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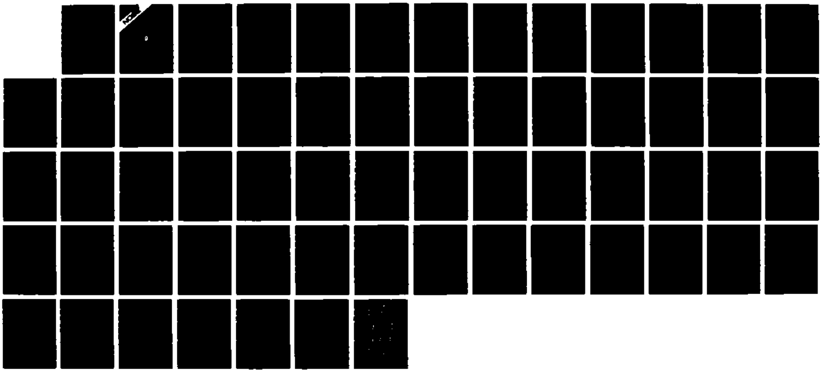

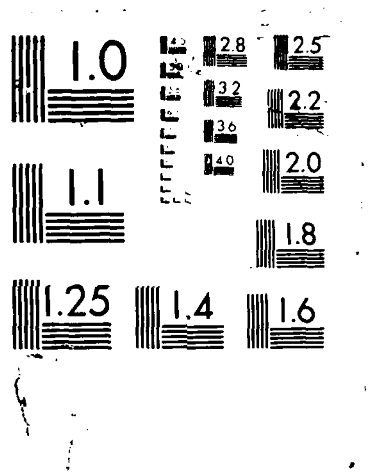
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A Piecewise Quadratic Strength Tensor Theory for Composites

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A piecewise quadratic strength tensor theory has been developed for composites with orthotropic, transversely isotropic, and isotropic material symmetries. Good correlations have been demonstrated between the theory and the biaxial fracture data of graphite/epoxy, graphite particulate, graphite/aluminum, glass/epoxy, and organic textolite composites.

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EXECUTIVE SUMMARY

OBJECTIVE

Develop the piecewise quadratic strength tensor theory for composite materials and demonstrate its applicability to the available biaxial fracture data on composites. The theory will have application to current composite structures of Naval Ocean Systems Center's interest such as transducers and future composite structures such as torpedo hull section, Vertical Launch ASROC (VLA) nosecaps, and tethered deep submergence structures. The theory can also be used with a wide variety of other NAVY structures such as aircraft and submarine substructures.

RESULTS

An extensive literature review has been conducted to search for the useful biaxial fracture data on composites. The data obtained pertain to a wide spectrum of composite material systems: graphite/epoxy, graphite particulate, graphite/aluminum, glass/epoxy, and organic textolite.

The general results pertaining to the proposed piecewise quadratic strength tensor theory have been reduced from an anisotropic material to an orthotropic, transversely isotropic, and isotropic material for composites applications. Also, these general results for a multiaxial stress state have been reduced to a biaxial stress state for correlation purposes.

Good correlations between the theory and the biaxial fracture data on the above-mentioned composite systems have been demonstrated and significant improvements of the proposed theory over the Tsai-Wu theory have been shown for the cases where the biaxial data have nonelliptical characteristics.

CONTENTS

1. INTRODUCTION

A multiaxial failure criterion* is a set of equations which, if satisfied by the six stress (or strain) components, imply failure. Mathematically, it can be written as

$$f(\sigma_k) = 1 \quad , \quad k=1,\ldots,6 \quad , \tag{1-1}$$

where σ_k is the contracted notation of the second rank stress tensor.** For a given stress state, σ_k, the function f is characterized by

$$\begin{aligned} &\text{if } f(\sigma_k) < 1 \quad , \text{ no failure occurs;} \\ &\text{if } f(\sigma_k) = 1 \quad , \text{ failure occurs; and} \\ &\text{if } f(\sigma_k) > 1 \quad , \text{ not admissible.} \end{aligned} \tag{1-2}$$

Geometrically, Eq. 1-1 defines a failure surface in the six-dimensional stress space and Eq. 1-2 states that a stress point can only reside either on or inside the failure surface and that failure occurs only if the stress point is on the surface. Because of this geometric interpretation, the failure criterion defined in Eq. 1-1 is also known as a failure surface.

The function f in Eq. 1-1 is usually obtained by a mathematical

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*To familiarize the reader with the general concepts of a failure criterion for composites, some content of this section has been reproduced from Reference 1.

**With reference to a rectangular Cartesian coordinate system (i.e., xyz or equivalently, $x_1x_2x_3$ system): $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$, $\sigma_4 = \tau_{xy}$, $\sigma_5 = \tau_{yz}$, $\sigma_6 = \tau_{zx}$.

correlation of experimental observations. It must, however, satisfy the following requirements:

R1. The function f must be a scalar function.

R2. The failure surface represented by Eq. 1.1 must be closed, surrounding the zero stress state.

The first requirement ensures that the failure criterion is valid for all coordinate systems; the second requirement ensures that the material strength is finite in all directions.

Unlike metals, composite materials are generally compressible and have different (uniaxial) ultimate strengths in tension and in compression. Moreover, they are often nonisotropic. In other words, a composite material may be anisotropic, orthotropic, transversely isotropic, or quasi-isotropic, which are defined below.

An anisotropic material has material properties that are different in all directions at a point of the body; there are no planes of material symmetry.

An orthotropic material has material properties that are different in three mutually perpendicular directions at a point in the body and, further, have three mutually perpendicular planes of material symmetry.

A transversely isotropic material possesses an axis of rotational symmetry such that the material properties on any plane perpendicular to the symmetry axis are practically invariant under any rotation of that plane about the axis, but the properties are different along the symmetry axis.

An isotropic material has material properties that are the same in every orientation at a point in the body. A quasi-isotropic material is essentially considered to be an isotropic material.

From the above discussions on the material characteristics of composites, it is clear that a failure criterion for composites must satisfy additional requirements:

R3. The failure criterion must be free of the restriction of incompressibility and depend on the hydrostatic state of stress (i.e., mean stress).

R4. The failure criterion must include the difference between the ultimate compressive and tensile strengths.

R5. The failure criterion must conform to the orthotropic, transversely isotropic, and isotropic material symmetries for orthotropic, transversely isotropic, and quasi-isotropic composites, respectively.

As is well-known, the Tsai-Wu's quadratic strength tensor theory (Reference 2) satisfies the above five minimum requirements and encompasses all other quadratic failure criteria used for composites. For a general anisotropic solid, Tsai and Wu's strength tensor theory can be written as*

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad , (i,j=1,\ldots,6) \quad , \tag{1-3}$$

where F_i and F_{ij} are the strength tensors of rank two and four, respectively. Without loss of generality, it is assumed that

$$F_{ij} = F_{ji} \quad . \tag{1-4}$$

In addition, constraints+ must be imposed on the strength tensor F_{ij} to ensure that the material strength is finite in all directions. More specifically, F_{ij} must be positive definite, that is,

$$F_{ij} \sigma_i \sigma_j > 0 \quad , \tag{1-5}$$

at all points σ_i in the six-dimensional stress space. Geometrically, Eq. 1-5 is a necessary and sufficient condition to ensure that the failure surface represented by the quadratic polynomial of Eq. 1-3 is closed and ellipsoidal.

In the biaxial stress plane, the Tsai-Wu criterion represents a single ellipse. In general, a single continuous ellipse cannot satisfactorily represent the biaxial data of composites in all four stress quadrants. To account for the nonelliptical characteristics of the biaxial fracture data of composites, Chamis (Reference 3) and Rosen (Reference 4) suggested to use the

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*Unless otherwise indicated, the usual summation convention over a repeated index is used throughout this report.

+Such constraints are usually referred to as the stability requirements or stability conditions.

Tsai-Wu quadratic criterion with different F_{ij} $(i \neq j)$ for different stress quadrants. Beyond having more coefficients for better data fit, there is no physical or mathematical justification (Reference 5). Another approach to improve the Tsai-Wu quadratic criterion for composites application was to include the cubic terms in Eq. 1-3 (References 6 and 7). Here, an enormous numbers of sixth order strength tensor components are needed. Moreover, having cubic stress terms, the failure surface becomes open-ended (Reference 2).

Without suffering any of these shortcomings, Tang and Kuei (e.g., Reference 8) improved Tsai and Wu's theory in correlating the biaxial strength data of (monotonous) polycrystalline graphite, which show similar nonelliptical characteristics to composite data. Recognizing the fact that such characteristics may be due to different fracture mechanisms being operative under different states of biaxial stresses with different combinations of tensile and compressive stresses (Reference 9), they added to the Tsai-Wu criterion the quadratic stress terms with the absolute value of the linear combination of stress components. The resulting piecewise quadratic strength tensor theory can be written as

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j + H_i \sigma_i |H_j \sigma_j| = 1 \quad , (i,j=1,\ldots,6) \quad , \qquad (1\text{-}6)$$

where H_i is a second rank tensor.

The above equation holds for a general anisotropic material. They then reduced all the results pertaining to the anisotropic material to a transversely isotropic and an isotropic graphite and demonstrated good correlations between the theory and the biaxial fracture data of graphite.

In view of such good correlations with biaxial graphite data which show similar nonelliptical characteristics to composite data, it is proposed that the piecewise quadratic strength tensor theory be developed for composites. The developments are presented in this report.

The next section records the general results pertaining to the proposed theory and the detailed derivation for reducing these results to an orthotropic material for composites applications. It also shows the detailed reduction of the results from an orthotropic material to a transversely isotropic and an isotropic material. Section 3 further reduces the results obtained in Section 2 for a multiaxial stress state to a biaxial stress state.

Section 4 describes the available biaxial fracture data on composites that can be used for correlation purposes. These data were obtained through an extensive literature review and covered a wide spectrum of composite material systems: graphite/epoxy, graphite particulate, graphite/aluminum, glass/epoxy, and organic textolite. Section 5 correlates the theoretical results developed in Section 3 with the data obtained in Section 4 and quantify the improvement of the proposed criterion over the Tsai-Wu theory. Finally, Section 6 concludes the applicability of the proposed theory to composites.

2. A MULTIAXIAL FAILURE THEORY FOR COMPOSITES

Presented below are the general results pertaining to the proposed multiaxial failure theory for composites, including general anisotropic, orthotropic, transversely isotropic, and isotropic materials. The results contain the explicit expressions for the failure criteria of materials with various material symmetries, the restrictions imposed on the components of the strength tensors occurring in these criteria, and the geometric meaning of these criteria. Along with the presentation, it should be clear that the proposed theory satisfies all the necessary requirements R1-R5 defined in the last section for a composite failure criterion.

2.1 Anisotropic Material

As introduced in the last section, the proposed multiaxial failure theory for a general anisotropic material was given in Eq. 1-6 and is recorded below for an easy reference:

$$f(\sigma_k) = F_i \sigma_i + F_{ij} \sigma_i \sigma_j + H_i \sigma_i |H_j \sigma_j| = 1 \quad , (i,j,k = 1,\ldots,6) \ . \qquad (2\text{-}1)$$

As in Tsai and Wu's theory, the strength tensor F_{ij} is assumed to be symmetric:

$$F_{ij} = F_{ji} \ . \qquad (2\text{-}2)$$

With this assumption, there are 6 independent strength tensor components for each of F_i and H_i, and 21 for F_{ij}. These numbers for a general anisotropic material can usually be reduced substantially for a material having a certain material symmetry. Such reductions will be shown in the next three subsections.

Equation 2-1 can be decomposed into two equations:

$$F_i \sigma_i + (F_{ij} + H_i H_j) \sigma_i \sigma_j = 1 \quad , \tag{2-3}$$

for all σ_i with

$$H_i \sigma_i \geq 0 \quad , \tag{2-4}$$

and

$$F_i \sigma_i + (F_{ij} - H_i H_j) \sigma_i \sigma_j = 1 \quad , \tag{2-5}$$

for all σ_i with

$$H_i \sigma_i < 0 \quad . \tag{2-6}$$

The stability conditions to ensure the closure of each of the failure surfaces represented by Eqs. 2-3 and 2-5, respectively, are

$$(F_{ij} + H_i H_j) \sigma_i \sigma_j > 0 \quad , \tag{2-7}$$

for all σ_i satisfying Eq. 2-4, and

$$(F_{ij} - H_i H_j) \sigma_i \sigma_j > 0 \quad , \tag{2-8}$$

for all σ_i satisfying Eq. 2-6. Clearly, Eqs. 2-7 and 2-8 are the stability conditions required to ensure the closure of the entire piecewise failure surface represented by Eq. 2-1.

