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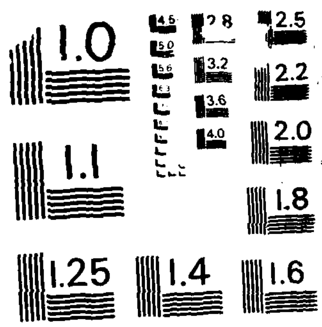
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The research under this grant focused on nonlinear behavior, coherence and chaos, in partial differential equations, especially those occurring nonlinear optics. Sixteen papers were published during the period of this grant.

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July 1, 1987

Kathleen L. Wetherell
Contracting Officer
Department of the Air Force
AFOSR
Bolling Air Force Base
D.C. 20332-6448

Dear Ms. Wetherell,

This is the final technical report for our Department of Defense grant, contract #AFOSR-83-0227, entitled "Nonlinear Behavior in Optical and Other Systems". This grant was used for research.

The following faculty and graduate students of both the Mathematics Department and the Applied Mathematics Program were involved:

Herman Flaschka
Christopher Jones
David McLaughlin
Jerome Moloney
Alan Newell
Hatsuo Adachihara

The following is a list of all papers whose research was carried out using grant funds. (See attached sheet.) Also enclosed are the covers of the preprints or reprints from many of these publications.

Sincerely,

Alan C. Newell
Head, Department of Mathematics

ACN:pm
Enc.

1. "Chaos and coherent structures in partial differential equations" by A.Aceves, H.Adachihara, C.Jones, J.C.Lerman, D.McLaughlin, J.Moloney, and A.C.Newell.

2. "On the infinitely many solutions of a semilinear elliptic equation" by C.Jones and T.Kupper.

3. "Instability of standing waves in nonlinear optical waveguides" by C.Jones and J.Moloney.

4. "The origin and saturation of modulational instabilities" by N.Ercolani, M.G.Forest, and D.W. McLaughlin.

5. "Coherence and chaos in the driven damped Sine-Gordon equation: measurement of the soliton spectrum" by E.A.Overman, D.W.McLaughlin, and A.R.Bishop.

6. "A quasi-periodic route to chaos in a near-integrable PDE" by A.R.Bishop, M.G.Forest, D.W.McLaughlin, and E.A.Overman.

7. "Weak limits of nonlinear conservation laws with oscillating data" by D.McLaughlin, G.Papanicolaou, and L.Tartar.

8. "Many=parameter routes to optical turbulence" by J.V.Moloney.

9. "Numerical evidence for nonstationary, nonlinear, slab-guided waves" by J.V.Moloney, J.Ariyasu, C.T.Seaton, and G.I.Stegeman.

10. "Stability of nonlinear stationary waves guided by a thin film bounded by nonlinear media" by J.V.Moloney, J.Ariyasu, C.T.Seaton, and G.I.Stegeman.

11. "New theoretical developments in nonlinear guided waves: non-kerr-like media" by J.V.Moloney, G.I.Stegeman, E.M.Wright, and C.T.Seaton.

12. "Saturation and power law dependence of nonlinear waves guided by a single interface" by J.V.Moloney, T.P.Shen, G.I.Stegeman, C.T.Seaton, and J.Ariyasu.

13. "Gaussian beam excitation of TE_0 nonlinear guided waves" by J.V.Moloney, E.M.Wright, G.I.Stegeman, and C.T.Seaton.

14. "Beam propagation method analysis of a nonlinear directional coupler" by J.V.Moloney, E.M.Wright, G.I.Stegeman, C.T.Seaton, and L.Thylen.

15. "Multi-soliton emission from a nonliear waveguide" by J.V.Moloney, A.D. Boardman, E.M.Wright, G.I.Stegeman, and C.T.Seaton.

16. "Nonlinear slab-guided waves in non-kerr-like media" by J.V.Moloney, T.P.Shen, A.A.Maradudin, R.F.Wallis, G.I.Stegeman, E.M.Wright, and C.T.Seaton.

