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MULTIPLE-MODEL PARAMETER-ADAPTIVE CONTROL
FOR IN-FLIGHT SIMULATION

THESIS

Thomas J. Berens
1Lt. USAF

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MULTIPLE MODEL PARAMETER ADAPTIVE CONTROL
FOR IN-FLIGHT SIMULATION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the Requirements for the Degree of
Master of Science in Electrical Engineering



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Preface

The purpose of this study was to modify an existing parameter estimation algorithm to a multiple-input multiple-output difference equation model. The immediate need for this procedure was for use in adaptive control of an in-flight aircraft simulator, but the approach should be valid for other applications.

The algorithm was used in conjunction with an adaptive control law in a computer simulation of an in-flight simulator aircraft.

Both the parameter estimation algorithm and the control law worked well. When sensor noise was added to the simulation, however, estimator performance suffered. Future study must modify the estimation algorithm to decrease noise sensitivity. The work should be continued, as it provides an alternative approach to fixed gain flight control systems which require gain scheduling.

This work was a direct extension of an earlier thesis by Capt. Luis A. Pineiro of the Flight Dynamics Lab who must be credited with the initial concept of the work. I am deeply indebted to Capt. Pineiro for many hours he spent explaining various aspects of parameter identification and working out software problems. Finally, I would like to thank my advisor, Col. Daniel Biezdad for his efforts and patience during a time when it appeared this work would never be completed.

Thomas J. Berens

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Abstract

Adaptive control of aircraft model-following systems has shown promising results for in-flight simulation, but the computational expense and slow convergence of conventional parameter estimation techniques for higher order models inhibits their direct use for in-flight simulation. Computer simulations of adaptive systems usually assume some knowledge of model parameters in order to maintain tracking fidelity at a reasonable computational cost as parameters change. This thesis incorporates a-priori information into a multiple-model estimation algorithm which assigns a probability weighting of each estimator within a "bank" of estimators. Final parameter estimates used in adaptive control are formed as a probabilistic weighted sum of individual estimates. Simulations of the system show excellent tracking performance throughout the flight envelope. A moving bank scheme for use over a wide range of flight conditions is recommended as a further area of study.

MULTIPLE-MODEL PARAMETER-ADAPTIVE CONTROL FOR IN-FLIGHT SIMULATION

I. Introduction

An in-flight simulator is an aircraft whose stability, feel, and flying characteristics can be changed to mimic another aircraft from the pilot's perspective. In-flight simulators are used in new aircraft system pre-production testing, in research and development, and in training test pilots [3, 14].

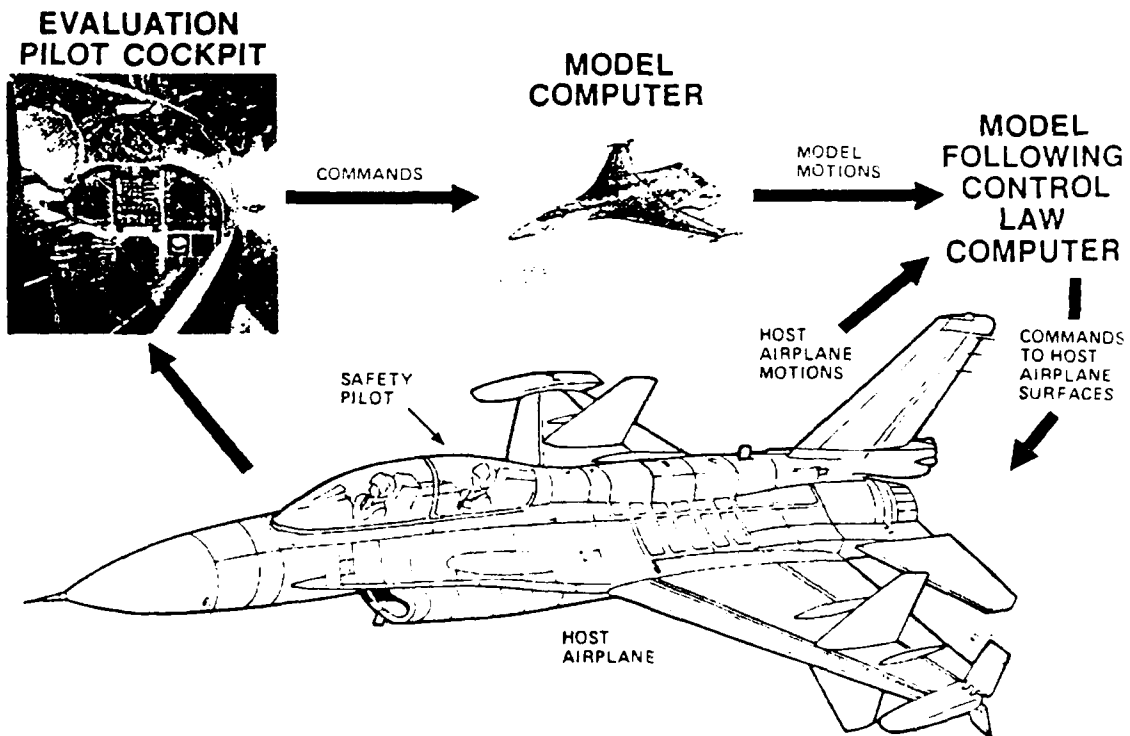


Figure (I-1) Vista Concept

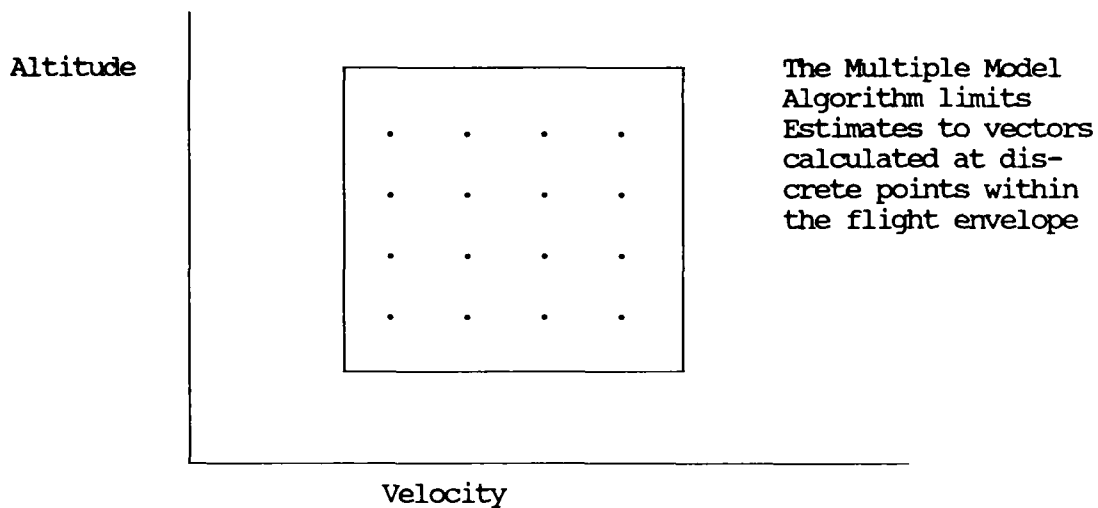


Figure (I-2) Flight Envelope

The U.S. Air Force currently has several types of in-flight simulators in its inventory and will soon replace its current fighter simulator, the NT-33. The NT-33 was developed in the 1950's and can no longer match the performance of new fighter aircraft. Its replacement, VISTA (Variable Stability In-flight Simulator Test Aircraft), is a

...modern, high performance fighter aircraft, modified with variable stability controls and a reprogrammable cockpit, offering the capability to perform large amplitude maneuvers over a wide range of operating conditions in an expanded flight envelope [4]

VISTA has spurred renewed interest in high performance adaptive control since it is an ideal testbed for new flight control methods. Adaptive control is well suited to in-flight simulation due to the

control since it is an ideal testbed for new flight control methods. Adaptive control is well suited to in-flight simulation due to the time-varying nature of the host aircraft's stability derivatives as the aircraft transitions within its flight envelope (see Figure I-2). This type of control implies that estimates of the stability derivatives are used in control law calculations [21-23] to reduce or minimize degraded tracking performance of the model aircraft by the host.

Recursive estimation techniques which calculate discrete parameter values from input and output samples are impractical with in-flight simulation when the model order results in a large number of parameter estimates [25] or when the input excitation is insufficient [6]. Pineiro [19,20] used an algorithm developed by Hagglund [7] to overcome the excitation problem, but limited the number of estimated parameters to avoid unacceptable computational expense and convergence times. In this thesis, known information about the parameters is combined via a multiple model estimation algorithm [1,2,15,26] which may be used in conjunction with Hagglund's algorithm to overcome these limitations without reducing the number of estimated parameters. Such a-priori data is usually available from flight and wind tunnel testing. Use of this data in a parameter-adaptive model-following system speeds convergence and significantly reduces the computational expense.

Section 2 of this thesis summarizes the model following system developed by Pineiro which implements an adaptive fast sampling MIMO control law for robust tracking and which partially estimates a set of linear difference equation model parameters. Section 3 shows how to estimate all parameters at a relatively small computational cost by

using a-priori information. The multiple model estimation algorithm is emphasized and a derivation is included. Section 4 discusses several modifications to the multiple model algorithm which improve stability, convergence speed, or computational cost. In Section 5 a simulation demonstrates the estimation of parameters which are blended via the multiple model algorithm, filtered [10], then fed to the control law. Finally, conclusions and recommendations for further study follow in Section 6.

I. Parameter-Adaptive Model-Following

Figure (II-1) shows a block diagram of a parameter-adaptive model-following system [20]. The plant is a MIMO state equation which simulates the longitudinal dynamics of an aircraft. The plant is controllable by a proportional-plus-integral (PI) control law.

This section describes two methods of calculating the gains of the control law [21-24]. The fixed gain method requires knowledge of plant parameters by the control law. When these parameters are unavailable, an adaptive method is used to estimate them from real time input-output measurements via a recursive parameter estimation algorithm. Several recursive parameter estimation algorithms are presented and design issues discussed.

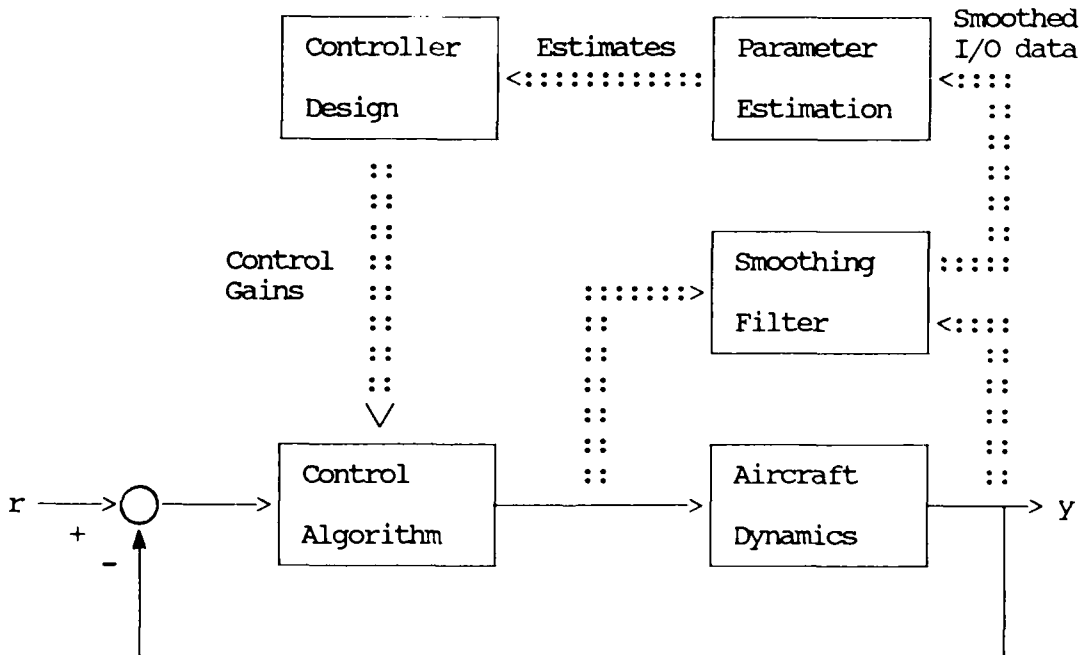


Figure (II-1) Adaptive Model Following Simulation Block Diagram

II.A. Fixed Gain Control Law

The aircraft plant is represented by a completely controllable and observable MIMO state space model

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2-1a)$$

$$y(t) = Cx(t) \quad (2-1b)$$

where

$$\begin{aligned} x(t) &\in \mathbb{R}^{n \times 1} & A &\in \mathbb{R}^{n \times n} & y(t) &\in \mathbb{R}^{m \times 1} \\ u(t) &\in \mathbb{R}^{m \times 1} & B &\in \mathbb{R}^{n \times m} \text{ with rank "m"} \end{aligned}$$

The A, B, and C matrices are partitioned according to the control input matrix, B, to yield

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \quad (2-2a)$$

and

$$y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (2-2b)$$

where

$$\begin{aligned} x_1(t) &\in \mathbb{R}^{p \times 1} & B_2 &\in \mathbb{R}^{(n-p) \times m} \text{ with rank "m"} \\ x_2(t) &\in \mathbb{R}^{(n-p) \times 1} \end{aligned}$$

A regular plant is defined as having a first Markov parameter, CB, of full rank. Regular plants with stable transmission zeroes will track input given the discrete output feedback control law (Figure (II-2)).

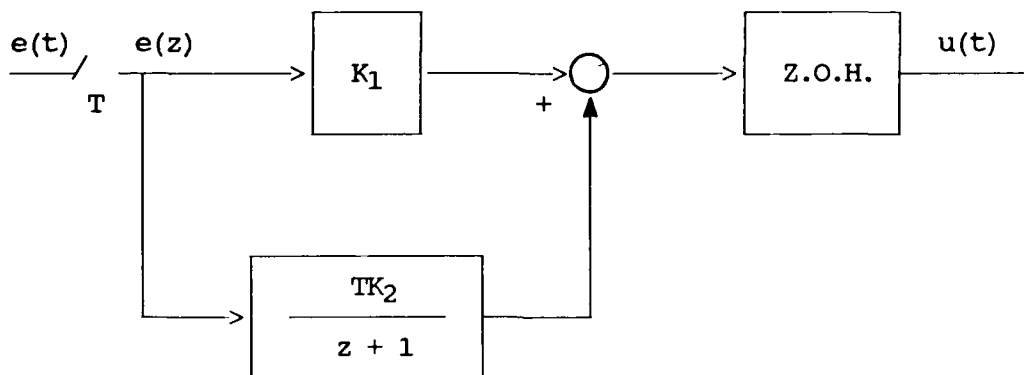


Figure (II-2) Discrete PI Control Law Block Diagram

$$u(k) = (1/T) [K_1 e_r(k) + K_2 Z(k)] \quad (2-3)$$

and

$$u(t) = u(k) \text{ for } t \in [kT, (k+1)T] \quad (2-4)$$

where

$*r(k)$ is the sampled reference tracking signal

k is an integer

T is the sampling period

K_1 is a control gain matrix [$K_1 \in R^{m \times m}$]

K_2 is a control gain matrix [$K_2 \in R^{m \times m}$]

$e_r(k)$ is the error vector [$e_r(k) = r(k) - y(k)$]

$Z(k)$ is the digital integral of the error vector

$$[Z(k+1) = Z(k) + T e_r(k)] \quad (2-5)$$

*For notational convenience, let $r(k) = r(kT)$. This notational format is used for all discrete variables.

