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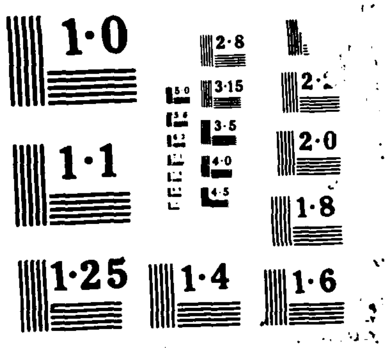
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A Reduced Navier Stokes (RNS) system that is more computationally efficient than full Navier Stokes solvers and more accurate and less cumbersome than matched viscous and inviscid methods has been shown to apply to a significant class of aerodynamic problems. The RNS system is a composite of the full Euler and boundary layer equations and is discritized to optimize the numerical representation of viscous and inviscid regions, respectively.

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COMPOSITE REDUCED NAVIER-STOKES PROCEDURES FOR FLOW PROBLEMS WITH STRONG PRESSURE INTERACTIONS

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1. PROGRESS REVIEW

The flow physics associated with strong viscous-inviscid interactions is generally quite complex. Computational methods to evaluate such aerodynamic phenomena have typically been associated with full, very time consuming, Navier-Stokes (NS) solvers or with approximate matched inviscid-interacting boundary layer (IBL) procedures. The purpose of the current AFOSR research program is the development and application of a global pressure relaxation or flux-split pressure differencing procedure for assessing steady and transient flows, respectively.

A reduced Navier-Stokes (RNS) system that is more computationally efficient than full NS solvers and more accurate and less cumbersome than matched viscous and inviscid (IBL) methods has been shown to apply to a significant class of aerodynamic problems. This RNS system is a composite of the full Euler and boundary layer equations and is discretized to optimize the numerical representation of viscous and inviscid regions, respectively.

Two primitive variable RNS systems have been considered. The first is for the pressure/velocity variables and represents a major upgrading of the parabolized Navier-Stokes or PNS methodology. This allows for full elliptic or upstream interactions, including flow separation, shock capturing, etc. The second is a composite pseudo-potential/vortical-velocity model that represents an extension of full potential methodology to viscous interacting flows. This formulation allows for the numerical advantage of potential flow methods and those of boundary layer methods to be included in a single and integrated, composite RNS solver.

The pressure/velocity formulation has been applied for two-dimensional steady and transient flows involving airfoil viscous/inviscid interaction, jet/base flow interaction, transonic and supersonic interactions. The composite velocity formulation has been applied for subcritical and transonic viscous airfoil and boattail aerodynamics. Both procedures are currently being developed and applied for three-dimensional flows over afterbodies and wing-body configurations, where axial and secondary flow separation and strong pressure interaction occurs. Numerous publications, presentations and dissertations have resulted from this research activity. Recent papers and presentations are detailed in the reference section. A review of progress during the year is given in the summaries and results that follow. Detailed information can be found in the references, most of which have been submitted for AFOSR review. A summary of highlights is detailed at the end of this report.

1.1 Pseudo-Potential/RNS Progress and Solutions

This methodology has been developed for the viscous RNS/potential and viscous RNS/Euler systems. The effect of Euler versus potential outer flow conditions on shock capturing, entropy/vorticity production and viscous interaction, in particular separation, has been evaluated. A major result obtained during the investigation relates to the appropriate 'conservation' differencing that (i) accurately convects the vorticity generated behind the embedded shock and (ii) reproduces the correct vorticity/entropy across the shock. A combined conservation/partial conservation form leads to the capturing of very sharp shocks with accurate entropy jumps and without any spurious generation of entropy in subsonic regions or near the leading edges. Recent solutions for the pressure and entropy distribution on a NACA 0012 airfoil are shown in figures 1a and 1b. As seen from the figures, the entropy jump is dominant only at the shock. The overshoot in entropy is

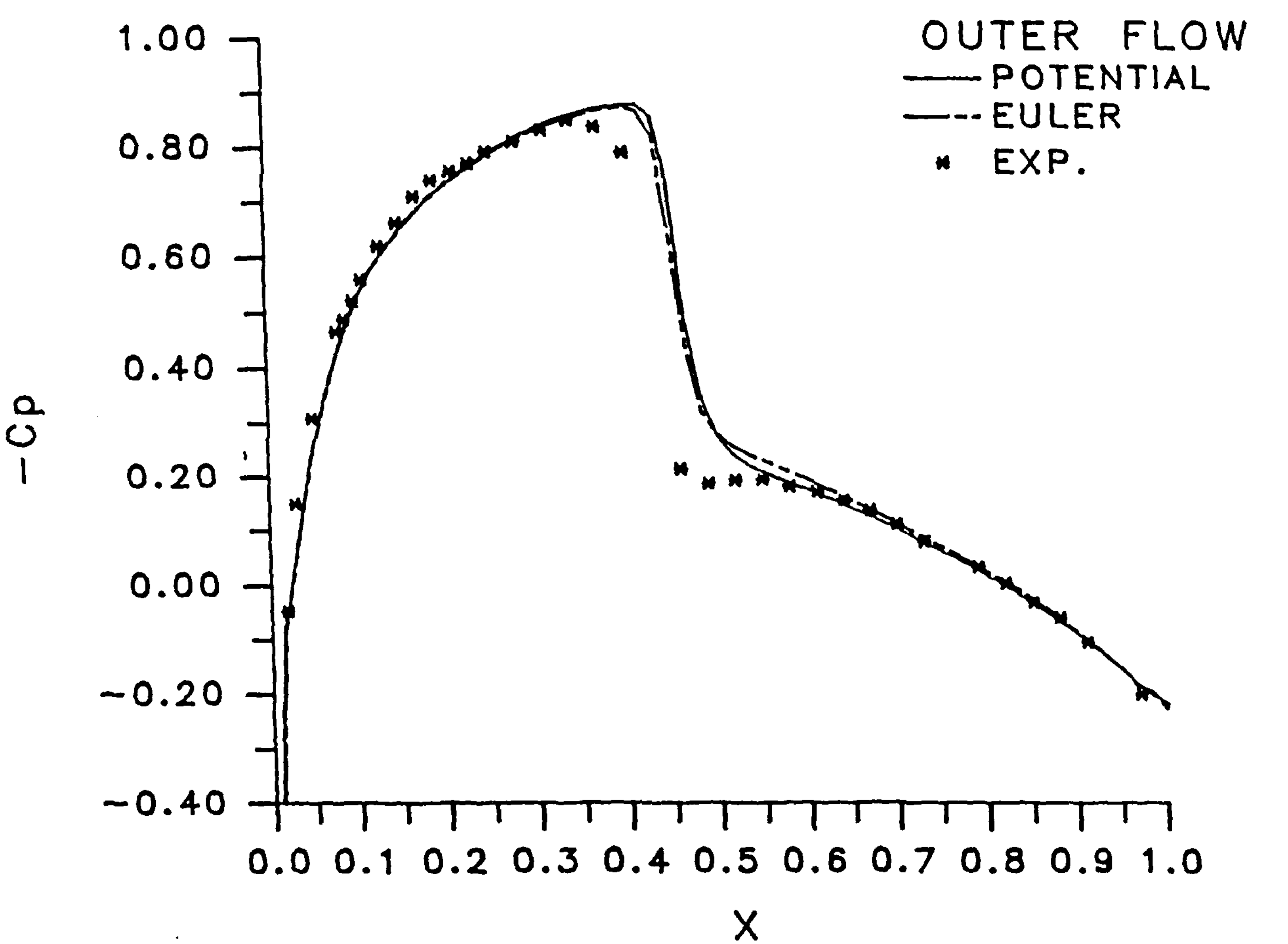


Figure 1a. R.N.S. Solution for an NACA0012 Airfoil, $M_\infty = 0.8$, $Re = 4.0 \times 10^6$; Comparison with Different Outer Flows

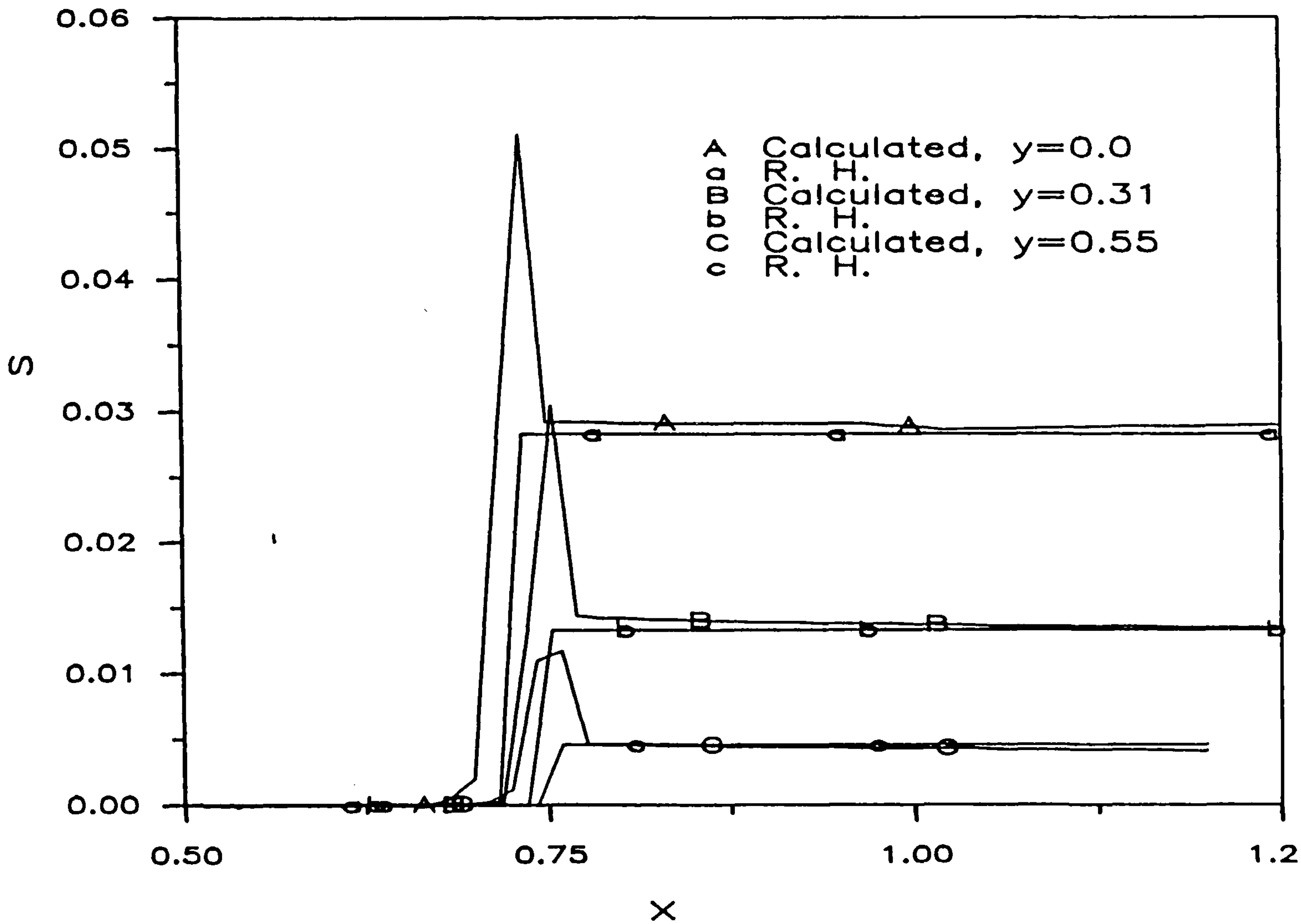


Figure 1b. Comparison of Entropy Jump Across the Shock with the Rankine–Hugoniot Values for Various Chord Lengths from the Surface, $M_\infty = 0.85$.

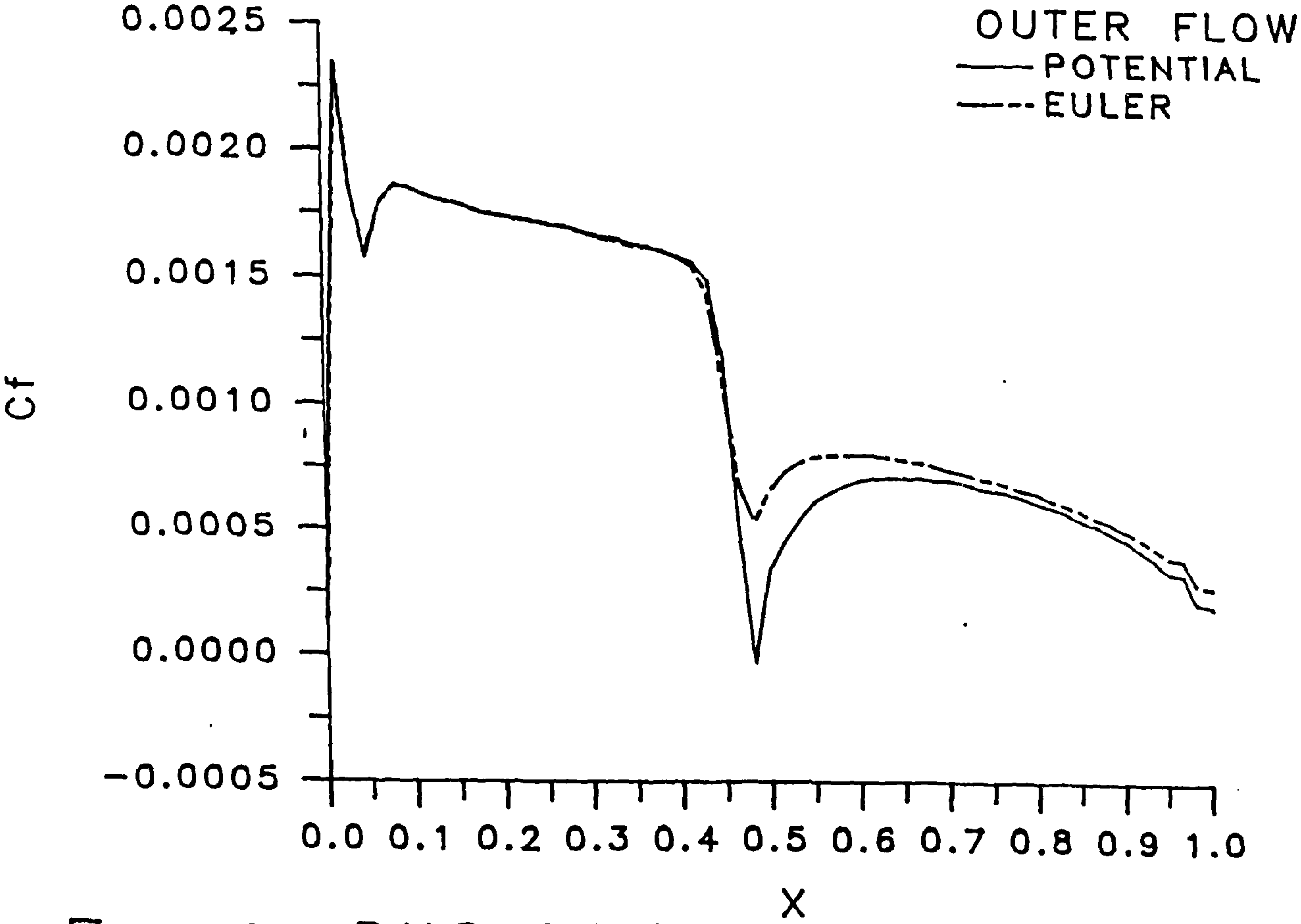


Figure 1c. R.N.S. Solution for an NACA0012 Airfoil, $M_\infty = 0.8$, $Re = 4.0 \times 10^6$; Comparison with Different Outer Flows

quite physical and typically occurs within the structure of a shock. In the present case, shock capturing is determined by the momentum shift principle of Osher-Enquist. The effect of outer potential/Euler conditions on the separation bubble is given in figure 1c. The skin friction is extremely sensitive to the outer flow formulation. A detailed paper on this aspect of the composite/RNS investigations was presented at a symposium at the Polytechnic University in August, 1987. The full paper will appear in a special issue of the journal, Computers and Fluids in August 1988.

This procedure has now been extended to three dimensional flows. The CV/RNS equations in a generalized non-orthogonal coordinate system are solved by the consistent coupled strongly implicit (CSIP) space marching procedure. The viscous analysis and early results will be described in section 1.5. Application of a three-dimensional preconditioned conjugate gradient-CSIP for the computation of inviscid flows past the afterbody configuration (figure 2a) has been completed. Both subsonic and transonic solutions have been obtained. The pressure distribution along the centerline on the top side of the body is depicted in figure 2b for a free stream mach number of 0.8. A comparison with a partial solution obtained elsewhere by a panel method is also shown for the boattail region. The mach contour along the top flap is shown in figures 2c. The flow is mildly supersonic on the boattail.

A Ph.D. student, Raymond Gordnier has been visiting at the AFWAL/WPAFB, with Professor Rubin during his sabbatical leave and now on a weekly basis, to continue the evaluation of the three-dimensional viscous afterbody interactions. This study should be completed later this year, see section 1.5.