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Physica 18D (1986) 85-112
North-Holland, Amsterdam

CHAOS AND COHERENT STRUCTURES IN PARTIAL DIFFERENTIAL EQUATIONS

Alejandro ACEVES, Hatsuo ADACHIHARA, Christopher JONES, Juan Carlos LERMAN,
David W. McLAUGHLIN, Jerome V. MOLONEY* and Alan C. NEWELL
Applied Mathematics Program, University of Arizona, Tucson, AZ 85721, USA

This paper addresses the possible connections between chaos, the unpredictable behavior of solutions of finite dimensional systems of ordinary differential and difference equations and turbulence, the unpredictable behavior of solutions of partial differential equations. It is dedicated to Martin Kruskal on the occasion of his 60th birthday.

1. Introduction

The chaos that occurs in p.d.e.'s
cannot be fathomed by legalese
so we apply Occam's razor
and using a laser
study structures in ring cavities

An appealing idea of modern dynamics is that the complicated and apparently stochastic time behavior of large and even infinite-dimensional nonlinear systems is in fact a manifestation of a deterministic flow on a low-dimensional chaotic attractor. If the system is indeed low dimensional, it is natural to ask whether one can identify the physical characteristics such as the spatial structure of those few active modes which dominate the dynamics. Our thesis is that these modes are closely related to and best described in terms of asymptotically robust, multiparameter solutions of the nonlinear governing equations. We find it hard to define this robust nature precisely, but loosely speaking the idea is that these solutions are very stable and resilient asymptotic states. They may be coherent lumps like solitons and solitary waves. They may have the form of coherent wave packets. They may have self-similar form. They need not necessarily be the asymptotic states which develop as t tends to infinity; structures which develop singularities in finite time like those involved in the collapse of Langmuir waves or in filamentation in nonlinear optics are also candidates. For example, singular solutions of the Euler equations may be useful in understanding the behavior of the Navier–Stokes equations at high Reynolds numbers. Singular solutions like defects and dislocations certainly do play important roles in the pattern formations arising in continuum and condensed matter physics. The key idea is that each of these structures is a natural asymptotic state that, by virtue of the various force balances in the governing equations, develops an identity which does not easily decay or disperse away.

One can envision two types of chaos occurring. The first is a *phase* or *weak* turbulence which arises when there is an endless competition between equally resilient, localized coherent structures which are infinite time asymptotic states and which are initiated at random at various parts of the physical domain. Examples of this type of turbulence are solitary waves in the one-dimensional complex envelope equation, Rayleigh–Bénard roll patterns with different orientations and the oscillatory skew varicose states in low Prandtl number convection. It is to be expected that such dynamics may be low dimensional. The second type of chaos is much more dramatic and, for want of a better word, may be described as an

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On the Infinitely Many Solutions of a Semilinear Elliptic Equation

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Instability of Standing Waves in Nonlinear Optical Waveguides

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PACS numbers: 02.78 42.20

Abstract

A new mathematical instability technique is presented and applied to determine the stability properties of a physically important class of standing waves in nonlinear planar optical waveguides. The method is illustrated by a case where soliton perturbation techniques or variational methods are inapplicable.

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North-Holland, Amsterdam

THE ORIGIN AND SATURATION OF MODULATIONAL INSTABILITIES

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Extended abstract

Although completely integrable, the periodic sine-Gordon (SG) equation,

$$u_{tt} - u_{xx} + \sin u = 0,$$

is a very complicated infinite dimensional Hamiltonian system. However, frequently its asymptotic states, and those of nearby equations, are dominated by only a few degrees of freedom. Here we explain the origin of this low dimensionality and give a recipe for determining the dominant degrees of freedom.

We consider the stability of an N-phase quasi-periodic solution u to (SG) in the space F of C^{∞}, periodic functions of x. Linear stability is treated in [1, 2] where we

(i) construct a countably infinite collection of solutions of the linearized problem which we conjecture to be a basis for the function space;
(ii) show that at most a finite number of elements ("modes") in this collection grow exponentially in time (the rest grow at worst polynomially);
(iii) provide a simple criterion which identifies the unstable modes;
(iv) compute the growth rates of these unstable modes.