If the matrices A, B, and C in eq. (2-1) are known, the control gain matrices K_1 and K_2 in eq. (2-3) are calculated by discretizing the plant state and output equations in block diagonal form [20] for the sampling period T as

$$\begin{bmatrix} \underline{x}_1(k+1) \\ \underline{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \underline{A}_1 & 0 \\ 0 & \underline{A}_4 \end{bmatrix} \begin{bmatrix} \underline{x}_1(k) \\ \underline{x}_2(k) \end{bmatrix} + \begin{bmatrix} \underline{B}_1 \\ \underline{B}_2 \end{bmatrix} r(k) \quad (2-6a)$$

and

$$y(k) = \begin{bmatrix} \underline{C}_1 & \underline{C}_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1(k) \\ \underline{x}_2(k) \end{bmatrix} \quad (2-6b)$$

where

$$\underline{x}_1(k) = \begin{bmatrix} z(k) \\ x_1(k) \end{bmatrix} \quad (2-6c)$$

$$\underline{x}_2(k) = x_2(k) \quad (2-6d)$$

$$\underline{C}_1 = [K_1^{-1}K_2, 0] \quad (2-6e)$$

$$\underline{C}_2 = C_2 \quad (2-6f)$$

$$\underline{A}_1 = \begin{bmatrix} I_m - TK_1^{-1}K_2 & 0 \\ TA_{12}C_2^{-1}K_1^{-1}K_2, I_{n-m} + A_{11}T - TA_{12}C_2^{-1}C_1 \end{bmatrix} \quad (2-6g)$$

$$\underline{B}_1 = \begin{bmatrix} 0 \\ TA_{12}C_2^{-1} \end{bmatrix} \quad (2-6h)$$

$$\underline{A}_4 = I_m - B_2K_1C_2 \quad (2-6i)$$

$$\underline{B}_2 = B_2K_1 \quad (2-6j)$$

The input-output relationship can be expressed in terms of the closed-loop transfer function, $G(z)$, where $y(z) = G(z)r(z)$ and z is the discrete transform operator. As the sampling frequency is increased $G(z)$ assumes the asymptotic form [20]

$$G(z) = G_1(z) + G_2(z) \quad (2-7a)$$

where z is the discrete transform operator and

$$G_1(z) = C_1(zI_n - I_n - TA_0)^{-1}TB_0 \quad (2-7b)$$

$$G_2(z) = C_2(zI_m - I_m - A_4)^{-1}B_4 \quad (2-7c)$$

with

$$A_0 = \begin{bmatrix} K_1^{-1}K_2 & 0 \\ A_{12}C_2^{-1}K_1^{-1}K_2, & A_{11}-A_{12}C_2^{-1}C_1 \end{bmatrix} \quad (2-7d)$$

$$B_0 = \begin{bmatrix} 0 \\ A_{12}C_2^{-1} \end{bmatrix} \quad (2-7e)$$

$$A_4 = -B_2K_1C_2 \quad (2-7f)$$

$$B_4 = B_2K_1 \quad (2-7g)$$

$G_1(z)$ and $G_2(z)$ are the slow and fast mode transfer functions respectively. The slow modes can be grouped into two sets Z_1 and Z_2 and are given by

$$Z_1 = \{z \in \mathbb{C} : \det\{zI_m - I_m + TK_1^{-1}K_2\} = 0\} \quad (2-8)$$

$$Z_2 = \{z \in \mathbb{C} : \det\{zI_{n-m} - I_{n-m} - TA_{11} + TA_{12}C_2^{-1}C_1\} = 0\} \quad (2-9)$$

The fast modes are given by

$$Z_3 = \{z \in \mathbb{C}: \det(zI_m - I_m + C_2B_2K_1) = 0\} \quad (2-10)$$

Because of the form of A_0 , B_0 , and C_1 , the eigenvalues of A_0 are uncontrollable or unobservable. Thus, as the sampling frequency increases, the slow transfer function asymptotically approaches zero and the overall system transfer function contains only the fast modes, as given by $G_2(z)$ which can be put in the equivalent form

$$G(z) = G_2(z) = (zI_m - I_m + C_2B_2K_1)^{-1}C_2B_2K_1 \quad (2-11)$$

The controller matrices K_1 and K_2 are then given by

$$K_1 = [C_2B_2]^{-1}S \quad (2-12a)$$

$$K_2 = qK_1 \quad (2-12b)$$

where q is any positive scalar greater than zero, and S is a diagonal tuning matrix. Both q and S are chosen by the designer to achieve the desired tracking characteristics.

II.B. Adaptive Control Law

If A , B , and C in eq. (2-1) are unknown, the control law gain matrices of eqs. (2-3) can be calculated adaptively by expressing eq. (2-8) as

$$y(k) = A_d(T)y(k-1) + H(T)u(k-1) \quad (2-13)$$

where $A_d(T) = \exp(AT)$ and $H(T)$ is defined as the step response matrix. $H(T)$ can be calculated in terms of the continuous time state space

system as:

$$H(T) = \int_0^T C \exp(At) B dt \quad (2-14)$$

Expanding $\exp(At)$ as a Taylor series about $t = 0$ and dropping the high order terms yields for small sampling periods $H(T) = TCB$, and the control gains can be expressed as

$$K_1 = H^{-1}(T)S \quad (2-17a)$$

$$K_2 = qK_1 \quad (2-17b)$$

The step response matrix $H(T)$ can be estimated from real-time input-output data by expressing eq. (2-13) in terms of an n th order autoregressive vector difference equation of the form

$$y(k) = B_1 u(k-1) - A_1 y(k-1) + \dots + B_n u(k-n) - A_n y(k-n) + e(k) \quad (2-16)$$

where the equation error vector $e(k)$ is assumed to be a zero-mean Gaussian white-noise vector with variance $v(k)$ and the matrices $A_i \in R^{m \times m}$ ($i=1, 2, \dots, n$), $B_i \in R^{m \times m}$ ($i=1, 2, \dots, n$) are the parameters of the n th order difference equation.

Eq. (2-16) can be expressed alternatively as

$$y(k) = \theta \phi^T(k) + e(k) \quad (2-17a)$$

where, for a SISO system,

$$\phi^T(k) = [-y^T(k-1), \dots, -y^T(k-n), u^T(k-1), \dots, u^T(k-n)]$$

$$e_{R^{1 \times 2(n)}(m)} \quad (2-17b)$$

and

$$\theta^T(k) = [A_1, \dots, A_n, B_1, \dots, B_n] \quad e_{R^{m \times 2(n)}(m)} \quad (2-17c)$$

The step response matrix is updated by invoking the certainty equivalence principle [20]. By definition of the step-response matrix it can be shown that [22,23]

$$H(T) \cong TCB = B_1 \quad (2-18)$$

All the parameters of eq. (2-16) must be estimated to identify B_1 .

The input-output relationship of eq. (2-6) is transformed into eq. (2-16) via the following steps:

Step 1: Obtain the Z-transform of (2-6)

$$zx(z) = A_d x(z) + B_d u(z) \quad (2-19a)$$

$$y(z) = Cx(z) \quad (2-19b)$$

Step 2: Obtain an input-output relationship independent of $x(z)$:

$$(zI - A_d)x(z) = B_d u(z) \quad (2-20)$$

$$x(z) = (zI - A_d)^{-1} B_d(z) u(z) \quad (2-21)$$

$$y(z) = G(z) u(z) \quad (2-22)$$

where

$$G(z) = C(zI - A_d)^{-1}B_d(z) \quad (2-23)$$

$G(z)$ is a polynomial matrix of the form:

$$G(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) & \dots & G_{1m}(z) \\ G_{21}(z) & G_{22}(z) & \dots & G_{2m}(z) \\ \vdots & \vdots & & \vdots \\ G_{m1}(z) & G_{m2}(z) & \dots & G_{mm}(z) \end{bmatrix} \quad (2-24)$$

where $G_{ij}(z)$ is the transfer function relating the output $y_i(z)$ to the control input $u_j(z)$ and is of the form

$$G_{ij}(z) = \frac{b_1 z^w + b_2 z^{w-1} + \dots + b_w z + b_{w+1}}{z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n} \quad (w < n) \quad (2-25)$$

by dividing each numerator and denominator in $G(z)$ by z^n , $G(z)$ is transformed into a delay operator of the form

$$G_{ij}(z) = \frac{b_1 z^{w-n} + b_2 z^{w-n-1} + \dots + b_{w+1} z^{-n}}{1 + a_1 z^{-1} + \dots + a_{n-1} z^{-n+1} + a_n z^{-n}} \quad (w < n) \quad (2-26)$$

Step 3: Redefine the input-output relationship obtained in step 2, expressed as eq. (2-22), in terms of polynomial matrices, $A(z)$ and $B(z)$, i.e.:

$$A(z)y(z) = B(z)u(z) \quad (2-27a)$$

where

$$A(z) = M + A_1 z^{-1} + \dots + A_n z^{-n} \quad (2-27b)$$

$$B(z) = M + B_1 z^{-1} + \dots + B_{w+1} z^{-n} \quad (2-27c)$$

$$A_{ij} = \begin{bmatrix} a_{ij}\{11\}(z) & a_{ij}\{12\}(z) & \dots & a_{ij}\{1m\}(z) \\ a_{ij}\{21\}(z) & a_{ij}\{22\}(z) & \dots & a_{ij}\{2m\}(z) \\ \vdots & \vdots & & \vdots \\ a_{ij}\{m1\}(z) & a_{ij}\{m2\}(z) & \dots & a_{ij}\{mm\}(z) \end{bmatrix} \quad (2-27d)$$

where $a_{ij}\{ij\}$ is the a_{ij} coefficient of $G_{ij}(z)$, eq. (2-26) and M is an $m \times m$ matrix of one's. B_{ij} are of the same form as eq. (2-24d).

Step 4: Obtain the inverse Z-transform of eq. (2-27) and rearrange terms to obtain the linear difference equation model given by eq. (2-16).

II.C. Recursive Parameter Estimation

The parameter vector, θ , of eq. (2-17) can be obtained from input-output measurements using a recursive parameter estimation algorithm. The recursive least squares (RLS) algorithm is a basic, widely used parameter estimation method [7,13]. The RLS algorithm assumes θ is a random vector with a Gaussian prior distribution of mean $\theta(k-1)$ and variance $P(k-1)$. Input-output measurements are correlated with θ , so at time kT the posterior probability density function, $p(\theta|y^k, u^k)$, can be formed, where

$$y^k = \{y(k), y(k-1), \dots, y(1)\}$$

$$u^k = \{u(k), u(k-1), \dots, u(1)\}$$

θ can be recursively updated for a SISO system as

$$\theta(k) = \theta(k-1) + (1/v(k))P(k)\phi(k)e(k) \quad (2-28)$$

$$P(k) = P(k-1) - \frac{P(k-1)\phi(k)\phi^T(k)P(k-1)}{v(k) + \phi^T(k)P(k-1)\phi(k)} \quad (2-29)$$

where

$v(k)$ = variance of $e(k)$
 $P(k)$ = estimated parameter covariance matrix $e \in R^{l \times l}$

The proof [13] is based on Bayes rule in the form

$$P(A|B,C) = P(B|A,C)P(A|C)/P(B|C) \quad (2-30)$$

Where $P(A|B,C)$ is the probability of the event A , conditioned on B and C . Assuming u^k is deterministic and applying this formula to the posterior density gives

$$\begin{aligned} p(\theta|y^k) &= p(\theta|y(k), y^{k-1}) \\ &= p(y(k)|\theta, y^{k-1})p(\theta|y^{k-1})/p(y(k)|y^{k-1}) \end{aligned} \quad (2-31)$$

The desired result is proved by induction.

Step 1:

$$\begin{aligned} p(\theta|y^0) &= (2\pi)^{-\dim \theta/2} (\det P(0))^{-1/2} \exp\{-1/2[\theta \\ &\quad - \theta(0)]^T P^{-1}[\theta - \theta(0)]\} \end{aligned} \quad (2-32)$$

by assumption.

Step 2: Assume that

$$\begin{aligned} p(\theta|y^{k-1}) &= (2\pi)^{-\dim \theta/2} (\det P(k-1))^{-1/2} \exp\{-1/2[\theta \\ &\quad - \theta(k-1)]^T P^{-1}(k-1)[\theta - \theta(k-1)]\} \end{aligned} \quad (2-33)$$

Now calculate $p(\theta|y^k)$ using eq. (2-31). From eq. (2-17)

$$e(k) = y(k) - \theta^T \phi(k) \quad (2-34)$$

Under the assumption that e^k is a sequence of independent random variables with zero means and variances $v(k)$,

$$p(y(k) | \theta, y^{k-1}) = (2\pi v(k))^{-1/2} \exp(-[1/2v(k)] [y(k) - \theta^T \phi(k)]^2) \quad (2-35)$$

Hence, from eq. (2-31),

$$p(\theta | y^k) = \text{Norm} \exp(-[1/2v(k)] [y(k) - \theta^T \phi(k)]^2 - 1/2 [\theta - \theta(k-1)]^T P^{-1}(k-1) [\theta - \theta(k-1)]) \quad (2-36)$$

where the θ independent normalization factor is not explicitly written out .

The exponent is now written as a quadratic form:

$$\begin{aligned} -2 \log p(\theta | y^k) &= \text{const} + [1/v(k)] y^2(k) \\ &\quad - [1/v(k)] y(k) \phi^T(k) \theta - [1/v(k)] \theta^T \phi(k) y(k) \\ &\quad + [1/v(k)] \theta^T \phi(k) \phi^T(k) \theta + \theta^T P^{-1}(k-1) \theta \\ &\quad - \theta^T P^{-1}(k-1) \theta(k-1) - \theta^T(k-1) P^{-1}(k-1) \theta \\ &\quad - 1) \theta + \theta^T(k-1) P^{-1}(k-1) \theta(k-1) \quad (2-37) \end{aligned}$$

Define

$$\underline{P}^{-1} = P^{-1}(k-1) + [1/v(k)] \phi(k) \phi^T(k); \quad (2-38)$$

thus the preceding expression is written as

$$\begin{aligned} &\text{const} + [1/v(k)] y^2(k) + \theta(k-1)^T P^{-1}(k-1) \theta(k-1) \\ &+ \theta^T \underline{P}^{-1} \theta - \theta^T [[1/v(k)] \phi(k) y(k) + P^{-1}(k-1) \theta(k-1)] \end{aligned}$$

$$\begin{aligned}
& - \{ [1/v(k)] \phi(k) y(k) + P^{-1}(k-1) \theta(k-1) \}^T \theta \\
& = \text{const}' + \{ \theta - \underline{P} [1/v(k)] \phi(k) y(k) - \underline{P} P^{-1}(k-1) \theta(k-1) \}^T \\
& \quad \underline{P}^{-1} [\theta - \underline{P} [1/v(k)] \phi(k) y(k) - \underline{P} P^{-1}(k-1) \theta(k-1)] \quad (2-39)
\end{aligned}$$

where const' is a new, θ independent normalization constant. Since

$$\underline{P} P^{-1}(k-1) = I - [1/v(k)] \underline{P} \phi(k) \phi^T(k), \quad (2-40)$$

The expression within the parentheses may be written as $\theta - \underline{\theta}$,

where

$$\underline{\theta} = \theta(k-1) + [1/v(k)] \underline{P} \phi^T(k) [y(k) - \theta^T(k-1) \phi(k)]. \quad (2-41)$$

This means that the posterior density at time kT , $p(\theta|y^k)$, is Gaussian with mean $\underline{\theta}$, given by eq. (2-41), and covariance matrix \underline{P} , given by eq. (2-42). Applying the matrix inversion lemma* to eq. (2-35) yields

$$\begin{aligned}
\underline{P} &= P(k-1) - P(k-1) \phi(k) \phi^T(k) P(k-1) / [v(k) \\
& \quad + \phi^T(k) P(k-1) \phi(k)] \quad (2-42)
\end{aligned}$$

Eq. (2-28) and eq. (2-29) can also be derived by minimization of the

*The Matrix Inversion lemma states that

$$[A + BCD]^{-1} = A^{-1} - A^{-1} B [DA^{-1} B + C^{-1}]^{-1} DA^{-1}$$

where A, B, C, and D are matrices of compatible dimensions, so that the product BCD and the sum A + BCD exists.

cost function [13,20].

$$J(\theta(k)) = \sum_{j=1}^k \{v^{-2}(j) [y(j) - \theta(j)\phi(j)]^2\} \quad (2-43)$$

II.D. Estimating v(k)

The variance of the prediction error is not explicitly updated by eq. (2-28) or eq. (2-29). Given the probability distribution of the prediction error is white, then an estimate of the variance can be made from the sequence e^{k-1} based on the central limit theorem:

$$v(k) = [1/k] \sum_{j=1}^k \{e(j)^2\} \quad (2-44)$$

If $v(k)$ is slowly time varying, then $v(k)$ can be tracked with a fading memory filter of the form

$$v(k) = \mu v(k-1) + (1 - \mu) e^2(k) \quad (2-45)$$

where $0 \leq \mu \leq 1$

The smaller the value of μ , the more information is forgotten during each update.

