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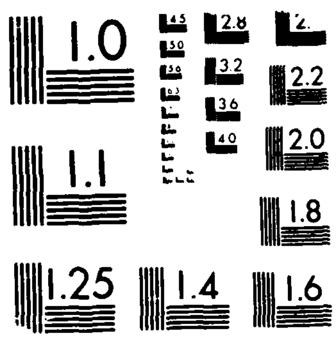

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AN INITIAL AMBIENT NOISE MODEL EVALUATION METHODOLOGY

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AN INITIAL AMBIENT
NOISE MODEL EVALUATION METHODOLOGY

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A. Introduction

I. Nature and Scope of Effort.

The Acoustic Model Evaluation Committee (AMEC) has been concerned with the evaluation of various acoustic models, the principal model types being transmission loss and ambient noise. Work on validation of transmission loss models has been done by the Panel On Sonar System Models (POSSM) which has run a number of well-known transmission loss models against the same set of environmental inputs (ref. e). Throughout this effort, a number of technical concerns have arisen regarding transmission loss model validation (refs. e and i). For ambient noise models, however, model validation seems to be restricted to either model development or the validation of a single model against a single type of environment (e.g., acoustic assessments). NORDA 320 has tasked the contractor "to assist in the formulation of an initial evaluation methodology for low frequency ambient noise models". In this report, we seek to establish a more general framework for ambient noise model evaluation.

This report addresses several considerations which will impact ambient noise evaluation, and provides an Initial Model Evaluation (IME) methodology for omni and horizontally directional ambient noise models. Inasmuch as ambient noise model evaluation for vertically directional models or temporal noise models involves both the criteria given in the IME and a number of other considerations unique to the models assumptions, it was considered better to address just the most widely used types (i.e. omni and horizontal directionality noise models) in this effort. Furthermore, consideration is restricted to the two major mechanisms of low-frequency noise: shipping and wind. Noise due to seismic, biologic and other meteorologic phenomena is assumed to be minor or negligiable at the frequencies under consideration. Since readers with different interests may read only those sections pertinent to them, there is some duplication of material between sections.

II. Fundamentals and Approaches.

If one were to do an ideal model evaluation, the model would be run on all types of inputs scenarios and its results would be compared with the "true" results for those sets of inputs. In ocean acoustics, this is blatently impossible. Environmental inputs are only known at certain points, and from there they are extrapolated throughout the ocean basin. Second, the physics of the model is a simplification of what we scientifically consider reality to be. A third source of difficulty is with the computer implementation of the model itself, which is a mathematical simplification of physics. Finally the output cannot be compared with any "truth", for all that is available is a set of acoustic measurements taken at a point, which are subject to a number of measurement and processing errors. Additionally, the number of possible cases necessary to thoroughly check this model would be phenomenal.

Out of this primeval chaos the acoustic modelers seeks to bring order. By judicious selection of cases, one may attempt to insure that the environmental and acoustical fields are well represented by the data, both qualitatively and quantitatively. Also, one may ask for less than perfect agreement. The comparison of model vs. data is often subjective, ranging from "It looks great" to "Oh no", thus a set of objective criteria must be structured. The extrapolation of model validity based on one or two acoustic sites to the entire ocean basin is a separate problem and this is often treated subjectively as well. Due to the lack of resources and desire to obtain measurements across the entire acoustic basin, such extrapolation is a necessary procedure, but it should be performed only when it can be reasonably justified. This is not to denigrate subjective analysis; it is a powerful tool when it incorporates the environmental and acoustic insights of an expert. In general, such learned subjective analysis is a prerequisite to a meaningful quantitative objective analysis.

If one is given a model, there are two broad types of approach to its evaluation. One may approach the model as a scientist, asking the question "How well does the model perform scientifically", implying both a valuation of performance in comparison with other models and a valuation of performance of the individual components. Or one may take the approach of the end user, and ask "How well does the model estimate its principal outputs" implying a comparison against the specific system requirements. This is not a dichotomy between pure science and applied engineering, rather it is a decision based on the use and possible uses of the model. A model such as FACT which is used by a number of organizations to compute transmission loss, and is a component of a number of large computer models, requires a scientific evaluation to determine the limits of performance for a wide variety of situations, thus the former approach is necessary. On the other hand, a model whose sole function is to serve in one specific system model, which is integrated into that model to the degree that it is untenable to remove it for future application, need not be evaluated in such a rigorous manner. If the overall system evaluation proves acceptable, under the full range of possible environment conditions for which the system is considered, than the individual components of the model may be considered to have performed acceptably, even though their performance by scientific standards may be rather poor. There are a number of instances in which peculiar types of model deficiencies in submodules tend to cancel themselves out, or have negligable effects, when incorporated in much larger models. For example, in a system performance model, both signal and noise are subject to transmission loss. Thus while a poor transmission loss module may lead to poor ambient noise predictions, the resulting SNR may be acceptable, due to certain types of errors canceling themselves out. Similarly in computing directional ambient noise the shipping field may be so evenly distributed that a range-averaging model (with little or no convergence zones) proves just as acceptable as a more precise model. These two approaches, of course, may be combined into a more flexible model evaluation methodology in which individual components such as transmission loss or ambient noise models are evaluated individually and then they are judged acceptable or unacceptable for the system model based on system requirements for accuracy of individual modules.

Evaluation obviously is based on the output of the model, such as components of transmission loss, transmission loss itself, and ambient noise. Thus the methodology developed will be dependent on the particular quantities produced. Modules which form the basic building blocks of larger models may produce outputs that are not easily identified with final system performance parameters, and thus whose influence may be difficult to determine on specific situations. Typical modules which produce only components of a measurable quantity are: transmission loss produced by normal mode calculations (assuming an infinite loss), bottom loss functions (as a submodule), or the vertical directionality patterns of the ship radiated noise level.

In evaluating a particular ambient noise model, one is evaluating sufficiency, accuracy, and stability of its specific outputs. Hence, depending upon the type of evaluation desired, one may work either from the final model output, or may demand an intermediate result from the individual modules. The types of outputs themselves differ considerably between the ambient noise models. Additional outputs needed for validation may include transmission loss curves, ship tracks, or decorrelation times. The plethora of possible outputs produced by some models tends to overshadow the outputs of primary importance.

One aspect of module evaluation that cannot be ignored is the performance of the computer system itself. As oceanographers and acousticians, we tend to judge performance on the ability to replicate various underwater acoustic or oceanographic phenomena.

However, uses of these models either as part of larger models, or by the fleet, often requires reasonable running times and hardware requirements. Computer parameters such as core requirements, execution time, ease of software modifications and adaptability to fleet computers do not properly play a part in the acoustic or oceanographic evaluation, but these considerations play an important role in comparing the computer performance of one model against another (refs. e and k).

III. Ambient Noise Models and Their Outputs

Although individual ambient noise models may differ significantly in their underlying science, order of computation, and methods of computation, all low frequency ambient noise models can be broken apart into component modules, such as those listed below:

- Ship distribution fields, either discrete or densities, along with allied ship acoustic information.
- The transmission loss module, which may be ray theory, normal mode theory, parabolic equation, or other type of model.
- Environmental field input module, containing wind speed, bathymetry, bottom loss, sound speed profiles, and any other environmental information needed to fully describe the ocean region for the transmission loss or ambient noise module.
- A module for beam patterns. This would convolute the ambient noise pattern with the beam pattern to produce the received beam noise.
- A summation module, which sums all noise sources according to their position.

Not all models have these functions broken out separately; indeed it is quite common for one computer subroutine to perform several functions. Rather, they have been broken apart in order to facilitate discussion and identify problem areas.

The shipping distribution module is essentially an input routine, which requires the ability, based on the geographic position of the receiver, to determine how much shipping information is needed and where the appropriate boundaries (either real, such as beaches, or acoustic, such as seamount chains) exist. Also, data on ship movement (if this is a temporal noise model), ship radiation pattern (which may be omni, dipole, horizontally directional, or vertically directional), and nature of the field (discrete or density) is needed. The transmission loss module will require a large number of environmental data items, and this is usually the principal data input.

The beam pattern module is normally quite simple, i.e., the beam patterns are specified either from a file or internally. But linear arrays are sometimes not linear and not all hydrophones continue to work at their original calibration setting, thus the beam pattern produced by an array in actual use may differ significantly from the theoretical beam pattern it was designed to have. Furthermore, differences in the acoustic intensity of both signal and noise along the length of a large aperture array may be significant, thereby completely altering the effect of signal processing based on the theoretical beam pattern. Most long arrays are designed on the assumption that all incoming acoustic energy is propagated in planar wave fronts of uniform intensity: measurement during exercises indicates that this is not always the case. Thus, the beam pattern module, while it may be simple, cannot be overlooked in terms of ambient noise model evaluation, for the real world may be incredibly complex.

The summation module is, fortunately, relatively straight-forward even for temporal noise models. However, the second order statistics of temporal noise models are highly sensitive to the tracks given for the discrete ships in the ocean area, and it may be difficult to separate problems in summation from problems with the tracks.

The type of ambient noise model varies according to application: Omni noise models are used for predicting performance of sonobuoy fields, horizontally directional models are used for arrays and fixed systems, vertically directional is necessary for vertical arrays, and temporal noise models find application to extremely-narrow-beam systems. Within temporal models, one may discriminate between analytic and replicative models, i.e., those which make the mathematics sufficiently simple to yield to analytic formulation and those which keep complexity but use multiple replications (ála Monte Carlo) to achieve statistical validity. Also with temporal models there are a number of types of output available: distribution, first moment statistics, second order statistics and specific probabilities (e.g., beam free time). The number of ambient noise models existing is extensive, hence no overview or synopsis of them will be attempted. It suffices to say that all models functionally fall into one of several categories: static models (omni, horizontal, vertical directionality), temporal analytic, or temporal replicative. Since the types of inputs for these models has been discussed above, we will proceed to the more significant problem of the types of outputs.

For convenience for the remainder of this report, we will define a number of terms relating to the outputs of the ambient noise models. Preliminary to this, we note that all these quantities are subject to the following general qualifications:

- noise level may be at fixed frequency or set of frequencies.
- acoustic levels may be either continuous wave or broadband, depending upon the transmission loss module and the ship spectra inputs.
- noise may be at a fixed point or set of points in the ocean region.

When making comparisons it is always necessary to consider these qualifiers and their value or values. In all equations given, the overscored quantity will represent the intensity of the level, and that without overscore represents the dB equivalent. For example, N(.) represent noise in dB while $\bar{N}(\cdot)$ the same level in intensity, so that

$$N(\cdot) = 10 \log \bar{N}(\cdot)$$

In regards to omni noise, we make the following symbolic definitions. Let:

N be the omni noise value produced from a stationary AN model

N(t) be the omni noise as a function of time $t \varepsilon (0,T)$ from a temporal AN model

F(x) be the distribution function of omni noise from a statistical AN model

M be the mean value of the distribution of omni noise from a statistical AN model

V be the variance of the distribution of omni noise from a statistical model

It should be noted that mean and variance are dependent upon the system of unit used, i.e., intensity or decibels. This depends upon the ambient noise model under consideration. For purposes of this discussion, it will be assumed that mean and variance are computed in intensity rather than dB. Use of the median rather than the mean eliminates this unfortunate ambiguity, for the median is equivalent whether calculations are performed in intensity or dB. While this is fine for first order statistics, it is not particularly helpful for the second order moments. In the case of a horizontally directional ambient noise model, the following symbolic definitions are made. Let:

N(θ) be the noise in the horizontal sector at azimuth θ. Since this quantity is dependent upon the sector width, let Δθ be the sector width in radians.

N(θ,t) be the directional noise at azimuth θ as a function of time, from a temporal model.

B(θ,t) be the noise measured on a beam pointed at angle θ (and, of course, its mirror image) as a function of time.

M(θ) the mean directional noise at azimuth θ, from a statistical AN model. As above, this may be in dB or intensity.

V(θ) the variance in the directional noise at azimuth θ, from a statistical AN model.

In the case of vertical directionality one may define the quantities

$$N(\phi),\ N(\phi,t),\ M(\phi),\ M(\phi,t), \Delta\phi$$

in a manner similar to the horizontal case above.

The temporal ambient noise models have quantities defined in a manner similar to the static case, but with a dependence upon time. However, those quantities of greatest interest in these models are not the noise quantities, but rather the beam noise statistics. Typical outputs for beam noise statistics are:

- the probability of the beam being free, or below a preset threshold. In this case the beam "sees" no ships, or if a threshold is used a small number of distant ships.
- the mean beam free period; i.e., the expected value of the interval in which the beam contains no ships.
- the mean time between beam free periods; i.e., the expected value of the intervals during which the beam contains ships.

- the distribution function of the beam free time, i.e., the one dimension distribution function of these time intervals.
- the auto correlation of beam noise, i.e., the correlation of noise upon a fixed beam at different times.
- the cross correlation between beams, that is the second order moment relating to the noise on two distinct beams at the same or different times.

Of the quantities defined above, there are several interrelationships. It is assumed that the ambient noise models under consideration take into account these fundamental relationships, hence it is generally unnecessary to test in order to insure that they hold. First we consider the relationship between the directional noise and the omni noise.

$$\overline{N} = \frac{1}{2\pi\Delta\theta} \int_{0}^{2\pi} \overline{N}(\theta)\, d\theta = \frac{1}{\pi\Delta\phi} \int_{-\pi/2}^{\pi/2} \overline{N}(\phi)\, d\phi$$

In order to relate the horizontal directional noise and the vertical directional noise fields, it is necessary to know the two dimension (spherical) noise field. Let $N(\theta,\phi)$ be the spherical noise field in intensity per steradian. Than the relations between these directionality functions is given by

$$\overline{N}(\theta) = \frac{\Delta\theta}{\pi} \int_{-\pi/2}^{\pi/2} \cos\phi \quad \overline{N}(\theta,\phi) \quad d\phi$$

$$\overline{N}(\phi) = \frac{\Delta\phi}{2\pi} \int_{0}^{2\pi} \cos\phi \quad \overline{N}(\theta,\phi) \quad d\theta$$

For temporal models there are further relationships between the data. As the time interval over which the measurements are made gets large, the averages of the temporal and statistical quantities should approach those outputs of the stationary model. This is based on the Ergodic Theorem, which is generally used in temporal models, although its underlying assumptions are rarely, if ever, fulfilled. Thus in the limit the temporal noise, both omni and directional should average out to the values calculated in the static case.

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{N}(t)dt = \bar{N}$$

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{N}(\theta,t)dt = \bar{N}(\theta)$$

$$\lim_{T\to\infty} \int_{-\infty}^{\infty} x \quad F(dx) = \bar{N}$$

where in the last equation, the distribution function F is an implicit function of T. Again this does not provide a method for checking models, for the models generally use these relationships in the calculation.

The final product of ambient noise model is often not the noise rose itself, but the noise as seen on the various beams of a surveillance system. Conversely acoustical measurements are not of the noise field per se, but of the noise seen on the beam of the array.

Measured noise fields are generally obtained by deconvolution of the measured beam noise. A number of the tests to be considered for the horizontal directionality of the noise roses apply equally well to the horizontal beam noise pattern. However, there are several concerns in regard to making comparisons based on the beam noise outputs as opposed to the noise rose itself.

First the beam patterns in most towed array or fixed system models assume that the particular beams are looking out in vertical planes at specified angles from the axis of the array. In reality the main response axis of these beam patterns lie in cones rather than vertical planes. Hence discrepancies in angle can be introduced and ships that the model places on one beam may actually be seen on a different beam due to this geometry. For RSR ray paths the arrival angle are generally within 30° of the horizontal, thus the effect of this conical pattern is minimal except for extremely narrow beam systems. For bottom systems however, the acoustic energy may have hit the bottom in the locality of the system, changing the ray paths and resulting in a greater angle of arrival. Similarly for all linear arrays, nearby targets can have bottom bounce propagation which could arrive at angles differing significantly from the horizontal. In light of these concerns, the beam noise patterns should be used with care in regards to end-fire and near end-fire beams. This conical main response axis pattern is significant if the end-fire beam is looking almost parallel to a shipping lane. It should also be pointed out that in the transformation from horizontal directional noise roses to beam noise patterns, some information is lost (the amount of information being dependent upon the axial symmetry of the linear array and the number of tows at different directions made with the array). Thus agreement between measured beam noise patterns and model beam noises level does not imply that there is a corresponding agreement between the actual noise roses and the model noise rose.

IV. Perturbations

Even when one is dealing with a static ambient noise model, the perturbations in the environment are significant. The static model takes a fixed shipping distribution, a fixed environment and fixed source levels, but even if the model were perfect, the result need not correspond to the acoustic measurements made during an exercise. There are a number of fluctuation mechanisms which can affect ambient noise:

- fluctuations in the sound speed profile due to weather patterns, mixing, currents, turbulence, internal waves or diurnal phenomena.
- changes in source and receiver positions due to local ship or array motions, doppler effects, changing aspect angles and movements in or out of the ocean area.
- movement of both target and background noise sources across the main response axis of the various beams and their side lobes.
- movement of a towed array away from its theoretical straight-line, level geometry.
- short-term variations in wind generated noise.
- changes in transmission loss for narrowband signals due to coherence pattern of multipath arrivals. As the various noise sources change position relative to the array, their coherence pattern in a multipath environment may quickly alter.

The time scale for these mechanisms vary significantly, and for any particular mechanism there is often a wide range of scale. Some of these, such as nearby ships crossing the main response axis of a beam, or fluctuation in transmission loss due to internal waves, turbulance or microstructure phenomena may be on the order of minutes. Others, such as diurnal phenomena have a period of a day, and weather phenomena may have periods of several days.

The effects of these different mechanisms are usually all lumped together into some sort of acoustic fluctuation, which is often described as a Gaussian process with zero mean and some standard deviation. This process is then assumed to quickly decorrelate itself, so that for many models the environment is assumed to be equivalent to a process utilizing several independent snapshots. As noted above, the time scales of these fluctuations vary from minutes to days, thus it is not possible to describe this in terms of a set of independent looks. For if one assumes a short time interval between the "snapshots", there will be a correlation (for the long term fluctuations are still slowly occurring). If one assumes a long time interval is necessary before independence may be achieved, then one is losing the radical effects of the short term fluctuation, for they would be averaged out or missed completely. While a static ambient noise model will give some sort of estimate of the mean ambient noise field, it is difficult to determine the second order statistics (e.g., variance and autocorrelation) from such a model. More relevant to the problem of model validation, however, is the effect of using measured data in such a static model. For the measured data itself is often obtained using integration over a period in order to smooth out small fluctuations. If acoustic measurements are integrated over too long of a time period, effects such as ships moving across the main response axis of the beam will be integrated out. Such loss of resolution of acoustic effects can only have a deleterious effect on a static ambient noise model. On the other hand, if very small integration times are used for acoustic data, the acoustic level cannot be established with as much precision.

During exercises acoustic measurements may have integration times from a few seconds to a significant fraction of a minute. When dealing with data collected by operational systems, integration time may be on the order of minutes. Thus there is a wide spread in integration times associated with measured data.

When computing the variance of measured ambient noise data after the fact, e.g., in a model evaluation rather than a reduction of the raw data, one may compute the variance of the measured noise values in a time period. Unfortunately, if the decorrelation times of the acoustic field do not compare well with the integration time used in data processing, than the estimate of variance may be significantly in error. Appendix A deals with this method of computation of variance, and the effects of a mismatch. The model given in Appendix A may be used to determine the validity of second order moments calculated in this manner and the effect of their use in ambient noise model evaluation.

B. Considerations and Comparisons

Before getting into the specific comparison methodology it is wise to state a few elementary assumptions. In the comparisons of model vs model or model vs data, it will be assumed that one is comparing equivalent data, thus individual comparisons require that both sets of noise values be for the same depth, same environment, and (in the case of horizontal directionality), the same angular grid. Interpolation of noise fields to allow comparisons between data at different depths or other conditions is possible. In light of the uncertainty of ambient noise model evaluation, however, it is far superior to not attempt such interpolation. Since we are not comparing data vs data, but a model against data or another model, the model can be re-run at the correct depth, environment, etc., to match the input parameters of the historical data. Thus all such interpolation of noise fields may be eliminated from consideration. Furthermore, if the deconvolved noise field is available, it is strongly suggested that this be used in the evaluation. The beam noise patterns may be used in addition to these, and would certainly be used when the deconvolved noise field data was questionable. Thus in the following test, although we may speak of just the noise rose, it should be envisioned that the test is applicable to both the directional noise field itself and the beam noise values. In making comparisons of ambient noise models one must carefully distinguish between stationary models (i.e., those which predict noise field independent of time) and temporal models (i.e., those which give time histories such as statistical beam noise models).

In evaluating the ambient noise model it is wise to consider the transmission loss as a module, thereby separating the evaluation of the transmission loss component from that of the ambient noise calculation itself. The overall accuracy, however, depends upon both components. It is preferable that the transmission loss module used in the ambient noise model be one which has already been subjected to a thorough validation. If the transmission loss module as a stand-alone model has not been through such an evaluation, than one must attempt to validate both the transmission loss and the remainder of the ambient noise model at the same time. This can be very difficult given the extreme variation in the environmental condition and resultant transmission loss properties. In the long run, it is much easier to perform a separate validation of the transmission loss model against a large number of realistic environmental inputs and examine the resultant calculations of this module separately before attempting to evaluate the ambient noise model as a whole.

For a stationary ambient noise model, one may compare the model with itself (using the laws of physics), with other models (using various criteria to judge which model performs adequately in various scenarios) and with measured data. Of these three types of validations, none are absolute in the sense of providing a perfect test. Both the model and that which it is being compared against differ from "truth", thus any sort of test has a certain level of uncertainty. By judicious use of different models, different inputs and measured data, it is possible to design a comprehensive methodology for stationary ambient noise model evaluation.

When one is dealing with temporal ambient noise models, however, there are additional considerations. One now deals with either distributions and their statistics (such as first or second moment of beam noise), or levels (such as the probability of the beam being free, the beam free time, mean time to be free). From SAI's review of temporal beam noise models (ref. k) it is seen that the models differ greatly in regards to underlying assumptions, applicable time scales and types of outputs. Thus it is rather difficult to give a general approach to model evaluation of temporal ambient noise models. Specific temporal beam noise model evaluations have been done however (e.g., ref. h).

V. Comparison of Model Against Itself

The starting point of any model evaluation should be the problems of model stability and model consistency. In this, one begins with basic concerns: items which should have been checked during model development. For a horizontal directionality noise model, the omni level should be the sum of the horizontal directional noise levels. The ambient noise model should work on elementary test cases, such as isovelocity and isogradient profiles, flat bottom, uniform ship distribution within the oceans basin, and infinite bottom loss. The transmission loss module should have reasonable accuracy, i.e., it should have been subjected to some test before the incorporation in the ambient noise model. Obviously, without valid results on tests such as these, it would make no sense to talk about performing an ambient noise model evaluation.

The first significant step in the model evaluation is the elementary tests. If the transmission loss is changed by a constant amount, then the ambient noise should reflect this difference. Similarly, if the radiated noise level of all ships is increased by a constant factor (i.e., XdB), than the output should also be changed by XdB (except for the wind generated components of the noise). The shipping field used, whether discrete or continuous, may be increased by constant factor (X%). Again the ambient noise output should reflect this increase. While it is not anticipated that these elementary tests will detect any problems in the models, they form a good base for initial evaluation. For the solutions may be computed exactly and they give the model evaluator familiarity with data preparation and model operation on the particular computer involved.

The next level of tests to be performed deals with exact solutions. A particularly vexing question for acoustic model evaluation is the role that "truth" must play. As pointed out in the POSSM report (ref. e) there are a number of basic problems in attempting to compare transmission loss models vs.

the "truth" of the real world. Since transmission loss is the most complex component of ambient noise model, the same considerations there apply to this model evaluation. The physics of the model i.e., the wave equation or ray acoustics, is not a correct representation of reality, however, close it may be . Second, experimental data is incomplete: one does not measure everything. For example the noise level for each particular ship in an ocean basin is not known, instead a class spectra is generally used. Phenomena such as ice noise, internal waves and turbulence are generally ignored. As Keller has pointed out (ref. d), the various mathematical formulation of acoustics (wave theory, ray theory, asymptotic expansions) all fail to reflect reality in certain instances so that special extensions to these theories are then necessary. Also the use of a CW model based on fixed frequency calculations in modeling broadband system performance is a consideration. As Lauer has noted (ref. e) use of incoherent information at a fixed frequency has not been proven equivalent to broadband acoustic propagation . Despite all these drawbacks, measured data must still serve as a backbone for noise model evaluation and indeed any acoustic model evaluation. Exact solutions of all problems may be used as supplemental checks and as basic checks or to investigate phenomena believed due to the physical basis of a model. Exact solutions however, have a tendency to be somewhat simplistic when compared with the oceanic environment. Nevertheless for questions of model sensitivity and computational stability these exact solutions are often adequate to determine the models performance. For ray trace models there are exact solutions which may be perturbed in special ways, but always leading to analytical results (refs. a, m & 1). For normal mode models sensitivity and stability may be investigated by probablistic methods (refs f, c and b). Thus, the ambient noise model under consideration should be exercised for some exact solution cases.

The cases mentioned in the elementary tests above should certainly be passed by any models; the exact solution tests used here would be more complex. Unless one is dealing with a strictly shallow-water model, ray theory may be used for the test case, realizing that ray theory has drawbacks at places such as caustics (cf. ref g). It is suggested that a flat bottom, parabolic profile environment with receiver on the sound channel axis be utilized as a test case. This case has the advantage that the profile is reasonably realistic, the ray paths are trivial to calculate, and the transmission loss along any ray path is reasonably tractable. An exact solution ambient noise test case is developed in Appendix B. A consideration in the testing of exact solutions, of course, is the concept of the spherical earth. While most transmission loss models assume a flat earth for the radial run, when an ambient noise model is exercised over a large area (such as the South Pacific Ocean), there is a significant difference between the area of a radial sector computed on a spherical earth and that of a flat earth. If the ship distribution is given in terms of density rather than discrete ships, this in turn affects the ambient noise received in that radial sector. For distances under a few thousand kilometers, this effect is negligable, but the world is round, thus one might insist that the exact solutions used reflect this spherical earth. The transmission loss modules used in most ambient noise models do not have a spherical earth correction in them. The exact solutions (in Appendix B), are based on a flat earth, for the purpose of an exact solution is to test how well the computer model reflects its input physical assumptions, not how well those assumptions reflect the real world. Purists may further argue about running the transmission loss from the receiver to various ranges, for the transmission loss is not quite reflexive but it is influenced by factor which is the square of the ratio of the sound velocity at the source to the sound at the receiver. As the ratio of these two quantities is very close to one, and ambient noise model results are rarely used past the numeric significance of 1/10dB, this consideration may safely be ignored.

In any comparison of model vs either itself, data, or another model, it is necessary to consider the sensitivity of the items being compared. In particular, one should understand the sensitivities prior to comparisons. For ambient noise model, it is wise to perform a perturbation analysis. As noted above, there are a number of inputs to an ambient noise model, hence a number of these parameters (those which are not known precisely) may be perturbed. However, we concentrate on just two types of perturbation: environmental inputs to the transmission loss module and shipping signatures/distribution.

In regard to the environmental inputs, the theory of geometrical acoustics is extremely sensitive to small perturbations, e.g., potential errors in measurements or changes in the environment over time. Similarly normal mode models can show extreme sensitivity in terms of small scale (e.g., coherence) phenomena. In order to perform a scientific model evaluation, it is extremely important when describing the acoustic field and its associate characteristics to give not only a mean value of the quantities of interest, but also to provide the statistics of those quantities. Since geometric acoustics is used in a number of transmission loss modules, one cannot gloss over the sensitivities of these models. Geometrical acoustics may be described by a series of non-linear differential equations and due to this non-linear form, standard perturbation methods for obtaining statistics do not work. That is, a small change in the environmental field may cause a large change in the acoustic field. With such a set of differential equations, movable singularities arise, hence theoretical bounds are generally difficult, and never global. Although geometrical acoustics is an elementary application of asymptotic expansion methods, when these expansions are carried through to investigate the statistical properties, divergence appears.

Another variation is to examine geometrical acoustics as a perturbation of a two point boundary value problem (the source and receiver being the two points). Unfortunately, due to the existance of caustics (which may be thought of as the locus of places where the Riemann surface of the wave function bends back upon itself) such boundary value perturbation methods will not work. Methods for perturbation of wave theory solutions, while more stable, tend to be very difficult (refs. f and c). In terms of the complexity of the real world, the state of our acoustic models, and the requirement for reasonable computer run time, it would appear that the best way to perform a perturbation analysis is to simply generate a number of perturbed environments (i.e., sound speed fields) and exercise the transmission loss module on these, noting the sensitivity by elementary statistics (ref. m). It should be noted that these perturbations should be such that not only is the difference in the sound speed profile in any given depth small, but the differences of both the first and second derivatives of the profile at any given depth are also small. This implies that microstructure, turbulence and mixing cannot be accurately modeled by this method, for their resulting statistics would be invalid.

The other consideration of sensitivity is the ships themselves. Shipping in an ocean can be described by a stochastic process. This process is such that, if an area in the ocean satisfies four assumptions, then the number of ships in that area is a random variable whose distribution may be approximated by a Poisson law. The four assumptions are:

- the area is small in comparison with the entire ocean
- the area is sufficiently far away from major ports that ships enter and leave the area independently (that is, the ships are not affected by docking schedules or by harbor waits)

- the area does not have a "weird" shape (i.e., it can be thought of as the union of a finite or countably infinite number of either circles or rectangles). This eliminates unusual types of sets which occur in measure theory. Any area which is of practial use will satisfy this contraint.
- the area is large enough so that if two ships are in it, their movement will not be dependent upon one another (i.e., both ships can exist in the area without taking evasive action to avoid imminent collision).

A Poisson distribution has the property that the mean is equal to the variance. Hence it is seen that for most open ocean areas the number of ships contained therein will fluctuate significantly with time. Also, the characteristics of these ships will change. It is possible to model the ship population as a sum of the different classes of ships, where each class has a Poisson law with some particular parameter for that area, and the distribution of distinct classes are independent. Now one is dealing with a (still Poisson) distribution of ships, but with different radiated levels for those ships. Numerous groups have measured ships spectra levels, and obtained differing results. Of particular significance are measurements made by NAVOCEANO on two sister ships (ref. j) which turned out to have significantly different spectra. Thus there can be significant changes in the horizontal directional noise rose due to such ship movement. Fortunately, for very large areas (e.g., the North East Pacific Ocean) the number of ships is so large that there are no "holes" in the directional noise pattern and, for reasonably wide beams, there are a reasonable number of ships on the beam. This implies a certain stability to the statistics which are lacking for low ship densitites.

The final type of test considered is that of field analysis. Given a point ambient noise model (i.e., a model that computes the noise at a particular point) one may run the model on a set of close points and intuitively one would expect the ambient noise to be 'close'. Specifically, if one assumes that ambient noise is computed at nine points (as in Figure 1), the bathymetry is flat in the region of these points, and there is no local shipping within the grid, then one should expect that the ambient noise at the middle point should not vary greatly from the values at the eight exterior points. One can construct cases where the ambient noise at the center is vastly different from that at the other points, e.g., by concentrating the shipping distribution at caustics or by changing the Sound Speed Profiles rapidly with range, but under 'normal' conditions there should be a relation. Appendix C develops bounds for the directional noise at the center point based on the bounded variation of shipping density, bounded variation in transmission loss, and change in environment. For various cases, these inequalities yield computable bounds which may then be checked against the model runs. This procedure may also be applied to field ambient noise models (which compute noise at many points), but one would not expect it to be a very significant test in this case.

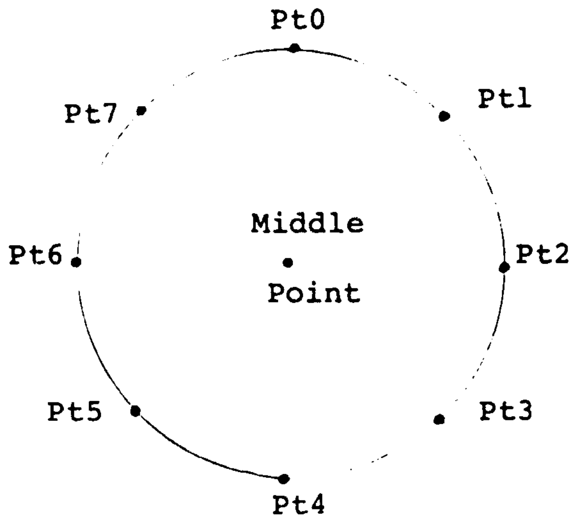


Figure 1 A Configuration for a Nine-Point

Ambient Noise Test

VI. Comparison of Model vs. Data

A major consideration in the comparison of a model against measured data is the credibility and applicability of the data. The first concern is to insure that all inputs are made for the same time and area. It will not be possible in all cases to have all data meet these specifications: in such cases it will generally suffice to insist that important inputs be at the same time and area and other inputs taken under similar environmental conditions. This entails a further test to verify similarities for data from different times (e.g., same season but different year) or different areas (e.g., similar ocean basins). One must be aware of the effect of using historical sound speed profiles over a broad ocean basin to drive the ambient noise model. For profiles measured at the time of the acoustic measurement collection will generally be available only at a few sites in the ocean basin. It is impossible during an acoustical exercise to obtain profiles for an entire ocean basin. Profiles along a radial run will generally be available, and these may be used for transmission loss perturbation runs (i.e., using both measured and historical profiles along that track) in order to investigate sensitivity to the sound speed profile measurements. A second measurement consideration deals with the shipping distribution. Shipping distributions during the exercises are obtained primarily using aircraft, and several hours are needed to measure the field. During these hours, however, the ships themselves may move a nontrivial distance, so that the horizontal noise field may be altered somewhat. Exercises in which ship course and speed were obtained as well as position alleviate this. Exercises in which the ocean basin could not be covered by aircraft for ship surveillance have the additional uncertainty of using historical ship distributions rather than the particular distribution of shipping near the time of the acoustic measurement. Over the course of an acoustical exercise, which may last several weeks, transmission loss may change dramatically due to a wide range of environmental anomalies: a transmission loss measurement made at the beginning of the exercise may not reflect the detailed structure present at the close of the exercise. Another general consideration before detailed comparisons are made is the nature of the acoustic measurement parameters. Parameters such as type of signal (CW or broadband), processing details, signal to noise ratio, and calibration accuracy allow one to perform a more intelligent assessment of the collected data. While such parameters are well known at the time of the exercise, over a period of years the exact details tend to be known only by those responsible for the data reduction and are buried in obscure Data Analysis plans. Effects of integration time of both the measurements and the environmental processes should also be considered at this point.

A general approach to model vs. data comparison is to first verify that the measured data is sufficiently stable to warrant comparison with models: If the data doesn't agree with itself, one can't expect it to agree with the model. Assuming self consistancy and stability of the data, then comparisons with omni measurements should be made, followed by directionality measurements. If the omni levels show serious disagreement (more than would be due to uncertainties in ship radiated noise level), it is questionable whether directional noise comparisons would be meaningful.

In regard to verifying the stationarity of omni level acoustic measurements, the basic statistics considered are the mean and variance of N(t). Examination of the variance of this process is, of course, dependent upon the integration time used in the measurement (see Appendix A). It is also valuable to produce a trace of measured noise with time: this may allow one to determine by "eyeball analysis" whether N(t) appears to be a random fluctuation about a level line or instead a fluctuation about some other sort of curve (see Figure 2). If the fluctuations are about a level line, and the variance of the measured noise is

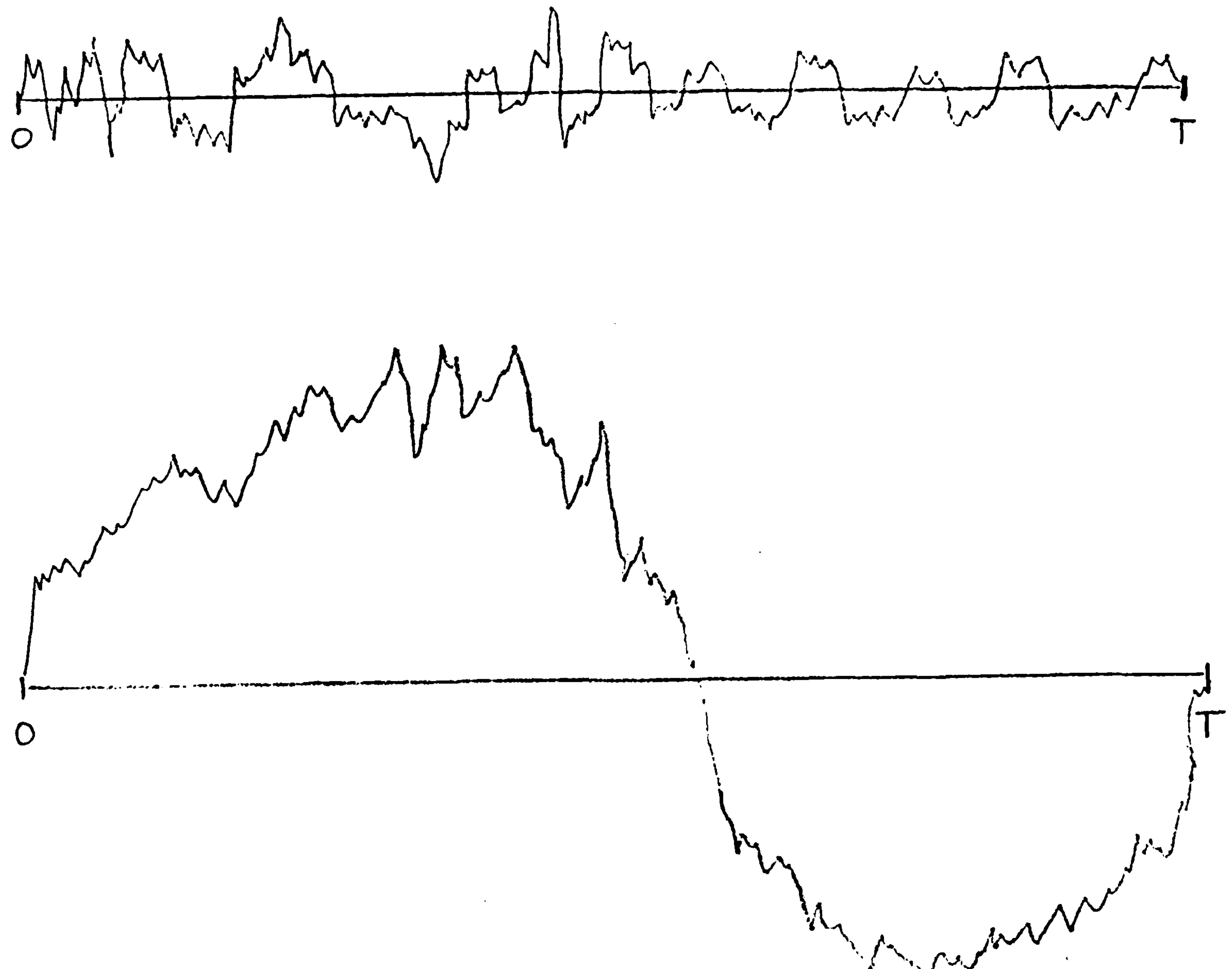


Figure 2. Time Series of Noise: a) following a level line, b) following a curve

sufficiently small, than a comparison with the model should be made. If one has confidence in the omni comparison, that is, the results either agree or the disagreement is due to identifiable casues, than directionality may be considered.