This work [1] shows that the classical modulational instability is merely a special case of a very general instability. Our analysis involves spectral theory and complex function theory. In addition, in [1] we use geometric methods to reveal the nonlinear phenomena which govern the origin and the saturation of the instability. We think that this geometric description makes the qualitative nature of the instability completely transparent.

Our analysis proceeds from the linear spectral problem which can be used to integrate (SG). Since $u(x, t)$ is periodic in x, a Floquet spectral theory is appropriate. The cornerstone of this spectral theory is the Floquet discriminant $\Delta : F \times C \to C$, a function of $\boldsymbol{u} = (u, u_t) \in F$ and $E \in C$. For fixed $\boldsymbol{u}$ the zeros of $[\Delta^2(\boldsymbol{u}, E) - 4]$ are the periodic and antiperiodic eigenvalues. An eigenvalue E_j is called a *simple point* or a *double point* according to its order as a zero of $\Delta^2 - 4$. Under the sine-Gordon flow, $\Delta(\boldsymbol{u}, E)$ is invariant and contains all of the sine-Gordon constants of motion. These invariants, together with a collection of dynamical variables $\{\mu_j\}$, constitute coordinates for $\boldsymbol{u}$. The dynamical variables $\{\mu_j\}$ are in 1-1 correspondence with the critical points of Δ, and satisfy O.D.E.'s in both x and t.

Our analysis of this instability has revealed several features [1] of the geometry of isospectral sets which were previously unknown (at least to us). Perhaps the most striking of these features is

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COHERENCE AND CHAOS IN THE DRIVEN DAMPED SINE-GORDON EQUATION: MEASUREMENT OF THE SOLITON SPECTRUM

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Received 14 November 1984
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A numerical procedure is developed which measures the sine-Gordon soliton and radiation content of any field (ϕ, ϕ_t) which is periodic in space. The procedure is applied to the field generated by a damped, driven sine-Gordon equation. This field can be either temporally periodic (locked to the driver) or chaotic. In either case the numerical measurement shows that the spatial structure can be described by only a few spatially localized (soliton wave-train) modes. The numerical procedure quantitatively identifies the presence, number and properties of these soliton wave-trains. For example, an increase of spatial symmetry is accompanied by the injection of additional solitons into the field.

1. Introduction

When an infinite-dimensional dynamical system (such as a nonlinear partial differential equation) is strongly perturbed, a rich variety of distinct responses is observed. These include temporal and spatial chaos, temporal chaos with spatial coherence, and intermittent bursts between chaotic and coherent states. We are particularly interested here in coherent, quasistable spatial structures in a chaotic field and their role in the transitions of a field into and out of chaos. Physical examples include large coherent eddies in a turbulent flow, dislocations in Couette flow, and solitary profiles in chaotic waves.

Support is gratefully acknowledged from the National Science Foundation, the U.S. Department of Energy, U.S. Air Force, and Los Alamos National Laboratory

The latter case is the easiest to study. It can be modeled [1–4] by a nonlinear, dispersive wave equation in one spatial dimension which is perturbed by dissipation and by a sinusoidal driver. In contrast with other examples, one dimensional waves (i) are relatively easy to integrate numerically; and (ii) possess coherent spatial structures (solitary waves) which are very well understood in the *absence* of perturbations.

A concrete example is the driven, damped sine-Gordon equation with periodic boundary conditions of period L:

$$\phi_{tt} - \phi_{xx} + \sin\phi = \Gamma\cos\omega t - \alpha\phi_t, \tag{1}$$

$$e^{i\phi(x+L,t)} = e^{i\phi(x,t)}, \qquad \text{(1.bc)}$$

$$\phi(x, t=0) = \phi_{in}(x), \qquad \text{(1.ic)}$$

$$\phi_t(x, t=0) = v_{in}(x).$$

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A QUASI-PERIODIC ROUTE TO CHAOS IN A NEAR-INTEGRABLE PDE

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Pattern formation and transitions to chaos are described for the damped, ac-driven, one-dimensional, periodic sine-Gordon equation. In a nonlinear Schrödinger regime, a generic quasi-periodic route to intermittent chaos is exhibited in detail using a range of dynamical systems diagnostics. In addition, a nonlinear spectral transform is exploited: (i) to identify and quantify ~~coordinatization~~ coordinates of space–time attractors in terms of a small number of soliton modes of the underlying integrable system; (ii) to use these analytic coordinates to identify ~~[illegible]~~ homoclinic orbits as possible sources of chaos; and (iii) to demonstrate the significance of ~~breather–breather collisions and~~ energy transfer between coherent and extended states in this chaotic system.

