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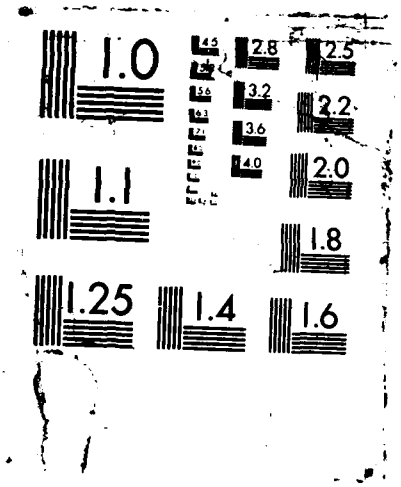
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DERIVATIVE ARRAYS, GEOMETRIC CONTROL THEORY, AND REALIZATIONS OF LINEAR DESCRIPTOR SYSTEMS

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Abstract

The relationship between numerical methods for realizations of $E(t)x'(t) + F(t)x(t) = f(t)$ based on derivative arrays and geometric control realization procedures based on Lie derivatives is examined.

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1 Introduction

The linear time varying descriptor system

$$E(t)x'(t) + F(t)x(t) = f(t) \tag{1}$$

plays an important role in the growing understanding of descriptor systems. It is a bridge between the well understood linear time invariant problem

$$Ex'(t) + Fx(t) = f(t) \tag{2}$$

and the general nonlinear implicit differential equation

$$G(x', x, t) = 0 \tag{3}$$

Many of the difficulties that separate (2) from (3) already occur in the linear time varying case (1). On the other hand, the linearity of (1) makes possible a more complete analysis. Understanding developed from analyzing (1) has proved useful in developing numerical and analytic techniques for (3) [6].

The first general numerical method for (1) is developed in [2],[3]. This method is based on working with arrays of derivatives of E, F, f. It has the advantage that all differentiations are performed directly on the coefficients of (1) without any time varying coordinate changes. Theoretical characterizations of solvability [4] for (1) and a numerical procedure for computing a state space realization [5] have come out of this approach.

Simultaneously, there has been progress in studying implicit differential equations using the ideas of geometric control theory. In particular, [11] considers realizations utilizing Lie derivatives.

Intuitively, there is a close connection between realizations and numerical methods. Many numerical ordinary differential equation (ODE) integrators are based on being able to estimate x' given x, t, whereas a realization is often in the form of an ordinary differential equation $x' = Q(x, t)$.

This paper is the beginning of our effort to unify these two approaches. There are several potential benefits to this effort. Hopefully, this will make the tools of geometric control theory available in trying to develop and analyze numerical methods for (3). Conversely, the results from the numerical theory will suggest better ways to compute the geometric objects. Also, as will be shown in this paper using ideas from the numerical theory, we

will be able to extend some of the geometric theory thereby obtaining new theoretical results.

Section 2 will quickly summarize the geometric method of realization given in [11]. Section 3 summarizes the key ideas from [4]. In Section 4, we develop the relationship between the ideas in Sections 2 and 3 and establish extensions to the theory. Finally, Section 5 contains concluding remarks and discussion.

2 Geometric Realizations

This section will give an outline of part of the method of realizations for (1) developed in [11]. An exposition of the underlying nonlinear control theory can be found in [9].

Consider a nonlinear state system, $x \in \mathcal{R}^n$;

$$x' = g_0(x) + \sum_{j=1}^{m} u_j g_j(x) \tag{4}$$

A submanifold $\mathcal{N} \subset \mathcal{R}^n$ is controlled invariant if there exists a smooth feedback $u_j = \alpha_j(x)$ such that when applied to (4) it makes the vector field for (4) tangent to $\mathcal{N}$. That is, solutions starting in $\mathcal{N}$, stay in $\mathcal{N}$. For a scalar function $T(x) = T(x_1, \ldots, x_n)$ from $\mathcal{R}^n$ to $\mathcal{R}$, and vector valued function $s(x)$ from $\mathcal{R}^n$ to $\mathcal{R}^n$, let L_sT be the derivative of T along the differential equation $x' = s(x)$. Thus

$$L_sT = \frac{d}{dt}T = \sum_{i=1}^{n} \frac{\partial T}{\partial x_i}\frac{dx_i}{dt} = \sum_{i=1}^{n} \frac{\partial T}{\partial x_i}s_i(x) = \nabla T \cdot s(x)$$

Notice that L_sT is again a scalar valued function of x so that $L_s^2T = L_s(L_sT)$ is well defined and similarly L_s^rT is well defined for nonnegative integers r.

