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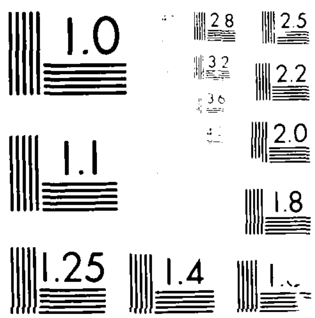
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VIBRATION CONTROL OF LARGE STRUCTURES

Submitted to:

Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, D.C. 20332

Attention: Dr. Anthony K. Amos

Submitted by:

W. D. Pilkey
Morse Professor

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SCHOOL OF ENGINEERING AND
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This is a study of vibration control for large space structures. Advantage is taken of the limiting performance characteristics of dynamic systems. This approach permits large problems with constraints to be analyzed. A modal formulation for the limiting performance was developed in order to enhance the applicability of limiting performance to large structural systems. One effort to develop an optimal control system is based on the limiting performance approach in combination with classical/optimal control theory. A limiting-performance/minimum-time solution was formulated to achieve the goal of rapid suppression of disturbances. Classical/optimal control studies show that a position loop might be useful in taking care of constraint controllers, such as proof-mass dampers. Finally, to derive feedback control law based on the limiting performance characteristics, parameter identification technique has been under investigation.

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TABLE OF CONTENTS

I. SUMMARY

This is a study of vibration control for large space structures. Advantage is taken of the limiting performance characteristics of dynamic systems. This approach permits large problems with constraints to be analyzed. A modal formulation for the limiting performance was developed in order to enhance the applicability of limiting performance to large structural systems. One effort to develop an optimal control system is based on the limiting performance approach in combination with classical/optimal control theory. A limiting–performance/minimum–time solution was formulated to achieve the goal of rapid suppression of disturbances. Classical/optimal control studies show that a position loop might be useful in taking care of constraint controllers, such as proof–mass dampers. Finally, to derive feedback control law based on the limiting performance characteristics, parameter identification technique has been under investigation.

II. RESEARCH OBJECTIVES

1. Modal Limiting Performance Formulation

Instead of evaluating Duhamel integrals to implement a linear programming formulation of the limiting performance problem, a modal approach was to be studied. The focus of the study was to develop a procedure for systemizing the coupling of vibration controllers to an existing (modal) structural model. A study of vibration control of structural systems was to be performed.

2. Limiting Performance Control of Large Space Structures

The limiting performance formulation was to be enhanced and expanded to treat control problems of large space structures (LSS) with proof-mass actuators. To take the characteristics of the LSS problem into consideration, a minimum-time solution , which was intended to dissipate the undesired vibration rapidly, was considered. Methods to incorporate a minimum time solution into the limiting performance formulation were of major concern.

3. Inertia Dampers

In addition to the limiting performance approach, inertia damper design problem was to be treated using the classical or optimal control laws. It was expected that the success of the control laws could be measured by the limiting performance control law. If the resulting control laws give characteristics which are close to those of the limiting performance, the control systems may be considered to be optimal. Since the inertia dampers were subject to constraints, a major consideration was related to the methodology for incorporating these constraints into the classical or optimal control laws.

4. Identification of Control Laws

In a preliminary study, identification of control laws based on the limiting performance characteristics was to be studied. This technique can handle the problem efficiently because once the limiting preformance characteristics are known, the identification process can be separated from the system dynamics.

III. RESEARCH PROGRESS AND STATUS

1. Background Information

Formulation of the Linear Programming Problem

A linear vibrating system with n degrees of freedom subjected to arbitrary excitation $\underline{f}(t)$ and control force $\underline{u}(t)$ is described by the equations

$$M\ddot{\underline{x}} + C\dot{\underline{x}} + K\underline{x} + V\underline{u} = \underline{f}(t) \tag{1}$$

where $\underline{x}$ is the displacement vector, M, C, K are the n x n mass, damping, and stiffness matrices, respectively; the n x j coefficient matrix V, associated with the j control forces $u_1, u_2, \ldots, u_j$, places these forces in the apppropriate rows in Eq. (1). In general, the measure of performance may be a linear combination of displacements, velocities, accelerations, control force components, and external forces. To define such a performance index, let

$$\underline{h} = P_1\ddot{\underline{x}} + P_2\dot{\underline{x}} + P_3\underline{x} + P_4\underline{u} + P_5\underline{f} \tag{2}$$

where the P_k are prescribed coefficient matrices. It is clear that $\underline{h}$ may be considered to be explicitly dependent on time t and the control forces $\underline{u}$.