1. Introduction

Solutions of nonlinear evolution equations often exhibit rich patterns in space and time which can have both coherent and chaotic components. Throughout this conference it has been apparent that significant progress is being made in understanding such "space-time complexity." We have heard about parallel developments in three main areas: (i) partial differential equations [1–3]; (ii) statistical mechanics [4]; and (iii) cellular automata [5] and coupled map lattices [6]. In each of these three areas it is useful to distinguish systems which are dissipation-dominated from nearly conservative ones. In the dissipation-dominated case (e.g. [3]), only a few stable or metastable states (space-time attractors) exist, and the organization into patterns occurs because the dissipation causes the system to quickly relax to these states. The attractors can usually be identified as local minima of a Lyapunov free energy functional, and pattern selection is frequently determined from consideration of boundary conditions. The phenomena are often generic for relatively large classes of equations and initial data.

In near conservative cases, the organizational process is more subtle. Typically, the underlying conservative problem has many exact solutions with a wide variety of spatial and temporal patterns. Of these, some will resonate with the perturbation and adjust to stable or metastable solutions of the near conservative system [1, 2, 7–9]. Thus, the organization occurs through a nonlinear resonance process [10, 11], and it can be quite sensitive to both the particular equation and initial data. Nevertheless, this organization can, in experimentally substantial parameter regimes, simplify the chaotic responses ~~for~~ of these infinite dimensional systems, ~~from those~~ which might be anticipated from ~~[illegible]~~ their low dimensional analogues (e.g. single nonlinear oscillators) [7, 8]. Concerning chaos in coupled oscillators, more is ~~usually~~ different!

compared with the response

In both the dissipation-dominated and near conservative cases, the solutions reside in an infinite dimensional phase space but may approach attractors which are low dimensional [1, 7–9]. Hence, the mathematical techniques developed in modern dynamical systems theory [12] are hopefully relevant. In particular, these techniques explain how motion near a low dimensional at-

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WEAK LIMITS OF NONLINEAR CONSERVATION LAWS WITH OSCILLATING DATA

D. McLaughlin,* University of Arizona

G. Papanicolaou,* Courant Institute, New York University

L. Tartar, C.E.N., Limeil, France

Abstract

We consider several examples of nonlinear evolution equations with initial data that are rapidly oscillating functions of the space variable. We obtain an effective system of nonlinear evolution equations for the various moments of the solution by a multiple scale method. We also show how in one case (the Carleman model) compensated compactness gives a very general way of obtaining the effective equations without the use of multiple scales.

1. Introduction

We are interested in the behavior of solutions of deterministic, nonlinear evolution equations or systems when the initial data are rapidly oscillating functions of the space variable. Such problems cannot be solved numerically in a direct way. They are frequently analyzed by obtaining a hierarchy of equations for the various moments of the solution. This hierarchy is infinite due to the nonlinearity of the equations and it is frequently rendered finite by various ad hoc closure procedures.

We shall examine closely two examples of equations with rapidly oscillating data in the following sections: the Carleman equations and the Broadwell equations. These examples are analyzed by the usual asymptotic methods of multiple scales and modulation theory. The form of the effective equations or of the hierarchy is then discussed and compared with the hierarchy obtained directly. In the case of the

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Many-parameter routes to optical turbulence

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(Received 21 November 1985)