To test stationarity of the horizontally directional noise measurement, using the continuous data $N(\theta,t)$ or its discretized equivalent, compute the statistics:

$$\mu(\theta) = \frac{1}{T}\int_0^T \bar{N}(\theta,t)\,dt$$

$$\sigma^2(\theta) = \frac{1}{T}\int_0^T (\bar{N}(\theta,t)-\mu(\theta))^2\,dt$$

These, the mean and variance of the data for a particular direction, may be handled in the same manner as the omni level statistics. However, suppose that rather than continuous (or near continuous) measurements, one has discrete noise measurements separated by periods of several hours. If there are two to four such measurements, a test for stationarity is to take the correlations of the noise roses pairwise. This may be done by defining:

$$p_i(\theta) = \bar{N}(\theta,t_i) - \bar{N}(t_i)$$

then defining

$$\rho_{ij}^2 = \frac{\left|\int_0^{2\pi} p_i(\theta)\,p_j(\theta)\,d\theta\right|^2}{\int_0^{2\pi} p_i^2(\theta)\,d\theta \int_0^{2\pi} p_j^2(\theta)\,d\theta}$$

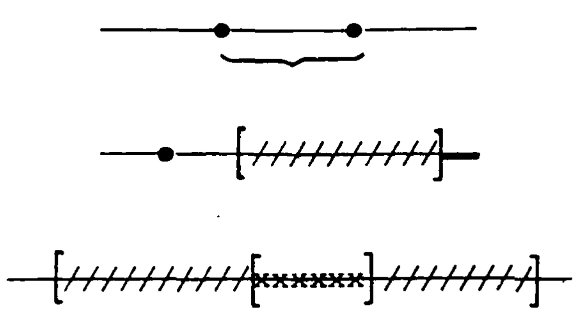
These give the correlations of noise over angle between discrete times. Then comparison of these correlation coefficients will determine if the data is stationary. If all correlations are high, than stationarity may be assumed. If some correlations are high (i.e., close to one) while others are low (i.e., close to zero) then one must determine if a certain set or sets of measurements have been subject to other environmental or acoustic influences, or whether stationarity simply does not exist. If there are more than four such discrete time measurements the above method requires a selection algorithm to process a large number of correlation pairs. Algorithms to construct maximum consistant subsets of data are available, however, it does not seem necessary to consider these until the occasion arises. An alternative is to consider the discrete measurements as a continuous measurement set, which implies that the noise field between the measurement is assumed to have not fluctuated significantly.

A consideration in stationarity of horizontal ambient noise measurements is the stability of the near field shipping distribution. If in the course of the acoustical measurements, a ship or ships has transited near the acoustic array, one would not expect the horizontally directional ambient noise field to be stationary with time. It is possible to construct a test to examine the princple transient noise components of a field (i.e., a component apparently due to a single noise source moving on a straight line track across the field), and to determine if this is a significant component of the total. If it is, this component may be removed; however, such removal may affect the integrity of the remaining data. Details of this test are given in Appendix D.

If one is dealing with a model for vertical directionality, then the comparison data must be subjected to stationarity tests as before. Whereas under uniform distribution of ships, transmission loss, and environment, the horizontal noise rose tends to a circular pattern (i.e., a constant level) the vertical pattern does not. In a completely omni environment, there will be a significant directionality pattern to vertical noise, due to the arrival structure at the receiver of the ray paths from surface shipping. Using ray theory, a narrow cone may be constructed in which ray paths from the surface will arrive at a receiver. While ray theory has several drawbacks, nevertheless this does imply a significant directionality pattern. The nature of acoustic waves, defraction in the ocean, surface scattering and defraction, effects of the slope bottoms, range dependent profiles, and bottom bounce propagation will tend to fill this "noise notch" up to some level, but the resultant pattern will still not be uniform. Thus even if ship distribution, sound speed profiles, bathymetry, etc, are completely wrong, the qualitative shape of the vertical directionality noise pattern which the model produces may be similar to the pattern produced by the model with the correct input. Therefore, tests based on mean, variance, and correlation such as those used above for horizontal stationarity are inapplicable to the vertical case. Quick analysis of vertical directionality and beam widths (Appendix E) suggest that there is no simple method for examination of such stationarity. Agreements between measurements does not indicate agreement of underlying environmental parameters.

The methodology for comparison will now be addressed. When dealing with point estimators, such as omni noise values, there are only two numbers: the model estimate and the measured estimate. Doing a point-to-point comparison, one is forced to rely on the difference of these numbers. This difference is then compared either with an absolute error tolerance, or a varying (dependent upon ocean environment, data stability, etc.) error estimate. In either case, one is actually comparing the distributions of errors which are represented by the point estimators.

If one has a number of data measurements, so that the data may be described by an envelope (or, better, a distribution function) then one may compare the model's point estimate against this envelope (or distribution), yielding point-to-envelope comparisons. The most elementary such comparison is to ask whether the point lies inside the interval or outside (Figure 3). Unfortunately, this test is by no means ideal: indeed with increasing volume of data, it is generally counterproductive. As the amount of data available for construction of the envelope increases, the underlying distribution of that envelope becomes better known (in the statistical sense), but the larger sample size often implies that the interval needed to contain the sampled points grows as well. This is a property of distributions with tails. If the underlying distribution can be described in terms of statistics, such as mean and standard deviation, then one can describe the distribution rather than the interval. In such a case methods of hypothesis testing may be used to determine (at a preset confidence level), whether the hypothesis of ascribing that point to the given distribution is acceptable. Standard hypothesis testing methodology makes the assumption of underlying normality for the distribution, which is often invalid for ambient noise acoustic levels, but in view of the uncertainty of the sample mean and standard deviation estimators in most situations of interest, the assumption of normality is a minor detail (unless one is working at an extreme value for the confidence level). The measured data may vary, and by perturbing the environmental and shipping inputs to the model, the model outputs will vary, thus it is possible to consider the comparison as being of envelope-to-envelope (or distribution to distribution). Again , if the underlying distributions are unknown, one is left with comparing only the envelopes, i.e., the amount of overlap (Figure 3), which is generally unsatisfactory for reasons given above. Thus if the underlying distributions can be estimated to any reasonable degree, it is far better to make the comparison based on hypothesis testing methodologies. Making the blatent



Point-to-point comparison

Point-to-envelope comparison

Envelope-to envelope comparison

Figure 3 Scaler Comparison Methodologies assumption that both distributions are Gaussian, one may than apply a simple statistical test to determine (at a preset confidence level) whether both sample distributions are representative of the same underlying distribution. Mathematical details of comparisons methodologies will be found in Appendix F.

The test becomes more complex when the outputs to be compared are not point estimates, but functions, such as horizontal ambient noise roses or beam noise levels. Since all such data is discretized, one may apply the above mentioned point tests to individual pairs, and then form some sort of overall estimation. This approach, however, is not ideal. Much is lost in terms of the order, shape, and correlation between levels in various directions. It is possible for example, to be comparing a single curve with an envelope of curves and have undesirable results. Consider a set of curves (either measured data or model outputs from perturbed environments) for directional ambient noise as in Figure 4. Note that all noise roses in this ensemble have a bulge in the Northeast direction. Compare this to a curve of uniform level fitting within the envelope bounds (Figure 5). Then this curve is completely contained within the envelope, but it does not acceptably represent the distribution, for it lacks the Northeastern bulge characteristic of all samples. Thus elementary tests are not always applicable. Mathematical methodologies for comparison of functions and distributions of functions will be found in Appendix F.

For these comparisons, whether they be point-to-point, point-to-envelope, or envelope-to-envelope, the following order is suggested:

- Omni noise values: N
- Horizontal directionality: $N(\theta)$
- Vertical directionality: $N(\phi)$
- Omni temporal noise: $N(t)$
- Horizontal temporal noise: $N(\theta,t)$
- Vertical temporal noise: $N(\phi,t)$
- Beam noise value: $B(\theta,t)$, $B(\phi.t)$
- Statistics

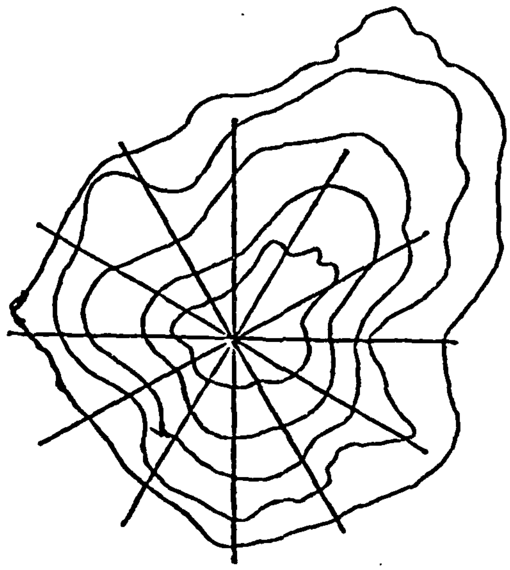


FIGURE 4 Ensemble of Ambient Noise Roses

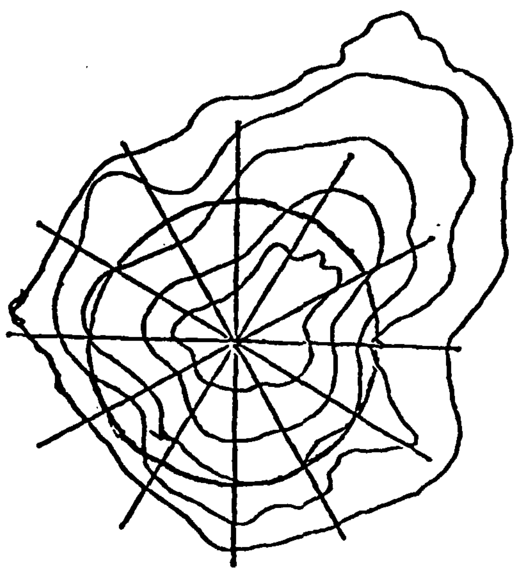
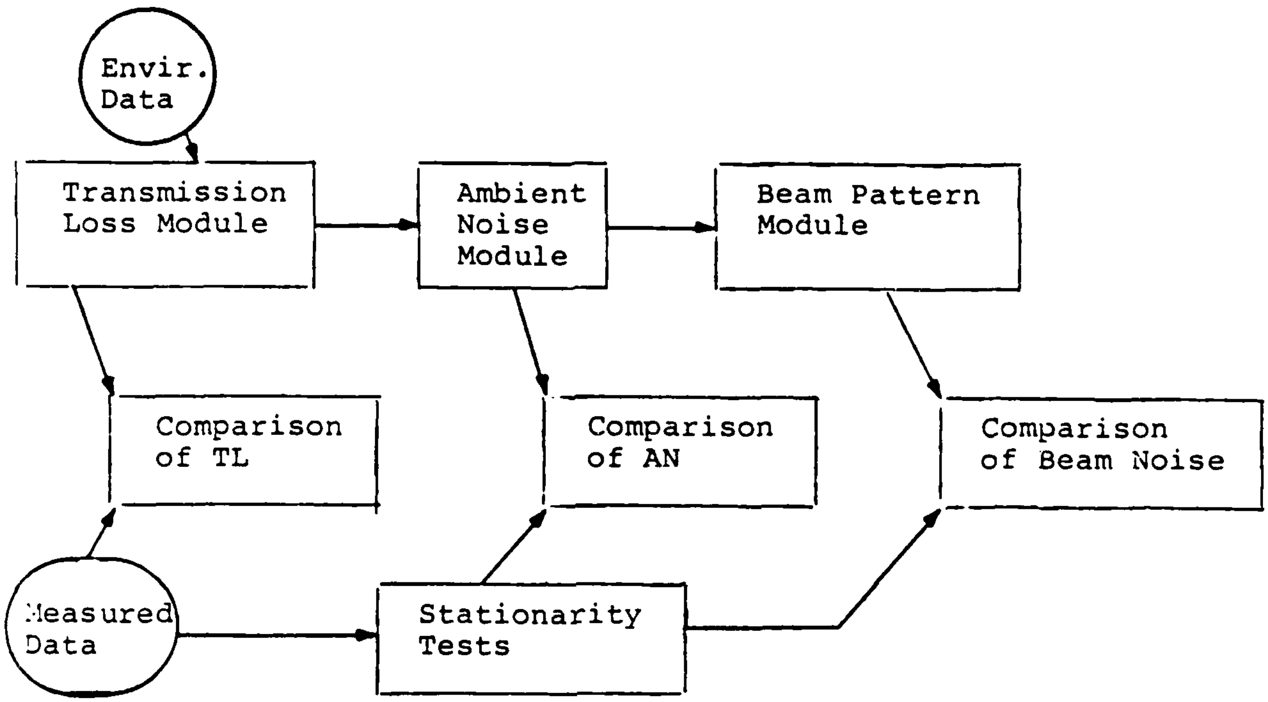


FIGURE 5 Constant Level (Omni) Rose Lying
Within Envlcope of Roses

Comparisons of omni and horizontal noise fields may be made in accordance with Appendix F, in light of the considerations noted above. Since vertical arrays tend to have rather large beam widths, and vertical directionality (as predicted by acoustic models) tends to be well structured into bands of surface noise, deep channel noise notches and locally generated noise contributions, the techniques used for horizontal noise cannot be meaningfully applied to the vertical case. For even if the transmission loss is significantly erroneous (in level, not in arrival angle structure), and the shipping distribution is widely in error, the vertical directionality patterns usually are qualitatively similar, and may be quantitatively close (Appendix E). In regard to the statistics, the comparison of the distribution functions should be done using the Levy metric, and comparison of moments of the distribution should be done using statistical tests.

Invariably, when comparing the model against measurements, disagreements will arise. If the model and the data were to agree perfectly, (e.g., to .01 dB) then one would have grave misgivings on the credibility of the test. It is to be expected with any non-trivial test that differences between the model and the measured data will be abundant. When these differences becomes excessive, it is necessary to determine the principal source or sources of the discrepancies. By working backwards in comparison of beam noise, ambient noise and transmission loss, the principal area of disagreement may often be quickly isolated (see flow chart)


Envir. Data
Transmission Loss Module
Ambient Noise Module
Beam Pattern Module
Comparison of TL
Comparison of AN
Comparison of Beam Noise
Measured Data
Stationarity Tests

One still must determine the precise area of disagreement; several possibilities are listed below:

- The transmission loss may have been measured during a period of fluctuation, thus not reflecting the mean environment.
- The transmission loss model may be inaccurate or erroneous
- The environmental data used in the transmission loss run may not correspond to the precise conditions at the time of transmission loss measurement.
- The positions of discrete ships, particularly nearby ones, may not correspond between model and reality.
- The radiated noise level of ships in the model are class approximations.
- The assumptions within the ambient noise model (e.g., radiated noise pattern of ships) may be erroneous.
- The theoretical beam patterns used by the model may grossly misrepresent the actual array performance.

This list, of course, is not complete but it does cover the principal areas. Again, the utility of using a validated transmission loss module with the proper environmental conditions is demonstrated in this problem of error tracing.

Before leaving the subject of model vs. measurement comparisons, a few words in regard to data sources are appropriate. There are a number of sources of acoustical measurements. However, for familiarity with the data collection techniques, data quality and intricacies of the experiments, it is felt that at the initial stage measurements should be selected from previous NORDA (earlier LRAPP) exercises. The SEAS data bank is designed to provide both environmental and acoustical data and can be quite useful for such an evaluation. Since most acoustic data is not yet in the SEAS data bank, it must be obtained from the NORDA contractors who collected it. The primary source of such acoustic data from previous LRAPP exercises is ARL/UT. The principle NORDA exercises for such data are as follows:

Exercise	Area
Church Anchor	Northeast Pacific
Church Opal	Northeast Pacific
Eastlant	Northeast Atlantic
Square Deal	Northeast Atlantic
Med ASW Augmentation, Task I	Mediterranean
Med ASW Augmentation, Tast V	Mediterranean
Church Gabbro	Caribbean
Church Stroke III	Gulf of Mexico

The data collected during these exercises includes acoustic as well as environmental data. The envirionmental data consists of XBT, AXBT, SVTD, SVD, STD, SVSTD and SVCTD measurements. Different exercises performed different measurements. For acoustic data, measurements were made by towed arrays and vertical arrays using stationary and towed sources as well as SUS. The SEAS Data Base Manual details what measurement results are available in the data base for each exercise and its format.

VII. Comparison of Model vs. Model

In comparing the model under consideration with another model (in some sense a "standard" model),the underlying principles and technique parallel the case of model vs. data comparisons. The types of tests mentioned in Section VI above, are again applicable to model vs model comparison. The significant property of model vs. model comparison is the flexibility, whereas measured data is a fixed set (i.e., certain sets of data exists, and these cannot be expanded quickly to cover other situations.) A model, however, can be exercised under whatever environmental conditions the modeler seeks: one is not limited to the existing acoustical data sets. This allows the development of methodology that will isolate the probable reasons for differences between the two ambient noise models as they are run on the same sets of environmental (and acoustical) inputs. If one notices a difference between the outputs of the two models, but is unsure as to the precise cause of this, particular environments may be developed which show certain characteristics very strongly (e.g., surface ducts, infinite bottom loss, no bottom loss, heavy surface reflection loss, double or triple sound channels, ice noise, unusual shipping patterns, extremely rough bathymetry, sea mount blockage, inverted profiles, no wind noise, extreme wind noise, etc.). These somewhat artificial but extremely valuable environments serve to isolate a peculiar phenomenon (of either the environment or model) in order to isolate the root cause of disagreement. This requires, however, that the model against which comparisons are being made has been, in some sense, "validated": thus when one does see agreement of model outputs, it is unlikely that both models are dead wrong, or, if disagreement, that neither of them matches "reality".

One may envision constructing a set of test environmental conditions which may not reflect any "real" environment yet would test all the significant characteristics which are found in normal open ocean, long range, low frequency ambient noise modelling. This set of test environments could form a library against which various ambient noise models can be exercised. From this, this modeler may be able to develop a list of strong and weak points for the various models. Again, before getting deeply involved in such methods, it is recommended that some sort of "reasonably believeable" model be available for each type of environmental phenomenon. This avoids myriad problems in determining how "truth" compares with the various models.

C. Initial Model Evaluation

VIII. The IME

As stated before, the initial model evaluation (IME) concentrates only on omni and horizontal directionality noise (including beam noise). Temporal and vertical directionality models are not considered. In developing this IME, the underlying philosophy has been that of Occam's Razor: "entia non sunt multiplicanda praeter necessitatem". This principle, that things should not be more complicated than needed, seems particularly relevant to this problem. The following methodology draws upon the tests described in previous sections and the appendicies: thus, the details will not be repeated.

A. Model vs. itself test.

1. If documentation is not available to show that elementary cases (isovelocity and isogradient environment) have been tried in the ambient noise model, test these two cases.
2. Change ship source levels, ship density, wind speed etc., to ensure that model reflects these changes.
3. Use exact solution for flat earth, parabolic profile, uniform bathymetry and infinite loss bottom for various ship distributions.
4. Perturb the sound speed profile. bottom loss functions, shipping distribution, etc., to see if the ambient noise model is stable. If the transmission loss submodule has been investigated in detail this test may be simplified considerably.
5. If the ambient noise model is for use in open ocean areas, perform the Nine-Point test in an appropriate region or regions.

B. Model vs. data tests.

6. Choose a data set or sets representative of the environmental and acoustical conditions for which the model is to be evaluated. The following steps in this section should be performed on each data set.

7. Test data for omni stationarity.
   a. If the variance of the measured omni noise is large, than the model will almost certainly lie within the $\mu \pm \sigma$ interval hence the exercise of the model is of little value.
   b. If the measured omni level noise is not a fluctuation of the mean level, but rather a fluctuation about another curve, this alternative curve must be explained. If the data cannot be explained do not try to validate the model using it.
   c. If the measured noise data is stationary, then it may be used for the test.
8. Test data for horizontally directional stationarity.
   a. Test for transits for nearby ships which could contaminate parts of the horizontal noise measurements.
   b. Test statistics (mean, variance, correlation) of horizontal noise measurement for stationarity.
   c. If ambient noise measurements are not available, but only beam noise values, use beams near center of the array in order to minimize problems with the conical beam patterns.
   d. If horizontal noise shows a stationarity either qualitatively (in terms of correlation) or quantitatively (in terms of variance) than comparisons with model should be undertaken. If neither of these holds, it is fruitless to attempt comparison.
9. Compare Omni noise measurement with model.
   a. Point-to-point comparison. The principal value of these tests is to have a single number indicitive of the closeness of model and measurement. If the number is large, determination of the cause or causes of this disagreement is required prior to proceeding. If the number is small, i.e., in good agreement, this does not indicate that model and data agree; the further tests will determine it.

b. Point (model run) to envelope (measurement) comparisons. In light of the extreme variability of acoustic measurements, it is extremely important in having a number of replications of the acoustic measurements (at least more than one). In this case, one may then compare the point prediction of the model to the envelope (or, if enough measurements exist, the distribution) of the measured data. This is perhaps the most relevant test for the omni noise estimate.

c. Point (mean measurement) to envelope (model with perturbed environment) comparison. In the case where only one measurement exists, or in the case where the point to envelope comparison, in b above, is inadequate, the mean measured data should be compared to the distribution obtained by running the model against the environment perturbed in a realistic fashion. This indicates whether the disagreement is due to model (or environmental) instabilities or whether true disagreement is present. This test is not necessary if a and b above are successful.

d. Envelope-to-Envelope Comparisons. If the model is exceptionally stable, and agreement in a and b above is also noteworthy, this test may be skipped. Otherwise the model should be used on a number of differing environments and the ensemble produced (including its underlying one dimensional probability distribution be compared to the ensemble of measured data.

e. Based on the above tests actually performed (between two and four of a thru d) the omni noise prediction of the model for this data set may be judged either acceptable or unacceptable. If they prove unacceptable, the root cause of the discrepancy should be identified (although in many cases it will take model vs. model comparisons to make a positive identification of the cause).

10. Compare Directional Noise Measurement with Model Predictions.

a. Even though the omni comparisons performed in 9 above may not be as good as anticipated or preferred, the directional tests should be performed as well. The division of the directional test into three areas (corresponding to level, variation and shape) enables one to test the directional noise levels for some property even if the omni values do not correspond well.

b. Point-to-Point Comparison. These comparisons should be on means of differences, variance of differences and correlation of noise roses.

c. Point (model)to envelope (measurements). With more than one set of measurements, the point-to-envelope comparisons should be performed. As in 9c, these are more relevant than the point-to-point comparisons. If a large number of measurements are obtained, distributional tests may be performed rather than simple envelope tests.

d. Point (measurements) to envelope (model with perturbed environment) comparisons. If the point-to-envelope comparisons made in c above prove inconclusive or poor, the model sensitivity to environmental and shipping perturbations should be used in this test, where the beam measurement is compared with the distribution of model outputs. As in 9d, this need not be done if b and c show adequate agreement.

e. Envelope-to-envelope Comparisons. If the above analysis is not conclusive (either for or against the validation of the model on this data set) an envelope-to-envelope comparison should be initiated. However, for directional noise, such a comparison involves a large amount of data, and detailed analysis. This test should be invoked only when the prior test are not sufficiently conclusive.

11. Analysis of Discrepancies. Before launching into the model vs. model comparisons, the discrepancies noted in the various measurement data sets vs. model comparison should be analyzed and their probable causes noted. This will allow a much more rigorous testing in the third section, and avoid tests which are immaterial (based on either extremely good or very poor results noted in the current analysis). Obvious model deficiencies noted at this point should be documented, and model successes noted as well.

C. Model vs. Model

12. Selection of Test Environments. Based on the problems noted previously, the capabilities of both the model under consideration and the "standard" model (which need not be perfect), a set of test environments should be selected. These environments need not be chosen from reality, but should cover the various environmental phenomena under consideration. It is assumed that the "standard" model has been subjected to an ambient noise model validation prior to this (in the sense of A and B above) so that questions of model stability, environmental anomolies, etc., may be treated in a cursory rather than a detailed fashion.

13. Examine the Omni Noise Prediction of the "Standard" Model for stationarity and believability. Since this model may be exercised at will on perturbed data, we may deal directly with one dimensional distributions rather than envelopes (i.e., intervals on the real line) If there are no outrageous outliers and the numbers seem reasonable, then comparison may continue for the omni values. If the "standard" model gives predictions that are suspicious (either from the point-of-view of stationarity or reasonableness), then comparison based on this case should be avoided.

14. Examine the Directional Noise Prediction of the "Standard" Model. As in 13 above, this should be done using the distributions rather than the envelopes. It is anticipated that if the "standard" model has passed the test of 13, it will pass these tests with no problem.

15. Compare Omni Noise Prediction of Both Models.

   a. Since we are dealing with two models, both may be run in order to get sufficient replications. Thus one may deal with point-to-point and envelope-to-envelope comparisons based directly upon the distribution. Point-to-envelope comparison (with either model providing the mean statistics for the point estimator) should be used only when the previous two (i.e., point-to-point and envelope-to-envelope) methods of comparison yield ambiguous results.

16. Compare Directional Noise Prediction of Both Models.

   a. As in the model vs. data comparison, above in 10, we compare the mean of the difference, the variance of the difference and the correlation of the values. As in 15 above, two comparison methods should be used: point-to-point and envelope-to-envelope comparison on the distribution. Point-to-envelope comparisons are not recommended in this case: the amount of information that they would contribute seems small in comparison with that obtained from the first two methods.

17. Overall Ambient Noise Evaluation: Using the results of all the above tests, lists of those good and bad model points should be drawn up. Quantitative expressions of model accuracy, reliability. predictability may be given as well (if the stability of the input environmental and acoustical data is adequate to warrant this). Based on these two lists and the applications envisioned for the ambient noise model, the model may be either validated, conditionally validated, or considered unsuitable at this time for those applications. Based on this model validation,model improvement (if necessary or desired) may be undertaken.

IX. Conclusions:

This IME, like any validation process, may be used or misused. Its proper use requires understanding of all model inputs and measurements. A benefit of such use is that many of the test procedures are also applicable to questions regarding the limits of model accuracy. Its misuse is accomplished when one forgets the general limitations, the implicit assumptions, and the nature of the real world. The tendency to use a model beyond its region of applicability is prevalent in any applications-oriented community, and for acoustic models the deterioration of performance due to such gradual over-extension is slow but significant. Another misuse is to accept a validation of the model for one use as implicitly validating it in every use for which the model was intended. Finally, it should always be remembered that there are cases for which no current ambient noise model will work, such as when shipping noise is bounced off the sides of canyons and thus arrives at the receiver from a number of (horizontal) directions (e.g., Rockall Basin).

Appendicies

APPENDIX A: On Integration Time and Second Order Statistics

Consider some function, such as an ambient noise measurement, which is measured by integration over some period. The same interval of data can be measured by integration over two or more different periods. This appendix will deal with the effects of using different periods of integration on some second order statistics.

Let X(t) denote the function which is to be measured. Since in the general case for ambient noise the environmental parameters involved are not all known precisely, consider X(t) as being an event in the sample space of some stochastic process, $X(t,\omega)$. Let X_i be the i-th data sample obtained when using an integration period of Δt. That is,

$$X_i = \frac{1}{\Delta t} \int_{T_o+(i-1)\Delta t}^{T_o+i\Delta t} X(t)dt \quad \text{for } i = 1,2,\ldots,N$$

where T_o is the point at which the initial sample measurement begins and N is the total number of samples. (Hence, $N\Delta t$ is the total length of integration).

Now, before dealing with the second order statistics using $\{X_i\}$ or other integration periods, some assumptions must be made about $\{X_i\}$ and, therefore, some restrictions must be placed on the underlying stochastic process, $X(t,\omega)$. In order to be able to deal with the second order statistics considered in this appendix and have those statistics be in a useful form, it is necessary to assume that for i=1,2,...,N, $EX_i^2 < \infty$ and that $\{X_i\}_{i=1}^{N}$ consists of identically distributed random variables. ($\{X_i\}$ need not be independent). These assumptions restrict the stochastic process $X(t,\omega)$ to the set of processes $Z(t,\omega)$ such that

$$EZ^2(t,\omega) < \infty$$

and

$$EZ(t,\omega)Z(\tau-\omega) = r(t-\tau)$$

where r, the covariance function, depends only on the difference $t-\tau$.

That is, $X(t,\omega)$ must be a weakly stationary stochastic process (see reference s). Thus, for any of the material in this appendix to be of any use, the analyst must understand the process that is being measured and be sure that it satisfies these restrictions.

So, assuming that the restrictions on $X(t,\omega)$ are met, then $\{X_i\}_{i=1}^{N}$ as defined above has the desired properties. Suppose that a new integration period was used to sample the same process for the same interval. That is, suppose $X(t)$ is sampled using an increased integration period of $m\Delta t$ while only taking n samples where n and m are integers such that $nm = N$, (Thus, the total integration time, $nm\Delta t$, remains $N\Delta t$). This type of change in the integration period is common. For example, the integration period is increased to improve the frequency resolution of the measurement.

This change in integration period changes the resulting sample. For the increased period of integration, $m\Delta t$, let Y_j be the j-th data sample obtained that is,

$$Y_j = \frac{1}{m\Delta t}\int_{T_o+(j-1)m\Delta t}^{T_o+jm\Delta t} X(t)dt \qquad \text{for } j = 1,2,\ldots,n.$$

Now, from the definitions of $\{X_i\}$, $\{Y_j\}$, N, m, and n, Y_j is also given by

$$Y_j = \frac{1}{m}\sum_{i=1}^{m} X_{m(j-1)+i} \qquad \text{for } j = 1,2,\ldots,n.$$

Therefore, the effects of using different periods of integration on second order statistics can be determined by examining the statistics based on $\{X_i\}$ and those based on $\{Y_j\}$.

Let μ denote the common means of $\{X_i\}$ and let σ^2 denote the common variance. (Although normality is not required, this would be the case if it is assumed that each X_i is $N(\mu,\sigma^2)$). The dependency is assumed to be a simple correlation. That is,

$$\text{Cov}(X_i, X_j) = \rho^{|i-j|}\sigma^2$$

for some constant ρ, $0 \leq \rho \leq 1$. (Note, this places an additional restriction on the covariance function, r, of the underlying stochastic process). The mean and variance of the sample $\{X_i\}_{i=1}^{N}$ are given by

$$m_1 = \frac{1}{N} \sum_{i=1}^{N} X_i$$

and

$$s_1^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - m_1)^2$$

respectively. Similarly, the common mean and variance of the sample $\{Y_j\}_{j=1}^{n}$ are,

$$m_2 = \frac{1}{n} \sum_{j=1}^{n} Y_j$$

and

$$s_2^2 = \frac{1}{n-1} \sum_{j=1}^{n} (Y_j - m_2)^2$$

Note that the second order statistics m_1, m_2, s_1^2, and s_2^2 are random variables. (They are statistical estimators based on random variables). Of interest to the analyst are the expected values of these statistics and how their expected values vary with changes in the period of integration.

Since $m_1 = m_2$, there is no difference in the computation of the expected values of the sample means. $Em_1 = \mu$, and so, m_1 and m_2 are unbiased estimates. This also shows that the sample mean is independent of the period of integration.

In general s_1^2 is not the same as s_2^2. The computations of Es_1^2 and Es_2^2 are rather lengthy, and so, only the main steps in computing Es_1^2 and an expression for Es_2^2 are given here. The entire derivations for both Es_1^2 and Es_2^2 are given in appendix G.

$$Es_1^2 = E\left[\frac{1}{N-1}\sum_{i=1}^{N}(X_i-m_1)^2\right]$$

$$= \frac{1}{N-1}\left\{E\left[\sum_{i=1}^{N}(X_i-\mu)^2 - \frac{1}{N}\sum_{i=1}^{N}(X_i-\mu)\sum_{i=1}^{N}(X_i-\mu)\right]\right\}$$

$$= \frac{1}{N-1}\left(N\sigma^2 - \frac{\sigma^2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\rho^{|i-j|}\right)$$

$$= \frac{\sigma^2}{N(N-1)}\left[N^2 - \left(2\sum_{\ell=1}^{N-1}\ell\rho^{N-\ell}+N\right)\right]$$

$$Es_1^2 = \frac{\sigma^2}{N(N-1)}\left[N(N-1) - 2\rho^{N-1}\sum_{i=1}^{N-1}\ell(\rho^{-1})^{\ell-1}\right] \quad \text{if } \rho\neq 1$$

Now, by differentiation of a geometric series it follows that,

$$\sum_{j=1}^{N} jx^{j-1} = \frac{Nx^{N+1} - (N+1)x^N+1}{(x-1)^2}$$

And so, after simplification,

$$Es_1^2 = \sigma^2 - \frac{2\rho\sigma^2}{N(N-1)(1-\rho)^2}\left[(N-1) - N\rho + \rho^N\right]$$

Which can also be written as,

$$Es_1^2 = \frac{\sigma^2}{N(N-1)(1-\rho)^2}\left[N(N-1) - 2(N^2-1)\rho + N(N+1)\rho^2 - 2\rho^{N+1}\right]$$

for $\rho\neq 1$. Of course,

$$Es_1^2 = 0 \quad \text{for } \rho=1$$

Es_2^2 is computed in a similar manner using the common variance of $\{Y_j\}$ and the covariance of $\{Y_j\}$. The variance of Y_j, σ_Y^2, is given by,

$$\sigma_Y^2 = \frac{\sigma^2}{m} + \frac{2\sigma^2\rho}{m^2(1-\rho)^2}\left[(m-1)-m\rho+\rho^m \right]$$

for $\rho \neq 1$ and by

$$\sigma_Y^2 = \sigma^2$$

for $\rho = 1$. The covariance is given by,

$$\text{Cov}\ (Y_i,Y_j) = \frac{\rho\sigma^2}{m^2}\ \rho^{m(|i-j|-1)}\left(\frac{1-\rho^m}{1-\rho}\right)^2$$

for $\rho \neq 1$ and by,

$$\text{Cov}\ (Y_i,Y_j) = \sigma^2$$

for $\rho=1$. The expected value of s_2^2 obtained using these expressions is,

$$Es_2^2 = \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{ n(n-1)m-2(n^2-1)\rho-n(n-1)m\rho^2+2n^2\rho^{m+1}-2\rho^{nm+1} \right\}$$

for $\rho \neq 1$ and

$$Es_2^2 = 0$$

for $\rho=1$

From these expressions it is clear that the expected value of s_1^2 will equal the expected value of s_2^2 only when $m=1$ (and hence, $n=N$ and $s_1^2=s_2^2$) or at isolated values of ρ (e.g. $\rho=1$). However, while these expressions give an exact mathematical representation of how changes in the sample integration period can effect the expected sample variance, the expressions do not provide the analyst with an intuitive understanding of how Es_2^2 is related to N, n, and m.

In order to help make the relationships between N,n,m and Es_1^2 clearer, Es_2^2 has been graphed as a function of ρ for various values of N, n, and m. These graphs are presented in figures A-1 to A-9. These figures are grouped into three sets of three figures.

The first set of figures, figures A-1 to A-3, show how Es_2^2 is effected by increasing m while keeping n fixed. That is, these figures show how Es_2^2 changes when the number of samples is fixed but the period of integration for each sample, $m\Delta t$, is incresed. (And, hence, the total integration time is increased). It is clear that for small values of ρ, Es_2^2 decreases and the integration period increase and that for large values of ρ, Es_2^2 increases as the integration period increases. These two results are expected if the expression for the common variance of Y_j is examined (since Es_2^2 is an unbiased estimator of σ_Y^2). For small values of ρ, σ_Y^2 increases as m increases. It can also be seen from figures A-1 to A-3 that as the sample size is increased the Es_2^2 increases for all values of ρ.

Figures A-4 to A-6 are included to make this even more apparent. For each of the figures the integration period ($m\Delta t$) is held constant while the sample size (n) is increased. The increase in sample variance is expected due to the dependence of $\{Y_j\}$.

Now, the third set of figures, figures A-7 to A-9, show how these factors interact when the total integration interval is held constant. That is, these figures show how the expected value of the second order statistic s_2^2 varies when the integration period, $m\Delta t$, is increased while decreasing the sample size, n, in order to sample the same interval, $N\Delta t$.

As a concrete example of what this third set of figures shows, consider the situation where X(t) is sampled for 5 minutes. Let the short period of integration be 5 seconds and the longer period of integration be 20 seconds. Using the shorter period of integration,

$$N = 300 \text{ sec}/5 \text{ sec.}$$
$$= 60$$

and so n = 60 since m = 1. The graph of Es_2^2 (Es_1^2) in this case is shown in figure A-8. (It is marked n=60, m=1). Using the longer period of integration,

$$N = 300 \text{ sec}/5\text{sec.}$$
$$= 60,$$
$$n = 300 \text{ sec}/20 \text{ sec},$$
$$= 15$$

and

$$m = 60/15$$
$$= 4$$

This graph of Es_2^2 is also shown in figure A-8. (It is marked n=15, m=4).

Now that the expressions for Es_1^2 and Es_2^2 have been derived and intuitively described the question arises of what use are these expressions. There are two uses. First, these expressions can be used to estimate unknown parameters. Usually, the exact values of n, m and N will be known. The values for Es_1^2, Es_2^2 , ρ, and σ^2 will not be known. Using s_1^2 and s_2^2 estimates can be made for ρ and σ^2. This can be done by first taking the quotient of the expressions for Es_2^2 and Es_1^2. This yields an expression where the unknowns are Es_1^2, Es_2^2 and ρ. If s_1^2 is substituted for Es_1^2 and s_2^2 for Es_2^2, then an estimate for ρ, $\hat{\rho}$, can be found using numerical methods. An estimate for σ^2, $\hat{\sigma}^2$, can then be found by using s_2^2 and $\hat{\rho}$ in the expression for Es_2^2 and solving for σ^2 (s_1^2 and the expression for Es_1^2 can also be used). This method for estimating σ^2 can also be used with estimated $\hat{\rho}$ obtained in other manners.

However, there are some problems associated with this method for estimating σ and ρ. To begin with it will almost surely be the case that $s_1^2, \neq Es_1^2$, and $s_2^2 \neq Es_2^2$. From figure A-7 it can be seen that for most values of ρ small variations in Es_1^2 and Es_2^2 may result in significant error in the estimate $\hat{\rho}$. This is compounded with the fact that error in the estimation $\hat{\rho}$ will lead to errors in $\hat{\sigma}^2$ (especially for large values of ρ e.g. $\rho > .9$ in figure A-7). Thus, this method of estimation is very unstable.

The second and more important use for these expressions is to make it graphically apparent that when the integration period of the sample is changed, the second order statistics can be drastically effected. Thus, when the analyst is making use of second order he should be aware of the dependence of the statistics on the sample period of intergration.

