AD-A191 421 NONLINEAR WAVE AND DIFFUSION EQUATIONS(U) OXFORD UNIV (ENGLAND) J B MCLEOD OCT 87 DAJA45-86-C-0040 1/1

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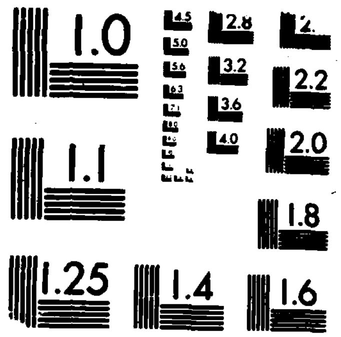
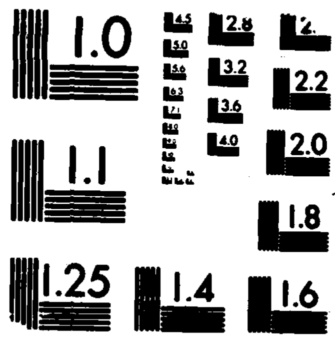


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

NONLINEAR WAVE AND DIFFUSION EQUATIONS

Principal Investigator : Dr J.B. McLeod

1st Periodic Report
October - November, 1986

Contract No. DAJA-86-C-0040

University of Oxford

2nd Periodic MAY 87
3rd Periodic OCT 87

The research reported in this document has been made possible through the support and sponsorship of the U.S. Government through its European Research Office of the U.S. Army. ~~This report is intended only for the internal management use of the Contractor and the U.S. Government.~~

Research Projects

Work done during the relevant period fell under the following headings:

Lens and Antenna Design

The problem here is, given two points, to design an optical lens which will have the property that it focusses all rays of light from the one point onto the other. The problem reduces to the study of an unusual pair of functional-differential equations, which have been studied by Dr McLeod and Professor A. Friedman (Purdue University). They showed that if the lens is symmetric, i.e. if the two faces of the lens are the same, then there exist precisely two lenses which will give the required focussing one in which the rays of light do not cross the axis of the lens, and the other in which they do.

There are many extensions of this sort of problem, some at least of which we have ear-marked for future research. For example, if one allows the two sides of the lens to be different, can one then focus two points onto two other points? If one replaces the lens (of constant refractive index) by a medium of variable refractive index, how much focussing can be done? The solution of these questions has immediate relevance to lens and antenna design.

In connection with this research, Dr McLeod visited for a month at the Center for Applied Mathematics at Purdue University.

Nonlinear Diffusion and Free Boundary Problems

During this period, much of the work in nonlinear diffusion centred on the fabrication of semiconductors. In particular, Dr J R King completed his thesis on the mathematical aspects of semiconductor process modelling, and he and C P Please wrote a paper on diffusion in crystalline silicon. There was also some related work on diffusion patterns as $m \to \infty$ for the equation $u_t = D(u^m Du)$.

Some preliminary work was done on free boundary problems in dislocation theory. Relevant equations were derived and partially analysed in joint work by Professor Head and Drs Howison, Ockendon, Titchener and Wilmott. This preliminary investigation will be followed up in future work.

Visits (wholly or partly supported by the Grant) were paid to Oxford by Professor J Chadam (Macmaster University), Professor M A Herrero (Madrid) and Professor A K Head (Melbourne). Dr S D Howison attended a conference at Lake Tahoe on Structure and Dynamics of Partially Solidified Systems, and presented a paper on some aspects of the Stefan model for phase transitions.

Papers in Preparation

The following papers either have been completed or are in an advanced stage of preparation. Copies should be available for the next Periodic Report.

Chadam, J., Howison, S.D. and Ortoleva, P.

Spherical crystals growing in a supersaturated solution, J. Crystal Growth, to appear.

Crowley, A.B. and Ockendon, J.R.

Modelling mushy regions, Appl. Sci. Res., to appear.

Elliott, C.M., Herrero, M.A., King, J.R. and Ockendon, J.R.

The mesa problem: diffusion patterns for $u_t = D(u^m Du)$ as $m \to \infty$, IMA J. Appl. Math., to appear.

Friedman, A. and McLeod, J.B.

Optimal design of an optical lens, Arch. Rational Mech. Anal., to appear.

Friedman, A. and McLeod, J.B.

An optical lens for focussing two pairs of points, in preparation.

Head, A.K., Howison, S.D., Ockendon, J.R. Titchener, J. and Wilmott, P.

A continuum model for dislocations in two dimensions, Phil. Mag., to appear.

Howison, S.D.

Some aspects of the Stefan model for phase transitions. Proc. Lake Tahoe Conf.

Structure and Dynamics of Partially Solidified Systems, Ed. D. Loper (1986).

King, J.R. and Please, C.P.

Diffusion in crystalline silicon, IMJ J. Appl. Math., to appear.

Administrative Action

Mr B van Brunt and Mr C J Mabb have been appointed as research assistants, initially for a period of one year. A programme of visitors and seminars is also being drawn up, and further details will be available for the next Report.

NONLINEAR WAVE AND DIFFUSION EQUATIONS

Principal Investigator: Dr. J.B. McLeod

2nd Periodic Report

December 1986 - May 1987

Contract No. DAJA-86-C-0040

University of Oxford.

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Research Projects

Work done during the relevant period falls under the following headings.