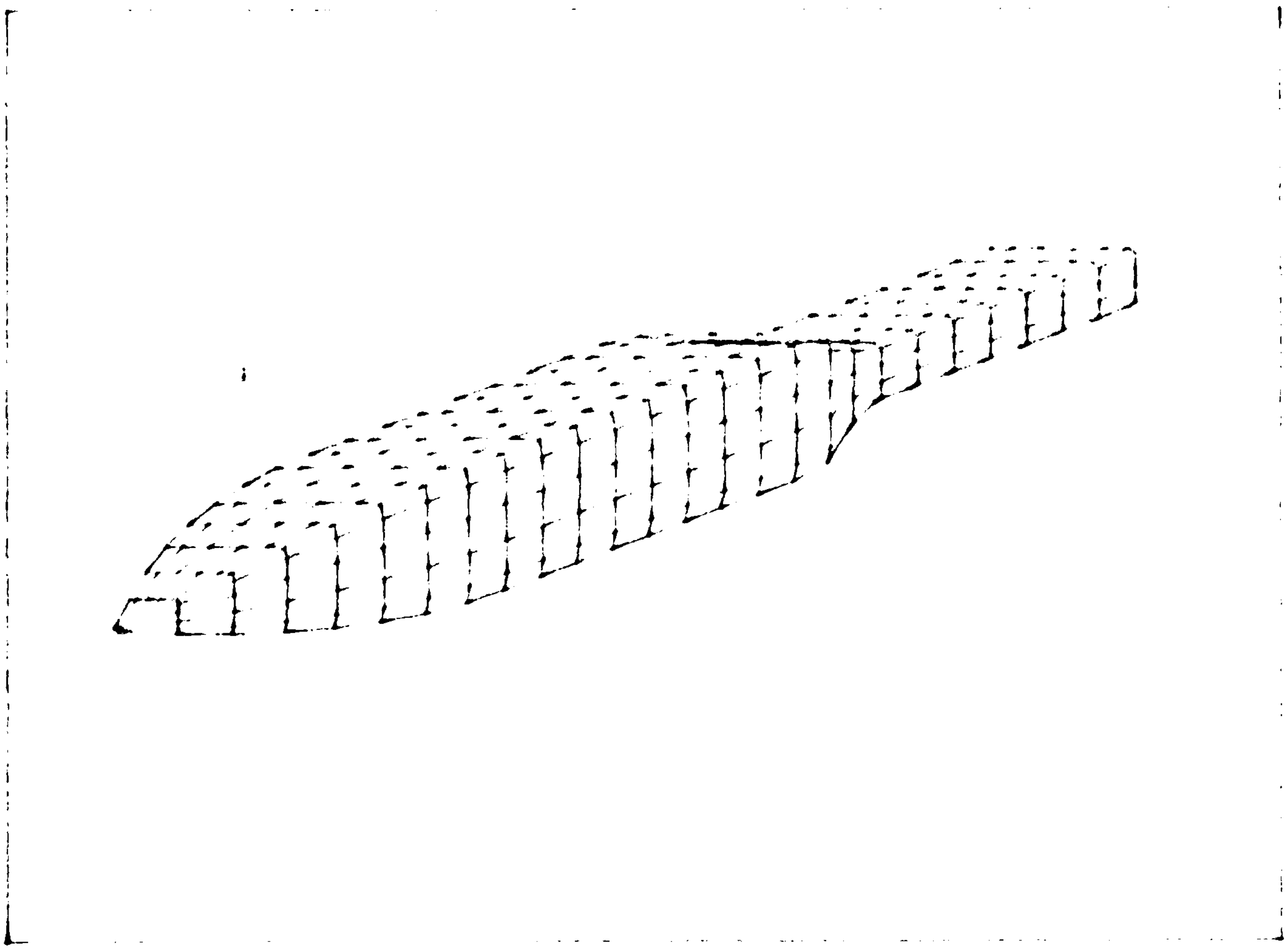


Fig. 2a. Arter Body Configuration

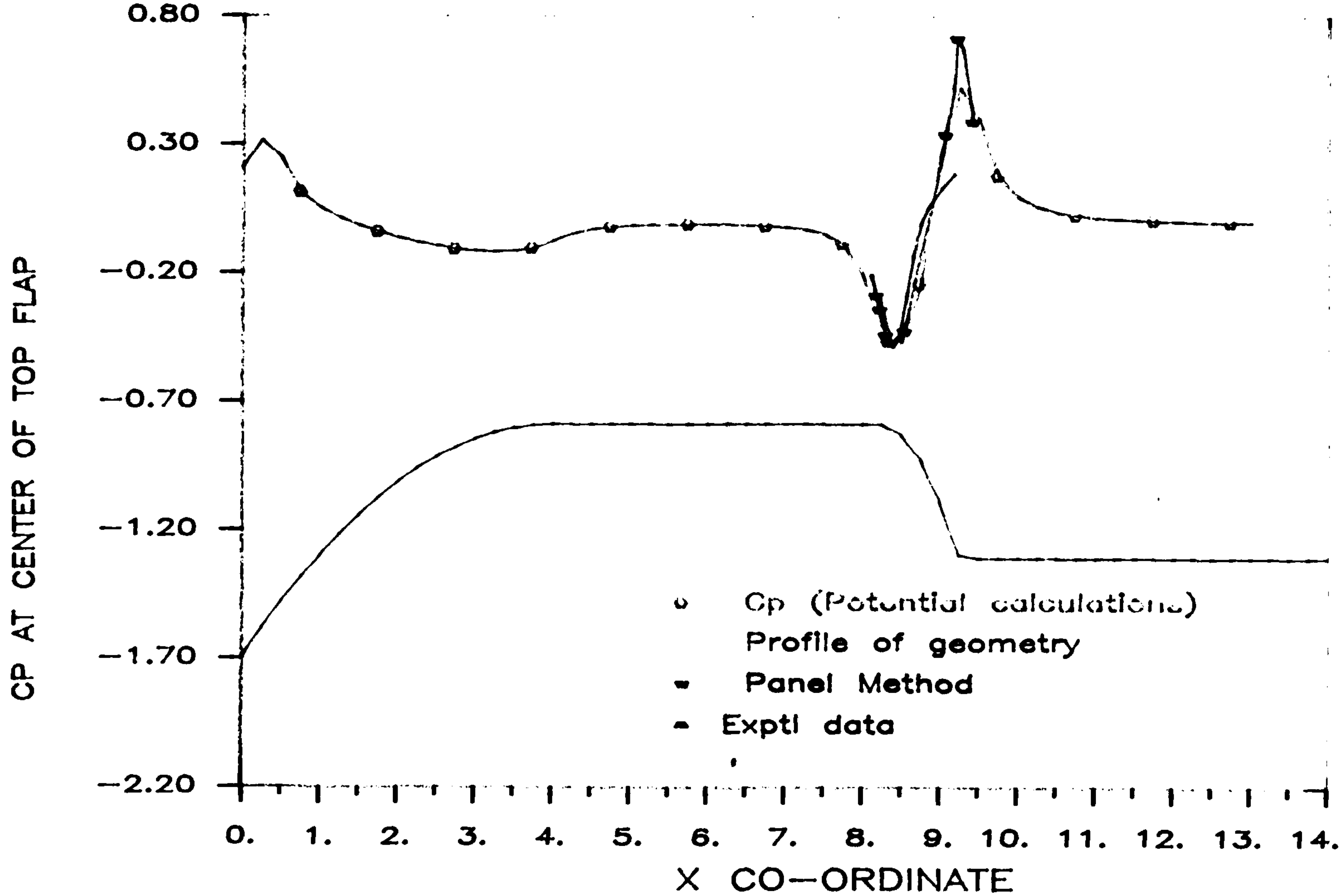


FIG. 2b. CP AT CENTER OF TOP FLAP (MACH # 0.8)

$M_\infty = 0.96$

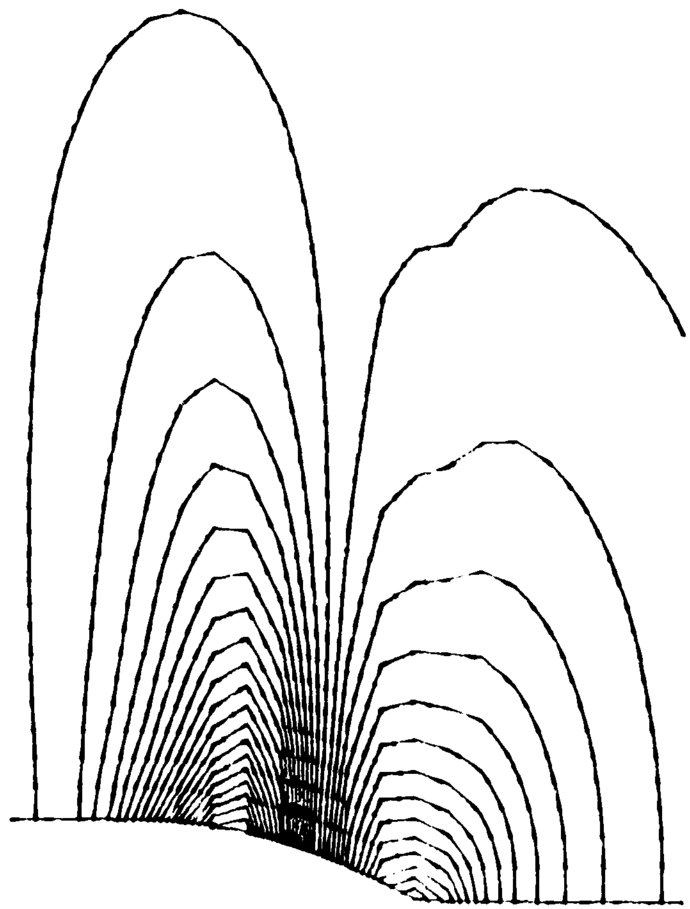


Fig. 2c. **Mach Contours (Max Mach#1.55)**

1.2 Unsteady RNS Flows

The RNS formulation has also been considered for the computation of unsteady flow past airfoils at incidence. For this purpose, time consistent CSIP, time consistent forms of global and alternating direction line relaxation and a sparse matrix direct solver (DS) have been considered. It should be emphasized that the present RNS formulation predicts the correct viscous-inviscid interaction at the trailing edge and does not require any second-order upwinding or fourth-order damping typically found in time dependent Navier-Stokes solvers. The RNS formulation also allows for larger time increments (Δt) and for more efficient placement of far field boundaries and application of boundary conditions. A paper containing additional details has appeared in the AIAA journal in July 1987.

A major result from the unsteady computations relates to a hystersis phenomena that has been observed as the airfoil oscillates between 0° and 13° incidence. The separation, lift and force patterns are quite different for the upward and downward motions. For example converged steady solutions presented in figure 3a are obtained at 5.7° incidence when this angle is approached from below. An unsteady solution is generated when this angle is approached with initial conditions generated from a vortical solution at a higher incidence. The skin friction for both the steady and unsteady conditions at the same incidence is depicted in figure 3a. Such phenomena have been observed experimentally but has not been reported in any earlier computations. Further analysis and grid resolution studies have validated these results. Unsteady flow solutions of the complete (ψ-ω) form of the Navier-Stokes equations using the DS have also been obtained for comparison purposes. These were presented in the last progress report. Additional information is given in section 1.6.

The lift coefficient and streamline patterns at a somewhat larger incidence and Mach number are shown in figures 3b,c. These results are in excellent agreement with full time-dependent Navier-Stokes solutions, that require considerably greater computational resources and times.

Additional results focus on an alternating direction predictor-corrector method for shedding phenomena on airfoils and the DS for transonic flows and for laminar flow breakdown on airfoils at incidence. More detailed solutions are contained in the Ph.D. dissertations of S. Ramakrishnan and E. Bender and will also be presented in papers at the National Fluid Dynamics Conference this June in Cincinnati.

1.3 Multigrid Acceleration of RNS Formulations

In the global pressure or pseudo-potential relaxation procedures, downstream information typically propagates upstream, one mesh point during each sweep. For flows with significant upstream influence and on finer meshes, the rate of convergence will necessarily be significantly reduced. In order to enhance the convergence rate, alternating direction (predictor-corrector) and uni-directional multigrid procedures have been developed and applied successfully. The latter is discussed here. Since the pressure or "potential" is the only variable that is being 'relaxed', a one-dimensional multigrid procedure, applied to the PV/RNS equations, has proven very effective for highly stretched grids and separated flow regions. Standard multigrid procedures fail in these circumstances. Both inviscid and viscous solutions have been computed to test the applicability of this procedure. Transonic flow past a biconvex airfoil, separated flow over a trough configuration and trailing edge flows with large pressure interaction have been evaluated, as these are representative of the interactive flow

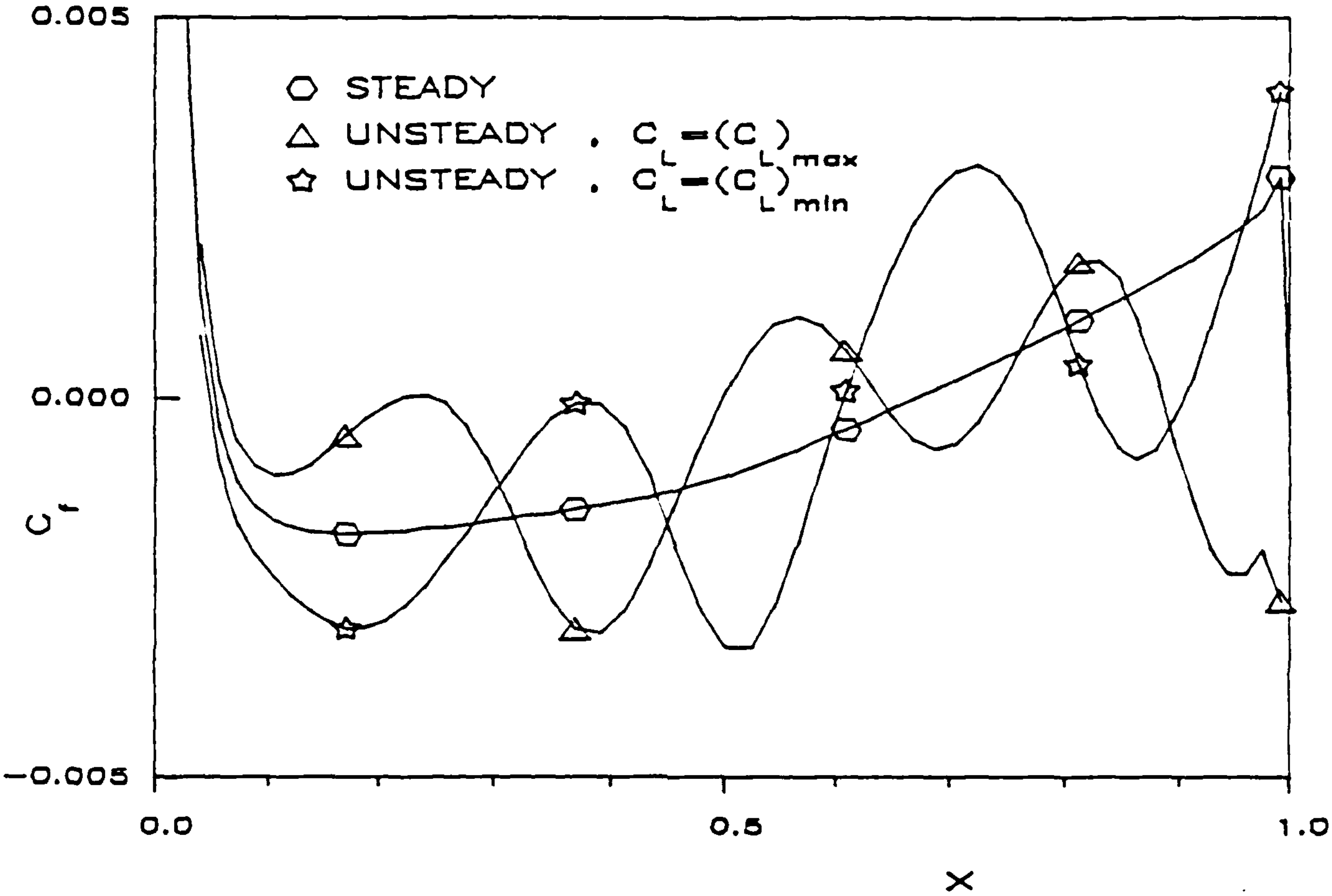


FIG. 3a. UPPER SURFACE WALL SHEAR DISTRIBUTION;
R = 10,000, $\alpha = 5.7°$.