FIGURE A-1: Es_2^2 as a Function of ρ with $n=15$

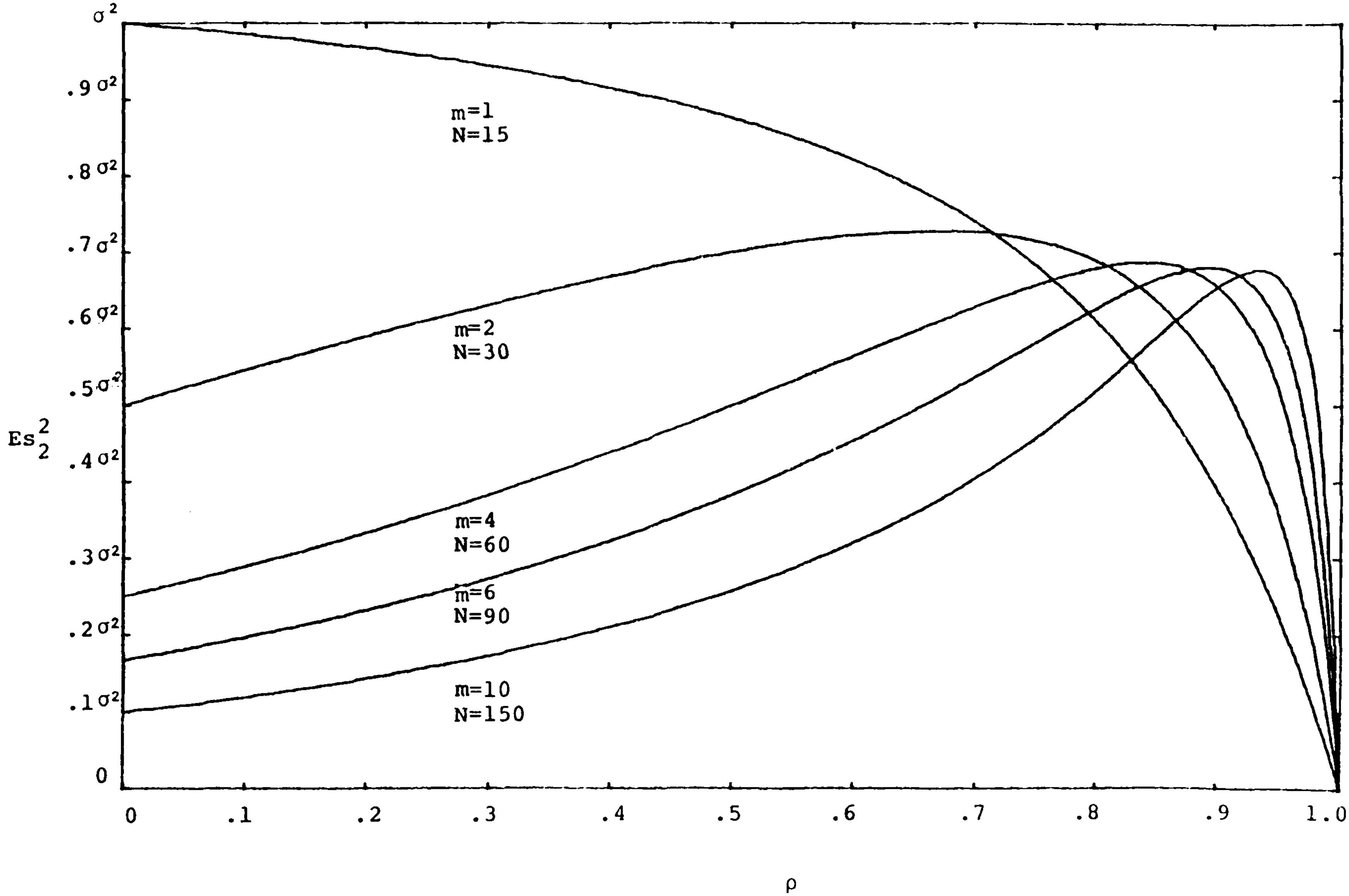


FIGURE A-2: Es_2^2 as a Function of ρ with m=30

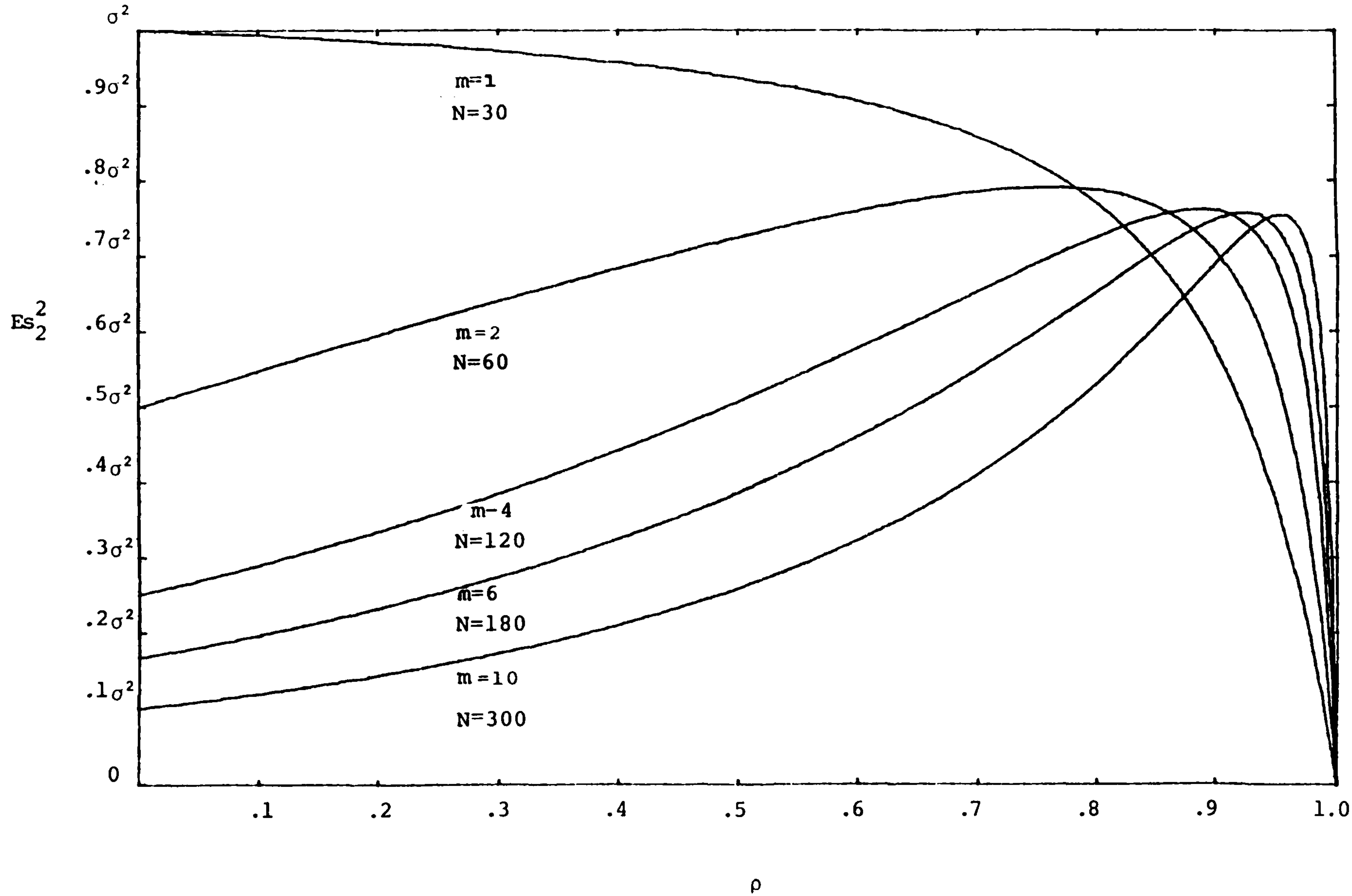


FIGURE A-3: Es_2^2 as a Function of ρ with m=45

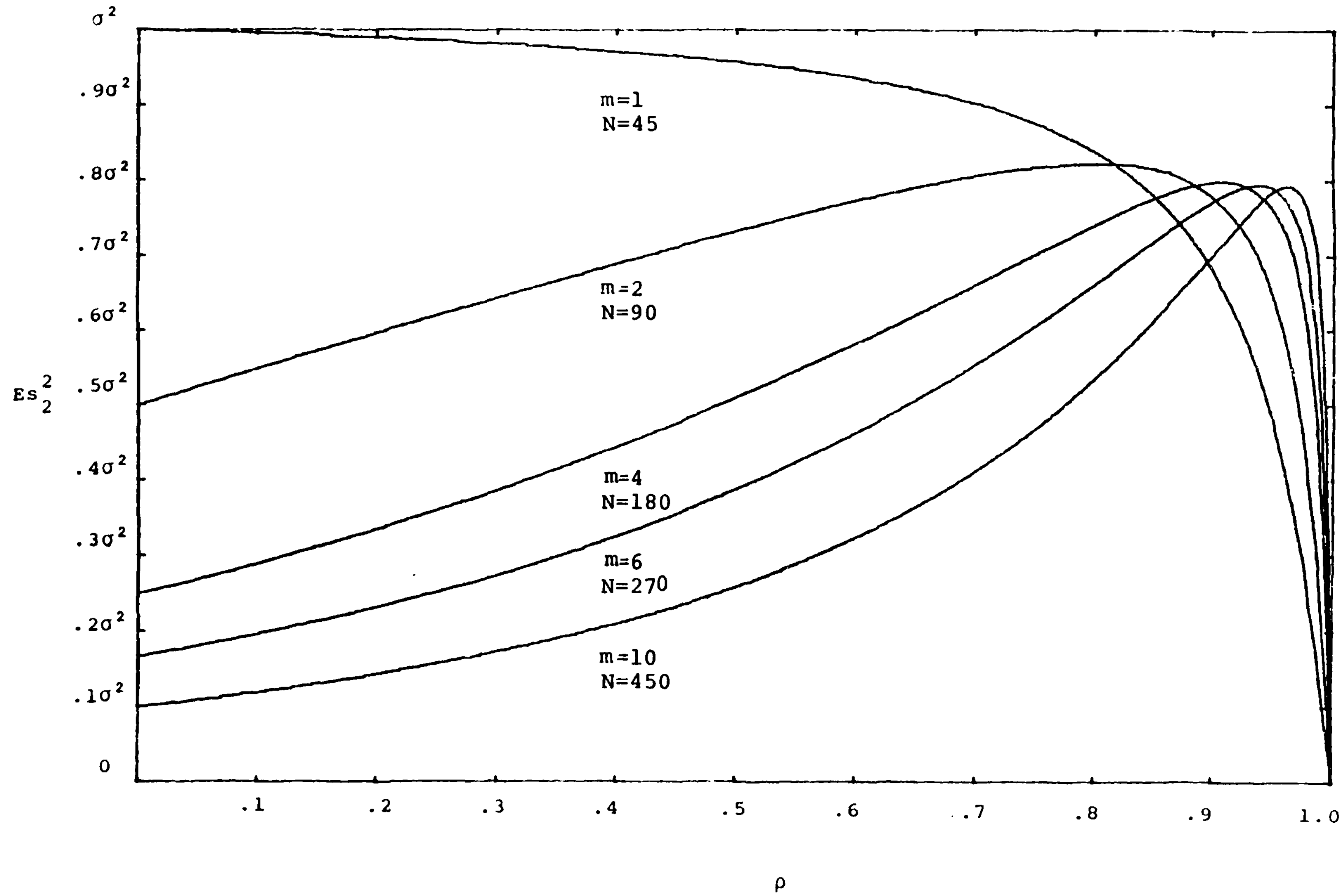


FIGURE A-4: Es_2^2 as a Function of ρ with m=1 (Es_1

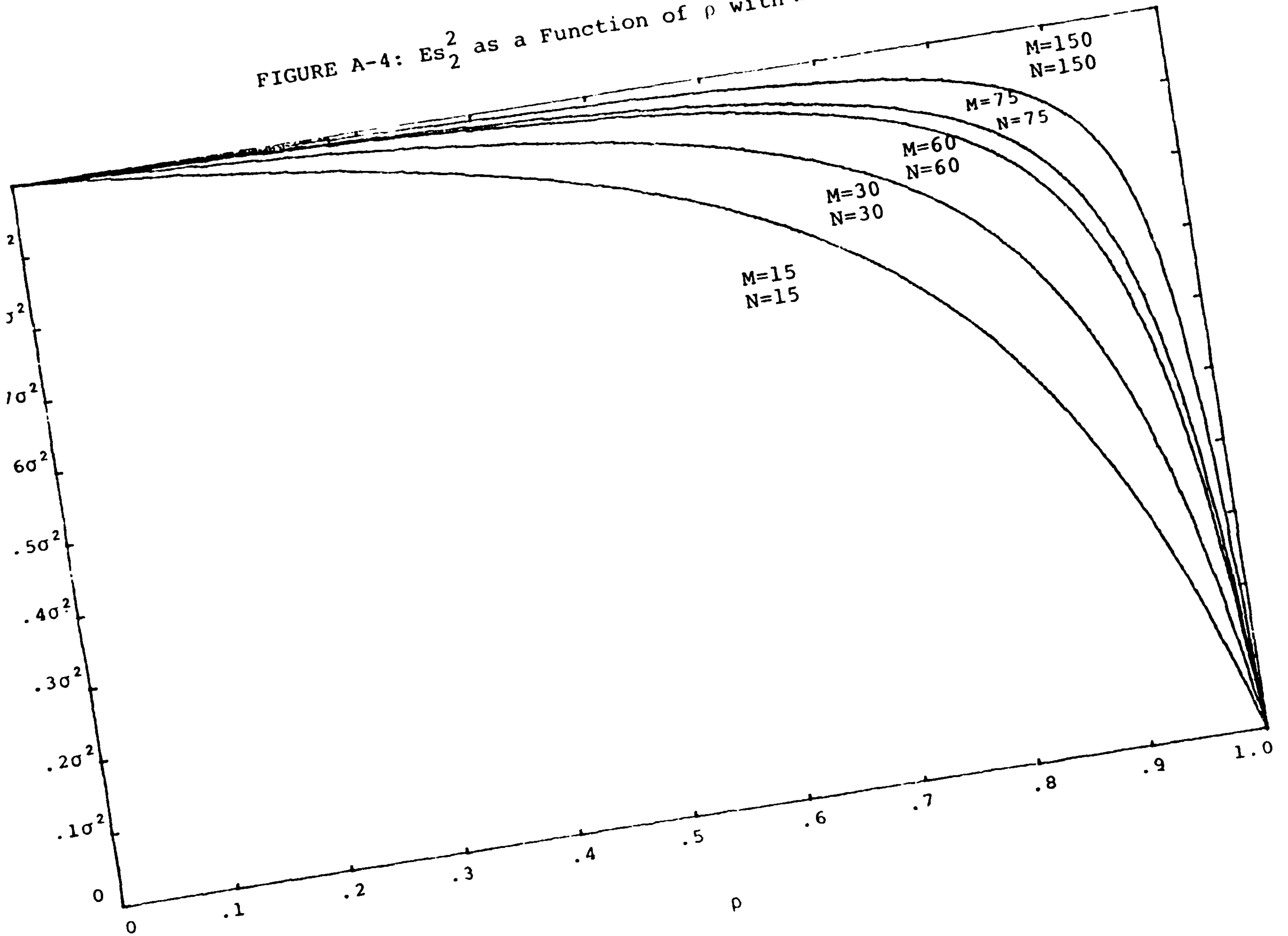


FIGURE A-5: Es_2^2 as a Function of ρ with m=5

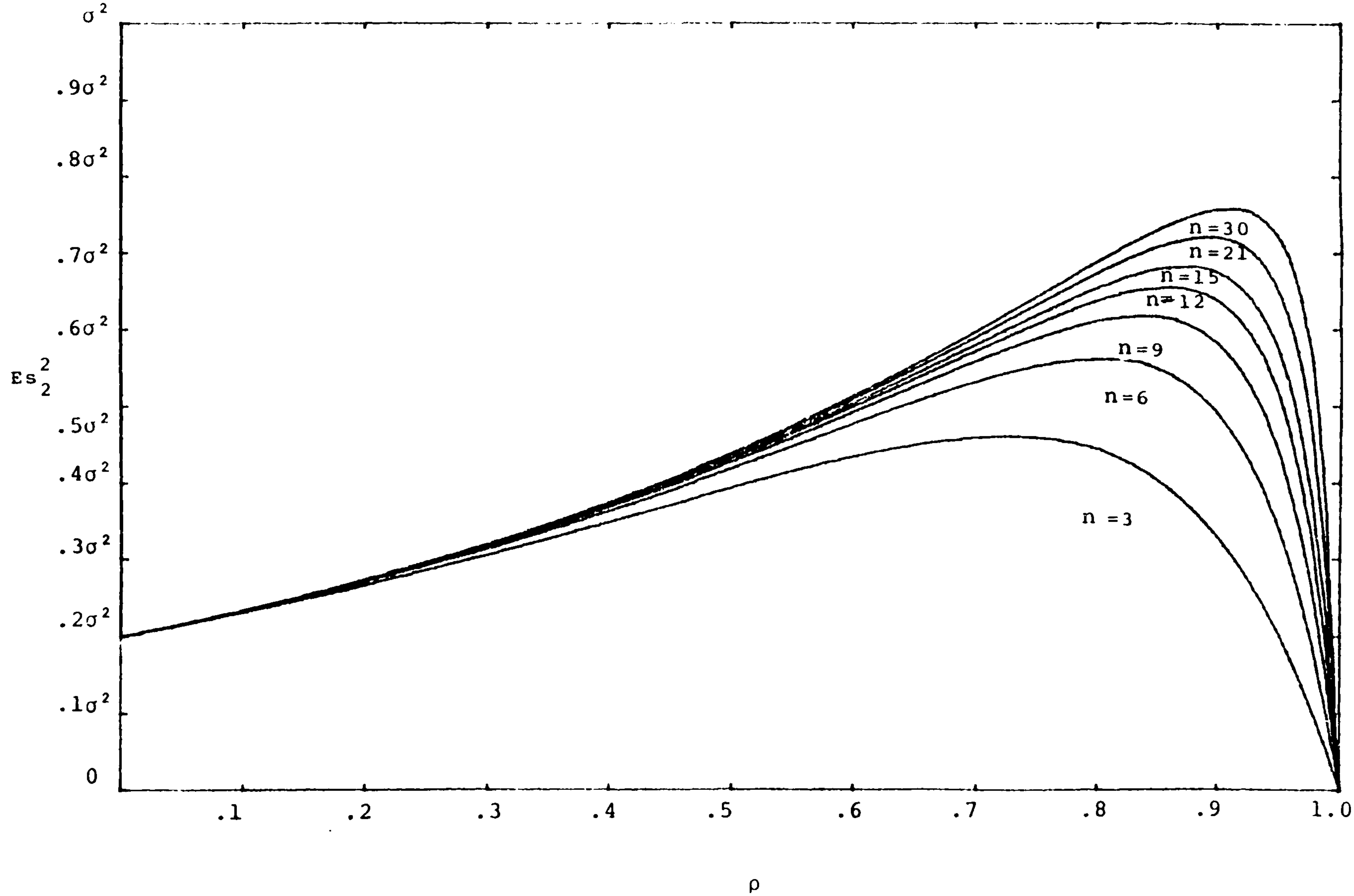


FIGURE A-6: Es_2^2 as a Function of ρ with $m=15$

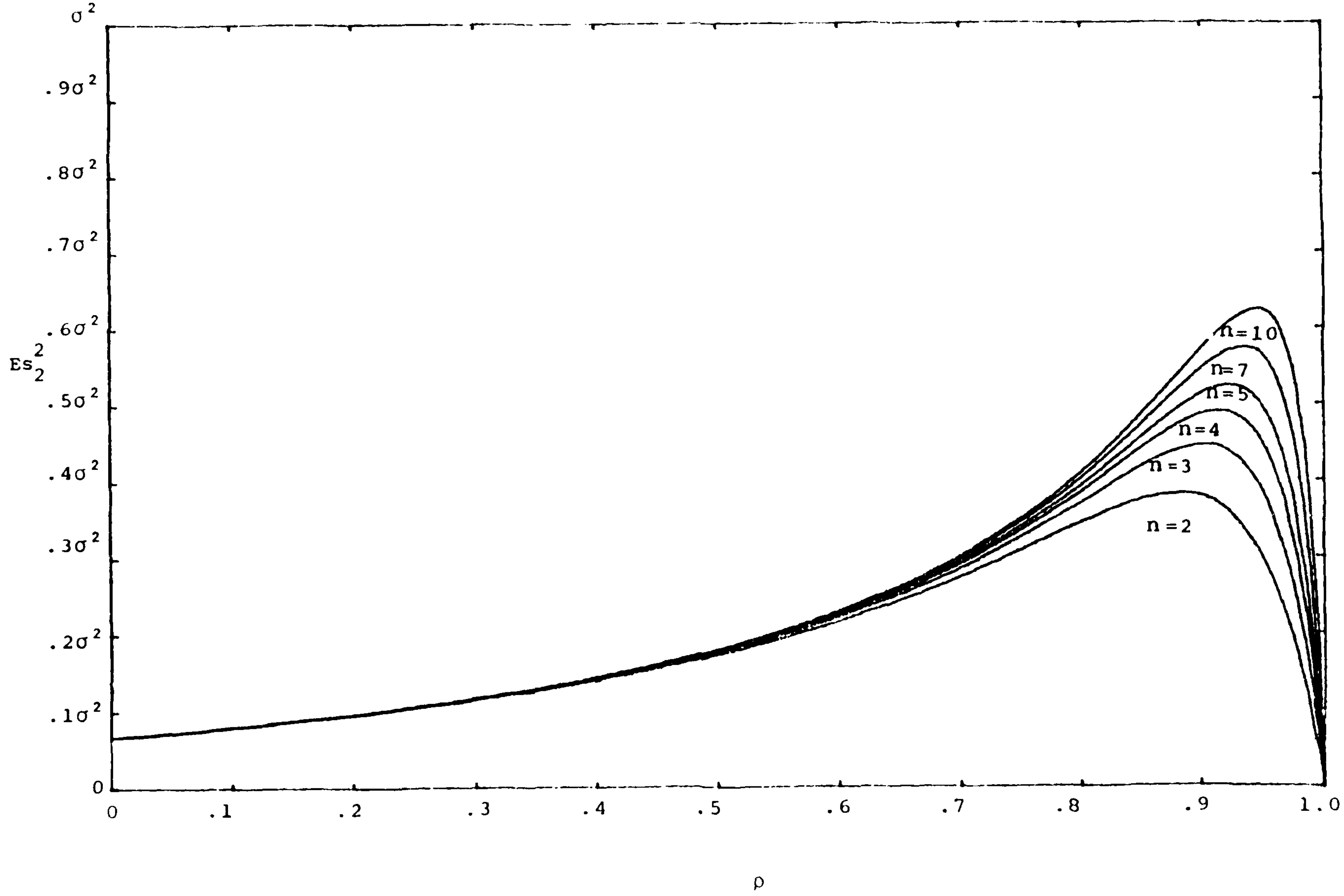


FIGURE A-7: Es_2^2 as a Function of ρ N=30

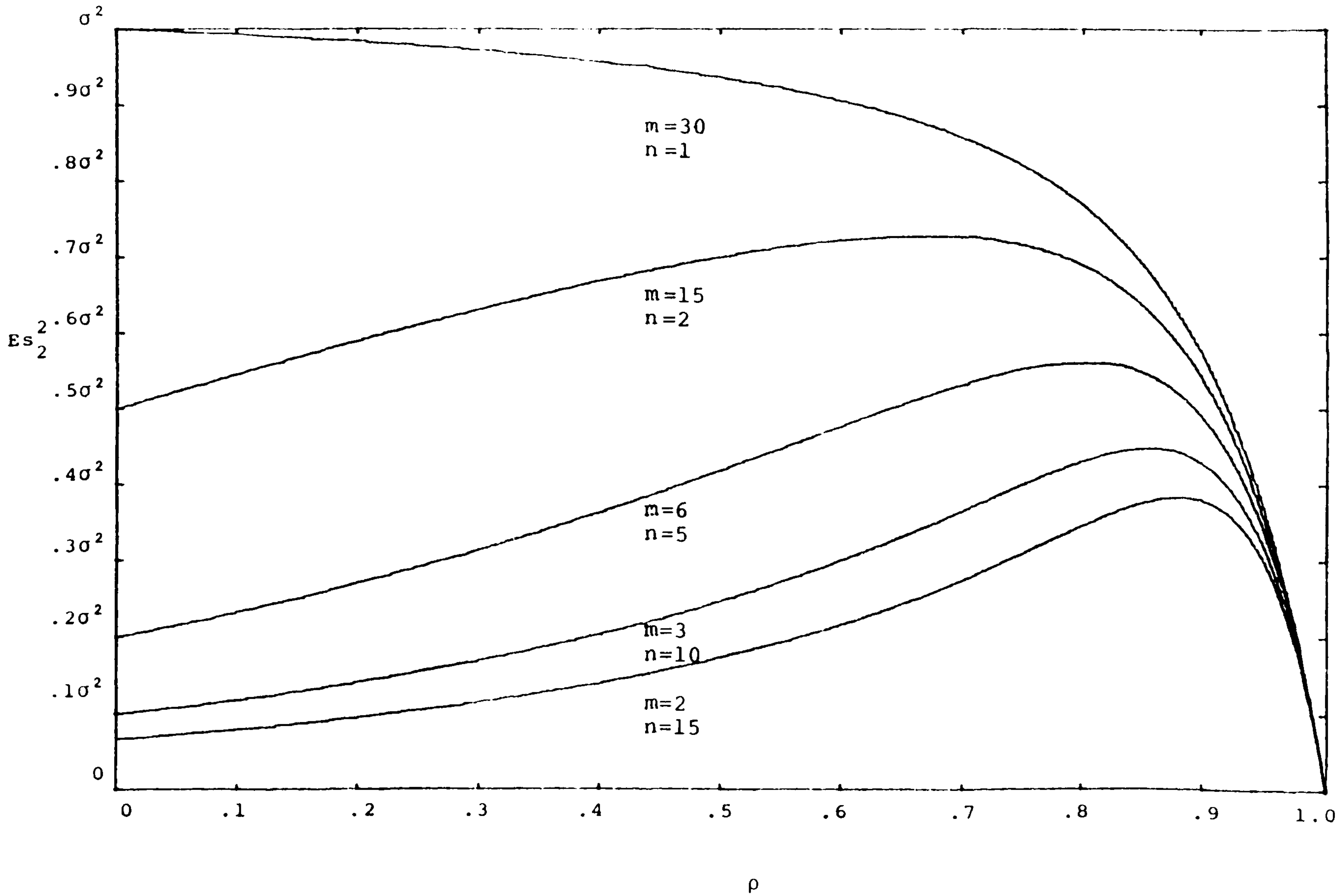


FIGURE A-8: Es_2^2 as a Function of ρ with N=60

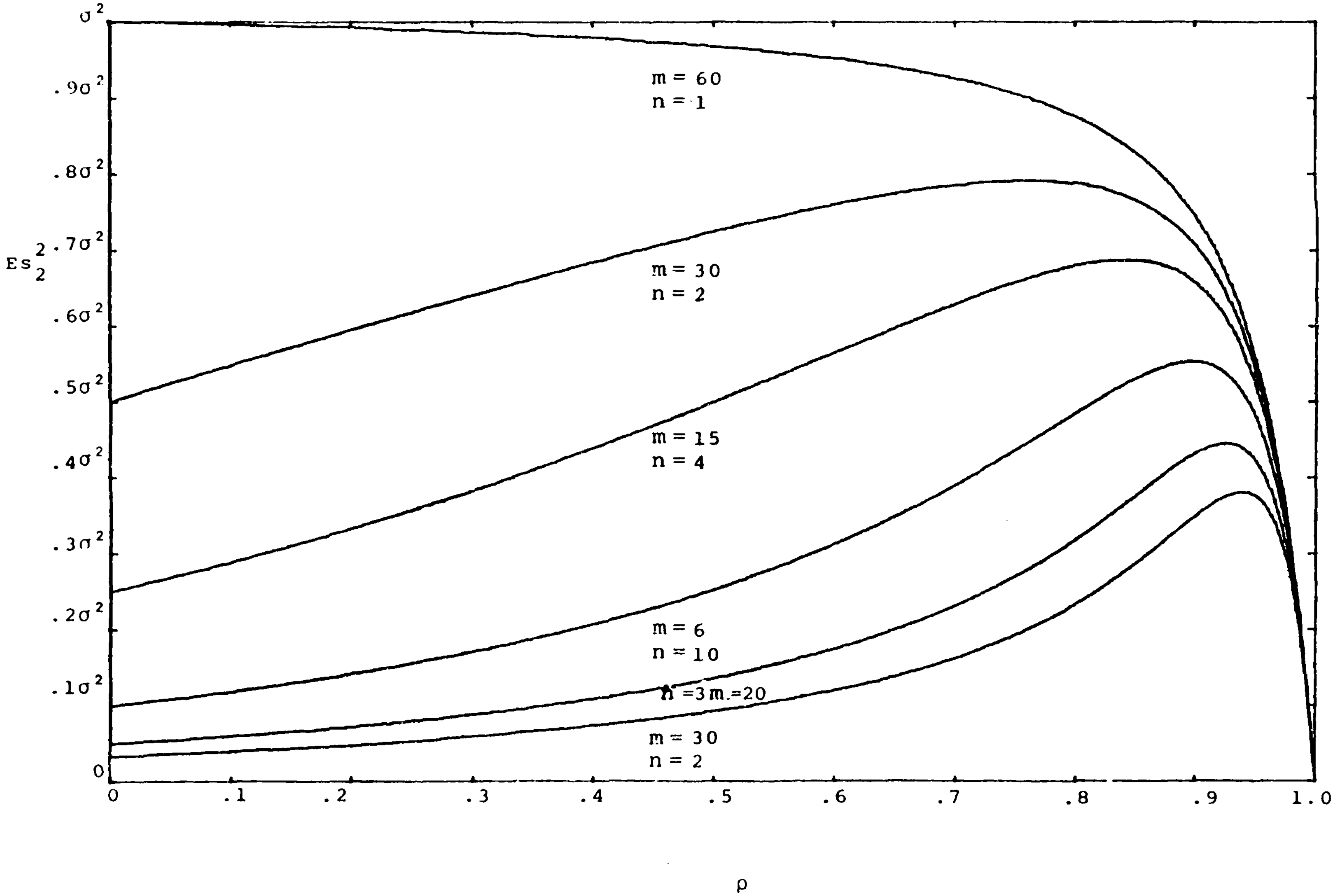
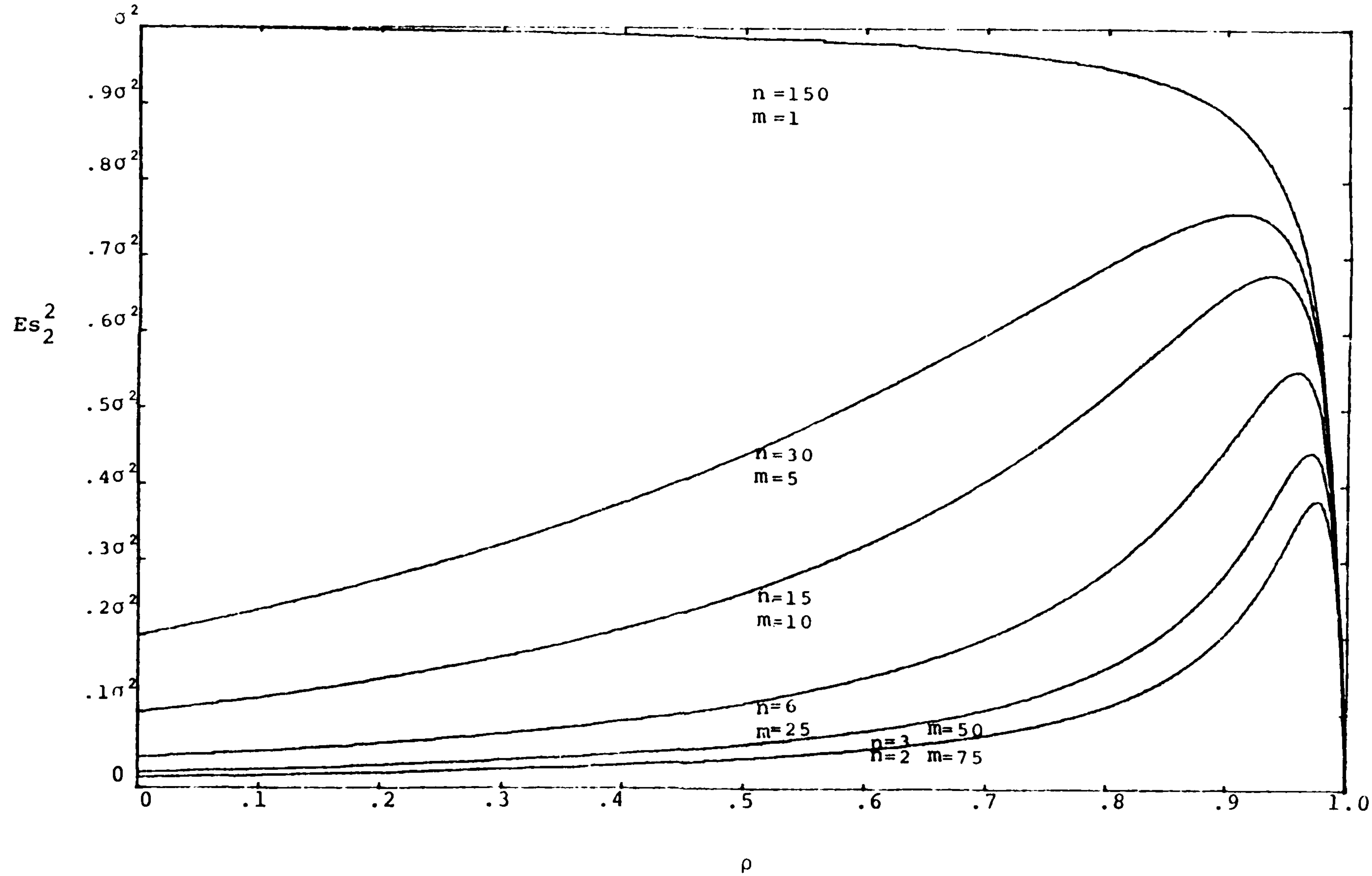


FIGURE A-9: Es_2^2 as a Function of ρ N=150



APPENDIX B

A Nontrivial Exact Solution for Ray Theory Models

1. Introduction
2. Environment
3. Propagation
4. Discrete Ships
5. Shipping Densitites

1. Introduction

The general approach will be to describe the nature of propagation in a specific environment, and show that for given ocean basin limits (acoustic, geographical or artificial) and ship distribution, one can (without the use of a large computer) obtain the horizontal directional ambient noise. This will allow a wide range of possible test cases for either discrete ships or densities. These equations are then used to obtain two test cases, which can be easily used in a computer model.

2. Environment

The purpose of this appendix is to provide a test case for ambient noise models which is capable of analytic solution, yet exercises more environmental acoustics than the traditional iso-velocity and isogradient test cases. To this end, the test environment used will have a range-independent sound velocity profile having a sound channel axis at depth z_o, of the form

$$c(z) = c(z_o)/\{1-\gamma^2(z-z_o)^2\}^{1/2} \qquad \gamma > 0$$

This parabolic profile applies to all depths z from the surface (0) to the bottom (z_{bott}), and requires that

$$\gamma|z-z_o|<1$$

for

$$o \leq z \leq z_{bott}$$

The bottom is assumed to be flat, below the conjugate depth, and has infinite loss. Thus

$$c_{bott} \equiv c(z_{bott}) > c(0) \equiv c_{surf}$$

There is no wind-generated noise, i.e. only surface shipping is calculated.

The receiver is placed on the sound channel axis, and range is measured from it. Thus the coordinates of the receiver are

$$(x,z) = (0, z_0)$$

The angle the raypath makes with the horizontal at the receiver is θ_0, measured clockwise (Figure B1).

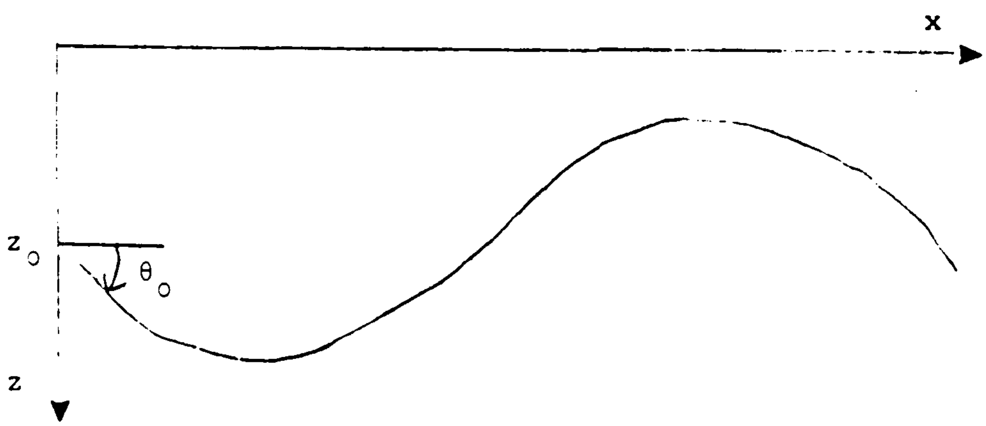


Figure B1. Raypath Geometry

3. Propagation

The raypaths may be specified by a system of differential equations (ref. n). Let $z(x;\theta_0)$, abbreviated as $z(x)$, be the equation of the raypath having angle θ_0 at the receiver as a function of range x, $\zeta(x)$ the partial derivative of $z(x)$ with respect to θ_0, $I(x)/I_0$ the (relative) intensity at range x and T the travel time along the raypath.

Then within the ocean region

$$\Psi \equiv 1 + \left(\frac{dz}{dx}\right)^2$$

$$c(x,z)\ \frac{d^2z}{dx^2} + \Psi\cdot\left(\frac{\partial c}{\partial z} - \frac{dz}{dx}\ \frac{\partial c}{\partial x}\right) = 0$$

$$c(x,z)\ \frac{d^2\zeta}{dx^2} - \Psi^{-1}\cdot\ \left(2\frac{dz}{dx}\ \frac{d^2z}{dx^2}\ c(x,z) + \Psi^2\ \frac{\partial c}{\partial x}\right)\frac{d\zeta}{dx}$$

$$+ \left(\Psi\cdot\left(\frac{\partial^2 c}{\partial z^2} - \frac{dz}{dx}\ \frac{\partial^2 c}{\partial x \partial z}\right) - \frac{d^2z}{dx^2}\ \frac{\partial c}{\partial z}\right)\zeta = 0$$

$$I(x)/I_o = \frac{\cos\ \theta_o}{x|\zeta|\cos\theta}$$

$$T = \int_o^x \Psi^{\frac{1}{2}}/c(x,z)dx$$

If the raypath hits the boundary of the region, these quantities are adjusted for the reflection (e.g. $\frac{dz}{dx}$) and for the boundary loss (e.g., I(x)). Attenuation loss may also be applied.

Applying these equations to the particular example yields three types of raypaths. The first type encompasses the raypaths which do not meet either boundary, i.e., the refracted paths. For these, the angle at the receiver θ_o must satisfy

$$-\ \bar{\theta}_o \leq \theta_o \leq \bar{\theta}_o$$

where

$$\bar{\theta}_o = \text{Arcsin}\ \ (\gamma z_o)\epsilon\ \ (0,\ \frac{\pi}{2})$$

is the angle of the ray which grazes the surface.