Geometrically, the failure surfaces represented by Eqs. 2-3 and 2-5, respectively, with the restrictions made by Eqs. 2-7 and 2-8 on the strength tensors F_{ij} and H_i are two ellipsoids in the two half spaces defined by Eqs. 2-4 and 2-6. Thus, the failure surface represented by Eq. 2-1 with the strength tensors F_{ij} and H_i satisfying the stability conditions given by Eqs. 2-7 and 2-8 is a piecewise ellipsoid in the six-dimensional stress space. Hence, the proposed quadratic strength tensor theory is referred to as the piecewise quadratic strength tensor theory.

Being in tensor form, the failure criterion defined in Eq. 2-1 is valid not only for the material axes but also for any different reference coordinate systems. The transformations of (σ_i, F_i, and H_i) and (F_{ij}) under the coordinate transformations follow the well-known tensorial laws (e.g. Reference 10) for the second rank and the fourth rank tensors, respectively. for example, if x'_i (i=1,2,3) denotes the reference coordinate system, x_i (i=1,2,3) designates the material axis system, and a_{ij} (i,j=1,2,3) represents the associated coordinate transformation defined by

$$x'_i = a_{ij} x_j \quad , \quad a_{ij} a_{kj} = 1 \quad , (i,j,k=1,2,3) \quad , \tag{2-9}$$

then, a second rank tensor f'_{um} and a fourth rank tensor f'_{umnp} referred to the x'_i system are related to f_{np} and f_{qrst} referred to the x_i system by

$$f'_{um} = a_{un} a_{mp} f_{np} \quad , \tag{2-10}$$

and

$$f'_{umnp} = a_{uq} a_{mr} a_{ns} a_{pt} f_{qrst} \quad , \tag{2-11}$$

respectively. All the latin indices in the last two equations take the values 1, 2, 3.

The major features of the piecewise quadratic strength tensor theory are as follows:

1. Equation 2-1 is in tensor form, hence, it is valid for all coordinate systems when it is valid for one coordinate system. This allows transformations of the strength criterion from one coordinate system to another through the well-known tensorial transformation laws.

2. The stability conditions imposed on the strength tensors assure the closure of the failure surface.

3. The effect of the hydrostatic state of stress on fracture can be considered since no restriction of incompressibility is contained in the theory.

4. Equation 2-1 contains linear terms, which take into account the effect of the difference in tensile and compressive strengths.

5. It is possible to derive* a complete set of independent strength tensor components for materials with different material symmetries, such as orthotropy, transverse isotropy, and isotropy.

6. Equation 2-1 contains the stress terms with the absolute value of the linear combination of stress components, which take into account the difference in failure mechanisms being operative under different multiaxial stress states.

7. The criterion has been obtained without using any constitutive equation of a brittle solid. More specifically, the linearly elastic stress-strain law has not been employed in the derivation of Eq. 2-1. Thus, if a nonlinear constitutive law is used to evaluate the stresses in Eq. 2-1, there will be no inconsistency.

8. The criterion is operationally simple because it represents a piecewise ellipsoid consisting of two ellipsoids and it contains no terms of the stress components higher than the quadratic ones.

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*The derivation will be given in the next three subsections for orthotropic, transversely isotropic, and isotropic materials, respectively.

2.2 Orthotropic Material

For an orthotropic material with the reference coordinate planes coinciding with the planes of material symmetry, based on the invariance requirements (References 11, 12) of orthotropy material symmetry, the stress dependent function f defined in Eq. 1-1 must be expressible as a polynomial in the seven quantities: $\sigma_1, \sigma_2, \sigma_3, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_4\sigma_5\sigma_6$. Thus,

$$f = f\,(\sigma_1, \sigma_2, \sigma_3, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_4\sigma_5\sigma_6) \quad . \tag{2-12}$$

Alternatively, f is expressible as

$$f = f\,(\sigma_1, \sigma_2, \sigma_3, \sigma_4^2, \sigma_5^2, \sigma_6^2, I_3) \quad , \tag{2-13}$$

where I_3 is a stress invariant given by

$$\begin{aligned} I_3 = {} & \sigma_1^3 + \sigma_2^3 + \sigma_3^3 + 3\sigma_1\,(\sigma_4^2 + \sigma_6^2) + 3\sigma_2\,(\sigma_4^2 + \sigma_5^2) \\ & + 3\sigma_3\,(\sigma_5^2 + \sigma_6^2) + 6\,\sigma_4\,\sigma_5\,\sigma_6 \quad . \end{aligned} \tag{2-14}$$

Hence, the explicit expression for the quadratic function f defined by Eq. 2-1 can be given in terms of the arguments occurring in Eq. 2-12 as

$$\begin{aligned} f = {} & F_1\,\sigma_1 + F_2\,\sigma_2 + F_3\,\sigma_3 + F_{11}\,\sigma_1^2 + 2F_{12}\,\sigma_1\,\sigma_2 + 2F_{13}\,\sigma_1\,\sigma_3 \\ & + F_{22}\,\sigma_2^2 + 2F_{23}\,\sigma_2\,\sigma_3 + F_{33}\,\sigma_3^2 + F_{44}\,\sigma_4^2 + F_{55}\,\sigma_5^2 + F_{66}\,\sigma_6^2 \\ & + (H_1\,\sigma_1 + H_2\,\sigma_2 + H_3\,\sigma_3)\,|H_1\,\sigma_1 + H_2\,\sigma_2 + H_3\,\sigma_3| = 1 \quad . \end{aligned} \tag{2-15}$$

Comparing Eq. 2-15 with Eq. 2-1, it can be seen that

$$F_4 = F_5 = F_6 = 0 \quad ,$$

$$F_{14} = F_{15} = F_{16} = F_{24} = F_{25} = F_{26} = F_{34} = F_{35} = F_{36}$$
$$= F_{45} = F_{46} = F_{56} = 0 \quad , \tag{2-16}$$

$$H_4 = H_5 = H_6 = 0 \quad .$$

With the above results, it is clear that there are only three independent strength tensor components for each of F_i and H_i (i.e., (F_1, F_2, F_3) and (H_1, H_2, H_3)), and only nine for F_{ij} (i.e., F_{11}, F_{22}, F_{33}, F_{44}, F_{55}, F_{66}, F_{12}, F_{23}, F_{13}). As noted earlier, the numbers of independent strength tensor components for a general anisotropic material has been substantially reduced due to orthotropy material symmetry.

The strength constants mentioned above are not free material parameters because they are restricted by the stability conditions Eqs. 2-7 and 2-8. For an orthotropic material, using Eq. 2-16, the independent restrictions on F_{ij} and H_i can be obtained as

$$F_{11} \pm H_1^{\,2} > 0 \quad ,$$

$$F_{22} \pm H_2^{\,2} > 0 \quad , \tag{2-17}$$

$$F_{33} + H_3^{\,2} > 0 \quad ,$$

$$F_{44} > 0 \quad ,$$

$$F_{55} > 0 \quad ,$$

$$F_{66} > 0 \quad ,$$

$$(F_{11} \pm H_1^{\,2})\ (F_{22} \pm H_2^{\,2}) - (F_{12} \pm H_1\ H_2)^2 > 0 \quad ,$$

$$(F_{22} \pm H_2^{\,2})\ (F_{33} \pm H_3^{\,2}) - (F_{23} \pm H_2\ H_3)^2 > 0 \quad , \qquad (2\text{-}17)$$

$$(F_{33} \pm H_3^{\,2})\ (F_{11} \pm H_1^{\,2}) - (F_{13} \pm H_1\ H_3)^2 > 0 \quad ,$$

$$\begin{aligned} &(F_{11} \pm H_1^{\,2})\ (F_{22} \pm H_2^{\,2})\ (F_{33} \pm H_3^{\,2}) \\ &+ 2\ (F_{12} \pm H_1\ H_2)\ (F_{23} \pm H_2\ H_3)\ (F_{13} \pm H_1\ H_3) \\ &- (F_{11} \pm H_1^{\,2})\ (F_{23} \pm H_2\ H_3)^2 - (F_{22} \pm H_2^{\,2})\ (F_{13} \pm H_1\ H_3)^2 \\ &- (F_{33} \pm H_3^{\,2})\ (F_{12} \pm H_1\ H_2)^2 > 0 \quad . \end{aligned}$$

2.3 Transversely Isotropic Material

For a transversely isotropic material with the x_3 - axis being parallel to the axis of rotational symmetry, based on the invariance requirements (e.g. Reference 11) of the transversely isotropy material symmetry, the function f in Eq. 1-1 must be expressible as a polynomial in the five quantities: I_1, I_2, I_3, σ_3, and $\sigma_5^{\,2} + \sigma_6^{\,2}$, where I_1 and I_2 are stress invariants given by

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \quad , \\ I_2 &= (\sigma_1^{\,2} + \sigma_2^{\,2} + \sigma_3^{\,2}) + 2\ (\sigma_4^{\,2} + \sigma_5^{\,2} + \sigma_6^{\,2}) \quad , \end{aligned} \qquad (2\text{-}18)$$

and I_3 is the stress invariant given by Eq. 2-14.

Thus,

$$f = f(I_1, I_2, I_3, \sigma_3, \sigma_5^2 + \sigma_6^2) \quad . \tag{2-19}$$

In light of this, the quadratic function f defined by Eq. 2-1 can be expressed from the arguments occurring in Eq. 2-19 as

$$f = a_0 I_1 + a_1 \sigma_3 + [b_0 I_1^2 + b_1 I_1 \sigma_3 + b_2 \sigma_3^2 + b_3 I_2 + b_4 (\sigma_5^2 + \sigma_6^2)] + (c_0 I_1 + c_1 \sigma_3)|c_0 I_1 + c_1 \sigma_3| \quad . \tag{2-20}$$

A comparison of Eq. 2-20 with Eq. 2-15 using Eqs. 2-18 and 2-14 leads to the following results for the nonvanishing strength tensor components:

$$F_1 = F_2 = a_0 \quad ,$$

$$F_3 = a_0 + a_1 \quad ,$$

$$F_{11} = F_{22} = b_0 + b_3 \quad ,$$

$$F_{33} = b_0 + b_1 + b_2 + b_3 \quad ,$$

$$F_{44} = 2b_3 = 2(F_{11} - F_{12}) \quad , \tag{2-21}$$

$$F_{55} = F_{66} = 2b_3 + b_4 \quad ,$$

$$F_{12} = b_0 \quad ,$$

$$F_{23} = F_{13} = b_0 + \frac{b_1}{2} \quad ,$$

$$H_1 = H_2 = c_0 \quad , \tag{2-21}$$

$$H_3 = c_0 + c_1 \quad .$$

Substituting these results into Eq. 2-15, the expression for f in Eq. 2-1 becomes

$$\begin{aligned} f = F_1 (\sigma_1 + \sigma_2) + F_3 \sigma_3 + F_{11} (\sigma_1^2 + \sigma_2^2 + 2\sigma_4^2) + F_{33} \sigma_3^2 \\ + F_{55} (\sigma_5^2 + \sigma_6^2) + 2F_{12} (\sigma_1 \sigma_2 - \sigma_4^2) + 2F_{13} (\sigma_1 + \sigma_2)\sigma_3 \\ + [H_1 (\sigma_1 + \sigma_2) + H_3 \sigma_3] \left| H_1 (\sigma_1 + \sigma_2) + H_3 \sigma_3 \right| = 1 \quad . \end{aligned} \tag{2-22}$$

From Eq. 2-21 or Eq. 2-22, it is clear that there are only two independent strength tensor components for each of F_i and H_i (i.e., (F_1, F_3) and (H_1, H_3)) and only five for F_{ij} (i.e., F_{11}, F_{33}, F_{55}, F_{12}, F_{13}). As noted earlier, the numbers of independent strength tensor components for a general anisotropic material has been further reduced due to transversely isotropy material symmetry.