$$\underline{h} = \underline{h}(t, \underline{u}) \tag{3}$$

The performance index ψ is then written in terms of $\underline{h}$ as

$$\psi = \max_i \max_t |h_i(t, \underline{u})| \tag{4}$$

where i varies over the rows of $\underline{h}$ and t over the time interval of concern. With these definitions, the optimization problem becomes minimization of ψ with respect to the control forces $u_1, u_2, \ldots, u_j$.

For any system governed by Eq. (1), the system responses are linear functions of $\underline{u}$. It is convenient at this point to discretize $\underline{u}$ in the time domain. Thus, if the time interval of interest is $0 \leq t \leq T_f$ and this interval is represented by N discrete instances of time, then $\underline{u}$ takes on N-1 discrete values. This piecewise constant discretized function is denoted $\bar{\underline{u}}$. Then, the discretized form of Eq. (2) becomes

$$\bar{\underline{h}} = W\bar{\underline{u}} + \underline{\underline{g}}(t) \tag{5}$$

where $\underline{\underline{g}}$ is an explicit, known function of time and W is obtained by solving Eq. (1) for the response variables in terms of $\underline{u}$, discretizing the results, and replacing the first four terms of Eq. (2) with these results.

To put the optimization procedure in standard linear programming form, define

$$\bar{\underline{u}} = \begin{bmatrix} \underline{u}(1) \\ \underline{u}(2) \\ \cdot \\ \cdot \\ \cdot \\ \underline{u}(N-1) \end{bmatrix} \tag{6}$$

where $\underline{u}(k)$ means the vector $\underline{u}(t)$ evaluated at the k-th subinterval of time. Thus, if the time discretization is uniform with each subinterval h seconds long, $\underline{u}(k) = \underline{u}((k-1)h)$. Next, let

$$\underline{z} = \begin{bmatrix} \psi \\ \\ \bar{\underline{u}} \end{bmatrix} \tag{7}$$

and

$$\underline{c}^T = [1 \ 0 \ . \ . \ . \ 0] \tag{8}$$

Then the linear programming problem is to minimize

$$\psi = \underline{c}^T \underline{z} \tag{9}$$

subject to the constraints

$$H\underline{z} \leq \underline{b} \tag{10}$$

The constraints (10) always include the conditions

$$\psi \geq |h_i(t,\underline{u})| \qquad (11)$$

Thus, minimizing ψ over the $\underline{u}$ such that conditions (11) hold makes ψ the least upper bound of the set $\{|h_i(t, \underline{u})|\}$ over time t and control forces $\underline{u}$. In addition to the constraints (11), Eq. (10) may also contain bounding constraints on response variables or control force magnitudes.

2. Modal Formulation [1]

A modal formulation of the limiting performance problem has been proposed as one of the tasks. Instead of evaluating Duhamel intergrals to accomplish the linear programming formulation, an alternative approach, a modal approach was explored. This approach is computationally convenient because it simplifies the steps necessary before linear programming optimization can be initiated. Also, modal truncation can be incorporated easily to handle large finite element models such as large space structure control problems. Furthermore, the practicality of determining limiting performance characteristics is enhanced because modal properties may be experimentally available. This modal formulation was applied to find the limiting performance characteristic of large structures, perhaps formed of substructures subject to transient distrubances. Modal formulations were delevoped for both systems with imbedded control forces and systems with generic control force connections between substructures.