II.E. Modifications to RLS for In-flight Simulation

The RLS algorithm given by eq. (2-28) and (2-29) assumes constant parameters which are actually a function of an aircraft's flight

condition. A simple method for tracking time-varying parameters is to exponentially discard old information by incorporating a constant forgetting factor into eq. (2-29) [7,13], i.e.

$$P(k) = (1/g)P(k - 1) - \frac{P(k - 1)\phi(k)\phi^T(k)P(k - 1)}{(1/g)v(k) + \phi^T(k)P(k - 1)\phi(k)} \quad (2-46)$$

where $0 < g \leq 1$. Choosing a value of g is a trade-off between tracking time varying parameters and discarding good information. A small value of g allows the RLS algorithm to quickly track changing parameters.

Accurate parameter estimation, however, requires sufficient input excitation [6]. If current measurements contain no new useful information and old information is being discarded, parameter estimates may "burst" from the best fitting parameter values. This phenomena is referred to in the literature as estimator wind-up [27].

Hagglund's algorithm [7] attempts to solve the wind-up problem by discounting past information in such a way that if the parameters were constant, a constant amount of information is retained [7]. Specifically, Hagglund's algorithm modifies the $P(k)$ update equation as

$$P(k) = P(k - 1) - \frac{P(k - 1)\phi(k)\phi^T(k)P(k - 1)}{[v^{-1}(k) - a(k)]^{-1} + \phi^T(k)P(k - 1)\phi(k)} \quad (2-47)$$

where $a(k)$ is a discounting factor dependent upon the amount of excitation present in the input and output signal. The entire algorithm is listed in Appendix C.

The RLS algorithm with Hagglund's modifications to eq. (2-28) converges slowly for high order models requiring a large number of parameter estimates. Fault detection is a method which may decrease convergence time. A fault is defined as a rapid change in system dynamics. After a fault the current parameter estimates no longer accurately describe aircraft dynamics. The estimation algorithm converges but only after an undesirable time delay which degrades control performance.

Hagglund's algorithm accounts for the loss of parameter knowledge by increasing the value of the estimated parameter error covariance when a fault is detected. The increased weight placed on new measurements allows the estimation algorithm to converge faster. A fault is expressed by modifying eq. (2-46) as

$$P(k) = P(k - 1) - \frac{P(k - 1)\phi(k)\phi^T(k)P(k - 1)}{[v^{-1}(k) - a(k)]^{-1} + \phi^T(k)P(k - 1)\phi(k)} + B(k) \quad (2-48)$$

where $B(k)$ is a matrix which depends upon the size of the fault which increases $P(k)$ after a large parameter change occurs. Criterion for determining when a fault has occurred and how to choose $B(k)$ is discussed by Hagglund [7].

Hagglund's algorithm alone is still inadequate for many higher order systems. Therefore, Pineiro assumed elements of the parameter vector $\theta(k)$ not used in the control law remain constant about a nominal flight condition. In such case these elements are not

estimated and eq. (2-17) must be partitioned as

$$y(k) = \theta_V^T(k) \phi_V(k) + \theta_C^T \phi_C(k) + e(k) \quad (2-50a)$$

$$\phi^T(k) = [\phi_V^T(k) ; \phi_C^T(k)] \quad (2-50b)$$

$$\theta^T(k) = [\theta_V^T(k) ; \theta_C^T(k)] \quad (2-50c)$$

where the subscript v denotes the identified parameters and c denotes the constant parameters. Note $\phi_C(k)$ corresponds to θ_C but is not constant. The relationship between $\theta_V(k)$ and B_1 is illustrated in the following example.

Example 2-1

Assume a linear difference equation of the form

$$y(k) = -A_1 y(k-1) - A_2 y(k-2) + B_1 u(k-1) + B_2 u(k-2) + e(k) \quad (2-51)$$

$$\begin{aligned} \text{where } y(k) &\in \mathbb{R}^{2 \times 1}, & A_1 &\in \mathbb{R}^{2 \times 2}, & A_2 &\in \mathbb{R}^{2 \times 2} \\ u(k) &\in \mathbb{R}^{2 \times 1}, & B_1 &\in \mathbb{R}^{2 \times 2}, & B_2 &\in \mathbb{R}^{2 \times 2} \end{aligned}$$

This equation can be partitioned in the form of eq. (2-51) to isolate B_1 for identification, i.e.

$$\phi_C^T(k) = [-y^T(k-1), -y^T(k-2), u^T(k-2)] \in \mathbb{R}^{1 \times 6} \quad (2-52a)$$

$$\phi_C^T(k) = u^T(k-1) \in \mathbb{R}^{1 \times 2} \quad (2-52b)$$

$$\theta_C^T(k) = [A_1, A_2, B_2] \in \mathbb{R}^{1 \times 2} \quad (2-52c)$$

$$\theta_V^T(k) = B_1 \quad (2-52d)$$

This section discussed several basic and important design issues in simulated adaptive control of an in-flight simulator. The section began by describing the plant and its control by a fixed gain PI control law. This simulation was then modified by replacing the fixed gain controller with an adaptive system. An adaptive system bases control gain calculations on input-output measurements rather than explicit knowledge of system parameters. Adaptive systems are more complex since they require a parameter estimation algorithm. A well known and widely used recursive parameter estimation algorithm, RLS, was derived. Modifications to the RLS algorithm, however, are required for use in in-flight simulation. Section 3 extends recursive parameter estimation to make better use of available a-priori information.

III. Parameter Estimation Using A-Priori Information

Standard recursive parameter estimation algorithms are not well suited to adaptive flight control. The computational expense and slow convergence of the RLS algorithm makes estimating all the parameters of higher-order linear difference equation aircraft dynamics models which require a large number of parameter estimates impractical for on-line use. Even modifying standard recursive parameter estimation techniques by adding a fault detection algorithm are found to be inadequate without assuming partial knowledge of model parameters [20].

This section develops the multiple model algorithm (MMA) which is well suited to adaptive flight control. It estimates parameters from a finite amount of a-priori information which speeds parameter estimator convergence and reduces computational expense. A-priori information available from wind tunnel and flight testing is in the form of models of aircraft at various flight conditions. The parameter vectors of eq. (2-16) are calculated for each model using the procedure described in Section 2 and are incorporated into the MMA as means of Gaussian distributions. The final model parameter vector used by the control law is a random variable whose conditional distribution function (conditioned on output prediction errors) is a weighed sum of these Gaussian distributions. Using Bayes rule, the weighting coefficients are updated recursively as functions of output prediction errors and estimates of the output prediction error variance.

III.A. Finite Set Parameter Estimation

Before discussing the MMA, consider the more general case of parameter estimation from a finite set of models. Assume $\theta(k)$ is limited to nm "candidate" model parameter vectors, $\theta_i(k)$, such that

$$\theta(k) = \sum_{i=1}^{nm} a_i(k) \theta_i(k) \quad (3-1)$$

where $\theta_i(k)$ represents the parameter vectors of linearized input-output models of a non-linear systems at nominal operating points.

Gain scheduling, a method often used to choose a dynamics model for flight control, is an example of parameter estimation from a set of candidate models. Gain scheduling chooses aircraft dynamics models based on dynamic pressure which is a function of an aircraft's airspeed and altitude. A disadvantage of gain scheduling is dynamic pressure is measured with external sensors.

Many quantities are available which require no external sensors such as:

- 1) The parameter covariance matrix (3-2)

$$P_i(k) = E\{(\theta(k) - \hat{\theta}(k))(\theta(k) - \hat{\theta}(k))^T\}$$

- 2) The variance of the prediction error (3-3)

$$v_i(k) = E\{e_i(k)e_i^T(k)\}$$

- 3) The Autocorrelation Function (3-4)

$$A_{ee}(k, j) = E\{e_i(k)e_i^T(k - j)\}$$

- 4) The Crosscorrelation Function (3-5)

$$A_{ue}(k, j) = E\{u(k)e_i^T(k - j)\}$$

III.B. Comparison of Parameter Fit Quantities

If each parameter vector of eq. (3-1) is estimated with a recursive parameter estimation routine such as RLS, the parameter variance can be used as a model fit indicator. Unfortunately, basing a model selection test on the parameter variance limits the designer. For example, when model parameters vary slowly with time, $P_i(k)$ may be modified nonlinearly, as discussed in Chapter 2, to prevent estimator wind-up and optimize parameter tracking performance. Another example occurs in fault detection schemes. Parameter estimates converge faster after a large parameter change when $P_i(k)$ is enlarged. Both these schemes destroy goodness of fit information contained within $P_i(k)$, since eq. (3-2) no longer holds true.

Prediction errors, $e_i(k)$, are not effected seriously by ad-hoc parameter tracking schemes. Also, a parameter estimation algorithm need not be used to calculate $\theta(k)$. This greatly reduces the computational cost involved in calculating $\theta(k)$. Estimates of the prediction error variance, however, may be masked by noise.

A. The Multiple Model Algorithm (MMA) - Derivation

The multiple model algorithm (MMA) provides a method of blending a-priori data with parameter estimation. Although similar to the RLS algorithm, this approach views $\theta(k)$ as a random variable whose probability density function (pdf) is approximated as a weighted sum of m Gaussian pdf's rather than a single Gaussian pdf.

$$p(\theta) = (2\pi)^{-\dim \theta/2-1/2} \sum_{i=1}^m a_i(0) [\det(P_i(0))]^{-1/2} \exp\{-1/2[\theta - \theta_i(0)]^T P_i^{-1}(0) [\theta - \theta_i(0)]\} \quad (3-6)$$

where $\theta_i(0)$ are parameter vectors such as those in eq. (3-1), which represent the means of each Gaussian pdf, and $P_i(k)$ represents the variance. The $a_i(0)$ are weighting coefficients where $0 \leq a_i(0) \leq 1$ and

$$\sum_{i=1}^m a_i(0) = 1 \quad (3-7)$$

Eq. (3-6) can be recursively updated from input-output data via the multiple model algorithm where:

$$p(\theta|y^{k-1}) = (2\pi)^{-\dim \theta/2-1/2} \sum_{i=1}^m a_i(k-1) [\det(P_i(k-1))]^{-1/2} \exp\{-1/2[\theta - \theta_i(k-1)]^T P_i^{-1}(k-1) [\theta - \theta_i(k-1)]\} \quad (3-8)$$

as

$$p(\theta|y^k) = (2\pi)^{-\dim \theta/2-1/2} \sum_{i=1}^m a_i(k) [\det(P_i(k))]^{-1/2} \exp\{-1/2[\theta - \theta_i(k)]^T P_i^{-1}(k) [\theta - \theta_i(k)]\} \quad (3-9)$$

where

$$P_i(k) = P_i(k-1) - P_i(k-1)\phi(k)[v(k) + \phi(k)^T P_i(k)\phi(k)]^{-1}\phi(k)^T P_i(k-1) \quad (3-10)$$

$$\theta_i(k) = \theta_i(k-1) + [1, v(k)][P_i(k)\phi(k)e_i(k)] \quad (3-11)$$

$$a_i(k) = C_S a_i(k-1) [v(k) + \phi(k)^T P_i(k)\phi(k)]^{-1/2} \exp\{-1/2e_i^T(k)[v(k) + \phi(k)^T P_i(k)\phi(k)]^{-1}e_i(k)\} \quad (3-12)$$

where C_S is a normalization factor, such that

$$\sum_{i=1}^{nm} a_i(k) = 1 \quad (3-13)$$

$$\theta(k) = \sum_{i=1}^{nm} a_i(k) \theta_i(k) \quad (3-14)$$

$$e_i(k) = y(k) - \theta_i^T(k) \phi(k) \quad (3-15)$$

The proof (Andersson) which is similar to the RLS proof in Section 2 is based on Bayes rule. Applying this formula to the posterior density gives

$$\begin{aligned} p(\theta|y^k) &= p(\theta|y(k), y^{k-1}) \\ &= p(y(k) | \theta, y^{k-1}) p(\theta|y^{k-1}) / p(y(k) | y^{k-1}) \end{aligned} \quad (3-16)$$

Now calculate $p(\theta|y^k)$ using eq. (3-16). From eq. (2-19) obtain

$$e(k) = y(k) - \theta^T \phi(k) \quad (3-17)$$

Under the assumption that $\{e(k)\}$ is a sequence of independent random variables with zero means and variances $v(k)$, obtain

$$\begin{aligned} p(y(k) | \theta, y^{k-1}) &= (2\pi v(k))^{-1/2} \exp\{-[1/2v(k)] [y(k) \\ &\quad - \theta^T \phi(k)]^2\} \end{aligned} \quad (3-18)$$

Combining eq. (3-8) and eq. (3-18) via Bayes rule yields:

$$\begin{aligned}
p(\theta|y^k) &= C[v(k)]^{-1/2} (2\pi)^{-\dim(\theta/2)-1/2} \sum_{i=1}^{nm} a_i(k) [\det(P_i(k-1))]^{-1/2} \\
&\quad \exp(-1/2[\theta - \theta_i(k-1)]^T P_i^{-1}(k) [\theta - \theta_i(k-1)]) \\
&\quad \exp(-[1/2v(k)][y(k) - \theta^T \phi(k)]^2) \tag{3-19}
\end{aligned}$$

Equation (3-9) is now equated with eq. (3-19) to find matrices, $\theta_i(k)$ and $P_i(k)$, and weighting factors, $a_i(k)$, of eq. (3-9). For notational convenience, the following substitutions are made: $P_i = P_i(k-1)$; $P'_i = P_i(k)$; $\theta'_i = \theta_i(k)$; $\theta_i = \theta_i(k-1)$; $a'_i = a_i(k)$; $a_i = a_i(k-1)$.