The independent stability restrictions on these strength constants can be obtained by substituting Eq. 2-21 into Eq. 2-17 as

$$F_{11} + F_{12} + 2H_1^2 > 0 \quad ,$$

$$F_{11} - F_{12} > 0 \quad , \tag{2-23}$$

$$F_{33} \pm H_3^2 > 0 \quad ,$$

$$(F_{33} \pm H_3^2)\,(F_{11} + F_{12} \pm 2H_1^2) - 2(F_{13} \pm H_1\, H_3)^2 > 0 \quad , \tag{2-23}$$

$$F_{55} > 0 \quad .$$

2.4 Isotropic Material

For an isotropic material, based on the invariance requirements (e.g., Reference 11) of the isotropy material symmetry, the stress dependent function f defined in Eq. 1-1 must be expressible as a polynomial in the stress invariants I_1, I_2, and I_3 defined in Eqs. 2-18 and 2-14. Thus,

$$f = f\,(I_1, I_2, I_3) \quad . \tag{2-24}$$

In light of this, the quadratic function defined by Eq. 2-1 can be expressed by

$$f = a_0\, I_1 + (b_0\, I_1^2 + b_3\, I_2) + c_0\, I_1\, |c_0\, I_1| \ . \tag{2-25}$$

Comparing Eq. 2-25 with Eq. 2-20, it can be seen that

$$a_1 = b_1 = b_2 = b_4 = c_1 = 0 \quad . \tag{2-26}$$

Substituting Eq. 2-26 into Eq. 2-21 the following relations for the nonvanishing strength tensor components can be obtained:

$$F_1 = F_2 = F_3 = a_0 \quad ,$$

$$F_{11} = F_{22} = F_{33} = b_0 + b_3 \quad ,$$

$$F_{44} = F_{55} = F_{66} = 2b_3 = 2(F_{11} - F_{12}) \quad , \tag{2-27}$$

$$F_{12} = F_{23} = F_{13} = b_0 \quad ,$$

$$H_1 = H_2 = H_3 = c_0 \quad .$$

Using Eqs. 2-27 and 2-25, the expression for f in Eq. 2-1 becomes

$$f = F_1 I_1 + F_{11} I_2 + F_{12} (I_1^2 - I_2) + H_1 I_1 |H_1 I_1| = 1 \quad . \tag{2-28}$$

An alternative expression for f can be obtained by substituting Eq. 2-27 into Eq. 2-15 or Eq. 2-22 as

$$\begin{aligned} f = {} & F_1 (\sigma_1 + \sigma_2 + \sigma_3) + F_{11} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_4^2 + \sigma_5^2 + \sigma_6^2)] \\ & + 2F_{12} [\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 - (\sigma_4^2 + \sigma_5^2 + \sigma_6^2)] \\ & + H_1 (\sigma_1 + \sigma_2 + \sigma_3) | H_1 (\sigma_1 + \sigma_2 + \sigma_3)| \quad . \end{aligned} \tag{2-29}$$

From Eqs. 2-27, 2-28, or 2-29, it is clear that there are only four independent strength tensor components: F_1, H_1, F_{11}, and F_{12}; one each for F_i and H_i and two for F_{ij}. Clearly, the numbers of independent strength tensor components for a general anisotropic material has been further reduced due to isotropy material symmetry.

The independent stability restrictions on these strength constants can be obtained by substituting Eq. 2-27 into Eq. 2-17 or 2-23 as

$$F_{11} + F_{12} + 2H_1^2 > 0 \quad ,$$

$$F_{11} - F_{12} > 0 \quad , \tag{2-30}$$

$$(F_{11} + H_1^2)(F_{11} + F_{12} + 2H_1^2) - 2(F_{12} + H_1^2)^2 > 0 \quad .$$

2.5 General Remarks

Along with the developments made so far, the following should be clear:

(1) In the proposed theory, there are 33, 15, 9, and 4 independent strength tensor components for anisotropic, orthotropic, transversely isotropic, and isotropic materials, respectively.

(2) Comparing Eq. 2-1 with Eq. 1-3, it can be seen that the proposed theory degenerates into the Tsai-Wu theory when

$$H_i = 0 \quad , \quad (i=1,\ldots,6) \quad . \tag{2-31}$$

Substitutions of Eq. 2-31 into various results pertaining to the proposed theory follow the corresponding results for Tsai and Wu's theory.

(3) In the Tsai-Wu theory, there are 27, 12, 7, and 3 independent strength tensor components for an anisotropic, orthotropic, transversely isotropic, and isotropic material, respectively.

3. BIAXIAL FAILURE CRITERION FOR COMPOSITES

To facilitate the correlations of the proposed theory with the biaxial fracture data on composites, the general results (including the explicit expressions for the failure criterion, the stability conditions on the strength tensor components, and the geometric meaning of the criterion) obtained in the last section for a general multiaxial stress state are reduced in this section for a biaxial stress state:*

$$\sigma_1 \neq 0, \quad \sigma_3 \neq 0, \quad \sigma_2 = \sigma_4 = \sigma_5 = \sigma_6 = 0 \quad , \tag{3-1}$$

which is a special case of the plane stress state:*

$$\sigma_1 \neq 0, \quad \sigma_3 \neq 0, \quad \sigma_6 \neq 0, \quad \sigma_2 = \sigma_4 = \sigma_5 = 0 \quad . \tag{3-2}$$

Moreover, to facilitate the comparison of the proposed theory with the Tsai-Wu theory in these correlations, the reduced results for orthotropic, transversely isotropic, and isotropic materials are also explicitly degenerated into those pertaining to the Tsai-Wu theory.

---

*Alternatively, in these stress states, we can use $\sigma_2 \neq 0$, $\sigma_3 = 0$ rather than $\sigma_2 = 0$, $\sigma_3 \neq 0$ without losing any generalities.

3.1 Anisotropic Material

Substituting Eq. 3-2 into Eq. 2-1, we obtain

$$
\begin{aligned}
f = {} & F_1 \sigma_1 + F_3 \sigma_3 + F_6 \sigma_6 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + F_{66} \sigma_6^2 \\
& + 2F_{13} \sigma_1 \sigma_3 + 2F_{16} \sigma_1 \sigma_6 + 2F_{36} \sigma_3 \sigma_6 \\
& + (H_1 \sigma_1 + H_3 \sigma_3 + H_6 \sigma_6) \left| H_1 \sigma_1 + H_3 \sigma_3 + H_6 \sigma_6 \right| = 1 \quad ,
\end{aligned}
\qquad (3\text{-}3)
$$

which is the failure criterion for an anisotropic material in general state of plane stress. Using Eq. 3-1, Eq. 3-3 can be simplified as

$$
\begin{aligned}
f = {} & F_1 \sigma_1 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + 2F_{13} \sigma_1 \sigma_3 \\
& + (H_1 \sigma_1 + H_3 \sigma_3) \left| H_1 \sigma_1 + H_3 \sigma_3 \right| = 1 \quad ,
\end{aligned}
\qquad (3\text{-}4)
$$

which is the desired biaxial failure criterion for an anisotropic material.

Utilizing Eq. 3-4, the stability conditions on the seven strength constants (F_1, F_3; F_{11}, F_{33}, F_{13}; H_1, H_3) appearing in Eq. 3-4 can be obtained as

$$F_{11} \pm H_1^2 > 0 \quad ,^{*}$$

$$(F_{11} \pm H_1^2)(F_{33} \pm H_3^2) - (F_{13} \pm H_1 H_3)^2 > 0 \quad . \qquad (3\text{-}5)$$

The above stability conditions pertaining to the biaxial criterion defined by

---

*Alternatively, this condition can be replaced by $F_{33} + H_3^2 > 0$.

Eq. 3-4 is a subset of the stability conditions given by Eqs. 2-7 and 2-8 for a general multiaxial stress state.

Geometrically, Eq. 3-4 with the strength constants satisfying the stability conditions given by Eq. 3-5 represents a piecewise ellipse in the biaxial stress plane. This piecewise ellipse is made of a single ellipse represented by

$$f = F_1 \sigma_1 + F_3 \sigma_3 + (F_{11} + H_1^2) \sigma_1^2 + (F_{33} + H_3^2) \sigma_3^2 + 2(F_{13} + H_1 H_3) \sigma_1 \sigma_3 = 1 \quad , \tag{3-6}$$

in the half plane

$$H_1 \sigma_1 + H_3 \sigma_3 \geq 0 \quad , \tag{3-7}$$

and another single ellipse represented by

$$f = F_1 \sigma_1 + F_3 \sigma_3 + (F_{11} - H_1^2) \sigma_1^2 + (F_{33} - H_3^2) \sigma_3^2 + 2(F_{13} - H_1 H_3) \sigma_1 \sigma_3 = 1 \quad , \tag{3-8}$$

in the half plane

$$H_1 \sigma_1 + H_3 \sigma_3 < 0 \quad . \tag{3-9}$$

3.2 Orthotropic Material

Substitution of the biaxial stress condition given by Eq. 3-1 into Eq. 2-15 leads to the biaxial failure criterion for an orthotropic material as

$$f = F_1 \sigma_1 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + 2F_{13} \sigma_1 \sigma_3 + (H_1 \sigma_1 + H_3 \sigma_3) |H_1 \sigma_1 + H_3 \sigma_3| = 1 \quad . \qquad (3\text{-}10)$$

It is interesting to note that Eq. 3-10 is identical to Eq. 3-4 for an anisotropic material. Consequently, the stability conditions on the seven strength constants (F_1, F_3,; F_{11}, F_{33}, F_{13}; H_1, H_3) appearing in Eq. 3-10 are also identical to Eq. 3-5 for an anisotropic material.

The results obtained so far in this subsection pertain only to the proposed theory. These results can be degenerated into those pertaining to Tsai and Wu's theory by utilizing the conditions:

$$H_1 = H_3 = 0 \quad , \qquad (3\text{-}12)$$

which is a subset of the conditions given by Eq. 2-31. Thus, for an orthotropic material, the biaxial failure criterion pertaining to the Tsai-Wu theory is

$$f = F_1 \sigma_1 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + 2F_{13} \sigma_1 \sigma_3 = 1 \quad , \qquad (3\text{-}13)$$

and the stability conditions on the five strength constants (F_1, F_3; F_{11}, F_{33}, F_{13}) appearing in Eq. 3-13 are

$$F_{11} > 0 \quad ,$$

$$F_{11} F_{33} - F_{13}^2 > 0 \quad . \tag{3-14}$$

As remarked earlier in Section 1, the surface represented by Eq. 3-13 with the strength constants satisfying the stability conditions given by Eq. 3-14 is a single ellipse in the biaxial stress plane.

3.3 Transversely Isotropic Material

Substitution of Eq. 3-1 into Eq. 2-22 leads to

$$f = F_1 \sigma_1 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + 2F_{13} \sigma_1 \sigma_3 + (H_1 \sigma_1 + H_3 \sigma_3) |H_1 \sigma_1 + H_3 \sigma_3| = 1 \quad , \tag{3-15}$$

which is identical to Eq. 3-10 for an orthotropic material. In view of this, other results obtained in the last subsection for an orthotropic material (i.e., the stability conditions given by Eq. 3-5 in the proposed theory and the results given by Eqs. 3-13 and 3-14 in the Tsai-Wu theory) are also applicable to a transversely isotropic material.

3.4 Isotropic Material

Substitution of the biaxial stress condition given by Eq. 3-1 into Eq.

2-29 leads to the biaxial failure criterion for an isotropic material as

$$f = F_1 (\sigma_1 + \sigma_3) + F_{11} (\sigma_1^2 + \sigma_3^2) + 2F_{12} \sigma_1 \sigma_3$$
$$+ H_1 (\sigma_1 + \sigma_3) \mid H_1 (\sigma_1 + \sigma_3) \mid = 1 \quad . \tag{3-16}$$

Using Eq. 3-16, the stability conditions on the four strength constants (F_1; F_{11}, F_{12}; H_1) appearing in Eq. 3-16 can be obtained as

$$F_{11} + F_{12} \pm 2H_1^2 > 0 \quad ,$$

$$F_{11} - F_{12} > 0 \quad , \tag{3-17}$$

which constitute only a subset of the stability conditions given by Eq. 2-30 for a general multiaxial stress state.

