141.11	unt.	1.24 . 14.7	tor tof t	50-461 F.	447,244	4.4.4	664°	. 6 .	HOU'	.510	1.054	4.460	
	13.160	1.111	96,260	1003111.	5174 ANT 1 *	48°°86	112.5	145.	124.*	1.417	uu.	1.22.1	1.8.9	

232222

3

22

TABLE I. Numerical values of parameters for cw HF laser. HF: H_2 : H_e : $0_2 = 0.12$:0.47:0.39:0.02

(d) v = 1, J = 8, $\lambda = 2.911 \times 10^{-6}$ m

~~~

ł

|          | i<br>i<br>i       |           |                                       | 1.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                       |          |              |                     |                                    |        |                          |                |
|----------|-------------------|-----------|---------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|----------|--------------|---------------------|------------------------------------|--------|--------------------------|----------------|
|          | i =<br>i =<br>i + |           |                                       | •                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | -<br>-<br>-<br>-<br>- |          |              | е т Р <sub>СС</sub> | γ. <sup>101</sup> - <sup>μ</sup> γ | ۲۰     | ر. ۲۰۰۱ <sup>- ۱</sup> ۵ | -              |
| E        | ×                 | E S       | 3                                     | -                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                       | 1        |              | -                   | ا `دەد                             |        | رس <sup>2</sup> / ۱۳۵۱ ه | c              |
|          | I                 |           | ж                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                       |          |              |                     |                                    |        |                          |                |
|          |                   | 460.      | • • • • • • • • • • • • • • • • • • • | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | N.S.L.                |          | ŤŦ.          | нь                  | 41 (1)                             | . 42   | 191.790                  | ss0.           |
| <i>.</i> | . 600             | 1.01      | • A IN 77 - 170+ 4901 1               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 48.1                  | 1 ** .   | . 4. 50      | 127.                | 41.01                              | 254.1  | 76.071                   | <b>6</b> [ ] . |
| 2017     | 1006.             | \$10.     | + Dite " - Divergent"                 | * • •                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                       | 44       | 444          | 515.                | 1267                               | 1.54 1 | 43.166                   | 611.           |
| au.      | 1.200             | 110.      | th the full of the t                  | <b>1</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 1 80.                 | 1567     | 1 41         | ť. e., "            | * [                                | 1. 164 | 912.5701                 | . NAM          |
| 1.116    | . 100             | 4.559     | • 11 m. 1 11 • 3m1 1 *                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 144                   | 4.1.1    | Г <b>н</b> . | Her.                | 1 . 1 .                            | 1.212  | 611.5                    | 144.           |
| 1. 116   | .604.             | 087.5     | • 1182° - 2000 1528°                  | 10.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                       | 114.     | 118.64.      | 1.4.                | 2577                               | 1.655  | 7.407                    | 1.140          |
| 1.116    | 0006.             | 025.1     | • का किंट्रे के स्थान के कि           | 11                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | ۲ <i>۲۵</i> .۴        | 1.44.    | 564.         | <u>сік.</u>         |                                    | 1.541  | 9.117                    | 201.1          |
| 1.116    | 1.700             | 1.140     | • Hout the grave deaters              | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | л.н.р                 | 1257     | 1            | 5 to .              |                                    | 1.1.1  | 85°.01                   | . 484          |
| 041-11   | 1001              | 455.9.11  | . 1. 11                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 154.51                |          | · · ·        | ×0.                 | 11                                 |        | ×.                       | 118 6 . 8      |
| 13,160   | .600              | (HP.111   | + 41 , 1 ° , 10+ 48355 °              | 1 100                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 405.44                | 5- I ° r |              | I. • •              | <b>1</b>                           |        | 14, 1                    | 11.302         |
| 1.41.11  | unp.              | 151.9.121 | • 1.11 (D+ 104V                       | e de la constante de la consta |                       | 414.4    | 664          |                     | H011.                              | 1.149  | . 10                     | 121.11         |
| 11.160   | 007.1             | 111.941   | • R. M Co+ 477 M.                     | -<br>-<br>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | M WA                  | ha N     | 1 n 1        | 1 B.S. 1            | , ואי                              | 1.1.1  | 4.01                     | 4.814          |

23

are too slow to affect this equilibrium. The convection, chemical pumping, and collisional deactivation terms are retained in Eq. (22c), which represents global conservation. Substitution of Eq. (22a) into Eq. (11f) indicates

$$\frac{2}{\pi} \frac{G_{J}(v')}{f_{J}N^{-}K_{J}(v')} = 1 + \left(\frac{R_{r}}{R_{t}} - 1\right)\left(1 - \frac{N_{J}}{f_{J}N^{-}}\right)$$
(23a)

where

$$K_{J}(v') = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{P_{v}L(v - v')}{1 + \Sigma L(v - v_{J,j})I_{J,j}} \frac{dv}{\Delta v_{h}}$$
(23b)

Equation (23) exhibits the variation of gain with frequency for a given transition v,J. The quantity  $K_J(v')$  is the normalized line shape and is seen to depend only on  $\Delta v_h$ ,  $\Delta v_d$ , and  $I_{J,j}$ . The coefficient of  $K_J(v')$  in Eq. (23a) is an amplitude function and requires a knowledge of the number densities N<sup>-</sup> and  $N_J^-$ .

Evaluation of the line shape can be simplified by introduction of the variables

$$X = 2 (ln2)^{1/2} (v - v_0) / \Delta v_d$$
 (24a)

$$X_{j} = 2(\ell n 2)^{1/2} (v_{j} - v_{o}) / \Delta v_{d}$$
 (24b)

$$Y_{J,j} = (\ell n 2)^{1/2} (\Delta v_h / \Delta v_d) \phi_{J,j}$$
(24c)

 $\phi_{J,j} = (1 + I_{J,j})^{1/2}$  (24d)

For cases where  $\Delta v_h \ll \Delta v_c$ , the value of  $K_J(v)$  at  $v_j$  is affected only by the laser intensity at  $v_j$  and is found from

$$K_{J}(v_{j}) \equiv K_{J,j} = \frac{(\ell n 2)^{1/2}}{\pi} \frac{\Delta v_{h}}{\Delta v_{d}} \int_{-\infty}^{\infty} \frac{\exp(-x^{2}) dx}{Y_{J,j}^{2} + (x_{j} - x)^{2}}$$
(25a)

$$= \frac{1}{\phi_{J,j}} \left\{ \exp(-X_j^2) - \frac{2Y_{J,j}}{\pi^{1/2}} \left[ 1 - 2 X_j D(X_j) \right] + O(Y_{J,j}^2) \right\}$$
(25b)

$$= \frac{1}{\phi_{J,j}} (\operatorname{erfc} Y_{J,j}) \exp(Y_{J,j}^{2}) [1 + O(X_{j}^{2})]$$
(25c)

where  $D(X_j)$  is the Dawson integral.<sup>14</sup> The quantity  $X_j$  can be replaced by X in Eqs. (25a) and (25b) when  $I_{J,j} = 0$ . In this case, Eqs. (25a)-(25c) provide expressions for the line shape  $K_J(v)$ .