The exponent terms of eq. (3-9) is:

$$\begin{aligned}
[\theta - \theta'_i]^T P'^{-1}_i [\theta - \theta'_i] &= \theta^T P'^{-1}_i \theta - \\
&2\theta'^T P'^{-1}_i \theta + \theta'^T P'^{-1}_i \theta \tag{3-20}
\end{aligned}$$

and the exponent terms of eq. (3-19) is:

$$\begin{aligned}
[1/v][y - \theta^T \phi]^2 &+ [\theta - \theta_i]^T P_i^{-1} [\theta - \theta_i] \\
&= [y^2 - 2y\theta^T \phi + (\theta^T \phi)^2][1/v] + \theta^T P_i^{-1} \theta - 2\theta_i^T P_i^{-1} \theta \\
&+ \theta_i^T P_i^{-1} \theta_i \tag{3-21}
\end{aligned}$$

inspection of eq. (3-20) and eq. (3-21) now gives:

$$[-2y\phi/v - 2\theta_i P_i^{-1}] \theta = -2\theta'^T P'^{-1}_i \theta \tag{3-22}$$

$$\theta^T[\phi\phi^T/v + P_i^{-1}]\theta = \theta^T P_i'^{-1}\theta \quad (3-23)$$

eq. (3-22) and eq. (3-23) must be satisfied for all θ , hence:

$$\theta'_i = P_i' [P_i^{-1}\theta_i + (1/v)\phi y] \quad (3-24)$$

$$P_i'^{-1} = P_i^{-1} + (1/v)\phi\phi^T \quad (3-25)$$

Replacing P_i^{-1} in eq. (3-24) by eq. (3-25) gives:

$$\theta'_i = \theta_i + (1/v)P_i'^{-1}\phi e \quad (3-26)$$

where, $e = y - \theta^T\phi$

Applying the matrix inversion lemma to eq. (3-25) gives:

$$P_i' = P_i - P_i\phi\phi^T P_i / [v + \phi^T P_i \phi] \quad (3-27)$$

Inserting eq. (3-23) and eq. (3-24) into eq. (3-20) and also using eq. (3-27) gives

$$\begin{aligned} [\theta - \theta_i]^T P_i'^{-1} [\theta - \theta_i] &= \theta^T P_i^{-1} \theta - 2\theta_i^T P_i^{-1} \theta + \\ &(1/v)(\theta^T \phi)^2 - (2/v)\phi^T \theta y + \theta_i^T P_i^{-1} \theta_i + (1/v)(2\theta_i^T + \\ &(1/v)\phi^T P_i y)\phi y - [1/(v + \phi^T P_i \phi)][(\theta^T \phi)^2 \\ &+ 2(1/v)\theta_i^T \phi \phi^T P_i y + (1/v)(\phi^T P_i \phi y)^2] \end{aligned} \quad (3-28)$$

The first five terms of the right hand side of eq. (3-28) are identical to eq. (3-21), except for the term $y^2(1/v)$ in eq. (3-21). Subtracting

the right hand side of eq. (3-28) for the right hand side of eq. (3-21) now gives the remaining part:

$$\begin{aligned}
 & (1/v)y^2 - (1/v)(2\theta_i^T + (1/v)\phi^T P_i y)\phi y + [v + \phi^T P_i \phi]^{-1} \{ (\theta^T \phi)^2 + \\
 & 2(1/v)\theta_i^T \phi \phi^T P_i \phi y + (1/v)^2 (\phi^T P_i \phi y)^2 \} \\
 & = (y - \theta_i^T \phi)^2 / [v + \phi^T P_i \phi] = e_i^2 / [v + \phi^T P_i \phi]
 \end{aligned}$$

Eq. (3-19) can now be written:

$$\begin{aligned}
 & C v^{-1/2} (2 \quad)^{-\dim\{\theta\}/2 - 1/2} \sum_{i=1}^m a_i \exp\{- (1/2) e_i^2 [v + \phi^T P_i \phi]\} \\
 & \det\{P_i^{-1/2}\} \exp\{-1/2[\theta - \theta'_i]^T P_i^{-1} [\theta - \theta'_i]\} \quad (3-29)
 \end{aligned}$$

Comparing with eq. (3-9) gives:

$$a'_i = C a_i [\det\{P'_i\} / \det\{P_i\}]^{1/2} \exp\{-1/2 e_i^2 [v + \phi^T P_i \phi]\} \quad (3-30)$$

Using

$$\begin{aligned}
 \det\{P'_i\} & = \det\{P_i - P_i \phi \phi^T P_i / [v + \phi^T P_i \phi]\} \\
 & = \det\{P_i\} [1 - \phi^T P_i \phi / [v + \phi^T P_i \phi]] \quad (3-31)
 \end{aligned}$$

gives

$$a'_i = C a_i [v / [v + \phi^T P_i \phi]]^{1/2} \exp\{- (1/2) e_i^2 / [v + \phi^T P_i \phi]\} \quad (3-32)$$

Hence, a'_i can be obtained by first computing

$$\underline{a}_i = a_i[v + \phi^T P_i \phi]^{-1/2} \exp\{-1/2 e_i^2 / [v + \phi^T P_i \phi]\} \quad (3-33)$$

for $i = 1, \dots, m$ and then setting

$$a'_i = \left[\sum_{j=1}^m a_j \right]^{-1/2} \underline{a}_i \quad (3-34)$$

This section developed the MMA as a method of incorporating available a-priori information into an adaptive control process. The MMA was shown to be similar to a multiple number of RLS estimators running in parallel. This observation is used in the following section to develop several mechanizations of the MMA used for simulated on-line adaptive flight control.

IV. Parameter Estimator Design - Implementing the MMA

Figure (IV-1) [15] shows a two-level mechanization of a MMA parameter estimator which incorporates a-priori information for use in in-flight simulation. A-priori information is stored within a parallel bank of secondary estimators. Each estimator initially contains a unique set of parameters modeling the aircraft operation at a nominal flight condition. Secondary estimates and modeling errors from each estimator are used to form a primary estimate of the best fitting model parameters. The primary estimates are then filtered and used to calculate the control law gains.

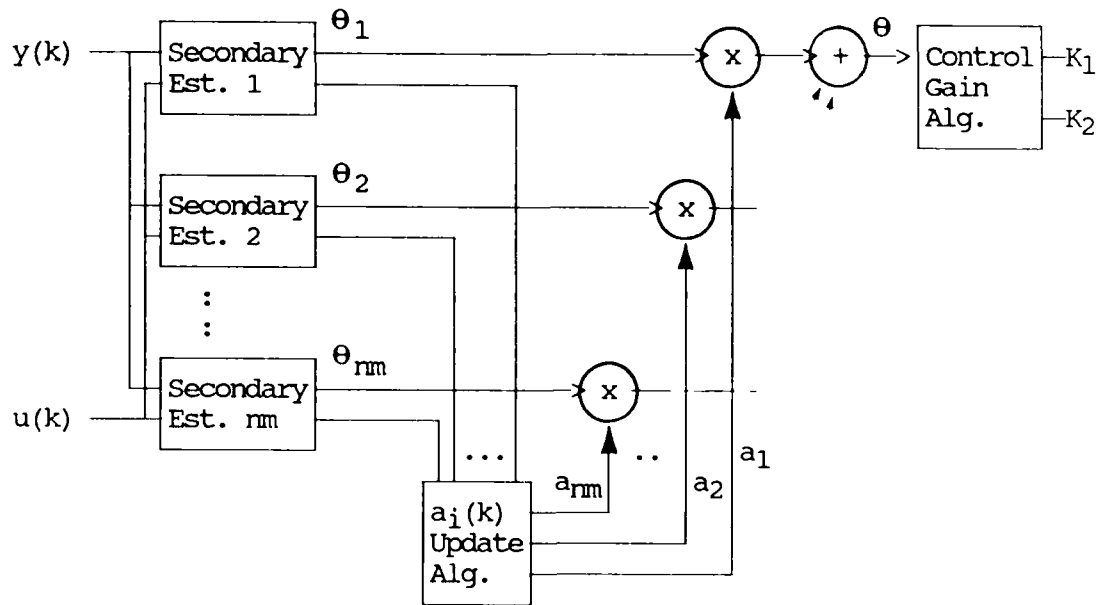


Figure (IV-1) MMA Mechanization

IV.A. Secondary Estimator Structure

Comparison of eq. (2-28) and eq. (2-29) with eq. (3-10) and eq. (3-11) shows the structure of the secondary parameter estimate ($\theta_i(k)$) update is that of the RLS algorithm. Therefore, eqs. (3-10) and (3-11) can also be substituted with Hagglund's Algorithm. The number of parameters which need be updated via a recursive parameter estimation routine is another design option. A comparison of estimating all, some (as in Pineiro's simulation), or none of the secondary parameters will be made.

IV.A.1. Full-Scale Secondary Parameter Estimation

A number of problems are associated with recursively updating all secondary parameters according to eq. (3-11) at every sampling period. As the number of inputs, outputs, and the order of the linear difference equation model increased, the computational effort increases. Any more than two inputs and two outputs is impractical due to long convergence time and computation time. Also, the more parameters which must be updated, the more excitation the system requires for parameter convergence. Another problem concerns convergence to the same operating point. Even though each estimator may begin at a different operating point set from a-priori information, the estimators will eventually converge to the same model [13] which makes the MMA redundant.

IV.A.2. Partial Secondary Parameter Estimation

The secondary estimators may be partially estimated as explained in

Section 2. This method requires much less computational effort than full-scale parameter estimation. Also, since fewer parameters must be estimated, they should converge faster. The common convergence problem of each estimator mentioned with full-scale estimation is also avoided since some parameters remained fixed.

IV.A.3. Fixed Secondary Parameter Estimation

The designer also has the option of leaving all parameters fixed at a nominal flight condition, skipping secondary parameter estimation altogether. This method is very simple, requiring relatively little computational effort. Its only major drawback is that it is only able to estimate parameters at discrete flight conditions. This is acceptable, however, given a robust control law.

IV.B. MMA Implementation

Output coupling effects upon parameter estimation were neglected in all simulations. Therefore the MMA weighting coefficient update eq. (3-12) is modified as a scalar update equation:

$$a_i(k) = C_S a_i(k-1) [\text{diag}\{v_i(k)\}]^{-1/2} \exp\{-1/2 \text{SUM}[e^2_{i,j}(k) / v_{i,j,j}(k)]\} \quad (4-1)$$

$$v(k) + \phi^T(k) P_i(k) \phi(k) = v_i(k) = \begin{bmatrix} v_{i,1,1}(k) & \cdots & v_{i,1,n}(k) \\ \vdots & & \vdots \\ v_{i,n,1}(k) & \cdots & v_{i,n,n}(k) \end{bmatrix} \quad (4-2)$$

$$e_i(k) = \begin{bmatrix} e_{i,1}(k) \\ \vdots \\ e_{i,n}(k) \end{bmatrix} \quad (4-3)$$

IV.C. Weighting Coefficient and Prediction Error Variance Estimation

Inspection of eq. (3-12) shows that the weighting coefficient update is a Gaussian distribution where:

$$a_i(k)/C_S a_i(k-1) = p(e_i) = {}^*N(0, v_i(k)) \quad (4-4)$$

in where

$$v_i(k) = v(k) + \phi^T(k) P_i(k) \phi(k) \quad (4-5)$$

Estimates of $v_i(k)$ for use in eq. (3-10) through eq. (3-12) can be based on either $e_i(k)$ or $P_i(k)$, for example

$$v_i^{est}(k) = (1/k) \sum_{ii=1}^k (e_i(ii) e_i^T(ii)) \quad (4-6)$$

$$v_i^{est}(k) = v(k) + \phi^T(k) P_i(k) \phi(k) \quad (4-7)$$

where

$$v(k) = (1/k) \sum_{ii=1}^k (e_j(ii) e_j^T(ii)) - \phi^T(k) P_j(k) \phi(k) \quad (4-8)$$

and j is the argument of the largest $a_i(k)$ and $v_i^{est}(k)$ is an estimate of $v_i(k)$. Eq. (4-6) is based on $e_i(k)$ and is similar to the estimate of $v(k)$ in Section 2. Eq. (4-7) requires a noise variance term, $v(k)$, which is calculated in eq. (4-8) and is the same for each weighting

* $N(m, Var)$ = Normal (Gaussian) distribution with mean m and variance, Var .

coefficient. Differences in $v_i(k)$ are based on differences in $P_i(k)$.

A physical interpretation of the two terms in eq. (4-7) is that of a separation of modeling and noise errors. Define the parameter error as

$$w_i(k) = \theta_i(k) - \theta(k) \quad (4-9)$$

then

$$e_i(k) = y(k) - \phi^T(k)\theta(k) - \phi^T(k)w_i(k) \quad (4-10)$$

The first two terms represent noise due to actuators, sensors and linear approximation errors. The third term is an additional model error term induced by the model not being the best fitting model. Therefore when neglecting cross-correlation terms and dropping the time index, k

$$E\{e_i e_i^T\} = E\{[y - \phi^T\theta][y - \phi^T\theta]^T\} + E\{[\phi^T w_i][\phi^T w_i]^T\} \quad (4-11)$$

$$= E\{[y - \phi^T\theta][y - \phi^T\theta]^T\} + E\{\phi^T w_i w_i^T \phi\} \quad (4-12)$$

$$= E\{[y - \phi^T\theta][y - \phi^T\theta]^T\} + \phi^T E\{w_i w_i^T\} \phi \quad (4-13)$$

$$= v(k) + \phi^T P_i \phi \quad (4-14)$$

An alternative to eqs. (4-6) and (4-7) is to implement a constant $v_i(k)$ in eqs. (3-10) - (3-12). Andersson [2] claims the MMA is relatively insensitive to $v_i^{\text{est}}(k)$ and a close guess may be good enough when $v(k)$ is constant. Andersson's simulations show acceptable results for an assumed value of $v(k)$

Though commonly used in literature, "noise variance" is a

misleading name for $v(k)$ since $v(k)$ is composed not only of sensor noise variance but also of linear approximation errors. In practical situations, sensor noise will be much larger than linear approximation errors and the linear approximation errors are neglected. In computer simulations which assume no sensor noise, however, linear approximations errors can not be neglected.

IV.D. Improving $v_i^{est}(k)$ Via Input/Output Filtering

Input-output filtering is necessary in adaptive control applications in order to estimate and track $v_i(k)$. Filters which estimate $v_i(k)$ may be limited by the assumption that $v_i(k)$ is relatively slowly time varying. Low-pass filtering has the effect of reducing the variation in time of $v(k)$. Since $v_i(k)$ is a function of $\phi(k)$, reducing the high frequency variation in $u(k)$ and $y(k)$ will reduce the variation of $v_i(k)$. A high-pass filter may be required to remove parameter estimate bias when $u(k)$ is non-zero mean [10,20]. Although high-pass filtering may not be useful in conditioning $v_i(k)$, its possible bias on $v_i(k)$ should be considered upon its implementation.

IV.E. Convergence Analysis

Although it is difficult to make any general conclusions about parameter estimate convergence time given certain assumptions a convergence analysis may be possible and useful when analysing simulation data. For example, a convergence analysis of the MMA for the assumption that the prediction error variance estimates are to

large and time invariant may provide information relating to the sensitivity of the algorithm to the value of the estimated prediction error variance.

Example 4-1

This example calculates an approximate convergence time for a SISO adaptive simulation which uses a 2-model MMA estimator. Assume the MMA weighting coefficients have reached steady state values $a_1(k-n) = 0.01$ and $a_2(k-n) = 0.99$. A maneuver is then simulated by changing the plant dynamics such that at time kT the weighting coefficients have changed to $a_1(k) = 0.99$ and $a_2(k) = 0.01$. In such case, the MMA weighting coefficient update equations are

$$a_1(k) = [a_1(k-1) / [a_1(k) + a_2(k)]] v_1^{-1/2} \exp(e_1^2(k) / v_1(k)) \quad (4-15a)$$

$$a_2(k) = [a_2(k-1) / [a_1(k) + a_2(k)]] v_2^{-1/2} \exp(e_2^2(k) / v_2(k)) \quad (4-15b)$$

If $E(e_1^2(k)) \gg v_1(k)$ and $E(e_2^2(k)) \gg v_2(k)$ then eq. (4-15) simplifies such that

$$a_1(k) = [a_1(k-1) / [a_1(k) + a_2(k)]] v_1^{-1/2} \quad (4-16a)$$

$$a_2(k) = [a_2(k-1) / [a_1(k) + a_2(k)]] v_2^{-1/2} \quad (4-16b)$$

and

$$\frac{a_1(k)}{a_2(k)} = \frac{a_1(k-1)v_2^{1/2}}{a_2(k-1)v_1^{1/2}} = \left[\frac{a_1(k-n)}{a_2(k-n)} \right] \left[\frac{v_2^{n/2}}{v_1^{n/2}} \right] \quad (4-17)$$

$$n/2 = \frac{\log[a_1(k)a_2(k-n)/a_2(k)a_1(k-n)]}{\log[v_2(k)/v_1(k)]} \quad (4-18)$$

IV.F. System Filters

Three filters are incorporated into the system shown in Figure (IV-1) to smooth variations in the parameter estimates. The first is a digital band-pass filter which filters $y(k)$ and $u(k)$. The non-linear second filter is added to the weighting coefficients within the MMA. It smooths sudden changes in the model probabilities which cause destabilizing rapid changes in the final parameter estimates. The third filter smooths estimates before entering the control law algorithm.