We are interested in the case when (4) satisfies constraints

$$H(x) = 0 \tag{5}$$

where H has values in $\mathcal{R}^p$. As in [11] consider p output equations

$$y = H(x) \tag{6}$$

Let $y_i = H_i(x)$ be the ith equation in (6). For $i = 1, \ldots, p$, let ρ_i be the smallest integer ρ such that $L_{g_j} L_{g_0}^{\rho} H_i(x) \not\equiv 0$ for some x and some $j \in \{1, \ldots, m\}$ Assume, as in [11], that ρ_i exists for $i = 1, \ldots, p$. Now let the $p \times m$ matrix $A(x)$ be defined by

$$A_{ij} = L_{g_j} L_{g_0}^{\rho_i} H_i(x) \tag{7}$$

The next assumption in [11] is that A has constant rank p. Then the maximal controlled invariant submanifold of $H(x) = 0$ exists and is given by

$$\mathcal{N}^* = \{x \mid L_{g_0}^k H_i(x) = 0,\ k = 0, \ldots, \rho_i,\ i = 1, \ldots, p\} \tag{8}$$

Furthermore the needed feedback is computed as follows. Let $\alpha(x)$ satisfy

$$A(x)\alpha(x) + b(x) = \gamma(x) \tag{9}$$

where

$$b_i(x) = L_{g_0}^{\rho_i + 1} H_i(x) \tag{10}$$

and γ is a vector of functions which are zero on $\mathcal{N}^*$. We shall take $\gamma = 0$. Let $\beta(x)$ be $m \times (m-p)$ with $\operatorname{rank}(\beta) = m - p$ such that $A\beta = 0$. Then the feedback law $u = \alpha + \beta v$ applied to (4) gives a control system with $m - p$ inputs v which leaves $\mathcal{N}^*$ invariant and thus can be reduced to a control system on $\mathcal{N}^*$:

$$x' = g_0(x) + \sum_{j=1}^{m} \alpha_j(x) g_j(x) + \sum_{k=1}^{m-p} v_k \left(\sum_{j=1}^{m} \beta_{jk}(x) g_j(x) \right) \tag{11}$$

Notice that this theory provides two key objects. The manifold $\mathcal{N}^*$ consists of solutions of (4) that satisfy the constraints $H(x) = 0$ for some control u while (11) gives a control system, defined on the whole space, which leaves $\mathcal{N}^*$ invariant and on $\mathcal{N}^*$ agrees with the solutions of (4),(5).

This theory is applied to implicit differential equations in [11] as follows. Suppose that we have the kth order descriptor system,

$$R_i[w, w', \ldots, w^{(k)}] = 0, \quad w \in \mathcal{R}^n, \quad i = 1, \ldots, \ell \tag{12}$$

This is rewritten as

$$\frac{d}{dt}\begin{bmatrix} w \\ w' \\ \cdot \\ \cdot \\ w^{(k)} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & \cdot & 0 \\ \cdot & \cdot & * & \cdot & \cdot \\ \cdot & \cdot & \cdot & * & 0 \\ \cdot & \cdot & \cdot & 0 & I \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{bmatrix} w \\ w' \\ \cdot \\ \cdot \\ w^{(k)} \end{bmatrix} + \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ I \end{bmatrix} \bar{v} \tag{13}$$

with output

$$z = R[w, w', \ldots, w^{(k)}] \tag{14}$$

Here the derivatives of w are considered to be variables. Note that w is a solution of (12) if and only if there is a $\bar{v}$ so that the solution of (13) makes the output (14) zero. Writing (13),(14) as (4),(6) using $x = (w, w', \ldots, w^{(k)})$, we have