2. Limit 
$$R_r >>1$$
,  $R_t >>1$ ,  $\left| \frac{R_r}{R_t} - 1 \right| \left( \frac{\Delta v_h}{\Delta v_d} \right) \ll 1$ 

Equations (22a)-(22c) can be further simplified if it is assumed that  $R_{r}$ >>1,  $R_{t}$ >>1 and

$$\left|\frac{\frac{R}{r}}{R_{t}}-1\right|\frac{\Delta v}{\Delta v}\frac{h}{d}\ll 1$$
(26)

Equations (22a), (22c), and (23a) become

$$N_{Jv} = p_{v} f_{J} N^{-} / [1 + \sum_{j} L(v - v_{J,j}) I_{J,j}]$$
(27a)

$$\frac{dN^{-}}{d\zeta} = \frac{dN_{T}}{d\zeta} - N_{T} - (1 + B)N^{-}$$
(27b)

$$\frac{2}{\pi} G_{J}(v) = f_{J} N^{T} K_{J}(v)$$
(27c)

where

$$B = \frac{S}{2} \sum_{J j} f_{J} K_{J} (v_{j}) I_{J,j}$$
(27d)

These equations can be evaluated without consideration of  $N_{\rm J}^{-}$ , which is found from

$$\frac{N_{J}}{f_{I}N} = 1 - \sqrt{\pi \ell n 2} \frac{\Delta v_{h}}{\Delta v_{d}} \frac{R_{t}}{R_{r}} \sum_{j} K_{J}(v_{j})I_{J,j}$$
(27e)

Note that  $N_J$  is reduced below its equilibrium value by an amount which, for  $K_J(v_j) = 1$ , is proportional to  $I_J \Delta v_h / \Delta v_d$ . Equations (27a)-(27c) are the same equations that result when rotational equilibrium is assumed.

Equations (20) and (26) are realistic for low pressure cw chemical lasers, because  $R_t \sim R_r \gg 1$  and  $\Delta v_h / \Delta v_d \sim 10^{-2} p(Torr) << 1$  in these lasers (Table I). With an increase in pressure, Eq. (26) remains valid if it is

assumed that  $R_r/R_t = 1$ . The assumption  $R_r/R_t = 1$  is consistent with estimates in the range  $0.5 \le R_r/R_t \le 10$  in Refs. 9 and 11 and with the simplified nature of the present model. The present results suggest that a first estimate for the performance of cw chemical lasers can be obtained by assuming translational nonequilibrium and rotational equilibrium [Eqs. (27a)-(27c)]. The quantity  $N_T$  is then found from Eq. (27e).

## B. Amplifier

We consider a multiline amplifier (Fig. 1b) in the limit given by Eqs. (20) and (26) and assume that the input intensity of each transition  $I_{J,j}$  is specified and is independent of  $\zeta$ . The quantity B in Eq. (27) is then a constant. For laminar mixing, integration of Eq. (27b), together with Eq. (27c), yield

$$\zeta_{\rm D}^{1/2} N^{-} = (2/\pi) \zeta_{\rm D}^{1/2} G_{\rm J}(v) / [f_{\rm J} K_{\rm J}(v)]$$

$$= \frac{2 + B}{(1 + B)^{3/2}} \left[ D[\sqrt{(1 + B)\zeta}] - \frac{[(1 + B)\zeta]^{1/2}}{2 + B} \right] .$$
(28)

where D( ) is again the Dawson integral. Equation (28) provides the variation of  $N^-$  with streamwise distance for various values of the parameter B. Numerical results are plotted in Fig. 2. The maximum points on these curves are denoted by subscript m and are included in Table II.

Equation (28) also provides the variation of gain with frequency, streamwise distance, and saturation. The line shape corresponding to a single laser transition at frequency  $X_j = 0$  and  $X_j = 0.40$  is shown in Figs. 3a and 3b, respectively. The ratio of homogeneous to inhomogeneous broadening is assumed to be  $\Delta v_h / \Delta v_d = 0.024$ , and the corresponding low saturation hole size,  $(\ln 2)^{1/2} \Delta v_h / \Delta v_d$ , is indicated. Hole size depends on  $Y_{J,j}$  [Eqs. (25a)-(25c)], and the significant increase of hole size with saturation is evident. The variation of local gain with frequency at  $\zeta = 0.305$ , for a single laser transition of  $X_j = 0.40$ , is shown in Figs. 4a and 4b for  $\Delta v_h / \Delta v_d = 0.024$  and 0.048, respectively. The station  $\zeta = 0.305$  corresponds to the streamwise location where the zero power gain is a maximum (Table II). Increased saturation is seen to depress the entire gain curve. This is a consequence of cross relaxation.



Fig. 2. Variation of Inversion Number Density with Streamwise Distance for Case of Uniformly Illuminated Amplifier with Laminar Mixing. Parameter B is a measure of saturation level.

|     | Pow                                                | er     | Feak Invers<br>Densi                              | ion Number<br>ty |
|-----|----------------------------------------------------|--------|---------------------------------------------------|------------------|
| В   | $\frac{2\zeta_{\rm D}^{1/2} P_{\rm e}}{2^{1/2}/3}$ | ۶e     | $\frac{\zeta_{\rm D}^{1/2}N_{\rm m}^{-}}{0.3528}$ | ζ <sub>m</sub>   |
| 0   | 0.0000                                             | 1.1301 | 1.0000                                            | 0.3051           |
| 1/9 | 0.0586                                             | 1.1050 | 0 <b>.9</b> 768                                   | 0.2926           |
| 1/4 | 0.1243                                             | 1.0763 | 0.9498                                            | 0.2788           |
| 3/7 | 0.1980                                             | 1.0427 | 0.9184                                            | 0.2629           |
| 2/3 | 0.2812                                             | 1.0036 | 0.8810                                            | 0.2438           |
| 1   | 0.3757                                             | 0.9566 | 0.8356                                            | 0.2222           |
| 1.5 | 0.4834                                             | 0.8992 | 0.7792                                            | 0.1958           |
| 7/3 | 0.6059                                             | 0.8273 | 0.7063                                            | 0.1635           |
| 4   | 0.7423                                             | 0.7341 | 0.6069                                            | 0.1237           |
| 7   | 0.6469                                             | 0.8491 | 0.5003                                            | 0.0860           |
| 9   | 0.8833                                             | 0.6149 | 0.4541                                            | 0.0714           |
| œ   | 1.000                                              | 0.5000 | 0.000                                             | 0                |

## TABLE II. Maxima for amplifier with uniform incident radiation and laminar diffusion.<sup>a</sup>

55.51

55555

<sup>a</sup>Eqs. (28)-(30).