Undamped Systems

For an undamped system, mode shapes ϕ_k and mode frequencies ω_k may be found from the eigenvalue problem

$$(-\omega_k^2 M + K)\ \underline{\phi}_k = \underline{0} \tag{12}$$

If mode shapes are dimensionalized such that

$$\underline{\Phi}^T M \underline{\Phi} = \underline{I} \tag{13}$$

$$\underline{\Phi}^T K \underline{\Phi} = \underline{\Omega}^2 = \mathrm{diag}(\omega_1^2,\ \omega_2^2,\ \ldots\ ,\ \omega_n^2) \tag{14}$$

where

$$\underline{\Phi} = \left[\ \underline{\phi}_1\ \underline{\phi}_2\ \ldots\ \underline{\phi}_n\ \right] \tag{15}$$

then the solution to Eq. (1) is given by

$$\underline{x}(t) = \sum_{k=1}^{n} \cos\omega_k t\ \underline{\phi}_k \underline{\phi}_k^T M\ \underline{x}(0) + \sum_{k=1}^{n} \frac{\sin\omega_k t}{\omega_k} \underline{\phi}_k \underline{\phi}_k^T M\ \underline{\dot{x}}(0)$$

$$+ \sum_{k=1}^{n} \underline{\phi}_k \frac{1}{\omega_k} \int_0^t \underline{\phi}_k^T\ [\underline{f}(\tau) - V\underline{u}(\tau)]\ \sin\omega_k(t-\tau)d\tau \tag{16}$$

No conditions have been placed on the control forces which replace the design elements: they may be linear or nonlinear. In the case where control forces have replaced passive elements such as springs or dashpots, it should be clear that the mode shapes and frequencies in Eq. (16) are those for a system without these elements included. The contributions to the displacement coordinate solution (16) from the replaced elements are contained in the control forces which have replaced them.

The advantage of using a modal formulation for the response variables is especially apparent when dealing with systems with control force connections between components. Mode frequencies ω_k in Eq. (16) are identically those of the components and mode shapes $\underline{\phi}_k$ are readily available from

$$\underline{\Phi} = \begin{bmatrix} \underline{\Phi}_1 & & & \\ & \underline{\Phi}_2 & & \\ & & \ddots & \\ & & & \underline{\Phi}_s \end{bmatrix} = \left[\underline{\phi}_1 \quad \underline{\phi}_2 \; . \; . \; . \underline{\phi}_n \right] \tag{17}$$

where $\underline{\Phi}_i$ is a matrix whose vectors are the modeshapes for the i-th component. Modal properties for each component may be obtained experimentally or determined computationally with general purpose (FE) computer programs.

If control forces are discretized in some fashion then the discretized response at time t=mT is obtained from Eq. (16)

$$\underline{x}(mT) = \underline{r}(mT) \tag{18}$$

$$- \sum_{k=1}^{n} \frac{1}{\omega_k} 2\underline{\phi}_k \underline{\phi}_k^T V \sum_{i=0}^{m-1} \underline{u}(iT) \left[\cos\,(m-i-1)\omega_k T - \cos(m-i)\omega_k T\right]$$

and r(mT) represents contributions from the disturbance forces $\underline{f}$ and initial conditions. Equation (16) may be differentiated and discretized to obtain velocity and acceleration expressions

$$\dot{\underline{x}}(mT) = \dot{\underline{r}}(mT) \tag{19}$$

$$- \sum_{k=1}^{n} \frac{1}{\omega_k} \underline{\phi}_k \underline{\phi}_k^T V \sum_{i=0}^{m-1} \underline{u}(iT)\,[\sin(m-i)\omega_k T - \sin(m-i-1)\omega_k T]$$

$$\ddot{\underline{x}}(mT) = \ddot{\underline{r}}(mT) - \sum_{k=1}^{n} \underline{\phi}_k \underline{\phi}_k^T V \underline{u}(mT)$$

$$- \sum_{k=1}^{n} \underline{\phi}_k \underline{\phi}_k^T V \sum_{i=0}^{m-1} \underline{u}(iT)\,[\cos(m-i)\omega_k T - \cos(m-i-1)\omega_k T] \tag{20}$$