Then

$$z = z_o + \gamma^{-1}\sin\theta_o \sin\{\gamma x \sec\theta_o\}$$

$$\zeta = \gamma^{-1} \cos\theta_o \sin\{\gamma x \sec\theta_o\} + x \tan^2\theta_o \cos\{\gamma x \sec\theta_o\}$$

Coherent summation of rays also requires the travel time, which is

$$T = \frac{c_o}{\cos\theta_o} \int_o^x \frac{dx}{c^2(z)}$$

$$= \frac{1}{c_o \cos\theta_o} \left\{ \frac{(2 - \sin^2\theta_o)}{2} x + \frac{\sin^2\theta_o \cos\theta_o \sin(2\gamma x \sec\theta_o)}{4\gamma} \right\}$$

The second type of raypath is the RSR, i.e., that which hits the surface but not the bottom. Let $\tilde{\theta}_o$ be the angle at the receiver of the ray which grazes the bottom, i.e.,

$$\tilde{\theta}_o = \text{Arcos}(c_o/c_{bott}) \in (0, \pi/2)$$

Then if θ_o satisfies either

$$- \tilde{\theta}_o < \theta_o < - \bar{\theta}_o$$

$$\bar{\theta}_o < \theta_o < \tilde{\theta}_o$$

the raypath will hit the surface but not the bottom. Define the auxillary raypath parameters

$$\Gamma_1 (\theta_o) = -\gamma^{-1}\cos\theta_o \arcsin(\gamma z_o \csc\theta_o)$$

$$\Gamma_2 (\theta_o) = -2\Gamma_1(|\theta_o|) + \gamma^{-1} \pi \cos\theta_o$$

where the (multiple-valued) arcsin function is taken to be the unique value in the range

$$\left[0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$$

Then Γ_1 is a negative quantity which represents the range at which the ray would hit the surface if it were extrapolated through the receiver to negative ranges.

The parameter Γ_2 is the period of the raypath (Figure B2).

Two more auxiliary raypath parameters are needed. Define

$$\Gamma_3(\theta_o) = \frac{1}{2c_o\cos\theta_o}\left\{(2-\sin^2\theta_o)\,\Gamma_1 - z_o\cos\theta_o\sqrt{\cos^2\bar{\theta}_o-\cos^2\theta_o}\right\}$$

$$\Gamma_4(\theta_o) = -2\Gamma_3(|\theta_o|) + \frac{(2-\sin^2\theta_o)}{2c_o\cos\theta_o}\cdot\frac{\pi\cos\theta_o}{\gamma}$$

Note that

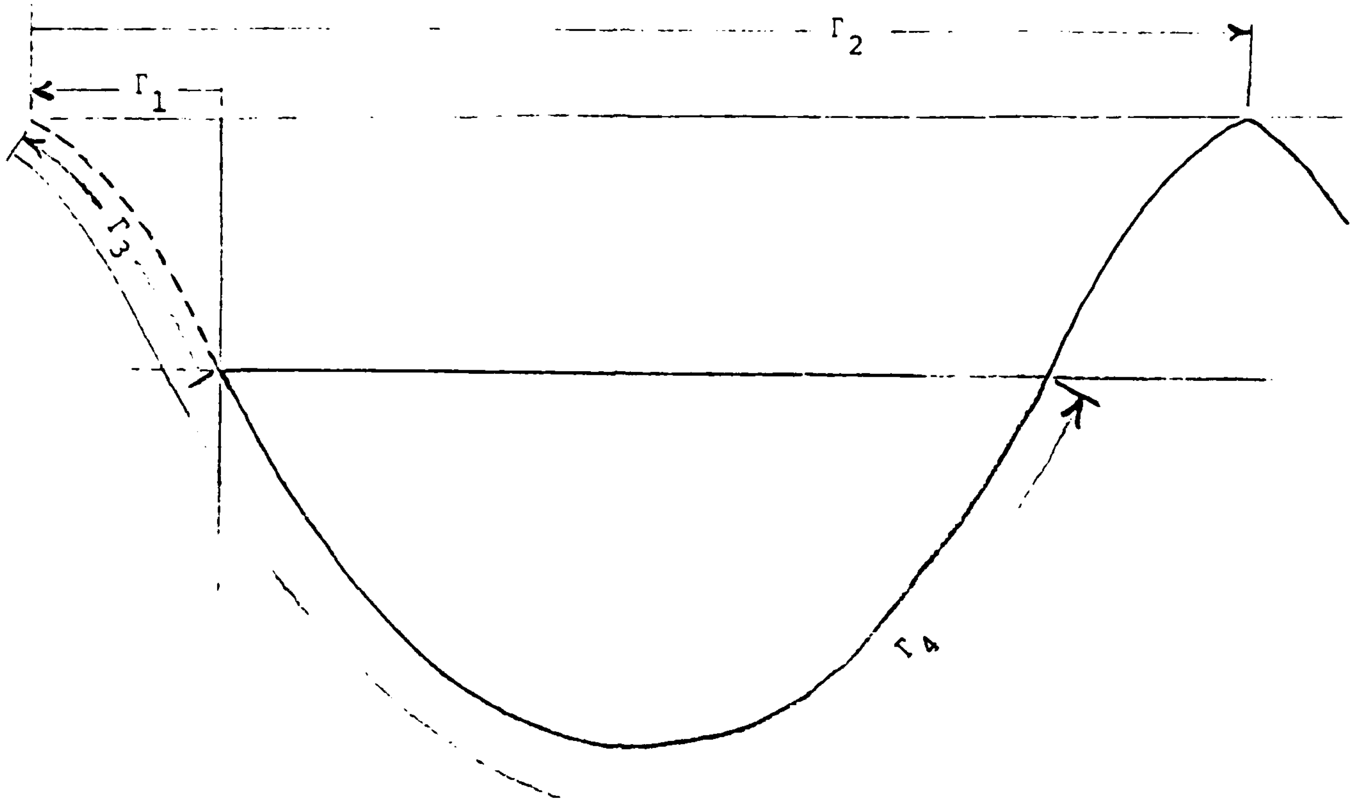
$$\Gamma_1(\theta_o),\ \Gamma_3(\theta_o) < 0$$

$$\Gamma_2(\theta_o),\ \Gamma_4(\theta_o) > 0$$

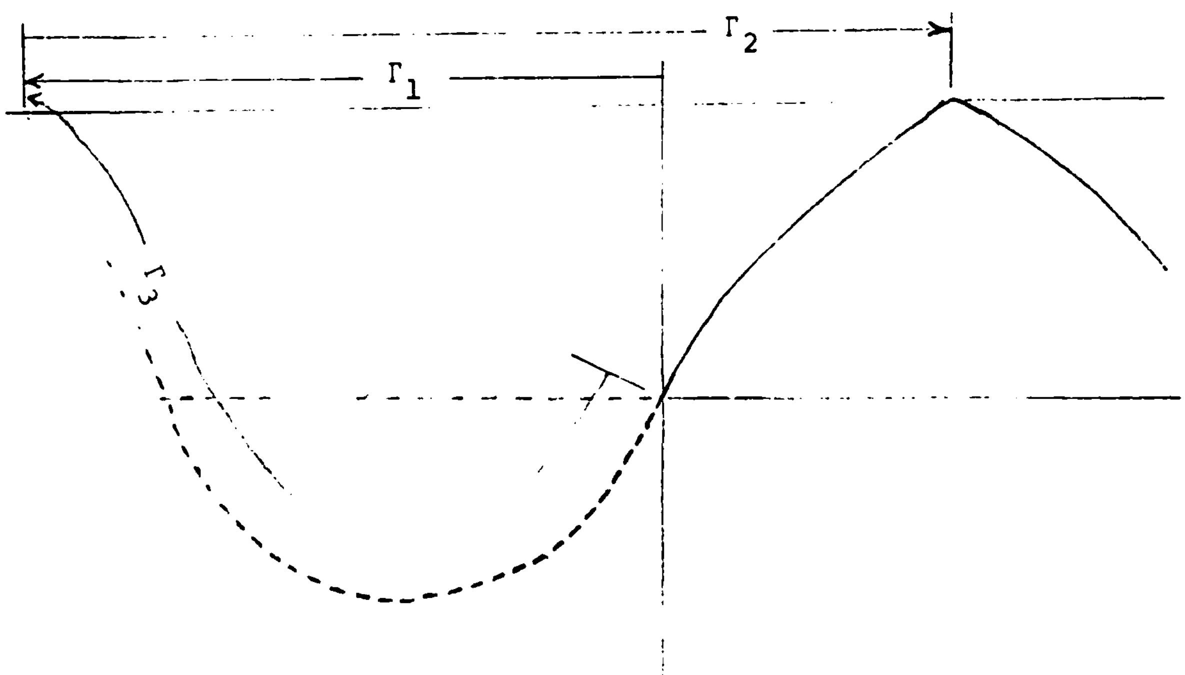
Then Γ_3 is a negative quantity which measures the travel time of the ray from the receiver back to the range corresponding to Γ_1. And Γ_4 is the travel time along the raypath for a period Γ_2.

Define the function

$$\Lambda(x) = \left[(x-\Gamma_1)\ /\ \Gamma_2\right]$$



a) For positive θ_o



b) For negative θ_o

Figure B2 The Parameters Γ_1 thru Γ_4

where $[\cdot]$ is the greatest integer function. The raypaths are then given by the equation:

$$z = z_o + \gamma^{-1}\sin\{\gamma(x - \Lambda(x)\Gamma_2)\ \sec\theta_o\}$$

Let x_k be the k-th positive range at which the ray hits the surface, i.e.

$$x_k = \Gamma_1 + k\Gamma_2$$

Since we are dealing with ship generated noise, travel time is only needed between the receiver and these ranges.

$$T(x_k) = \Gamma_3 + k\ \Gamma_4$$

Note that

$$\Lambda\ (x_k) = k$$

and at any range $x > 0$, $\Lambda(x)$ gives the number of reflections the ray has at the surface in the interval $[0,x]$. With the travel time and the number of surface reflections, the phase for a CW signal may be calculated for the raypath.

To compute the quantity $\zeta(x)$, note that within the region

$$\frac{\partial z}{\partial \theta_o} = -\frac{\partial x}{\partial \theta_o} \Big/ \frac{dz}{dx}$$

At the surface, $\frac{\partial z}{\partial \theta_o}$ and $\frac{dz}{dx}$ have discontinuities, for as the ray goes through the reflection these two derivatives change sign. Thus at the surface

$$\left|\frac{\partial z}{\partial \theta_o}\right| = -\mathrm{sgn}\theta_o\ \frac{\partial x}{\partial \theta_o} \Big/ \left|\frac{dz}{dx}\right|$$

since

$$\frac{\partial x_k}{\partial \theta_o} < 0 \text{ for } \theta_o > \bar{\theta}_o$$

$$\frac{\partial x_k}{\partial \theta_o} > 0 \text{ for } \theta_o < -\bar{\theta}_o$$

Now if $\theta(x)$ represents the angle the ray makes at range x (measured downward from the horizontal), then

$$\frac{dz}{dx} = \tan\theta(x)$$

$$\theta(0) = \theta_o$$

and by Snell's law

$$\cos\theta(x_k) = \frac{c_{surf}}{c(z_o)} \cos\theta_o$$

thus

$$|\tan\theta_{(x_k)}| = \sqrt{\cos^{-2}\theta(x_k) - 1}$$

$$= \sqrt{\frac{c^2(z_o)}{c^2_{surf}} \sec^2\theta_o - 1}$$

also

$$\frac{\partial x_k}{\partial \theta_o} = \frac{d\Gamma_1}{d\theta_o} + k\frac{d\Gamma_2}{d\theta_o}$$

Now

$$\frac{d\Gamma_1}{d\theta_o} = -\left(\Gamma_1(\theta_o) \tan\theta_o + \frac{z_o \cot\theta_o \cos\theta_o}{\sqrt{\cos^2\bar{\theta}_o - \cos^2\theta_o}}\right)$$

and

$$\frac{d\Gamma_2}{d\theta_o} = -2\,\text{sgn}\,\theta_o \frac{d\Gamma_1}{d\theta_o}(|\theta_o|) - \pi\gamma^{-1}\sin\theta_o$$

Thus $\frac{\partial x_k}{\partial \theta_o}$ is known.

Let $\lambda(\theta)$ be the ratio of the intensity of the reflected ray to that of the incident ray at the surface, for incident angle θ (measured from horizontal). Then

$$0 < \lambda(\theta) \leq 1$$

Attenuation is usually given in terms of loss αR, in dB. Let β be the corresponding intensity factor, i.e.,

$$10^{(\alpha x/10)} = \beta^x$$

Then the intensity of the raypath is

$$I(x)/I_o = \frac{\cos\,\theta_o}{x|\zeta|\cos\theta} \quad \beta^x \lambda^{\Lambda(x)}$$

so that

$$I(x_k)/I_o = \frac{c_{surf}}{c(z_o)} \quad \frac{|\tan\,\theta_k|}{\left|\frac{\partial x_k}{\partial \theta_o}\right|} \quad \beta^{x_k} \lambda^k$$

Where all quantities on the right-hand side are known.

The third type of raypath is that which, if extended, would strike the bottom. With the infinite bottom loss, this restricts the case to RAP propagation, nearby ships and negative angles:

$$-\frac{\pi}{2} < \theta_o \leq -\tilde{\theta}_o$$

The equations used above may still be used, provided that the range is restricted.

Thus RSR (and RAP) propagation occurs when θ_o is in the set

$$\Xi_1 = (-\tilde{\theta}_o, -\bar{\theta}_o) \cup (\bar{\theta}_o, \tilde{\theta}_o)$$

and RAP propagation only may occur when θ_o is in the set

$$\Xi_2 = (-\frac{\pi}{2}, -\tilde{\theta}_o]$$

Define

$$\Xi = \Xi_1 \cup \Xi_2$$

4. Discrete Ships

To construct a test case for discrete ships, a small number of ships may be positioned at various ranges (preferably chosen so that the angles θ_o of the significant raypaths are known). Then the intensity and phase (if desired) of the raypaths for each ship may be calculated. The distinct ships are summed incoherently.

5. Shipping Densities

If the shipping for the model is in the form of densities rather than discrete ships, the test cases should also use ship densities.

Consider the ambient noise coming in along a horizontal radial sector of width Δ (in radians). Let x be range measured along that radial. Let Ω be the maximum range for that radial. Let $f(\theta,\psi)$ be the density function for shipping, i.e., the expected number of ships per one square unit of area, where the unit is matched with that of the range x. Although f is specified as a function of latitude and longitude, we need it along the radial as a function of range. Let f(x) be this ship density along the radial (figure B3)

Let $\Theta(x)$ be the set of receiver angles θ_o of rays which hit the surface at range x(and, of course, go to the receiver). Let $X(\theta_o)$ be the set of ranges $x > 0$ at which the ray having angle θ_o at the receiver hits the surface, i.e.

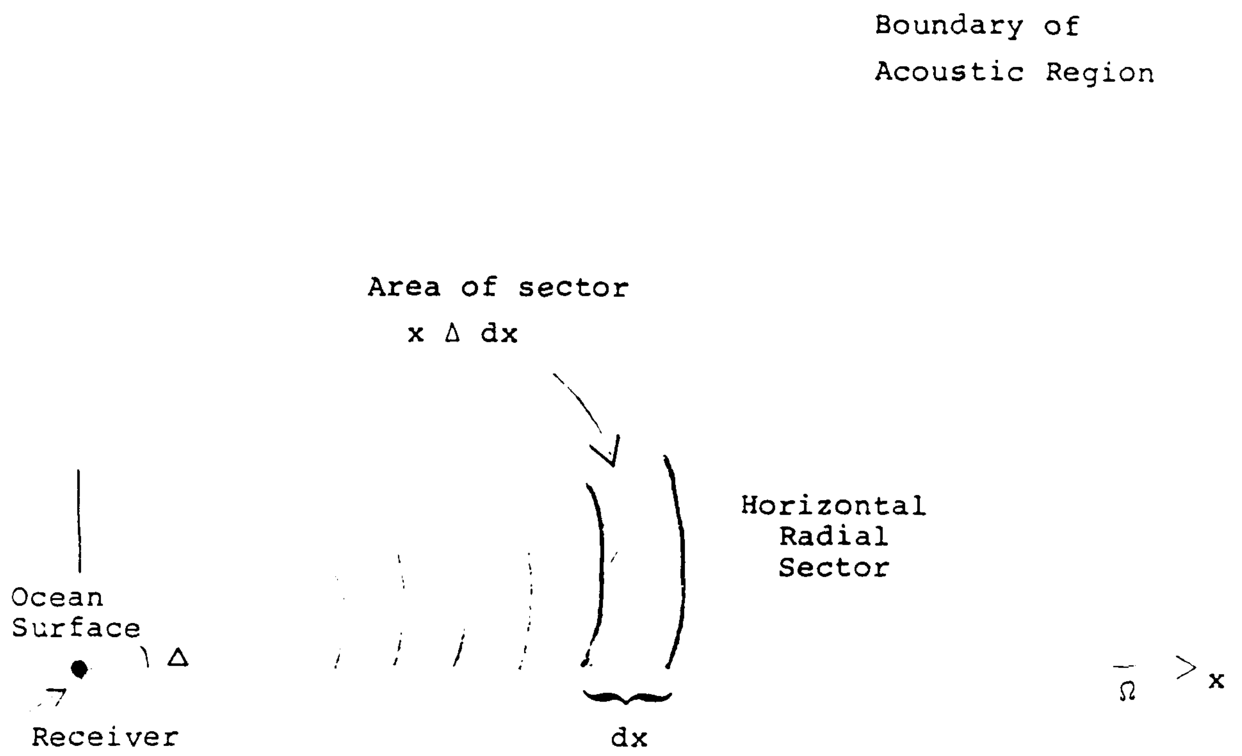


Figure B3 Geometry, Looking Down on Ocean

$$\Theta(x) = \{\theta_o : z(x;\theta_o) = 0\}$$

$$X(\theta_o) = \{x: z(x;\theta_o) = 0\}$$

Notice that $x \varepsilon X(\theta_o)$ only if $x = x_k(\theta_o)$ for some $k > 0$. Thus

$$X(\theta_o) = \{x_k : k>0, x_k \leq \Omega\}$$

Let k_∞ be the largest such index k, i.e.

$$k_\infty (\theta_o) = \max\{k: x_k \varepsilon X(\theta_o)\}$$

Hence

$$k_\infty(\theta_o) = \begin{cases} \Lambda(\Omega) & \text{if } \theta_o \varepsilon \Xi_1 \\ 1 & \text{if } \theta_o \varepsilon \Xi_2 \end{cases}$$

Let s be the level (in intensity) of the 'average' ship in this basin. Then the ambient noise received on this sector is given by the incoherent integral of intensity over range:

$$AN = s\Delta \int_o^\Omega TL(x)\ f(x) x\ dx$$

In the computer model one tends to separate the ship level and add it (in dB) in after the major computation. But for the purpose of this example, we leave the noise in the above form. The transmission loss at range x may be computed by coherent, semi-coherent or incoherent addition of raypaths. Contributions from discrete ships should be added incoherently, and those of an individiual ship added coherently. But when working with ship densities (e.g. 1.7 ships), the model cannot do both. Also, coherent summation tends to produce much fine structure in the transmission loss curve: structure that is much smaller scale than the uncertainties in ship positions (historical ship distribution are given by 1^o square).

The test cases given will be for an ambient noise model using incoherent summation in the transmission loss module. Thus TL(x) is assumed to be incoherent. A different formulation is needed for coherent addition. By breaking transmission loss into an intensity summation over all raypaths, and changing the variable of integration, we have

$$AN = s\Delta \int_0^{\Omega} TL(x) f(x) x \, dx$$

$$= s\Delta \int_0^{\Omega} \sum_{\theta_o \varepsilon \Theta} TL(x;\theta_o) \; f(x) x \, dx$$

$$= s\Delta \int_{\Theta} \sum_{x \varepsilon X} TL(x;\theta_o) \; f(x) x \left| \frac{dx}{d\theta_o} \right| \; d\theta_o$$

$$= s\Delta \int_{\Theta} \sum_{x \varepsilon X} \frac{\beta^{x} \lambda^{\Lambda(x)-1} \cos\theta_o}{x |\zeta| \cos\theta} \; x f(x) \left| \frac{\partial z}{\partial \theta_o} \right| |\cot\theta| \, d\theta_o$$

$$= s\Delta \int_{\Theta} \sum_{k=1}^{k_\infty} \frac{\beta^{x_k} \lambda^{k-1} \cos\theta_o}{x_k |\zeta| \cos\theta_{surf}} \; x_k \; f(x_k) \, |\zeta| \, |\cot\theta_{surf}| \; d\theta_o$$

$$= s\Delta \int_{\Theta} \sum_{k=1}^{k_\infty} \frac{\cos\theta_o}{|\sin\theta_{surf}|} \, f(x_k) \; \beta^{x_k} \lambda^{k-1} d\theta_o$$

$$= s\Delta \int_{\Theta} \frac{\cos\theta_o}{|\sin\theta_{surf}|} \sum_{k=1}^{k_\infty} \beta^{x_k} \lambda^{k-1} f(x_k) \, d\theta_o$$

This is the type of approach taken by Talham (Ref. 6), and used in FANM. Unlike these, however, the shipping density f need not constant over a ray cycle. The above derivation assumes that

a) $\frac{\partial x}{\partial \theta_o} \neq 0$ for any $x \in X(\theta_o)$

b) $\frac{\partial x}{\partial \theta_o} \neq \infty$ except on a θ_o-set of measure zero

c) the ship density $f(x)$ is continuous

For the parabolic profile, the first two hold for all $\theta_o \varepsilon\ \Xi$. The third may be relaxed to include simple functions (i.e. finite linear combinations of step functions).

The intensity does not appear explicitly, which seems to violate common sense. However the spreading effects are used, for the measure of integration (shipping density over x_k times Lebesque measure over θ_o) implicitly contains the geometric spreading.

From this equation for ambient noise a number of test cases may be developed for various ship densities, etc. Two will be given.

Test case 1'

Assumptions:

a) The maximum range Ω is large (e.g. over 1500km)

b) The surface loss is significant, so that RSR rays from the maximum range are attenuated signficantly.

c) Assumptions a and b may be quantitatively expressed as

$$\lambda^m << 1$$

where

$$m = k_\infty(\bar{\theta}_o) = \left[\frac{\Omega\gamma}{2\pi\cos\bar{\theta}_o} - \frac{1}{4}\right]$$

is the minimum number of surface reflections at maximum range Ω.

d) The frequency is low, so that the surface reflection loss is more significant than the attenuation over a ray cycle. Using the attenuation coefficient of Thorp (Ref p) it is seen that at 100hz the attenuation loss over 70kyd is only 1.6%, hence this should not be a serious restriction.

e) The ship density is constant along the radial out to range Ω.

With these assumptions, three approximations will be made. The first is that the attenuation over any ray cycle having $\theta_o \varepsilon \Xi_1$ will be approximated by a constant. This constant may be taken as the geometric mean of the endpoints, viz

$$\nu = \left(\beta^{\Gamma_2(\bar{\theta}_o)} \cdot \beta^{\Gamma_2(\tilde{\theta}_o)}\right)^{1/2}$$

The error committed in making this approximation depends on the values of c_{surf}, $c(z_o)$ and c_{bott}, and the relative loss due to reflection versus attenuation. Usually this will be small.

The second approximation is obtained by running the summation to infinity. This increases the ambient noise by a value between 0 and

$$-10\log_{10}(1-\lambda^m)$$

dB, which may be made very small by proper choice of λ.

The third approximation is short-range attenuation. The attenuation between the receiver and the first surface bounce for RSR raypaths may be approximated by the geometric mean of the four endpoints:

$$\nu' = \left(\beta^{-\Gamma_1(\tilde{\theta}_o)}\ \beta^{-\Gamma_1(\bar{\theta}_o)}\ \beta^{\Gamma_2(\bar{\theta}_o) + \Gamma_1(\bar{\theta}_o)}\ \beta^{\Gamma_2(\tilde{\theta}_o) + \Gamma_1(\tilde{\theta}_o)}\right)^{1/4}$$

$$= \sqrt{\nu}$$

For RAP paths of greater angles, i.e., those in Ξ_2, it is assumed that the attenuation may be ignored. Since we are dealing with close-to-vertical raypaths, their length is very small.

Since the attenuation involved in this partial ray cycle is small, the effect should be minimal. Note that if the model is run with $\beta = 1$ (i.e., no attenuation), then two of these assumptions are superfluous.

These ambient noise integral may be expressed as an integral over Ξ_1 and one over Ξ_2, call the portions AN_1 and AN_2. Then

$$AN_1 = s\Delta \int_{\Xi_1} \frac{\cos\theta_0}{|\sin\theta_{surf}|} \sum_{k=1}^{k_\infty} \beta^{\Gamma_1 + k\Gamma_2} \lambda^{k-1} f \, d\theta_0$$

$$\approx s\Delta \int_{\Xi_1} \frac{\cos\theta_0}{|\sin\theta_{surf}|} \nu' \sum_{k=1}^{k_\infty} \nu^{k-1} \lambda^{k-1} f \, d\theta_0$$

$$\approx s\Delta \int_{\Xi_1} \frac{\cos\theta_0}{|\sin\theta_{surf}|} \frac{\nu' f}{1-\nu\lambda} \, d\theta_0$$

$$= \frac{s\Delta\nu' f}{1-\nu\lambda} \int_{\Xi_1} \frac{\cos\theta_0}{\sqrt{1-n^2\cos^2\theta_0}} \, d\theta_0$$

where

$$n = c_{surf}/c(z_0) > 1$$

Now Ξ_1 is the union of two intervals, so the integral is split into pieces. Now the first is

$$\int_{\bar{\theta}_o}^{\tilde{\theta}_o} \frac{\cos\theta_o}{\sqrt{1-n^2\cos^2\theta_o}}\, d\theta_o$$

$$= \frac{1}{n} \ln \left| n \sin\theta_o + \sqrt{1-n^2\cos^2\theta_o} \right| \Bigg|_{\bar{\theta}_o}^{\tilde{\theta}_o}$$

$$= \frac{1}{n} \ln \left| \frac{\sin\tilde{\theta}_o + n^{-1} \sin\tilde{\theta}_{surf}}{\sin\bar{\theta}_o} \right|$$

The second integral, over $(-\tilde{\theta}_o, -\bar{\theta}_o)$ has the same value. Therefore,

$$AN_1 = \frac{s\Delta\nu' f}{(1-\nu\lambda)n} \ln \left| \frac{\sin\tilde{\theta}_o + n^{-1}\sin\tilde{\theta}_{surf}}{\sin\bar{\theta}_o} \right|$$

The second component, due to local shipping is

$$AN_2 = s\Delta \int_{\Xi_2} \frac{\cos\theta_o}{|\sin\theta_{surf}|} \beta^{|\Gamma_1(|\theta_o|)|} f\, d\theta_o$$

$$\approx s\Delta \int_{-\pi/2}^{-\tilde{\theta}_o} \frac{\cos\theta_o}{\sqrt{1-n^2\cos^2\theta_o}} f\, d\theta_o$$

$$= \frac{s\Delta f}{n} \ln \left| \frac{1 + n^{-1}}{\sin\tilde{\theta}_o + n^{-1}\sin\tilde{\theta}_{surf}} \right|$$

Therefore we have a relatively simple expression for the ambient noise for this case.

Test Case β'

Most ambient noise models using shipping densities assume the density is uniform over a small range, and at the end of that range it jumps to another value, remaining uniform on the next small interval. Such a function is, of course, a finite linear combination of step functions, i.e., a Simple function. As noted above, the ambient noise integral is valid when f(x) is a Simple function. Thus given a Simple function for the shipping, any surface reflection and the attenuation appropriate for the frequency, the ambient noise integral may be evaluated. This may be done analytically, but the number of logarithms and inverse trigonometric functions required for the general expression is incredible. Thus it is easier to evaluate the exact ambient noise integral using numeric quadrature rather than using closed-form expressions. Unlike case α', no approximations are made (except the error introduced by the quadrature which may be made arbitrarily small). Using a shipping density on a grid (the standard method of feeding the models), one can thus transform the two-dimensional Simple function to one-dimensional Simple functions along various great circle radials from the receiver, and obtain the ambient noise along each radial by numeric quadrature. This appears to be an ideal exact solution to use for model evaluation.

APPENDIX C: Nine Point Field Analysis

The ambient noise at a point is the acoustic energy arriving at that point. As such, ambient noise is subject to the laws of physics. Hence, the values of the ambient noise at the points which make up an ambient noise field should satisfy the mathematical representations of the physical laws (e.g., the values of the ambient noise field will be a solution to the wave equation).

Ambient noise models produce estimated values for points within an ambient noise field. In order for these estimates to be considered reasonable, it is a necessary condition that they also satisfy the mathematical representations of the physical laws. In particular given the ambient noise at each point in the array of points shown in Figure C-1, the ambient noise at point m will in some ways be related to the ambient noise at the points 0 through 7.*

The purpose of this Appendix is to determine what some of these relationships are. To fulfill this purpose several situations will be examined. First, the relationships between the directional ambient noise at two points will be considered. Using the information gathered from that situation, the case of directional ambient noise at nine points will be considered. Next, the relationships between the omnidirectional ambient noise at two points will be examined. Finally, some bounds for the omnidirectional ambient noise at nine points will be determined.

There are several approaches that can be taken when considering the problem of determining some of the relationships between the directional ambient noise at two points. The approach that will be used here is to first consider the problem in an idealized case.

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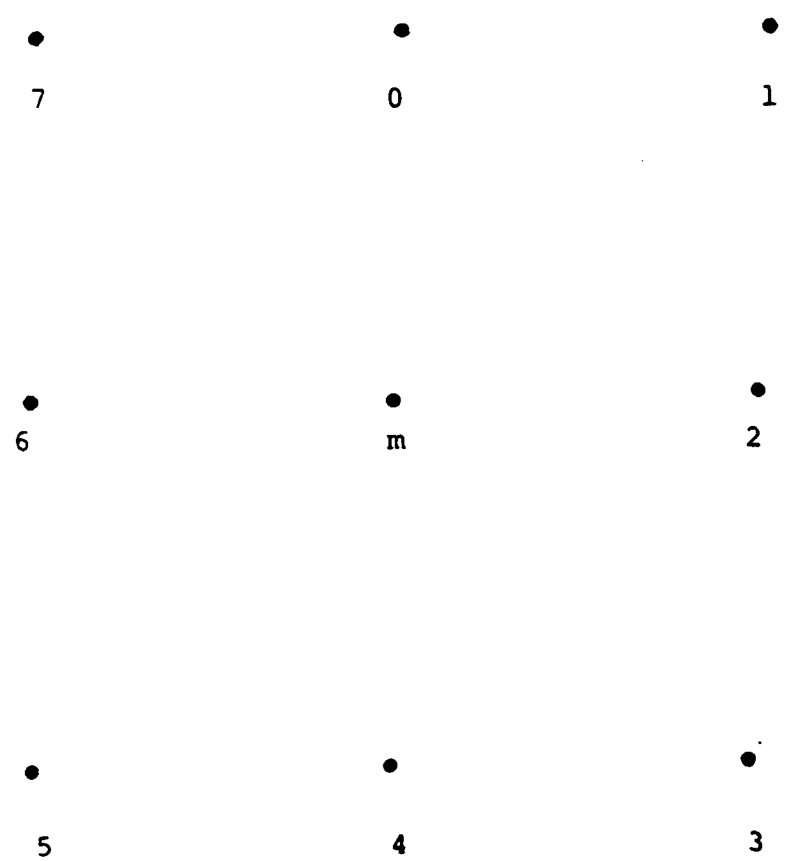
*Note: This Appendix will only consider horizontal ambient noise. For vertical ambient noise one can formulate a similar problem, however, different tools must be developed to solve it. The technique given in ref. q should provide a basis for some vertical directionality tests when the shipping field is smooth (i.e., when range-averaging applies).

Figure C-1: Array of Points



From this a general solution can then be used to determine what the relationships will be as various assumptions are made about the environment.

For the idealized case imagine the earth covered by water with depth excess everywhere so that there are no acoustic obstructions (such as continents, islands, sea mounts, etc.). Choose two points and denote them by p_m and p_1. Select a direction θ, $\theta \varepsilon, [0,2\pi)$ (i.e., an angle measured from north).

The ambient noise at the point p_1 in the direction θ, denoted $AN_1(\theta)$, will be given by

$$AN_1(\theta) = \int_{\ell_1(\theta)} TL_1(p)\,dF(p)$$

In this equation, $TL_1(p)$ is the transmission loss between the points p and p_1. The symbol

$$\int_{\ell_1(\theta)} dF(p)$$

is used to indicate that a Stieltjes integral with respect to the distribution of noise sources, F, is taken along $\ell_1(\theta)$. $\ell_1(\theta)$ is the directed great circle passing through p_1 and oriented in the direction θ. (See Figure C-2.)

The equation for the ambient noise at the point p_m in the direction θ, denoted $AN_m(\theta)$, is

$$AN_m(\theta) = \int_{\ell_m(\theta)} TL_m(p)\,dF(p)$$

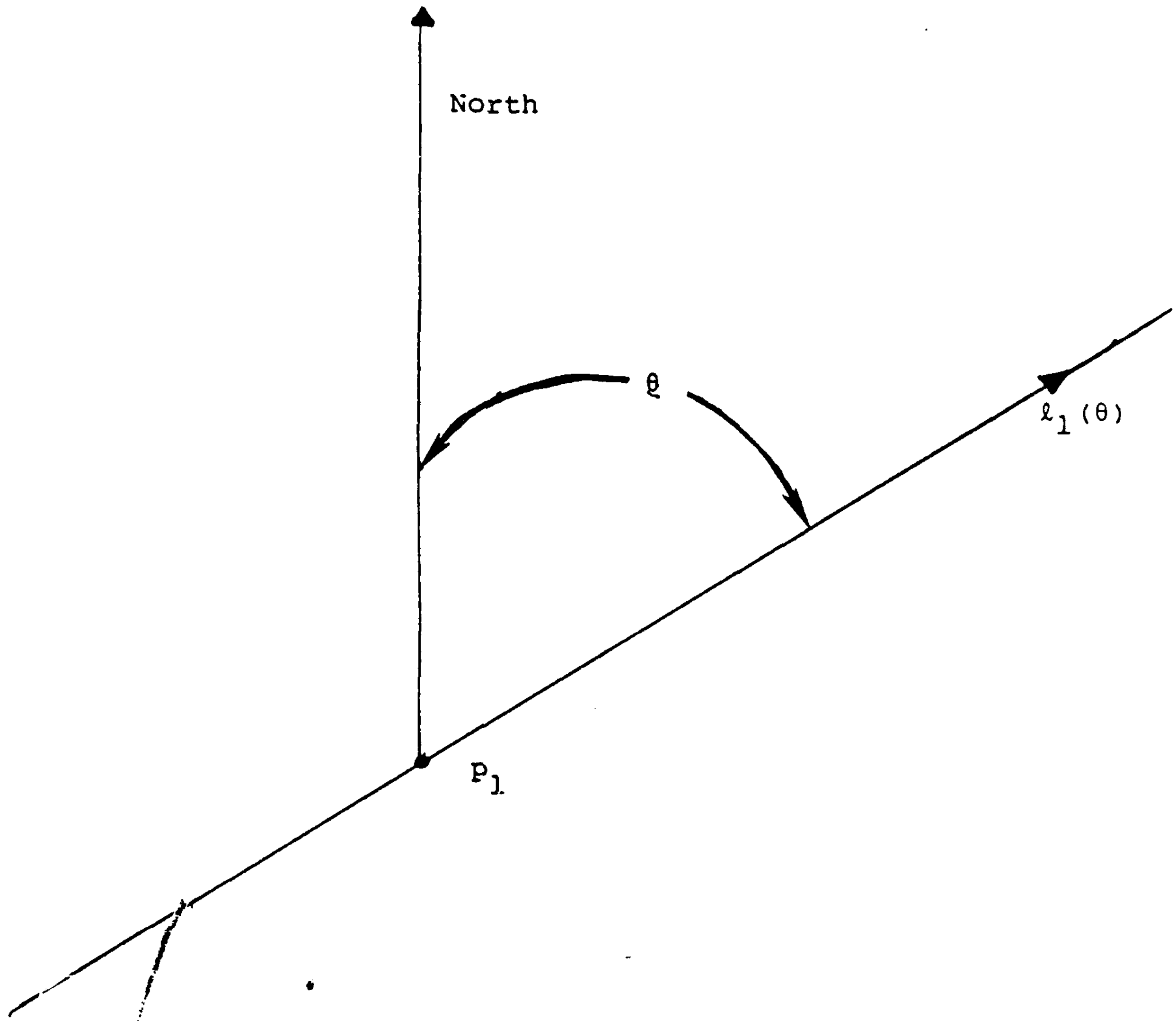


Figure C-2: p_1 and $\ell_1(\theta)$

where $\ell_m(\theta)$ and $TL_m(p)$ are defined in the same ways as their counterparts for p_1. (See Figure C-3).

Thus, are exact expression for the relationship between the ambient noise levels at p_1 and p_m in the direction θ is,

$$AN_1(\theta) - AN_m(\theta) = \int_{\ell_1(\theta)} TL_1(p)\,dF(p) - \int_{\ell_m(\theta)} TL_m(p)\,dF(p)$$

The magnitude of this difference is governed by the physics of the situation (e.g., the distance between p_1 and p_m, the distributions of noise sources, the environment giving rise to TL_1 and TL_m etc.).

In ambient noise models $AN_1(\theta)$ and $AN_m(\theta)$ are estimated rather than known. Similarly, the environment is not known exactly. What will be known when using noise models is the distribution of noise sources and reasonable upper bounds for the difference between the two transmission loss functions. Using the equation above and these two known factors, bounds will be palced on the difference between the ambient noise estimates.

First, for convenience of notation several definitions will be made. For any two points p' and p'' let $\rho(p', p'')$ be the great circle distance between the two points. Let R be defined as the linear transformation on the surface of the sphere with the three properties:

(i) R preserves distances,

(ii) R: $\ell_m(\theta) \rightarrow \ell_1(\theta)$ preserving the orientation,

and (iii) $R(p_m) = p_1$.

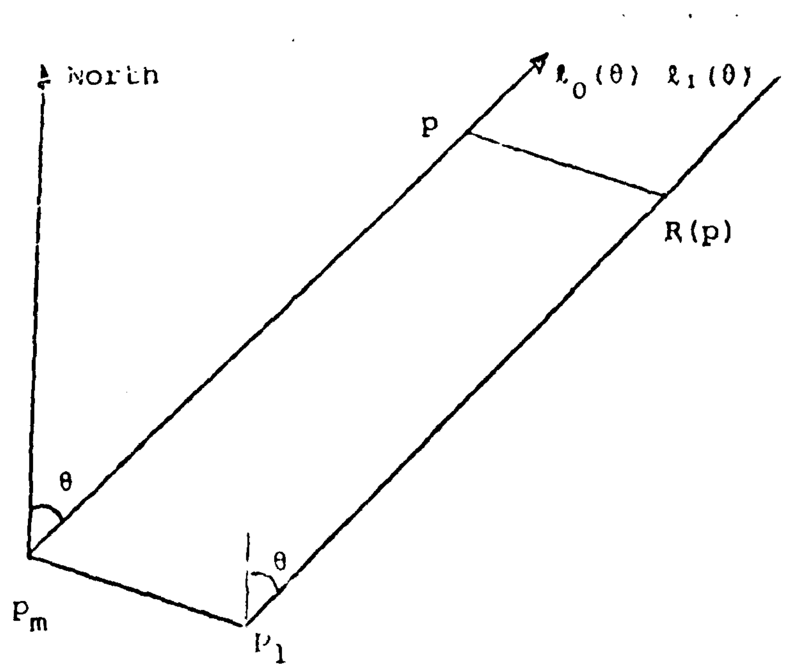


Figure C-3: Two Point Directional Noise in Idealized Ocean

This transformation will be unique. Also, it will have an inverse transformation. Let R^{-1} denote this inverse. Next, define the function $\Delta_{p_i} F$ by

$$\Delta_{p_i} F(p) = F(p) - F(R^{-1}(p))$$

Note that while $F(p) \geq 0$ for all p_i, $\Delta_{p_i} F$ may be either positive or negative. Hence, $\Delta_{p_i} F$ will induce a signal measure when integration is done with respect to $\Delta_{p_i} F$.