IV.F.1. Input-Output Filter

The input-output signals are filtered by a band-pass filter. The low frequency components of these signals must be removed to reduce parameter estimate bias, while high frequency signal components are removed to smooth input excitation [6,10,11,20]. The filter is mechanized as a sixth-order butterworth digital band-pass filter.

IV.F.2. Weighting Coefficient Filter

The weighting coefficients, $a_i(k)$, are filtered to limit the rate at which they can change in a given sampling period. This corresponds to a limit of the amount of old information which is thrown away from $a_i(k)$ during an update. Large variations in $a_i(k)$ in turn cause large variations in the parameter estimates which is destabilizing.

IV.F.3. Rate Limiting Filter Design

A rate limiting filter, which satisfies the listed design criteria and requires no renormalization, limits the change from $a_i(k-1)$ to $a_i(k)$ such that:

$$a_{i,fil}(k) = a_i(k-1) + c[d_{a_i}(k)] \quad (4-19)$$

where

$$d_{a_i} = a_i(k) - a_i(k-1) \quad \text{and}$$

$$c = \begin{cases} 1 & \text{if } a_{max} \leq a_j \\ a_{max}/a_j & \text{if } a_{max} \geq a_j \end{cases}$$

and

j = argument of the maximum d_{a_i}

a_{max} = a constant representing the largest value of $d_{a_i}(k)$ permitted.

Summary

Implementing the MMA requires developing a specific mechanization and choosing appropriate filters. The MMA is very flexible in that it contains many application dependent methods of calculating key variables such as secondary parameter estimates and prediction error variances. The following section will experimentally examine many of these issues.

V. Experimental Design

V.A. Objectives

The objective of this section is to experimentally determine which of the mechanizations developed in Section 4 works best for in-flight simulation by judging them on their effect on three main design criteria for the MMA. Convergence to the best fitting model is the first objective. Even though the control law is robust, the controller's tracking ability will degrade given biased estimates. Fast parameter tracking is the second objective. Slow parameter changes which occur when the aircraft's flight condition changes must be followed by the estimator. The third and final objective is for the parameter estimates to be insensitive to sensor noise. In other words, the first two objectives hold when a realistic amount of sensor noise is added.

V.B. Computer Implementation

Simulations are run on a VAX 11/780 digital computer using Fortran subroutines interfaced with the Matrix_x computer-aided control design software package [9].

V.C. Set-Up

The simulations approximate the motion of an *AFTI F-16 aircraft by linearized longitudinal equations of motion given in state space

* The VISTA, currently under development, will be represented by the AFTI F-16 which has many of the control performance features of the VISTA. The AFTI F-16 is discussed in appendix.

form in appendix B. Assume at time $t=0$ the host is tracking *itself at 0.60 Mach, 10,000 ft. MSL. The reference tracking signals are from a real-time, non-linear simulation [19,20] with elevators and flaperons as control surfaces with flight path angle and pitch rate as outputs. At time $t = 6$ seconds the linearized equations are changed to approximate 0.31 Mach, 10,000 ft. MSL to represent a worst case parameter change. The MMA uses 2 estimators. The linear difference equation parameters of estimator 1 are initialized at 0.60 Mach, 10,000 ft. MSL (appendix B). The parameters of estimator 2 are initialized at 0.31 Mach, 10,000 ft. MSL. The weighting coefficients, $a_i(k)$, initially are each 0.5. These quantities are fixed until the algorithm is turned on at $t = 2$ seconds.

V.D. Sensor Noise

Sensor noise is considered in some simulations. The form and magnitude of the noise is discussed in appendix B.

V.E. Tuning Hagglund's Algorithm

Since fault detection is not being used the asymptotic covariance variable, a , is the only important tuning parameter which needs discussion. Hagglund's algorithm (appendix C) seeks to force the parameter covariance matrix, $P_i(k)$, to converge to " aI ", where " I " is the identity matrix, when parameters remain time invariant. Therefore since an initial estimate of the parameter vector is available a-

* The model aircraft in each simulation was the same as the host aircraft to avoid implementing model-following equations. Model-following theory for in-flight simulation is well established in the literature [20].

priori, a value of "a" should be based upon it. Note also since "a" directly effects the asymptotic parameter covariance it can be used to adjust the sensitivity of the estimator. Reducing the value of "a" reduces the parameter estimate noise while increasing "a" allows the estimator to respond more quickly to changes in aircraft dynamics.

V.F. Parameter Estimator Variations

Table (V-1) lists ten possible variations of the estimator shown in Figure (IV-1). For the advantages shown, this experiment is limited to method 1 of estimating the prediction error variance in conjunction with partial secondary estimation with Hagglund's algorithm and no secondary parameter estimation.

Table (V-1) Comparison of Secondary Parameter Estimation Methods

Secondary Estimation	Method 1 for Estimating $v_j(k)$	Method 2 for Estimating $v_j(k)$
Full Scale RLS	-High Computation Cost -Parallel Estimators Converge to Same Operating Point	-High Computation Cost -Parallel Estimators Converge to Same Operating Point
Full Scale Hagglund	-High Computation Cost -Impractical for High Order -Parallel Estimators Converge to Same Operating Point	-High Computation Cost -Same as Method 1 and -Modifications to P destroy goodness of fit information
Partial RLS	-Acceptable Computation Cost Continuous Envelope Coverage	-P only Partially Estimated
Partial Hagglund	-Same as Partial RLS but better Wind-up Resistance	-Same as Partial RLS but better Wind-up Resistance
No Update	-Small Computation Costs -Discrete Operating points only.	-Not Possible, P not estimated

V.G. Estimator Set-up

The experiment consists of four simulations of the system, each with a different method of parameter estimation:

Simulation 1 - Partial secondary parameter estimation via Hagglund's algorithm, error variance estimation via method 1, no noise.

Simulation 2 - Partial secondary parameter estimation via Hagglund's algorithm, error variance estimation via method 1, with noise.

Simulation 3 - No secondary parameter estimation, error variance estimation via method 1, no noise

Simulation 4 - No secondary parameter estimation, error variance estimation via method 1, with noise.

V.H. Multiple Model Estimator Performance

Simulation data shows the MMA is useful for adaptive control for in-flight simulation by meeting the stated objectives. Plots of parameter estimates show parameter convergence and tracking after a jump change in host aircraft dynamics. Output data plots show almost perfect tracking of the model aircraft output by the host aircraft.

V.I. Comparative Estimator Variation Performance

Examination of experimental data provides several important guidelines for using MMA variations shown in Table (V-1).

- 1) Prediction error variance estimation method 1 (section 3) is preferable to method 2 in most instances. This is especially true if Hagglund's algorithm is used for secondary parameter estimation.
- 2) Secondary parameter estimates don't converge in general to the best fitting model when partial parameter estimation is used.
- 3) The key to MMA performance is the prediction error variance estimates, $v_i(k)$. If prediction error distributions of each model have similar variances, the weighting coefficients, $a_i(k)$, may converge to the wrong model.

- 4) The MMA's tolerance to sensor noise is dependent on input excitation. Sensor noise variance must remain smaller than model error variance to ensure probability convergence.
- 5) Proper input/output filtering is essential.
- 6) Weighting coefficients, $a_i(k)$, tend to converge to unity for which ever model has the smallest prediction error variance estimates.
- 7) Input excitation has a direct effect on the prediction error variance due to model error.

Guideline 1

Plots of elements of $P_i(k)$ illustrate the problem associated with simultaneously using method 2 to estimate $v_i(k)$ when using Hagglund's algorithm for secondary parameter estimation. Notice the plots of element (1,1) of various $P_i(k)$ matrices, which illustrate that in general, the $P_i(k)$ matrix whose argument corresponds to the best fitting model has the smallest elements. Hagglund's algorithm, however, forces the diagonal elements of $P_i(k)$ to converge to a predetermined constant (see discussion of Hagglund's Algorithm parameter "a" in Appendix C). If the diagonal elements of the estimated $P_i(k)$ matrix converge to this constant in both models, the portion of the prediction error variance due to modeling errors will be biased, degrading the performance of the MMA.

Guideline 2

When using partial parameter estimation, secondary estimates of B_1 do not converge to the B_1 associated with the best fitting model. Instead, they tend to vary around the B_1 associated with same model as the fixed parameters. This behavior is illustrated in plots of

secondary B_1 estimates from Simulations 1 and 2.

The bias in B_1 means parameter estimates will be limited to points on the flight envelope for which the secondary models are calculated. A robust control law, however, should still perform adequately provided the models are not spread too far apart within the flight envelope. Model placement is an important issue and is recommended as an area of continued research.

Guideline 3

The weighting coefficients of the MMA converge to the model with the smallest prediction error variance estimate. Convergence behavior is difficult to determine when prediction variance estimates are of similar magnitude. Notice the plots of the prediction error variance in Simulation 2 and 4. Around 7 seconds, when little input excitation occurs and the variance of each model is similar, the probability calculation does not converge.

Guideline 4

The prediction error variance due to modelling error must be larger than that due to sensor noise in order for the weighting coefficients calculations to converge to the best fitting model. If the prediction errors are due more to sensor noise than modeling error, then the estimated prediction error variance will be almost the same for each model. Then, from guideline 3, probability calculations are suspect. This behavior is illustrated in prediction error variance estimates in Simulation 2 and 4.

Guideline 5

Proper input/output filtering is essential to proper MMA operation. Prediction error noise spikes may cause spikes in prediction error variance estimates which may cause weighting coefficients to diverge. This behavior is observed in simulations (not shown) where input and output measurements are not filtered. Heavy filtering, however, discards good information by reducing the difference between filtered prediction error variances. Filtering also causes a "coloring" of the prediction errors [15]. No quantitative method for choosing an input-output filter has been developed so trial and error is used to choose the band width of the band-pass filter used in these simulations.

Guideline 6

Weighting coefficients tend to converge very quickly, with the weighting coefficient of the best fitting model converging to unity. This behavior is demonstrated in simulations (not shown) where the actual operating point is between the two points used to calculate initial secondary parameter estimates.

Guideline 7

The theoretical dependence of the input on the prediction error variance due to modeling error is discussed in Section 4. Comparison of the input signal and the prediction error variance plots verifies this relationship.

Table (V-2) List of Plots - Experimental Results

<u>PAGE</u>	<u>UPPER</u>	<u>LOWER</u>
P-1	Secondary estimated B ₁ , element 1,1 - Sim. 1	Secondary estimated B ₁ , element 1,1 - Sim. 2
P-2	Primary estimated B ₁ , element 1,1 - Sim. 1	Primary estimated B ₁ , element 1,1 - Sim. 2
P-3	Primary estimated B ₁ , element 1,1 - Sim. 3	Primary estimated B ₁ , element 1,1 - Sim. 4
P-4	Primary estimated B ₁ , element 2,2 - Sim. 1	Primary estimated B ₁ , element 2,2 - Sim. 2
P-5	Primary estimated B ₁ , element 2,2 - Sim. 3	Primary estimated B ₁ , element 2,2 - Sim. 4
P-6	P _i , element 1,1 - Sim. 1	P _i , element 1,1 - Sim. 2
P-7	P _i , element 2,2 - Sim. 1	P _i , element 2,2 - Sim. 2
P-8	v _i , element 1,1 - Sim. 1	v _i , element 1,1 - Sim. 2
P-9	v _i , element 1,1 - Sim. 3	v _i , element 1,1 - Sim. 4
P-10	v _i , element 2,2 - Sim. 1	v _i , element 2,2 - Sim. 2
P-11	v _i , element 2,2 - Sim. 3	v _i , element 2,2 - Sim. 4
P-12	a ₁ , Sim. 1	a ₁ , Sim. 2
P-13	a ₁ , Sim. 3	a ₁ , Sim. 4
P-14	Flight path angle Commanded and actual Sim. 1	Flight path angle Commanded and actual Sim. 2
P-15	Flight path angle Commanded and actual Sim. 3	Flight path angle Commanded and actual Sim. 4
P-16	Pitch rate Commanded and actual Sim. 1	Pitch rate Commanded and actual Sim. 2
P-17	Pitch Rate Commanded and actual Sim. 3	Pitch Rate Commanded and actual Sim. 4

VI. Conclusion

The purpose of this thesis is to use the MMA for parameter identification used in adaptive control of an in-flight simulator. The thesis is an extension of Pineiro's thesis [20].

Pineiro sought to control an in-flight simulator via an adaptive model-following PI control law [20-24]. The control law bases its control gains upon the parameters of a linear difference equation model which describes the input/output relationship of the host aircraft's dynamics. The parameters of this model are estimated from input and output measurements via a recursive parameter estimation algorithm.

This thesis differed from Pineiro's in that the recursive parameter estimation algorithm is replaced by the MMA to increase convergence speed to the best fitting model parameters. Slow convergence is a critical problem for recursive parameter estimation algorithms when used with higher order systems such as those typically required to describe aircraft dynamics.

A computer simulation is performed which shows excellent results. The MMA converge quickly to the best fitting model. The host aircraft tracks the model aircraft accurately. Degraded but acceptable performance results when sensor noise was added.

Experimental data also reveals useful information on tuning the MMA for actual use. The MMA is flexible and can be implemented in several ways. Implementation options are discussed and performance compared.

Recommendations for Further Study

Optimal placement within the flight envelope is one example research area which needs to be investigated. The computational cost of adding more a-priori models to the simulation is a trade-off of more accurate parameter estimation. A moving bank scheme [16] which varies the models used in the MMA may be used to reduce the number of models needed to cover the entire envelope.

More research also is required to decrease the MMA's sensitivity to sensor noise. One possibility is to turn the MMA off during noisy periods and when the system is receiving little input excitation.

Appendix A - The AFTI/F-16

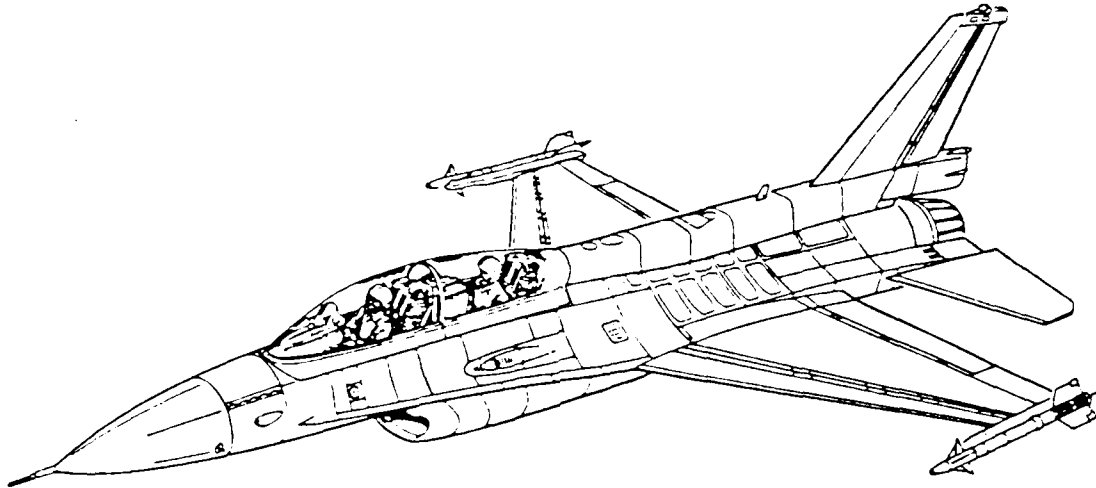


Figure (A-1) AFTI F-16

The AFTI (Advanced Fighter Technology Integration)/F-16 aircraft, shown in Figure (A-1), is an F-16A air superiority fighter modified to be a test bed for evaluating new aircraft technologies. These modifications include the addition of two vertical canards which are mounted on the engine inlet, control surfaces that allow independent motion of the trailing edge flaps and independent motion of the horizontal tail halves, and a redundant, digital fly-by-wire flight control system.