Substitution of Eq. 3-12 into Eqs. 3-16 and 3-17 results in the biaxial failure criterion pertaining to the Tsai-Wu theory as

$$f = F_1 (\sigma_1 + \sigma_3) + F_{11} (\sigma_1^2 + \sigma_3^2) + 2F_{12} \sigma_1 \sigma_3 = 1 \quad , \tag{3-18}$$

and the appropriate stability conditions on the three strength constants (F_1; F_{11}, F_{12}) as

$$F_{11} + F_{12} > 0 \quad ,$$

$$F_{11} - F_{12} > 0 \quad . \tag{3-19}$$

4. BIAXIAL FRACTURE DATA FOR COMPOSITES

An extensive literature review has been conducted on the biaxial fracture data on composites. Table 4-1 collects the references that contain enough biaxial data for our correlation purposes, which cover a wide spectrum of composite material systems: graphite/epoxy (Reference 13), graphite particulate (Reference 9), graphite/aluminum (Reference 14), glass/epoxy (References 15, 16, 17), and organic fiber-reinforced textolite (Reference 18). Table 4-1 also contains the secondary information on the nations where and the years when these data were reported.

These biaxial data were all obtained from tubular specimens subjected simultaneously to an axial load and internal and/or external fluid pressure, except those on graphite/aluminum unidirectional composite which were obtained from flat cruciform specimens under biaxial in-plane loadings. In presenting the data for tube specimens, the circumferential (i.e., tangential) direction will be designated as the direction for x (i.e., x_1) axis and the direction along the axis of the tube as the direction for z (i.e., x_3) axis. For the flat cruciform specimens, the fiber and its perpendicular directions will be identified as the directions for x and z axis, respectively.

4.1 Graphite/

Figure 4- illustrates the extensive experimental results for 0-deg graphite/epoxy amina, which is orthotropic Morganite II/epoxy (Reference 13).

Table 4-1. References Containing Sufficient Biaxial Data on Composites for Correlation Purposes

COMPOSITE MATERIAL SYSTEM	REFERENCE	NATION	YEAR
Graphite/Epoxy	Wu and Scheublein (Reference 13)	USA	1974
Graphite Particulate	Weng (Reference 9)	USA	1968
Graphite/Aluminum	Zimmerman and Adams (Reference 14)	USA	1984
Glass/Epoxy	Protasov and Kopnov (Reference 15)	USSR	1965
	Hotter, Schelling, and Krauss (Reference 16)	Germany	1974
	Teters, et. al. (Reference 17)	USSR	1981
Organic Textolite	Maksimov, Sokolov, and Plume (Reference 18)	USSR	1979

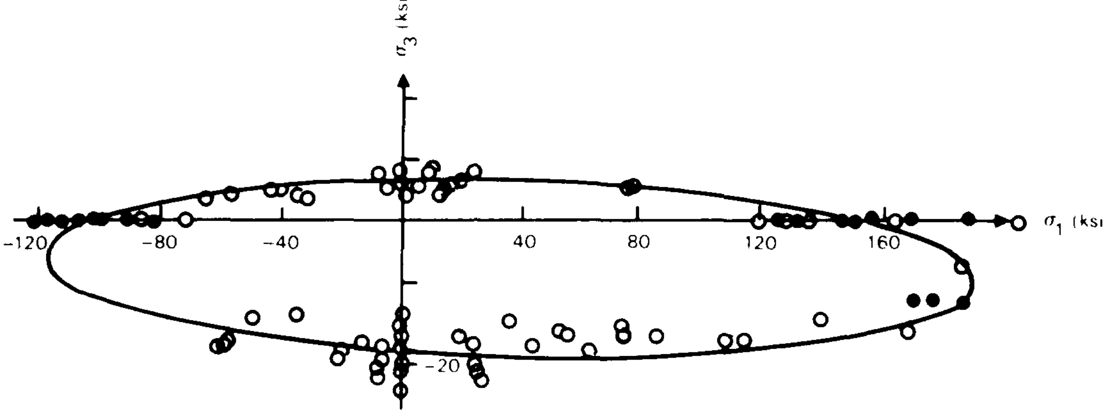


Figure 4-1. Biaxial fracture data of graphite/epoxy lamina (Morganite II) (Reference 13).

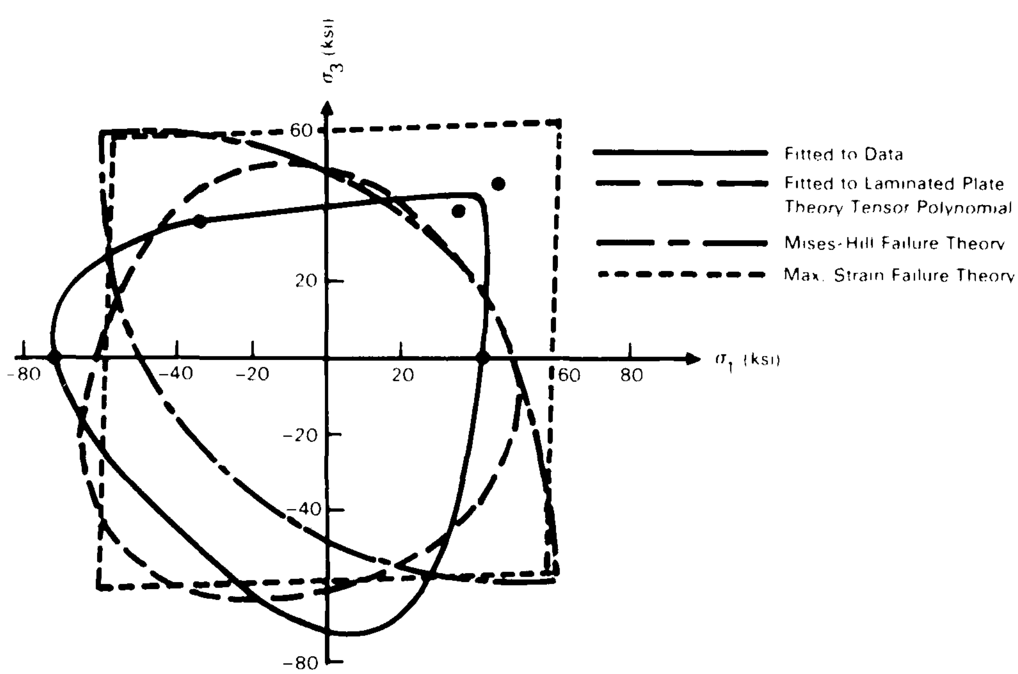


Figure 4-2. Observed and predicted biaxial strengths for $[0/90/0/90]_S$ graphite/epoxy laminate (Reference 13).

Figure 4-2 presents the observed and predicted biaxial strengths for a symmetrical cross-ply graphite/epoxy laminate with lamination geometry: 0, 90, 0, 90, 90, 0, 90, 0, or equivalently, $[0/90/0/90]_s$ (Reference 13). Due to this lamination geometry, the laminate is considered isotropic. In Figure 4-2, solid circles represent the actual experimental data, whereas open circles represent the inferred data from isotropic material symmetry. The latter data have been deemed necessary for the correlations performed in Reference 13. However, they will not be used in the next section for our correlations.

4.2 Graphite Particulate

Figure 4-3 shows the biaxial fracture strength of the JT-50 composite and their comparison with the Tsai-Wu criterion (Reference 9). The JT-series composites are graphite-based refractory composites, which are a class of particulate composite materials produced by varying the proportion of the carbon matrix and the metallic additives. Resulting from the fabrication processes, the JT-50 graphite particulate posseses transversely isotropy material symmetry. In the biaxial strength test results shown in Figure 4-3, the longitudinal axis of the test specimens was oriented parallel to the symmetry axis (i.e., the x_3 axis) of the material.

4.3 Graphite/Aluminum

Figure 4-4 presents the biaxial fracture data for a unidirectional graphite/aluminum composite, which is orthotropic (Reference 14). As mentioned earlier, these data have been generated by flat cruciform, rather

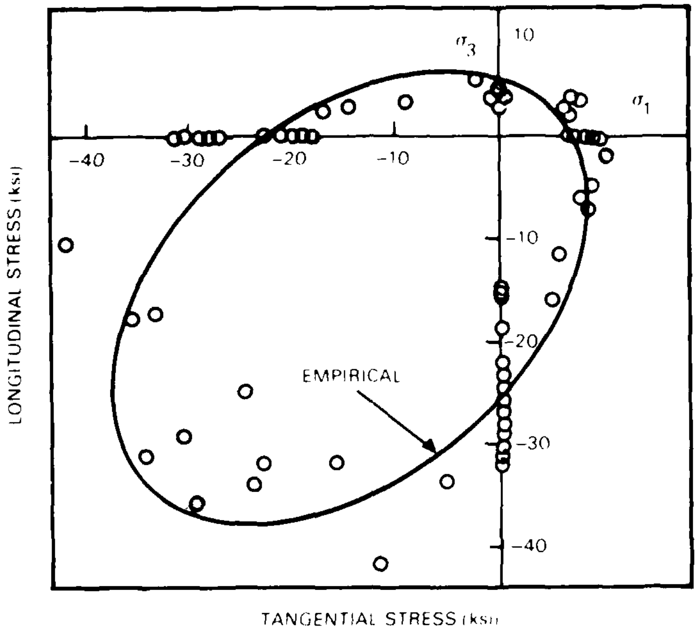


Figure 4-3. Biaxial fracture strengths for JT-50 composite material (Reference 9).

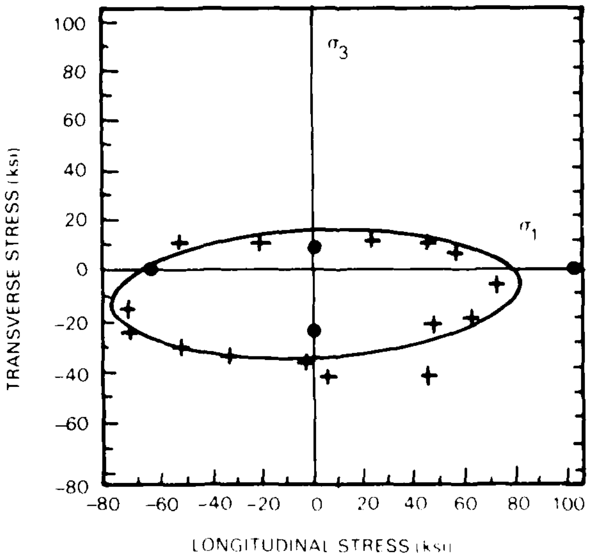


Figure 4-4. Biaxial fracture data of graphite/aluminum lamina (Reference 14).

than filament-wound tube, specimens under longitudinal (along the fiber direction) and transverse biaxial loadings.

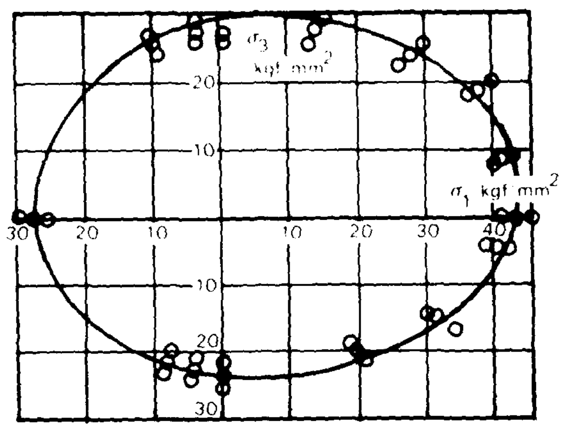
4.4 Glass/Epoxy

Presented below in this subsection are tubular tests results on several glass fiber-reinforced plastics, which are all orthotropic laminates.