$$g_0(x) = \begin{bmatrix} w' \\ \vdots \\ w^{(k)} \\ 0 \end{bmatrix}, \quad g_j(x) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ e_j \end{bmatrix} \tag{15}$$

where $\{e_j\}$ is the standard basis for $\mathcal{R}^n$, and

$$H(x) = R[w, w', \ldots, w^{(k)}] \tag{16}$$

Then

$$A(x) = \frac{\partial R_i}{\partial w_j^{k-\rho_i}}(x) \tag{17}$$

Assuming that the rank of $A(x)$ is ℓ, there then exists a feedback $u = \alpha(x) + \beta(x)v$ so that $x' = g_0(x) + \sum u_j g_j(x)$ leaves $\mathcal{N}^*$ controlled invariant and hence will generate a realization in $\mathcal{N}^*$ coordinates.

We shall illustrate these ideas with an example of a simple singular system. This example will be referred to again later.

Example 1 Consider (2) with

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

so that

$$\begin{aligned} x_2' &= x_1 + u_1 \\ 0 &= x_2 + u_2 \end{aligned}$$

and $f = u$. Letting $w = [x, u] = [x_1, x_2, u_1, u_2]$, $w' = [x', u']$ we get that (13),(14) are

$$\frac{d}{dt}\begin{bmatrix} w \\ w' \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} w \\ w' \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} v \tag{18}$$

and

$$R_1(w, w') = x_2' - x_2 - u_1 \tag{19}$$

$$R_2(w, w') = x_2 + u_2 \tag{20}$$

Also

$$g_0(w, w') = \begin{bmatrix} w' \\ 0 \end{bmatrix}, \quad g_j = \begin{bmatrix} 0 \\ e_j \end{bmatrix}, \quad j = 1, \ldots, 4$$

To simplify our notation, let 0_r be an r dimensional zero vector and let ; separate the blocks of n entries. Then

$$L_{g_j} L^0_{g_0} R_1 = L_{g_j} R_1 = [-1\ 0\ -1\ 0; 0\ 1\ 0\ 0][0_4; e_j^T]^T = \delta_{2j}$$

Hence $\rho_1 = 0$ since $L_{g_2} R_1 = 1 \neq 0$. However,

$$L_{g_j} R_2 = [0\ 1\ 0\ 1; 0_4][0_4; e_j^T]^T = 0$$

and

$$L_{g_0} R_2 = [0\ 1\ 0\ 1; 0_4][x_1', x_2', u_1', u_2'; 0_4]^T = x_2' + u_2'$$

Hence

$$L_{g_j} L_{g_0} R_2 = [0_4; 0\ 1\ 0\ 1][0; e_j^T]^T$$

is 1 if $j = 2, 4$ and zero otherwise. Hence $\rho_2 = 1$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Furthermore $\mathcal{N}^*$ is given by

$$\begin{aligned} P_1 &= x_2' - x_1 - u_1 = 0 \\ P_2 &= x_2 + u_2 = 0 \\ L_{g_j} P_2 &= x_2' + u_2' = 0 \end{aligned} \tag{21}$$

or equivalently,

$$x_1 = -u_1 - u_2' \tag{22}$$
$$x_2 = -u_2 \tag{23}$$

which is exactly the solution manifold of (2) for this example. Continuing, we compute that

$$b = \begin{bmatrix} -x_1' - u_1' \\ 0 \end{bmatrix}$$

Let $\gamma = 0$. Then one solution for α, β is

$$\alpha = \begin{bmatrix} 0 \\ x_1' + u_1' \\ 0 \\ -x_1' - u_1' \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Applying the feedback $v = \alpha + \beta\bar{v}$ gives the explicit control system (11) which after deleting the redundant equations is

$$\begin{aligned} x_1'' &= \bar{v}_1 \\ x_2'' &= x_1' + u_1' \\ u_1'' &= \bar{v}_2 \\ u_2'' &= -x_1' - u_1' \end{aligned} \tag{24}$$

This explicit system, for the correct choices of $\bar{v}_1, \bar{v}_2$ includes the solutions (22),(23) of the original implicit descriptor system if the initial conditions satisfy the $\mathcal{N}^*$ conditions (21).