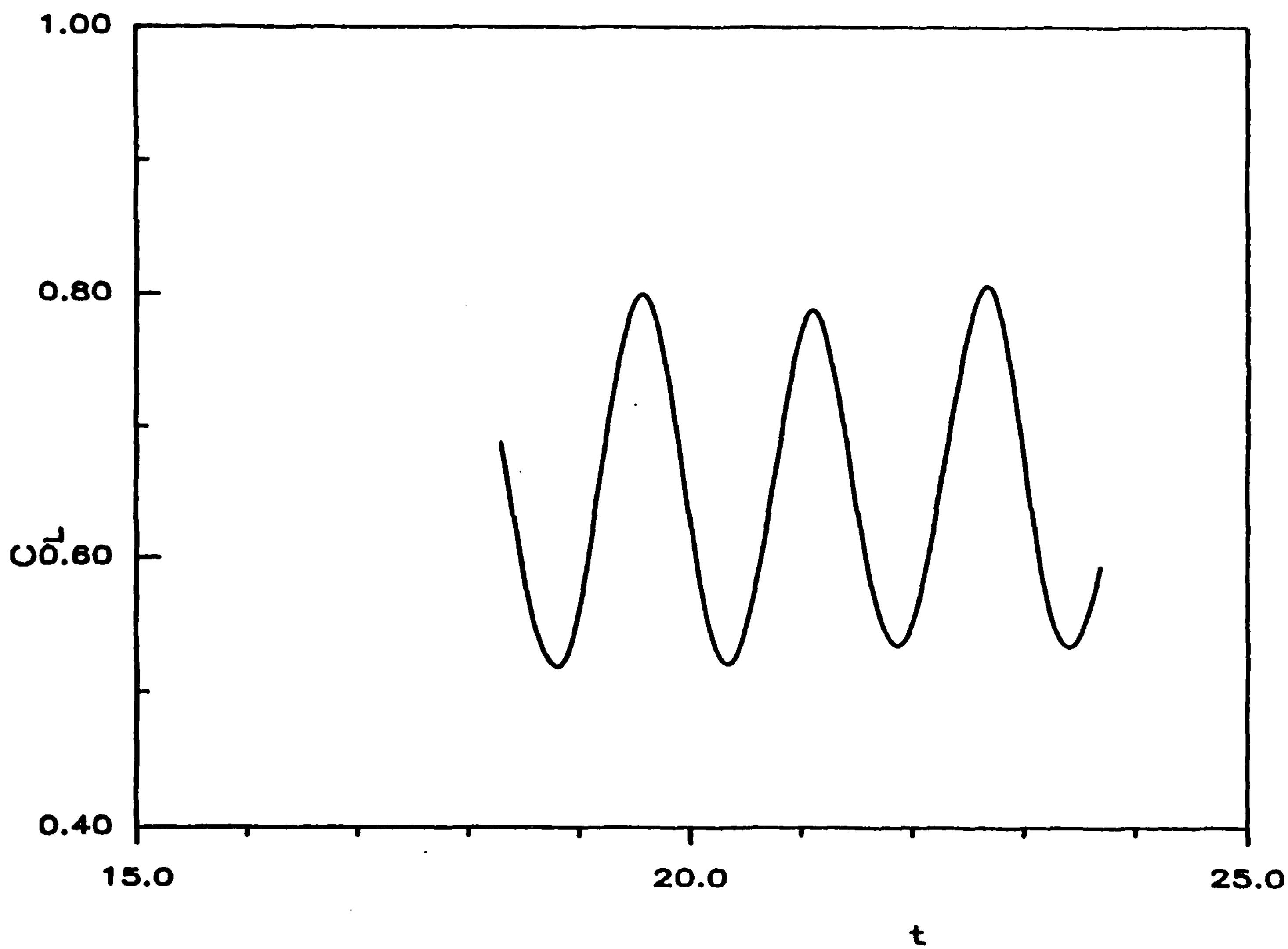


FIG. 3b. Lift Coefficient Time History for 12% Joukowski Airfoil; $\alpha = 15^{o}$; $R = 1000$; $M_\infty = 0.4$

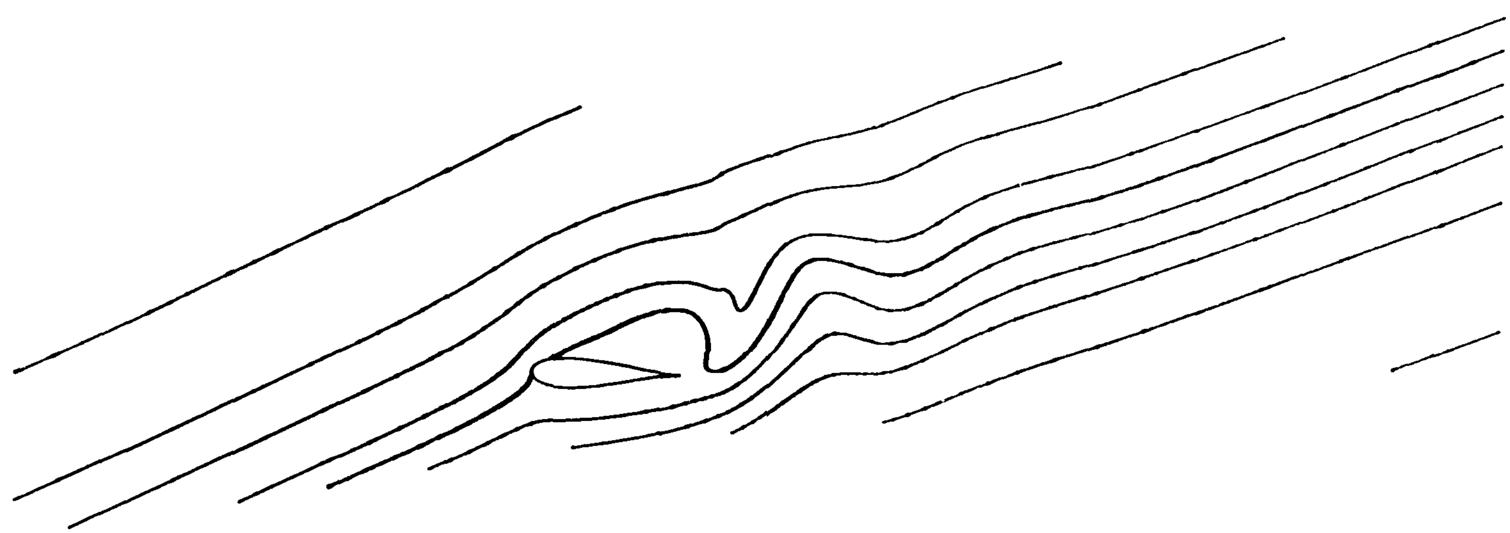


FIG. 3c. STREAMLINE PATTERN AT MINIMUM LIFT; $\alpha = 15°$; $M_\infty = 0.4$, $R = 1000$.

phenomena that will arise in the more complex configurations under consideration. In each case, the multigrid procedure accurately captures the physical phenomena at a significantly reduced computational cost, figure 4a. In most cases the standard multigrid approach either failed or provided marginal improvements in the convergence rate. For a deep trough (never considered by previous investigators), separated flow solutions were obtained for a moderate Reynolds number ($8x10^4$). A fine grid solution for a moderate trough is depicted in figure 4a. On the deep trough, for Re = 10^5, for a very fine mesh, a breakdown of the laminar flow solution in the separated flow region is observed. As seen from figure 4c, the recirculation bubble becomes locally unstable and the computation diverges locally (not globally). A similar phenomena has also been observed previously by the present investigators for the sine-wave airfoil (see section 1.6). This is indicative of an apparent laminar flow instability that is seen numerically on the fine meshes possible with the multigrid procedure, e.g. 500 mesh points across the trough. This instability may signal inadequate resolution, e.g., a small secondary vortex forms, or incipient laminar flow transition, or the occurrence of dynamic stall, which has also been predicted by large Re asymptotic methods. The present results, which were presented at the AIAA meeting in Honolulu, June 1987, provide a severe warning for coarse mesh calculations. Recent results obtained with full NS solvers tend to confirm the RNS predictions and lend credence to the RNS approximation even for these flow conditions.

1.4 Solution of RNS Equations at High Supersonic Mach Numbers

The global pressure relaxation has also been used for the solution of high supersonic flows past sharp nosed bodies. For mach numbers greater