Fig. 4. Variation of Gain with Frequency at Streamwise Station  $\zeta = 0.305$  in a Uniformly Illuminated Amplifier with a Single Transition at X<sub>1</sub> = 0.4. Laminar mixing and homogeneous widths  $\Delta v_h / \Delta v_d = 0.024$ , 0.048 are assumed [Eq. (23a)]; (a) Case  $\Delta v_h / \Delta v_d = 0.024$ .



Fig. 4. Variation of Gain with Frequency at Streamwise Station  $\zeta = 0.305$  in a Uniformly Illuminated Amplifier with a Single Transition at  $X_j = 0.4$ . Laminar mixing and homogeneous widths  $\Delta v_h / \Delta v_d = 0.024$ , 0.048 are assumed [Eq. (23a)]; (b) case  $\Delta v_h / \Delta v_d = 0.048$ .

Reference 15 reports experimental measurements of the decrease in line center gain at streamwise station x = 0.4 resulting from amplifier radiation at frequency  $X_j = 0.40$ . Figure 3a and Table Id can be used to provide theoretical estimates for line center gain variation with amplifier input intensity. The resultant estimates are included in Fig. 5 and indicate good agreement with experiment. [The authors of Ref. 15 interpreted the present line shape estimates (Fig. 3) as line center gain estimates (Fig. 4) and incorrectly concluded that the present model does not properly evaluate saturation effects.]

The downstream end of the positive gain region is denoted  $\zeta_e$  and is found by equating N<sup>-</sup> to zero. The result is

$$[(1 + B)\zeta_e]^{1/2} = (2 + B)D[\sqrt{(1 + B)\zeta_e}]$$
(29)

The net output power is

$$2\zeta_{D}^{1/2}P_{e} = [B/(1+B)]\zeta_{e}^{1/2}[1-(2/3)\zeta_{e}]$$
(30)

Corresponding values of B,  $\zeta_e$ , and P<sub>e</sub> are included in Table II. For a saturated (B +  $\infty$ ) amplifier

$$\zeta_e = 1/2 ; 2\zeta_D^{1/2} P_e = 2^{1/2}/3$$
 (31)

Equations (28)-(31) are identical in form to the corresponding results presented in Ref. 1. The parameter B in these equations replaces the parameter  $K_2$  in Ref. 1. The previous results neglect hole burning effects on  $K_{J_1}$ .

The single-line amplifier solution provides a convenient basis for investigating the saturation process in cw chemical lasers. The effect of saturation on normalized line shape is determined by the parameter  $I_{J,j}$ . For an inhomogeneous medium, the intensity  $I_{J,j} = 1$  results in a reduction of  $K_{J,j}$  by a factor of  $2^{-1/2}$  [Eqs. (24d) and (25b)]. The effect of saturation on power extraction is characterized by the parameter B, as indicated in Table II. In the limit  $B + \infty$ , all available power is extracted by a single line. For the case of a single lasing transition, a power extraction saturation intensity  $\overline{I}_{S,P}$  can be defined by Eq. (27d)



Fig. 5. Effect of Input Intensity on Line Center  $(v = v_0)$  Gain at Fixed Streamwise Station for Uniformly Illuminated Amplifier. Circles denote experimental data from Ref. 15 that were taken at x = 0.4 cm with a gain medium at p = 6 Torr and a laser transition corresponding to v, J = 1,8 and  $X_J = 0.4$ . The symbol  $\blacklozenge$  denotes present estimate for gain based on line center values from Fig. 4a and  $\overline{I}_{S,L} = 46 \text{ W/cm}^2$ . The estimate for  $I_{S,L}$  is obtained by interpolation from Table Ia by recalling  $\overline{I}_{S,L} \sim p^2$  and by assuming p = 6 Torr and T = 900 K. Note also, Table Id indicates  $\Delta v_h / \Delta v_d = 0.025$ , which is consistent with use of Fig. 4a.

$$\bar{I}_{S,P}/\bar{I}_{S,L} = [\beta^2 + (\beta^4 + 4\beta^2)^{1/2}]/2$$
(32)

where

$$\beta = 2B/(Sf_{J}\phi_{J,j}K_{J,j})$$

Values of  $\overline{I}_{S,P}$ , corresponding to B = 1.5 and B = 7.0, are included in Table I and are seen to be large when S is small. For cases with N strong laser transitions and  $\Delta v_h \ll \Delta v_c$ , the power extraction saturation intensity is approximated by Eq. (32) with  $\beta$  replaced by  $\beta/N$ .

## C. Oscillator

We now consider a Fabry-Perot oscillator in the limit given by Eqs. (20) and (26) and further assume that  $\Delta v_h \ll \Delta v_c$  and  $Y_{J,j} \ll 1$ . These assumptions simplify the line shape [Eq. 25b)]. We also assume that, within each v,J lasing band, one laser frequency corresponds to  $v_o$ ; the other laser frequencies are then symmetric about  $v_o$ , and the present summation with respect to j, rather than thermal velocity, needs no modification.<sup>6</sup> The intensity  $I_{J,j}$  then represents the sum of the intensities in the +y and -y directions. The case  $\Delta v_c \ll \Delta v_h$  is treated in Ref. 7.