3 Derivative Arrays

As in [4] we assume that E, F, f are infinitely differentiable to avoid technical difficulties. The system (1) is solvable on an interval $\mathcal{I}$ if for every f, there is a solution x defined on all of $\mathcal{I}$ and for a given f, solutions are uniquely determined by their value at any $t_0 \in \mathcal{I}$. See [4] for a more careful

definition. Let $c^{[i]} = \frac{c^{(i)}(t)}{i!}$ for $c = E, F, f, x$. Then for any $j > 0$;

$$\begin{bmatrix} E^{[0]} & 0 & \cdot & 0 \\ E^{[1]} + F^{[0]} & 2E^{[0]} & \cdot & \cdot \\ * & * & * & 0 \\ E^{[j-1]} + F^{[j-2]} & 2E^{[j-2]} + F^{[j-3]} & * & jE^{[0]} \end{bmatrix} \begin{bmatrix} x^{[1]} \\ \cdot \\ \cdot \\ x^{[j]} \end{bmatrix}$$

$$= \begin{bmatrix} f^{[0]} \\ \cdot \\ \cdot \\ f^{[j-1]} \end{bmatrix} - \begin{bmatrix} F^{[0]} \\ \cdot \\ \cdot \\ F^{[j-1]} \end{bmatrix} x^{[0]} \tag{25}$$

where $x^{[0]} = x$. We rewrite (25) as

$$\mathcal{E}_j \mathbf{x}_j = \mathbf{f}_j - \mathcal{F}_j x^{[0]} \tag{26}$$

The matrix $\mathcal{E}_j$ is said to be 1-full if there is a nonsingular matrix Θ such that

$$\Theta \mathcal{E}_j = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}$$

where I is $n \times n$. From [4] we have the following fundamental result;

Theorem 3.1 *The system (1) with E,F infinitely differentiable, is solvable if and only if there is a j such that*

(i) *$\mathcal{E}_j$ has constant rank on $\mathcal{I}$*

(ii) *$\mathcal{E}_j$ is 1-full for every $t \in \mathcal{I}$*

(iii) *$\mathcal{R}(\mathcal{E}_i) + \mathcal{R}(\mathcal{F}_i) = \mathcal{C}^n$ for all $t \in \mathcal{I}$, $1 \leq i \leq j$.*

Furthermore, if (i)-(iii) hold for j_0, then they hold for any $j > j_0$. Also, $j = n + 1$ satisfies (i)-(iii).

The result we shall need from [3],[4] is;

Theorem 3.2 *Suppose that (1) is solvable and j satisfies (i)-(iii) of Theorem 1. Then the manifold of consistent initial conditions at time t is characterized by*

$$\mathcal{E}_j(t)\mathbf{x}_j = \mathbf{f}_j(t) - \mathcal{F}_j(t)x(t) \tag{27}$$

is consistent. That is, $\mathbf{f}_j - \mathcal{F}_j x \in \mathcal{R}(\mathcal{E}_j)$.

A numerical method utilizing these Theorems is developed in [3]. But for our purposes here, it suffices to recall that we can multiply (26) by a nonsingular Θ to get

$$\begin{bmatrix} I_n & 0 & 0 \\ 0 & \hat{M} & M \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}_j = \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix} - \begin{bmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{bmatrix} x \tag{28}$$

The first row of (28) is

$$x' = -Q_1(t)x + q_1(t) \tag{29}$$

which is an explicit ordinary differential equation whose solutions include the solutions of the original implicit differential equation (1). The third row of (28),

$$Q_3(t)x = q_3(t) \tag{30}$$

characterizes the solution manifold.

4 Discussion

We begin our discussion by considering the simple solvable system in Example 2.

Example 2. In (2), let

$$E = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If the procedure in Section 2 is applied, we get $\rho_1 = \rho_2 = 0$ and

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

which does not have full row rank.

Example 2 shows that the procedure of Section 2 can experience difficulty with solvable systems. Of course, for this example, a constant coordinate change would convert it to Example 1. However, the general case

is much more complicated. There may not exist any smooth coordinate changes that put a solvable system into a nice structural form. Even if there are such coordinate changes, they will be time varying, even nonlinear in the case of (4), and thus sometimes difficult to compute. In general then, if time varying coordinate changes are not computed and the method of Section 2 is being used, we would expect the matrix A to have varying rank or be rank deficient, unless the system (1) has a well defined structural index.[8] This does happen with some systems of physical importance, such as some of those in mechanics [1],[7],[10] but is not a general characteristic of solvable systems.