Now, using the equation for the difference in the ambient noise levels,

$$AN_i(\theta) - AN_m(\theta) = \int_{\ell_i(\theta)} TL_i(p)dF(p) - \int_{\ell_m(\theta)} TL_m(p)dF(p)$$

$$= \int_{\ell_i(\theta)} TL_i(p)\, d\left\{ F(p) - F(R^{-1}(p)) + F(R^{-1}(p)) \right\} - \int_{\ell_m(\theta)} TL_m(p)dF(p)$$

$$AN_i(\theta) - AN_m(\theta) = \int_{\ell_i(\theta)} TL_i(p)\, d\Delta_{p_i} F(p) + \int_{\ell_i(\theta)} TL_i(p)\, dF(R^{-1}(p)) - \int_{\ell_m(\theta)} TL_m(\theta) dF(p)$$

$$= \int_{\ell_i(\theta)} TL_i(p)\, d\Delta_{p_i} F(p) + \int_{\ell_m(\theta)} TL_i(R(p))dF(p) - \int_{\ell_m(\theta)} TL_m(\theta) dF(p)$$

$$AN_i(\theta) - AN_m(\theta) = \int_{\ell_i(\theta)} TL_i(p)\, d\Delta_{p_i} F(p) + \int_{\ell_m(\theta)} \left\{ TL_i(R(p)) - TL_m(p) \right\} dF(p)$$

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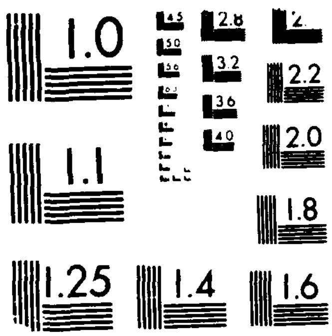

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This final equation expresses the difference between the ambient noise levels at the two points in terms of the variations in the noise source distribution in the direction θ and the transmission loss functions. That is, the first term on the right hand side of the equation is the difference in the ambient noise that is due to variations in the distribution of noise sources along $\ell_1(\theta)$ and $\ell_m(\theta)$. The second term is the difference in the ambient noise that is due to the variations between the transmission loss functions TL_1 and TL_m.

The difference in the estimates for the directional ambient noise levels at p_1 and p_m should then be bounded by

$$|AN_1(\theta)-AN_m(\theta)| \leq \int_{\ell_1(\theta)} TL_1(p)\,d|\Delta_{p_1} F(p)| + \int_{\ell_m(\theta)} |TL_1(R(p))-TL_m(p)|\,dF(p)$$

Since this is an idealized situation, it is not worthwhile to examine this bound more closely. Instead, a general realistic solution for the problem of comparing ambient noise at two points will now be considered.

The most general case is the same as the idealized case except that acoustic obstructions are considered. For this situation define the points $\hat{p}_m$, $\hat{p}_1$, and p^* (see Figure C-4). $\hat{p}_m$ is the point at which $\ell_m(\theta)$ first meets an acoustical obstruction. $\hat{p}_1$ is defined in the same way for $\ell_1(\theta)$. p^* is the point on $\ell_m(\theta)$ which is furthest from p_m such that if p is between p_m and p^* then there is no acoustical obstruction between p and p_1 and for which

$$\rho(p_1, R(p)) \leq \rho(p_1, \hat{p}_1)$$

The expression of the difference between the ambient noise levels at the two points in this situation depends upon the relative positions of p_m, $\hat{p}_1$, and p^*. There are three possible expressions:

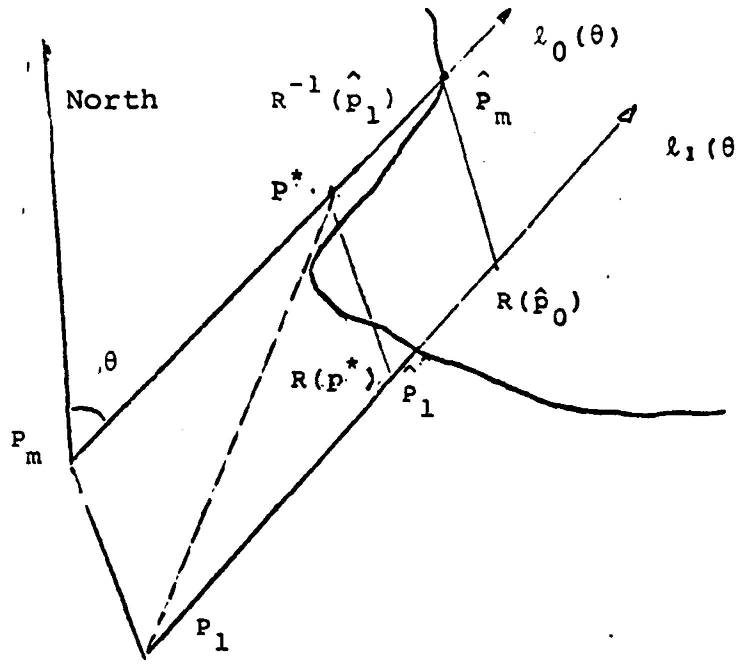


Figure C-4: Two Point Directional Noise in an Idealized Ocean

Case 1: $\rho(p_m,p^*)<\rho(p_1,\hat{p}_1)$ and $\rho(p_m,p^*)<\rho(p_m,\hat{p}_m)$

In this case the difference in ambient noise is given by

$$AN_1(\theta)-AN_m(\theta) = \int_{\substack{p_1 \\ [\ell_1(\theta)]}}^{\hat{p}_1} TL_1(p)\,dF(p) - \int_{\substack{p_m \\ [\ell_m(\theta)]}}^{\hat{p}_m} TL_m(p)\,dF(p)$$

where $\int_{\substack{p' \\ \ell(\theta)}}^{p''}$ indicates that the Stieltjes line integral is to be taken along the line $\ell(\theta)$ from the point p' to the point p''. This can be broken down to

$$AN_1(\theta)-AN_m(\theta) = \int_{\substack{R(p^*) \\ [\ell_1(\theta)]}}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{\substack{p_1 \\ [\ell_1(\theta)]}}^{R(p^*)} TL_1(p)\,dF(p) - \int_{\substack{p_m \\ [\ell_m(\theta)]}}^{p^*} TL_m(p)\,dF(p)$$

$$- \int_{p^*}^{\hat{p}_o} TL_m(p)\,dF(p)$$

which reduces to

$$AN_1(\theta)-AN_m(\theta) = \int_{\substack{R(p^*) \\ [\ell_1(\theta)]}}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{\substack{p_1 \\ [\ell_1(\theta)]}}^{R(p^*)} TL_1(p)\,\underset{p_1}{d\Delta F(p)} + \int_{\substack{p_m \\ [\ell_m(\theta)]}}^{p^*} \left\{TL_1(R(p))-TL_m(p)\right\}\,dF(p$$

$$- \int_{\substack{p^* \\ [\ell_m(\theta)]}}^{\hat{p}_m} TL_m(p)\,dF(p)$$

(See Appendix G for a complete derivation of this equation.) As in the idealized situation, this equation indicates the source of the difference between the two ambient noise levels in terms of quantities that can be bounded when using ambient noise models. The second term on the right hand side of the equation is the difference due to the variation in the distribution of noise sources along $\ell_1(\theta)$ and $\ell_m(\theta)$. The third term is the difference which is due to the variation between the transmission loss functions TL_1 and TL_m. The first and fourth terms together are the difference that is caused by acoustical obstructions.

The difference in the estimates of the ambient noise levels at the two points as produced by ambient noise models can, thus, be bounded by

$$|AN_1(\theta)-AN_m(\theta)| \leq \int_{\substack{R(p^*) \\ [\ell_1(\theta)]}}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{\substack{p_1 \\ [\ell_1(\theta)]}}^{R(p^*)} TL_1(p)\,d|\Delta F(p)|_{p_1}$$

$$+ \int_{\substack{p_m \\ [\ell_m(\theta)]}}^{p^*} |TL_1(R(p))-TL(p)|\,dF(p) + \int_{\substack{p^* \\ [\ell_m(\theta)]}}^{\hat{p}_m} TL_m(p)\,dF(p)$$

This bound will be considered further after the second and third cases have been presented.

Case 2: $\rho(p_1,\hat{p}_1) = \rho(p_m,p^*)$ and $p^*<p_m$

As in Case 1 the difference between the ambient noise levels at the two points is:

$$AN_1(\theta)-AN_m(\theta) = \int_{p_1 [\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p) - \int_{p_m [\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

which in this case reduces to

$$AN_1(\theta)-AN_m(\theta) = \int_{p_1 [\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,d\Delta_{p_1} F(p) + \int_{p_m [\ell_m(\theta)]}^{p^*} \left\{TL_1(R(p))-TL_m(p)\right\} dF(p)$$

$$-\int_{p^* [\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

Note that in this case $R(p^*) = \hat{p}_1$ and $p^* = R^{-1}(\hat{p}_1)$. The terms in this equation are due to the same factors as the terms in the analagous equation in Case 1. In this case, however, one of the terms due to the acoustical obstructions is zero.

Here, the bound for the difference in the estimated ambient noise levels is,

$$|AN_1(\theta)-AN_m(\theta)| \leq \int_{p_1 [\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,d|\Delta F(p)| + \int_{p_m [\ell_m(\theta)]}^{p^*} |TL_1(R(p))-TL_m(p)|\,dF(p)$$

$$-\int_{p^* [\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

Case 3: $\rho(p_m, \hat{p}_m) < \rho(p_1, \hat{p}_1)$ and $p^* = \hat{p}_m$

As in the other two cases the difference in the ambient noise at the two points is,

$$AN_1(\theta) - AN_m(\theta) = \int_{p_1\,[\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p) - \int_{p_m\,[\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

which reduces to

$$AN_1(\theta) - AN_m(\theta) = \int_{R(p^*)\,[\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{p_1\,[\ell_1(\theta)]}^{R(p^*)} TL_1(p)\,d\Delta F_{p_1}(p) + \int_{p_m\,[\ell_m(\theta)]}^{p^*} \left\{TL_1(R(p)) - TL_m(p)\right\} dF(P)$$

This is similar to the analogous equations derived in Cases 1 and 2. Like Case 2, one of the terms due to acoustical obstruction is zero in this Case.

The bound for the ambient noise estimates similar to those presented in the previous Cases is,

$$|AN_1(\theta) - AN_m(\theta)| \le \int_{R(p^*)\,[\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{p_1\,[\ell_1(\theta)]}^{R(p^*)} TL_1(p)\,d|\Delta F_{p_1}(p)| + \int_{p_m\,[\ell_m(\theta)]}^{p^*} |TL_1(R(p)) - TL_m(p)|\,dF(p)$$

Therefore, in this general case the difference in the estimates of the ambient noise levels at the two points as produced by ambient noise models can be bounded by

$$|AN_1(\theta)-AN_m(\theta)| \le \int_{\substack{R(p^*)\\ [\ell_1(\theta)]}}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{\substack{p_1\\ [\ell_1(\theta)]}}^{R(p^*)} TL_1(p)\,d|\Delta F_{p_1}(p)| + \int_{\substack{p_m\\ [\ell_m(\theta)]}}^{p^*} |TL_1(R(p))-TL(p)|\,dF(p)$$

$$+\int_{\substack{p^*\\ [\ell_m(\theta)]}}^{\hat{p}_m} TL_m(p)\,dF(p)$$

with the possibility that one or both of the terms due to the acoustical obstructions could be zero.

Now, when actually estimating a bound for the difference in the ambient noise estimates, each of the terms in the equation above would be bounded. Further, the bound for each term that would be used would not be the bound for a particular pair p_m and p_1. Rather it would generally be a bound for all possible points p_1 given that $\rho(p_m, p_1) < r$ for some fixed p_m and some constant r.

Thus, for the term representing the difference in the estimates due to the variations in the distribution of noise sources,

$$\int_{\substack{p_1\\ [\ell_1(\theta)]}}^{R(p^*)} TL_1(p)\,d|\Delta F_{p_1}(p)| = \left[\max_{p\in[p_1,R(p^*)]}\left\{TL_1(p)\right\}\right] \int_{\substack{p_1\\ [\ell_1(\theta)]}}^{R(p^*)} |\Delta F_{p_1}(p)|$$

$$= \left[\operatorname*{Max}_{p\in[p_1,R(p^*)]}\left\{TL_1(p)\right\}\right]\left(\operatorname*{TV}_{[p_1,R(p^*)]}(\Delta F_{p_1}(p))\right)$$

where $\underset{[p_1, R(p^*)]}{TV_{p_1}}(\Delta F(p))$ is the total variation in $\Delta F(p)$ along the great circle arc between p_1 and $R(p^*)$. Let ε_ρ denote the bound for this term on the set $A = \{p_1 | \rho(p_m, p_1) < r\}$. ε_ρ is given by

$$\varepsilon_\rho = \max_{p_1 \varepsilon A} \left\{ \left[\max_{p \varepsilon [p_1, R(p^*)]} \left(TL_1(p) \right) \right] \left[\underset{[p_1, R(p^*)]}{TV_{p_1}} (\Delta F(p)) \right] \right\}$$

For the term representing the difference in the estimates due to the variation in the transmission loss functions,

$$\int_{\substack{p_m \\ [\ell_m(\theta)]}}^{p^*} |TL_1(R(p)) - TL_1(p)| dF(p)$$

$$\leq \left[\max_{p \varepsilon [p_m, p^*]} \left\{ |TL_1(R(p)) - TL_m(p)| \right\} \right] \int_{\substack{p_m \\ [\ell_m(\theta)]}}^{p^*} dF(p)$$

$$= |F(p^*) - F(p_m)| \left[\max_{p \varepsilon [p_m, p^*]} \left\{ |TL_1(R(p)) - TL_m(p)| \right\} \right]$$

Let ε_t denote the bound for this term on the set A. ε_t is given by

$$\varepsilon_t = \max_{p_1 \varepsilon A} \left\{ |F(p^*) - F(p_m)| \left[\max_{p \varepsilon [p_m, p^*]} \left\{ |TL_1(R(p)) - TL_m(p)| \right\} \right] \right\}$$

These two expressions, ε_ρ and ε_t, are generally estimated rather than computed.

The two terms representing the difference in the estimates due to acoustical obstructions are lumped together. Let ε_a denote the bound for these two terms on the set A. Since ε_ρ and ε_t are not tight bounds and since the difference in the estimates due to the acoustical obstructions is usually trivial when compared to the error in ε_ρ and ε_t, ε_a will almost always be set equal to zero. When not zero, ε_a is estimated instead of computed.

Thus, if no restrictions are placed on the environment, then

$$|AN_l(\theta) - AN_m(\theta)| \leq \varepsilon_\rho + \varepsilon_t + \varepsilon_a$$

Other bounds can be obtained from this by making assumptions about the environment. If it is assumed that p_m is sufficiently distant from all acoustic obstructions, then

$$|AN_l(\theta) - AN_m(\theta)| \leq \varepsilon_\rho + \varepsilon_t$$

If in addition to this it is assumed that $TL_l = TL_m$ for all $p_l \varepsilon A$, then

$$|AN_l(\theta) - AN_m(\theta)| \leq \varepsilon_\rho$$

Note that this assumption will greatly effect the estimates $AN_l(\theta)$ and $AN_m(\theta)$. However, since internal model consistency is being tested and not model accuracy, this effect is not important. If instead of a uniform transmission loss, it is assumed that the distribution of noise sources is a constant distribution, then

$$|AN_l(\theta) - AN_m(\theta)| \leq \varepsilon_t$$

Even though this is an unreasonable assumption for a real-world situation, it is nonetheless useful when testing internal model consistency.

This completes the discussion of the two point directional ambient noise case. The nine point directional field analysis will be considered next.

The question was posed at the beginning of this appendix of how the ambient noise at the point m in figure C-1 is related to the ambient noise at the points 0 through 7. (Denote these points by p_m, p_0, p_1, ... p_7, respectively.) When testing internal model consistency, this question is equivalent to asking what can be said about the expression

$$|AN_m(\theta) - \sum_{i=1}^{7} \omega_i AN_i(\theta)|$$

for appropriate choices of ω_i, i = 0, 1, ..., 7.

One way to answer this is to use the bounds derived above. Let r be the distance between p_m and p_i for even i. (See figure C-5.) Let $\varepsilon_\rho(r)$ and $\varepsilon_t(r)$ denote the bounds derived above for the distance r. Define ω_i, i = 0, 1, ..., 7 by

$$\omega_i = \begin{cases} [\Omega(\varepsilon_\rho(r) + \varepsilon_t(r))]^{-1} & \text{if i is even} \\ [\Omega(\varepsilon_\rho(\sqrt{2}r) + \varepsilon_t(\sqrt{2}r))]^{-1} & \text{if i is odd} \end{cases}$$

where $\Omega = 4\left[\frac{1}{\varepsilon_\rho(r) + \varepsilon_t(r)} + \frac{1}{\varepsilon_\rho(\sqrt{2}r) + \varepsilon_t(\sqrt{2}r)}\right]$ to ensure that $\sum_{i=0}^{7} \omega_i = 1$. For this choice of ω_i

$$|AN_m(\theta) - \sum_{i=0}^{7} \omega_i AN_i(\theta)| \leq \frac{8}{\Omega}$$

(see appendix G for a demonstration of this.) This bound can be improved. Let ε_i be the bound for the difference in the estimates of the ambient noise levels at p_m and p_i (i.e., the bound for the two specific points p_m and p_i). Define ω_i, i = 0, 1, ..., 7 by

$$\omega_i = [\varepsilon_i \Omega']^{-1} \qquad \text{for } i = 0, 1, \ldots, 7$$

when $\Omega' = \sum_{i=0}^{7} \varepsilon_i^{-1}$. For this choice of ω_i

$$|AN_m(\theta) - \sum_{i=0}^{7} \omega_i AN_i(\theta)| \leq \frac{8}{\Omega'}$$

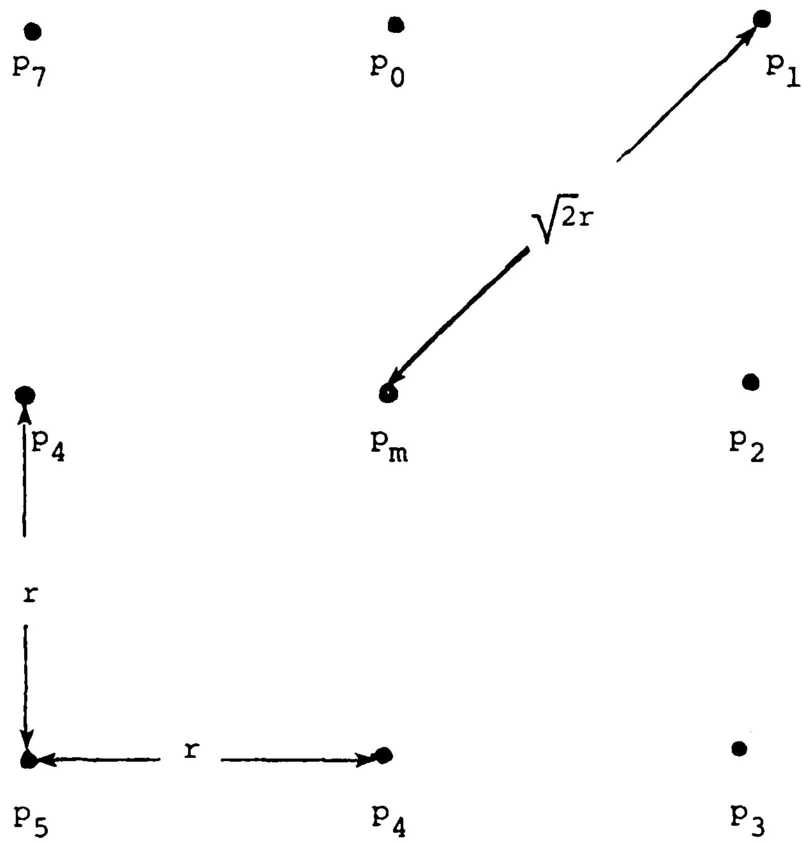


Figure C-5: Distances with the Array of Points

This is an improvement over the previous bound since $\Omega \leq \Omega'$ (however, more work is required to obtain $\{\omega_i\}$ in this case).

There are other methods of finding bounds for the level of ambient noise at p_m. Assume that the transmission loss function for each point is monotonic. (This assumption is made in many of the current ambient noise models.) Also, assume that the distribution of noise sources is identically zero for an area around the array of points.

If the distribution of noise sources is continuous, then upper and lower bounds for the ambient noise at p_m in the direction θ are

$$AN_m(\theta) \leq w_1 AN_k(\theta) + w_2 AN_{(k+1)\bmod 8}(\theta)$$

and

$$AN_m(\theta) \geq w_1 AN_{(k+4)\bmod 8}(\theta) + w_2 AN_{(k+5)\bmod 8}(\theta)$$

where $k = \left[\frac{4\theta}{\pi}\right]$ (i.e., the greatest integer less than or equal to $\frac{4\theta}{\pi}$), $w_2 = \frac{4\theta}{\pi} - k$, and $w_1 = 1 - w_2$.

For a discrete distribution of noise sources similar bounds can be determined. Let the distance from the edge of the array of points to the nearest ship be kr (see figure C-6). Let α be an angle such that

$$|\alpha| \leq 2 \arcsin(2k)^{-1}$$

The upper and lower bounds for the ambient noise at p_m in the direction θ are

$$AN_m(\theta) \leq \max_{\theta_1,\theta_2} \left[w_1 AN_k(\theta_1) + w_2 AN_{(k+1)\bmod 8}(\theta_2)\right]$$

and

$$AN_m(\theta) \geq \max_{\theta_1,\theta_2} \left[w_1 AN_{(k+4)\bmod 8}(\theta_1) + w_2 AN_{(k+5)\bmod 8}(\theta_2)\right]$$

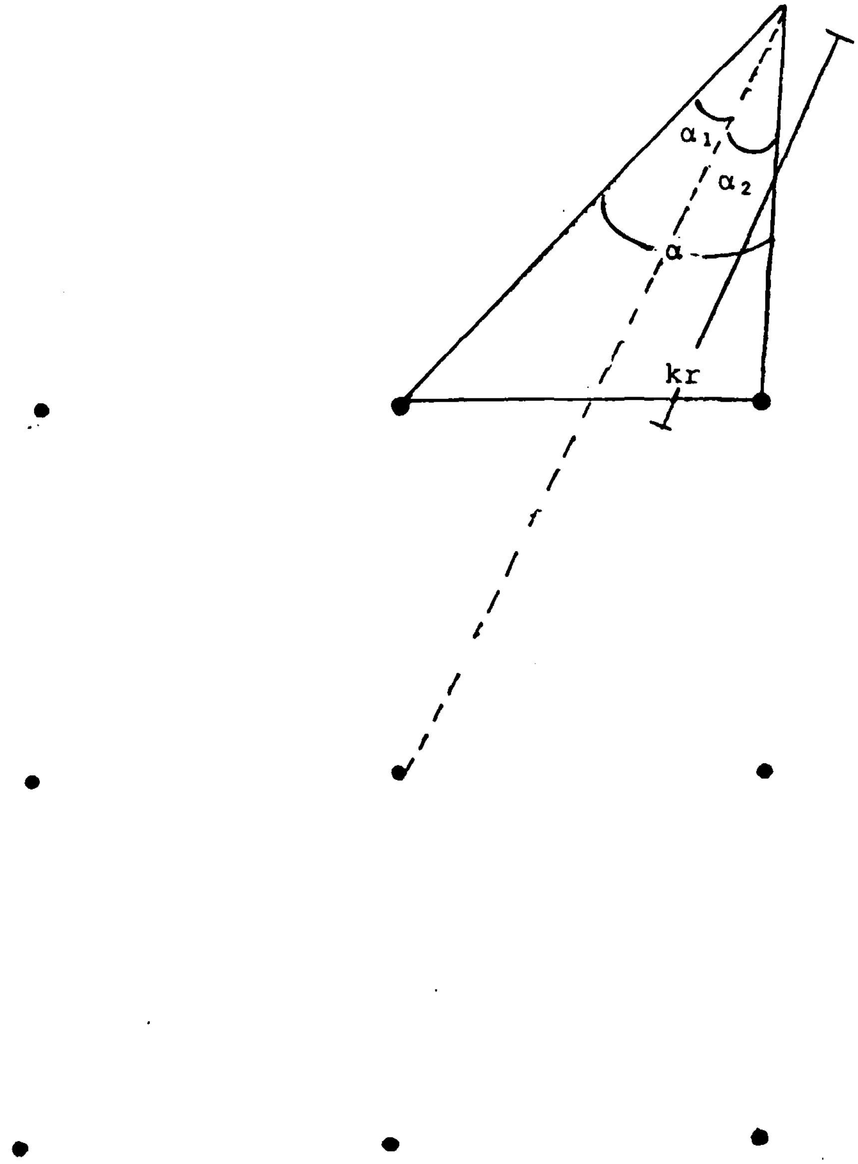


Figure C-6: Discrete Distribution

where w_1, w_2, and k are defined the same as for the continuous distribution of noise sources. $\max_{\theta_1,\theta_2}$ is used to indicate that the maximum is taken over the values $\theta_1 \varepsilon(\theta, \theta + \alpha_1)$ and $\theta_2 \varepsilon(\theta - \alpha_2, \theta)$ where

$$|\alpha_1| + |\alpha_2| \leq |\alpha|$$

as shown in figure C-6. $\min_{\theta_1,\theta_2}$ is defined analogously.

To obtain different bounds different arrangements for the points p_i can be used (see figure C-7). For this arrangement, the difference in the transmission loss functions with range should be compared with the difference in the distribution of noise sources. That is, δTL should be compared to δF where

$$\delta TL = \max_i \left\{ \max_{p \varepsilon \ell_m(\theta)} \left| TL_i(R_i(p)) - TL_m(p) \right| \right\}$$

and

$$\delta F = \max_i \left\{ \max_{p \varepsilon \ell_i(\theta)} \left| \underset{p_i}{\Delta} F \right| \right\}$$

where $\ell_i(\theta)$, $TL_i(p)$, R_i, and $\underset{p_i}{\Delta} F$ are defined in the same fashion as $\ell_1(\theta)$, $TL_1(p)$, R, and $\underset{p_1}{\Delta} F$. If $\delta TL < c\delta F$ (c a constant to account for units of measure) then a bound is found using $\varepsilon_\rho(r)$ and $\varepsilon_t(r)$ and points along radials near θ, i.e.,

$$\left| AN_m(\theta) - \frac{w_1}{2} AN_k(\theta) - \frac{w_2}{2} AN_{(k+1)\text{mod}8}(\theta) - \frac{w_1}{2} AN_{(k+4)\text{mod}8}(\theta) - \frac{w_2}{2} AN_{(k+5)\text{mod}8}(\theta) \right| \leq \varepsilon_\rho(r) + \varepsilon_t(r)$$

where w_1 and w_2 are computed as before. If $\delta TL > c\delta F$, then a bound is found using $\varepsilon_\rho(r)$ and $\varepsilon_t(r)$ and points on radials nearly perpendicular to θ, i.e.,

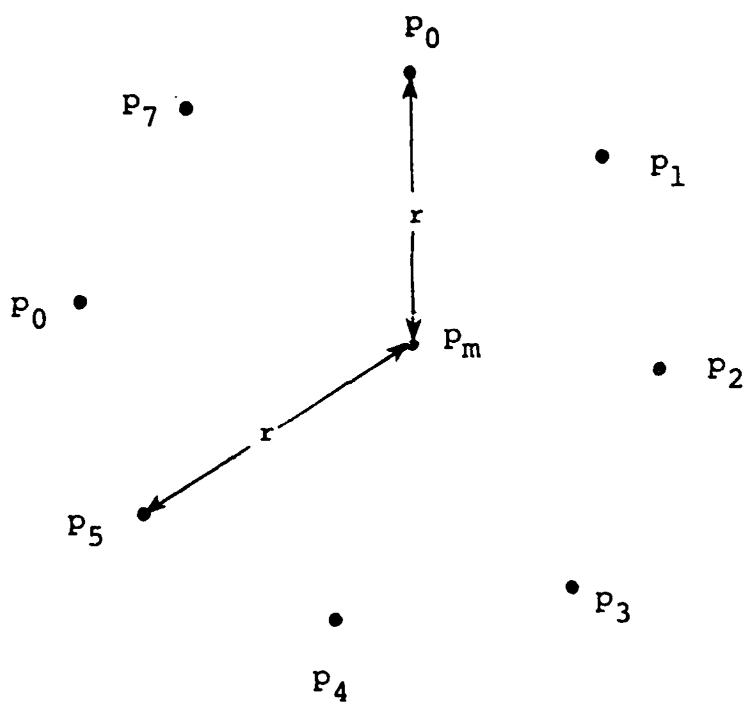


Figure C-7: Alternative 9 Point Test

$$\left|AN_m(\theta) - \frac{w_1}{2} AN_{(k+2)mod8}(\theta) - \frac{w_2}{2} AN_{(k+3)mod8}(\theta)\right.$$

$$\left. - \frac{w_1}{2} AN_{(k+6)mod8}(\theta) - \frac{w_2}{2} AN_{(k+7)mod8}(\theta)\right| \leq \varepsilon_\rho(r) + \varepsilon_t(r)$$

As an extension of this arrangement, consider the case where the ambient noise at p_m is combined into sections with width of $\frac{\pi}{2N}$. Choose 4N points equally spaced on the circle of radius r centered at p_m. Compute the ambient noise at these points (see figure C-8). The previous two equations simplify to

$$\left|AN_m(\theta) - \frac{1}{2} AN_j(\theta) - \frac{1}{2} AN_{(j+2n)mod4n}(\theta)\right| \leq \varepsilon_\rho(r) + \varepsilon_t(r)$$

and

$$\left|AN_m(\theta) - \frac{1}{2} AN_{(j+n)mod4n}(\theta) - \frac{1}{2} AN_{(j+3n)mod4n}(\theta)\right|$$

$$\leq \varepsilon_\rho(r) + \varepsilon_t(r)$$

where

$$j = \left[\left(\frac{4N\theta}{2\pi} + .5\right) \text{mod } 4N\right] .$$

Note that the bound is the same for all sets equations. These bounds can be improved and other arrangements of points can be considered. The alternate bounds ε_i developed previously may be used. This is significant since now the bounds will depend upon the Total Variation in transmission loss in the one direction and on the difference between transmission loss curves in the perpendicular direction. However, the bounds and arrangements presented above represent the basic techniques that can be used. Omnidirectional ambient noise will now be considered.

The problem of finding the relationship between the omnidirectional ambient noise at two points is similar to the problem of finding the relationship between the directional ambient noise at the two points. Hence, it will only be covered briefly. The method used to solve this problem will be the same as was used for the directional problem. That is, first an ideal case will be considered. Then a general case will be examined. Finally, a nine point field analysis will be developed.

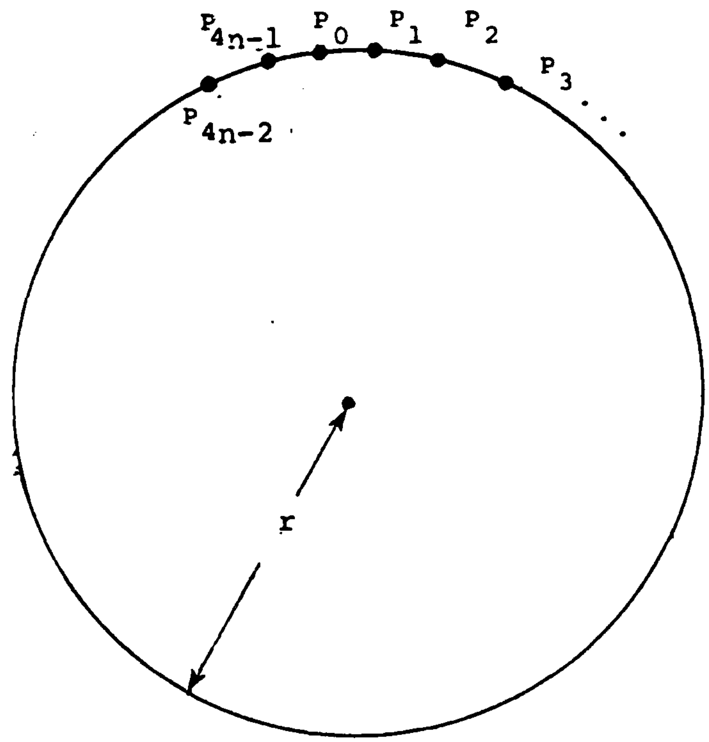


Figure C-8: N Point Test

In the ideal ocean with depth excess everywhere, as described above, the omnidirectional ambient noise at the point p_1, denoted by AN_1, is

$$AN_1 = \iint TL_1(p)dF(p)$$

where TL_1 and F are as previously defined. This integral is a two dimensional Stieltjes integral taken with respect to the distribution of noise sources F over the entire sphere.

As in the directional case one way to compare the ambient noise levels at the points p_1 and p_m is to compute the difference in noise levels. This difference is given by

$$AN_1 - AN_m = \iint TL_1(p)dF(p) - \iint TL_m(p)dF(p)$$

which can be expressed as

$$AN_1 - AN_0 = \iint TL_1(p)d \underset{p_1}{\Delta} F(p) + \iint \{TL_1(R(p)) - TL_m(p)\}dF(p)$$

where R is the linear transformation determined by p_1, p_m and the great circle containing p_m and p.

In the general case there are acoustical obstructions to be considered once more. To take these obstructions into account define the following sets:

$A = \{p|$ the great circle arc pp_m is unobstructed or the great circle arc pp_1 is unobstructed$\}$,

$A_0 = \{p|$ the great circle arc p_1p is obstructed and $p \epsilon A\}$,

$A_1 = \{p|$ the great circle arc p_mp is obstructed and $p \epsilon A\}$,

$A_2 = \{p| \ p \epsilon A \setminus (A_0 \cup A_1)$ and $R(p) \not\epsilon A \setminus A_0\}$,

and

$A_3 = A \setminus (A_0 \cup A_1 \cup A_2)$

Using these sets the difference in the levels of omnidirectional ambient noise at p_1 and p_m is given by

$$AN_1 - AN_m = \iint_{A\setminus A_0} TL_1(p)\,dF(p) - \iint_{A\setminus A_1} TL_m(p)\,dF(p)$$

which is equivalent to

$$AN_1 - AN_m = \iint_{A_1 U A_2} TL_1(p)\,dF(p) + \iint_{A_3} TL_1(p)\,d \underset{p_1}{\Delta} F(p)$$

$$+ \iint_{A_3} \{TL_1(R(p)) - TL_m(p)\}\,dF(p) - \iint_{A_0 U A_2} TL_m(p)\,dF(p)$$

Using this expression an upper bound on the difference is

$$|AN_1 - AN_m| = \iint_{A_1 U A_2} TL_1(p)\,dF(p) + \iint_{A_3} TL_1(p)\,d\,|\underset{p_1}{\Delta} F(p)|$$

$$+ \iint_{A_3} |TL_1(R(p) - TL_m(p)|\,dF(p) - \iint_{A_0 U A_2} TL_m(p)\,dF(p)$$

As with the directional case, define the three quantities ε_ρ, ε_t and ε_0. Let ε_ρ be the bound for the difference in the omnidirectional ambient noise level due to variations in the distribution of noise sources (the second term in the above equation). Let ε_t be the bound for the difference in the omnidirectional ambient noise levels due to the variations in the transmission loss functions (the third term above). Finally, let ε_0 be a bound for the difference in the noise levels due to the acoustical obstructions (the first and fourth terms above).

These three bounds can be used in an omnidirectional nine point test similar to the directional test previously described (see figure C-5). This test has exactly the same form as the directional test. However, the omnidirectional bounds just given are used in place of the directional bounds.

In conclusion, the estimated ambient noise levels must satisfy the mathematical expression which represent the laws of physics. Using these mathematical expressions, bounds on the variation from point to point in the estimated ambient noise field can be established. These bounds can be expressed in terms of quantities that can be either computed or estimated. Finally, these bounds can be used to test the physical consistency of ambient noise models.

APPENDIX D: On Transits of Nearby Ships

This appendix is concerned with the effects on the ambient noise field of a ship passing near to an array. These effects will be considered from both a theoretical and practical point of view.

Obviously, if a ship is of sufficient size or it passes sufficiently close to an array, it will be possible to identify its passage using only a visual inspection of the data. However, in marginal cases visual inspection will be inadequate. That is,in some cases it will not be clear that a ship has passed. Thus, since low frequency ambient noise is the summation of ships at various distances, this appendix will deal only with the problem of determining the significant ships (assuming none are found by cursory visual inspection.)

In the theoretical problem it is necessary to make some assumptions concerning the ship making the transit and the environment. It is necessary to assume thatthe ship is on a great circle course relative to the array (whether towed or fixed) and that the ship moves with a constant velocity along this course. The ship must also be a reasonably uniform radiator of noise. For the environmental requirements it will be assumed that the transmission loss will be a monotonic function. Hence, the received level of noise will be a maximum at the ship's closest point of approach.

To represent the problem mathematically, it is necessary to specify the coordinate system that will be used. Polar coordinates relative to the array will be used. Also since the distances involved are much less that the radius of the earth, it will not be necessary to use spherical geometry. Finally, ambient noise will be considered rather than beam noise so that there will be no ambiguity with beams.

As notation for this problem let t_0 denote the time at which CPA occurs. The location of the ship at CPA to the array can be specified by two quantities: a and b. a is the distance from the array to the ship at CPA. b is the angle relative to the array of the CPA. Denoting the velocity of the ship by v, the position of the ship can be specified for any time, t, using these constants. In the polar coordinate system, the angle, θ, of the ship relative to the array at the time t is given by

$$\theta = b + \arctan[v(t-t_0)/a]$$

The distance ρ from the ship to the array at time t is,

$$\rho(\theta) = a \sec(\theta - b)$$

(Note that the unit used for v is determined by the units selected for a and t.)

If the range of time is large, then the noise from the ship will be detected at a great distance, as the ship passes CPA, and as the ship moves off to a great distance. A graph of the maximum noise level is given in figure D-1.

The ambient noise due to a single ship making a nearby transit is given by

$$\hat{AN}(\theta) = \frac{s}{I(\rho(\theta))}$$

where s is the source level of the ship and I is the transmission loss function in intensity. (the expression $I(\rho(\theta))$ is used to show that I is a function of range which in this case is a function of θ.)