The threshold gain is denoted  $G_c$  and is a constant. The gain for each lasing transition is then, assuming  $\overline{\sigma}_{r,I} = \overline{\sigma}_r$ 

$$G_{J,j} = (\pi/2) f_{J} K_{J,j} N = G_{c}$$
 (33)

Equations (25b) and (33) yield

$$I_{J,j} = \left[\frac{f_{J}e^{-X_{j}^{2}}N^{-}}{(2/\pi)G_{c}}\right]^{2} - 1$$
(34a)

2

where  $N^{-}$  is obtained from Eq. (22c), namely

$$\frac{dN^{-}}{d\zeta} = \frac{dN_{T}}{d\zeta} - N_{T} - N^{-} - \frac{SG_{c}}{\pi} \sum_{J \ j} \left\{ \left[ \frac{f_{J}e^{-X_{J}^{-}N^{-}}}{(2/\pi)G_{c}} \right]^{2} - 1 \right\}$$
(34b)

Equation (34a) provides the dependence of  $I_{J,j}$  on  $f_J$  and  $X_j$  at each streamwise station. The number of rotational levels and longitudinal modes that reach threshold at each streamwise station is found from the requirement that  $I_{J,j} > 0$  in Eq. (34a), namely

$$f_{\rm I} > (2/\pi)G_{\rm c}/N^{-1}$$
 (35a)

$$x_j^2 < \ln[N_f_J/(\frac{2}{\pi}G_c)]$$
(35b)

The number of lasing transitions increases as  $G_c/N^-$  decreases. Equation (34b) cannot, in general, be integrated in closed form. Two subcases are treated: a saturated multiline oscillator and a partially saturated single line oscillator.

## 1. Saturated multiline oscillator

Assume that  $G_c \ll 1$  and S = O(1). In this limit,  $N \ll 1$  and  $N / G_c^{1/2} = O(1)$ . The laser is saturated, and there is a large number of laser transitions. Equations (34a) and (34b) become

$$(N^{-})^{2} = \frac{4G_{c}}{\pi S} \frac{dN_{T}/d\zeta - N_{T}}{\sum_{J} f_{J}^{2} \sum_{j} e^{-2X_{J}^{2}} }$$
(36a)

$$I_{J,j} = \frac{\pi}{SG_{c}} \frac{(dN_{T}/d\zeta - N_{T})f_{J}^{2}e^{-2X_{j}^{2}}}{\sum_{J} f_{J}^{2} \sum_{j} e^{-2X_{j}^{2}}}$$
(36b)

Corresponding number densities are

$$\frac{N_{J_{\nu}}}{P_{\nu}f_{J}N^{-}} = \frac{1}{1 + L(\nu - \nu_{J,j})I_{J,j}}$$
(36c)

$$\frac{N_{J}}{f_{J}N} = 1 - \frac{dN_{T}/d\zeta - N_{T}}{\bar{f}_{r}R_{r}f_{J}N} \sum_{j} \frac{f_{J}^{2}}{f_{J}^{2}}$$
(36d)

For cases where the rotational energy levels and longitudinal mode frequencies are closely spaced, the summations in Eqs. (36a)-(36d) can be replaced by

$$\sum_{j} e^{-2X_{j}^{2}} = \frac{1}{2} \left(\frac{\pi}{2\ell n 2}\right)^{1/2} \frac{\Delta v_{d}}{\Delta v_{c}} \left[1 + 0\left(\frac{\Delta v}{\Delta v_{d}}\right)\right]$$
$$\sum_{j} f_{j}^{2} = \frac{1}{2(\overline{f}_{j})^{2}} \left(\frac{\pi \vartheta_{R}}{2}\right)^{1/2} \left[1 + 0(\vartheta_{R})\right]$$

Equations (36) and (26b) provide the net inversion N<sup>-</sup> and the lasing intensity  $I_{J_{-1}}$  at each streamwise station. The quantities

$$I_{J,j}/I_{J} = e^{-2X_{j}^{2}/\sum_{j}} e^{-2X_{j}^{2}}$$
 (37a)

$$I_{J}/I = f_{J}^{2}/\sum_{J} f_{J}^{2}$$
(37b)

are plotted in Figs. 6a and 6b for  $G_c + 0$ . Longitudinal mode intensity  $I_{J,J}$  is inversely proportional to  $\Delta v_d / \Delta v_c$ , which is a measure of the number of longitudinal modes with a significant amount of power. The dependence of  $I_J$  on  $\vartheta_R$  is indicated in Fig. 6b. The number of active lasing modes decreases as  $G_c$  increases. Equation (36c) indicates that  $N_{Jv}$  departs from the equilibrium value  $p_v f_J N^-$  only in the vicinity of each lasing frequency  $v_{J,j}$ . The departure from equilibrium is large because  $I_{J,j}$  is large. The departure of  $N_J^-$  from the equilibrium value  $f_I N^-$  is of order  $1/(R_N^-)$ .

Because the laser is saturated, the output power and mode length are  $2\zeta_D^{1/2}P_e = 2^{1/2}/3$  and  $\zeta_e = 1/2$ , respectively, for laminar mixing. Thus, for the saturated multiline oscillator, rotational and translational nonequil-ibrium impact spectral output but not output power.

## 2. Single line oscillator

The parameters S and  $G_c$  determine the degree of saturation. In order to investigate their influence, we consider a case where line selection results in a single laser transition. For convenience, consider laminar mixing,  $f_J = 1$ , and  $X_J = 0$ . Lasing is initiated at station  $\zeta_i$ , where the gain first reaches the threshold value. Integration of Eq. (34b) with  $I_{J,i} = 0$  indicates

$$\zeta_{D}^{1/2} N_{i}^{-} = (2/\pi) \zeta_{D}^{1/2} G_{c} = 2D(\zeta_{i}^{1/2}) - \zeta_{i}^{1/2}$$
(38)



Fig. 6. Performance of Saturated Multiline Fabry-Perot Oscillator in the Limit  $R_r = R_t >> 1$ ,  $\Delta \circ_h << \Delta \lambda_c$ ,  $Y_{J,j} << 1$ ,  $G_c + 0$ , and S = 0(1) [Eqs. (36) and (37)]. (a) Variation of longitudinal mode intensity  $I_{J,j}$  with frequency  $X_j$  and (b) variation of intensity  $I_J$  with rotational level J.

which agrees with Eq. (28) for B = 0. Equation (38) provides  $\zeta_i$  and  $\zeta_D^{1/2} N_i^{-1}$  for a given value of  $\zeta_D^{1/2} G_c$ . Downstream of  $\zeta_i$ 

$$\frac{d\zeta_{\rm D}^{1/2}N^{-}}{d\zeta} = \frac{1}{2\zeta^{1/2}} - \zeta_{\rm D}^{1/2} - \zeta_{\rm D}^{1/2} N^{-} - \frac{S}{2} \zeta_{\rm D}^{1/2} N_{\rm I}^{-} \left[ \left( \frac{N^{-}}{N_{\rm I}} \right)^{2} - 1 \right]$$
(39)

which is integrated with the initial condition  $N^- = N_i^-$  at  $\zeta = \zeta_i$ . Lasing terminates at the downstream station  $\zeta_e$ , where  $N^- = N_i^-$ . The net output power is

$$2\zeta_{\rm D}^{1/2} P_{\rm e} = \left[\zeta^{1/2} - (2/3)\zeta^{3/2}\right]_{\zeta_{\rm i}}^{\zeta_{\rm e}} - \int_{\zeta_{\rm i}}^{\zeta_{\rm e}} \zeta_{\rm D}^{1/2} N^{\rm -} d\zeta \qquad (40)$$

Equations (39) and (40) have been evaluated for several values of S and  $\zeta_D^{1/2}G_c$ . Output power is given in Fig. 7 and decreases as S and  $G_c$  are reduced.