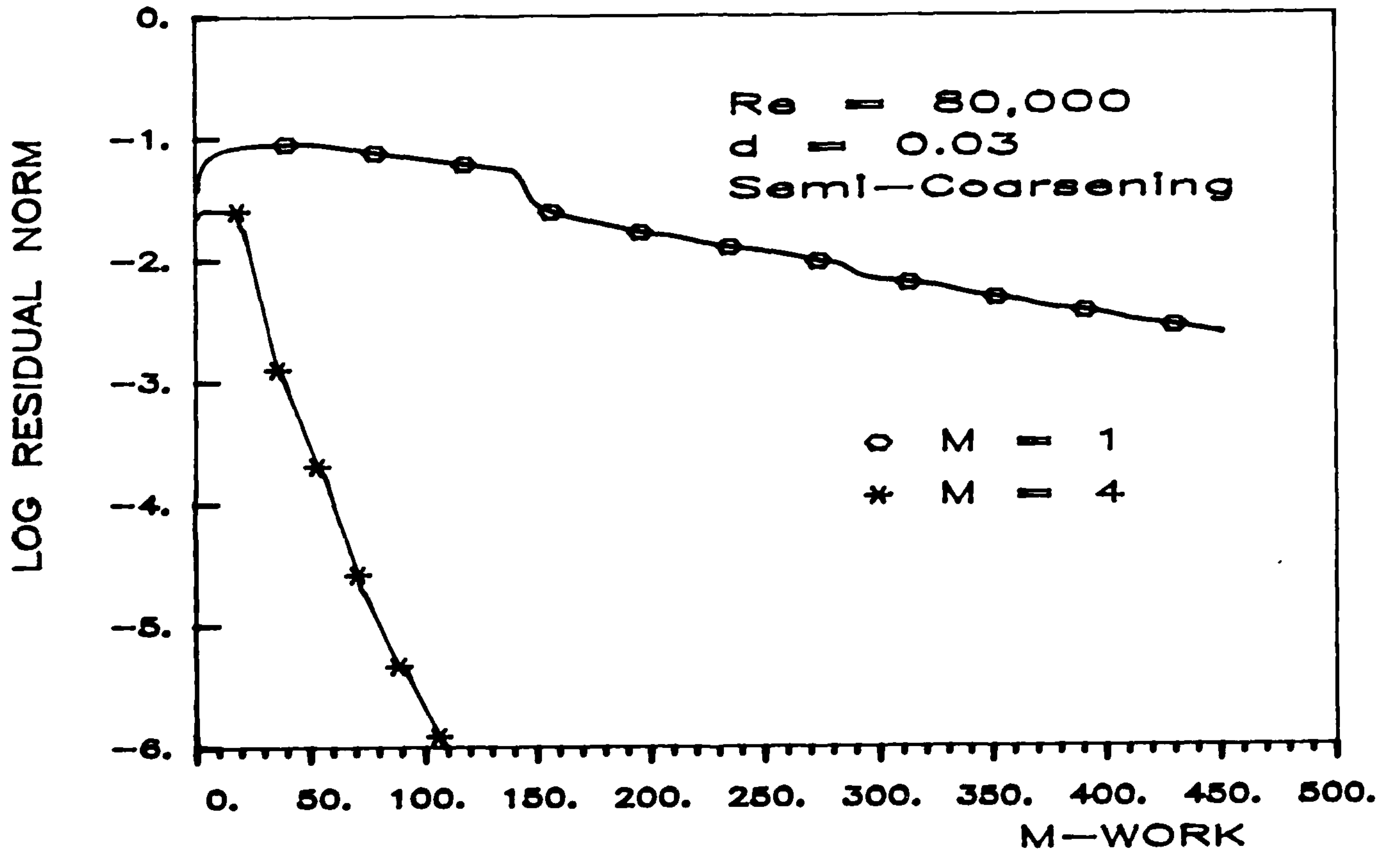


Fig. 4a. Carter—Wornom Trough : MG Convergence Rate

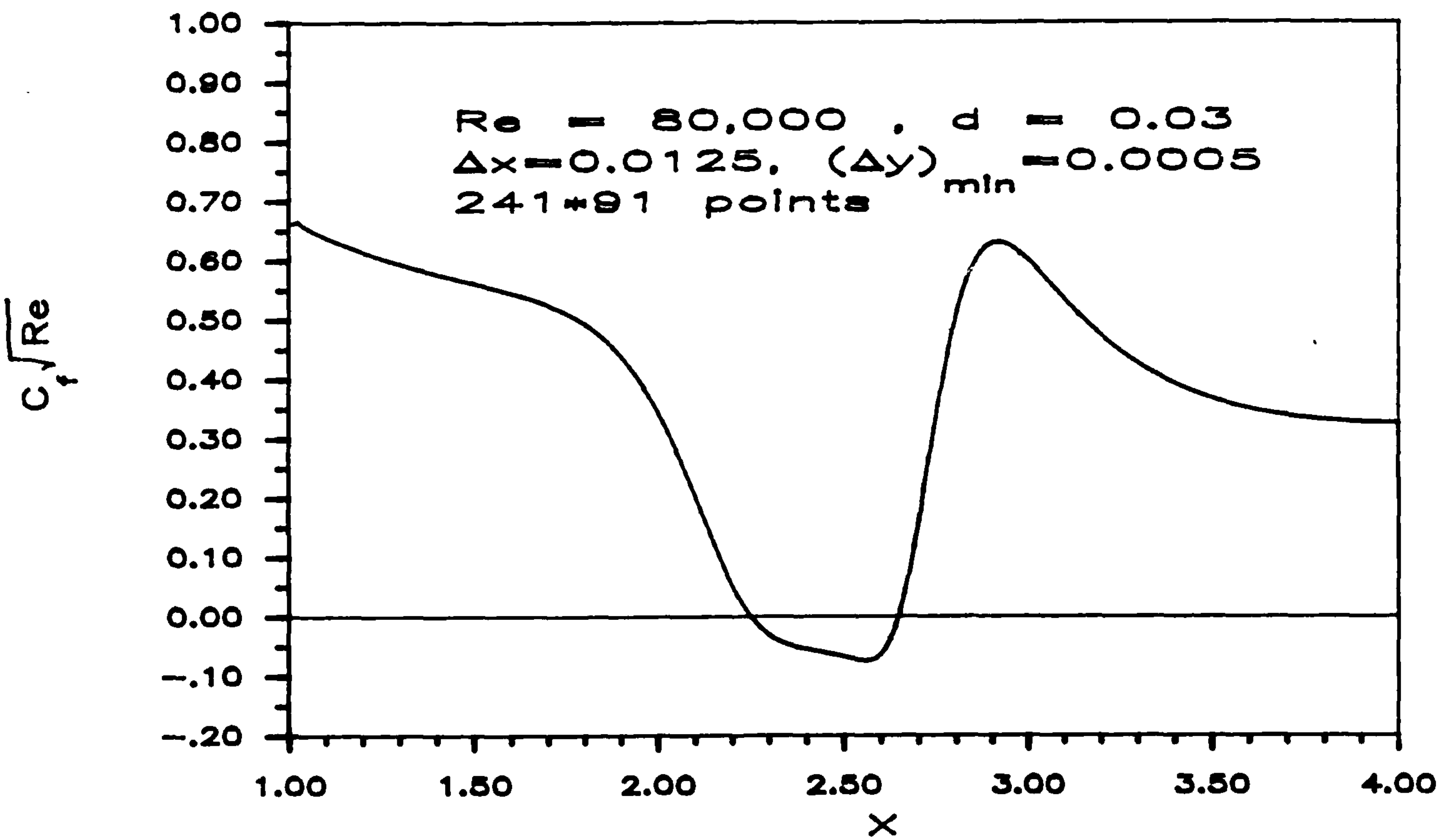


Fig. 4b. Carter—Wornom Trough : Skin Friction

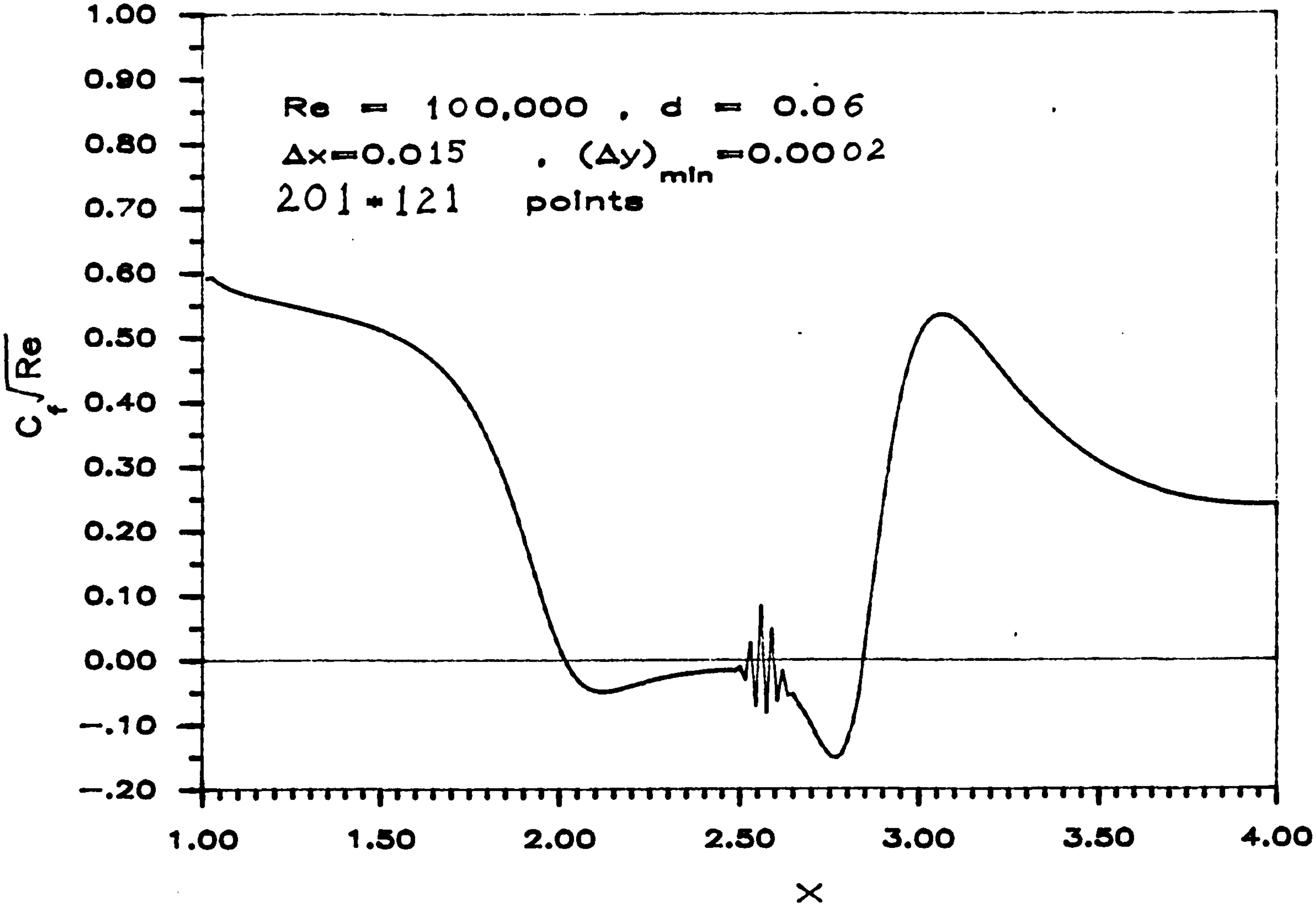


Fig. 4c. **Carter—Wornom Trough : Skin Friction**

than 3, all bow shock waves are fitted and imbedded shocks are captured. Two axisymmetric configurations have been considered, i.e., a cone-cylinder boattail and a simple forebody configuration of a fighter aircraft. The calculated mach contours, pressure coefficient and skin friction for the forebody are shown in figures 5a,b. At the higher mach numbers, the separated flow region that forms on the boattail, due to shock-boundary layer interaction, weakens and the shock becomes highly oblique. The shock that develops near the canopy interacts directly with the bow shock. This shock-shock interaction is captured reasonably by the present technique. The present formulation is highly efficient for large mach number flows. This is evident from figure 5c where the rates of convergence for the global relaxation procedure are presented. The method converges very quickly, even without the multigrid accelerator. Additional results for an axisymmetric surface cavity have also been obtained, see figure 5d, and turbulent flow computations have been initiated.

1.5 Three-Dimensional Space Marching

The consistent (CSIP) algorithm has also been used for the computation of three-dimensional flow configurations with the CV/RNS formulations. For the problems that have been considered, the axial flow direction (in an appropriate body-fitted coordinate system) is discretized in a time-like or marching character, and the cross-plane difference system is prescribed with the familiar five-point discretization. The global pressure relaxation algorithm described in previous discussions is directly applicable to these problems. The flow over a three dimensional rectangular wing of zero thickness and the flow past a centerbody of rectangular or elliptical cross-section have been considered to date. The rectangular elliptical cross

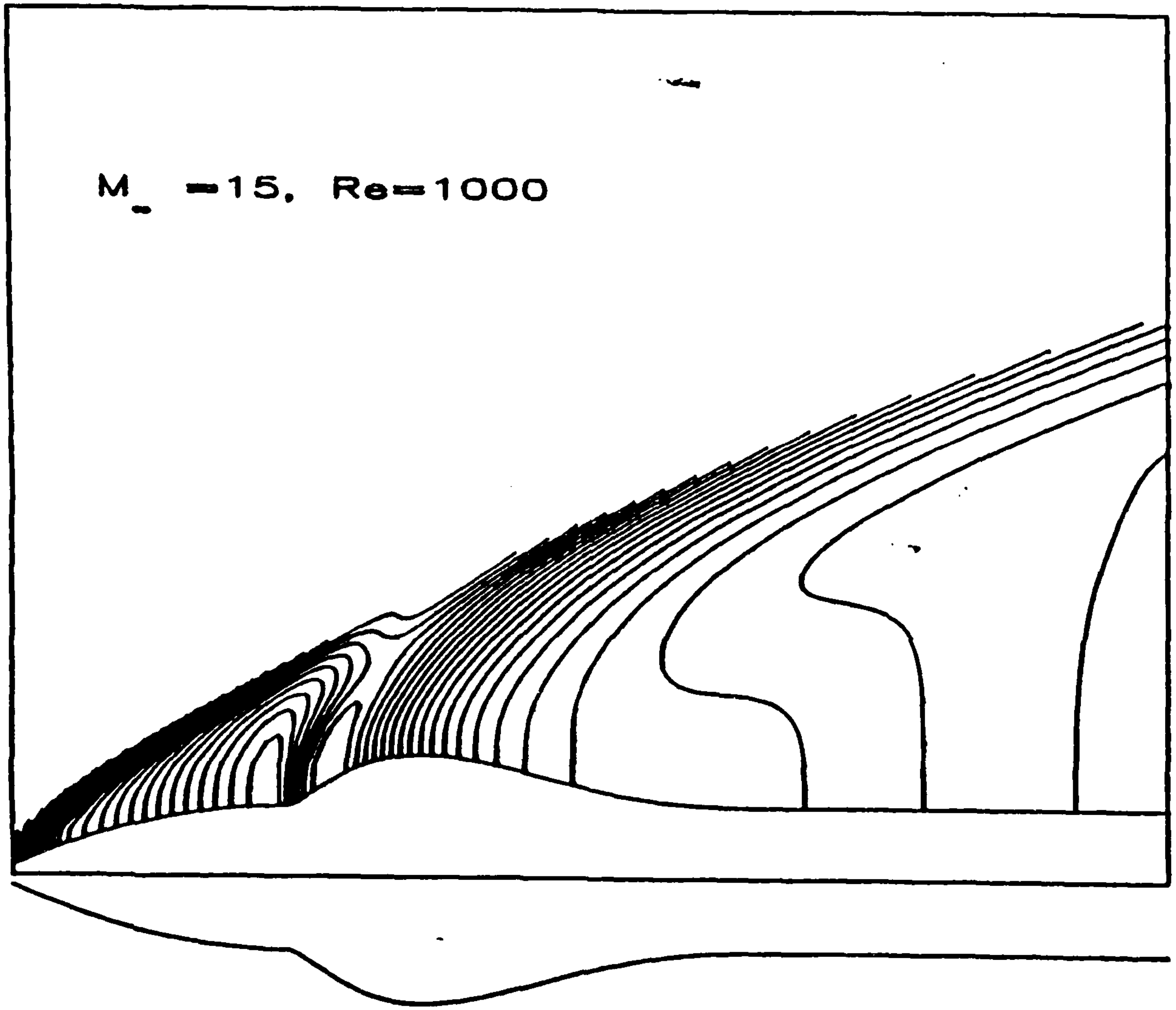


Fig. 5a. **Aircraft Forebody : Pressure Contours**

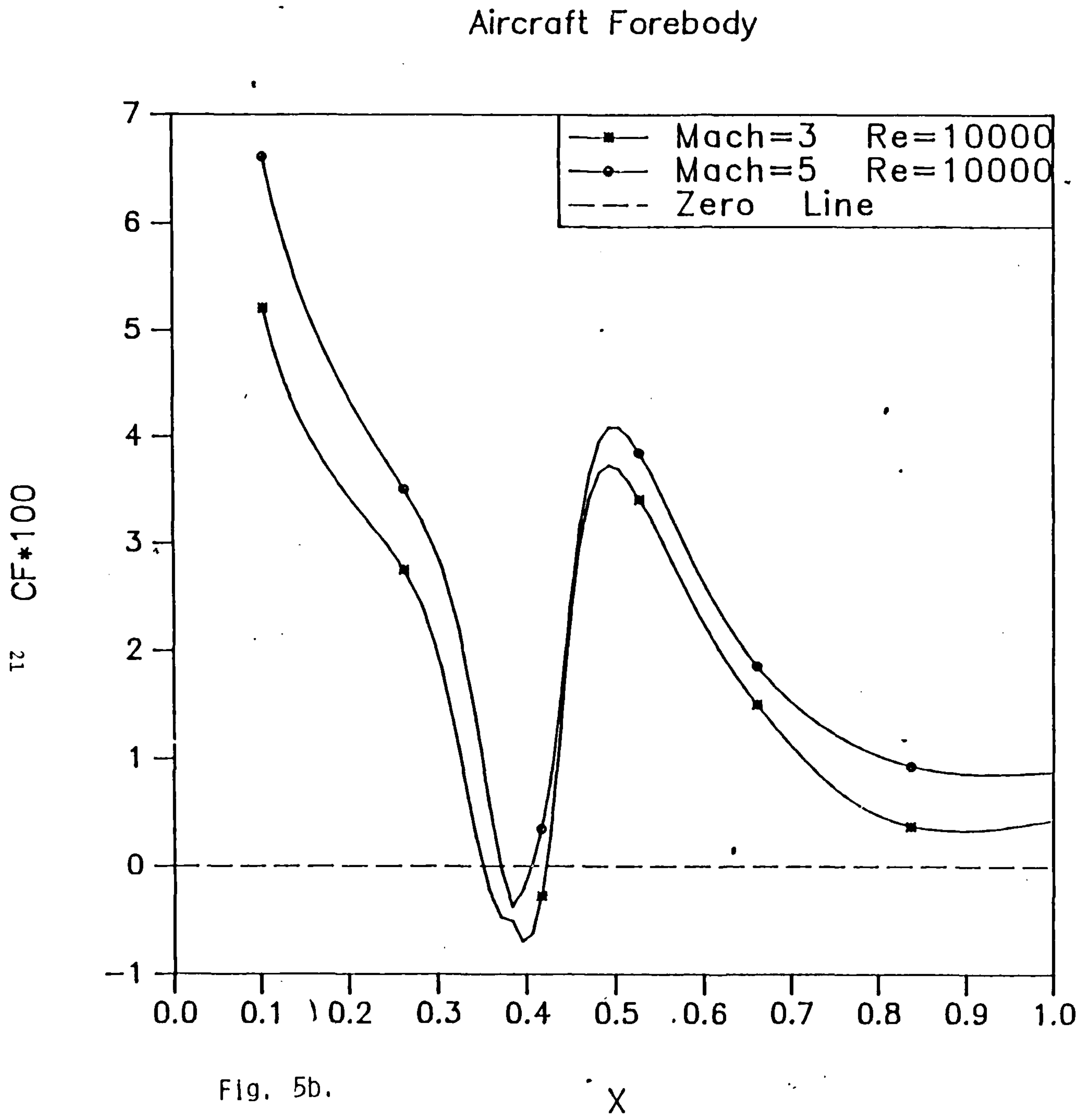

Aircraft Forebody
Mach=3 Re=10000
Mach=5 Re=10000
Zero Line
CF*100
X
7
6
5
4
3
2
1
0
-1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 5b.

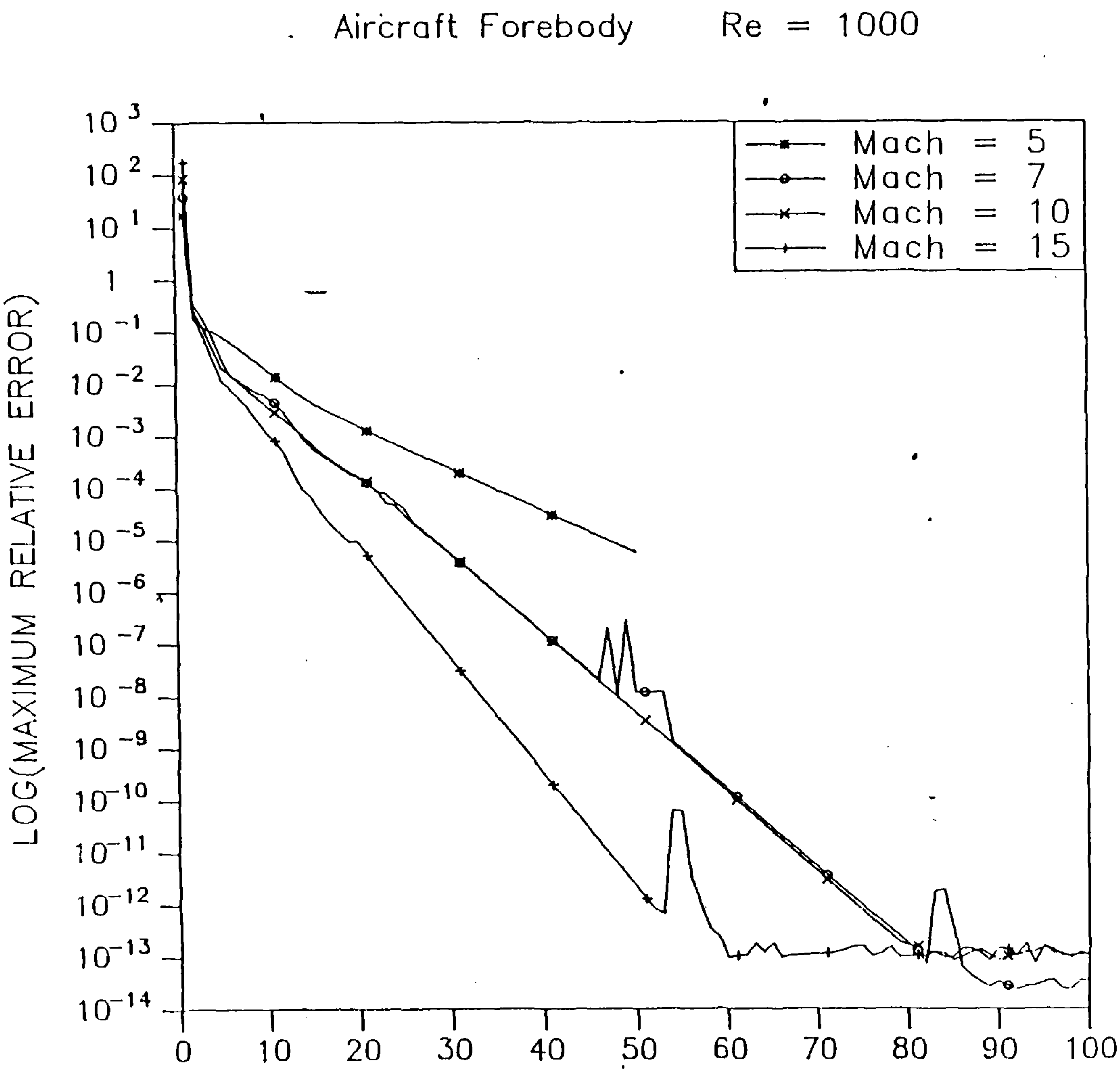


Fig. 5c. NO. OF ITERATION

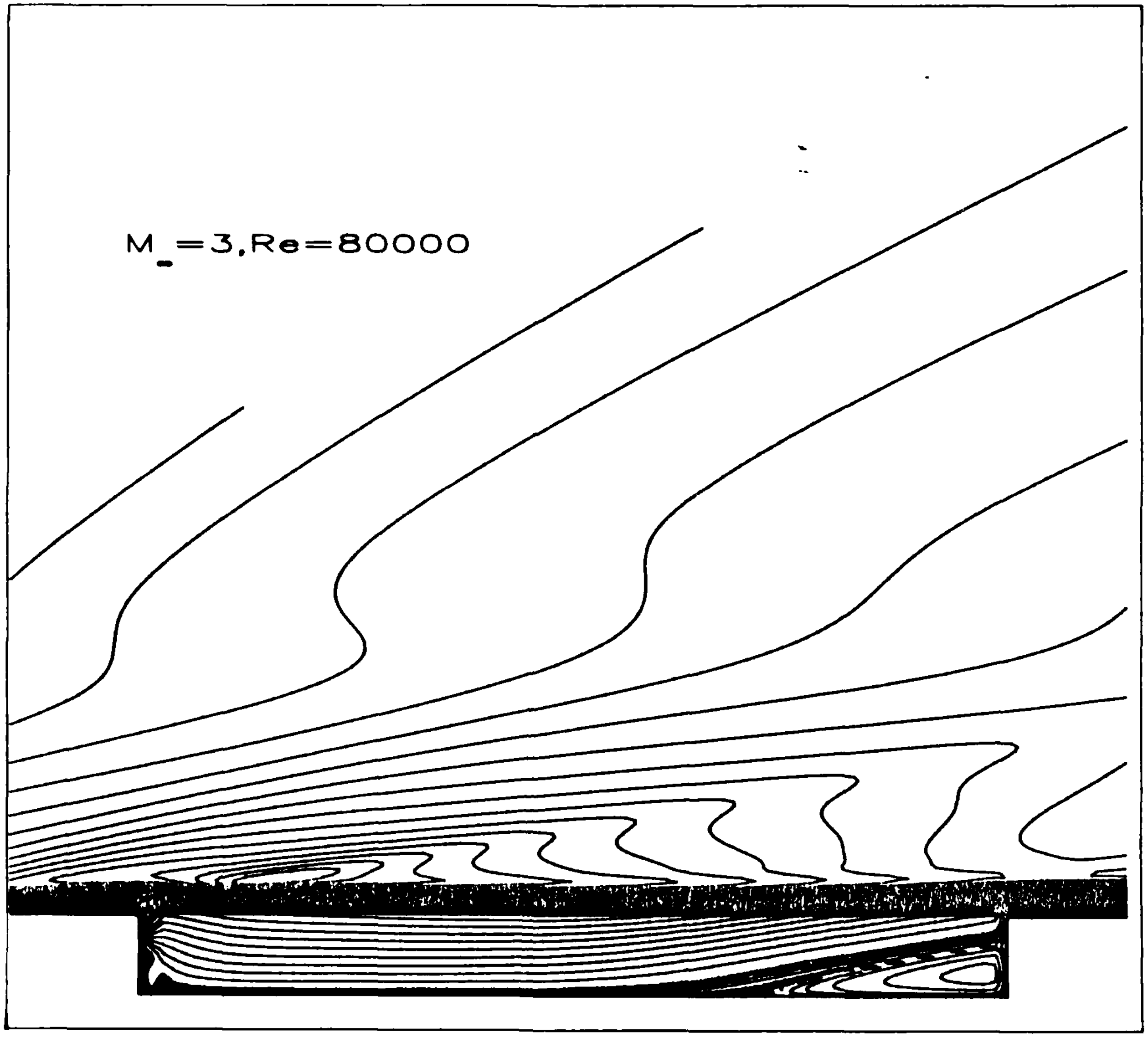


Fig 5d. Cone–Cylinder–Groove : Isomach Contours

section has been examined with the PV/RNS formulation and the effect of finite span on the trailing edge interaction has been assessed in an earlier report. Current research relates to variable camber or thickness effects, see figures 6a,b. The CV/RNS formulation has now been applied for an rectangular/elliptical cylinder configuration. The skin friction along the top and the side symmetry plane are presented in figures 6c,d. The composite velocity/vorticity formulation has been modified to remove the singular behavior that results for zero incidence and zero cross flow conditions. The improved code has been tested for axisymmetric geometries and is currently being applied to the CV/RNS computation of flow past the afterbody configuration discussed previously. These results will appear in a Ph.D. dissertation of R. Gordnier to be completed later this year. A detailed paper should be available by early 1989.