The approach of Section 3, like that of Section 2, produces a description of the solution manifold (30), and a differential equation defined on all of $\mathcal{R}^n$ (29) whose solutions include those of the singular system. However, in Section 3, no attempt is made to take the minimum number of differentiations. Rather a sufficient number are taken. Then all needed algebra is performed pointwise so that it is never necessary to differentiate computed quantities which is numerically unstable.

In this section, we shall discuss the adaptation of these ideas to the approach of Section 2. Our long range goal is two fold. First, we want to extend the theory of Section 2. Secondly, we wish to utilize the insight gained from the method of Section 2, in order to extend the approach of Section 3 to general nonlinear descriptor systems. Any approach which generates an explicit differential equation from an implicit one is potentially useful in making possible the application of explicit differential equation numerical integration schemes to implicit differential equations.

Consider again the linear time varying singular system (1). Differentiating the equation $j-1$ times yields

$$\begin{bmatrix} E & 0 & \cdot & \cdot & 0 \\ E'+F & E & \cdot & \cdot & \cdot \\ E''+2F' & 2E'+F & E & \cdot & \cdot \\ * & * & * & * & 0 \\ E^{(j-1)}+(j-1)F^{(j-2)} & * & * & * & E \end{bmatrix} \begin{bmatrix} x' \\ x'' \\ \cdot \\ \cdot \\ x^{(j)} \end{bmatrix}$$

$$= -\begin{bmatrix} F \\ F' \\ \cdot \\ \cdot \\ F^{(j-1)} \end{bmatrix} x + \begin{bmatrix} f \\ f' \\ \cdot \\ \cdot \\ f^{(j-1)} \end{bmatrix} \tag{31}$$

where the ij-entry of (31) for $i > j$ is $\begin{pmatrix} i-1 \\ j-1 \end{pmatrix} E^{(i-j)} + \begin{pmatrix} i-1 \\ j \end{pmatrix} F^{(i-j-1)}$.

We write (31) as

$$\hat{\mathcal{E}}_j \hat{\mathbf{x}}_j = -\hat{\mathcal{F}}_j x + \hat{\mathbf{f}}_j \tag{32}$$

Let D_j =Diag$\{1, 1, \frac{1}{2!}, \ldots, \frac{1}{(j-1)!}\}$, $\bar{D}_j$ =Diag$\{1!, 2!, \ldots j!\}$. Then

$$D_j \hat{\mathcal{F}}_j = \mathcal{F}_j,\ D_j \hat{\mathbf{f}}_j = \mathbf{f}_j,\ D_j \hat{\mathbf{x}}_j = \mathbf{x}_j,\ D_j \hat{\mathcal{E}}_j \bar{D}_j = \mathcal{E}_j$$

Thus Theorems 3.1,3.2 hold using (32) instead of (26).

Now consider (1). Again let $u = f$ and $w = [x, u]$, and rewrite (1) as

$$\tau' = 1 \tag{33}$$

$$R(\tau, w, w') = E(\tau)x' + F(\tau)x - u = 0 \tag{34}$$

If we were following Section 2 we would let $k = 1$ since there are only first derivatives in the R equation. However, we know that in general additional derivatives are needed. Consider then the system

$$\frac{d}{dt}\begin{bmatrix} \tau \\ w \\ w' \\ \vdots \\ w^{(j)} \end{bmatrix} = \begin{bmatrix} 1 \\ w' \\ \vdots \\ w^{(j)} \\ 0 \end{bmatrix} + \sum_{i=1}^{2(j+1)n} v_i \begin{bmatrix} 0_1 \\ e_i \end{bmatrix} \tag{35}$$

$$z = R[\tau, w, \ldots, w^{(j)}] = E(\tau)x' + F(\tau)x - u = 0 \tag{36}$$

Let

$$y = \begin{bmatrix} 1 \\ w' \\ \vdots \\ w^{(j)} \\ 0 \end{bmatrix}, \quad g_i = \begin{bmatrix} 0_1 \\ e_i \end{bmatrix}$$