Oscillator solutions where hole burning effects are neglected correspond to  $S \neq \infty$ ,  $I_{J,j} \neq 0$ , and  $SI_{J,j} = finite$ . The number density in the lasing region is a constant given by  $N^- = N_i^-$ , where  $N_i^-$  and  $\zeta_i$  are obtained from Eq. (38). The local lasing intensity for laminar mixing is

$$2\zeta_{\rm D}^{1/2} \frac{\rm dP}{\rm d\zeta} = \frac{1}{2\zeta^{1/2}} - \zeta^{1/2} - \zeta_{\rm D}^{1/2} N_{\rm i}^{-}$$
(41)

The end of the lasing region occurs when the intensity goes to zero, or

$$z_{e} = \{ \left[ \left( \zeta_{D}^{1/2} N_{i}^{-} \right)^{2} + 2 \right]^{1/2} - \zeta_{D}^{1/2} N_{i}^{-} \right]^{2} / 4$$
(42)

The net output power is found from Eq. (40) and is included in Fig. 7. Neglect of hole burning is seen to overestimate output power.



Fig. 7. Performance of Single Line Fabry-Perot Oscillator in Limit  $R_r = R_t >> 1$ and  $Y_{j,j} << 1$ .

V. DISCUSSION

The parameters that characterize laser performance are liscussed further, and numerical estimates are provided.

The parameters introduced in the present study may be expressed in the form

$$R_{t} \sim \frac{k_{tr}}{k_{cd}} \frac{n_{2}(J,v)}{n_{2}(J,v)} \sim \frac{Particle \ Collision \ Rate}{Particle \ Deactivation \ Rate}$$
(43a)

$$I_{J,j} \sim \frac{(\bar{\sigma}_{v,J}/\epsilon_{v,J})\bar{I}_{J,j}}{k_{tr}} \frac{n_2(J,v)}{n_2(J,v)} \sim \frac{Particle Stim. Emission Rate}{Particle Collision Rate}$$
(43b)

$$S \sim \frac{k_{tr}}{k_{cd}} \frac{\Delta v_h n_2(J,v)}{n_2} \sim \frac{\text{Resonant Particle Collision Rate}}{\text{Net Deactivation Rate}}$$
 (43c)

$$B \sim \frac{\sum_{j} \sum_{i=1}^{n} \frac{v_{j}J_{j}}{\varepsilon_{v,j}} \overline{I}_{J,j} \Delta v_{n} n_{2}(J,v)}{n_{2}} \sim \frac{\text{Net Stim. Emission Rate}}{\text{Net Deactivation Rate}}$$
(43d)

$$G_{c}S \sum_{J j} \sum_{J,j} \frac{\sum_{j} \frac{I_{J,j}}{\varepsilon_{J,j}}}{k_{cd}^{n_{2}}} \sim \frac{\frac{Net \ Stim. \ Emission \ Rate}{Net \ Deactivation \ Rate}}{Net \ Deactivation \ Rate}$$
(43e)

The parameter  $R_t$  may be viewed as the number of translational collisions an upper level particle undergoes before it is collisionally deactivated. The parameter  $I_{J,j}$  represents the ratio of particle stimulated emission rate to particle collisional deactivation rate. Similarly, S represents the ratio of the collisional deactivation rate of particles resonant with  $I_{J,j}$  to the net upper level particle collisional deactivation rate. Finally, B and  $G_c S \sum_{j=1}^{r} I_{J,j}$  apply to amplifiers and oscillators, respectively, and represent the ratio of net stimulated emission rate to net collisional deactivation rate. The latter ratio characterizes laser output power and efficiency.

Numerical estimates for parameters and rate coefficients are included in Table I for a fixed stoichiometry, 300 < T, K < 1200, 0.132 < p, atm < 13.2 (0.10 < p, Torr < 10.0), and v,J = 0, 6; 0, 9; 1,5, and 1, 8. The pressure dependence of quantities in Table I is given by

$$\frac{S}{p} \sim \frac{k_{cd}}{p} \sim \frac{k_{tr}}{p} \sim \frac{\Delta v}{p} \sim \frac{\overline{I}S, L}{p} \sim 1$$
(44)

and permits interpolation of data in Table I with regard to pressure. Values of the power saturation intensity  $\overline{I}_{S,P}$  are given for B = 1.5 and 7.0. These values of  $\overline{I}_{S,P}$  correspond, roughly, to achieving one-half saturated power output and one-half zero power inversion number density, respectively. Note that  $\overline{I}_{S,P}$  is relatively insensitive to pressure, in contrast to  $\overline{I}_{S,L}$ .

## VI. CONCLUDING REMARKS

In the present model we have assumed that convection, chemical pumping, and collisional deactivation rates are small compared with translational relaxation, rotational relaxation, and stimulated emission rates. Similar approximations are introduced in Ref. 11. It is expected that corresponding simplifications can be introduced into numerical codes in order to facilitate solutions.

The further assumption  $|R_r/R_t - 1| (\Delta v_h/\Delta v_d) \ll 1$  resulted in a system of equations for cw chemical laser performance that are independent of  $N_J$ . This result suggests that a reasonable first estimate for cw chemical laser performance (other than evaluation of  $N_J$ ) can be obtained by assuming rotational equilibrium and translational nonequilibrium, as was done in Refs. 6-8. The physical basis for the latter approximation is as follows. CW chemical lasers operate at pressures of the order of 1 Torr and are inhomogeneously broadened. The modification of line shape (i.e., hole burning), induced by lasing, is more important than lasing induced departures from rotational equilibrium.

The present results may be contrasted with those for a pulsed chemical laser. A pulsed chemical laser generally operates at pressures of the order of one atmosphere, in order to achieve high energy density and is homogeneously broadened. As a consequence, its performance is insensitive to translational nonequilibrium, and spectral output is determined from considerations of rotational nonequilibrium.