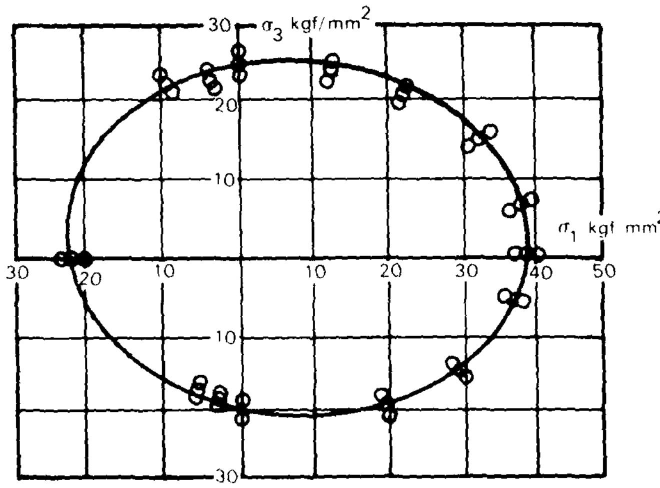
Figure 4-5 shows the biaxial fracture data of laminates made of satin- and linen-weave glass fabric and epoxy-phenolic resin (Reference 15). The fill direction coincided with the direction of the tubular test piece axis (i.e., the x_3 axis), while the warp fibers ran in the circumferential (i.e. x_1) direction.

Figure 4-6 presents the biaxial strength data for a unidirectional glass fiber reinforced laminate (Reference 16). The tube specimens used were circumferentially wound. Figure 4-7 presents the biaxial strength data for a multilayer (i.e. [90/$\pm$30/90]) glass fiber reinforced laminate (Reference 16). The low failure points for biaxial compression were possible due to buckling failure.

Figures 4-8 and 4-9 illustrate the biaxial experimental results for cross-ply and helically wound glass fiber reinforced tubular specimens, respectively (Reference 17). The layer orientation of the cross-ply tube was as follows: the first layer was oriented along the circumference (i.e., x_1 axis), the second layer along the tube axis (i.e., x_3 axis), and the third layer along the circumference. The reinforcement orientation of the helically



(a)



(b)

Figure 4-5 Biaxial fracture data of (a) satin-weave and (b) linen-weave glass-reinforced plastics (Reference 15).

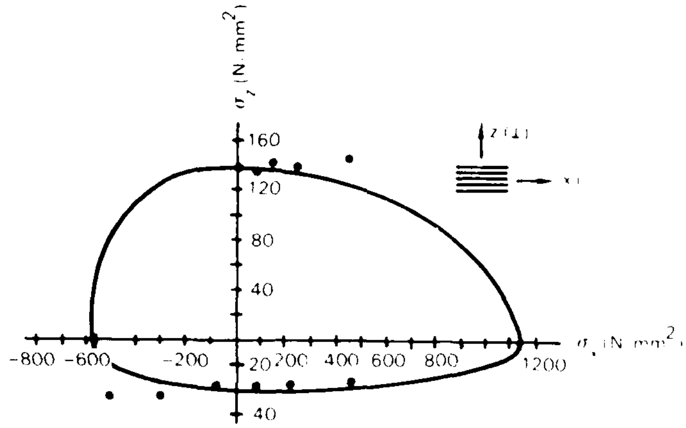


Figure 4-6. Biaxial fracture data of a unidirectional glass/epoxy laminate (Reference 16).

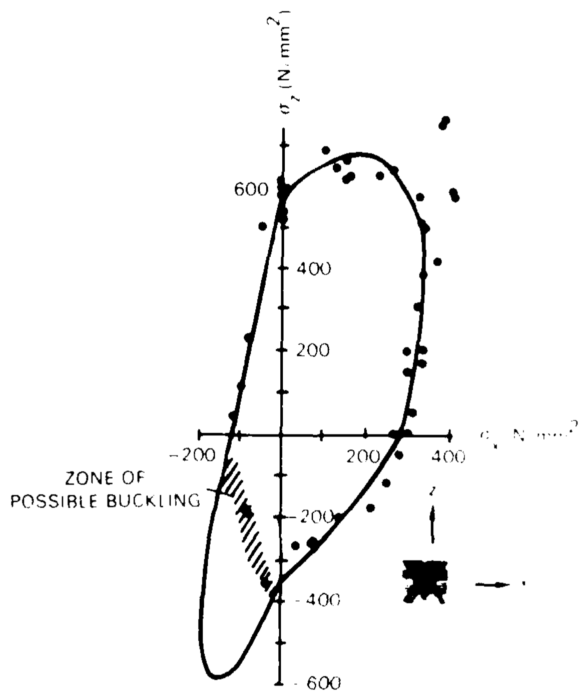


Figure 4-7. Biaxial fracture data of a [90/±30/90] glass epoxy laminate (Reference 16).

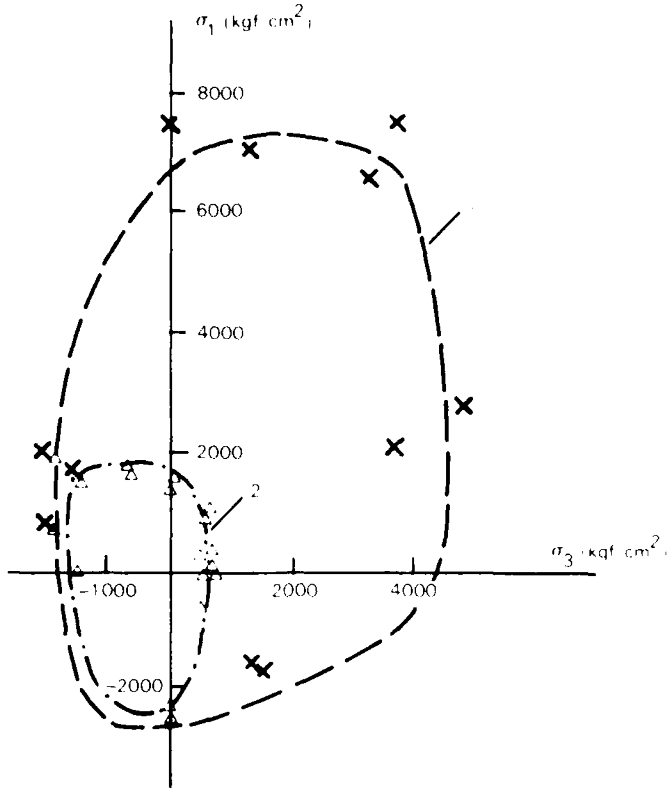


Figure 4-8. Experimental surfaces of destructive stresses (1) and stresses, at which the photoemission (2) commences for a cross-ply glass/epoxy laminate (Reference 17).

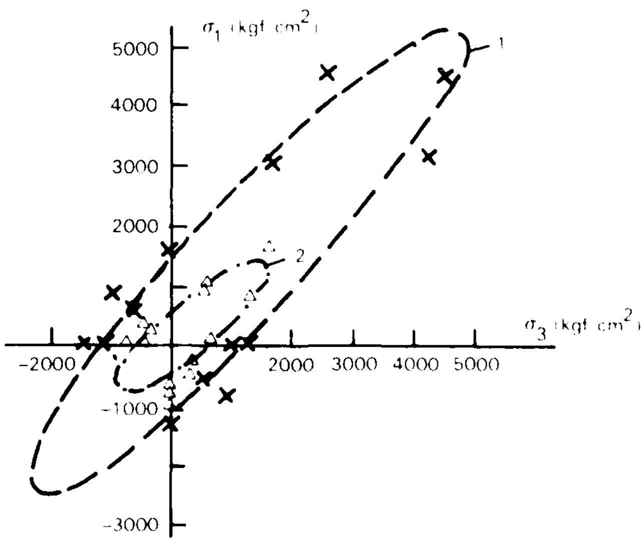


Figure 4-9. The surface of strength in an ultimate state of the material 1 (X), and the surface, describing the state of stress, in which light emission 2 (△) starts for a helically wound glass/epoxy laminate (Reference 17)

wound tube was at an angle of $\pm 45°$ to the tube axis. Stresses at both the ultimate failure and the onset of the light emission (i.e., photoemission) were presented in Figures 4-8 and 4-9. However, for our purpose, only the stresses at the ultimate failure will be considered in the next section.

4.5 Organic Textolite

Figure 4-10 shows the biaxial fracture data for an orthotropic organic textolite, as well as a glass textolite and three hybrid textolites with different values of the relative concentrations (μ'_{gf} μ'_{of}) of layers of glass and organic fabric (Reference 18). Only the organic textolite has sufficient data for our application and will be considered in the next section. The reinforcement of the organic textolite was a satin-weave fabric composed of high-modulus organic fiber yarn. The fill direction of the reinforcing fabric coincided with the longitudinal (i.e., x_3) direction of the tubular specimen, while the warp ran in the tangential (i.e., x_1) direction.

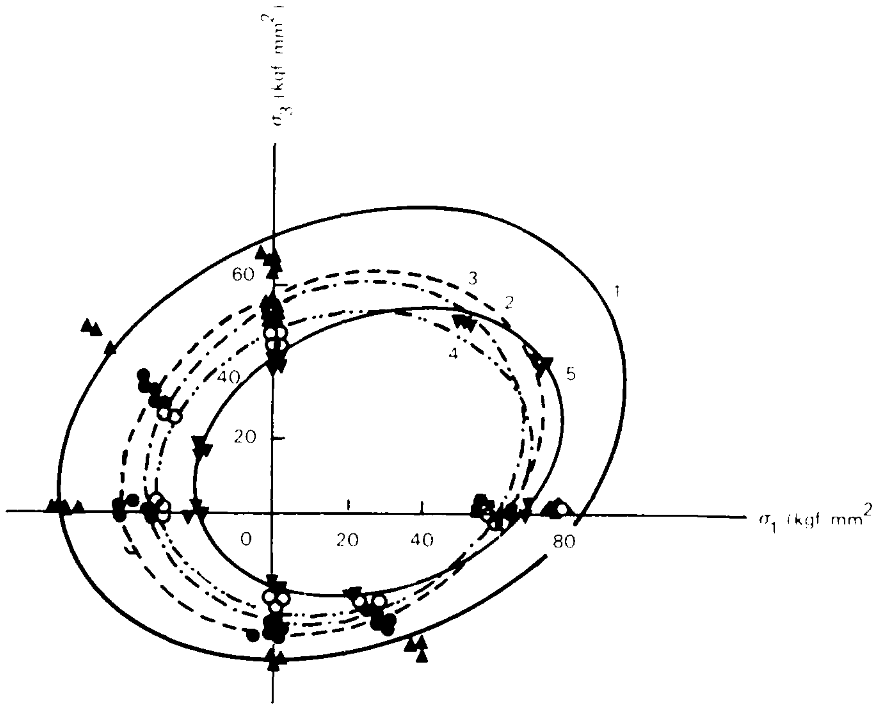


Figure 4-10. Biaxial fracture data for glass textolite (1), hybrid textolite with μ'_{gf}/μ' of = 0.7/0.3 (2), 0.5/0.5 (3), 0.3/0.7 (4), and organic textolite (5) (Reference 18).

5. CORRELATIONS OF THE THEORY WITH BIAXIAL FRACTURE DATA OF COMPOSITES

To demonstrate the applicability of the proposed piecewise quadratic strength tensor theory to composites, comparisons are made in this section between the proposed theory and the available strength data of five composite material systems: graphite/epoxy (Reference 13), graphite particulate (Reference 9), graphite/aluminum (Reference 14), glass/epoxy (References 15, 16, 17), and organic textolite (Reference 18). Also, to show the improvements of the proposed theory over the Tsai-Wu theory, comparisons are made between the two theories. In these comparisons, Eq. 3-16 of the proposed theory and Eq. 3-18 of the Tsai-Wu theory are used for $[0/90/0/90]_S$ graphite/epoxy laminate which is isotropic, while Eq. 3-10 (or the identical equation, Eq. 3-15) of the proposed theory and Eq. 3-13 of the Tsai-Wu theory are used for other composites which are either orthotropic or transversely isotropic.

Tables 5-1 and 5-2, respectively, present the strength constants* for the five composite systems least-square-fitted by the above-mentioned equations pertaining to the Tsai-Wu theory and the proposed theory. In these fittings, individual data points were used directly+ and the appropriate stability restrictions on the fitted strength constants were not violated. These restrictions included Eq. 3-17 of the proposed theory and Eq. 3-19 of the Tsai-Wu theory for the isotropic graphite/epoxy laminate and Eq. 3-5 of the proposed theory and Eq. 3-14 of the Tsai-Wu theory for other nonisotropic composites.