The unaugmented AFTI/F-16 is statically unstable in the longitudinal axis for subsonic flight since the center of gravity is located behind the aircraft's center of gravity. The instability, manifested by an unstable short period root, allows the aircraft to withstand higher load factors and reduces drag. Additionally, the aircraft has a lightly damped dutch roll mode. Therefore the flight control system has a twofold purpose: to stabilize the aircraft longitudinally and to improve the dutch roll damping.

The AFTI/F-16 can perform conventional maneuvers, unconventional maneuvers, or both simultaneously. Conventional maneuvers include pitching longitudinally, rolling laterally, and turning with zero sideslip. Unconventional maneuvers require decoupling of the aircraft's forces and moments and include pitch-pointing, yaw pointing, and lateral and longitudinal translation.

Control surfaces which maneuver the AFTI/F-16 are shown in Figure (A-1). The horizontal tail halves perform a dual function in that they can be deflected symmetrically as elevators to pitch the aircraft, or they can be deflected asymmetrically to augment rolling. Likewise, the flaperons can be deflected symmetrically to function as conventional flaps or asymmetrically to function as ailerons to control roll. The canards act either as a speed brake or to provide side forces, while the rudder is used for yawing.

Primary mission tasks involve flight maneuvers which are divided into four flight control configuration modes. The first is referred to as the normal mode and is used for takeoff, cruise, landing, and air refueling tasks. The second is the air-to-air gunnery mode, used for precise target pointing and evasive maneuvering. The third is the air-to-surface gunnery mode, used for precise pointing needed for strafing ground targets. The final configuration is the air-to-surface bombing mode used for precise velocity control required for accurate bombing. The aircraft models used in this thesis were obtained when the aircraft was in the normal mode.

Appendix B - Model Data

Host Aircraft Model - The AFTI F-16

These are the continuous time state space model approximating the equations of motion used in the simulation:

$$\begin{aligned}
 u(t) &= [u_1(t) \ u_2(t)]^T & u_1(t) &= \text{elevator deflection (degrees)} \\
 & & u_2(t) &= \text{flaperon deflection (degrees)} \\
 y(t) &= [y_1(t) \ y_2(t)]^T & y_1(t) &= \text{flight path angle (degrees)} \\
 & & y_2(t) &= \text{pitch rate (degrees/sec)} \\
 x(t) &= [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T & x_1(t) &= \text{pitch angle (degrees)} \\
 & & x_2(t) &= \text{perturbation velocity (ft/sec)} \\
 & & x_3(t) &= \text{angle of attack (degrees)} \\
 & & x_4(t) &= \text{pitch rate (degrees/sec)}
 \end{aligned}$$

$$C = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{output matrix}$$

Nominal Flight Condition = 10,000 ft., MACH .31

$$A = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ -3.1884D+1 & -1.2236D-2 & 1.7789D+1 & -4.5361D+1 \\ -1.2770D-2 & -2.9100D-4 & -4.8935D-1 & 0.999914 \\ 7.5310D-4 & 6.0000D-5 & 1.78948 & -3.8710D-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00000 & 0.00000 \\ 1.66435 & -4.29844 \\ -7.70780D-2 & -6.91360D-2 \\ -3.25199 & 3.25307D-1 \end{bmatrix}$$

Nominal Flight Condition = 10,000 ft., MACH .60

$$A = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ -3.2046D+1 & -1.5269D-2 & 2.3608D+1 & -3.6205D+1 \\ -7.0900D-3 & -2.0800D-4 & -6.4800D-1 & 0.999924 \\ 5.8700D-4 & 1.3220D-4 & 2.28298 & -4.1422D-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00000 & 0.00000 \\ 2.01664 & -1.30775 \\ -9.31110D-2 & -1.37395D-1 \\ -4.72071 & -2.30380D-1 \end{bmatrix}$$

Model Structure

$$y(k) = -A_1y(k-1) - A_2y(k-2) - A_3y(k-3) - A_4y(k-4) \\ + B_1u(k-1) + B_2u(k-2) + B_3u(k-3) + B_4u(k-4) + e(k)$$

Parameters at Nominal Flight Condition: .60 MACH; 10,000 ft. MSL

<u>Matrix</u>	Element <u>(1,1)</u>	Element <u>(1,2)</u>	Element <u>(2,1)</u>	Element <u>(2,2)</u>
B1	1.411739D-3	2.386221D-3	-1.324101D-1	-1.311730D-2
A1	-3.982372	0.0	0.0	-3.982372
B2	-4.241853D-3	-7.145616D-3	3.956153D-1	3.907266D-2
A2	5.946657	0.0	0.0	5.946657
B3	4.232493D-3	7.129810D-3	-3.940006D-1	-3.879344D-2
A3	-3.946199	0.0	0.0	-3.946199
B4	-1.402381D-3	-2.370406D-3	1.307954D-1	1.283808D-2
A4	9.819136D-1	0.0	0.0	9.819136D-1

Parameters at Nominal Flight Condition: .31 MACH; 10,000 ft. MSL

<u>Matrix</u>	Element <u>(1,1)</u>	Element <u>(1,2)</u>	Element <u>(2,1)</u>	Element <u>(2,2)</u>
B1	1.000519D-3	1.504922D-3	-5.588077D-2	-2.567509D-3
A1	-3.988486	0.0	0.0	-3.988486
B2	-3.001350D-3	-4.508592D-3	1.672050D-1	7.644987D-3
A2	5.965235	0.0	0.0	5.965235
B3	2.996832D-3	4.501846D-3	-1.667678D-1	-7.587455D-3
A3	-3.965013	0.0	0.0	-3.965013
B4	-9.960008D-4	-1.498176D-3	5.544359D-2	2.509977D-3
A4	9.882635D-1	0.0	0.0	9.882635D-1

Noise

Sensor noise was generated by a random number generator, where the random number, r_i , fit the bound $0 < r_i < 1$. In other words, $p(r_i) = *U(0,1)$, where $E\{r_i\} = .5$, $E\{r_i^2\} = 1/3$, $\text{Var}\{r_i\} = 1/12$ [17].

A pdf with statistics similar to a gaussian distribution can be generated as

$$W = \sum_{j=1}^{12} (r_j) - 6$$

where $E\{W\} = 0$, $E\{W^2\} = \text{Var}\{W\} = 1$

The variance of W was calculated from the following theorem [17]:

If $\{r_1, r_2, \dots, r_i\}$ is a sequence of independent random variables, then

$$\text{Var}\left(\sum_{j=1}^i (r_j)\right) = \sum_{j=1}^i (\text{Var}(r_j))$$

Simulated sensor noise is made by scaling W with the desired noise variance.

Noise Figures

Zero-mean pseudo gaussian noise of the following strength was added to the state vector:

<u>Variance</u>	<u>State</u>
7.438×10^{-11}	x_1
0.000	x_2
4.896×10^{-10}	x_3
3.404×10^{-9}	x_4

* $p(x) = U(L,U)$ is a uniform distribution, where $p(x) = 0$ for $x < L$, $x > U$ and $p(x) = [U-L]^{-1}$ for $L \leq x \leq U$.

Appendix C - Hagglund's Algorithm for Tracking Slow Parameter

Changes

The goal of the estimator developed by Hagglund is to weight the incoming information so that the covariance matrix becomes proportional to the identity matrix. The diagonal elements of $P(k)$ may be interpreted as approximations of the variances of the corresponding parameters. Therefore, $P^{-1}(k)$ is a measure of "goodness of fit" which should be kept constant. In other words, if no information is in coming nothing is forgotten but if the information content of the current measurement is large, old information is discarded quickly for fast parameter adaptation.

If information is to be discounted according to the new principle the $P(k)$ update equation must be modified. The RLS $P(k)$ update, eq. (2-31), can be alternately expressed as a $P^{-1}(k)$ update

$$P^{-1}(k) = P^{-1}(k - 1) + v^{-1}(k) \phi(k) \phi^T(k) \quad (C-1)$$

Eq. (C-1) discounts old information exponentially in all directions. Hagglund's algorithm modifies eq. (C-1) as

$$P^{-1}(k) = P^{-1}(k - 1) + v^{-1}(k) \phi(k) \phi^T(k) - a(k) \phi(k) \phi^T(k) \quad (C-2)$$

$$= P^{-1}(k - 1) + [v^{-1}(k) - a(k)] \phi(k) \phi^T(k) \quad (C-3)$$

where $a(k)$ is a discounting factor. The new information is proportional to $\phi(k) \phi^T(k)$ and it may be said that the new information

is coming in the direction of $\phi(k)$ while old information is discounted in the same direction. Eq. (C-2) can be transformed via the matrix inversion lemma into a $P(k)$ update equation:

$$P(k) = P(k - 1) - \frac{P(k - 1)\phi(k)\phi^T(k)P(k - 1)}{[v^{-1}(k) - a(k)]^{-1} + \phi^T(k)P(k - 1)\phi(k)} \quad (C-4)$$

Since the form of the covariance matrix update has changed, the equation for updating the parameter estimates will also change. Using eq. (C-4) along with the basic definition of the parameter update equation in the least-squares algorithm, Hagglund derives the following parameter estimate update equation:

$$\theta(k) = \theta(k - 1) + (1/v(k))P(k)\phi(k)e(k) \quad (C-5)$$

It remains to be shown how to select an appropriate discounting factor $a(k)$. Eq. (C-2) and eq. (C-3) show that $a(k)$ must be positive or information would be added instead of removed but if $a(k)$ is too large the covariance matrix could become non-positive definite. Hagglund performed a stability investigation showing that the covariance matrix remains positive definite if $a(k)$ is chosen such that

$$0 \leq a(k) \leq [\phi^T(k)P(k - 1)\phi(k)]^{-1} \quad (C-6)$$

Furthermore, in order to obtain a diagonal P-Matrix of the form "aI"

where "a" is the desired variance of the parameter estimates and "I" is the identity matrix, Hagglund shows that a(k) must be selected so that

$$a(k) = \frac{\phi^T(k)P(k-1)P(k)P(k-1)\phi(k)}{\phi^T(k)P(k-1)P(k-1)\phi(k)} \quad (C-7)$$

substituting eq. (C-4) into eq. (C-7) yields the following desired value of a(k)

$$a_d(k) = v^{-1}(k) + \frac{\text{delta}_d(k)}{\text{delta}_d(k)\phi^T(k)P(k-1)\phi(k) - 1} \quad (C-8)$$

$$\text{delta}_d = \frac{1}{\phi^T(k)P^2(k-1)\phi(k)} \left[\frac{\phi^T(k)P^3(k-1)\phi(k)}{\phi^T(k)P^2(k-1)\phi(k)} - a \right] \quad (C-9)$$

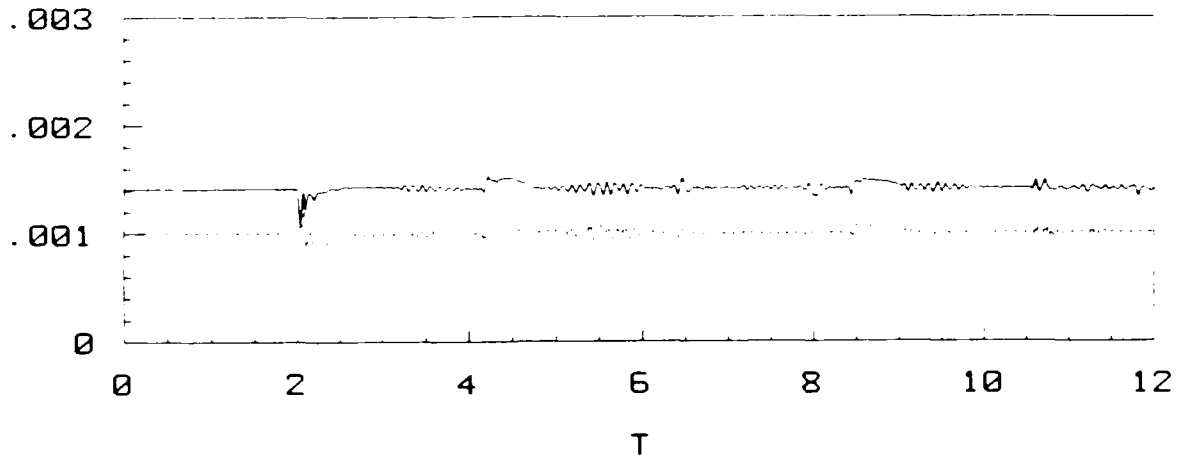
Eq. (C-7) can not always be used, however, due the restrictions given in eq. (C-6). Hagglund shows that by incorporating the bounds of a(k) in eq. (C-6) in conjunction with eq. (C-8) the choice of a(k) becomes

$$a(k) = \begin{cases} 0 & \text{if } a_d(k) \leq 0 \\ a_d(k) & \text{if } 0 < a_d(k) \leq [1/\underline{n}(k)] \\ 1/\underline{n}(k) & \text{if } 1/\underline{n}(k) < a_d(k) \leq v^{-1}(k) + 1/\underline{n}(k) \\ 0 & \text{if } a_d(k) > v^{-1}(k) + 1/\underline{n}(k) \end{cases} \quad (C-10)$$

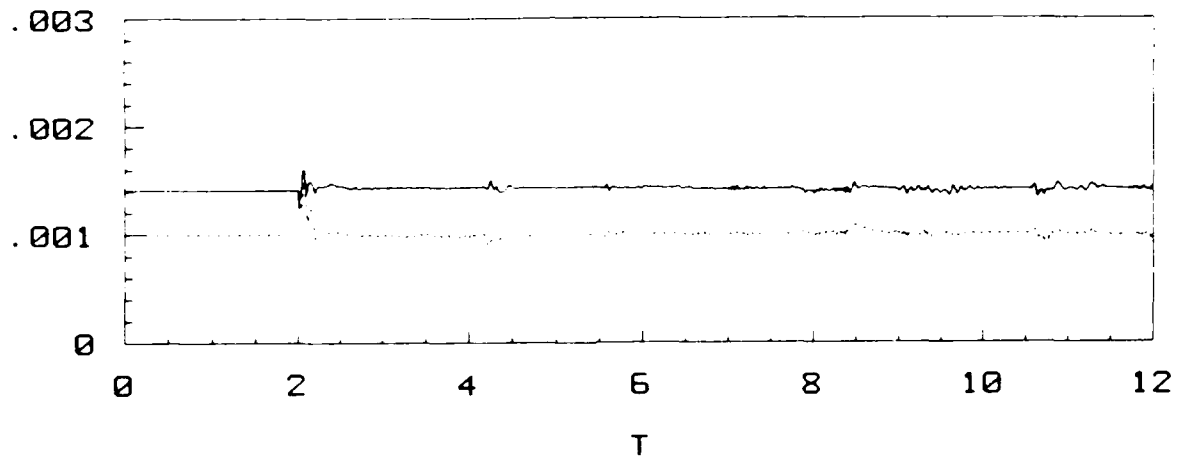
where $\underline{n}(k) = \phi^T(k)P(k-1)\phi(k)$