The output of an externally pumped passive nonlinear-optical ring resonator is known to exhibit chaotic dynamics under variation of one or more external control parameters. The dynamics of the internal transverse laser beam profile is described, in the good-cavity limit, by an infinite-dimensional discrete-time map in function space. We find that the asymptotic dynamical behavior of the internal resonator field is in marked contrast to earlier plane-wave predictions. Instead, self-focusing, self-defocusing nonlinearities, and linear diffraction, combined with the pump and feedback of the ring resonator, give rise to strong spatial modulation of the transverse profile. For a self-focusing nonlinearity, spatially coherent robust transverse solitonlike structures persist even though the beam is undergoing temporally chaotic motion. Using attractor embedding techniques on our numerical solutions, we isolate distinct routes to optical turbulence under variation of a number of different control parameters. In particular, we provide examples of Ruelle-Takens-Newhouse sequences, intermittency, and a nongeneric bifurcation involving period doubling of invariant circles. A main result of the paper is that many few-dimensional attractors can coexist in different regions of the infinite-dimensional phase space at fixed values of the external control parameters. These attractors are accessed by varying the systems initial conditions and can, under variation of an appropriate control parameter, independently undergo transition to chaos. Interaction between neighboring attractors appears to be responsible for numerically observed departures from generic behavior. We note striking similarities between some of our numerically generated instability sequences and recent experimental observations in low-aspect-ratio fluid systems.

MS code no. AL3060 1986 PACS number(s): 42.65.−k, 42.60.Da, ~~42.50.Tj~~
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I. INTRODUCTION

The transition to turbulence in physical systems is an old subject with much of the earlier focus being on fluid dynamical systems. Understanding fully developed turbulence and its onset is still a major unsolved problem,[1] although spectacular success has been achieved over the past few decades in understanding the onset of low-level turbulence in low-aspect-ratio fluids.[2] Experiments on Rayleigh-Bernard convection[3] and Taylor-Couette flow[4] have essentially confirmed the mathematical scenarios proposed as a result of the work of Newhouse, Ruelle, and Takens,[5] Feigenbaum,[6] and others.[7] The difficulty with these mathematical models, however, is that they rely on topological ideas or on the study of the universal properties of few-dimensional maps. Bifurcation parameters appearing in these models cannot easily be identified with physical stress (control) parameters such as a Reynolds or Prandtl number for a fluid flow. An alternative approach is based on a Galerkin approximation, whereby the geometry of the system is exploited and the full fluid equations expanded in a Fourier series.[5] Truncation of the Fourier series to a few spatial modes leads to a system of coupled nonlinear ordinary differential equations that can then be integrated directly. Such a procedure led Lorentz[9] to observe, for the first time, chaotic dynamics in a coupled system of three ordinary differential equations. However, it is not clear to what extent truncated models mimic the true physical situation. Cases in point are the above fluid experiments where the number of degrees of freedom available to the system are reduced by designing low-aspect-ratio experiments. While truncated models can exhibit complex bifurcation structure, the nature of the transition to chaos becomes sensitive to the level of truncation.[8] There is in fact evidence to show that, in some instances, the original partial differential equations from which these models are derived exhibit no turbulence over the same parameter range.

The nonlinear-optical model considered in the present paper has the advantage that, unlike the Navier-Stokes equations of fluid dynamics, the full partial differential equations are directly amenable to numerical study over a reasonably wide parameter range. *Ad hoc* truncations to plane waves or TEM resonator modes are thereby avoided. Our results show that such truncations are inappropriate and would lead to erroneous results. The numerical studies show that low-level turbulence appears to be intrinsic to nonlinear-optical systems, unlike fluids as noted above. We observe a close analogy between our numerically generated optical results and experimentally observed instabilities in the latter low-aspect-ratio fluid systems.

Haken,[10] in 1975, was the first to point out the close similarity between the Lorenz equations and the single-mode laser equations for a bad cavity. Originally, the physical constraints required in order to make this model valid were thought to be too severe to lead to a successful experimental verification. Recently, however, experiments using far and mid infrared lasers[11] suggest that the proper bad-cavity conditions and required gain can be achieved to test the Haken model. No direct comparison between the model and experiment has yet been made.

Numerical evidence for nonstationary, nonlinear, slab-guided waves

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Numerical studies of the propagation of waves guided by a thin film bounded by two self-focusing media reveal a whole new class of nonstationary solutions that can, in their own right, propagate as guided-wave fields, some with solitonlike components.