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*In presenting the strength constants for the isotropic graphite/epoxy laminate, the results, $F_{12} = F_{13}$, of Eq. $(2\text{-}27)_{10,11}$ have been used.

+Rather than using the mean values for the data under the same loading conditions.

Table 5-1. Fitted Strength Constants of the Tsai-Wu Criterion

CONSTANT / COMPOSITE	F_1 10^{-2} $(ksi)^{-1}$	F_3 10^{-2} $(ksi)^{-1}$	F_{11} 10^{-4} $(ksi)^{-2}$	F_{33} 10^{-4} $(ksi)^{-2}$	F_{13} 10^{-4} $(ksi)^{-2}$
Graphite/Epoxy Lamina	-0.3360	6.956	0.6132	52.43	0.1836
$[0/90/0/90]_s$ Graphite/Epoxy Laminate	0.7119	0.7119	2.751	2.751	-1.972
JT-50 Graphite Particulate	4.177	3.381	28.40	25.27	-9.849
Graphite/Aluminum Lamina	-0.0935	3.015	1.651	14.14	-0.0650
Satin-Weave Glass/Epoxy	-0.7834	-0.5236	4.027	7.699	-0.2239
Linen-Weave Glass/Epoxy	-1.319	-0.7454	5.841	10.737	0.6448
Unidirectional Glass/Epoxy Laminate	-0.2281	-10.496	0.5108	79.62	-1.950
$[90/\pm30/90]$ Glass/Epoxy Laminate	-1.382	-0.5628	9.522	2.488	-2.508
Cross-ply Glass/Epoxy Laminate	-1.213	-1.558	2.217	4.999	-0.4808
Helically Wound Glass/Epoxy Laminate	-0.8847	-0.2221	20.59	22.69	-20.03
Organic Textolite	-1.758	-2.198	6.394	3.2783	-1.107

Table 5-2. Fitted Strength Constants of the Proposed Theory

COMPOSITE \ CONSTANT	F_1 10^{-2} (ksi)$^{-1}$	F_3 10^{-2} (ksi)$^{-1}$	F_{11} 10^{-4} (ksi)$^{-2}$	F_{33} 10^{-4} (ksi)$^{-2}$	F_{13} 10^{-4} (ksi)$^{-2}$	H_1 10^{-2} (ksi)$^{-1}$	H_3 10^{-2} (ksi)$^{-1}$
Graphite/Epoxy Lamina	-0.6722	-0.6707	0.6075	85.37	2.289	0.4925	7.902
$[0/90/0/90]_s$ Graphite/Epoxy Laminate	2.068	2.068	[illegible]	2.862	-1.211	-1.373	-1.373
JT-50 Graphite Particulate	7.613	-4.124	36.58	292.6	-58.28	-3.331	17.09
Graphite/Aluminum Lamina	-0.2902	-2.631	1.430	59.84	1.515	0.3344	7.681
Satin-Weave Glass/Epoxy	0.3341	0.0232	4.727	7.626	0.8501	-1.611	-1.008
Linen-Weave Glass/Epoxy	-0.1471	-0.0011	6.785	10.845	2.164	-1.766	-1.437
Unidirectional Glass/Epoxy Laminate	-0.5559	-14.66	0.5479	72.01	-2.490	0.3539	5.188
[90/±30/90] Glass/Epoxy Laminate	1.920	-0.5295	29.016	1.996	-2.268	-5.208	0.1021
Cross-ply Glass/Epoxy Laminate	[illegible]	[illegible]	10.627	[illegible]	[illegible]	-3.231	0.5694
Helically Wound Glass/Epoxy Laminate	[illegible]	[illegible]	20.59	[illegible]	[illegible]	[illegible]	-0.9592
[illegible] Textolite	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

5.1 Graphite/Epoxy

Figures 5-1 and 5-2 present the correlations of the proposed theory and the Tsai-Wu theory with the biaxial fracture data of a graphite/epoxy lamina and a $[0/90/0/90]_S$ graphite laminate (Reference 13), respectively. For the graphite/epoxy lamina, both theories correlate equally well with the biaxial data due to the elliptical characteristics of the data. For the graphite/epoxy laminate, the proposed theory correlates better than the Tsai-Wu theory with the data and predicts significantly different results, at least in the third (i.e., compression-compression) stress quadrant, from the Tai-Wu theory.

5.2 Graphite Particulate

Figure 5-3 presents the correlations* of the proposed theory and the Tsai-Wu theory with the biaxial fracture data of JT-50 composite material (Reference 9). Significant improvements of the proposed theory over Tsai and Wu's theory can be seen in the correlations.

5.3 Graphite/Aluminum

Figure 5-4 compares the predictions of the proposed theory and the Tsai-Wu criterion with the biaxial fracture data of a graphite/aluminum lamina (Reference 14). Significant improvements of the proposed theory over the Tsai-Wu theory can be found in the comparisons.

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*Similar correlations have been reported in Reference 8, where mean data, rather than individual data, were used to fit the proposed criterion.

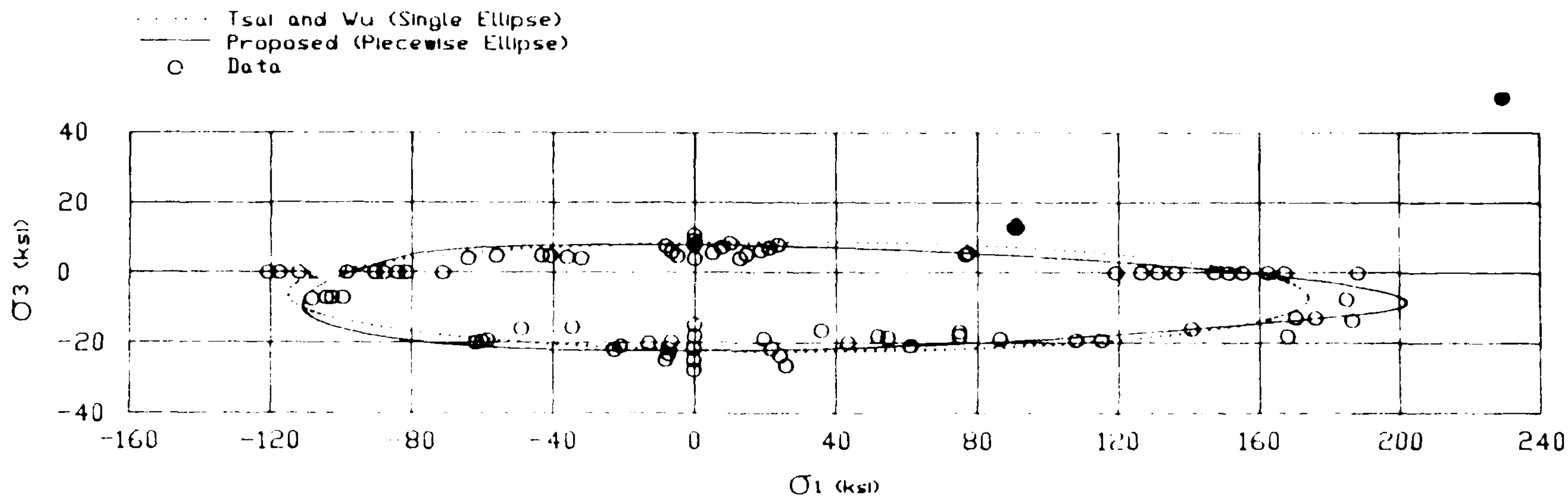


Figure 5 1 Correlations of the quadratic strength tensor theories with the biaxial fracture data of a graphite epoxy lamina (Reference 13)

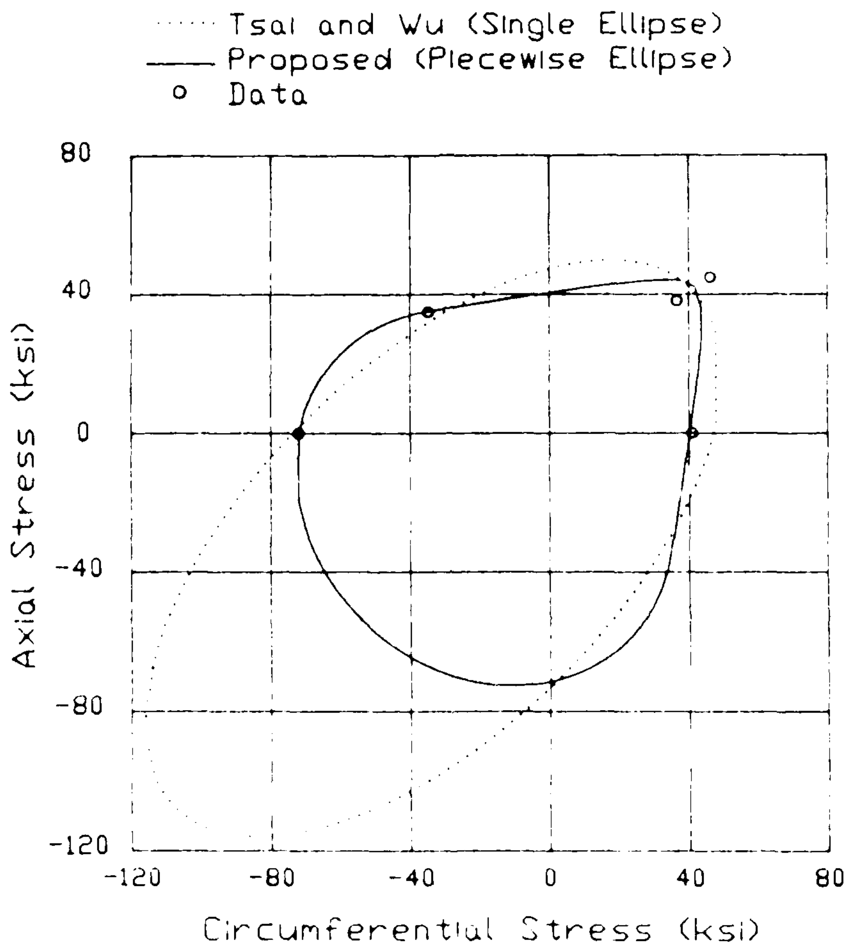


Figure 5-2 Correlations of the quadratic strength tensor theories with the biaxial fracture data of a (0 90 0 90)s graphite epoxy laminate (Reference 13)

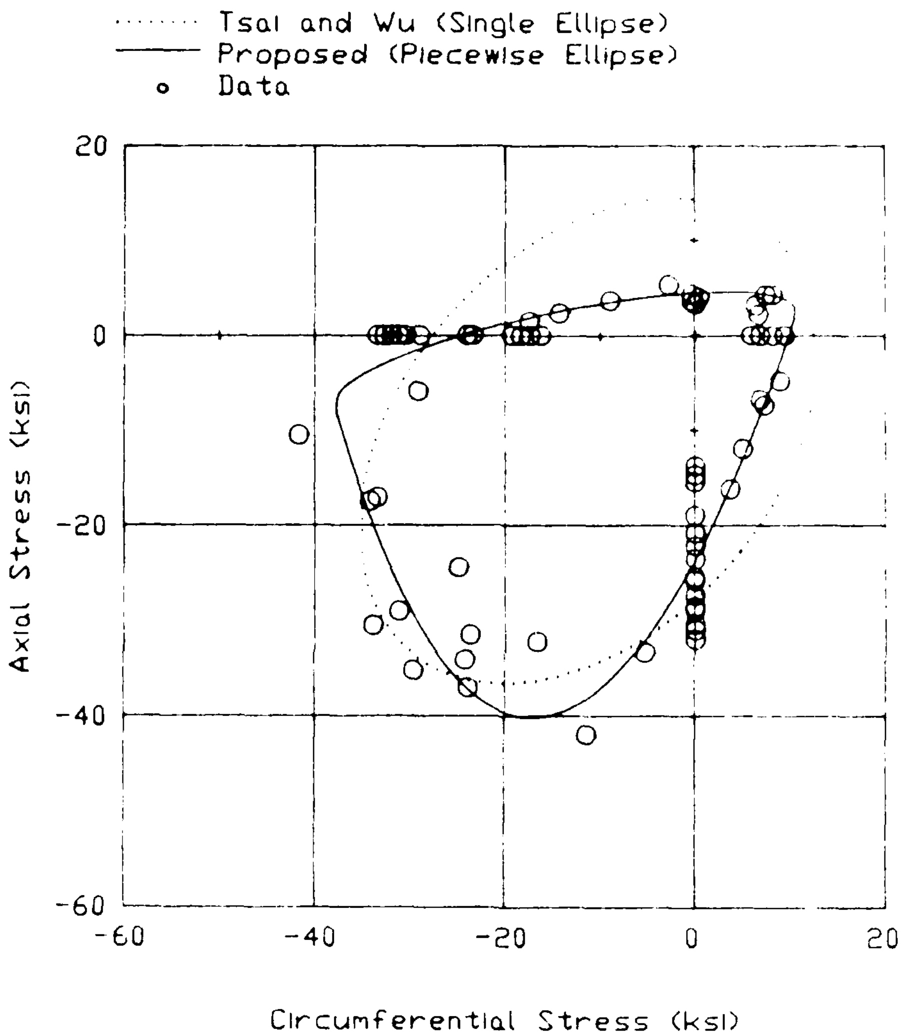


Figure 5-3 Correlations of the quadratic strength tensor theories with the biaxial fracture data of JT-50 composite material (Reference 9).