1.6 Application of the Sparse Matrix Direct Solver (DS)

During the last contract period an innovative approach to the solution of compressible flow problems, based on a direct sparse matrix solver, had been initiated. The solution algorithm is highly flexible and robust. The most significant outcome of the research has been the development of a domain-decomposition-iterative procedure based on the DS. The computational domain is decomposed into several overlapping subdomains. The DS is then used to solve the problem in each sub-domain. Although a related procedure has recently been presented by others for simple and highly elliptic equations, the present decomposition is found to be more suitable for the solution of the Navier-Stokes equations at high Reynolds number. This procedure honors the asymptotic character of the flow.

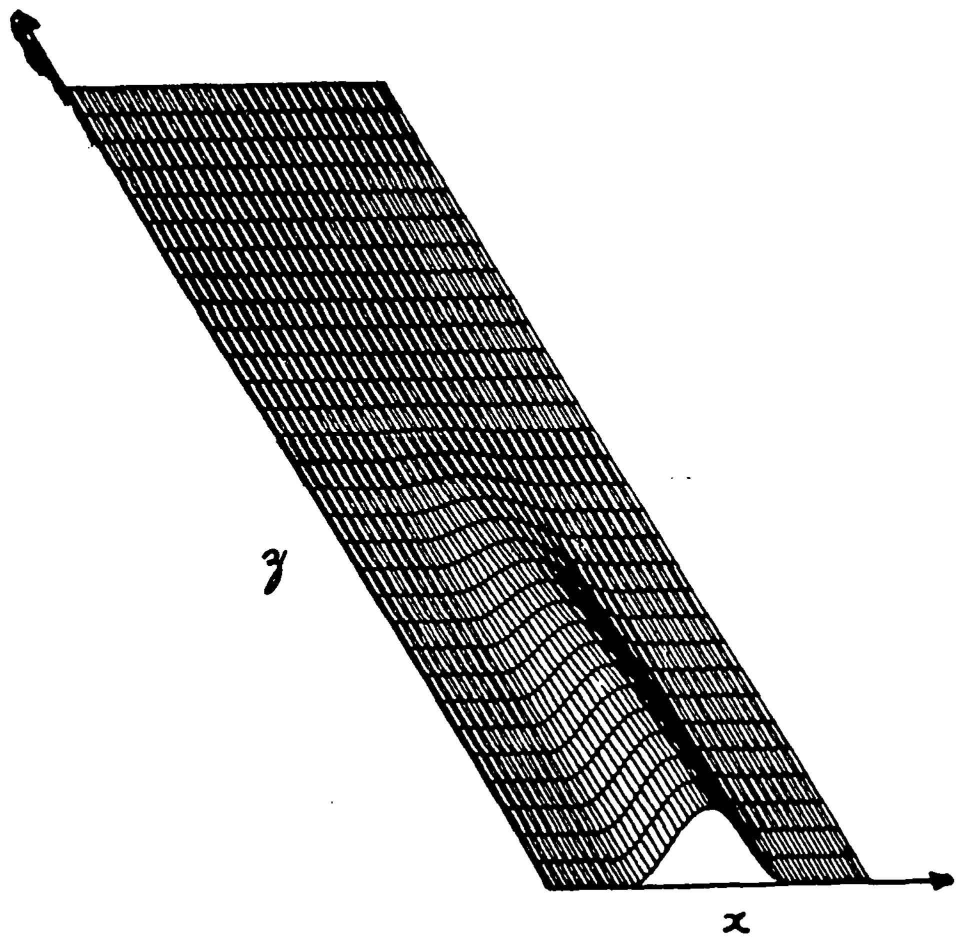


FIG. 6a. 3-D VARIABLE CAMBERED WING.

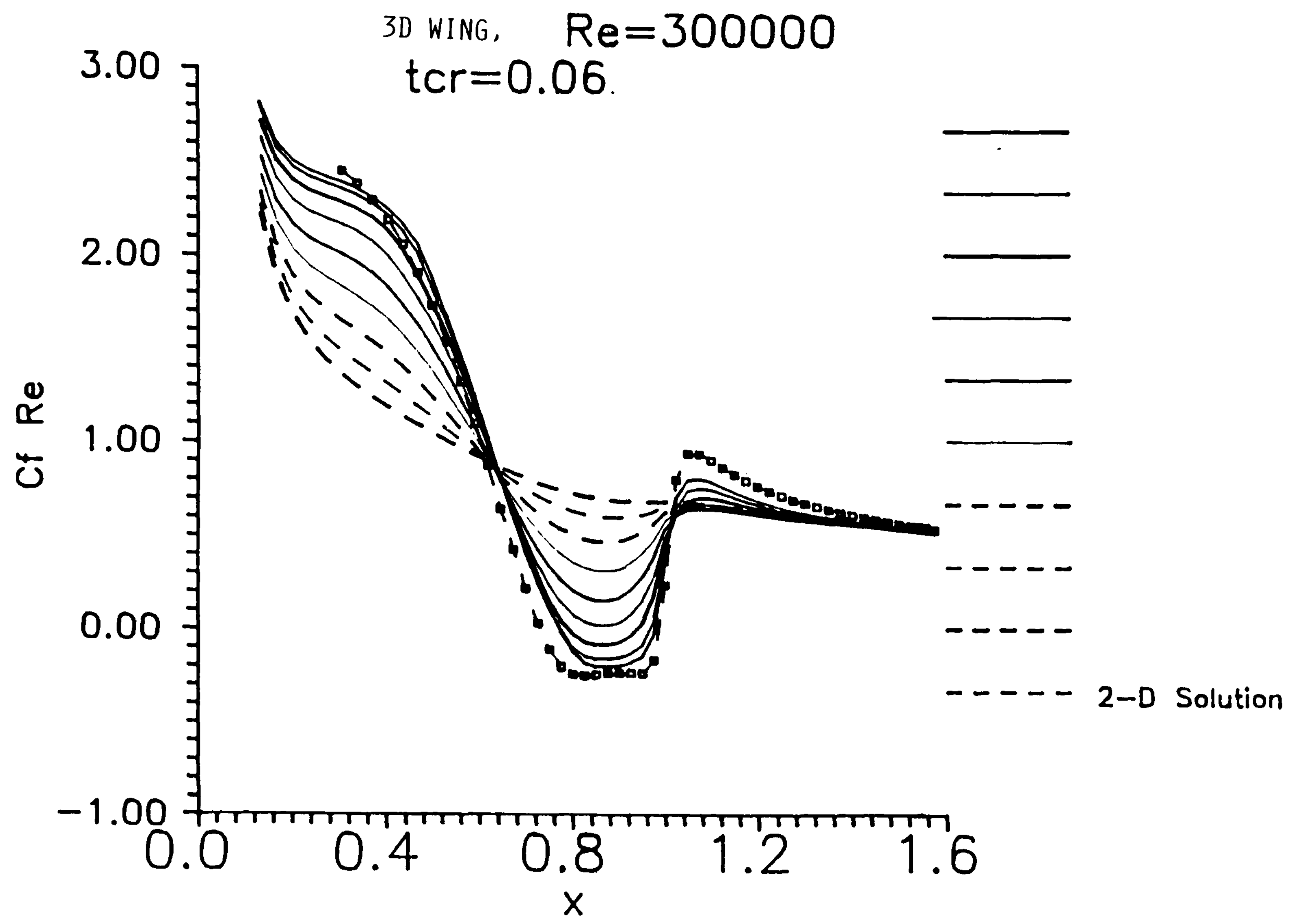


FIG. 6b. STREAMWISE VARIATION OF SKIN FRICTION.

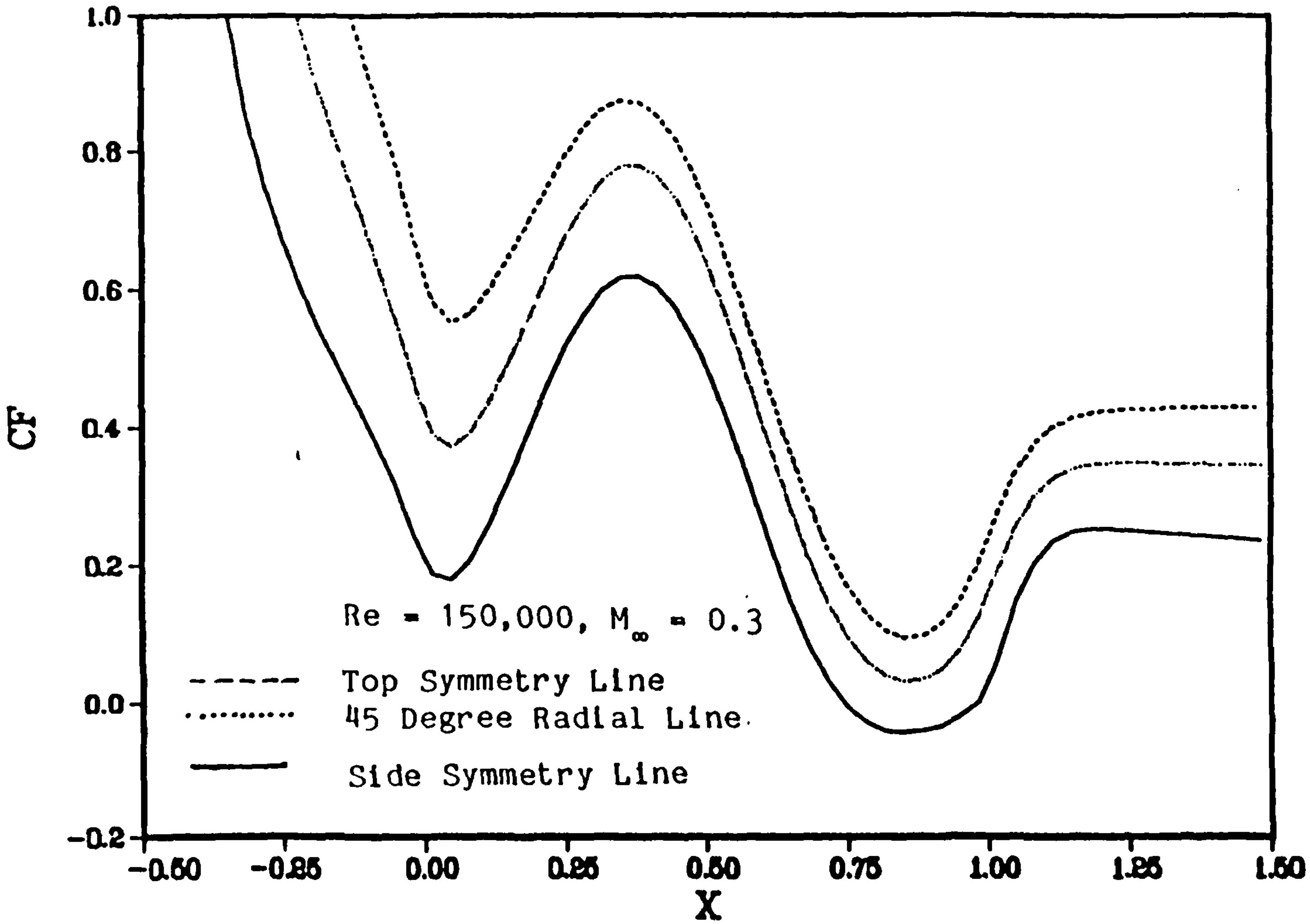


FIG. 6c. Flow Along a Cylinder with Elliptical Cross Section and a Sinusoidal Thickness Variation in the Axial Direction

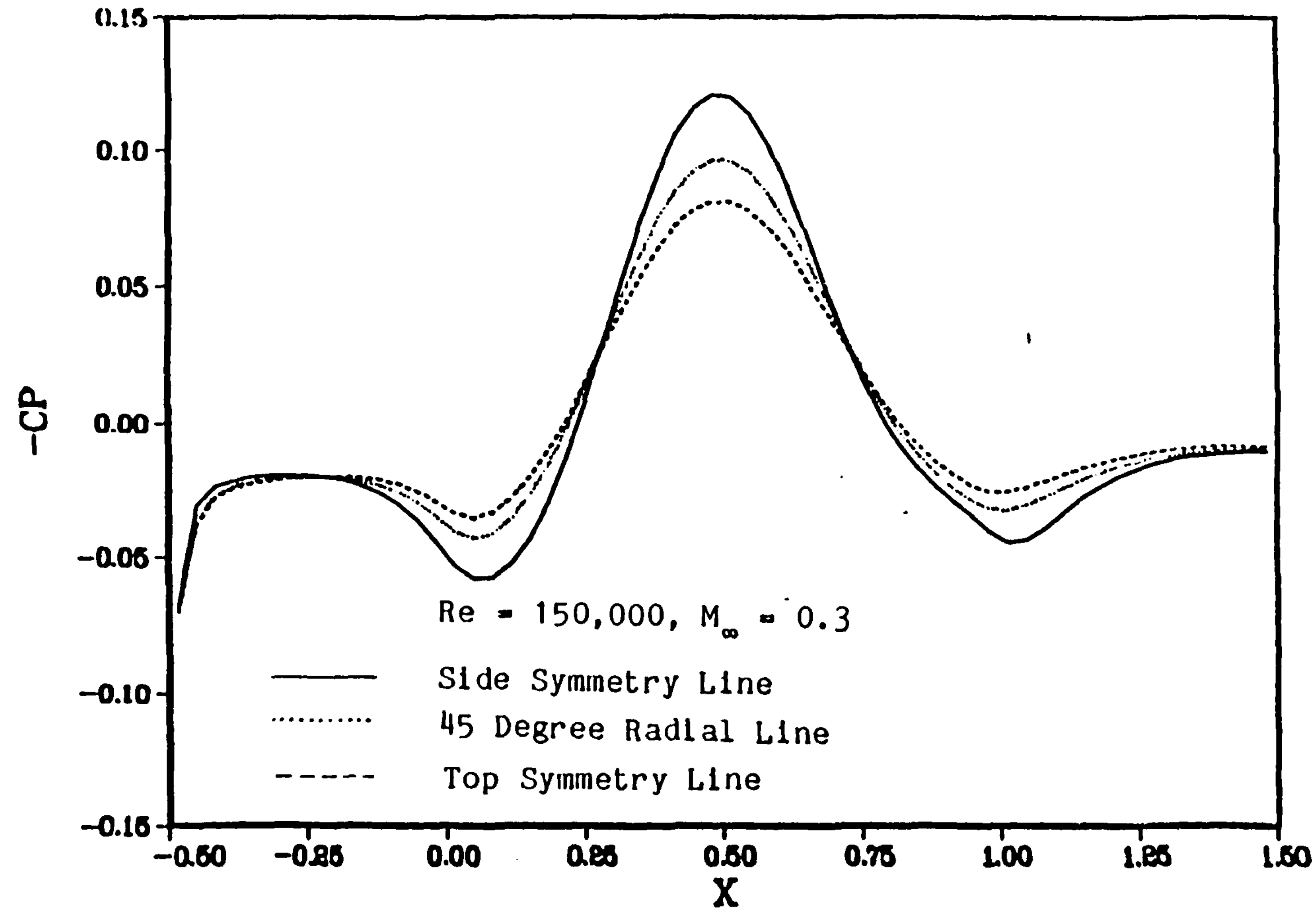


Fig. 6d. Flow Along a Cylinder with Elliptical Cross Section and a Sinusoidal Thickness Variation in the Axial Direction

The technique has been applied to the computation of flow past the sine-wave airfoil at high Reynolds number discussed previously. Earlier calculations on coarser meshes had predicted a considerable dependence of the solution on the grid distribution; in particular, sensitivity on the size and structure of the separation bubble and possible laminar flow breakdown with NS and RNS solvers. Recent calculations with the full (ψ-ω) form of the Navier-Stokes equations and with second order central differencing on a very fine mesh of 355x101 have been possible using the domain decomposition procedure. The skin friction at Re = 5×10^5 is shown in figure 7a. Solutions using seven domains converged in 35-30 iterations. Results for a variety of Re are given in figure 7b. A paper on this investigation will be presented at the National Fluid Dynamics Conference in Cincinnati in June 1988.