— Estimator 1
- - - Estimator 2

P-1



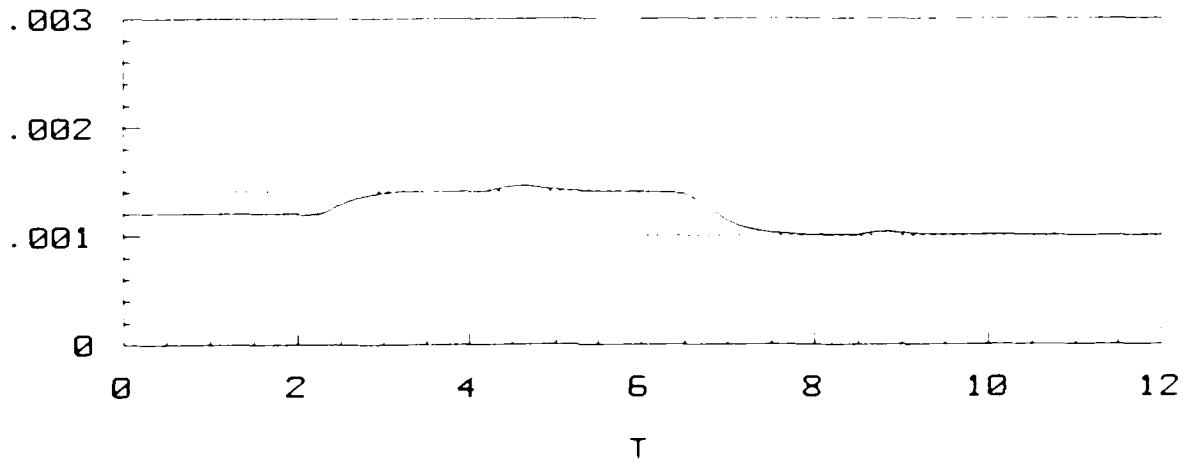
SECONDARY ESTIMATES OF $B_1(1,1)$, SIM 1



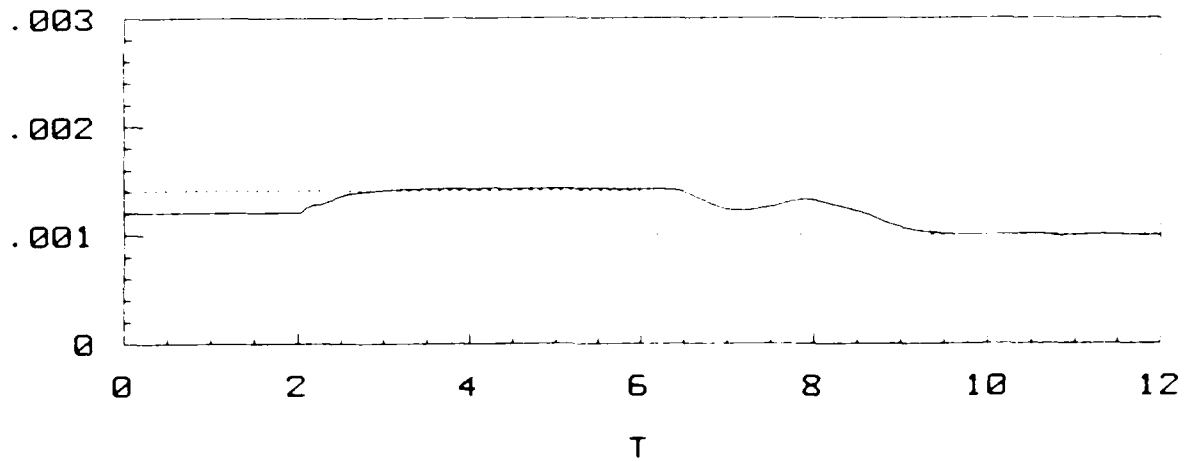
SECONDARY ESTIMATES OF $B_1(1,1)$, SIM 2