This expression can be made more specific by making additional assumptions regarding the transmission loss. If, in addition to being monotonic, the transmission loss is assumed to be due to spreading loss (i.e. $I(\rho) = \rho^m$ for some fixed m) then the ambient noise due to the ship is

$$\hat{AN}(\theta) = \frac{s}{a^m \sec^m(\theta - b)}$$

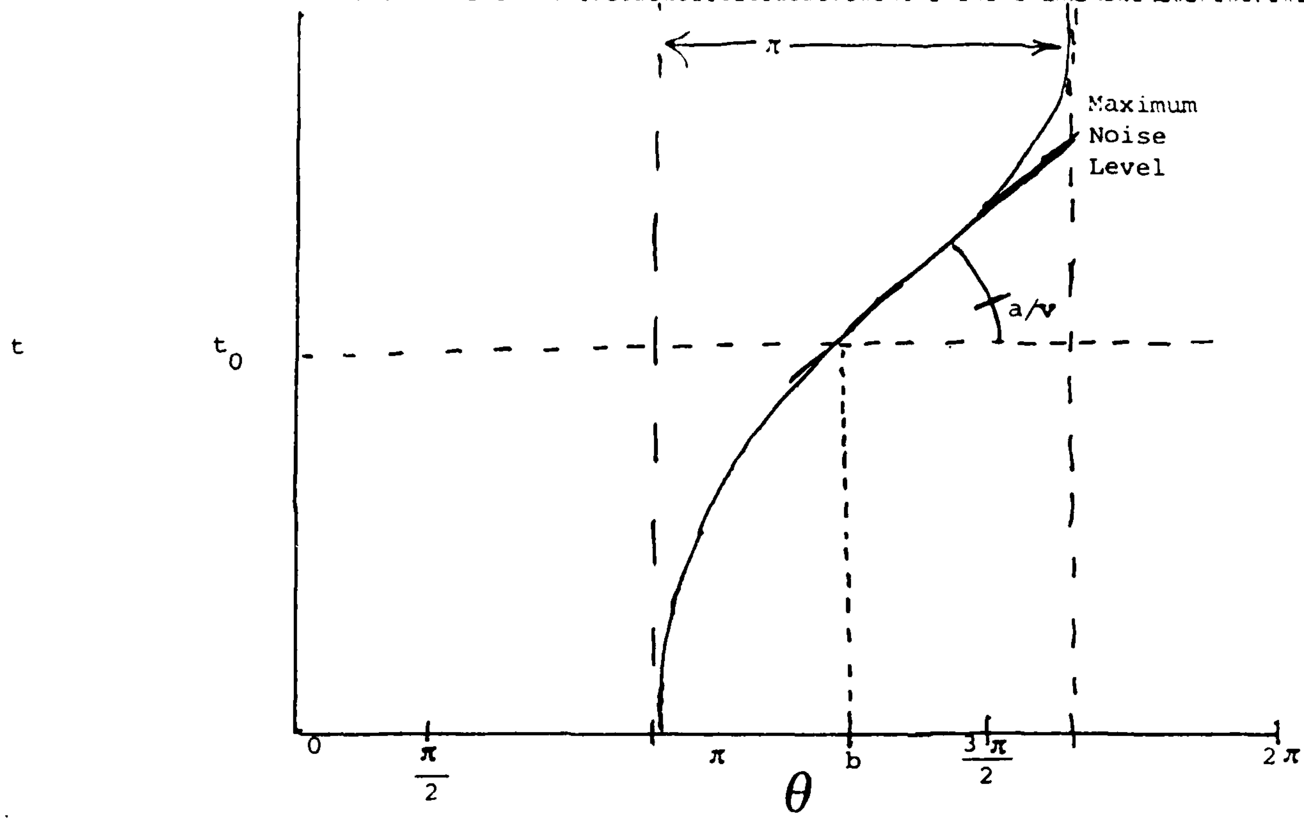


Figure D-1: Maximum Noise Level as a Function of Angle Versus Time

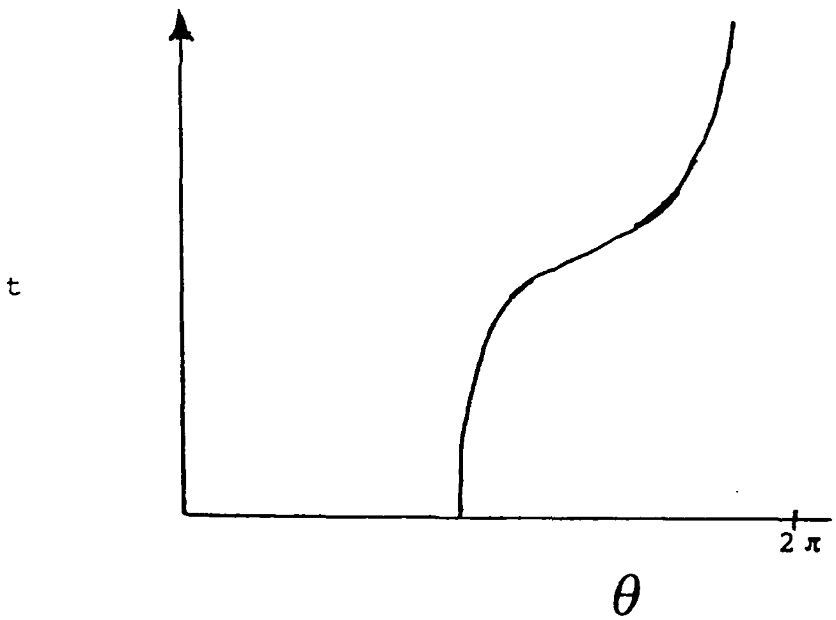


Figure D-2: Maximum Noise Level

Furthermore, if m=2, then the ambient noise reduces to the simple expression

$$\hat{A}N(\theta) = \frac{s}{a^2 + v^2(t-t_0)^2}$$

However, even if these additional assumptions are not made the maximum noise due to the ship will still have a graph similar to the one shown in figure D-2. Observe, that if θ in this graph were considered to be frequency and not angle, then the graph would represent a standard signal processing problem (e.g. a Doppler shift or Frequency Tracker problem.) These kinds of problems can be solved using well established algorithms such as Maximum Likelihood Estimators or Least Squares methods.

Thus, for the theoretical problem of a ship making a transit near to an array several assumptions (which may be unrealistic) must be made about the ship and the environment. The problem can then be reduced to a standard signal processing problem. This signal processing problem can be solved using well known methods.

In practice many of the assumptions made above will not hold. It will not be possible to use ambient noise. Instead, noise measured on beams will be used. These beam measurements are subject to some ambiguity in direction. Also, there is the problem of cones with the endfire beams. These beam measurements will not be made over an infinite period of time. Thus, it is possible that a ship making a nearby transit will be measured for only part of that transit (e.g. this will occur when the measurements stop before the ship reaches its CPA or if the measurements begin after the ship has left its CPA.) Finally, the transmission loss will almost certainly not be a simple spreading loss and will usually not be monotonic.

The practical situation will also differ from the theoretical in that more information will be known. In the practical case there will generally be some data on the nearby ships. For towed arrays the tow ship will have radar to locate ships within the vicinity. In some cases there will also be aircraft surveillance to extend this coverage out to a few hundred miles. Buoys and fixed arrays will not have this kind of coverage. However, with the buoys there is normally little or no horizontal directionality. The buoys will use the noise level without even trying to handle the direction problem. Fixed arrays may have over-flights, but the watch officers are very good at visual spotting even without the aid of aircraft data.

Therefore, in practical situations the problem of determining the effects on ambient noise of ships making transits near an array involves two cases. One case is when omnidirectional (or vertical) noise is considered. The other is when horizontal noise for ships not found by other methods (e.g. radar, aircraft, or visual inspection) is considered.

First, the omnidirectional case will be covered. In the theoretical part of this appendix the ambient noise for a single nearby ship was found to be

$$\hat{AN}(t) = \frac{s}{a^2 - v^2(t-t_o)^2}$$

when the transmission loss was due to spherical spreading. In this case it will only be assumed that the transmission loss is of the form

$$I(\rho) = f(\rho^2)$$

i.e. the transmission loss is not spherical spreading but some function of ρ^2. Using this transmission loss the ambient noise due to the ship is

$$\hat{AN}(t) = \frac{s}{f(a^2 + v^2(t-t_o)^2)}$$

which is symmetric about $t = t_0$. If the transmission loss function, f, is far from monotonic then little can be done. However, for monotonic transmission loss there will be a relative maximum in the ambient noise at CPA. Such maxima are best found using visual inspection

In the horizontal case the ambient noise data will generally be in discrete form. That is, the noise measured in the direction of the beam at θ_j during the interval $[T_0 + (i-1)\Delta t, T_0 + (i)\Delta t]$ will be

$$T_{ij} = T_i(\theta_j) = \int_{T_0 + (i-1)\Delta t}^{T_0 + (i)\Delta t} BN_j(t)dt$$

where BN_j is the noise along beam j, T_0 is the time at which the noise measurements began, and Δt is the integration time. The transit of a nearby ship will show up as a relative maxima moving from beam to beam (see figure D-3).

In order to make this pattern of movement easier to detect the discrete data should be averaged. This can be done using either

$$A_{kj} = \frac{1}{N}\sum_{i=1}^{N} T_{(n(k-1) + i)j} = \frac{1}{N}\sum_{i=1}^{N} T_{(n(k-1) + i)}(\theta_j)$$

or

$$A_{kj} = \frac{1}{N}\sum_{i=1}^{N} T_{((k-1) + i)\ j} = \frac{1}{N}\sum_{i=1}^{N} T_{((k-1) + i)}(\theta_j)$$

where N in an integer. N should be chosen large enough to eliminate small fluctuations in $\{T_{ij}\}$. N should also be small enough so that near CPA the ship will spend at least 2N periods in the beams. That is,

$$2N\Delta t v \leq 2R \sin\Delta\theta \sim 2R\,\Delta\theta$$ (see figure D-4)

or

$$N \leq \frac{R\Delta\theta}{v\Delta t}$$

Once N is chosen, A_{kj} (the groups of N T_{ij}'s) can be examined. This examination cannot be made using the noise level. This is because the transmission loss is not necessarily monotonic. Also, the range of the ship need not change significantly over the period of the measurements. And so, instead of using the absolute level of noise, the relative level of noise will be examined. That is, define the matrix $\{a_{ij}\}$ where the elements of the matrix are given by

$$a_{kj} = \begin{cases} 1 \text{ if} & A_{kj} \geq A_{k-1j} \text{ and } A_{kj} \geq A_{k+1j} \\ 0 \text{ otherwise} & \end{cases}$$

and $a_{0j} = 0$ for all j.

For a ship making a transit this matrix should show a pattern of zeros and ones similar to a discrete version of figure D-1. (see figure D-5).

The best way to determine if such a pattern exists is a visual inspection of the data. Since a visual inspection is to be used, the type of average above will depend on the amount of data. That is, if there is a great deal of data which would produce a large $\{a_{ij}\}$ matrix, the first method of averaging should be used. Otherwise, the second method of averaging should be used.

In conclusion, the effects of a nearby ship making a transit of an array can be determine both theoretically and practically. The theoretical method reduces to a signal processing problem after making some assumptions. The method used in practice will generally involve reducing the measured data to a symbolic format and visually inspecting it for the patterns expected from the theory.

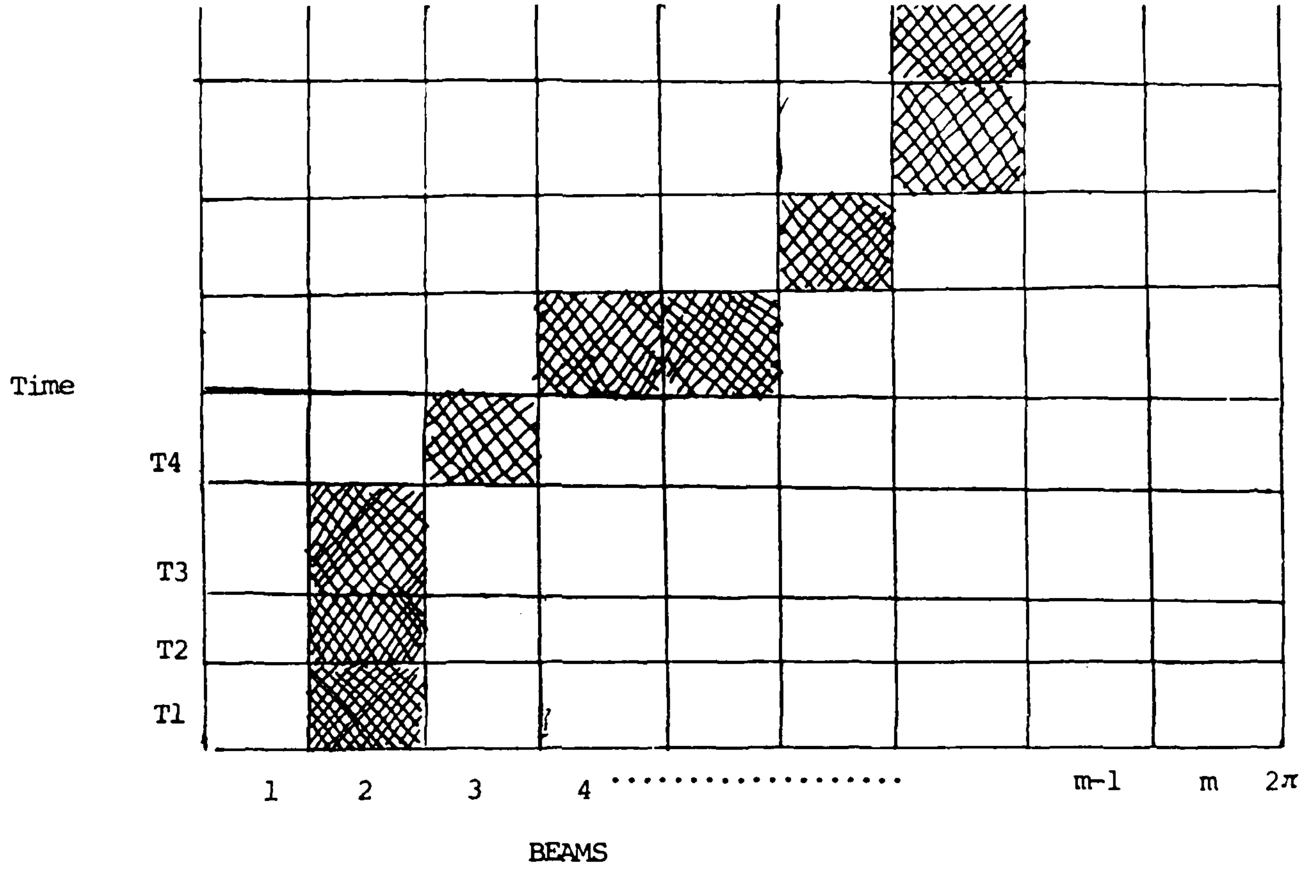


Figure D-3: Maximum Noise Level as a Funcuion of Beam Versus Time

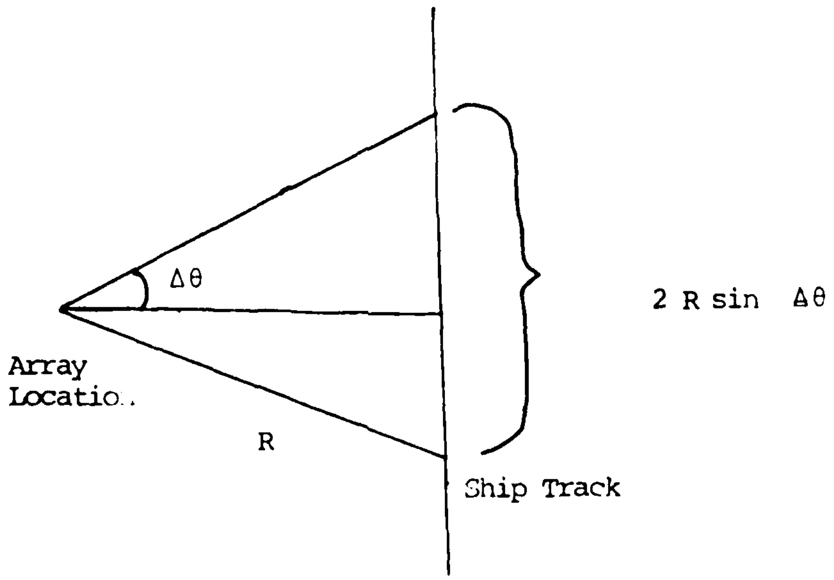


Figure D-4: Geometry for Determining a Maximum Value for N.

$$
\left\{ a_{ij} \right\} = \left\{ \begin{array}{c}
010000\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots00 \\
010000\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots00 \\
001000\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots00 \\
000110\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots00 \\
000001\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots00 \\
000001\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots00 \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots
\end{array} \right\}
$$

Figure D-5: a_{ij} matrix

Appendix E Vertical Deconvolution

The purpose of this Appendix is to discuss the usefulness of comparing measurements of vertical directionality of ambient noise due to surface shipping with model results. The discussion will show that the agreement of model results with measurements does not necessarily indicate that it is a good model. First, the nature of this ambient noise will be discussed considering the effects of varying shipping distributions and transmission loss. The noise directionality of vertical arrays and the beam widths at the 3 dB down points will be compared to the properties of the shipping noise. Conclusions will then be drawn concerning the applicability of testing vertically directional models using this approach.

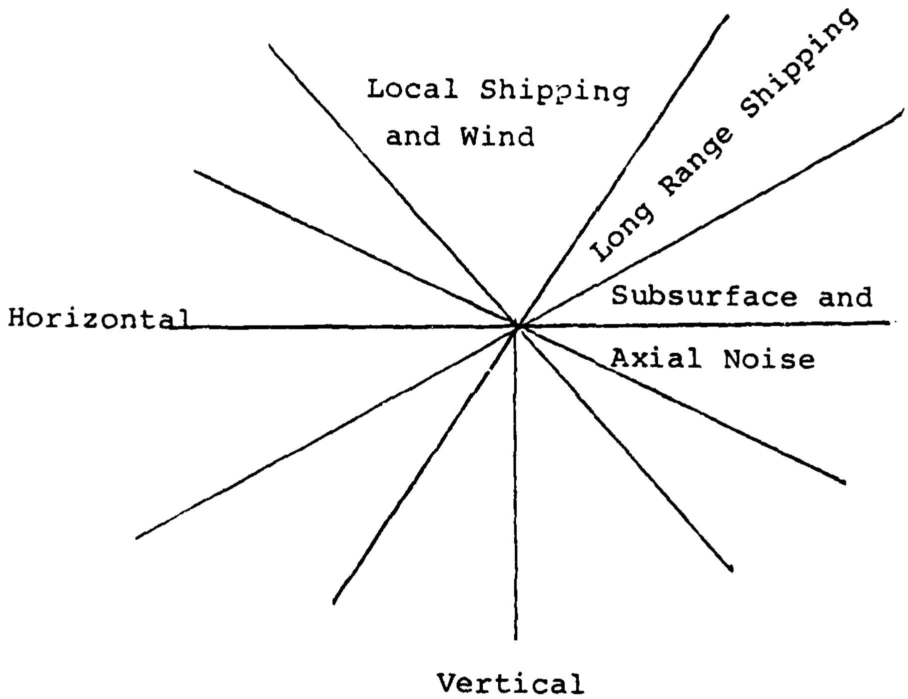
Ambient noise at an array due to surface shipping can be modelled in the following fashion. For weak (or absent) dependence of sound speed with range, distant shipping contributes to ambient noise at an array site as shown in Figure E-1. If for a range-independent sound speed profile and a flat bottom one considers ray paths which originate at the surface and reach the receiver after transitting a distance such that bottom bounce paths have died out, the primary propagation paths for this shipping noise will be received at the array in the solid angle limited by the surface grazing ray (θ_{sg}) and the bottom grazing ray (θ_{bg}). The ambient noise received in this solid angle from surface ships can be described with the following equation

$$AN = \int_{0}^{\Omega} TL\ (\rho) \cdot \tilde{f}(\rho)\ d\rho \qquad \text{(E-1)}$$

where Ω is the distance to which surface shipping is considered, $TL(\rho)$ is the transmission loss as a function of distance and $\tilde{f}(\rho)$

Figure E-1

Vertical Distribution of Ambient Noise At Array



is the shipping distribution function. For arbitrary shipping distributions $\tilde{f}(\rho) = 2\pi\rho f(\rho)$ where f is the shipping density per unit area.

Now the widths which current vertical arrays have are roughly the same as the angle $|\theta_{bg} - \theta_{sg}|$. This means, that the shipping noise will arrive on essentially one beam. This means that there is very little information available and it is not possible to discriminate where on the surface the shipping noise is coming from. In other words, there is insufficient data to perform any deconvolution.

Now looking again at equation E-1, it can be seen that the contribution to the ambient noise from the shipping cannot be discriminated from the effect of the transmission loss on the ambient noise. Shipping distributions, whether historical or measured are usually inaccurate due to the dynamics of the field, and the shipping radiated noise levels used are class averages, not measurements. Therefore, when comparing model predictions with vertical array measurements two situations can exist. If the model and the measurements disagree the shipping distribution, or levels, may be the cause and it can be adjusted until the model and measurements agree The second situation is if the model and measurements do agree it is hard to tell whether the model really is good or whether the errors in the transmission loss and shipping levels have just offset each other. This problem is due to the lack of information on the received noise. A vertical array doesn't have the advantage that a linear horizontal array has in that very fine resolution beams can be used to obtain more information from received energy. The beams on vertical arrays are so broad that very little information can be gathered on energy received in the shipping lobes. A further complication is the fact that arrays are designed specifically to discriminate against energy being received in these lobes.

From this argument it can be seen that the accuracy of the transmission loss cannot be determined by the ambient noise received even if the shipping distribution is known. As long as the shipping distribution is reasonably smooth, the transmission loss could be very erratic with many convergence zones or it could be very smooth, but due to the integration and only one beam measurement being used, the two very different transmission losses could not be discriminated.

Thus it can be seen that it is not reasonable to try to prove or disprove the validity of a vertical directionality model solely by comparing its results with measured beam noise data.

APPENDIX F: Horizontal Comparison Methodologies

Given two sets of horizontal ambient noise data, the problem arises of how to compare the two sets of data. This appendix describes some solutions to this problem. First, the general types of methodologies covered in this appendix are discussed. Next, the factors that should be considered when selecting a particular method of comparison are given. Then the methodologies themselves are presented. Finally, a summary of the methodologies and when they should be used is provided.

The methods of comparing ambient noise data that are given in this appendix are all quantitative in nature. That is, the two sets of noise data are reduced to a set of parameters using well defined mathematical tests. These parameters have been chosen so that their values indicate the similarities and differences between the original two sets of horizontal ambient noise data. In some cases the parameters may also be used to determine possible sources of the differences in sets.

The mathematical tests used to determine the parameters are of two types: point tests and functional tests. The point tests treat the sets of data as individual points. These tests are mostly statistical in nature. They can be found in almost any text on statistics. However, anyone using these statistical comparison tests should be very familiar with the underlying statistical tests. They should be aware of the mathematical limitations and the implicit assumptions of the statistics.

The functional tests on the other hand treat the sets of data as values derived from some ambient noise functions. These noise functions are considered as members of some family of functions. The functional tests compare the ambient noise functions as members of this family.

Regardless of the type of test, pointwise or functional, the test may be absolute or relative. Absolute tests are those which depend solely on the data in the sets that are being compared. For example, if each set contained only one value, then an absolute test would be whether the two values are equal or not. A relative test is one which depends upon data not in the sets themselves. An example of a relative test would be to compare the difference in the values in the sets to some quantity which had been derived empirically. This kind of test results whenever concepts such as "large" or "small" are quantified. In the example just given the difference might be considered "large" (significant) if it is greater than the quantity derived from past values.

For each of the above types of tests there are many useful tests that fall into that particular type. However, not all of these tests should be used with each pair of sets of ambient noise data. It would be pointless, for example, to use the functional comparison tests when there is only one value in each set. Thus, it is important to consider what factors will determine whether a particular test is used in a given comparison methodology.

One important factor in selecting comparison tests for a particular methodology is the amount of data in each set of data. This amount can vary from a single value in each set (as in the case when two omnidirectional noise levels are compared) to an entire set of functions in each set (as in the case when comparing replications of two different directional ambient noise models). This factor is of such importance that the individual methodologies presented in this appendix will be characterized by the amount of data that they require. Table F-1 lists the possible quantities of data, examples of how these quantities could be obtained, and which of the methodologies presented later in this appendix to use in each case.

However, while the amount of data is important, another factor to consider when selecting tests for a particular comparison methodology is the quality of the data. This factor, rather than effecting which comparison test is used in a methodology, effects

TABLE F-1: Data Quantity Categories

Amount of Data In First Set	Example	Amount of Data In Second Set	Example	Methodology
A single point	An omnidirectional noise level	A single point	An omnidirectional noise level	1
A single point	An omnidirectional noise level	A set of points	Replications of a non-directional ambient noise model	2
A set of points	Replications of an omnidirectional noise level measurement	A set of points	Replications of a non-directional ambient noise model	3
An ambient noise function	Directional ambient noise levels	An ambient noise function	A directional ambient noise model	4
An ambient noise function	Directional ambient noise levels	A set of ambient noise functions	Replications of a directional ambient noise model	5
A set of ambient noise functions	Replications of directional ambient noise levels	A set of ambient noise functions	Replications of a directional ambient noise model	6

the constants used in the relative tests in that methodology. Consider two cases: measured ambient noise compared to computed ambient noise and computed ambient noise from one model compared to ambient noise computed by another model. In the first case data that is different in kind is being compared. In the second case while the data is from two different models it is still the same kind of data. Thus, the constants for relative comparison tests would be different for these two cases.

Now, the comparison methodologies themselves can be presented.

Methodology 1: Point-to-Point Comparison

Denote the two noise values by y and x. Since there is so little data available in this case there is only one reasonable test It is,

$$\nu_1 = \begin{cases} 1 \text{ if } D < |x-y| \\ 0 \text{ if } d < |x-y| \leq D \\ -1 \text{ if } |x-y| \leq d \end{cases}$$

where d and D are constants. This is a *relative* test. That is, for different kinds of data the values of d and D are different. However, for each kind of data $|x-y| > D$ indicates a significant difference in the values while $|x-y| < d$ indicates no significant difference. As noted previously, the values for d and D for each kind of data should be determined empirically.

Methodology 2: Point-to-Envelope Comparison

In this method of comparison two parameters will be used to indicate the similarities between the two sets of data. Denote by y the single value in the first set of data. Denote by $\{x_j\}_{j=1}^{n}$ the values in the second set of data. For the first comparison test in this method let,

$$\bar{x} = \max_{1 \leq j \leq n} \{x_j\}$$

and

$$\underline{x} = \min_{1 \leq j \leq n} \{x_j\}$$

Now, the first test is,

$$\nu_2 = \begin{cases} 1 \text{ if } \underline{x} \leq y \leq \bar{x} \\ 0 \text{ otherwise} \end{cases}$$

This is an absolute test. It shows, in a rough way, whether y is similar to $\{x_j\}$. It has two advantages over the other test in this method. ν_2 can be determine even when there is very little data (e.g., even when n = 2). Also, this test can be used without making any assumptions about the statistical distribution of $\{x_j\}$ (i.e., if $\{x_j\}$ is considered as a random sample from some population no assumption is placed on the distribution of that population).

There are also some disadvantages to using this comparison test. These disadvantages are illustrated in Figure F-1. In the case shown in Figure F-1a, y is similar to many elements of the set $\{x_j\}$, and yet, it does not lie in the interval $(\underline{x},\bar{x})$. Hence, it may be the case that $\nu_2=0$ even when y and $\{x_j\}$ are similar. The converse of this problem is shown in Figure F-1b. It is also a problem with this test but it is not as important as the one shown in Figure F-1a.

There are two ways to overcome these disadvantages. The best way is to use this comparison test only when there is too little data to justify using the other test (e.g., $n \leq 7$). In this way the problems cannot arise. The other solution is to use the comparison test ν_1 with y and each element in the set $\{x_j\}$ when $\nu_2=0$. If $\nu_1=1$ for several elements of $\{x_j\}$ then the situation shown in Figure F-1a has arisen.

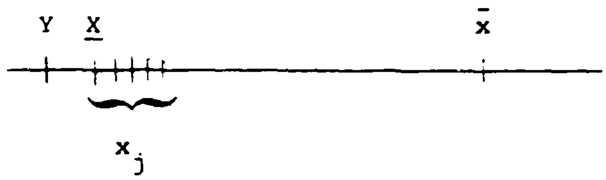
The second test in this methodology is a statistical test. It uses the statistics,

$$m_x = \frac{1}{n} \sum_{j=1}^{n} x_j$$

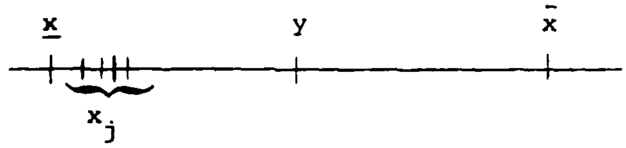
and

$$s_x^2 = \frac{1}{n-1} \sum_{j=1}^{n} (x_j - m_x)^2$$

Figure F-1: Disadvantages in ν_2



a. $\nu_2 = 0$



b. $\nu_3=1$

Also, for this test it is assumed that $(x_j)_{j=1}^{n}$ is a random sample from a $N(\mu_x, \sigma_x^2)$ population.

This test, ν_3 uses the notion of tolerance limits. If, given the assumption of normality, the constants μ_x and σ_x^2 were known, then it would be possible to find a constant, k such that 100P percent of the population would be in the interval $(\mu_x - k\sigma_x, \mu_x + k\sigma_x)$. Here, however, the values for μ_x and σ_x^2 are not known and are only estimated by m_x and s_x^2. Nevertheless, it is still possible to find a constant, $K(\gamma,P)$, for which it can be assumed with 100γ percent confidence that 100P percent of the population will be in the interval $[m_x - K(\gamma,P)s_x,\ m_x + K(\gamma,P)s_x]$ $(0 \le \gamma, P \le 1)$. Tabulated values for $K(\gamma,P)$ can be found in references r and t.

Using this idea let,

$$L_x(\gamma,P) = m_x - K(\gamma,P)s_x$$

and

$$U_x(\gamma,P) = m_x + K(\gamma,P)s_x$$

The comparison test ν_3 is,

$$\nu_3 = \begin{cases} 1 \text{ if } L_x(\gamma,P) \le y \le U_x(\gamma,P) \\ 0 \text{ otherwise} \end{cases}$$

This test is not subject to the problems shown in Figure F-1. Avoiding these problems is paid for by the requirements for more data and the assumption of normality. When these requirements are satisfied, if $\nu_3 = 1$, then y and $\{x_j\}$ may all be perturbations of the same value. If $\nu_3 = 0$, then either y is probably not from the same population as $\{x_j\}$ or if y is from the same population, then it is probably an outlier.

The values of γ and P to be used depend on the nature of the data. The values that are of most use to the analyst will be determined empirically.

Methodology 3: Envelope-to-Envelope Comparison

In this comparison methodology four parameters will be used to indicate the similarities between the two sets of data. Two additional comparison tests will be introduced. Denote the first set of data values by $\{x_j\}_{j=1}^{n_1}$. Denote the other set of values by $(y_i)_{i=1}^{n_2}$. Let $\underline{x}$, $\bar{x}$, m_x, s_x^2, μ_x, and σ_x^2 be defined as in methodology 2. Similarly define

$$\bar{y} = \max_{1 \le i \le n_2} \{y_i\},$$

$$\underline{y} = \min_{1 \le i \le n_2} \{y_i\},$$

$$m_y = \frac{1}{n_2} \sum_{i=1}^{m_2} y_i,$$

$$s_y^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_i - m_y)^2,$$

and μ_y and σ_y to be the parameters that uniquely determine the normally distributed population to which $\{y_i\}$ belongs. Also, let ℓ be a set function on the set of intervals where $\ell(I)$ is the length of interval I.

The first comparison parameter is given by the value of ℓ for the intervals,

$$I_1 = \{z \varepsilon R | \underline{x} \le z \le \bar{x}\}$$

and

$$I_2 = \{z \varepsilon R | \underline{y} \le z \le \bar{y}\}$$

where

$$\nu_4 = \frac{\ell(I_1 \cap I_2)}{\min\{\ell(I_1),\ \ell(I_2)\}}$$

This test measures the overlap of the sets $\{x_j\}$ and $\{y_i\}$. The greater the overlap in the intervals is, the greater the similarity between the two sets of data (see Figure F-2). $\nu_4 \in (0,1)$. $\nu_4 = 0$ when the intervals do not overlap (i.e., when $\bar{x} < \underline{y}$ or $\bar{y} < \underline{x}$) (see Figure F-2a). $\nu_4 = 1$ when one interval contains the other (i.e., when $\underline{x} \le \underline{y} \le \bar{y} \le \bar{x}$ or $\underline{y} \le \underline{x} \le \bar{x} \le \bar{y}$) (see Figure F-2c). It would be expected that if $\{x_j\}$ and $\{y_i\}$ were from the same population (regardless of the population's distribution) that ν_4 would probably have a value near to 1. Alternatively, if the two sets are from nearly independent populations then values for ν_4 nearer to zero would be expected.

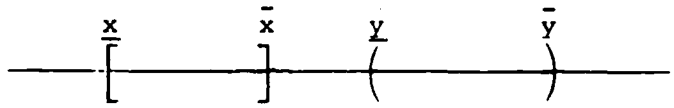
The advantages and disadvantages of this test are analogous to those of ν_2 viz., the test can be done with little data and it needs no assumptions about distributions, but it gives incomplete information for some situations. An example of this problem is shown in Figure F-3. In this figure while the intervals are very different the value of ν_4 is the same for both Figure F-3a and F-3b, namely 1.

To distinguish between these two cases a second comparison test is needed. Let,

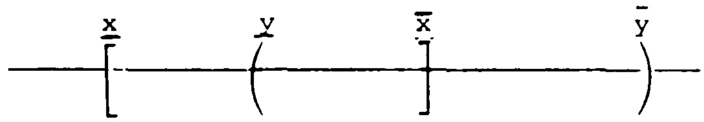
$$\nu_5 = \frac{\ell(I_1 \cap I_2)}{\ell(I_1 \cup I_2)}$$

where I_1 and I_2 are defined as before.

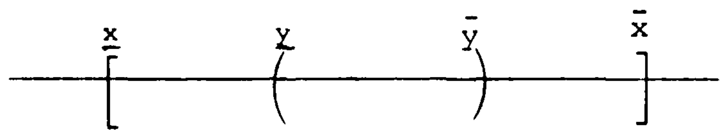
Figure F-2: Range of Values for ν_4



a. $\nu_4=0$

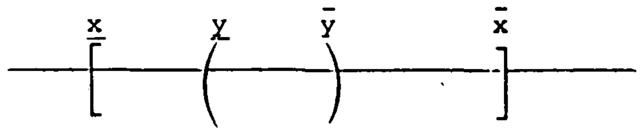


b. $0 < \nu_4 < 1$

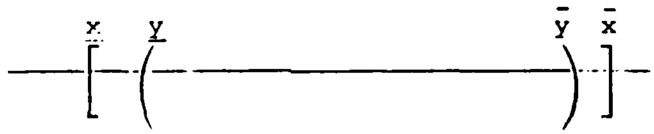


c. $\nu_4=1$

Figure F-3: Disadvantages in Using ν_4 Only



a. $\ell(I_1) >> \ell(I_2)$



b. $\ell(I_1) \doteq \ell(I_2)$

This test compares the overlap in the two intervals relative to the total length of the two intervals when taken as a single interval (or in the case of disjoint intervals, when taken as two intervals). $\nu_5 \varepsilon (0,1)$. $\nu_5 = 0$ when the intervals are disjoint and $\nu_5 = 1$ when the intervals are identical. This comparison test can be used to distinguish between the examples given in figure F-3. For figure F-3a, ν_5 is nearly zero while for figure F-3b, ν_5 is nearly 1. This is not to say that ν_5 should be used in place of ν_4 but rather that ν_5 should be used in conjunction with ν_4. Figure F-4 shows why this is the case. In both figure F-4a and figure F-4b ν_5 is the same. On the other hand, ν_4 is distinct in these cases.

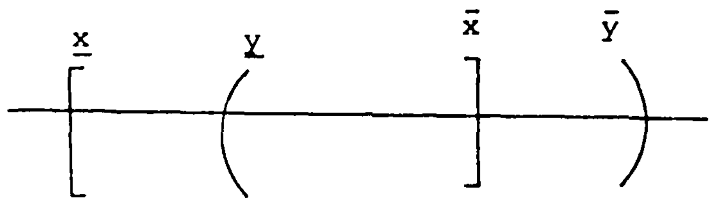
Therefore, when comparing the two intervals, $\{z \varepsilon R | \underline{x} \leq z \leq \bar{x}\}$ and $\{z \varepsilon R | \underline{y} \leq z \leq \bar{y}\}$ the recommended procedure is to compute both ν_4 and ν_5. ν_4 will indicate how much of the smaller interval is contained in the larger and ν_5 will indicate how the intersection of the intervals compares to the union of the intervals. Used together, each of the parameters compliments the weakness in the other.

Before describing the remaining two parameters that are used in this comparison methodology a note of caution is needed. The tests ν_4 and ν_5 when using the intervals I_1 and I_2 are subject to the same kind of problems as ν_2 in methodology 2. That is, outliers can strongly affect both ν_4 and ν_5. The remaining two parameters, which are based on statistical methods, are not as strongly affected by outliers. However, these methods make assumptions concerning the distributions of the data points. Thus, the relative importance of the non-statistical and the statistical methods will depend on the quantity and quality of the data. In some cases where the data is sparse or does not fit the assumed distribution, the tests as described above should be given the most weight. When there is sufficient data, the statistical comparison parameters defined next should be given more weight.

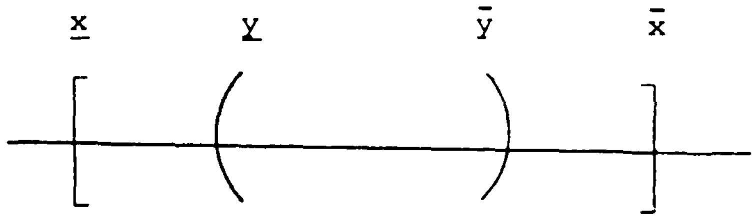
The statistical comparison parameters are computed using ν_4 and ν_5 and tolerance intervals. Let $L_X(\gamma,P)$ and $U_X(\gamma,P)$ be defined as before (i.e., the endpoints for the interval for which

Figure F-4: Disadvantages in Using ν_5 Only

a.



b.



there is a 100γ percent confidence that 100P percent of the normal population lies within the interval). Define,

$$L_y(\gamma,P) = m_y - K(\gamma,P)s_y$$

and

$$U_y(\gamma,P) = m_y + K(\gamma,P)s_y$$

where $K(\gamma,P)$ is the same as in computing $L_x(\gamma,P)$ and $U_x(\gamma,P)$. Now, let,

$$I_1 = \{z\varepsilon R | L_x(\gamma,P) \leq z \leq U_x(\gamma,P)\}$$

and

$$I_2 = \{z\varepsilon R | L_y(\gamma,P) \leq z \leq U_y(\gamma,P)\}$$

This method of comparing the sets of ambient noise data is to compute ν_4 and ν_5 for these intervals. These two parameters will provide the same type of information as the first two, but in this case they will be less strongly effected by outliers. The price paid for this improvement is the requirement for more data. As with the choice of which intervals to use, the choice of values γ and P will depend on the data and on the analyst's judgement. Other comparison tests of a statistical nature are given in Appendix H.