In matrix notation, Eqs. (18) to (20) have the form

$$\underline{x}(mT) = \underline{r}(mT) + Q\bar{\underline{u}} \tag{21}$$

$$\dot{\underline{x}}(mT) = \dot{\underline{r}}(mT) + R\bar{\underline{u}} \tag{22}$$

$$\ddot{\underline{x}}(mT) = \ddot{\underline{r}}(mT) + S\bar{\underline{u}} \tag{23}$$

where Q, R, and S are dependent upon modal properties, V, and the discrete time interval chosen. With the use of these relations, the vector $\underline{h}$ as a function of control forces and time is given by

$$\underline{h} = (P_1 S + P_2 R + P_3 Q + P_4)\,\bar{\underline{u}} + P_5 \bar{\underline{f}}$$
$$+ P_1 \ddot{\underline{r}}(mT) + P_2 \dot{\underline{r}}(mT) + P_3 \underline{r}(mT) \tag{24}$$

Once response variables and $\underline{h}$ are expressed as functions of $\bar{\underline{u}}$, the performance index and all constraints may be written in terms of the vector $\underline{z}$ for each discrete instance in time.

Damped Systems

A similar analysis may be obtained for damped systems. When viscous damping is present, mode shapes $\underline{\psi}_k$ and eigenvalues λ_k may be found from the eigenvalue problem

$$(\lambda_k^2 M + \lambda_k C + K)\underline{\psi}_k = \underline{0} \qquad 1 \leq k \leq 2n \tag{25}$$

If mode shapes are dimensionalized such that

$$\underline{\psi}_k^T (2\lambda_k M + C)\underline{\psi}_k = 1 \tag{26}$$

then the solution to Eq. (1) is given by

$$\underline{x}(t) = \underline{r}(t) - \sum_{k=1}^{2s} \underline{\psi}_k e^{\lambda_k t} \int_0^t e^{-\lambda_k t} \underline{\psi}_k^T V\underline{u}(\tau)d\tau$$

$$-2 \sum_{k=2s+1}^{n+s} \int_0^t \mathrm{Re}(e^{\lambda_k(t-\tau)} \underline{\psi}_k \underline{\psi}_k^T) V\underline{u}(\tau)d\tau \tag{27}$$

where it is assumed that the first 2s eigenvalues are real and λ_{n+s+1}, ..., λ_{2n} are the complex conjugates of λ_{2s+1}, ..., λ_{n+s}. r(t) denotes the part of the response independent of the control forces.

If control forces are discretized in the same fashion as for the undamped case, the discretized response at time t=mT is obtained from Eq. (27)

$$\underline{x}(mT) = \underline{r}(mT) + \sum_{k=1}^{2s} \frac{e^{\lambda_k t}}{\lambda_k} \underline{\psi}_k \underline{\psi}_k^T V \sum_{i=0}^{m-1} \underline{u}(iT) \left[e^{-\lambda_k (i+1)T} - e^{-\lambda_k iT} \right]$$

$$+ 2 \sum_{k=2s+1}^{n+s} \mathrm{Re}\left\{ \frac{1}{\lambda_k} \underline{\psi}_k \underline{\psi}_k^T V \sum_{i=0}^{m-1} \underline{u}(iT) \left[e^{\lambda_k (m-(i+1))T} - e^{\lambda_k (m-i)T} \right] \right\} \quad (28)$$

Velocity and acceleration expressions may be found by differentiating Eq. (27) and discretizing the result. The remaining steps necessary to define the linear programming problem are not presented since they are identical to those followed for the undamped case.

Application

A three DOF system was treated to demonstrate the concepts presented in modal formulation.

3. Limiting-Performance/Minimum-Time Formulation [3]

A minimum time solution has been superimposed on the conventional limiting performance response to achieve the rapid suppression of the disturbances in the minimum time. Since the min-max norm of the limiting performance gives a unique solution only until the peak value of the performance index is achieved, an additional measure of performance is desired to obtain a unique solution after the peak value. Two different approaches were studied to accomplish the limiting-performance/minimum-time solution. One approach uses additional constraints and the other is based on the performance index. The two methods were applied to the control of a simple model of a contilever beam with a proof-mass damper and the two methods led to the identical minimum time solutions.