Self-focusing effects in slab-waveguiding structures have been predicted theoretically,[1-5] and in a few cases observed experimentally,[6,7] to lead to a number of novel phenomena. Guided-wave dispersion relations based on the nonlinear wave equation were obtained initially for Kerr-type media[1-5] and subsequently for more general nonlinearities.[8,9] The salient theoretical predictions were that both the propagation wave vector and field distributions vary with guided-wave power and that new branches with power thresholds can exist. These in turn have led to the identification[10] of a number of potential all-optical devices, such as optical limiters, lower-threshold structures, and bistable and switching devices.

The stability to propagation of nonlinear single-interface[11,12] and thin-film guided[13,14] waves has been studied recently. For nonlinear waves guided by a single interface between a self-focusing medium and a power-independent medium, Akhmediev *et al.*[11,12] found the solutions to be stable only if $dP/d\beta > 0$, where P is the guided-wave power and β is the effective index. Although similar results were found for TE_0 waves guided by thin films bounded by two self-focusing media,[13,14] the TE_1 (Ref. 15) case was more complex. In this Letter we report further numerical investigations of the TE_0 unstable branches that reveal a whole new class of nonstationary guided waves. Under appropriate conditions, these solutions exhibit solitonlike components that propagate away from the film structure.

This study deals with an asymmetric slab-waveguide geometry for which all-optical switching and bistability have been identified as possible device applications.[10] The TE_0 field is written as

$$E(r, t) = \tfrac{1}{2}\hat{j}W(x, z)\exp[i(\beta k_0 z - \omega t)] + \text{c.c.}, \quad (1)$$

with $k_0 = \omega/c$; β is the effective index. Both cladding $(-d > x)$ and substrate $(x > d)$ layers exhibit a positive (self-focusing) nonlinearity with refractive indices given by $n^2(x, |W|^2) = n_\gamma^2 + \alpha_\gamma |W|^2$, $\gamma = c, s$ $[|x| \geq d]$ for the cladding and the substrate, respectively, with $n_c = n_s = 1.55$ and $\alpha_c = 2\alpha_s$. The film medium is $2d$ thick, with $2k_0 d = 12.2$, and has an intensity-independent refractive index $n_f = 1.57$. These parameters correspond to those of the film and the cladding used experimentally in Ref. 6. In order to incorporate propagation effects, we must study a mixed-type linear–nonlinear Schrödinger equation, which is more complex than the stationary nonlinear wave equation. It is of the form

$$2i\beta k_0 \frac{\partial W}{\partial z} = -\frac{\partial^2 W}{\partial x^2} + k_0^2[\beta^2 - n^2(x, |W|^2)]W. \quad (2)$$

We have assumed in deriving Eq. (2) that the field propagates close to the axis of the guide (along z) and that the field variation along the z axis is small over one wavelength. Note that the usual stationary nonlinear waves are z-independent solutions $[W(x, z) = W_0(x)]$ of Eq. (2). The properties of Eq. (2) are well known for a homogeneous isotropic nonlinear Kerr medium, and a whole class of N-soliton and radiation solutions is known to exist.[16,17] It is this type of solution that we seek in the present case. In numerically solving Eq. (2) we have used the beam-propagation method,[17] incorporating a split-step fast Fourier transform with 512 transverse sampling points and a step length of 0.01 free-space wavelength. In all examples we have propagated the waves a minimum of 200 wavelengths along the z axis, which required 2–3 h of CPU time on a Data General MV10000 computer.

Figure 1 summarizes the stability properties of nonlinear guided waves initially launched at points on the nonlinear dispersion curves obtained for the guided-wave power P versus effective index β. Unstable stationary waves are defined as waves whose field distributions change with propagation distance. As expected, the TE_0-like dispersion curves in Fig. 1 show similar stability properties to earlier results[13,14] obtained for $2k_0 d = 1.256$. Loss of stability always occurs on the upper branch (II), except for the region in

Stability of nonlinear stationary waves guided by a thin film bounded by nonlinear media

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The stability of stationary, TE_0-type, nonlinear, thin-film guided waves was investigated numerically for both symmetric and asymmetric planar waveguides with nonlinear cladding and substrate layers. It is found that large regions of the dispersion curves are unstable at high powers.