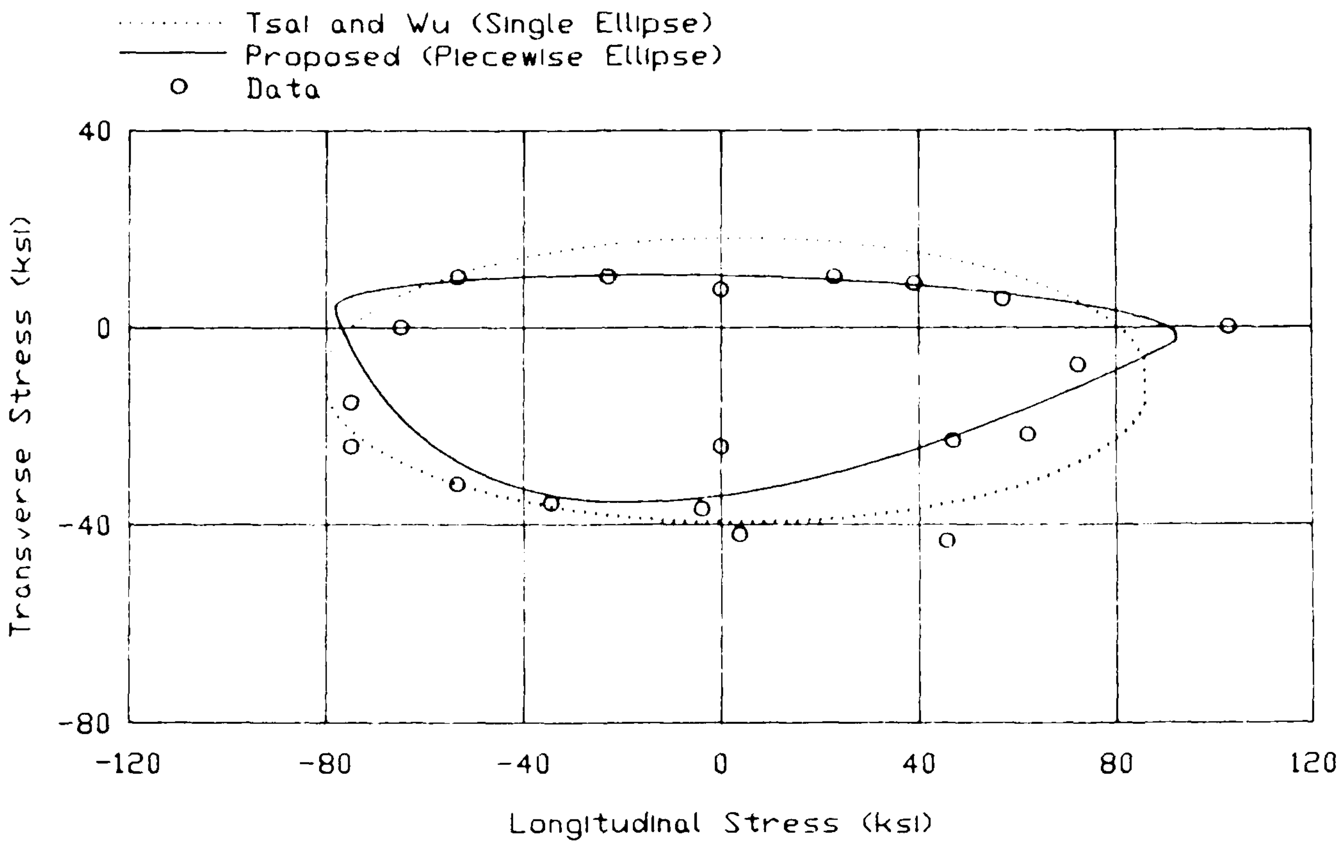


Figure 5-4. Correlations of the quadratic strength tensor theories with the biaxial fracture data of a graphite aluminum lamina (Reference 14).

5.4 Glass/Epoxy

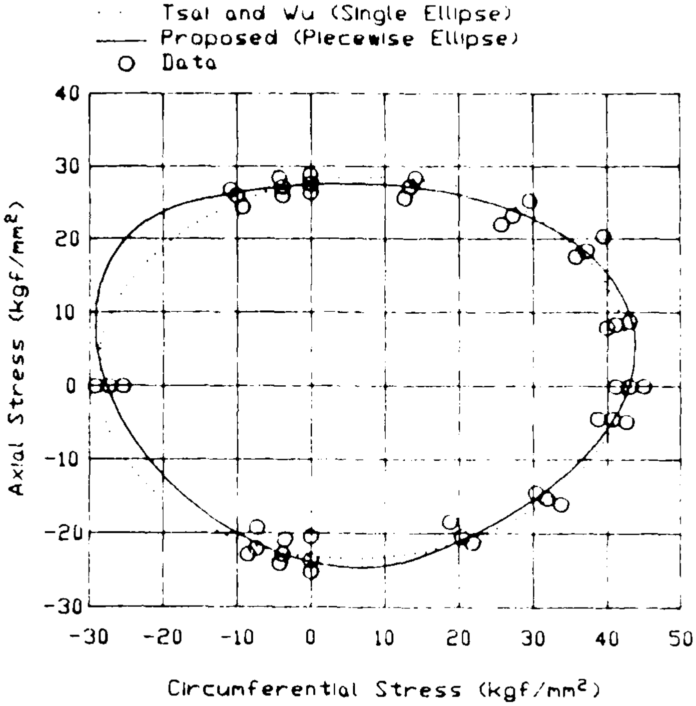
Figure 5-5 compares the predictions of the proposed and the Tsai-Wu theories with the biaxial fracture data of satin-weave and linen-weave glass/epoxy (Reference 15). Both theories compare equally well with the data due to the elliptical natures of the data.

Figure 5-6 presents the correlations of the proposed and the Tsai-Wu theories with the biaxial fracture data of a unidirectional glass/epoxy laminate (Reference 16). Both theories correlate almost equally well due to the elliptical characteristics of the data.

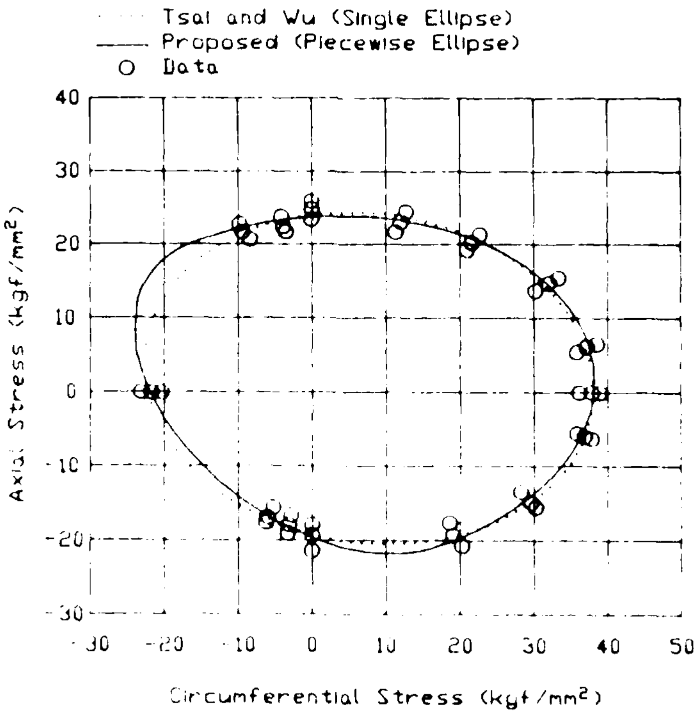
Figure 5-7 presents the correlations of the proposed and the Tsai-Wu theories with the biaxial fracture data of a [90/±30/90] glass/epoxy laminate (Reference 16). Significant improvements of the proposed theory over the Tsai-Wu theory can be seen in the correlations.

Figure 5-8 presents the comparisons of the proposed theory and the Tsai-Wu criterion with the biaxial fracture data of a cross-ply glass/epoxy laminate (Reference 17). Again, significant improvements of the proposed theory over the Tsai-Wu criterion can be found in the comparisons.

Figure 5-9 shows the correlations of the proposed and the Tsai-Wu theories with the biaxial data of a helically wound glass/epoxy laminate (Reference 17). The two theories correlate equally well with the data.



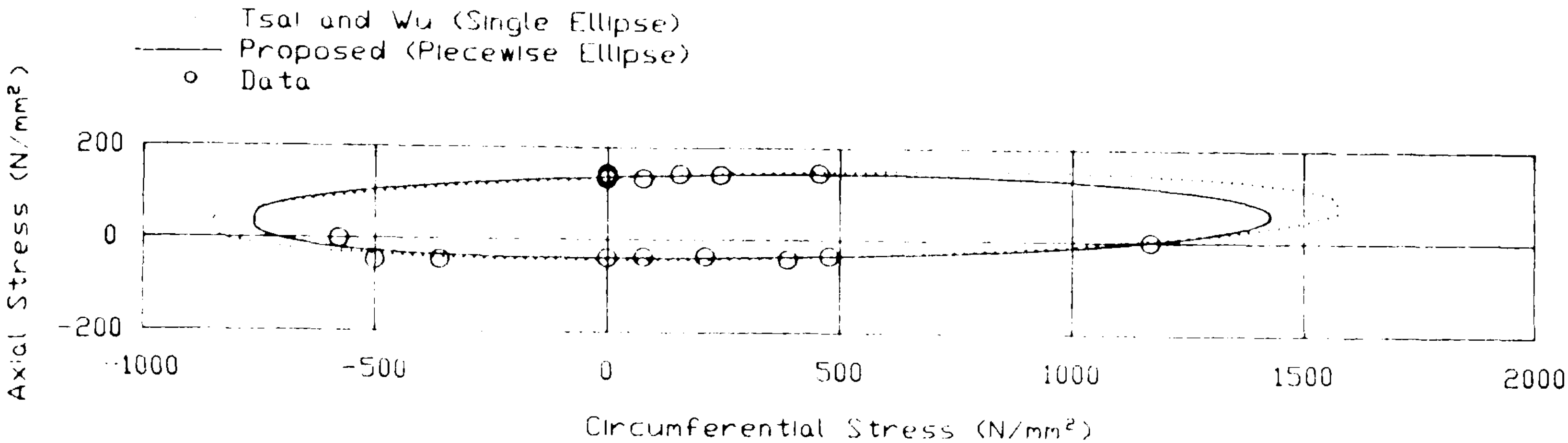
(a)



(b)

Note: 1 kgf/mm² = 1.422 ksi

Figure 5-5 Correlations of the quadratic strength tensor theories with the biaxial fracture data of (a) satin-weave and (b) linen-weave glass-reinforced plastics (Reference 15)



Note: 1 N/mm^2 = 1.450×10^{-1} ksi

Figure 5.6 Correlations of the quadratic strength tensor theories with the biaxial fracture data of a unidirectional glass epoxy laminate (Reference 16)