Methodology 4: Directional Noise Point-to-Point Comparison

The problem dealt with in this section is how to compare two horizontal ambient noise functions, $x(\theta)$ and $y(\theta)$. The problem is complicated by the fact that $x(\theta)$ and $y(\theta)$ are not known explicitly. Instead, there are several values of at which both $x(\theta)$ and $y(\theta)$ are known. Denote these values by θ_i, i = 1,2,...,n.

This is the first case in which both statistical and functional methods of comparison can be used. In the statistical comparisons $\{x(\theta_i)\}_{i=1}^n$ and $\{y(\theta_i)\}_{i=1}^n$ are considered as sets of points and the comparison tests are similar to those in Methodology 3. There are three statistical comparison parameters in this method. In the functional comparisons $\{x(\theta_i)\}_{i=1}^{n}$ and $\{y(\theta_i)\}_{i=1}^{n}$ are considered as values of $x(\theta)$ and $y(\theta)$. These values are used to obtain approximations for $x(\theta)$ and $y(\theta)$. The two approximation functions are then compared using functional methods. There are four functional comparison parameters used in this method.

For the statistical comparison parameters define,

$$D_i = x(\theta_i) - y(\theta_i) \qquad i = 1,2,\ldots,n,$$

$$m_D = \frac{1}{n}\sum_{i=1}^{n} D_i$$

$$s_D^2 = \frac{1}{n-1}\sum_{i=1}^{n} (D_i - m_D)^2,$$

$$m_x = \frac{1}{n}\sum_{i=1}^{n} x(\theta_i),$$

$$m_y = \frac{1}{n}\sum_{i=1}^{n} y(\theta_i),$$

and

$$r_{xy} = \frac{\sum_{i=1}^{n}\left[(x(\theta_i) - m_x)(y(\theta_i) - m_y)\right]}{\left[\sum_{i=1}^{n}(x(\theta_i)-m_x)^2 \sum_{i=1}^{n}(y(\theta_i) - m_y)^2\right]^{1/2}}$$

The first statistical comparison test is,

$$\nu_6 = \begin{cases} 1 \text{ if } M<|m_D| \\ 0 \text{ if } m<|m_D| \leq M \\ -1 \text{ if } |m_D| \leq m \end{cases}$$

where m and M are constants. This is a relative comparison test, and so, the values of m and M will depend on the type of data being compared. Whatever the data, M will be selected so that if $M < |m_D|$ then the mean difference between $\{x(\theta_i)\}$ and $\{y(\theta_i)\}$ is significant (i.e., there is significant difference between $x(\theta)$ and $y(\theta)$). m will be selected so that if $|m_D| \leq m$ then $\{x(\theta_i)\}$ and $\{y(\theta_i)\}$ are similar (hence, $x(\theta)$ and $y(\theta)$ are probably similar).

The second statistical comparison test is,

$$\nu_7 = \begin{cases} 1 \text{ if } S < s_D \\ 0 \text{ if } s < s_D \leq S \\ -1 \text{ if } s_D \leq s \end{cases}$$

where s and S are constants. When $\nu_7 = 1$, then either $x(\theta)$ differs from $y(\theta)$ in shape or in magnitude of variation or both. When $\nu_7=1$, then the magnitude of variation in $\{x(\theta_i)-y(\theta_i)\}_{i=1}^{n}$ is small (i.e., $x(\theta)$ and $y(\theta)$ can be considered as having the same shape and magnitude of variation from their mean level of noise).

$$\nu_8 = \begin{cases} 1 \text{ if } R < r_{xy} \\ 0 \text{ if } r < r_{xy} \leq R \\ -1 \text{ if } r_{xy} \leq r \end{cases}$$

where r and R are constants. When $\nu_8=1$ then the functions $x(\theta)$ and $y(\theta)$ are similar shapes. When $\nu_8=-1$, then the two curves have different shapes. It should be noted, however, that in this test (and, in fact, all three statistical comparison tests) the conclusions about $x(\theta)$ and $y(\theta)$ are infered from information about $\{x(\theta_i)\}$ and $\{y(\theta_i)\}$.

These three parameters are most useful when all three are considered together. Table F-2 lists some possible values for ν_6, ν_7 and ν_8. It also gives interpretations of these values. The remaining values for ν_6, ν_7, and ν_8 do not provide as clear a description of the relationship between the curves. For example, when $\nu_6 = 1$, $\nu_7 = 1$, and $\nu_8 = 0$ the curves may be related as in case 1 but with more random error, or they may be dissimilar as in case 2 but with a higher chance correlation.

The remaining noise comparison tests in this section compare $x(\theta)$ and $y(\theta)$ as functions (even though their values are known only on the finite set $\{\theta_i\}_{i=1}^n$). However, before these tests can be described two things must be specified. First, since only $\{x(\theta_i)\}$ and $\{y(\theta_i)\}$ are known, the method used to approximate $x(\theta)$ and $y(\theta)$, respectively, using these values must be specified. Second, a method of quantifying the differences between functions must be adopted.

Horizontal ambient noise functions belong to the class of bounded piecewise continuous functions on the interval $[0,2\pi]$. Given the values of one of these functions at a finite set of points, there are many types of approximation that can be employed. For example, step functions, linear splines, cubic splines, or polynomial interpolation may be used. It is recommended that linear splines be used as the method of approximation. This method will generally be superior to step function approximation and easier to compute than other methods of approximation. The other methods, involving more complex computations, in this case do not guarantee any improvement in the approximation. However, any reasonable method of approximation can be used.

Now, using linear spline approximation (or, in fact, any type of approximation given above) the set of all possible approximation functions becomes a subspace of each of the L_p spaces on the interval $[0,2\pi]$. If the L_p norm for some acceptable value of p is restricted

TABLE F-2: Values for ν_7, ν_8, and ν_9 and Their Interpretation

Case	ν_7	ν_8	ν_9	Meaning	Comment
1	1	1	1	Both curves have similar shapes but they differ significantly in their mean level of noise and in the magnitude of their variation from the mean level.	Most probably this will be the result of using a non-sense transmission loss function in a noise model. Although it is much less likely, this may also be a result of a systematic difference between the curves e.g. $x=ky$, k a constant.
2	1	1	-1	The two curves are completely dissimilar. That is they differ in level of noise, magnitude of variation and shape.	The two curves are independent of each other.
3	1	-1	1	The two curves have the same shape and magnitude of variation but they differ significantly in their levels of noise.	This may indicate a systematic bias between the two curves, e.g. $x=y+h$, h a constant.
4	1	-1	-1	The two curves are dissimilar in shape and level of noise. However, they both have nearly the same magnitude of variation from their mean levels of noise.	Most probably the two curves are independent. Other tests, such as given later in this section and in appendix H, may help indicate if this is correct.
5	-1	1	1	Both curves have similar shapes and mean levels of noise. Their magnitude of variation are significantly different.	This probably indicates that for the true ambient noise function $a(\theta)=\ell+\nu(\theta)$ (where ℓ is the mean level and $\nu(\theta)$ is the magnitude of the variation) $x(\theta)$ and $y(\theta)$ are of the form $x(\theta)=\hat{\ell}+k_x\ \hat{\nu}(\theta)$ and $y(\theta)=\hat{\ell}+k_y\ \hat{\nu}(\theta)$ (where $\hat{\ell}$ is an estimate of ℓ, $\hat{\nu}$ is an estimate of ν and k_x and k_y are constants).

TABLE F-2: Values for ν_7, ν_8, and ν_9 and Their Interpretation (Cont).

Case	ν_7	ν_8	ν_9	Meaning	Comment
6	-1	1	-1	Both curves are similar in the mean level of ambient noise.	This is probably similar to the situation in case 5 except $x(\theta)=\hat{\ell}+\hat{\nu}_x(\theta)$ while $y(\theta)=\hat{\ell}+\hat{\nu}_y(\theta)$.
7	-1	-1	1	The curves are of the same shape, and have similar mean levels of noise, and similar magnitudes of variation from the mean levels.	The two ambient noise curves are reasonable agreement i.e. $x(\theta)\doteq y(\theta)$.
8	-1	-1	-1	The curves have the similar level and magnitude but differ in shape.	This will result when both curves are nearly constant at the same level but with small variations.

to the subspace, then the subspace of all possible linear spline approximation functions on the interval $[0,2\pi]$ is a metric space with the restricted L_p norm as the metric. The distance between two elements (functions) in this metric space will then be a measure of the difference between the functions.

For the remainder of this section this method of determining the difference between an individual functions will be used with $p=2$. That is, the distance between two functions, say ϕ and ψ, will be,

$$\rho(\phi,\psi) = \left[\frac{1}{2\pi}\int_{0}^{2\pi} (\phi(\theta) - \psi(\theta))^2 \, d\theta\right]^{1/2}$$

Note, however, that the analyst is free to choose any metric and is not restricted to the L_2 norm (nor, in fact, to any L_p norm).

For the functional comparison tests, let $\hat{x}$ be the linear spline approximation to x and $\hat{y}$ be the linear spline approximation to y. The first functional comparison test is

$$\nu_9 = \begin{cases} 1 \text{ if } D < \rho(\hat{x},\hat{y}) \\ 0 \text{ if } d < \rho(\hat{x},\hat{y}) \le D \\ -1 \text{ if } \rho(\hat{x},\hat{y}) \le d \end{cases}$$

where d and D are constants that depend on the type of data. This test compares the distance in the function space between the two functions to background data. If $\nu_9 = 1$ it will be assumed that x and y are definitely distinct. If $\nu_9 = -1$ then it will be assumed that x and y are similar. When $\nu_9 = 0$ the test in inconclusive. Note that although the test is made using $\hat{x}$ and $\hat{y}$, the inference is made about the relationship between x and y.

For the remaining comparison parameters make the following definitions. Let o_z be the ambient noise function with a constant level of z in all directions. Also, define,

$$\omega_x = \min_{z \varepsilon R}\{\rho(\hat{x}, o_z)\}$$

and

$$\omega_y = \min_{z \varepsilon R}\{\rho(\hat{y}, o_z)\}$$

$\bar{x}$ is defined to be the element of $\{o_z | z \varepsilon R\}$ such that $\rho(\hat{x},\bar{x})=\omega_x$. Similarly, $\bar{y}$ is the element of $\{o_z\}$ such that $\rho(y,\bar{y})=\omega_y$. Since $\{o_z | z \varepsilon R\}$ is locally compact these elements exist and are unique.

Now, the second functional comparison test in this method is,

$$\nu_{10}(z) = \begin{cases} 1 \text{ if } D < \omega_z \\ 0 \text{ if } d < \omega_z \leq D \\ -1 \text{ if } \omega_z \leq d \end{cases}$$

where d and D are the same constants as were used in ν_9. This comparison test indicates the relationship between a function and the constant noise function that best approximates it (in terms of the metric ρ). If $\nu_{10} = 1$ then the function varies greatly from its best constant approximation. If $\nu_{10} = -1$ then the function itself is nearly constant. The two comparison parameters that are of interest in this method are $\nu_{10}(\hat{x})$ and $\nu_{10}(\hat{y})$.

The third functional comparison test is

$$\nu_{11} = \begin{cases} 1 \text{ if } D < \rho(\bar{x},\bar{y}) \\ 0 \text{ if } d < \rho(\bar{x},\bar{y}) \leq D \\ -1 \text{ if } \rho(\bar{x},\bar{y}) \leq d \end{cases}$$

where d and D are the same constants as were used in ν_9. This test compares the best constant approximations to the functions $\hat{x}$ and $\hat{y}$. That is, this comparison test indicates what the (approximate) difference in level of noise is between x and y.

The final comparison test used in this methodology is

$$\nu_{12} = \begin{cases} 1 \text{ if } D < \rho(\hat{x}-\bar{x},\hat{y}-\bar{y}) \\ 0 \text{ if } d < \rho(\hat{x}-\bar{x},\hat{y}-\bar{y}) \\ -1 \text{ if } \rho(\hat{x}-\bar{x},\hat{y}-\bar{y}) \leq d \end{cases}$$

where, once again, d and D are the constants from the ν_9 test. This test compares the variations from the constant approximations to x and y.

These five functional parameters ν_9, $\nu_{10}(\hat{x})$, $\nu_{10}(\hat{y})$, ν_{11} and ν_{12}, provide information about the relationship between the functions $\hat{x}$ and $\hat{y}$. (Hence, they provide indirect information about the relationships between x and y.) Table F-3 gives a summary of the functional comparison parameters, their meanings, and how to use the tests in conjunction with each other.

The final aspect of the directional point-to-point comparison methodology to be considered here is the problem of using the statistical parameters versus the functional parameters. That is, if only one set of parameters is to be computed which set should it be. Or, if both sets are computed which set should be given more weight. Unfortunately, there is no clear answer to these questions. There are several reasons for not using or minimizing the importance of the statistical comparison tests (e.g., the assumption of normality, the assumption of independence, etc.) but there are also some reasons for minimizing the use of the functional comparison parameters. As a result the most reasonable way to determine how much each set of parameters should be weighed is to base the decision on empirical data. Thus, as in the choice of the best constants for the relative comparison tests, the final solution of this problem will depend on the judgement of the analyst as based on empirical evidence.

Methodology 5: Directional Point-to-Envelope Comparison

Let $\{x_j(\theta)\}_{j=1}^{n1}$ be the set of horizontal ambient noise functions and let $y(\theta)$ be the function that is to be compared with $\{x_j(\theta)\}$. Once again both $y(\theta)$ and $\{x_j(\theta)\}$ are known at only a finite number of points.

TABLE F-3: Functional Comparison Tests

Test	Value	Interpretation	Comments
ν_{10}	1	$\hat{x}$ and $\hat{y}$ are very distinct	The remaining tests can be to determine how $\hat{x}$ and $\hat{y}$ differ.
	0	$\hat{x}$ and $\hat{y}$ are neither clearly distinct nor clearly similar.	The remaining test can be used to determine which aspects of the two curves are similar and which aspects are distinct.
	-1	$\hat{x}$ and $\hat{y}$ are similar.	The remaining test can be used to describe $\hat{x}$ and $\hat{y}$ (e.g. shape, level of noise, magnitude of variation from this level, etc.)
$\nu_{11}(\hat{z})$	1	$\hat{z}$ is not well approximated by a constant noise level.	This test should be used with both $\hat{x}$ and $\hat{y}$. It gives a general indication of the shape of the curves (relative to constant level curves).
	0	$\hat{z}$ is only moderately well approximated by a constant noise level.	See comment for $\nu_{11}=1$
	-1	$\hat{z}$ is nearly a constant level noise function.	See comment for $\nu_{11}=1$
ν_{12}	1	The constant approximations to $\hat{x}$ and $\hat{y}$ differ significantly.	This may indicate two different conditions. $\hat{x}$ and y may be completely different. This will be indicated by greatly differing values for ω_x and ω_y or by $\nu_{13}=1$. Alternatively $\hat{x}$ and $\hat{y}$ may have similar shapes but may be systematically different (e.g. $\hat{x}=k+\hat{y}$, k a constant). If this is the case, then $\omega_x \doteq \omega_y$ and $\nu_{13}=-1$.

TABLE F-3: Functional Comparison Tests (Cont.)

Test	Value	Interpretation	Comments
	0	The constant approximations to to x and y are somewhat similar but not significantly so.	As in the case where there are also two possible conditions. The values of ω_x, and ω_y, and ν_{13} should be examined (see comments for $\nu_{12}=1$).
	-1	x and y can be approximated by the same constant noise function	This would be expected if $\nu_{10}=1$. If $\nu_{10}\neq 1$ then while $\hat{x}$ and $\hat{y}$ are similar in their constant approximations, they differ in either their shape or in their magnitude of variation from a constant. If $^{\omega}x$ and $^{\omega}y$ are significantly *different* then it cannot be determined whether $\hat{x}$ and $\hat{y}$ differ in shape or in their magnitude of variation from constants. If $^{\omega}x={}^{\omega}y$ then the curves differ in shape if $\nu_{13}=1$ and they differ in magnitude of variation if $\nu_{13}=-1$.
ν_{13}	1	Either the shape or the magnitude of variation from constant level are different for $\hat{x}$ and $\hat{y}$.	See the comments for ν_{12}
	0	An intermediate case between $\nu_{13}=1$ and $\nu_{13}=-1$.	See the comments for ν_{12}
	-1	$\hat{x}$ an $\hat{y}$ are similar in both shape and magnitude of variation from their constant approximations.	See the comments for ν_{12}

For statistical comparison test the quantities δ_{ij}, given by

$$\delta_{ij} = x_j(\theta_i) - y(\theta_i)$$

are computed. The most apparent method of statistical comparison is analysis of variance using $\{\delta_{ij}\}$. However, the utility of analysis of variance methods is questionable in this case. And so, the statistical tests used when comparing one noise function to a set of functions has been relegated to appendix H.

For the functional comparison tests, the problems of methods of approximation and individual comparison arise once more. As a solution to these problems, let $\hat{y}$ be the linear spline approximation to y and let $\hat{x}_j$ be the linear spline approximation to x_j for $j = 1,2,...n$. (The analyst may use any method of approximation. Here, linear spline approximation is used because it is simple to compute while still providing a reasonable approximation.) Let the metric ρ be defined as before. That is, ρ is the L_2 norm restricted to the subspace of all bounded piecewise linear functions. As with the method of approximation, the analyst may select another definition of ρ.

The first functional comparison test is a generalized, directional form of ν_2. For this test define,

$$x^+(\theta) = \max_{1 \leq j \leq n_1} \{\hat{x}_j(\theta)\}$$

$$x^-(\theta) = \min_{1 \leq j \leq n_1} \{\hat{x}_j(\theta)\}$$

$$\Theta = \{\theta \varepsilon [0,2\pi] \,|\, x^-(\theta) \leq \hat{y}(\theta) \leq x^+(\theta)\}$$

and

$$\bar{\Theta} = [0,2\pi] \setminus \Theta$$

Also, let μ be the Lebesgue measure. The first comparison test is

$$\nu_{13} = \begin{cases} 1 \text{ if } \mu(\Theta) \le d \\ 0 \text{ otherwise} \end{cases}$$

where d is a constant which reflects the quality of the data. d is chosen so that this test indicates when the set of points where $x^-(\theta) \ge \hat{y}(\theta)$ or $x^+(\theta) \le \hat{y}(\theta)$ is of no consequence. If this test were made with y and $\{x_j(\theta)\}$ and if these functions were based on perfect data, then it would be reasonable to set d = 0. However, y and $\{x_j\}$ are not used and the data will vary in type and quality. Thus, in most cases d will be greater than zero. The exact value of d for each type of situation will be based on the analyst's judgement and previous data of the same type.

This test provides only a crude comparison between y and $\{x_j\}$. It is conceivable that y could be very distinct from $\{x_j\}$ and yet $\nu_{13} = 0$. Also, it could be the case that $\nu_{13} = 1$ and the function y could be very similar to many elements of $\{x_j\}$. Examples of these two situations are shown in figure F-5. Therefore, while this test is useful in a general way, the remainder of the functional comparison tests give a much more reliable description of the relationships between y and $\{x_j\}$.

These comparison tests will be presented in three steps. First, tests which characterize the set $\{x_j\}$ within the space of functions will be given. Next, the tests which compare y to the $\{x_j\}$ will be defined. Finally, a procedure for using two types of tests together will be presented.

$\{x_j\}$ can be characterized by the distance between its elements, the differences between levels of noise of the elements, and the magnitude of variation from these levels of noise. Let

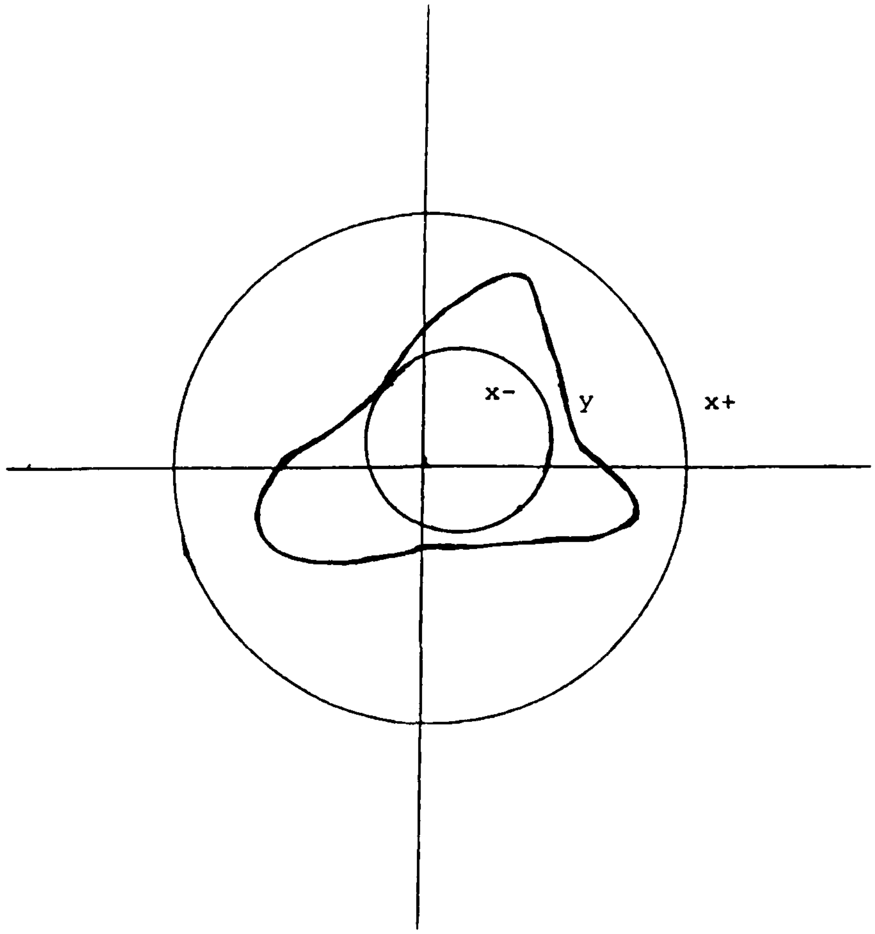
$$m_x^+ = \max_{i \ne j} \{\rho(\hat{x}_i, \hat{x}_j)\}$$

and

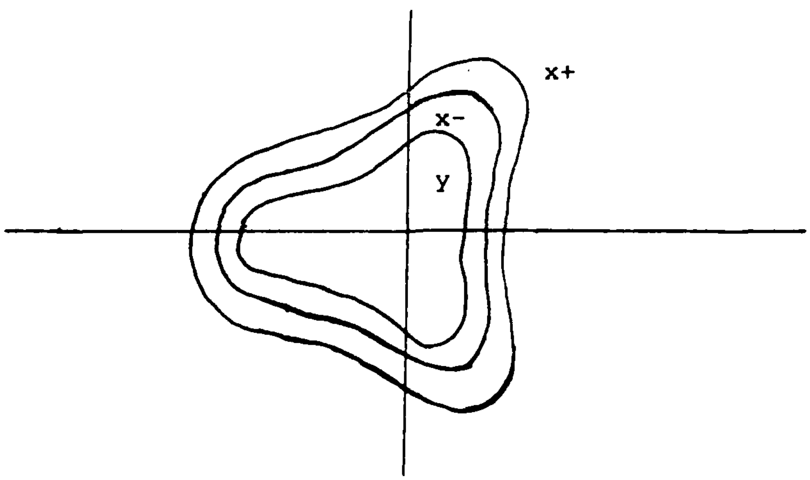
$$m_x^- = \min_{i \ne j} \{\rho(\hat{x}_i, \hat{x}_j)\}$$

These two constants give the maximum and minimum distances, in terms of the metric ρ, between the elements of $\{\hat{x}_j\}$. Recall that

Figure F-5: Disadvantages in the ν_{13} Comparison Test



a. $\nu_{13}=1$ and y and $\{x_j\}$ are dissimilar



b. $\nu_{13}=0$ and y and $\{x_j\}$ are similar the distance between two functions of this metric space is a measure of how the functions differ. The most reasonable comparison test based on these two constants is

$$\nu_{14} = m_x^+$$

(m_x^- will be used in later tests). The interpretation of this test will depend not so much on the absolute magnitude of m_x^+ but rather on how this value compares with the other measures of dispersion. It should be noted that this test may be affected by outlying functions in the set $\{x_j\}$. Thus, the analyst's judgment will be required to both compare this test with the other tests and to weigh all of the dispersion tests in light of the quality of the data.

The set can also be characterized by the level of noise of each of its elements. The levels of noise that will be used for the elements of $\{x_j\}$ will be the functions that are the best constant approximation to each of the elements of $\{\hat{x}_j\}$. To find the best constant approximating function let,

$$\omega_j = \min_{z \varepsilon R} \{\rho(\hat{x}_j, o_z)\} \qquad \text{for } j = 1,2,\ldots n_1$$

where o_z is the constant ambient noise function with level z in all directions. For each j the best approximating function is the constant function, $\bar{x}_j$, such that $\omega_j = \rho(\hat{x}_j, \bar{x}_j)$. (As previously noted, $\bar{x}_j$ will exist and be unique for all j.) ω_j can then be interpreted as a measure of how good the constant function approximation is.

The parameters that characterize the differences between levels of noise of $\{x_j\}$ are

$$\bar{x}^+ = \max_{1 \le j \le n_1} \{\rho(\bar{x}_j, 0_o)\}$$

and

$$\bar{x}^- = \min_{1 \le j \le n_1} \{\rho(\bar{x}_j, 0_o)\}$$

where 0_o is the constant noise function with a zero noise level in all directions. These parameters are the maximum and minimum levels, respectively, of the set of constant approximation functions, $\{\bar{x}_j\}$.

$$\mathrm{Cov}(Y_i, Y_j) = E[(Y_i - \mu)(Y_j - \mu)]$$

$$= E\left[\left(\frac{1}{m}\sum_{h=1}^{m} X_{m(i-1)+h} - \frac{1}{m}\sum_{h=1}^{m} \mu\right)\left(\frac{1}{m}\sum_{\ell=1}^{m} X_{m(j-1)+\ell} - \frac{1}{m}\sum_{\ell=1}^{m} \mu\right)\right]$$

$$= \frac{1}{m^2} E\left\{\left[\sum_{h=1}^{m}\left(X_{m(i-1)+k} - \mu\right)\right]\left[\sum_{h=1}^{m}\left(X_{m(j-1)+\ell} - \mu\right)\right]\right\}$$

$$= \frac{1}{m^2}\sum_{h=1}^{m}\sum_{\ell=1}^{m} E\left(X_{m(i-1)+h} - \mu\right)\left(X_{m(j-1)+\ell} - \mu\right)$$

$$= \frac{\sigma^2}{m^2}\sum_{h=1}^{m}\sum_{\ell=1}^{m} \rho^{\left(m(i-1)+h\right) - \left(m(j-1)+\ell\right)} \qquad \text{for } i > j$$

$$= \frac{\sigma^2}{m^2}\sum_{h=1}^{m}\sum_{\ell=1}^{m} \rho^{m(i-j)+h-\ell}$$

$$= \frac{\sigma^2}{m^2}\sum_{h=1}^{m}\sum_{\ell=1}^{m} \rho^{m[(i-j)-1]+h+(m-\ell)}$$

$$= \frac{\sigma^2}{m^2}\rho^{m[(i-j)-1]}\sum_{h=1}^{m}\sum_{k=1}^{m} \rho^{h+k-1}$$

$$= \frac{\rho\sigma^2}{m^2}\rho^{m[(i-j)-1]}\sum_{h=1}^{m}\sum_{k=1}^{m} \rho^{h-1+k-1}$$

$$= \frac{\rho\sigma^2}{m^2}\rho^{m[(i-j)-1]}\sum_{h=1}^{m} \rho^{h-1}\sum_{k=1}^{m} \rho^{k-1}$$

$$= \frac{\rho\sigma^2}{m^2}\rho^{m[(i-j)-1]}\left(\frac{1-\rho^m}{1-\rho}\right)\left(\frac{1-\rho^m}{1-\rho}\right)$$

$$\mathrm{Cov}(Y_i, Y_j) = \frac{\rho\sigma^2}{m^2}\rho^{m[(i-j)-1]}\left(\frac{1-\rho^m}{1-\rho}\right)^2 \qquad \text{for } i > j$$

Similarly, for j>i

$$\mathrm{Cov}\left(Y_i, Y_j\right) = \frac{\rho\sigma^2}{m^2} \; \rho^{m\left[(j-i)-1\right]} \left(\frac{1-\rho^m}{1-\rho}\right)^2$$

Hence, for $1 \le i, j \le n;\ i \ne j$,

$$\mathrm{Cov}\left(Y_i, Y_j\right) = \frac{\rho\sigma^2}{m^2} \; \rho^{m(|i-j|-1)} \left(\frac{1-\rho^m}{1-\rho}\right)^2$$

Now, the derivation for Es_2^2 can be given.

$$s_2^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(Y_i - m_2\right)^2$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} \left(Y_i - \mu\right)^2 - \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(Y_i - \mu\right)\left(Y_j - \mu\right) \right]$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} \left(Y_i - \mu\right)^2 - \frac{1}{n} \sum_{i=1}^{n} \sum_{j=i} \left(Y_i - \mu\right)\left(Y_j - \mu\right) - \frac{1}{n} \sum_{i=1}^{n} \sum_{j\ne i} \left(Y_i - \mu\right)\left(Y_j - \mu\right) \right]$$

$$s_2^2 = \frac{1}{n-1} \left[\frac{n-1}{n} \sum_{i=1}^{n} \left(Y_i - \mu\right)^2 - \frac{1}{n} \sum_{i=1}^{n} \sum_{i\ne j} \left(Y_i - \mu\right)\left(Y_j - \mu\right) \right]$$

Taking the expected value yields

$$Es_2^2 = \frac{1}{n-1} \left[\frac{n-1}{n} \sum_{i=1}^{n} E\left(Y_i - \mu\right)^2 - \frac{1}{n} \sum_{i=1}^{n} \sum_{j\ne i} E\left(Y_i - \mu\right)\left(Y_j - \mu\right) \right]$$

$$= \frac{1}{n-1} \left\{ (n-1)\sigma_Y^2 - \frac{1}{n} \sum_{i=1}^{n} \sum_{j\ne i} \left[\frac{\rho\sigma^2}{m^2} \rho^{m(|i-j|-1)} \left(\frac{1-\rho^m}{1-\rho}\right)^2 \right] \right\}$$

$$Es_2^2 = \frac{1}{n-1} \left\{ (n-1) \left[\frac{\sigma^2}{m} + \frac{2\rho\sigma^2}{m^2(1-\rho)^2} \left[(m-1) - m\rho + \rho^m \right] \right] - \frac{\rho^{1-m}\sigma^2 (1-\rho^m)^2}{nm^2(1-\rho)^2} \left[2 \sum_{i=1}^{n} \sum_{j<i} \left(\rho^m\right)^{i-j} \right] \right\}$$

$$Es_2^2 = \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{n(n-1)m(1-\rho)^2 + 2n(n-1)\rho\left[(m-1)-m\rho + \rho^m\right]\right.$$

$$\left. -2\rho^{1-m}(1-\rho^m)^2 \sum_{\ell=1}^{n-1} \ell(\rho^m)^{n-\ell}\right\}$$

$$= \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{n(n-1)m - 2n(n-1)m\rho + n(n-1)m\rho^2 + 2n(n-1)(m-1)\rho\right.$$

$$\left. -2n(n-1)m\rho^2 + 2n(n-1)\rho^{m+1} - 2\rho^{mn-2m+1}(1-\rho^m)^2 \sum_{i=1}^{m-1} \ell(\rho^{-m})^{\ell-1}\right\}$$

$$= \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{n(n-1)m - 2n(n-1)\rho - n(n-1)m\rho^2 + 2n(n-1)\rho^{m+1}\right.$$

$$\left. -2^{nm-2m+1}(1-\rho^m)^2\left[\frac{(n-1)(\rho^{-m})^n - (n)(\rho^{-m})^{n-1} + 1}{(\rho^{-m}-1)^2}\right]\right\}$$

$$= \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{n(n-1)m - 2n(n-1)\rho - n(n-1)m\rho^2 + 2n(n-1)\rho^{m+1}\right.$$

$$\left. -2\rho^{nm+1}(1-\rho^m)^2\left[\frac{(n-1)\rho^{-nm} - n\rho^{m-mn} + 1}{(1-\rho^m)^2}\right]\right\}$$

$$= \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{n(n-1)m - 2n(n-1)\rho - n(n-1)m\rho^2 + 2n(n-1)\rho^{m+1}\right.$$

$$\left. -2(n-1)\rho + 2n\rho^{m+1} - 2\rho^{nm+1}\right\}$$

$$Es_2^2 = \frac{\sigma^2}{n(n-1)m^2(1-\rho)^2}\left\{n(n-1)m - 2(n^2-1)\rho - n(n-1)m\rho^2 + 2n^2\rho^{m+1} - 2\rho^{nm+1}\right.$$

In appendix C, case 1, an expression for the difference between the ambient noise levels at two points was presented. Its derivation is

$$AN_1(\theta) - AN_m(\theta) = \int_{p_1\,[\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p) - \int_{p_m\,[\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

$$= \int_{R(p^*)\,[\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{p_1\,[\ell_1(\theta)]}^{R(p^*)} TL_1(p)\,dF(p)$$

$$- \int_{p_m\,[\ell_m(\theta)]}^{p^*} TL_m(p)\,dF(p) - \int_{p^*\,[\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

$$= \int_{R(p^*)\,[\ell_1(\theta)]}^{\hat{p}_1} TL_1(p)\,dF(p)$$

$$+ \int_{p_1\,[\ell_1(\theta)]}^{R(p^*)} TL_1(p)\,d\left\{F(p) - F(R^{-1}(p)) + F(R^{-1}(p))\right\}$$

$$- \int_{p_m\,[\ell_m(\theta)]}^{p^*} TL_m(p)\,dF(p) - \int_{p^*\,[\ell_m(\theta)]}^{\hat{p}_m} TL_m(p)\,dF(p)$$

APPENDIX G: Derivation of Equations

At several points in the body of this report equations were used without being derived. This was done to make the text clear, since only the final equations were needed. In these cases lengthy derivations would only obscure the text. However, how these equations are obtained is not always obvious. Therefore, some of the equations and their complete derivations have been included in this appendix.

The first equation, which comes from Appendix A, is for Es_1^2, It uses the identity

$$\sum_{\ell=1}^{N} \ell x^{N-1} = \frac{Nx^{N+1} - (N+1)x^N + 1}{(x-1)^2}$$

The derivation for Es_1^2, is:

$$Es_1^2 = E\left(\frac{1}{N-1}\sum_{i=1}^{N}(X_i - m_1)^2\right)$$

$$= E\left\{\frac{1}{N-1}\sum_{i=1}^{N}\left[(X_i-\mu)^2 - 2(X_i-\mu)(m_1-\mu) + (m_1-\mu)^2\right]\right\}$$

$$= E\left\{\frac{1}{N-1}\left[\sum_{i=1}^{N}(X_i-\mu)^2 - 2(m_1-\mu)\sum_{i=1}^{N}(X_i-\mu) + N(m_1-\mu)^2\right]\right\}$$

$$= E\left\{\frac{1}{N-1}\left[\sum_{i=1}^{N}(X_i-\mu)^2 - \frac{2}{N}\sum_{i=1}^{N}(X_i-\mu)\sum_{i=1}^{N}(X_i-\mu) - \frac{1}{N}\sum_{i=1}^{N}(X_i-\mu)\sum_{i=1}^{N}(X_i-\mu)\right]\right\}$$

$$= E\left\{\frac{1}{N-1}\left[\sum_{i=1}^{N}(X_i-\mu)^2 - \frac{1}{N}\sum_{i=1}^{N}(X_i-\mu)\sum_{i=1}^{N}(X_i-\mu)\right]\right\}$$

$$= \frac{1}{N-1}\left\{\sum_{i=1}^{N}E(X_i-\mu)^2 - \frac{1}{N}E\left[\sum_{i=1}^{N}(X_i-\mu)\sum_{i=1}^{N}(X_i-\mu)\right]\right\}$$

$$= \frac{1}{N-1}\left\{N\sigma^2 - \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}E\left[(X_i-\mu)(X_j-\mu)\right]\right\}$$

$$Es_1^2 = \frac{\sigma^2}{N(N-1)} \left\{ N^2 - \left[2\sum_{i=1}^{N} \sum_{j\le i} \rho^{i-j} - \sum_{i=1}^{N} \sum_{j=i} \rho^{i-j} \right] \right\}$$

$$= \frac{\sigma^2}{N(N-1)} \left\{ N^2 - \left[2\sum_{\ell=1}^{N} \ell\rho^{N-\ell} - N \right] \right\}$$

$$= \frac{\sigma^2}{N(N-1)} \left\{ N(N+1) - 2\rho^{N-1} \sum_{\ell=1}^{N} \ell(\rho^{-1})^{\ell-1} \right\}$$

$$= \frac{\sigma^2}{N(N-1)} \left\{ N(N+1) - 2\rho^{N-1} \left[\frac{N\rho^{-N-1} - (N+1)\rho^{-N} + 1}{(\rho^{-1} - 1)^2} \right] \right\}$$

$$= \frac{\sigma^2}{N(N-1)} \left\{ N(N+1) - \frac{2}{(1-\rho)^2} \left[N - (N+1)\rho + \rho^{N+1} \right] \right\}$$

$$= \frac{\sigma^2}{N(N-1)(1-\rho)^2} \left\{ N(N+1)(1-\rho)^2 - 2N + 2(N-1)\rho - 2\rho^{N+1} \right\}$$

$$Es_1^2 = \frac{\sigma^2}{N(N-1)(1-\rho)^2} \left\{ N(N-1) - 2(N^2-1)\rho + N(N+1)\rho^2 - 2\rho^{N+1} \right\}$$

The other expression for Es_1^2 can be obtained in the same fashion except the step

$$Es_1^2 = \frac{\sigma^2}{N(N-1)} \left\{ N^2 - \left[2\sum_{i=1}^{N} \sum_{j\le i} \rho^{i-j} - \sum_{i=1}^{N} \sum_{j=i} \rho^{i-j} \right] \right\}$$

is replaced by the step

$$Es_1^2 = \frac{\sigma^2}{N(N-1)} \left\{ N^2 - \left[2\sum_{i=1}^{N} \sum_{j<1} \rho^{i-j} + \sum_{i=1}^{N} \sum_{j=i} \rho^{i-j} \right] \right\}$$

and the derivation completed.