| AF             | PENDIX A. PA                        | RTIAL LIST OF SYMBOLS                                           |
|----------------|-------------------------------------|-----------------------------------------------------------------|
| В              |                                     | parameter defining amplifier saturation, Eq. (27)               |
| c              |                                     | speed of light in vacuum                                        |
| D(             | )                                   | Dawson integral, Ref. 14                                        |
| Ē              | , f <sub>j</sub>                    | fraction of particles in rotational energy level, Eq. (6)       |
| ເງ             | (v), G <sub>J,v</sub>               | normalized gain, Eq. (11)                                       |
| g              | .J <sup>(v)</sup>                   | gain, Eq. (8)                                                   |
| Ī              | .1                                  | intensity for longitudinal mode J,j                             |
| Ī              | ,I                                  | net intensities                                                 |
| Ī              | ,L' <sup>Ī</sup> S,P                | line shape and power saturation intensities, Eqs. (12) and (32) |
| IJ             | .1                                  | nondimensional intensity, Eq. (11d)                             |
| J              |                                     | rotational energy level                                         |
| j              |                                     | longitudinal mode number                                        |
| к <sub>ј</sub> | (v),K<br>J.i                        | line shape, Eqs. (23) and (25)                                  |
| k c            | d, <sup>k</sup> tr, <sup>k</sup> rr | deactivation, translational and rotational relaxation rates     |
| L(             | u - u')                             | Lorentzian distribution, Eq. (8)                                |
| NJ             | , N <sub>J</sub> ,N,N <sub>T</sub>  | normalized particle number densities, Eq. (11)                  |
| 'nv            | (J,v),n <sub>v</sub> (J),n          | particle number densities, Eq. (1)                              |
| n <sub>T</sub> |                                     | total number of lasing species, $n_1 + n_2$                     |
| Ρ              |                                     | normalized output power, Eq. (17)                               |
| P,             | Po,Pv                               | velocity distribution functions, Eqs. (3)-(5)                   |
| р              |                                     | pressure                                                        |
| ₽ <sub>t</sub> | , <sup>R</sup> r                    | collisional rate ratios, Eq. (lle)                              |
| S              |                                     | collisional deactivation rate parameter, Eq. (11h)              |
| Т              |                                     | temperature                                                     |
| u              |                                     | streamwise velocity                                             |
| vу             |                                     | thermal velocity in y direction                                 |
| Х,             | ×j                                  | normalized frequency, Eq. (24)                                  |
| x              |                                     | streamwise distance, Fig. 1                                     |
| ×D             |                                     | characteristic diffusion distance, Fig. l                       |
| ۲ <sub>J</sub> | đ                                   | homogeneous width parameter, Eq. (24)                           |
| У              |                                     | transverse distance, Fig. l                                     |
| εv             | J                                   | energy per mole of photons                                      |
| ζ              |                                     | normalized streamwise distance k <sub>cd</sub> x/u              |
|                |                                     | 45                                                              |

PERSONAL POSSESSION PERSONAL

CONTRACTOR DESCRIPTION DESCRIPTION

لتنتند

| ς <sub>D</sub>   | normalized diffusion distance, $k_{cd}x_D/u$             |
|------------------|----------------------------------------------------------|
| θ <sub>R</sub>   | characteristic rotational temperature parameter, Eq. (6) |
| λ                | wavelength                                               |
| ν                | frequency                                                |
| vo               | line center frequency                                    |
| Δνd              | Doppler width [full-width, half-maximum (FWHM)]          |
| Δνh              | homogeneous width (FWHM)                                 |
| ۵۷               | longitudinal mode separation, c/2L                       |
| σ <sub>v.J</sub> | cross section for stimulated emisssion, Eq. (B-8)        |
| <sup>ф</sup> Ј,ј | $(1 + I_{J,j})^{1/2}$                                    |

Ŋ

CESSESS 1333

- 1552222

1222253

5533733

## Subscripts

| e | end of lasing region                           |
|---|------------------------------------------------|
| i | start of lasing region                         |
| J | rotational level                               |
| j | longitudinal mode                              |
| m | maximum value                                  |
| r | reference value or rotational relaxation value |
| v | vibrational level                              |

## Superscripts

| + | sum of numbe | r densities, | Eq. (11)       |    |
|---|--------------|--------------|----------------|----|
| - | difference o | f number den | sities, Eq. (1 | 1) |

APPENDIX B. PARAMETER EVALUATION

B.1 Dopper width [full-width, half-maximum (FWHM)]  

$$\Delta v_{d} = 2 \left(\frac{2kT\ell n2}{Mc^{2}}\right)^{1/2} v_{o} \text{ sec}^{-1}$$

$$= (831.6/\lambda)(T/300)^{1/2}(20/M)^{1/2}$$
(B-1)

where  $\lambda$  is wavelength in meters and M is molecular weight.

B.2 Homogeneous width (FWHM)

$$\pi T^{1/2} \frac{\Delta v_h}{p} = \sum_i \frac{p_i}{p} \overline{\gamma}_i \qquad \frac{(K)^{1/2}}{atm-sec} \qquad (B-2)$$

Part 5155

アナイイングイン

للاعتلاط الملكات

where  $\bar{\gamma}_i$  is  $2\pi c T^{1/2} \gamma_i$  and  $\gamma_i$  is tabulated in Ref. 7. Values of  $\bar{\gamma}_i$  are given in Table III.

B.3 Gas kinetic collision rate (HF +  $M_i$  + HF +  $M_i$ )

$$T^{1/2} \frac{k_{gk}}{p} = \sum_{i} \frac{p_i}{p} a_i \qquad \frac{(K)^{1/2}}{atm-sec} \qquad (B-3)$$

where

いらくくらい

$$a_{i} = T^{1/2} \frac{(k_{gk})_{i}}{p_{i}} = \frac{N_{A}^{3/2}}{R_{o}} \left[ 8\pi k \left(\frac{1}{M_{HF}} + \frac{1}{M_{i}}\right) \right]^{1/2} \left[\frac{d_{HF} + d_{i}}{2}\right]^{2}$$

Values of  $a_i$  are included in Table III. For a billiard ball model,  $\overline{\gamma}_i = a_i$  and  $\pi \Delta v_h = k_{gk}$ .