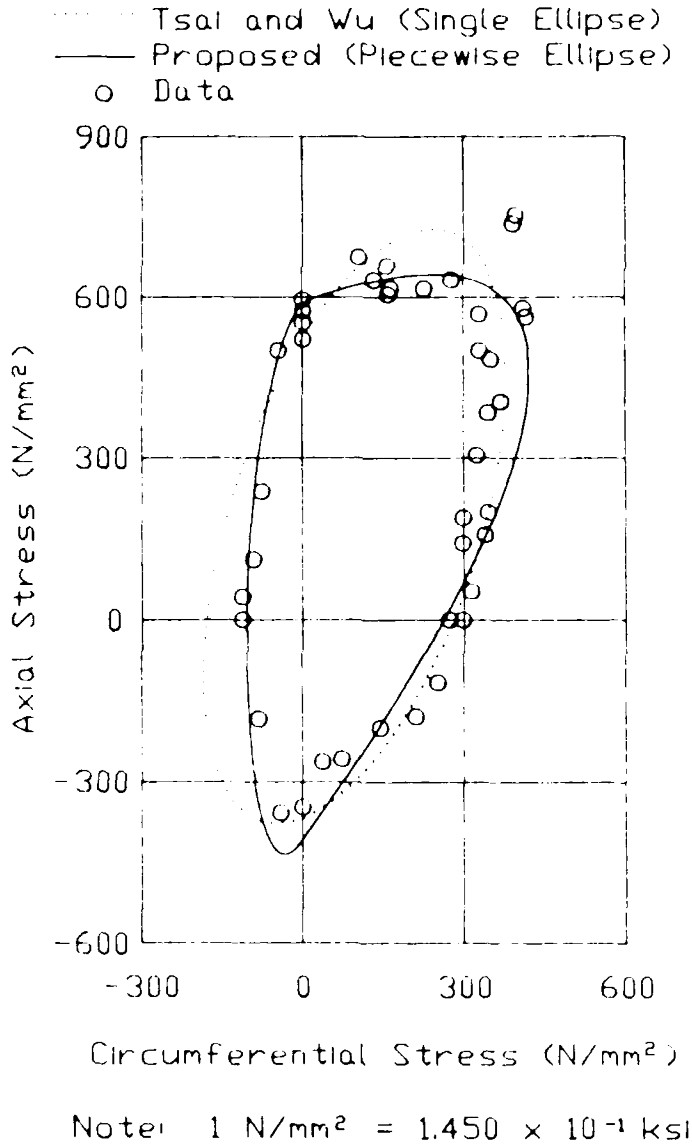
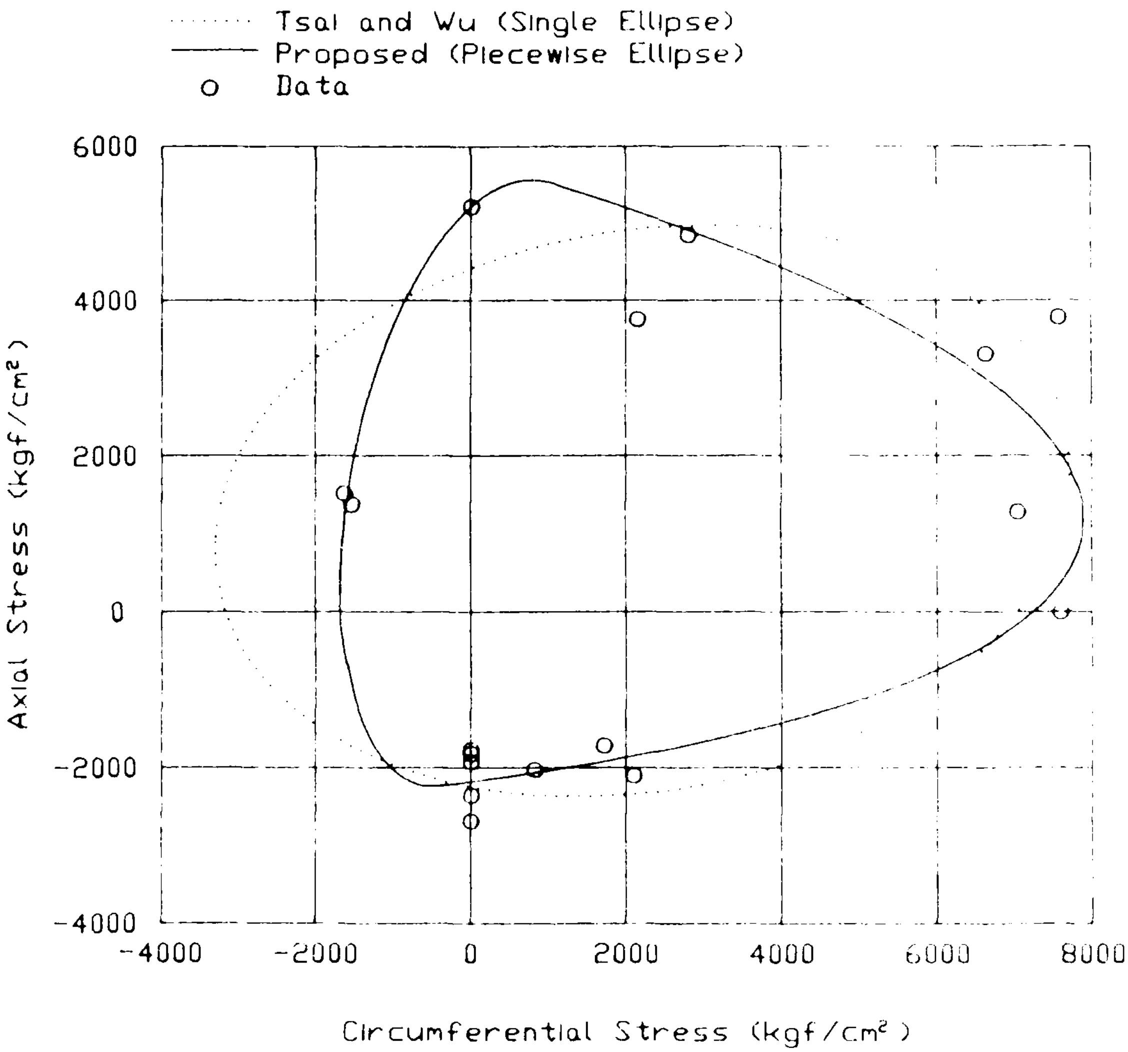


Figure 5-7 Correlations of the quadratic strength tensor theories with the biaxial fracture data of a (90/±30/90) glass/epoxy laminate (Reference 16)



Note: 1 kgf/cm^2 = 1.422 × 10^{-2} ksi

Figure 5-8 Correlations of the quadratic strength tensor theories with the biaxial fracture data of a cross-ply glass/epoxy laminate (Reference 17)

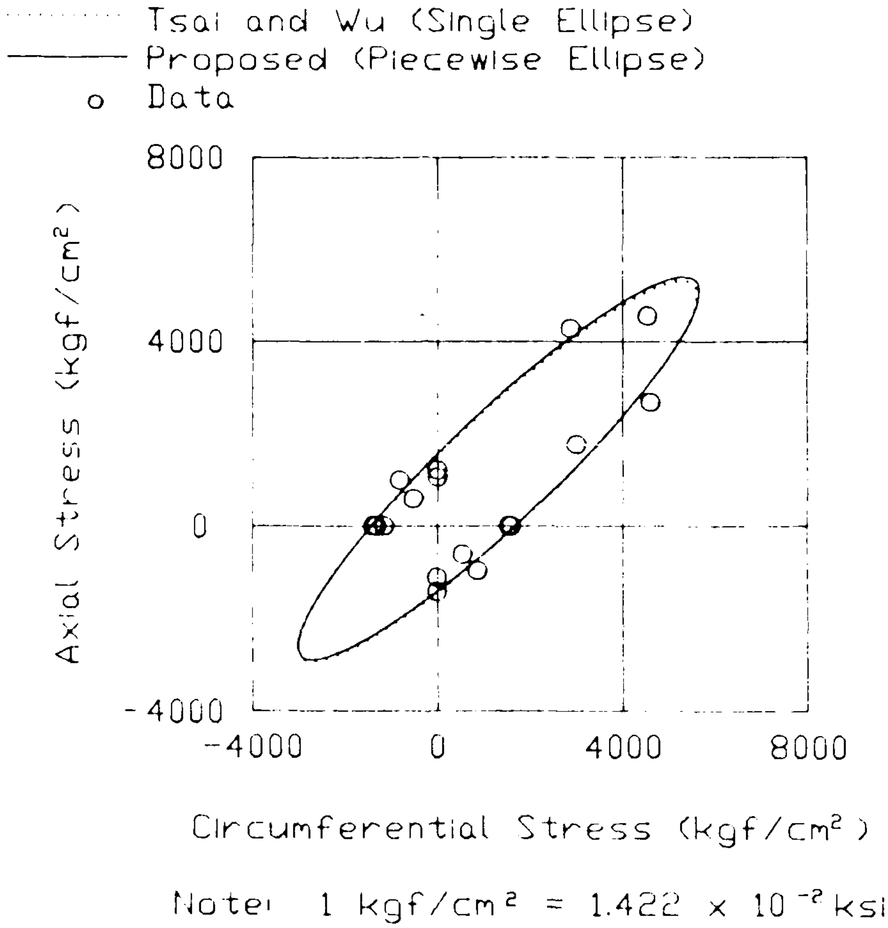
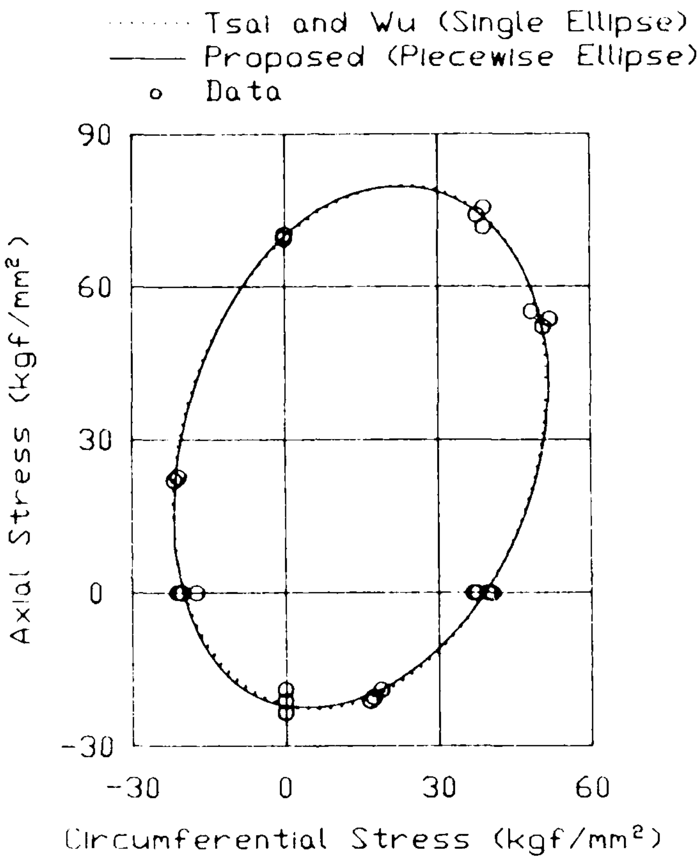


Figure 5-9. Correlations of the quadratic strength tensor theories with the biaxial fracture data of a helically wound glass/epoxy laminate (Reference 17)

5.5 Organic Textolite

Figure 5-10 shows the comparisons of the proposed and the Tsai-Wu theories with the biaxial fracture data of an organic textolite (Reference 18). Both theories correlate equally well with the biaxial data due to the elliptical characteristics of the data.



Note: 1 kgf/mm² = 1.422 ksi

Figure 5-10 Correlations of the quadratic strength tensor theories with the biaxial fracture data of an organic textolite (Reference 18).

6. CONCLUSIONS

Good correlations between the theory and the biaxial fracture data have been demonstrated for all five composite material systems: graphite epoxy, graphite particulate, graphite aluminum, glass epoxy, and organic textolite. Furthermore, significant improvements of the proposed theory over the Tsai-Wu theory have been shown for the cases where the biaxial data possesses nonelliptical characteristics. From these results, the following conclusions are reached:

1. The proposed piecewise quadratic strength tensor theory is applicable to the composites.

2. The proposed theory can significantly improve Tsai-Wu's quadratic strength tensor theory for composite applications.

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