Es_2^2 is obtained in a similar way. However, before deriving the expression for Es_2^2, the equation for $Cov(Y_i, Y_j)$ must be justified. $Cov(Y_i, Y_j)$ is given by,

$$= \int_{R(p^*)}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{p_1}^{R(p^*)} TL_1(p)\,d\underset{p_1}{\Delta F}(p)$$

$[\ell_1(\theta)]$ $[\ell_1(\theta)]$

$$+ \int_{p_1}^{R(p^*)} TL_1(p)\,dF(R^{-1}(p)) - \int_{p_m}^{p^*} TL_m(p)\,dF(p)$$

$[\ell_1(\theta)]$ $[\ell_m(\theta)]$

$$- \int_{p^*}^{\hat{p}_m} TL_m(p)\,dF(p)$$

$[\ell_m(\theta)]$

$$AN_1(\theta) - AN_m(\theta) = \int_{R(p^*)}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{p_1}^{R(p^*)} TL_1(p)\,d\underset{p_1}{\Delta F}(p)$$

$[\ell_1(\theta)]$ $[\ell_1(\theta)]$

$$+ \int_{p_m}^{p^*} TL_1(R(p))\,dF(p) - \int_{p_m}^{p^*} TL_m(p)\,dF(p)$$

$[\ell_m(\theta)]$ $[\ell_m(\theta)]$

$$- \int_{p^*}^{\hat{p}_m} TL_m(p)\,dF(p)$$

$[\ell_m(\theta)]$

$$AN_1(\theta) - AN_m(\theta) = \int_{R(p^*)}^{\hat{p}_1} TL_1(p)\,dF(p) + \int_{p_1}^{R(p^*)} TL_1(p)\,d\underset{p_1}{\Delta F}(p)$$

$[\ell_1(\theta)]$ $[\ell_1(\theta)]$

$$+ \int_{p_m}^{p^*} \left\{ TL_1(R(p)) - TL_m(p) \right\} dF(p) - \int_{p^*}^{\hat{p}_m} TL_m(p)\,dF(p)$$

$[\ell_m(\theta)]$

which is the expression that was given.

The other expression presented without derivation in appendix C was

$$|AN_m(\theta) - \sum_{i=0}^{7} \omega_i AN_i(\theta)| \leq \Omega$$

To see that this is indeed true consider,

$$|AN_m(\theta) - \sum_{i=0}^{7} \omega_i AN_i(\theta)| = |\sum_{i=0}^{7} \omega_i AN_m(\theta) - \sum_{i=0}^{7} \omega_i AN_i(\theta)|$$

$$\text{since} \quad \sum_{i=0}^{7} \omega_i = 1$$

$$= |\sum_{i=0}^{7} \omega_i (AN_m(\theta) - AN_i(\theta))|$$

$$\leq \sum_{i=0}^{7} \omega_i |AN_i(\theta) - AN_m(\theta)|$$

$$\leq \sum_{i=0}^{3} \omega_{2i} (\varepsilon_\rho(r) + \varepsilon_t(r)) + \sum_{i=0}^{3} \omega_{2i+1} (\varepsilon_\rho(\sqrt{2}r) + \varepsilon_t(\sqrt{2}r))$$

$$\leq \sum_{i=0}^{3} \frac{\varepsilon_\rho(r) + \varepsilon_t(r)}{\Omega(\varepsilon_\rho(r) + \varepsilon_t(r))} + \sum_{i=0}^{3} \frac{\varepsilon_\rho(\sqrt{2}r) + \varepsilon_t(\sqrt{2}r)}{\Omega(\varepsilon_\rho(\sqrt{2}r) + \varepsilon_t(\sqrt{2}r)}$$

$$\leq \frac{4}{\Omega} + \frac{4}{\Omega}$$

$$|AN_m(\theta) - \sum_{i=0}^{7} \omega_i AN_i(\theta)| \leq \frac{8}{\Omega}$$

APPENDIX H: Alternative Comparison Methodologies

This appendix presents ambient noise comparison tests that can be used in place of the tests given in appendix F. The alternative tests are listed by the methodology in which they would be included. The reasons that these tests were not used in appendix F are given along with the tests.

Methodology 1: Point to Point Comparison

There is so little data available in this case there is only one reasonable method of comparison. This comparison test is the one given in appendix F.

Methodology 2: Point to Envelope Comparison

In this method of comparison there is an alternative to the test υ_3. This test uses the statistical notion of hypothesis testing. It is assumed that $\{x_i\}_{i=1}^n$ is from a population with a $N(\mu_x, \sigma_x^2)$ distribution. This comparison parameter tests the null hypothesis,

$$H_0: \mu_x = y$$

against the alternative hypothesis,

$$H_1: \mu_x \neq y.$$

The test statistic that is used is,

$$t = \frac{\sqrt{n}}{s_x} (m_x - y)$$

This statistic has a Student's t distribution with n-1 degrees of freedom. The alternative comparison test to υ_3 is

$$\hat{\upsilon}_3 = \begin{cases} 1 \text{ if } |t| < t_{[1 - \frac{\alpha}{2}](n - 1)} \\ 0 \text{ otherwise} \end{cases}$$

In this test α is the probability of Type I error, i.e., the probability of rejecting the null hypothesis when it is true. $t_{[\cdot](\nu)}$

is the value of the t statistic with ν degrees of freedom such that $P(t \leq t_{[\varepsilon](\nu)}) = \varepsilon$.

$\hat{\nu}_3$ would be the ideal test to use if y were known without any error. The value of $\hat{\nu}_3$ in this case would indicate whether $\{x_i\}$ is from the $N(y,\sigma^2)$ population ($\hat{\nu}_3 = 1$) or from a normal population with mean μ_x and $\mu_x \neq y$ ($\hat{\nu}_3 = 0$). However, y will not be an exact value, and so, the test based on tolerance limits will be more reliable.

Methodology 3: Envelope to Envelope Comparison

There are several alternative comparison tests for this methodology. Two of the alternative tests use the statistical notion of confidence intervals. The remainder use hypothesis testing.

For the confidence interval tests it is assumed that $\{x_i\}_{i=1}^{n_1}$ comes from a $N(\mu_x,\sigma_x^2)$ population and that $\{y_j\}_{j=1}^{n_2}$ comes from a $N(\mu_y,\sigma_y^2)$ population. For this discussion a confidence interval for a set will be a pair of real numbers L and U such that there is a *100λ percent confidence that the mean of the population* to which the set belongs is in the interval [L,U]. The parameter λ is to be selected by the analyst. For the confidence interval comparison test define

$$L_x(\lambda) = m_x - \frac{s_x}{\sqrt{n_1}} t_{\left[\frac{1+\lambda}{2}\right](n_1 - 1)}$$

$$U_x(\lambda) = m_x + \frac{s_x}{\sqrt{n_1}} t_{\left[\frac{1+\lambda}{2}\right](n_1 - 1)}$$

$$L_y(\lambda) = m_y - \frac{s_y}{\sqrt{n_2}} t_{\left[\frac{1+\lambda}{2}\right](n_2 - 1)}$$

$$U_y(\lambda) = m_y + \frac{s_y}{\sqrt{n_2}} t_{\left[\frac{1+\lambda}{2}\right](n_2 - 1)}$$

where $t_{\left[\frac{1+\lambda}{2}\right](n_1 - 1)}$ is the value of the Student's t statistic as described above. The 100λ percent confidence interval for the mean of the x population is

$$I_1 = \left[L_x(\lambda), U_x(\lambda)\right]$$

and the 100λ percent confidence interval for the mean of the y population is

$$I_2 = \left[L_y(\lambda), U_y(\lambda)\right].$$

As alternative tests to the comparison tests in appendix F, ν_4 and ν_5 can be computed using the confidence intervals defined above.

The reason that these intervals were not presented in appendix F is that they do not present as much information as the interval tests given there. When $\nu_4 \doteq 0$ and $\nu_5 \doteq 0$, then it is reasonable to assume that the x and y populations are dissimilar. However, when ν_4 and ν_5 are not near zero, it is not necessarily true that $\{x_i\}$ and $\{y_j\}$ come from the same population.

For the alternative comparison tests that use hypothesis testing, it will also be assumed that $\{x_i\}$ is from a $N(\mu_x, \sigma_x^2)$ population and that $\{y_j\}$ is from a $N(\mu_y, \sigma_y^2)$ population. There are two tests that compare the means of these populations. One test is used when $\sigma_x^2 = \sigma_y^2$ and the other is used when $\sigma_x^2 \neq \sigma_y^2$. If it is not reasonable to assume that $\sigma_x^2 = \sigma_y^2$ or $\sigma_x^2 \neq \sigma_y^2$ based on the data alone, then hypothesis testing can be used to determine which condition probably holds.

The null hypothesis for comparing σ_x^2 and σ_y^2 is

$$H_0: \sigma_x^2 = \sigma_y^2$$

and the alternative hypothesis is

$$H_1: \sigma_x^2 \neq \sigma_y^2$$

The test statistic to be used is

$$F = \frac{s_x^2}{s_y^2}$$

This statistic has an F distribution. The test to determine whether σ_x^2 is probably equal to σ_y^2 or not is

$$\nu_{22} = \begin{cases} 1 \text{ if } F_{(\alpha/2)[n_1-1,\ n_2-1]} < F < F_{(1-\alpha/2)[n_1-1,\ n_2-1]} \\ 0 \text{ otherwise} \end{cases}$$

where $F_{(\varepsilon)[\nu_1,\ \nu_2]}$ is the value of the cumulative F distribution with ν_1, ν_2 degrees of freedom at which $P(F \leq F_{(\varepsilon)[\nu_1,\ \nu_2]}) = \varepsilon$. α is the probability of Type I error and is selected by the analyst. If $\nu_{22} = 1$, then it is reasonable to assume that $\sigma_x^2 = \sigma_y^2$. If $\nu_{22} = 0$, then it is reasonable to assume that $\sigma_x^2 \neq \sigma_y^2$.

When $\sigma_x^2 = \sigma_y^2$ the alternative test to compare the means of the populations has null and alternative hypothesis

$$H_0: \quad \mu_x = \mu_y$$

and

$$H_1: \quad \mu_x \neq \mu_y ,$$

respectively. The test statistic is

$$t = \frac{(m_x - m_y)}{s\sqrt{n_1^{-1} + n_2^{-1}}}$$

where s^2 is the common variance given by

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The comparison test is

$$\nu_{23} = \begin{cases} 1 \text{ if } |t| < t_{(1-\alpha/2)(n_1 + n_2 - 2)} \\ 0 \text{ otherwise} \end{cases}$$

where $t_{(1-\alpha/2)(n_1 + n_2 - 2)}$ is the value of the Student's t statistic as described above and α is the probability of Type I error.

When $\sigma_x^2 \neq \sigma_y^2$ the alternative test to compare the means of the populations has the same null and alternative hypotheses as ν_{23} (i.e., H_0: $\mu_x = \mu_y$ and H_1: $\mu_x \neq \mu_y$). However, the test statistic here is

$$t' = \frac{(m_x - m_y)}{\sqrt{S_x^2 n_1^{-1} + S_y^2 n_2^{-1}}}$$

This statistic has a distribution which is approximately a Student's t distribution. The comparison test itself uses the quantities defined by

$$w_x = \frac{s_x^2}{n_1}$$

$$w_y = \frac{s_y^2}{n_2}$$

$$t_1 = t_{[1-\alpha/2](n_1 - 1)}$$

and

$$t_2 = t_{[1-\alpha/2](n_2 - 1)}$$

where $t_{[1-\alpha/2](n_1 - 1)}$ is the Student's t statistic as described above. The comparison test is

$$\nu_{24} = \begin{cases} 1 \text{ if } |t'| < \dfrac{w_x t_1 + w_y t_2}{w_x + w_y} \\ 0 \text{ otherwise} \end{cases}$$

When $\nu_{24} = 1$, it can be assumed that $\mu_x = \mu_y$. Otherwise, $\mu_x \neq \mu_y$. It should be noted that this statistical test is only approximately a t test. In practice, it is notoriously unreliable.

Since ν_{24} is not reliable, it was not included in appendix F. ν_{23} was not included in appendix F because it will seldom be the case that $\mu_x = \mu_y$. Without ν_{23} and ν_{24} there is no reason to use ν_{22}. Therefore, no comparison tests using hypothesis testing were mentioned.

Methodology 4: Directional Noise Point to Point Comparison

In this section one alternative statistical comparison test, one alternative functional comparison test, and several variations of the functional tests will be presented. However, these alternative tests are not the most important part of this section. The point that should be emphasized here is that it is the fact that some kind of functional comparison is made which is important. The particular functional comparisons used are relatively unimportant. The alternative tests should be considered with this fact in mind.

The statistical comparison test once again uses hypothesis testing. It is assumed that $\{D_i\}_{i=1}^n$ (the differences in $x(\theta_i)$ and $y(\theta_i)$ for each i) is from a normal population, $N(\mu_D, \sigma_D^2)$. The null hypothesis is

$$H_0: \quad \mu_D = 0$$

i.e., the mean difference between the paired sets $\{x(\theta_i)\}$ and $\{y(\theta_i)\}$ is zero. The alternative hypothesis is

$$H_1: \quad \mu_D \neq 0$$

The test statistic is

$$t = \sqrt{n}\,\frac{m_D}{s_D}$$

which has a Student's t distribution. The comparison test is

$$\nu_{25} = \begin{cases} 1 \text{ if } |t| < t_{[1-\alpha/2](n-1)} \\ 0 \text{ otherwise} \end{cases}$$

where $t_{[1-\alpha/2](n-1)}$ is the value of the Student's t statistic as described under methodology 1 of this appendix. α is the probability of Type I error. This test was not given in appendix F because in most cases there is no reason to assume that $\{D_i\}$ is from a normal population.

For the alternate functional comparison test define the function $\hat{x} + t$ by

$$[\hat{x} + t](\theta) = \hat{x}[(\theta + t) \bmod 2\pi]$$

where $-\pi \leq t \leq \pi$. The alternative comparison parameter is

$$\nu_{26} = \min_t \{\rho(\hat{x} + t, \hat{y})\}$$

This test indicates whether $\hat{x}$ and $\hat{y}$ are similar but slightly rotated relative to each other. It should only be used when $\nu_9 = -1$ (i.e., when $\hat{x}$ and $\hat{y}$ are similar to begin with). ν_{26} may indicate small errors in direction (e.g., when the bearing of an array is incorrectly recorded by a few degrees when measurements are being made every degree). However, since errors of this type will seldom be found, the effort necessary to compute ν_{26} is usually not merited.

The variations in the functional comparison tests alluded to at the beginning of this section arise from the method used to approximate $x(\theta)$ and $y(\theta)$ and the metric used when comparing functions. As mentioned before, these considerations are secondary to the fact that a functional approach is used at all. Nonetheless, it will be worthwhile to make a brief mention of them here.

The two functions $x(\theta)$ and $y(\theta)$ are known at a fixed, finite set of values, $\{\theta_i\}_{i=1}^{n}$. Using the values $\{x(\theta_i)\}_{i=1}^{n}$ the function $x(\theta)$ can be approximated in many ways. (The function $x(\theta)$

will be used here but the discussion will apply to $y(\theta)$ and, in fact, the sets of functions that arise in methodologies 5 and 6.) For convenience of notation, define the quantities θ_0, θ_{n+1}, $x(\theta_0)$, $x(\theta_{n+1})$ by $\theta_0 = \theta_n$, $\theta_{n+1} = \theta_1$, $x(\theta_0) = x(\theta_n)$, and $x(\theta_{n+1}) = x(\theta_1)$. The method of approximation assumed in appendix F was linear spline approximation. That is,

$$x(\theta) = \frac{x(\theta_{i+1}) - x(\theta_i)}{\theta_{i+1} - \theta_i} (\theta - \theta_i) + x(\theta_i)$$

where i is chosen such that $\theta_i \leq \theta \leq \theta_{i+1}$. Also mentioned in appendix F was step function approximation. In this case

$$x(\theta) = x(\theta_i)$$

where i is chosen such that $\frac{\theta_{i-1} + \theta_i}{2} \leq \theta < \frac{\theta_i + \theta_{i+1}}{2}$. The step function approximation has the advantage that it is simple to use in computations. The linear spline approximation has the advantage that it is continuous while still being relatively easy to manipulate. Cubic spline, polynomial interpolation, and other methods all have their own advantages and disadvantages. However, as can be seen from the simple example above, the different approximation methods will produce approximation functions with different properties. Hence, while the analyst is free to chose any kind of approximation, some consideration should be given to the results of this choice.

Everything that has been said concerning the choice of the method of approximation can be applied to the choice of the metric ρ. In appendix F ρ was

$$\rho(\phi, \psi) = \left[\frac{1}{2\pi}\int_0^{2\pi} (\phi(\theta) - \psi(\theta))^2 \, d\theta\right]^{1/2}$$

Other possible choices for ρ are

$$\rho(\phi, \psi) = \frac{1}{2\pi}\int_0^{2\pi} |\phi(\theta) - \psi(\theta)| \, d\theta$$

and

$$\rho(\phi, \psi) = \max_{\theta \varepsilon [0,2\pi]} \{|\phi(\theta) - \psi(\theta)|\}$$

Each of these metrics will produce different results. It is the fact that a metric is used which is important. The selection of that metric depends only on the analyst's requirements.

Methodology 5: Directional Point to Point Comparison

The only alternative comparison test that will be present here is an analysis of variance test. For variations of the functional comparison tests see methodology 4.

In appendix F it was mentioned that analysis of variance could be used with $\{x_j(\theta_i)\}_{j=1}^{n}$ and $y(\theta_i)$ where $i = 1,2,...,n$. For this purpose the quantities δ_{ij} were formed where

$$\delta_{ij} = x_j(\theta_i) - y(\theta_i)$$

The additional quantities needed for analysis of variance are,

$$\bar{\delta}_{\cdot j} = \frac{1}{n} \sum_{i=1}^{n} \delta_{ij}$$

and

$$\bar{\delta} = \frac{1}{nn_1} \sum_{j=1}^{n_1} \sum_{i=1}^{n} \delta_{ij} = \frac{1}{n_1} \sum_{j=1}^{n_1} \bar{\delta}_{\cdot j}$$

The parameter that would be of interest to the analyst is

$$\nu_{27}^2 = \frac{n}{n_1 - 1} \sum_{j=1}^{n_1} (\bar{\delta}_{\cdot j} - \bar{\delta})^2$$

which is the mean residual error. ν_{27}^2 reflects the variation in the differences between each of the functions x_j and the function y. That is, when ν_{27}^2 is small the difference between x_j and y is similar for all j.

The reason that this comparison method was not included in appendix F is that the underlying assumptions of analysis of variance will seldom be met by the data. That is, for analysis of variance it is assumed that the elements of $\{\delta_{ij}\}_{i=1}^{n}$ are independent and normally distributed for each j. The data from ambient noise functions will seldom be normally distributed and almost never independent. Thus, this method of comparison has very limited use.

Methodology 6: Directional Envelope to Envelope Comparison

In this section an analysis of variance test will be given for $\{x_j(\theta_i)\}_{j=1}^{n_1}$ and $\{y_k(\theta_i)\}_{k=1}^{n_2}$ where $i = 1,2,\ldots,n$. Also, an alternative functional comparison test will be presented. For additional variations to the function given in appendix F, see methodology 4 in this appendix.

In appendix F the quantities δ_{ijk} were formed for use in an analysis of variance comparison. They were defined by

$$\delta_{ijk} = x_j(\theta_i) - y_k(\theta_i)$$

For the comparison test compute the sums,

$$\bar{\delta}_{\cdot jk} = \frac{1}{n} \sum_{i=1}^{n} \delta_{ijk}$$

$$\bar{\delta}_{\cdot\cdot k} = \frac{1}{nn_1} \sum_{j=1}^{n_1} \sum_{i=1}^{n} \delta_{ijk}$$

$$\bar{\delta}_{\cdot j \cdot} = \frac{1}{nn_2} \sum_{k=1}^{n_2} \sum_{i=1}^{n} \delta_{ijk}$$

and

$$\bar{\delta}_{\cdot\cdot\cdot} = \frac{1}{nn_1n_2} \sum_{k=1}^{n_2} \sum_{j=1}^{n_1} \sum_{i=1}^{n} \delta_{ijk}$$

The parameters that would be of interest to the analysis are

$$\nu_{28}^2 = \frac{nn_2}{n_1 - 1} \sum_{j=1}^{n_1} (\bar{\delta}_{\cdot j \cdot} - \bar{\delta})^2$$

and

$$\nu_{29}^2 = \frac{nn_1}{n_2 - 1} \sum_{k=1}^{n_2} (\bar{\delta}_{\cdot \cdot k} - \bar{\delta})^2$$

which are, respectively, the mean residual error due to $\{x_j\}$ and the mean residual error due to $\{y_k\}$. These parameters are interpreted in the same fashion as ν_{27}^2. Also, the reasons for not using ν_{28}^2 and ν_{29}^2 are the same as for not using ν_{27}^2. That is, the data will almost never meet the requirements of normality and independence.

In the case where the sets of functions $\{x_j(\theta)\}$ and $\{y_k(\theta)\}$ were compared in appendix F it was noted that there was no test which indicated the distance between these two sets and which was both easy to compute and comprehensive. ν_{22} was given as one way to compare the two sets. A second method to compare the distance between the two sets is

$$\nu_{30} = \text{med}\{\rho(x_j, y_{kj})\} + \text{med}\{\rho(x_{jk}, y_k)\}$$

where x_{jk} and y_{kj} are as they were defined in methodology 6 of appendix F and $\text{med}\{z_\ell\}$ is the median of $\{z_\ell\}$. This comparison test has the advantage over ν_{22} that it is less strongly effected by outliers (since it is a rank test). Although ν_{30} is more difficult to compute than ν_{22}, it is still relatively simple to compute.

This concludes the listing of alternative comparison methodologies to those of appendix F.

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The comparison parameter that is used to indicate the dispersion of $\{\bar{x}_j\}$ is

$$\nu_{15} = \bar{x}^+ - \bar{x}^-$$

This parameter is similar to ν_{14} in that it is the maximum difference between the constant approximating functions. Its value, when compared to the value of ν_{14}, is an indicator of the difference in $\{x_j\}$ that can be attributed to variation in the noise levels of the elements of $\{x_j\}$. Like ν_{14} it is also effected by extreme functions.

A final characteristic of $\{x_j\}$ is the variation of the elements of this set from their best constant approximations (i.e., the magnitude of variation of $\bar{x}_j$ from $\hat{x}_j$ for each j). For the comparison parameter for this type of dispersion define the constants,

$$\omega_x^+ = \max_{1 \leq j \leq n_1} \{\omega_j\}$$

and

$$\omega_x^- = \min_{1 \leq j \leq n_1} \{\omega_j\}$$

the comparison parameter itself is

$$\nu_{16} = \omega_x^+ - \omega_x^-$$

This is the maximum difference in the magnitude of variation in $\{x_j\}$.

These last three parameters, ν_{14}, ν_{15}, and ν_{16} provide a general characterization of $\{x_j\}$. Table F-4 can be used to interpret their values. An "L" indicates a value that is large when compared to the value of ν_{14}. An "S" indicates a value that is small when compared to the value of ν_{14}. Clearly, the analyst's evaluation of the relative sizes of these values and of their size compared to background data will determine the interpretation of the results of these three tests in any particular situation.

TABLE F-4: Summary of [illegible], [illegible], and [illegible] values

v_{15}	v_{16}	Meaning
L	L	The functions in the set $\{\bar{x}_j\}$ are widely [illegible] in the space of functions. That is, the [illegible] vary in the level of their constant [illegible] and in their magnitude [illegible] variation from these levels.
L	[illegible]	This indicates that [illegible] of [illegible]. However, [illegible] of variat[illegible] x [illegible]
	[illegible]	[illegible]

[illegible]erefore, the analyst will only be able to derive meaning from [illegible]ese tests after a body of data has been used.

[illegible]ow that a general picture of $\{x_j\}$ has been built up, y [illegible] compared to $\{x_j\}$. For the first comparison test let

$$y^{-} = \min_{1 \le j \le n_1} [illegible](x_j, y)$$

[illegible] itself is

$$\begin{cases} [illegible] \text{ if } m_x^{+} [illegible] y^{-} \\ [illegible] \text{ if } [illegible]_x [illegible] y^{-} [illegible] m_x^{+} \\ [illegible] m_x^{-} \end{cases}$$

[illegible] distance between y and the elements of $\{x_j\}$ [illegible] If [illegible] = 1 then the distance [illegible] greater than the maximum dis- [illegible] x_j [illegible] Thus, y is significantly differ- [illegible] It [illegible] = 0 or [illegible] =-1, then y is [illegible] the elements of $\{x_j\}$. The next te[illegible] [illegible] the set $\{x_j\}$ as a whole.

[illegible] average distance between y [illegible]

[illegible]

$$\begin{cases} [illegible] A [illegible] \\ [illegible] A \\ [illegible] \end{cases}$$

[illegible] are constants that depend on the type of data. The [illegible] of values for [illegible] and [illegible] and their inter- [illegible] given in table F-5.

TABLE F-5: Interpretation of ν_{17} and ν_{18}

ν_{17}	ν_{18}	Interpretation*
1	Any	y and $\{x_j\}$ are completely dissimilar. That is, y is not similar to any element of $\{x_j\}$
-1	1	While y is similar to some element of $\{x_j\}$ it is dissimilar to most of the elements.
-1	-1	y is similar to most of the elements of $\{x_j\}$

*Note: As in Table F-2, the intermediate values for ν_{17} and ν_{18} are not shown here. Their meanings are similar to those given in the Table.

Two additional comparison tests can be used to partially determine the source of differences between y and $\{x_j\}$ indicated by table F-5. These tests indicate how the best constant approximation to y and the magnitude of variation in y compare with $\{\bar{x}_j\}$ and $\{\omega_j\}$. Let

$$\omega_y = \min_{z \varepsilon R} \{\rho(\hat{y}, o_z)\}$$

and let $\bar{y}$ be the element of $\{o_z\}$ such that $\omega_y = \rho(\hat{y}, \bar{y})$. Then the test that indicates the differences in the constant approximations to y and to the elements of $\{\bar{x}_j\}$ is

$$\nu_{19} = \begin{cases} 1 \text{ if } \bar{x}^- \leq \rho(\bar{y}, 0_o) \leq \bar{x}^+ \\ 0 \text{ otherwise} \end{cases}$$

where 0_o is as previously defined. If $\nu_{19} = 0$, then y and the elements of $\{x_j\}$ vary in their level of noise. On the other hand, if $\nu_{19} = 1$, then the differences between y and $\{x_j\}$ are only partially explained as distinct levels of noise.

The comparison test that indicates the difference in the magnitude of variation in y and $\{x_j\}$ is

$$\nu_{20} = \begin{cases} 1 \text{ if } \omega_x^- \leq \omega_y \leq \omega_x^+ \\ 0 \text{ otherwise} \end{cases}$$

If $\nu_{20} = 0$, then the variation in $\hat{y}$ from $\bar{y}$ is different from the variation in the set $\{\hat{x}_j\}$. Thus, the distance between y and $\{x_j\}$ is due to the difference in variation. If $\nu_{20} = 1$, then this distance is only partially explained by ν_{20}.

The procedure for using all of these comparison parameters together to derive information about y and $\{x_j\}$ is as follows. First, ν_{14}, ν_{15}, and ν_{16} should be computed. These parameters will indicate the dispersion in the set $\{x_j\}$ as indicated in table F-4. Next, ν_{17} and ν_{18} should be computed. Their values will indicate the similarity between y and $\{x_j\}$. They should be interpreted using table F-5 and the values from the first three comparison parameters. Finally, ν_{19} and ν_{20} should be computed. These two parameters will indicate sources of differences between y and $\{x_j\}$.

Methodology 6: Directional Envelope to Envelope Comparison

Let the two sets of ambient noise functions be denoted by $\{x_j(\theta)\}_{j=1}^{n_1}$ and $\{y_k(\theta)\}_{k=1}^{n_2}$. Once again the points at which all of the functions are known is finite. Denote this set by $\{\theta_i\}_{i=1}^{n}$.

As in the directional point to envelope comparison methodology there are some difficulties in using statistical tests with $\{x_j(\theta_i)\}$ and $\{y_k(\theta_i)\}$. Hence, these statistical tests are also relegated to appendix H. It should be mentioned, however, that these statistical comparisons use analysis of variance methods with the quantities

$$\epsilon_{ijk} = x_j(\theta_i) - y_k(\theta_i)$$

The functional comparison tests in this methodology are a combination of generalized version of the comparison tests from the omnidirectional envelope to envelope comparison methodology and tests from the directional point to envelope comparison methodology. Once more linear spline approximation and the restricted L_2 norm will be used The comparison tests will be presented in three steps. First, the sets $\{x_j\}$ and $\{y_k\}$ will be described as sets of functions in the space of functions. Next the two sets will be compared. Finally, a procedure for using these two types of comparison tests will be given.

As in the fifth methodology, the two sets of functions can be individually characterized by the distance between their elements, the differences between the levels of noise of the elements of the sets, and the magnitude of variation from these levels of noise. Let m_x^+ and m_x^- be defined as in the previous section. Define

$$m_y^+ = \max_{i \neq j} \rho(y_i, y_j)$$

and

$$m_y^- = \min_{i \neq j} \rho(y_i, y_j)$$

which are the maximum and minimum distances between the elements [illegible] y_k [illegible] y_k is the linear approximation to y_k.)

The comparison test used with these variables is

[illegible] m_z^+

[illegible] parameters that are of interest [illegible] the analyst [illegible]. The interpretation of these parameters is the [illegible] the previous section.

[illegible] can also be characterized by the level of noise [illegible] elements. The level of noise [illegible] each function [illegible] constant approximation [illegible] that function (with [illegible]). To find these approximations let

$$\min_{z \in R} \rho(x_j, z) \qquad \text{for } j = 1, 2, \ldots, n_1$$

[illegible]

$$\min_{z \in R} \rho(y_k, z) \qquad \text{for } k = 1, 2, \ldots, n_2$$

[illegible] is the constant ambient noise function with level [illegible] functions. For $j = 1, 2, \ldots, n_1$ the best approximating constant function to x_j is $\bar{x}_j$ where [illegible] $= \rho(x_j, \bar{x}_j)$. For $k = 1, 2, \ldots, n_2$ the best constant approximation function to y_k is $\bar{y}_k$ where [illegible] $= \rho(y_k, \bar{y}_k)$. Each of these approximation functions exists and is unique since the space of linear approximation functions is locally compact.

The parameters that characterize the differences between the levels of noise within each set are $\bar{x}^+$ and $\bar{x}^-$ which are defined as before and $\bar{y}^+$ and $\bar{y}^-$ which are given by

$$\bar{y}^+ = \max_{1 \le k \le n_2} \{\rho(\bar{y}_k, 0_o)\}$$

and

$$\bar{y}^- = \min_{1 \le k \le n_2} (\bar{y}_k, 0)$$

[illegible] is the constant noise function with a zero level in all [illegible]. $\bar{y}^+$ and $\bar{y}^-$, like $\bar{x}^+$ and $\bar{x}^-$, are the maximum and minimum levels, respectively, of the set of constant approximation functions, $\bar{y}_k$. For indicating difference in the levels of [illegible] noise within each of the two sets $\{x_j\}$ and $\{y_k\}$ the com[illegible]

$$[illegible] = \bar{z}^+ - \bar{z}^-$$

[illegible] maximum difference between the constant approximations [illegible]. The parameters that will be of interest to the [illegible] and [illegible] and their relation to the parameters [illegible] respectively. These parameters should be in[illegible] the parameter [illegible] was in the previous section.

The final characteristic of the two sets that will be [illegible] is the variation of the elements of the sets from their [illegible] noise approximations respective. Define

$$\omega_x^+ = \max_{1 \le j \le n_1} (\omega_j^*)$$

$$\omega_x^- = \min_{1 \le j \le n_1} (\omega_j^*)$$

$$\omega_y^+ = \max_{1 \le k \le n_2} (\omega_k^*)$$

and

$$\omega_y^- = \min_{1 \le k \le n_2} (\omega_k^*)$$

The comparison parameter that is used to find the maximum difference in the magnitude of variation from constant functions within each of the sets $\{x_j\}$ and $\{y_k\}$ is

$$\omega_\Delta(z) = \omega_z^+ - \omega_z^-$$

These six comparison parameters, $\nu_{14}(x)$, $\nu_{15}(x)$, $\nu_{16}(x)$, $\nu_{14}(y)$, $\nu_{15}(y)$, and $\nu_{16}(y)$, provide a general description of the sets $\{x_j\}$ and $\{y_k\}$. They are exactly the parameters ν_{14}, ν_{15}, and ν_{16} from methodology 5. Thus, table F-4 can be used to interpret their values. Also, these parameters, when compared pairwise (i.e., $\nu_{14}(x)$ to $\nu_{14}(y)$, $\nu_{15}(x)$ to $\nu_{15}(y)$, or $\nu_{16}(x)$ to $\nu_{16}(y)$), will provide some information on how the two sets are dispersed, relative to one another, within the space of functions.

Thus, a general description of $\{x_j\}$ and $\{y_k\}$ can be found. Now there are many tests which will describe the similarities between $\{x_j\}$ and $\{y_k\}$. However, none of these tests are both comprehensive and easy to compute. The two which are easiest to compute while still providing useful information are presented here. Another comparison test is given in appendix H.

For the first comparison test let

$$\mu = \min_{j,k} \{\rho(\hat{x}_j, \hat{y}_k)\}$$

The comparison test is

$$\nu_{21} = \begin{cases} 1 \text{ if } M < \mu \\ 0 \text{ if } m < \mu \leq M \\ -1 \text{ if } \mu \leq m \end{cases}$$

where m and M are constants. This test indicates whether at least one pair of elements, one element from $\{x_j\}$ and one element from $\{y_k\}$, are similar. If $\nu_{21} = 1$ then it can be assumed that $\{x_j\}$ and $\{y_k\}$ are dissimilar as sets. However, if $\nu_{21} = 0$ or $\nu_{21} = -1$, it can only be assumed that one pair of elements from the sets is similar. The next test will provide more information about all the elements in both sets.

For this second comparison test for each j define $\hat{y}_{kj}$ to be the element of $\{\hat{y}_k\}$ that minimizes $\rho(\hat{y}_k, \hat{x}_j)$. Similarly, for $1 \leq k \leq n_2$ define x_{jk} to be the element of $\{\hat{x}_j\}$ that minimizes $\rho(\hat{y}_k, \hat{x}_j)$. The comparison test is

$$\nu_{22} = \frac{1}{n_1} \sum_{j=1}^{n_1} \rho(\hat{x}_j, \hat{y}_{kj}) + \frac{1}{n_2} \sum_{k=1}^{n_2} \rho(\hat{x}_{jk}, \hat{y}_k)$$

This could be modified so that its form would be the same as other tests (e.g., ν_{21}). However, in order to choose reasonable constants for the test, n_1 and n_2 must be large. Thus, since n_1 and n_2 will generally be small the best way to evaluate the results of this test is to make use of the analyst's judgment based on past data.

There are additional comparison tests which are analogous to ν_{19} and ν_{20}. ν_{19} was used to indicate differences in the best constant approximations. In this comparison methodology to indicate the differences in the best constant approximations to the elements of $\{x_j\}$ and $\{y_k\}$ let $\bar{x}^+$ and $\bar{x}^-$ be defined as in methodology 5. Recall that

$$\bar{y}^+ = \max_{1 \leq k \leq n_2} \{\rho(\bar{y}_k, O_0)\}$$

and

$$\bar{y}^- = \min_{1 \leq k \leq n_2} \{\rho(\bar{y}_k, O_0)\}$$

where O_0 is the constant noise function with a zero noise level in all directions. Define the intervals

$$I_1 = \{z \varepsilon R \,|\, \bar{x}^- \leq z \leq \bar{x}^+\}$$

and

$$I_2 = \{z \varepsilon R \,|\, \bar{y}^- \leq z \leq \bar{y}^+\}$$

Now, the comparison tests ν_4 and ν_5 can be used. In this method of comparison, these tests will indicate how the levels of noise of the elements of $\{x_j\}$ and $\{y_k\}$ compare.

ν_{20} was used to indicate the magnitude of variation from constant levels of noise. For the test in this method comparable to ν_{20} define the intervals

$$I_1 = \{z \varepsilon R \,|\, \omega_x^- \leq z \leq \omega_x^+\}$$

and

$$I_2 = \{z \varepsilon R | \omega_y^- \leq z \leq \omega_y^+\}$$

Once again the comparison tests ν_4 and ν_5 can be used. Here, these tests indicate the degree to which the magnitude of variation from constant noise levels of the elements of $\{x_j\}$ compares with the magnitude of variation of the elements of $\{y_x\}$.

The procedure for using these comparison parameters together in order to derive information about $\{x_j\}$ and $\{y_k\}$ is as follows. First, the parameters $\nu_{14}(x)$, $\nu_{15}(x)$, $\nu_{16}(x)$, $\nu_{14}(y)$, $\nu_{15}(y)$, and $\nu_{16}(y)$ should be computed. These parameters will indicate how the sets $\{x_j\}$ and $\{y_k\}$ are dispersed in the function space. Next, ν_{21} and ν_{22} should be computed. These tests will provide an indication of the similarities between the two sets of functions. Finally, the comparison tests ν_4 and ν_5 should be used with intervals described above in order to compare the levels of noise and magnitudes of variations of the sets.

Summary

Six comparison methodologies have been presented in this appendix. The selection of methodology depends upon the quantity of data. The tests within each methodology depend on the quality of the data. It should be noted for the functional tests that the important point is not which functional comparison test is used, but rather that a functional test is used at all. The comparison tests described are listed by methodology in table F-6. Alternative tests are given in appendix H.

TABLE F-6: Summary of Tests

Methodology	Computed Values	Comparison Parameter	Notes
1	ν_1	ν_1	Difference in noise values
2	$\underline{x}\ \overline{x}$	ν_2	Simple point to interval test
	$L_x(\gamma,P)$	ν_3	Tolerance limits point to interval test
	$U_x(\gamma,P)$		
3	I_1, I_2	$\nu_4\quad \nu_5$	Simple overlap comparison tests
	I_1, I_2	$\nu_4\quad \nu_5$	Tolerance limits overlap comparison tests
4	m_d	ν_6	Difference in mean level test
	s_d	ν_7	Variation from mean level test
	r_{xy}	ν_8	Correlation (shape) test
	$\rho(\hat{x},\hat{y})$	ν_9	Functional distance test
	ω_x	$\nu_{10}(x)$	Functional variation from constant level tests
	ω_y	$\nu_{10}(y)$	
	$\rho(x,y)$	ν_{11}	Functional difference in level test
	$\rho(\hat{x}-\overline{x},\hat{y}-\overline{y})$	ν_{12}	Functional variation comparison test

TABLE F-6: Summary of Tests (Cont.)

Methodology	Computed Values	Comparison Parameter	Notes
5	$\bar{\sigma}$	[illegible]	Lebesgue Measure test
	m_x^+	[illegible]	Set diameter test
	$\bar{x}^+, \bar{x}^-$	[illegible]	Maximum difference in constant level test
	$\tilde{\omega}_x^+, \tilde{\omega}_x^-$	[illegible]	Maximum difference in magnitude of variation test
	m_x^+, m_x^-, y^-	[illegible]	Minimum distance between the sets of data
	d	[illegible]	Average distance between the sets of data
	$x^+, x^-, \hat{y}$	[illegible]	Level of noise point to interval test
	w_x^+, w_x^-, w_y	[illegible]	Variation of noise point to interval test
6	m_x^+	[illegible]	Set diameter test

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