|                | Gas kinetic rate                                  | Homogeneous widths    |                                                                     |  |
|----------------|---------------------------------------------------|-----------------------|---------------------------------------------------------------------|--|
| Mi             | $a_i \times 10^{-11}$ (K) <sup>1/2</sup> /atm-sec | Rotational<br>Level J | $\overline{\gamma}_{i} \times 10^{-11}$ (K) <sup>1/2</sup> /atm-sec |  |
| HF             | 0.77                                              | 6 + 10                | 9.1 + 3.3                                                           |  |
| DF             |                                                   | 6 + 10                | 5.7 + 3.0                                                           |  |
| н <sub>2</sub> | 1.95                                              | 5 + 9                 | l.i → 0.73                                                          |  |
| N <sub>2</sub> | 1.00                                              | 6 + 8                 | 1.2 + 0.87                                                          |  |
| Н <sub>е</sub> | 1.28                                              | A11                   | 0.16                                                                |  |
| Ar             | 0.86                                              | 6 + 8                 | 0.16 + 0.49                                                         |  |
| 0 <sub>2</sub> | 0.92                                              |                       |                                                                     |  |
| F <sub>2</sub> | 0.86                                              |                       |                                                                     |  |

TABLE III. Homogeneous width and gas kinetic collision rate data for HF +  $M_i$  + HF +  $M_i$ .<sup>a</sup>

**I E S** 

New York

VARIA VICTORIA

<sup>a</sup>Refs. 2 and 6.

## B.4 Vibrational deactivation rate

We consider vibrational deactivation of the form

$$HF(v + 1) + M_{1} \xrightarrow{\overline{c}d} HF(v) + M_{1} \qquad (B-4)$$

The net deactivation rate  $k_{cd}$  is then

$$\frac{k_{cd}}{p} \equiv -\frac{1}{p} \frac{dln \ HF(v+1)}{dt} = \sum_{i} \frac{p_i}{p} \frac{k_{cd}}{R_o T} \qquad \frac{1}{atm-sec} \qquad (B-5)$$

Values of  $\vec{k}_{cd}^{i}/(R_{o}^{T})$  are given in Table IV.

## B.5 Rotational equilibrium

For rotational equilibrium, the fraction of particles with rotational energy  $J(J + 1)kT_R$  is given by

$$\frac{n_{v}(J)}{n_{v}} \equiv \overline{f}_{J} = \frac{(2J+1)\exp[-J(J+1)\theta_{R}]}{\sum_{j} (2J+1)\exp[-J(J+1)\theta_{R}]}$$
(B-6a)

$$= \theta_{R} (2J + 1) \exp[-J(J + 1)\theta_{R}] [1 + O(\theta_{R})]$$
 (B-6b)

where the summation is from J = 0 to  $J = \infty$ ,  $\theta_R = T_R/T$ , and  $T_R = 30.16$  K for HF. The maximum value of  $\overline{f}_J$  and the corresponding value of J, associated with a fixed value of  $\theta_R$ , are denoted  $\overline{f}_{J,m}$  and  $J_m$ , respectively. Typical values are included in Table V. For a fixed value of  $\theta_R$ , the quantity  $\overline{f}_{J,m}$  provides a convenient value for the reference quantity  $\overline{f}_r$ . When  $\overline{f}_r = \overline{f}_{J,m}$ 

$$f_{J} \equiv \frac{\tilde{f}_{J}}{\tilde{f}_{r}} = \frac{(2J+1)\exp[-J(J+1)\theta_{R}]}{(2J_{m}+1)\exp[-J_{m}(J_{m}+1)\theta_{R}]}$$
(B-6c)

and  $f_J < 1$  for  $J \neq J_m$ .

B.6 Photon energy

Photon energy, per mole, is

TABLE IV. Vibrational deactivation rates for reaction

 $\frac{\kappa}{k} \frac{1}{\frac{cd}{d}} HF(v + 1) + M_{1} \xrightarrow{cd} HF(v) + M_{1} \cdot^{a}$ 

|             | **************************************                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                         |                 | d (utmisse)             |                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|-----------------|-------------------------|------------------------|
|             | ( m.).( m.)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | ж<br>(Ю)                | ALM K           | чин қ                   | 12001 K                |
| -           | $(a_1, a_2, b_1, b_2, b_2, b_1, b_2, b_2, b_2, b_1, b_2, b_2, b_2, b_2, b_2, b_2, b_2, b_2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 101 + 7.5               | $1.15 + 10^{7}$ | 4.76 - Hi <sup>f</sup>  | 5.78 + 10 <sup>1</sup> |
|             | $(1)_{0,0,1}$ $(1)_{0,1}$ $(1)_{0,1}$ $(1)_{0,1}$ $(1)_{0,1}$ $(1)_{0,1}$ $(1)_{0,1}$ $(1)_{0,1}$ $(1)_{0,1}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | н. 1, - 1 <sup>пћ</sup> | $v_1 + 10^7$    | 5.64 + 10 <sup>7</sup>  | 1 . 27 . 4             |
|             | $(v + 1)^{2,2}$ $(v - 1)^{1-1} + 1^{2} + 1^{2} + 1^{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                         | 201 - 65.9      | 7.56 - 10 <sup>3</sup>  | 1 • 70-1               |
| -<br>-<br>- | (c + b) + c + b + c                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 1.14 - 10'              | 101 + 67.1      | 5.11 . 101              | 1.45 - 1               |
|             | $(\zeta_{1}, \zeta_{2}, \zeta_{2}, \zeta_{3}, \zeta_{3},$ | 01 . 11 .               | 101 - 61 -      | 9,44, - 10 <sup>3</sup> | 1 . 07 . 1             |

"Reserves were the where Price State (in <sup>1</sup> stim)/(mode K)

MARCHINE STREET, MARCHINE MARCHINE MARCHINE MARCHINE

ESSENT CONTRACTOR DOUGHTS REPORTED

| θR    | J <sub>m</sub> | f <sub>J,m</sub> |
|-------|----------------|------------------|
| 1/10  | 2              | 0.2654           |
| 1/20  | 3              | 0.1889           |
| 1/40  | 4              | 0.1353           |
| 1/60  | 5              | 0.1106           |
| 1/80  | 6              | 0.0957           |
| 1/100 | 7              | 0.0854           |

TABLE V. Maximum value of  $\overline{f}_J$  and corresponding value of J for fixed  $\theta_R$ .

<u>к.</u>

$$\varepsilon_{v,J} = N hv = 0.1196/\lambda J/mole$$
 (B-7)

1.1.1.1.1.1.1.1

where  $\boldsymbol{\lambda}$  is in meters.