Unique properties[1-7] have been predicted for waves guided by thin films when one or more of the guiding media exhibit a field-dependent refractive index. Self-focusing bounding media lead to multiple new branches with power thresholds, as well as field distributions whose maxima shift from the film to the bounding media with increasing power.[1-7] In fact, this geometry has been identified as an excellent candidate for all-optical switching, with our without bistability.[5-7] To date, however, theoretical analysis has been based solely on steady-state solutions to a nonlinear wave equation which contains an intensity-dependent refractive index. The salient question is whether these wave solutions are stable on propagation, and the consequences of possible unstable regions to proposed devices. For the related problem of self-focusing of plane waves in infinite media, Kolokolov[8] has shown that the solutions are stable for $dP/d\beta > 0$ where P is the power and β is the power-dependent refractive index. Numerical propagation studies of nonlinear waves guided by the interface between a self-focusing and a power-independent medium by Akhmediev and co-workers[9,10] have led to a similar conclusion. Recently, a theory based on phase portraits has been developed[11] for the stability of TE_0 waves guided by a thin film bounded by self-focusing media, and the important conclusion of that work is that the waves are unstable on negatively sloped branches ($dP/d\beta < 0$) of the nonlinear dispersion curve, and that they are stable on positively sloped regions provided that self-focusing occurs in only one nonlinear medium. In this letter we report a test of this conclusion via a numerical investigation of the stability of TE_0 solutions for films bounded by self-focusing media.

The geometry analyzed consists of a film ($|x| < d$, refractive index n_0) bounded by two nonlinear media with low power indices n_1 and n_2, as shown in Fig. 1. Since the numerical analysis is performed in the slowly varying phase and amplitude approximation, we write the optical field as

$$E(r) = \tfrac{1}{2}W(x,z)e^{i(\beta k_0 z - \omega t)} + \text{c.c.}, \tag{1}$$

where β is the effective guided wave index, $k_0 = \omega/c$ and the variation in the amplitude term $W(x,z)$ along the propagation direction z is assumed to be small over one wavelength. The refractive index in the various media is given by

$$n^2[|W(x,z)|^2] = n_\gamma^2 + \alpha_\gamma |W(x,z)|^2 \qquad \gamma = 1,2 \quad (|x| > d) \tag{2a}$$

and

$$n^2[|W(x,z)|^2] = n_0^2 \quad (|x| < d)\,. \tag{2b}$$

Substituting into the nonlinear wave equation and retaining only the first derivative of $W(x,z)$ with respect to z leads to

$$2i\beta k_0 \frac{\partial W(x,z)}{\partial z} + \frac{\partial^2 W(x,z)}{\partial x^2} - k_0^2 \times \{\beta^2 - n^2[|W(x,z)|^2]\} W(x,z) = 0\,. \tag{3}$$

Analytical stability analysis of Eq. (3) is complicated by the fact that it is a partial differential equation and, furthermore, is a Hamiltonian system. The usual stability analysis for dissipative dynamical systems does not apply (unless one deliberately introduces losses into the problem). One reason for the difficulty in studying the stability of Eq. (3) is that many of the eigenvalues of the linearization of (3) lie on the imaginary axis which is precisely the condition for instability in a dissipative dynamical system. Although it has been possible[11] to perform a stability analysis for TE_m with $m = 0$, it is necessary to proceed numerically for $m \neq 0$. First, the steady-state solutions with $W(x,z) \to W(x)$ were obtained in the usual way[1-7] to obtain a field solution corresponding to a particular point on one of the nonlinear guided wave solution branches. This distribution was then assumed to be launched at $z = 0$ and Eq. (3) was solved numerically for

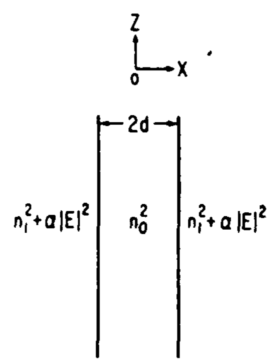

FIG. 1 Nonlinear waveguide geometry studied for wave stability.