Problem Statement

A linear vibrating system with n degrees of freedom subject to arbitrary external excitations $\underline{f}(t)$ and control forces $\underline{u}(t)$ is expressed in the first order system of differential equations

$$\dot{\underline{s}}(t) = A\underline{s}(t) + B\underline{u}(t) + C\underline{f}(t) \tag{29}$$

where $\underline{s}(t)$ is an n-dimensional state vector, A, B, and C are n x n, n x nu and n x nf constant coefficient matrices. The quantities nu and nf are the number of control forces and excitations, respectively.

Constraints are imposed on the dynamic system under study. The format of the constraints is

$$\underline{y}_L \leq Q_1\underline{s} + Q_2\underline{u} + Q_3\underline{f} \leq \underline{y}_U \qquad \text{for } t_o \leq t \leq t_f \tag{30}$$

where $\underline{y}_L$ and $\underline{y}_U$ are nc-dimensional lower and upper constraint vectors, Q_1, Q_2 and Q_3 are nc x n, nc x nu and nc x nf constant coefficient matrices, and t_o and t_f are given initial and final time.

The problem is to find an optimal control $\underline{u}(t)$ which will transfer an initial state $\underline{s}(t_o) = \underline{s}_o$ to a desired final state $\underline{s}(t_f) = \underline{s}_f$ in the minimum time while extremizing a given performance index of the form

$$\text{Minimize} \quad J = \{ \max_{t_o \leq t \leq t_f} |\underline{p}_1^T\underline{s} + \underline{p}_2^T\underline{u} + \underline{p}_3^T\underline{f}| \} \tag{31}$$

where $\underline{p}_1$, $\underline{p}_2$ and $\underline{p}_3$ are given n, nu and nf constant coefficient vectors.

Formulation Using Constraints

In addition to the constraints in Eq. (30), an additional constraint set, referred to as steady-state constraints, is imposed on the time from the assumed minimum time (t_t) to the final time (t_f).

$$\underline{y}_{SL} \leq Q_{1S}\,\underline{s} + Q_{2S}\,\underline{u} + Q_{3S}\,\underline{f} \leq \underline{y}_{SU} \qquad \text{for } t_t \leq t \leq t_f \tag{32}$$

where $\underline{y}_{SL}$ and $\underline{y}_{SU}$ are ncs-dimensional coefficient vectors representing lower and upper bounds of steady-state constraints; Q_{1S}, Q_{2S}, and Q_{3S} are ncs x n, ncs x nu, and ncs x nf coefficient matrices. The steady-state constraints represent the desired final states of the system.

To place the optimization procedure into the standard linear programming form, the system in Eq. (29) is discretized using uniform time intervals to obtain a set of state difference equations

$$\underline{s}(k+1) = G\underline{s}(k) + H[B\underline{u}(k) + C\underline{f}(k)] \tag{33}$$

where

$\underline{s}(k)$ = state vector at time $t = t_k$

$\underline{u}(k)$, $\underline{f}(k)$ = control and external excitation vector at $t = t_k$, assumed to be constant over the interval $t_k \leq t < t_{k+1}$

$$G = e^{Ah}$$

$$H = \int_0^h e^{A(h-\tau)} d\tau$$

h = time interval = $t_{k+1} - t_k$

$k = 1, 2, \ldots, N-1$

The state vector, at any time $t = t_k$, can be expressed as a function of the initial state $\underline{s}(1)$, the control history $\underline{u}(1)$, $\underline{u}(2)$, ..., $\underline{u}(N-1)$ and the external excitation $\underline{f}(1)$, $\underline{f}(2)$, ..., $\underline{f}(N-1)$.

$$\underline{s}(k+1) = G^k \underline{s}(1) + \sum_{j=1}^{k-1} G^{k-j} H[B\underline{u}(j) + C\underline{f}(j)] + H[B\underline{u}(k) + C\underline{f}(k)] \tag{34}$$

$k = 1, 2, \ldots, N-1$

The constraints in Eq. (30) and (32) are discretized similarly

$$\underline{y}_L(k) \le Q_1\underline{s}(k) + Q_2\underline{u}(k) + Q_3\underline{f}(k) \le \underline{y}_U(k) \quad \text{for } k = 1,2,\ldots,N-1 \tag{35}$$

$$\underline{y}_{SL}(k) \le Q_{1S}\underline{s}(k) + Q_{2S}\underline{u}(k) + Q_{3S}\underline{f}(k) \le \underline{y}_{SU}(k) \tag{36}$$
$$\text{for } k = NT, NT+1, \ldots, N-1$$

where NT is discretized assumed minimum time.