Lens and Antenna Design

As was mentioned in the last report, the problem is to design an optical lens which will give as perfect focussing as possible. In the previous report, we reported on work by Dr. McLeod and Professor A. Friedman (Purdue) on designing a symmetric lens (i.e. one where the two faces of the lens are the same), which would focus light from one given point perfectly onto a second given point.

With the ultimate aim of discovering just how flexible such a system can become, we have discussed the problem of whether one can use a lens with two different faces to focus light from a given pair of points perfectly onto another given pair, where all the four points lie on a common line (the axis of the lens). Again, in general, this can be done, and preprints are enclosed of the work on both the symmetric and non-symmetric lenses.

Dr. McLeod visited Purdue University, and also the Universities of Wisconsin, Minnesota and Leiden (Holland).

Free Boundary Problems

In the area of Stefan-like free boundary problems, the major unsolved problem remains the stabilisation of ill-posed (i.e. supercooled or superheated) Stefan problems by surface energy effects. A conjecture about this stabilisation for the simpler, one phase Hele-Shaw problem is made in [1]: it is that the cusps which would form in the absence of surface energy evolve into rapidly growing thin fingers or "cracks". Much work remains to be done to validate this conjecture.

Other work in free boundary problems has concerned preparation for the triennial international conference, to be held in Bavaria in June, and for a minisymposium at ICIAM in Paris in June. Further studies have been made of free boundary problems in inviscid hydrodynamics in connection with the cooling of turbine blades and in the modelling of dislocation distributions in elasticity theory [2].

In the area of nonlinear diffusion, two problems have been solved which arise in models of semiconductor fabrication. The first [3] considers the limit as $m \to \infty$ in the porous medium equation; this limit reveals more mathematical structure and enables the problem to be written as a variational inequality. The second [4] considers the asymptotic solution for a more realistic model of dopant diffusion.

Several free boundary and nonlinear diffusion problems arose at the 1987 Study Group with Industry [5].

References

1. Howison S.D, Lacey A.A., Ockendon J.R., Hele-Shaw Free Boundary Problems with Suction. Preprint (1987).

2. Head A.K., Howison S.D., Ockendon J.R., Titchener J.B. & Wilmott P., A continuum model for 2-D dislocation distributions. Phil Mag. (1987) (to appear).

3. Elliot C.M., Herrero M.A., King J.R. & Ockendon J.R., The Mesa Problem. IMA J. App. Math. (1986) 37, 147-154.

4. King J.R., Please C.P., Diffusion of Dopant in Crystalline Silicon - An Asymptotic Analysis. IMA J. App. Math. (1987) (to appear).

5. Twentieth Study Group with Industry. Oxford 1987.

NONLINEAR WAVE AND DIFFUSION EQUATIONS

Principal Investigator : Dr. J.B. McLeod

3rd Periodic Report

May - October, 1987

Contract No. DAJA-86-C-0040

University of Oxford

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RESEARCH PROJECTS

Work done during the relevant period falls under the following headings.

Profile of Blow-up and Quenching for Nonlinear Diffusion Equations

Equations of the general type

$$u_t = \Delta u + f(u)$$

can exhibit blow-up of solutions (when, at some finite time T, $u(x,T)$ becomes infinite for some value of x), or quenching of solutions (when, at some finite time T, a positive solution becomes identically zero). The question arises as to what is the nature of the profile of the solution just before blow-up or quenching, and the expected answer is that, in suitably scaled variables, the solution should approach some similarity solution. Whether a certain scaling is appropriate is therefore linked to the existence or non-existence of a similarity solution under that scaling and specifically to the existence or non-existence of solutions of equations such as

$$u'' + \left(\frac{n-1}{x} - \frac{x}{2}\right)u' = f(u), \quad x > 0 ,$$

where $f(u)$ may be

$$\frac{u}{p-1} - u^p \qquad (1<p<\infty) ,$$

$$1 - e^u ,$$

$$u^q - \frac{u}{1-q} \qquad (0<q<1) .$$

Dr. McLeod has been working on this problem with Professors A. and J. Friedman, and a preprint of their results is available (Ref. 1).

During the period under review, Dr. McLeod has visited the Institute for Mathematics and its Applications at the University of Minnesota, and also the University of Pittsburgh, where investigations began into problems of combustion and ionisation. These will be discussed further in future reports.

Stefan Problems

A major effort has been to assist in the organisation of the Symposium on Free Boundary Problems, Theory and Applications, held at Irsee in Bavaria in June, 1987. Two contributions from that symposium are available as preprints (Refs. 2,3). Further research has continued into the structure of models for super-cooled Stefan problems (Refs. 4,5), mushy regions (Ref. 6) and alloy solidifcation (Ref. 7).

Semiconductor Fabrication

Much research has emanated from the models in Ref. 8 (see Ref. 9), where several hierarchies of nonlinear diffusion equations are linked. This work led to a paper (Ref. 10) being given at the Irsee Symposium referred to above, and also to a mini symposium at the ICIAM meeting in Paris in June, 1987. This last was chaired by A.B. Tayler, and C. Please, A. Crowley and J. Ockendon were among the contributors. The related problem of implantation into n-baths has been discussed with Dr. J. Zavada, ERO, London.

Publications and References

1. A. Friedman, J. Friedman and B. McLeod, Concavity of solutions of nonlinear ordinary differential equations, Technical Report, Center for Applied Mathematics, Purdue University.

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