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NEW THEORETICAL DEVELOPMENTS IN NONLINEAR GUIDED WAVES: NON-KERR-LIKE MEDIA

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Abstract: Guided waves with unique, power-dependent properties arise when one or more of the media bounding a guiding film exhibits an intensity-dependent refractive index. Previous theoretical work on this problem has been based formalism-limited to Kerr-type nonlinear media in which the change in refractive index is quadratic in the optical field. In this paper a formalism recently reported by Langbein et al is used to investigate nonlinear guided wave solutions in more realistic material systems. It is shown numerically that saturation of the optically induced change in the refractive index can dramatically alter, and in some cases eliminate the more interesting power-dependent features of the solutions. Nonlinear wave solutions are also investigated for a larger class of media that are characterized by refractive indices which depend on the optical field raised to some arbitrary power.

SATURATION AND POWER LAW DEPENDENCE OF NONLINEAR WAVES GUIDED BY A SINGLE INTERFACE

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The nonlinear waves guided by the interface between a linear and a non-Kerr-like, nonlinear medium were investigated numerically using the formalism recently developed by Langbein et al. [Optics Comm. 53 (1985) 417]. The power dependence of the propagation constant and field distributions were calculated for both power-law and saturable field-dependent dielectric constants.

1. Introduction

It has been shown theoretically that the interface between two dielectric media can guide TE polarized waves, provided that a power threshold is exceeded, and that one of the two media exhibits a dielectric constant which depends quadratically (Kerr-like medium) on the local optical field [1–5]. To date, these investigations have been based on the nonlinear wave equation which can be solved analytically [1,6] for a relative dielectric constant of the form $\epsilon = \epsilon_\gamma + \epsilon_\gamma^{NL}(E)$ with $\epsilon_\gamma^{NL}(E) = \alpha_\gamma E^2$. This approach has also been used to investigate nonlinear waves guided by thin films. Recently, Langbein et al. reported [7] a new technique for analysing nonlinear guided wave phenomena which does not require analytical field solutions to the nonlinear wave equation in order to evaluate the power dependence of the propagation wavevector. Their approach, which requires numerical integration in some cases, is valid for arbitrary forms of ϵ^{NL}. This important development now allows realistic materials to be considered theoretically. For example, all materials exhibit a saturation in the optically induced refractive index change and it has already been shown that saturation has a large effect on nonlinear waves guided by thin dielectric films [8]. Furthermore, the dependence of the dielectric constant on the field is not quadratic in, for example, semiconductor materials [9–11]. In this paper we investigate the nonlinear waves guided by a single interface for saturable nonlinear media, and non-quadratic power law media.

2. Review of general theory

The details of the technique can be found in ref. [7] and here we briefly summarize the salient points. Writing the TE solution fields in the γ'th medium in the general form

Gaussian Beam Excitation of TE_0 Nonlinear Guided Waves

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Abstract: Selective, efficient excitation of TE_0 nonlinear guided waves is demonstrated numerically. For a thin film bounded by two self-focusing media, three different field distributions corresponding to the same power can be excited independently by suitable Gaussian input beams.

BEAM PROPAGATION METHOD ANALYSIS OF A NONLINEAR DIRECTIONAL COUPLER

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Abstract: The beam propagation method is employed to analyse nonlinear directional coupler operation for various combinations of nonlinear materials and initially mismatched guides. A mixed focusing/defocusing configuration is found to give optimal results.

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MULTI-SOLITON EMISSION FROM A NONLINEAR WAVEGUIDE

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Abstract: We demonstrate numerically that external excitation of a nonlinear waveguide can produce sequential threshold behavior via multi-soliton emission from the waveguide. This behavior is similar to that predicted to occur at a nonlinear interface.

NONLINEAR SLAB-GUIDED WAVES IN NON-KERR-LIKE MEDIA

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