The objective function of Eq. (31), which reflects the min-max norm, is discretized and converted into a constraint set. Since J is the maximum value of $|\underline{p}_1^T\underline{s} + \underline{p}_2^T\underline{u} + \underline{p}_3^T\underline{f}|$

$$|\underline{p}_1^T \underline{s}(k) + \underline{p}_2^T \underline{u}(k) + \underline{p}_3^T \underline{f}(k)| \leq J \tag{37}$$

or

$$\begin{aligned} -J + [\underline{p}_1^T \underline{s}(k) + \underline{p}_2^T \underline{u}(k) + \underline{p}_3^T \underline{f}(k)] &\leq 0 \\ -J - [\underline{p}_1^T \underline{s}(k) + \underline{p}_2^T \underline{u}(k) + \underline{p}_3^T \underline{f}(k)] &\leq 0 \end{aligned} \tag{38}$$

Define

$$\underline{z} = \begin{bmatrix} J \\ \underline{\underline{u}} \end{bmatrix} \tag{39}$$

where

$$\underline{\underline{u}} = [\underline{u}(1), \underline{u}(2), \ldots, \underline{u}(N - 1)]^T \tag{40}$$

and

$$\underline{c}^T = [1 \quad 0 \; \ldots . \; 0] \tag{41}$$

Then the linear programming problem is to minimize

$$J = \underline{c}^T \underline{z} \tag{42}$$

subject to the constraints

$$H\underline{z} \leq \underline{b} \tag{43}$$

where H and $\underline{b}$ represent constraints of Eqs. (35), (36), and (38).

While extremizing the given performance index, the linear programming routine will check the feasibility of the solution with the given initial states, external forces, and the two sets of constraints. The minimum time is the smallest time which will make the solution feasible with the given conditions. If the response behaves monotonically, i.e., increases or decreases without fluctuation, $t_f = t_t$ can be used. However, if the response varies sinusoidally a very careful selection of t_f is required. For the sinusoidal response, $t_f - t_t$ must be at least a half period of the response. The search for the smallest feasible time, t_{min}, can be done efficiently using the bisection method.

Formulation Using Performance Index

To achieve alternative limiting-performance/minimum-time solution, the performance index is given in Eq. (31) is modified. Two sets of performance indices are considered. One set of them, referred to as the transient performance index, is given by

$$J_t = \max_{t_o \leq t < t_t} |\underline{p}_1^T \underline{s} + \underline{p}_2^T \underline{u} + \underline{p}_3^T \underline{f}| \tag{44}$$

where t_t is the time limit for the transient period. The other set, referred to as the steady-state performance index is defined as

$$J_s = \max_{t_t \leq t < t_f} |\underline{p}_1^T \underline{s} + \underline{p}_2^T \underline{u} + \underline{p}_3^T \underline{f}| \tag{45}$$

Now, the "global" performance index is defined by

$$J = J_t + J_s \tag{46}$$

Note that the vectors $\underline{p}_1$, $\underline{p}_2$, and $\underline{p}_3$ are not changed in Eqs. ([illegible]) and (45).