## B.7 Gain coefficient

The gain coefficient for a P-branch transition v + l, J - l + v, J can be expressed<sup>2</sup>

$$\frac{\pi}{2} \Delta v_{h} \frac{\overline{\sigma}_{v,J}}{\varepsilon_{v,J}} = \frac{B(v,J,-1)}{4\pi} \qquad \frac{cm^{2}}{J-sec} \qquad (B-8)$$

where B(v,J,-1) is the Einstein coefficient.<sup>2</sup> For HF, the latter can be approximated by

$$B(v, J, -1) = 3.79 \times 10^{13} \frac{2J(1 + v)}{2J + 1} (1 + 0.063J) (1 - \frac{0.01v^3}{1 + v})$$

which is believed to be correct to within about 10% for  $1 \leq J \leq 16$  and  $v \leq 6$ .

## REFERENCES

- H. Mirels, R. Hofland, and W. S. King, "Simplified Model of CW Diffusion Type Chemical Laser," AIAA J. 11 (12), 156-164 (February 1973).
- G. Emanuel, "Numerical Modeling of Chemical Lasers," <u>Handbook of Chemical Lasers</u>," edited by R. W. F. Gross and J. F. Bott (John Wiley and Sons, 1976), pp. 488-496.
- R. J. Hall, "Rotational Nonequilibrium and Line-Selected Operation in cw DF Chemical Lasers," IEEE J. of Quantum Electron. <u>QE-12</u>, 453-462 (August 1976).
- L. H. Sentman and W. Rushmore, "Computationally Efficient, Rotational Nonequilibrium cw Chemical Laser Model," AIAA J. <u>19</u> (10), 1323 (October 1981).
- 5. T. T. Yang, "Modeling of cw HF Chemical Laser with Rotational Nonequilibrium," J. de Phys. Colloque C9 <u>41</u> (11), C9-51 (November 1980).
- H. Mirels, "Inhomogeneous Broadening Effects in cw Chemical Lasers," AIAA J. <u>17</u> (5), 478-489 (May 1979).
- H. Mirels, "Inhomogeneous Broadening Effects in Multimode cw Chemical Lasers," Appl. Opt. 20 (2), 362-373 (15 January 1981).
- H. Mirels, "Multimode Low Pressure cw Chemical Laser Performance Including Source Flow Effects," Appl. Opt. 20 (14), 2379-2388 (15 July 1981).
- 9. N. Cohen, J. F. Bott, M. A. Kwok, and R. L. Wilkins, <u>The Status of Rota-tional Nonequilibrium in HF Chemical Lasers</u>, Report No. TR-0086(6603)-2 (The Aerospace Corporation, El Segundo, CA, May 1986).
- 10. T. Kan and G. J. Wolga, "Influence of Collisions on Radiative Saturation and Lamb Dip Formation in CO<sub>2</sub> Molecular Lasers," IEEE J. of Quantum Electron. QE-7 (4), 141-150 (April 1971).
- A. A. Stepanov and V. A. Shcheglov, "Dynamic Saturation of Optical Transitions in High Power Molecular Lasers," Soviet J. Quantum Electron. <u>12</u> (5), 619-624 (May 1982).
- D. L. Bullock, M. M. Valley, and R. S. Lipkis, <u>Advanced Chemical Laser</u> <u>Optics Study</u>, Final Report, Contract No. F29601-79-C-0011 (TRW, 15 July 1982).
- 13. D. L. Bullock, J. de Phys. C9 37 (1980).
- M. Abramowitz and I. A. Stegun, Handbook of Matnematical Functions, AMS 55 (National Bureau of Standards, June 1964), pp. 297-303.

15. R. W. F. Gross and J. C. Coffer, "Saturation Processes in Doppler-Broadened HF Vibrational Transition," in <u>Gas Flow and Chemical Lasers</u>, edited by M. Onorato (Plenum Press, 1984), pp. 127-139.

16. N. Cohen and J. F. Bott, <u>Review of Rate Data For Reactions of Interest in</u> <u>HF and DF Lasers</u>, Report No. TR-0083(3603)-02 (The Aerospace Corporation, El Segundo, CA, October 1982).

## LABORATORY OPERATIONS

The Aerospace Corporation functions as an "architect-engineer" for national security projects, specializing in advanced military space systems. Providing research support, the corporation's Laboratory Operations conducts experimental and theoretical investigations that focus on the application of scientific and technical advances to such systems. Vital to the success of these investigations is the technical staff's wide-ranging expertise and its ability to stay current with new developments. This expertise is enhanced by a research program aimed at dealing with the many problems associated with rapidly evolving space systems. Contributing their capabilities to the research effort are these individual laboratories:

<u>Aerophysics Laboratory</u>: Launch vehicle and reentry fluid mechanics, heat transfer and flight dynamics; chemical and electric propulsion, propellant chemistry, chemical dynamics, environmental chemistry, trace detection; spacecraft structural mechanics, contamination, thermal and structural control; high temperature thermomechanics, gas kinetics and radiation; cw and pulsed chemical and excimer laser development including chemical kinetics, spectroscopy, optical resonators, beam control, atmospheric propagation, laser effects and countermeasures.

<u>Chemistry and Physics Laboratory</u>: Atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiative signatures of missile plumes, sensor out-of-field-of-view rejection, applied laser spectroscopy, laser chemistry, laser optoelectronics, solar cell physics, battery electrochemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, thermionic emission, photosensitive materials and detectors, atomic frequency standards, and environmental chemistry.

<u>Computer Science Laboratory</u>: Program verification, program translation, performance-sensitive system design, distributed architectures for spaceborne computers, fault-tolerant computer systems, artificial intelligence, microelectronics applications, communication protocols, and computer security.

<u>Electronics Research Laboratory</u>: Microelectronics, solid-state device physics, compound semiconductors, radiation hardening; electro-optics, quantum electronics, solid-state lasers, optical propagation and communications; microwave semiconductor devices, microwave/millimeter wave measurements, diagnostics and radiometry, microwave/millimeter wave thermionic devices; atomic time and frequency standards; antennas, rf systems, electromagnetic propagation phenomena, space communication systems.

<u>Materials Sciences Laboratory</u>: Development of new materials: metals, alloys, ceramics, polymers and their composites, and new forms of carbon; nondestructive evaluation, component failure analysis and reliability; fracture mechanics and stress corrosion; analysis and evaluation of materials at cryogenic and elevated temperatures as well as in space and enemy-induced environments.

<u>Space Sciences Laboratory</u>: Magnetospheric, auroral and cosmic ray physics, wave-particle interactions, magnetospheric plasma waves; atmospheric and ionospheric physics, density and composition of the upper atmosphere, remote sensing using atmospheric radiation; solar physics, infrared astronomy, infrared signature analysis; effects of solar activity, magnetic storms and nuclear explosions on the earth's atmosphere, ionosphere and magnetosphere; effects of electromagnetic and particulate radiations on space systems; space instrumentation.

. . .

END DATE FILMED 5-88 DTIC