Additional issues relating to non-linear iteration and sensitivity to initial conditions have also been investigated. A paper on this study will be presented at the International Conference on Computational Engineering Science to be held in Atlanta in April 1988. Furthermore, the application of the DS to the computation of transonic inviscid flow has also been completed. A modified Newton method and several other techniques have been investigated to explore the sensitivity to initial conditions. Recent computations of transonic flows by M. Hafez using a direct solver required the application of a continuation technique for the solution of flows with strong sharp imbedded shock. For such cases, it has been determined by the present investigators that the choice of initial conditions is critical. In a majority of the transonic flow cases, the direct application of Newton's method will lead to divergence. A number of modifications have been carried out in order to eliminate this sensitivity to initial conditions. The

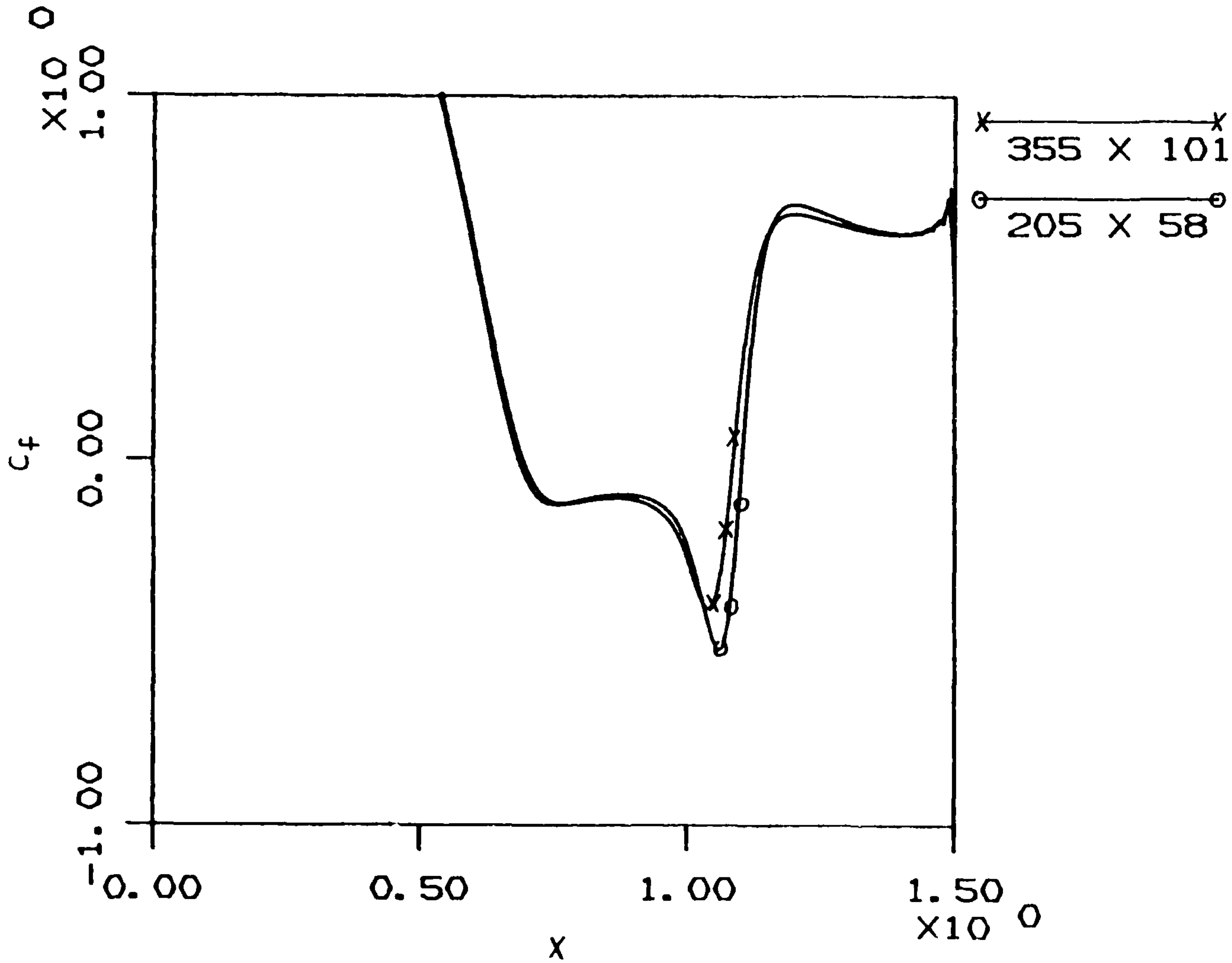
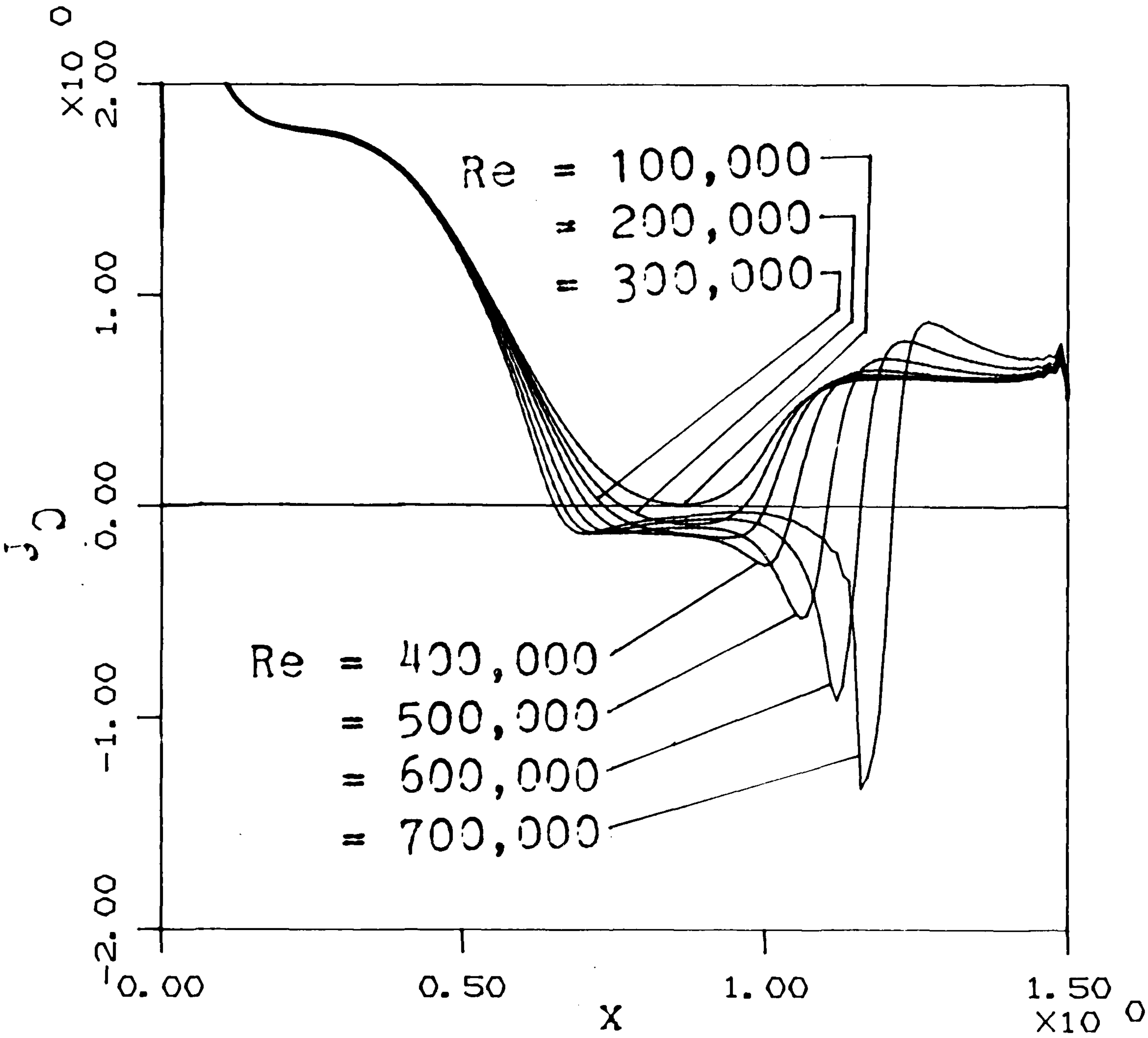


FIG. 7a.



Re = 100,000
= 200,000
= 300,000
Re = 400,000
= 500,000
= 600,000
= 700,000
2.00
1.00
0.00
-1.00
-2.00
X10 0
C f
0.00
0.50
1.00
1.50
X10 0
X

FIG. 7b.

solution for the flow over a biconvex airfoil at M = 0.92 has been obtained very efficiently. This flow exhibits a complex fishtail shock that has been computed with arbitrary initial conditions, see figures 7c,d. These new features include the modification of the Newton's method and/or the addition of a transient artificial viscosity that vanishes in the converged state. More complete results will appear in the Ph.D. thesis of E. Bender later this year. Several technical papers are in progress and should be completed by early 1989.

Application of the sparse matrix DS for three dimensional space marching is also under investigation. The DS is used, at each marcing step, to solve for the discrete algebraic system in the secondary flow plane. Preliminary solutions have been obtained for flow past a finite plate and along an axial (90°) corner. A paper on the three-dimensional wing-bump solutions presented previously has been presented at a CFD meeting in Sydney, Australia in August 1987. A second paper will appear in the near future.

1.7 RNS Relaxation and Flux Vector Splitting

The relation between global pressure relaxation and a new form of flux vector splitting has recently been clarified by the present investigators. A paper on this subject is scheduled to appear in Computers and Fluids. A preliminary copy is appended herein.

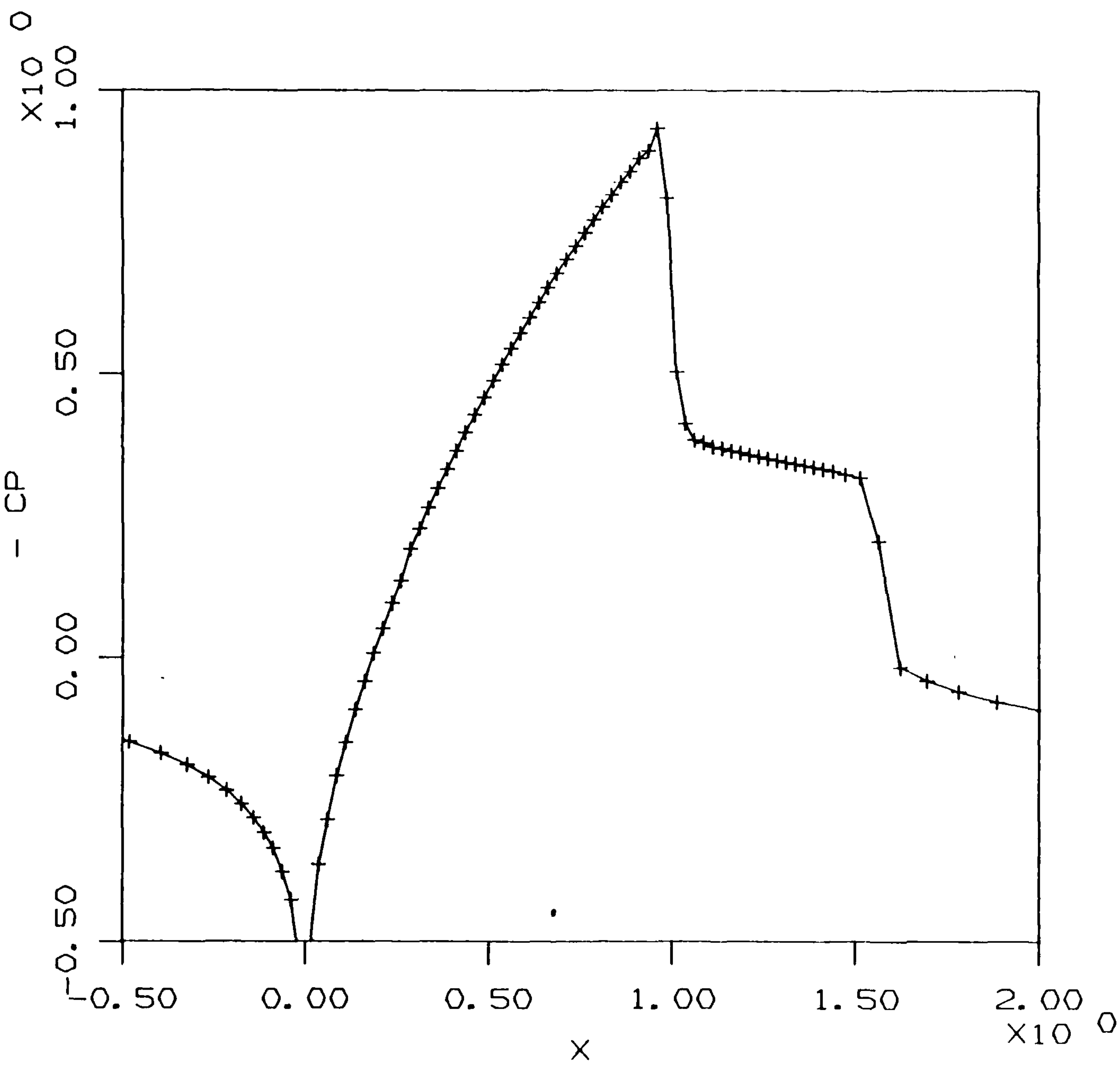


Fig. 7c. PRESSURE COEFF. - MACH NO. = 0.92

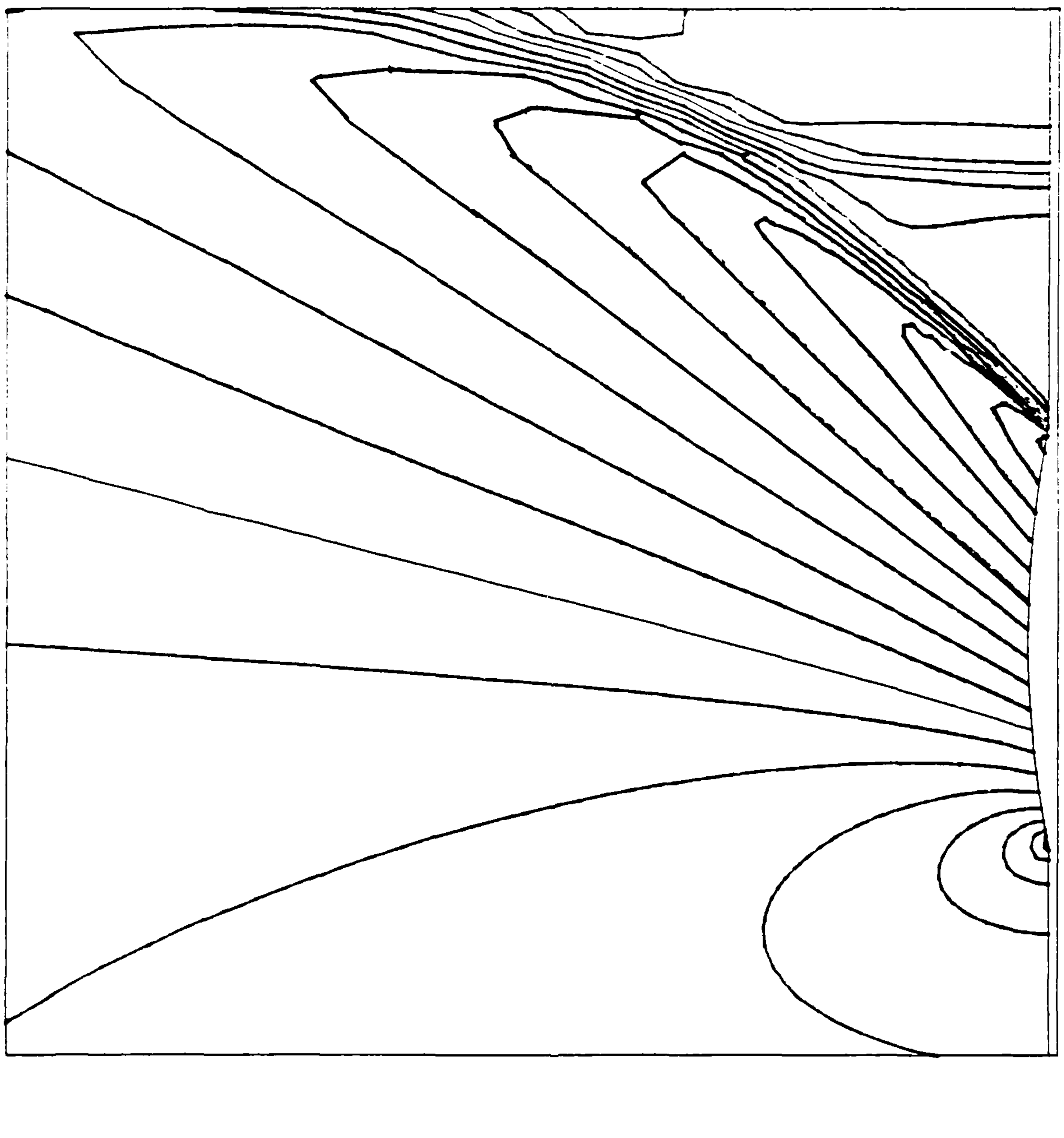


FIG. 7d. MACH 0.92

1.8 Highlights of Research Progress

1. An effective computational methodology based on PV and CV/RNS equations for the solution of inviscid and viscous flows with strong pressure interaction has been established. The procedures require reduced storage for steady flows and are computationally efficient for steady and transient flows. Several steady state and transient flow applications have been completed for airfoil and transonic flows. An alaternating direction (predictor-corrector) algorithm has been applied successfully with larger values of Δt than allowed by NS solvers for airfoil interaction with shedding.