Consider now the linear programming formulation. The objective functions of Eqs. (44) and (45) which reflect the min-max norm can be converted into a constant set as follows:

$$|\underline{p}_1^T \underline{s} + \underline{p}_2^T \underline{u} + \underline{p}_3^T \underline{f}| \leq J_t$$

and (47)

$$|\underline{p}_1^T \underline{s} + \underline{p}_2^T \underline{u} + \underline{p}_3^T \underline{f}| \leq J_s$$

To place this optimization problem into a standard linear programming form, define

$$\underline{z}' = \begin{Bmatrix} J_t \\ J_s \\ \underline{\underline{u}} \end{Bmatrix} \tag{48}$$

$$\text{where } \underline{\underline{u}} = [\underline{u}(1) \quad \underline{u}(2) \quad \ldots \quad \underline{u}(N-1)]^T \tag{49}$$

and

$$\underline{c}'^T = [1 \quad 1 \quad 0 \; \ldots . \; 0] \tag{50}$$

Then the linear programming problem is to minimize

$$J = \underline{c}'^T \underline{z}' \tag{51}$$

subject to the constraints

$$H'\underline{z}' \leq \underline{b}' \tag{52}$$

where H'and $\underline{b}'$represent constraints of Eqs. (30) and (47).

The minimum time (t_{min}) is the smallest time which will make the global performance index of Eq. (46) stay within a desired value. Since the performance index can be computed for each iteration, an interpolation method such as the secant method can be employed to find t_{min} efficiently.

4. Suboptimal Feedback Control for the Proof-Mass Actuator Using a Position-Loop [2, 3]

In addition to the various limiting performance approaches, the problem of the optimal design of the proof-mass dampers in a large structural system using the linear control laws and the optimal control theory was studied. Initially, linear control laws to control the proof-mass dampers were investigated. The results showed that, due to the constraints of the proof-mass dampers, although adequate damping could be achieved at high frequencies, very little damping could be obtained at frequencies of one Hz or less. While maintaining the adequate damping at higher frequencies, to improve damping at lower frequencies, both the limiting performance technique and optimal control technique were investigated and compared. The limiting-performance/minimum-time solutions were obtained for the control of a proof-mass damper attached to the end of a clamped-free beam which represented a small scale preliminary model of large flexible space structures and used as a measure of the success of the optimal control law. The system was composed of a proof-mass actuator, a servo-accelerometer attached to the free end of the beam and a proximeter to measure the position of the proof-mass along its track. Using the optimal control theory and the full state feedback, a control law which was simple but stable both in the linear and nonlinear region was obtained. To take the constraints into consideration, a position loop was added to the system and the constraints of rattlespace and control force of a proof-mass actuator were able to be treated by only limiting the desired relative position of the proof-mass. The resulting suboptimal control law showed adequately damped response at low frequencies which was very close to that of limiting-performance/ minimum-time control.

A Mathematical Model

A perfect model is rarely obtained for any physical process. One of the most prominent sources of modeling error in large flexible space structures is the deletion of modes in the formation of the design model. The low-frequency modes, which are more accurately known, are retained and the high-frequency modes are deleted. Assume a perfect actuator and consider only the first mode of the beam to be perfectly known, identified, and observed. The residual unknown modes are represented by the second mode. This mode usually has eigenfrequency that is not far from the closed-loop bandwidth and must be considered in the control system design. Modern control theory is used to obtain optimal first mode performance and a stable second mode. It is assumed that higher residual modes have eigenfrequncies above the main control system bandwidth. They can be damped by passive dampers and do not interfere with the low-frequency control law. Assume that the plant's variations in time are much slower than its characteristic time constant, so it can be treated as a time invariant system. A simplified model for the first two modes of the beam's free end is treated. The system is subjected to the following control force (u) and rattlespace (d) constraints

$$-u_{max} \leq u \leq u_{max} \tag{53}$$

$$-d_{max} \leq d \leq d_{max} \tag{54}$$

Suboptimal Control Considering Constraints

It is apparent from the results of the minimum time solution that the proof-mass relative motion along its finite length track is rather simple. This kind of motion can be implemented by using a direct position control. The system is expanded by considering a position loop added to the plant. The input to the system is the desired relative position of the proof-mass d_c, where d is its actual relative position. Introduction of a position loop into the plant and the proper choice of its gains make it possible to reduce the problem to a constrained input problem.

For the performance index

$$J = \phi + \int_{t_0}^{t_f} L(s,u)\, dt \tag{55}$$

the optimal control can be treated through the calculus of variations and Hamilton's principle. The Hamiltonian is defined by

$$H = L + \lambda^T \dot{\underline{s}} \tag{56}$$

where λ is the costate vector.