Then

$$\begin{aligned} R &= 0 \\ L_{g_0} R &= 0 \\ &\vdots \\ L_{g_0}^{j-1} R &= 0 \end{aligned} \tag{37}$$

is the same as

$$\hat{\mathcal{E}}_j(\tau)\hat{\mathbf{x}}_j(t) + \hat{\mathcal{F}}_j(\tau)x(t) - \hat{\mathbf{u}}_j = 0 \tag{38}$$

Thus Theorems 3.1, 3.2 show that the extended family of Lie derivatives (37) will determine a solution manifold and a flow in all of $\mathcal{R}^n$. However, this fact is expressed in terms of Section 3. We wish to exploit the information in these theorems to extend the approach of Section 2.

If a solution of (35) is to leave (36) invariant for a control v, we need that

$$\frac{d}{dt}(L_{g_0}^r R) = L_{g_0}^{r+1} R + (L_{\bar{g}} L_{g_0}^r R)v = 0 \tag{39}$$

for $r = 0, \ldots, j-1$, $(j \leq n)$ where the ith column of $L_{\bar{g}} L_{g_0}^r R$ is $L_{g_i} L_{g_0}^r R$. The equations (39) may be rewritten as

$$\hat{b} + \hat{A}\hat{\alpha} = 0 \tag{40}$$

and $\hat{A} = [\hat{\mathcal{F}}_j, \hat{\mathcal{E}}_j, -I]P$ where P is the permutation matrix that lists the $x^{(i)}$ first and then the $u^{(i)}$. Notice that $\hat{A}$ has full row rank by Theorem 3.1. As before, define the feedback $v = \hat{\alpha} + \hat{\beta}\hat{v}$, by $\hat{A}\hat{\alpha} = -\hat{b}, \hat{A}\hat{\beta} = 0$. Using these theorems, observe that not only is there such a feedback, but that in $\hat{\beta}$ we may take u arbitrary and then x is determined by $\mathbf{u}_j, x_0$.

Alternatively, we may let $w = x$, and rewrite (1) as

$$\frac{d}{dt}\begin{bmatrix} \tau \\ x \\ \vdots \\ x^{(j)} \end{bmatrix} = \begin{bmatrix} 1 \\ x' \\ \vdots \\ x^{(j)} \\ 0 \end{bmatrix} + \sum_{i=1}^{(j+1)n} v_i \begin{bmatrix} 0_1 \\ e_i \end{bmatrix} \tag{41}$$

$$z = R[\tau, x, \ldots, x^{(j)}] = \mathcal{E}(\tau)x' - F(\tau)x - u(\tau) = 0 \tag{42}$$

Again (37) becomes (38). This time $\hat{A} = [\hat{\mathcal{F}}_j, \hat{\mathcal{E}}_j]$ and the equations (39) are

$$\hat{b} + \hat{u}_j = [\hat{\mathcal{F}}_j, \hat{\mathcal{E}}_j]\hat{\alpha} \tag{43}$$

From Theorem 3.1 we again have that $\hat{A}$ has full row rank and hence $\hat{\alpha}$ exists. Furthermore, in this case, the second n components of β depend only on the first n components of β

Another variation of the preceding approach would be to consider

$$E(t)x' + F(t)x = B(t)u \tag{44}$$

In (35),(36) and (41),(42), we have introduced the variable τ to make the resulting systems time invariant, and thus more closely resemble those in Section 2. However, this is not necessary.

Which of these alternatives is the best way to view (1) remains to be determined.

5 Conclusion

In this paper we have briefly discussed two approaches concerning descriptor systems and observed that they deal with similar derivative arrays. The approach of Section 2 is part of an evolving elegant differential geometric control theory. The approach of Section 3 is computationally simpler in that the only needed symbolic manipulations are derivatives of known functions. Numerical linear algebra routines may then be used pointwise. The assumptions of Section 3 are also somewhat more general than those of Section 2.

In subsequent papers, we hope to exploit this relationship more fully, especially in the nonlinear case.

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