2. A consistent version of the coupled strongly implicit procedure (CSIP) for unsteady and three-dimensional space marching flows has been developed. A number of PV and CV/RNS solutions have been presented with this algorithm. Three-dimensional applications using the CV/RNS development have been demonstrated for an afterbody configuration and with the PV/RNS development for a wing of variable camber.

3. A hysteresis phenomena for unsteady flow over airfoils has been predicted and has now been confirmed with more detailed and refined computations.

4. For separated flow computations, the strong influence of grid on the resolution of flow behavior has been established. A laminar flow 'instability' has been observed for fine meshes and large Reynolds number. Further RNS and NS analyses have demonstrated that the appearance of this "shock like" reattachment is probably physical and not numerical. Grid refinement, temporal accuracy and large Re asymptotic theories have reinforced this result. Artificial viscosity in conventional NS solvers suppresses this behavior as well as the true

diffusional influences on the flow. This has been shown independently by F. Marconi of Grumman and by researchers at NASA Langley for flow over conical and airfoil geometries.

5. A uni-directional multigrid pressure relaxation algorithm has been developed for highly stretched grids associated with thin viscous layers and separated flows. Conventional procedures fail in these cases. This procedure is now being considered for three-dimensional computations.

6. The RNS philosophy for high supersonic flows has been confirmed. Flows with shock-shock interaction and cavities have been successfully computed. A flux vector split analysis has demonstrated the relationship of pressure splitting to the flux eigenvalues. This procedure appears quite promising for hypersonic flows.

7. The application of a sparse matrix direct solver (DS) for the solution of separated flow and transonic flow problems has been investigated. The application of the DS can markedly improve the computational efficiency of the solution procedure. Overlapping grid DS procedures appear to be quite promising. A variety of solutions have now been obtained.

8. Applications to three-dimensions have been initiated. The influence of moderate bump on a wing geometry has led to the first 3D separation solutions. Further results have been obtained for juncture and afterbody geometries.

2. AFOSR PUBLICATIONS, PRESENTATIONS, ACTIVITIES AND INTERACTIONS

1/87 - 1/88

A. Publications and Proceedings

1. Khosla, P.K., Rubin, S.G.: Consistent Strongly Implicit Iterative Procedures for Two Dimensional Unsteady and Three Dimensional Space Marching Flow Calculations. Computers & Fluids, 15, 4, pp. 361-378, October 1987.

2. Lai, H.T. and Khosla, P.K.: Global Pressure Relaxation Procedure for Steady State Euler Equations. Computers & Fluids, 15, 2, pp. 215-228, July 1987.

3. Ramakrishnan, S.V. and Rubin, S.G.: Transient Flow Past a Finite Flat Plate at Incidence. AIAA Paper 87-0192, February, 1987.

4. Ramakrishnan, S.V. and Rubin, S.G.: Numerical Solution of Unsteady Compressible Reduced Navier-Stokes Equations. AIAA Journal, 25, 7, pp. 905-913, July 1987.

5. Bender, E.E. and Khosla, P.K.: Solution of Two-Dimensional Navier-Stokes Equations Using Sparse Matrix Solvers. AIAA Paper 87-603, February, 1987.

6. Himansu, A. and Rubin, S.G.: Multigrid Acceleration of a Relaxation Procedure for the RNS Equations. AIAA Paper 87-1145 CP, June 1987.

7. Gordnier, R.E. and Rubin, S.G.: Transonic Flow Solution Using a Composite Velocity Procedure for Potential, Euler and RNS Equations. To appear in a special issue of Computers and Fluids, July 1988.

8. Rubin, S.G.: PNS Methodology Hypersonic to Low Speed Flows. AIAA Special Publication, June 1987.

9. Reddy, D.R. and Rubin, S.G.: Consistent Boundary Conditions for RNS Schemes Applied to 3D Internal Flows. AIAA Paper 88-0714, January 1988.

10. Cohen, R. and Khosla, P.K.: Three-Dimensional RNS Solutions for Subsonic Unseparated Flows Using Global Pressure Relaxation. Proceedings 2nd International Conference on Numerical Methods in Fluid Mechanics, Sydney, Australia, August, 1987.

To Appear

11. Bender, E. and Khosla, P.K.: Application of Sparse Matrix Solvers and Newton's Method to Fluid Flow Problems. Proceedings of National Congress on Fluid Dynamics, Cincinnati, Ohio, June, 1988.

12. Bender, E.E. and Khosla, P.K.: A Modified Newton's method for the Computation of Fluid Flows. Proceedings of International Conference on Computational Engineering Science, Atlanta, Georgia, April 10-14, 1988.

13. Khosla, P.K., Rubin, S.G. and Himansu, A.: RNS Solutions for Two Dimensional Transient and 3-D Steady Flow. Proceedings of 11th ICMFD Conference, Williamsburg, Virginia, June, 1988.

14. Rubin, S.G.: RNS Pressure Relaxation and Flux Vector Splitting. Accepted for Computers and Fluids, 1988.

15. Lai, H.T. and Khosla, P.K., "Global Pressure Relaxation for Transonic Turbulent Flows," accepted for Computers and Fluids, 1988.

16. Ramakrishnan, S.V. and Rubin, S.G.: ;Numerical Solutions of Unsteady Compressible RNS Equations. Proceedings 1st National Congress on Fluid Dynamics, Cincinnati, Ohio, June, 1988.

Presentations

1. Himansu, A. and Rubin, S.G.: Multigrid Acceleration of a Relaxation Procedure for the RNS Equations. 8th AIAA Computational Fluid Dynamics Conference, Hawaii, June, 1987.

2. Reddy, D.R. and Rubin, S.G.: Consistent Boundary Conditions for RNS Schemes Applied to Three-Dimensional Internal Flows, 25th Aerospace Sciences Meeting, Reno, Nevada, January, 1988.

3. Cohen, R. and Khosla, P.K.: Three-Dimensional RNS Solutions. Second International Conference on Numerical Methods in Fluid Mechanics, Sydney, Australia, August, 1987

Seminars and Short Courses

1. Rubin, S.G., Ballistics Research Laboratory, December 11, 1987.
2. Rubin, S.G., NASA Lewis Research Center, May 3, 1987.
3. Rubin, S.G., Grumman Aerospace Corp., March 26, 27, 1987.
4. Rubin, S.G., AFWAL, WPAFB, March 4, 1987.
5. Khosla, P.K., NASA Lewis Research Center, September 10, 1987.

Student Presentations

1. Himansu, A.: " Multigrid Acceleration of Relaxation Procedure for the RNS Equations". AIAA Student Paper Competition, Region III, Dayton, Ohio, March 1987. (Won Second Prize)
2. Pordal, H.S.: "A Comparison of Conjugate Gradients and Minimum Residual Algorithms for the Acceleration of Relaxation for Three-Dimensional Inviscid Flow Past an Afterbody Configuration". AIAA Student Paper Competition, Region III, Dayton, Ohio, March 1987.
3. Himansu, A.: "Multi-Grid Acceleration of a Global Pressure Relaxation Procedure". M.S. Thesis, January, 1987 (Now Ph.D. Student).
4. Ramakrishnan, S.V.: "Numerical Solution of Unsteady Compressible Reduced Navier-Stokes Equations". Ph.D. Dissertation, February 1988 (now at Rockwell International, Sunnyvale, California).

5. Pordal, H.S.: "Conjugate Gradient Type Methods for Three-Dimensional Potential Flow Calculations." M.S. Thesis, October 1987 (now Ph.D. Student).

6. Bender, E.: "Use of Direct Sparse Matrix Solvers and Newton's Iteration for the Numerical Solution of Fluid Flows". Ph.D. Dissertation, April, 1988 (scheduled).

7. Lai, H.T.: (see publications), Post-Doctoral Fellow and Ph.D. graduate (now at Sverdrup, Inc., Cleveland, Ohio).

B. Committees and Assignments

Rubin, S.G.

AIAA Session Chairman and Organizing Committee, Fluid & Plasma Conference, Honolulu, 1987

AIAA Session Chairman, CFD Conference, Honolulu, 1987.

NASA Aerospace Research & Technology Subcommittee (ARTS), 1986-88.

NASA Aerospace Advisory Committee, CFD Validation Subcommittee, 1986-87.

Case Institute/NASA Lewis Institute for Computational Mechanics in Propulsion (ICOMP) Advisory Committee, 1987-89.

WPAFB - IPA - Visiting Scientist, 1987.

Editor-in-Chief - International Journal, Computers and Fluids.

NASA Review Committee of Aeronautics Advisory Committee - Supersonic Aircraft Drag Reduction, 1988-89.

Khosla, P.K.

Editorial Advisory Board, Computers and Fluids.

C. Interactions

During the period of the current AFOSR contract the principal investigators have interacted technically with several outside researchers and organizations. The CSIP developed by the PI's has been applied to internal flow problems by D. Reddy of Sverdrup, Inc. in conjunction with the NASA Lewis Research Center. A paper on this subject was presented at the AIAA meeting in Reno in January 1988. This procedure is also being considered for internal flows by R. Pletcher of Iowa State University. The RNS model and CSIP algorithm are continuing to be applied to hydrodynamics problems by Raven and Hoekstra at MARIN, The Netherlands and to aerodynamics problems by D. Reddy at Sverdrup, M. Rosenfeld of The Technion (Israel) and the NASA Ames Research Center, M. Barnett at United Technologies Research Center (see AIAA paper, Reno 1988) and C. Fletcher of the University of Sydney in Australia. Roger Cohen, a Fulbright Research Fellow from the University of Sydney has completed his work on these techniques during a two-year visit at the University of Cincinnati. He has developed the three dimensional code for the rectangular wing with spanwise ripples. The results of these studies were presented at the International CFD Conference in Sydney during August 1987. His Ph.D. dissertation should be completed during 1988 and a second paper with P. Khosla is in progress.

During 1987, Professor Rubin spent a sabbatical lave on a joint AFOSR(IPA)-University Resident Research Program at AFWAL, WPAFB. He worked with Dr. J. Shang and Dr. D. Rizzetta of the Computational Aerodynamics Group to develop the RNS procedure for high speed three-dimensional flows. The earlier AFOSR supported work on PNS and RNS computations have led to two results that are now default options in the latest version of the AFWAL PNS code as updated by Lockheed. The minimum step size restrictions

$\Delta\xi = 2y_{M=1/\pi}$ and the sublayer apoproximations of Rubin and Lin are new features of the code. Moreover, the boundary region or cross flow diffusion terms have been added to account for large curvature or separation effects. The result of this activity is several papers on high-speed RNS techniques and flux vector splitting that will appear in the near future.

The work on the application of sparse-matrix direct-solvers has generated significant interest around the country. L. Wigton at Boeing, D. Venkata at NASA Ames, M. Salas at NASA Langley, C. Merkel at Penn State, M. Hafez at University of California (Davis) are just a few of the researchers who are actively engaged in pursuing similar ideas and who have requested further information on our investigations. Papers on this subject, by Professor Khosla and E. Bender, are to be presented at conferences in Atlanta this spring and in Cincinnati this summer.

Professor Rubin presented a two-day seminar at Grumman Aerospace on RNS methods. A two-dimensional/axisymmetric code has been provided to the Grumman group for evaluation. Further discussions are ongoing with Dr. R. Melnik relating to a possible research project on three dimensional aerodynamic flows. As a result of these discussions, a project on the effects of artifical diffusion and cross flow (boundary region) diffusion was initiated. This has led to some important conclusions that were presented by Dr. F. Marconi of Grumman at the 1988 Reno meeting.

Professor A. Polak is continuing the application of the RNS code for the investigation of roughness effects at supersonic speeds. The geometry under consideration is a hollow cylinder at $M_\infty = 3$ for aircraft considerations. Professor P. Disimile is investigating the roughness problem in collaboration with the experimental aerodynamics group at WPAFB. He has a mini-grant to aid in this project. The RNS code as modified by

Professor Polak, will provide the computational support for this work. In addition, W. Sturek at BRL has expressed particular interest for application to roughness problems in ballistics. A seminar by S.G. Rubin was presented at BRL this winter. Additional computations by several Ph.D. students have demonstrated the strong convergence properties of the RNS formulation for high supersonic Mach numbers and strong pressure interactions, including shock waves and separation.

A new high Mach number RNS initiative with members of the staff at Sverdrup, Inc. is under discussion. Professor Rubin will visit ICOMP and NASA Lewis and present a seminar at Sverdrup this spring. Other interactions with AFWAL, General Dynamaics and Grumman, as they relate to hypersonic aerodynamics, are under discussion.

RNS PRESSURE RELAXATION AND FLUX VECTOR SPLITTING

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1. Introduction

The present author and co-workers have previously presented composite primitive variable[1-4] and velocity/potential[5-8] formulations for the computation of viscous/inviscid interacting flows. These procedures have been designed to emulate large Reynolds number (Re) asymptotic behavior, but, at the same time, to allow for the efficient solution of a single composite system of equations. This methodology is intermediate to that of full time-dependent Navier-Stokes solvers and to that of interacting or inverse matched boundary layer-inviscid solvers. The resulting system of reduced Navier-Stokes or RNS equations is a composite of the full Euler and second-order boundary layer systems. The neglected Navier-Stokes or diffusive terms are higher-order in Re for appropriate 'streamline' coordinates. Although, these terms can be retained through an explicit deferred-corrector, it must be emphasized that <u>the boundary conditions and the discrete approximation to the differential equations are dictated solely by the form of the lowest-order implicit RNS operator.</u>

The RNS model, which is valid throughout the entire Mach number (M) range, represents an enhancement of the hypersonic merged layer[9,10] and thin layer approximations[11], wherein the axial (x) or streamwise pressure gradient (p_x) is prescribed, or of the PNS model[12,13], where p_x is approximated with the truncated form ωp_x. The parameter $\omega(M)$ is a function of the local Mach number and is such that $0 \leq \omega \leq \omega_m < 1$, where $\omega_m(0) = 0$, $\omega_m(1) = 1$. These systems of equations are all 'hyperbolic', i.e., exhibit initial-value character in the axial or x direction. Forward marching or backward (upwind) differencing is applied in order to obtain a solution in a single x sweep. These methods are inadequate however, when axial flow separation, transonic regions with strong shock waves, pressure-viscous layer interaction, large axial curvature or subsonic to moderate supersonic free streams are present.

With appropriate treatment of the elliptic character of the equations, the RNS approximation, which allows for the full implicit pressure gradient (p_x), has been shown to accurately represent axial reversed flow and strong pressure-viscous interaction and to provide a more efficient method than full time-dependent Navier-Stokes solvers for this important class of flow problems. Although the upwind character of the solution procedure is retained, the RNS solver is no longer of initial value type and therefore the upwinding, or more appropriately, flux-vector splitting, must reflect the pressure (or potential) and/or reversed flow boundary value character.

A global pressure relaxation or multi-sweep procedure for the RNS model has been presented[1-4]. Viscous RNS and inviscid ($Re = \infty$) or Euler solutions have been obtained for a variety of flows with strong pressure interaction[1,2], with captured shocks[4,14] and with flow reversal[2,3]. Both steady[1-4] and transient[14] algorithms have been developed and a uni-directional (x) multi-grid accelerator[15] has been effective for improving the convergence rate of the relaxation procedure.