In the case of constrained input, it was shown by Pontryagin et al. that "the Hamiltonian must be minimized over all admissible control for optimal values of the state and costate". In the case under consideration, the Hamiltonian is a quadratic function of the control d_c. There will be only one location which satisfies

$$\frac{\partial H}{\partial d_c} = 0 \tag{57}$$

This point will be denoted as d_c^* .d derived from the solution of the Riccati equation . For constrained input, Pontryagin's Minimum Principle leads to the following solution

$$d_c = \begin{cases} d_{max} & \text{for} \quad d_c^* \geq d_{max} \\ d_c^* & \text{for} \quad |d_c^*| < d_{max} \\ -d_{max} & \text{for} \quad d_c^* \leq -d_{max} \end{cases} \tag{58}$$

Interest is in the steady state behavior of the process where the optimal feedback gain matrix is obtained from the solution of the Algebraic Riccati Equation to minimize the following quadratic performance index

$$J = 0.5 \int_0^{\infty} [\underline{s}^T Q \underline{s} + R^2 d_c] \, dt \tag{59}$$

The resulting linear position command is represented by

$$\overset{*}{d}_c = -(K_x x_1 + K_v \dot{x}_1 + K_d d + K_w \dot{d}) \tag{60}$$

5. <u>System Identification of Suboptimal Feedback Control Parameters Based on the Limiting Performance Characteristics</u>

Since the limiting performance gives the best possible or "limiting" response of a system, it would appear to be reasonable to base a control system on the limiting performance characteristics. However, due to uncertainties in control problems, open loop control such as the limiting performance control may not be applicable in practice, unless real-time computi... power for the limiting performance is available. To overcome this difficulty, parameter identification to find suboptimal feedback control laws based on the limiting performance characteristics is under study.

Consider a linear, time invariant dynamic system represented by a set of difference equations

$$\underline{s}(k + 1) = G\underline{s}(k) + HB\underline{u}(k) \tag{61}$$

From the limiting performance characteristics, the optimal time responses $\underline{s}^*(k)$ and $\underline{u}^*(k)$ are obtained.

Consider a linear controller,

$$\underline{u}(k) = K\underline{s}(k) \tag{62}$$

where K is an m x n feedback gain matrix.

Since m controllers are considered and optimal control forces are available for each controller, it is possible to proceed controller by controller. Also, the system is assumed to be subject to the constraints. For controller i, a suboptimal linear control law to be determined is described by

$$Us_i(k) = \begin{cases} Umax_i & \text{for } Us_i^*(k) > Umax_i \\ Us_i^*(k) & \text{for } |Us_i^*(k)| < Umax_i \\ Umin_i & \text{for } Us_i^*(k) < Umin_i \end{cases} \tag{63}$$

where

$Us_i(k)$ = suboptimal control force for controller i

$$Us_i^*(k) = \underline{k}_i\underline{s}^*(k) \tag{64}$$

$\underline{k}_i$ = i^{th} row of K matrix

It was assumed that controller i had constraint

$$Umin_i \leq U_i(k) \leq Umax_i \tag{65}$$

Define

$$re_i(k) = U_i^*(k) - Us_i(k) \tag{66}$$

where $U_i^*(k)$ is an optimal control force for controller i.

Then a constant feedback gain matrix K in Eq.(62) is selected to minimize

$$RE_i = \sum_{k=1}^{N-1} [re_i(k)]^2 \tag{67}$$

Once the optimal limiting performance characteristics are known, it is not necessary to perform a complete structural analysis for this identification procedure. Thus, this identification method handles problems efficiently.

It is expected that this pramether identification technique will give a stable feedback control law for a large structural system. To ensure stability the Routh-Hurwitz criterion is considered to be incorporated as constraints when a least squares fit is performed. An efficient computational scheme to achieve a stable feedback control based on the limiting performance characteristics is under study.

PUBLICATIONS

Robertson, B. P. and Pilkey, W. D. , "Limiting Performance for the Control of Large Vibrating Structures by a Modal Approach", International Journal of Analytical and Experimental Modal Analysis, 1987.

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