The RNS discretization reflects first or second-order upwinding only in the axial or x direction. The normal (y) gradients are approximated with 'central' discretizations. These are two-point or trapezoidal for first-order terms, e.g. v_y, p_y in the continuity and normal momentum equations, respectively. This allows for consistent non-reflective boundary conditions and reduces numerically induced oscillations that may appear with full three-point central discretizations for these terms. This also allows for more efficient application of far-field boundary conditions and more accurately represents the behavior of thin viscous layers, see references (1-4, 14) for further details.

Other investigators concerned with Euler, thin layer Navier-Stokes or full Navier-Stokes equations have also presented algorithms that are designed to relieve the 'artificial-viscosity' difficulties associated with central-difference discretizations. These are 'upwind' approximations that are associated with the movement of the physical forward and backward moving waves, i.e., flux-difference splitting, or with the movement of the discrete forward and backward moving particles, i.e., flux-vector splitting.

In previous PNS or RNS investigations, the choice of pressure/convective upwinding has resulted from characteristic or stability considerations. However, these results should also be derivable from a form of flux-vector splitting, since the characteristic analysis is simply a reflection of the eigenvalue or flux behavior. Therefore, in the present note, the pressure-velocity decomposition, that has been applied for the initial value PNS or boundary value RNS methodology, is re-examined from the point of view of flux vector splitting. The results represent a variation of the methods previously presented by Steger-Warming[16] and Van Leer[17].

2. Analysis

For the purpose of the present analysis we consider the one-dimensional Euler system given by

$$Q_t + \{E(Q)\}_x = Q_t + \{E^+(Q)\}_x + \{E^-(Q)\}_x = 0 \ , \qquad (1a)$$

where Q is the solution vector $Q = [\rho, \rho u, \rho e]^T$, e is the specific internal energy, ρ the density and u the velocty. $E(Q) = E^+(Q) + E^-(Q)$ and is such that all eigenvalues $\lambda^\pm$ of dE^+/dQ, dE^-/dQ are non-negative and non-positive, respectively.

In order to reflect the initial value characteristic behavior for supersonic flows, we also require that $E^+(Q) = E(Q)$, $E^-(Q) = 0$ for $M = u/a \geq 1$, where a is the sound speed ($a^2 = \gamma p/\rho$ for a perfect gas), γ is the ratio of specific heats and p is the pressure ($p = (\gamma-1)\rho e$). Other desirable properties[17] are that $E^\pm(Q)$, $dE^\pm(Q)/dQ$ and $\lambda^\pm$ be continuous for all M, or for all $(\frac{u}{u_{ref}})$ for incompressible flow, and that $dE^\pm/dQ$ have at least one zero eigenvalue for $|M| < 1$. This latter condition allows for a steady state shock structure with a maximum of two interior zones[17] and therefore results in more accurate shock capturing. In the present analysis, we have abandoned both the uniqueness condition[17] that E(Q) is the lowest degree polynomial in M and the symmetry condition for $M \rightarrow -M$[17]. The objective herein is to identify the flux-splitting as strongly as possible with the respective convective (axial) and acoustic disturbances in order to maximize the 'marching' or relaxation bias of the RNS formulation. This is in keeping with the methodology of interacting boundary layer calculations, in which the axial pressure gradient provides the only boundary value contribution for attached flows. This is also true for the Euler system, where the acoustic upstream influence is also pressure based. For regions of reversed flow, the convective fluxes must be appropriately upwinded in order to reflect the direction of the particle flow. The specification of the acoustic and mass flux-splitting is non-unique[17].

Two new flux vector splittings that satisfy many of the properties on $E^{\pm}$ delineated previously are introduced here. In one model, the continuity condition on $E^{\pm}$ at separation and reattachment points is no longer satisfied. For a second more satisfactory model, all of the properties including continuity are satisfied; however, the additional requirement that $E^-(Q) = E(Q)$, $E^+(Q) = 0$ for $M \leq -1$ is not enforced. Since we are not concerned here with reversed supersonic regions, this additional condition does not appear to be a severe limitation. Morever, it is possible to 'blend' the two forms of flux splitting in order to move smoothly from one to the other and thereby satisfy all conditions. This would lead to a 'central' difference approximation over a limited range where $-|\bar{M}| \leq M \leq 0$ or $-|\bar{u}| \leq u \leq 0$.

In order to evaluate the pressure/convective form of flux-vector splitting, we consider the simplified, partially conservative, Euler system (1). The differential system is given by

$$Q_t + (dE^+/dQ)Q_x^+ + (dE^-/dQ)\,Q_x^- = 0 \tag{1b}$$

or in scalar form

$$\rho_t + (\rho u)_x = 0 \tag{1c}$$

$$(\rho u)_t - u^2\rho_x + 2u(\rho u)_x + \omega(\gamma-1)(\rho e)_x + (1-\omega)(\gamma-1)(\rho e)_x = 0 \tag{1d}$$

$$(\rho e)_t + u(\rho e)_x + e(\rho u)_x - eu\rho_x + \frac{p}{\rho}(\rho u)_x - \frac{up}{\rho}\rho_x = 0 \qquad (1e)$$

where $\omega = \omega(M)$.

In the RNS methodology for attached flows, all spatial gradients, save the pressure gradient $(1-\omega)(\gamma-1)(\rho e)_x$ term in (1d), are associated with $E^+(Q)$. Therefore

$$\frac{dE^+(Q)}{dQ} = \begin{vmatrix} 0 & 1 & 0 \\ -u^2 & 2u & \omega(\gamma-1) \\ -uh & h & u \end{vmatrix}, \quad \frac{dE^-}{dQ} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & (1-\omega)(\gamma-1) \\ 0 & 0 & 0 \end{vmatrix} \qquad (2)$$

where $h = e + p/\rho$ and $a^2 = (\gamma-1)h$ for a perfect gas. The values of $E^+(Q)$, $E^-(Q)$ can be approximated by the following generalized expressions:

$$E^+(Q) = [\rho u,\ \rho u\tilde{u} + (\gamma-1)[\omega\rho e - \int_0^{\omega(x)} \rho e\, d\xi],\ \rho\tilde{u}e + \int_0^{\tilde{u}(x)} p d\xi + \int_0^{\rho\hat{u}(x)} h\, d\xi]^T \qquad (3a)$$

$$E^-(Q) = [0,\ \rho u\hat{u} + (\gamma-1)[(1-\omega)\rho e + \int_0^{\omega(x)} \rho e\, d\xi],\ \rho\hat{u}e + \int_0^{\hat{u}(x)} p d\xi - \int_0^{\rho\hat{u}(x)} h d\xi]^T \qquad (3b)$$

where $\tilde{u} = (u,0)_{max}$ and $\hat{u} = -(-u,0)_{max}$. Note that the model equations are not in full conservation form and this leads to the integral flux contribution in $E^{\pm}(Q)$. Also note that the contribution in $E^{\pm}(Q)$ from the pressure gradient p_x or $(\gamma-1)(\rho e)_x$ in (1d) is not given by $\rho u^2+\omega p$ and $(1-\omega)p$, respectively. The parameter ω is functionally dependent on M and has a discontinuity in slope at M=1. However, with the definitions (3) for $E^{\pm}(Q)$,

ω_x contributions will not appear in $dE^{\pm}(Q)/dQ$. These terms are not present in $dE(Q)/dQ$. Therefore, for the RNS splitting given by (2, 3), the continuity of $dE^{\pm}(Q)/dQ$ is maintained at sonic points to insure smooth shock capturing. The present form of $E^{\pm}(Q)$ leads directly to the RNS pressure decomposition that has been applied in earlier analysis. The discretizations for $\{E^{\pm}(Q)\}_x$ can now be given by

$$[\omega_i p_i - \omega_{i-1} p_{i-1} - \int_{\omega_{i-1}}^{\omega_i} p \, d\xi]/(x_i - x_{i-1}) \tag{4a}$$

for $E^-(Q)$, and

$$[(1-\omega_{i+1})p_{i+1} - (1-\omega_i)p_i - \int_{\omega_i}^{\omega_{i+1}} p \, d\xi]/(x_{i+1} - x_i) \tag{4b}$$

for $E^+(Q)$.

For $\int_{\omega_{i-1}}^{\omega_i} p \, d\xi - (\omega_i - \omega_{i-1}) \hat{p}$ and $\hat{p} \in [p_{i-1}, p_i]$, we obtain for $\{E^+(Q)\}_x$.

$$\begin{pmatrix} \omega_i \\ \dfrac{\omega_i + \omega_{i-1}}{2} \end{pmatrix} (p_i - p_{i-1}) \quad \text{for} \quad \hat{p} = \begin{pmatrix} p_{i-1} \\ \dfrac{p_i + p_{i-1}}{2} \end{pmatrix}$$

and for $\{E^-(Q)\}_x$,

$$\begin{pmatrix} 1 - \omega_{i+1} \\ 1 - \dfrac{\omega_i + \omega_{i+1}}{2} \end{pmatrix} (p_{i+1} - p_i) \quad \text{for} \quad \hat{p} = \begin{pmatrix} p_i \\ \dfrac{p_i + p_{i+1}}{2} \end{pmatrix}.$$

These are precisely the discrete approximations specified in previous RNS computations.

The eigenvalue ($\lambda^{\pm}$) analysis for the matrices (2) leads to the following for $u \geq 0$;

$$\lambda^{+}_{1,2,3} = (u + a\omega^{1/2},\ u - a\omega^{1/2},\ u)\ , \tag{5a}$$

all of which are non-negative for $u \geq 0$, and

$$\lambda^{-}_{1,2,3} = (0,\ 0,\ 0)\ , \tag{5b}$$

all of which are zero; therefore, in regions of attached flow upstream influence is of purely diffusive character. These results require that the function ω is given by $0 \leq \omega \leq \omega_m = M^2$. For $\omega = \omega_m$

$$\lambda^{+}_{1,2,3} = (2u,\ 0,\ u) \tag{5c}$$

and therefore $dE^{+}(Q)/dQ$ has one zero eigenvalue for all $|M| < 1$.

For separated flows ($u < 0$), one choice of flux splitting is to reverse the roles of all fluxes so that $E^{\pm}(Q) \leftrightarrows \overline{E^{\mp}}(Q)$ and hence $\lambda^{\pm} \rightleftarrows \overline{\lambda^{\mp}}$. As noted previously, this satisfies all of the positivity (negativity) conditions. Although, the eigenvalues of $\partial E^{\pm}(Q)/dQ$ are continuous, both $E^{\pm}(Q)$ and $dE^{\pm}(Q)/dQ)$ are discontinuous when u changes sign. Although this splitting has been applied successfully for several time dependent computations[14], it is sensitive to the temporal increment and marginally stable for Newton linearized steady state relaxation. An alternate and more desirable form is obtained with the fluxes (2,3), but with $\omega = 0$ for all $u \leq 0$. The flux expressions (3) for $u < 0$ become

$$E^{+}(Q) = \left[\rho u,\ \ 0,\ \int_0^{\rho u(x)} h\, d\xi\right]^T \tag{6a}$$

$$E^-(Q) = [0,\ \rho u^2+(\gamma-1)\rho e,\ \rho ue + \int_0^u p\, d\xi - \int_0^{\rho u} h\, d\xi] \qquad (6b)$$

so that for $u < 0$

$$\frac{dE^+(Q)}{dQ} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & h & 0 \end{vmatrix}, \quad \frac{dE^-(Q)}{dQ} = \begin{vmatrix} 0 & 0 & 0 \\ -u^2 & 2u & \gamma-1 \\ -uh & 0 & u \end{vmatrix} \qquad (6c)$$

In this form $E^{\pm}(Q)$ and $dE^{\pm}(Q)/dQ$ are continuous at $u = 0$, as well as at $M=1$, as are the eigenvalues $\lambda^{\pm}$, which for $u < 0$ are all zero or non-positive, respectively:

$$\lambda^+_{1,2,3} = (0,\quad 0,\quad 0) \qquad (7a)$$

$$\lambda^-_{1,2,3} = (0,\quad 2u,\quad u) \qquad (7b)$$

Identical results for the eigenvalues $\lambda^{\pm}$ are obtained with equations (1) given in full conservation form in terms of entropy, or in terms of total energy ($e_T = e + u^2/2$), if the up_x work term in the resulting energy equation is split in the same manner as is p_x in the momentum equation. If the up_x term in the total energy equation is unsplit, i.e., constant stagnation enthalpy or fully backward differenced, an alternate form of flux splitting results and the eigenvalues $\lambda^{\pm}$ for $u \geq 0$ become

$$\lambda^+_{1,2,3} = [u,\ \frac{u}{2}\,(2 + (\gamma-1)(1-\omega) \pm a(\{(\gamma-1)M(1-\omega)\}^2 + 4\omega)^{1/2}] \qquad (8a)$$

and

$$\lambda^-_{1,2,3} = [0,\ (1-\omega)u(\gamma-1),\ 0] \qquad (8b)$$

where again $0 \leq \omega \leq \omega_m$, but now $\omega_m = \frac{\gamma M^2}{1+(\gamma-1)M^2}$ and $\lambda_3^+ = 0$ for $M < 1$ for $\omega = \omega_m$. This is a slightly different condition than obtained in (5) and reflects the modification of the up_x flux splitting. This condition was obtained previously by Vigneron et al.[12], where it was also assumed that $E^-(Q) = 0$, so that an initial value or fully backward differenced or forward marched PNS solver results. Of course, as noted previously, this truncated method is in considerable error for low speeds, for separated flows and for attached flows, where upstream pressure interaction is non-negligible, e.g. strong shocks, large curvature, surface discontinuities, etc. The RNS formulation retains the full $E^-(Q)$ contribution for all forms of the differential approximation. The diffusion terms that are omitted from the full Navier-Stokes system can also be recovered with a global deferred-corrector, when such effects are not dominant[14].

For reversed flows, full shifting of the $E^\pm(Q)$ terms is also applicable for the total energy system; however, once again the alternative splitting wherein for $u \leq 0$ we specify only $\omega = 0$, is preferable. This leads to the desired continuity of $E^\pm(Q)$ and $dE^\pm(Q)/dQ$. For the total energy forms of the equations the eigenvalues $\lambda^\pm$ for $u < 0$ are,

$$\lambda^+_{1,2,3} = (0, \quad -(\gamma-1)u, \quad 0) \tag{9a}$$

$$\lambda^-_{1,2,3} = (0, \quad \gamma u, \quad 2u) \tag{9b}$$

When the total energy is one of the dependent variables, an additional condition is required to maintain the full upwind form. Since

$p = (\gamma-1)\rho e_T - \frac{\gamma-1}{2}\rho u^2$, the $(\rho u)_x$ flux arising from p_x also changes sign with u; therefore, these terms which appear in the momentum and energy equations must also be given in $\tilde{u}$, $\hat{u}$ form. The results (9) are then recovered. This additional condition does not arise when the temperature or the specific internal energy is one of the dependent variables; the results (7) are then recovered.

3. Stability

It can be shown[19] that both the full shift and $\omega = 0$ shift flux-splittings are unconditionally stable for steady state line relaxation. These methods are sensitive, however, to the assumed initial conditions for pressure, etc. that are required for steady-state calculations, for terms entering from $E^-(Q)$. It has been shown[19] that this sensitivity can be controlled by (i) an improved initial guess, e.g., several inviscid relaxation steps, (ii) local non-linear iteration at each axial location or (iii) underrelaxation of the velocities in separated regions, by the inclusion of temporal damping, i.e., finite Δt, in the early stages of the relaxation process. After a few steps, e.g., 5 to 20, full relaxation without local iteration can be restored. It can be shown that the discretization for the elliptic pressure term arising from $E^-(Q)$ corresponds to complete overrelaxation. While this is desirable close to convergence, it is undesirable in the initial stages of the relaxation process. This difficulty can be tempered by a pressure correction term developed by Israeli[18]. The correction, which vanishes in the steady state introduces pressure underrelaxation and also residual smoothing for multi-grid transfers. A combination of the pressure smoothing with full overrelaxation has provided a useful approach for the semi-coarsening or uni-directional multi-grid accelerator[15,19].

4. Conclusion

Global pressure relaxation for a pressure split form of the Euler or reduced Navier-Stokes (RNS) equations has been shown to be equivalent to a form of flux-vector splitting that satisfies the major eigenvalue and continuity constraints on the fluxes and flux derivatives. The upwinding is applied only in the axial or 'mass-flow' direction. In keeping with the asymptotic form of the RNS system, two-point or trapezoidal discretization is used for appropriate normal gradients in order to allow for the accurate evaluation of shear layers and a consistent specification of far field boundary conditions. If the pressure gradient parameter $\omega(M)$ is given by its maximum value in subsonic regions, one eigenvalue of λ^+ is always zero and therefore shock resolution is improved. For regions of reversed flow convective upwinding is combined with the condition $\omega = 0$. This insures that the fluxes, flux derivatives and eigenvalues remain continuous throughout the flow. This form of flux vector splitting is specifically designed to maintain a bias in the direction of the convective fluxes and therefore abandons the ($\pm$) symmetry[17] of the earlier forms of flux vector splitting. The upwinding leads to a relaxation method that is solely acoustic driven throughout subsonic regions, but also includes convective relaxation in regions of reversed flow.

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