— Estimated Parameter
- - - Actual Parameter

P-2



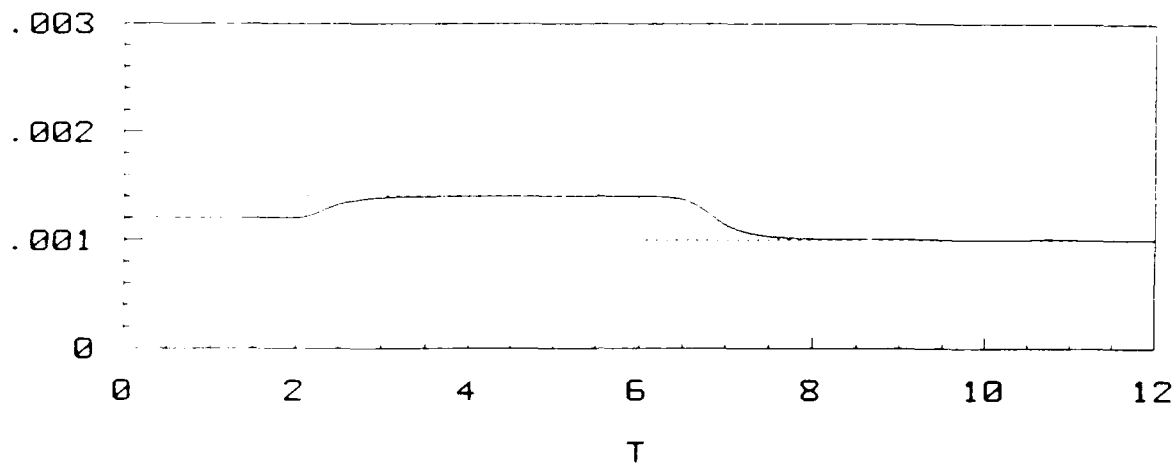
PRIMARY ESTIMATE OF $B1(1,1)$, SIM 1



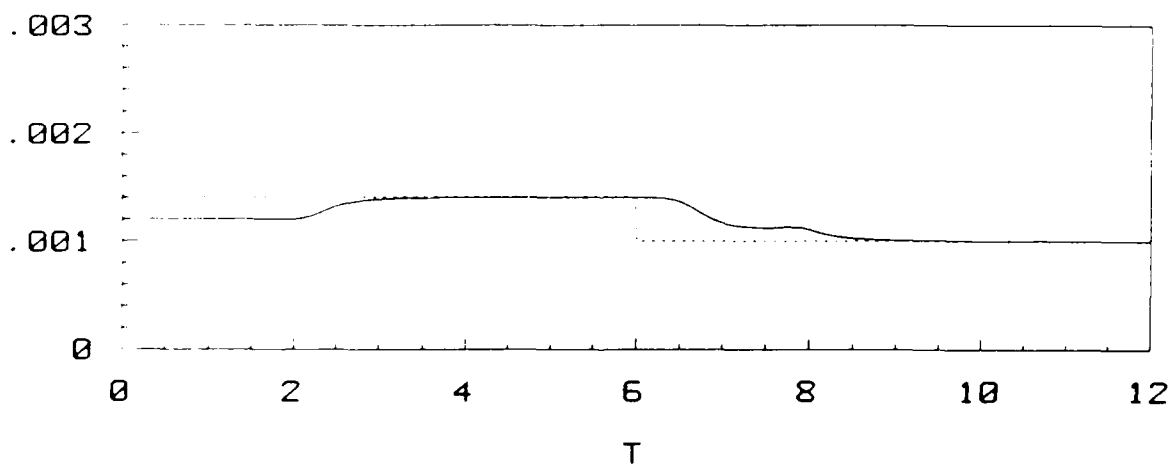
PRIMARY ESTIMATE OF $B1(1,1)$, SIM 2

— Estimated Parameter
- - - Actual Parameter

P-3



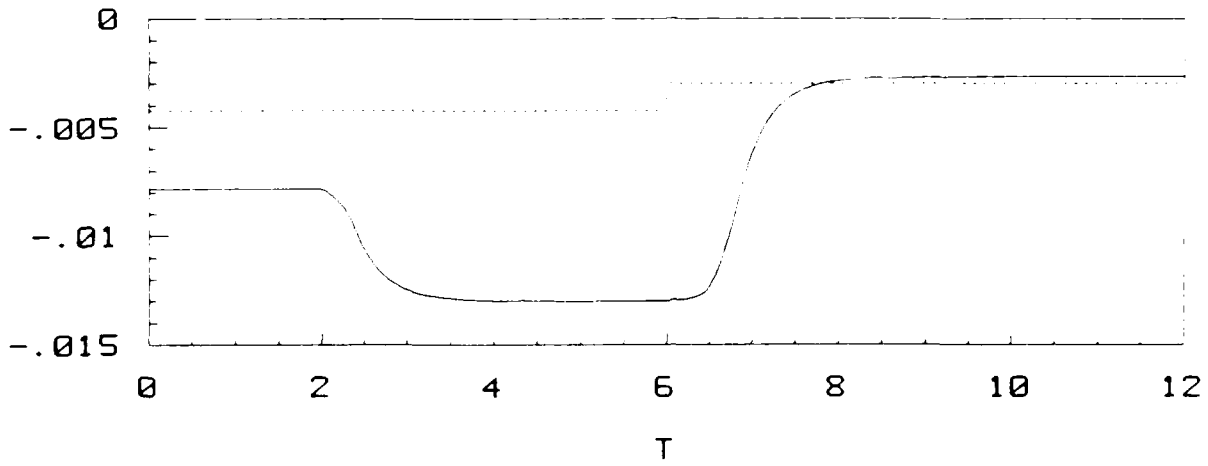
PRIMARY ESTIMATE OF $B1(1,1)$, SIM 3



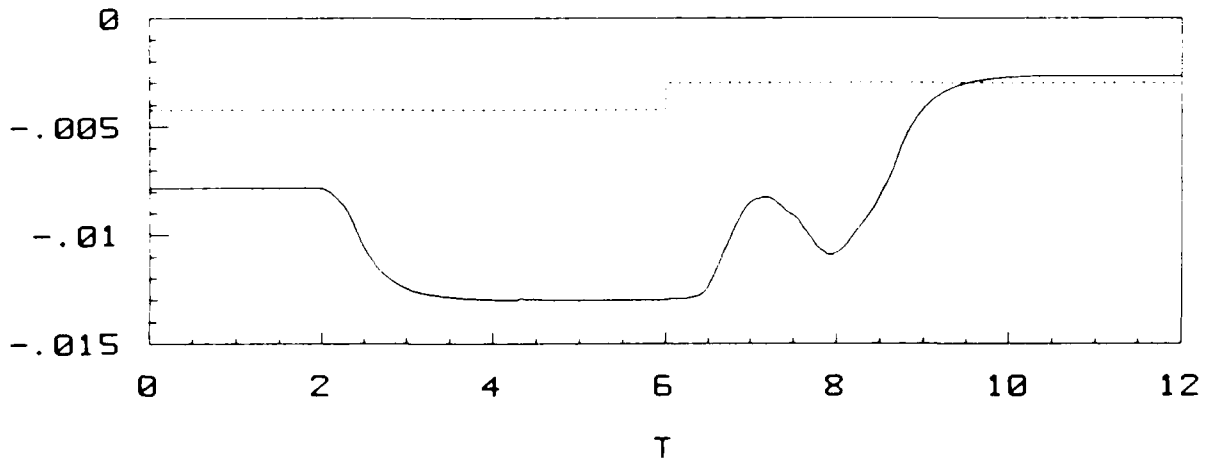
PRIMARY ESTIMATE OF $B1(1,1)$, SIM 4

— Estimated Parameter
- - - Actual Parameter

P-4



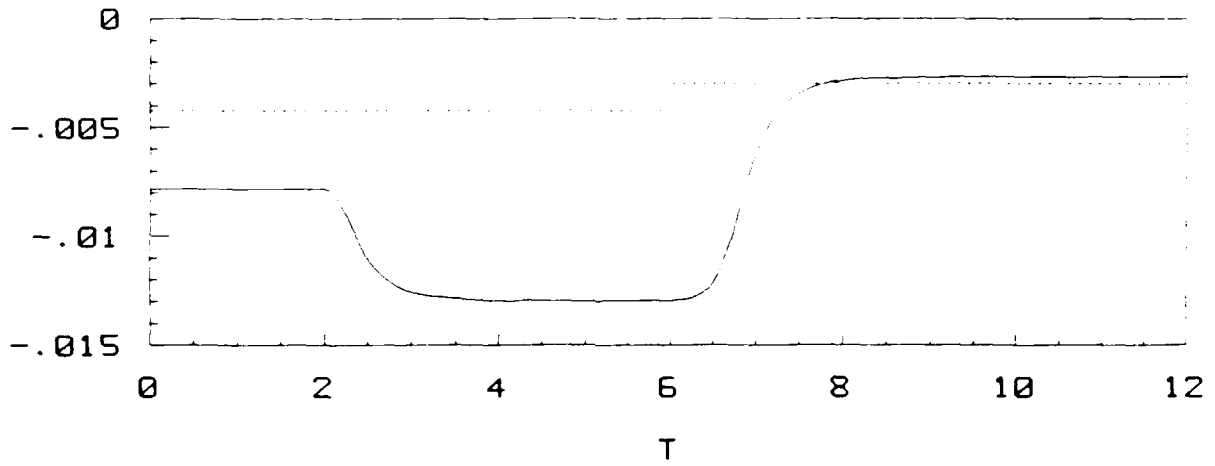
PRIMARY ESTIMATE OF $B_1(2,2)$, SIM 1



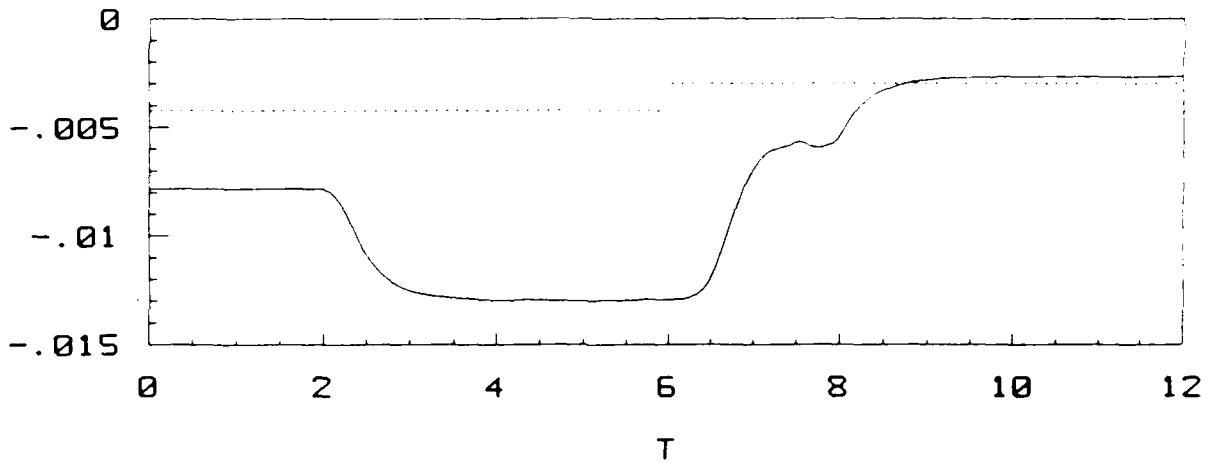
PRIMARY ESTIMATE OF $B_1(2,2)$, SIM 2

— Estimated Parameter
- - - Actual Parameter

P-5



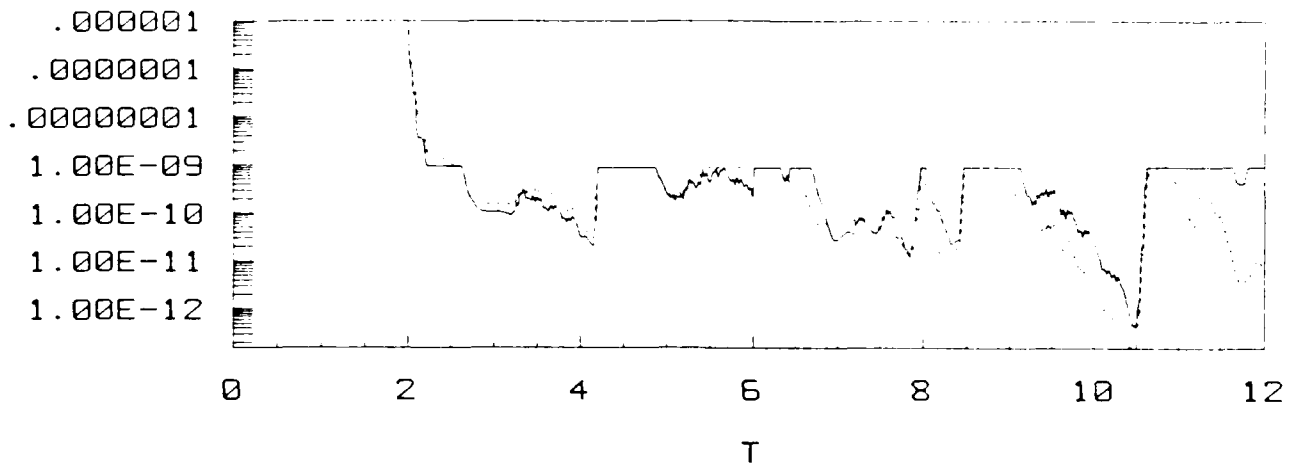
PRIMARY ESTIMATE OF $B_1(2,2)$, SIM 3



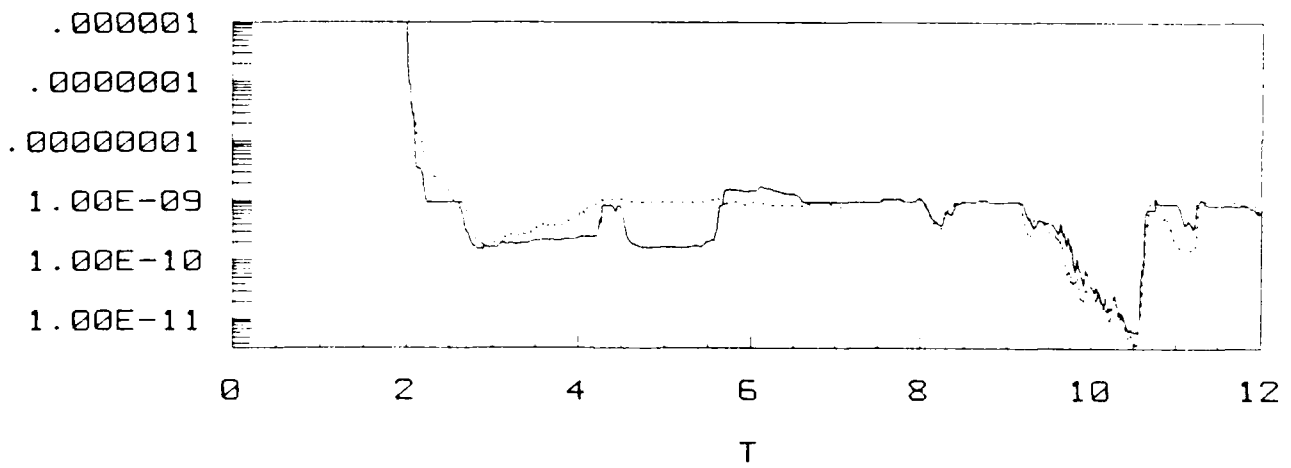
PRIMARY ESTIMATE OF $B_1(2,2)$, SIM 4

-- Estimator 1
- - - Estimator 2

P-6



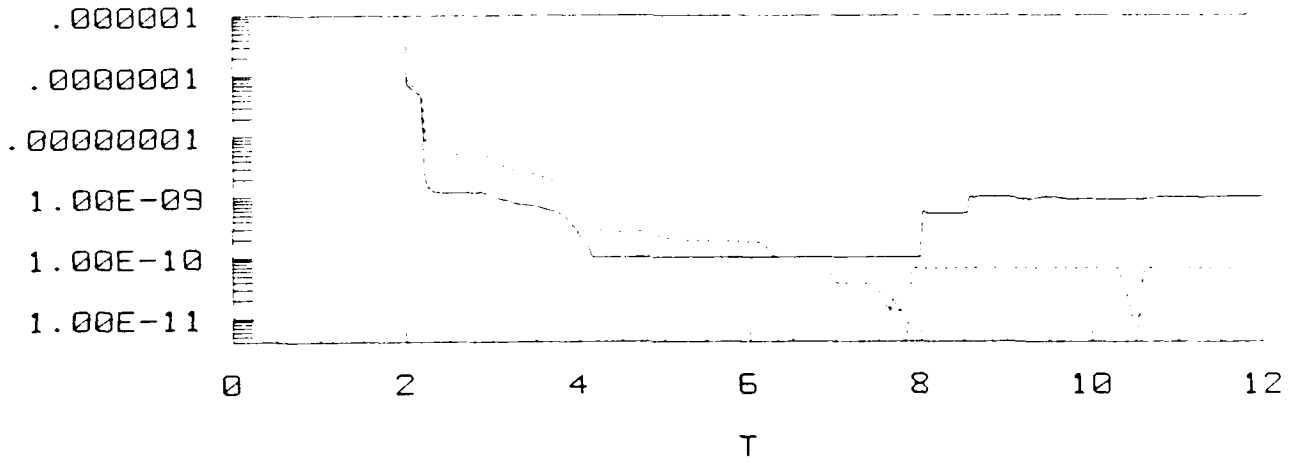
ESTIMATES OF $P_1(1,1)$, SIM 1



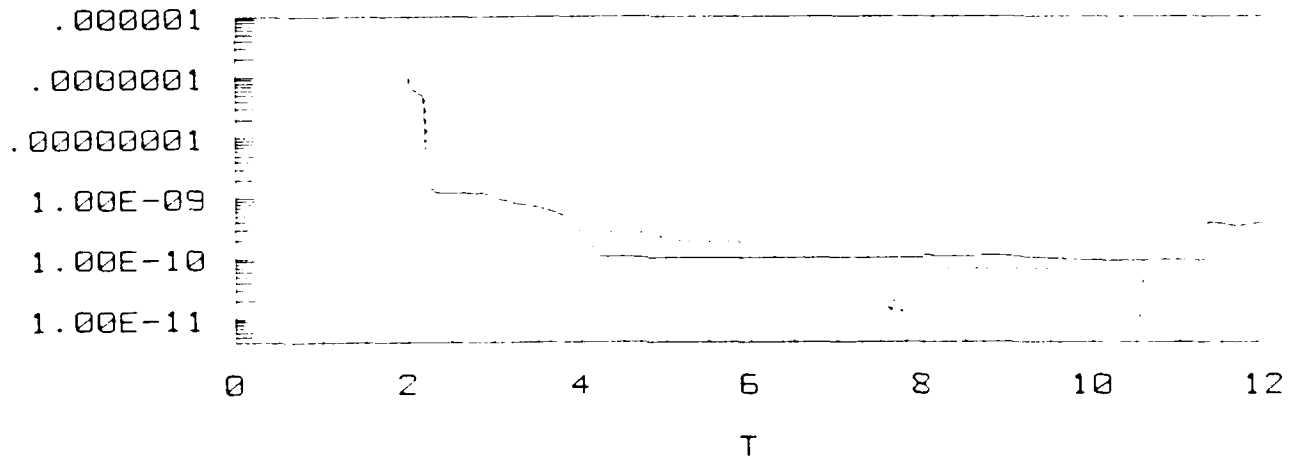
ESTIMATES OF $P_1(1,1)$, SIM 2

- - - Estimator 1
- - - Estimator 2

P-7

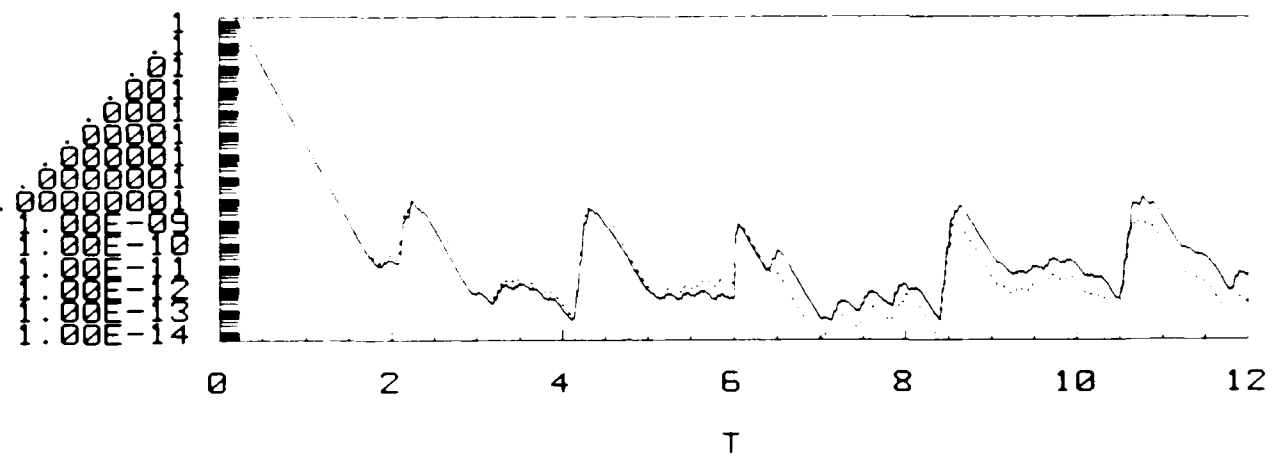


ESTIMATES OF P1(2,2), SIM 1

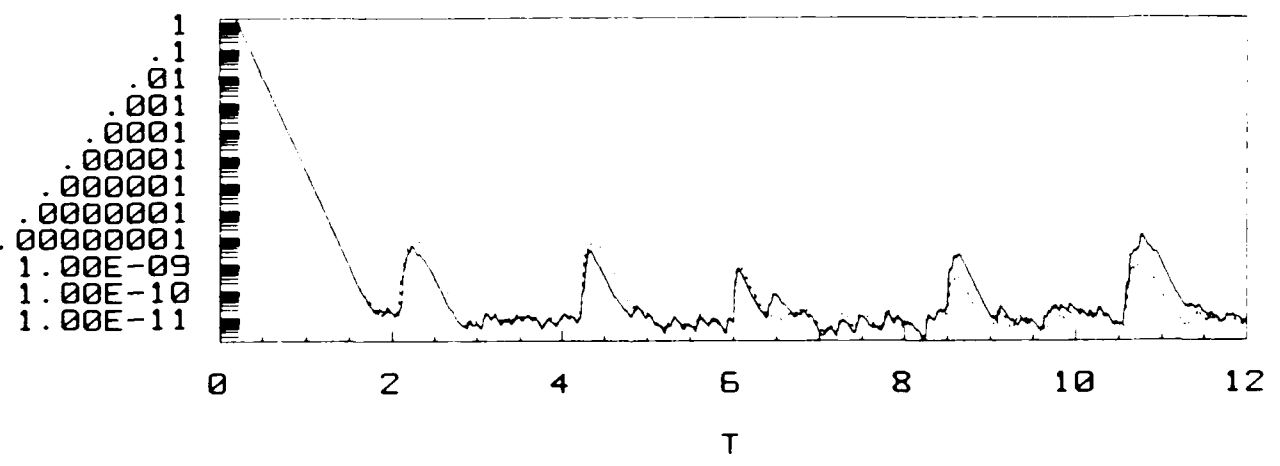


ESTIMATES OF P1(2,2), SIM 2

— Estimator 1
- - - Estimator 2



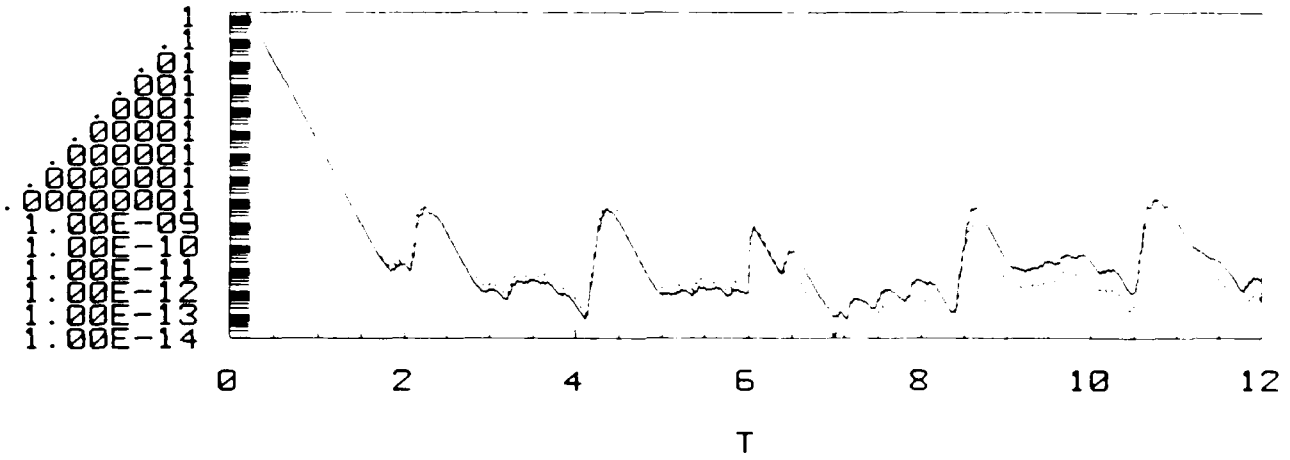
U1(1,1), U2(1,1), SIM 1



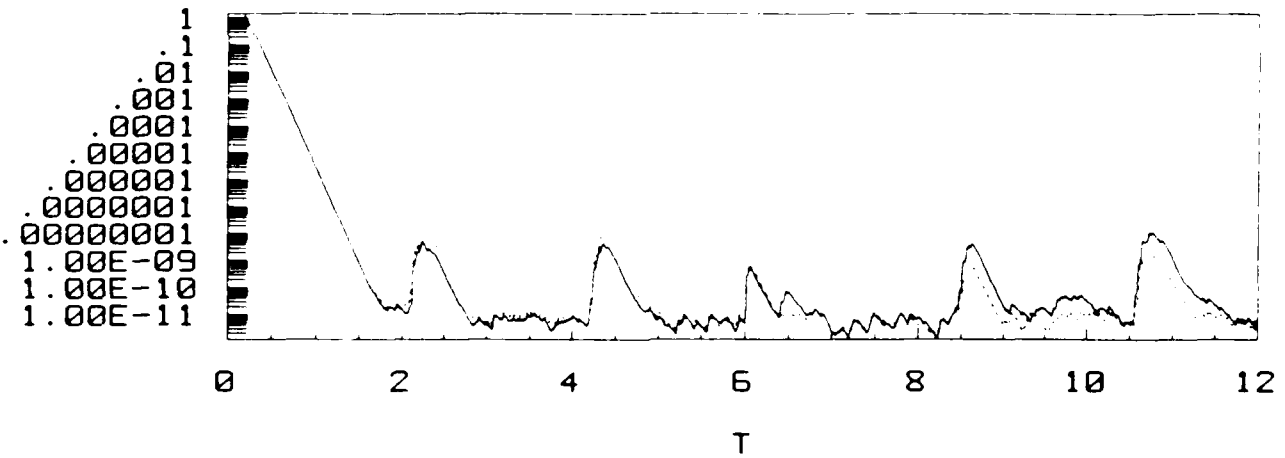
U1(1,1), U2(1,1), SIM 2

— Estimator 1
- - - Estimator 2

P-9



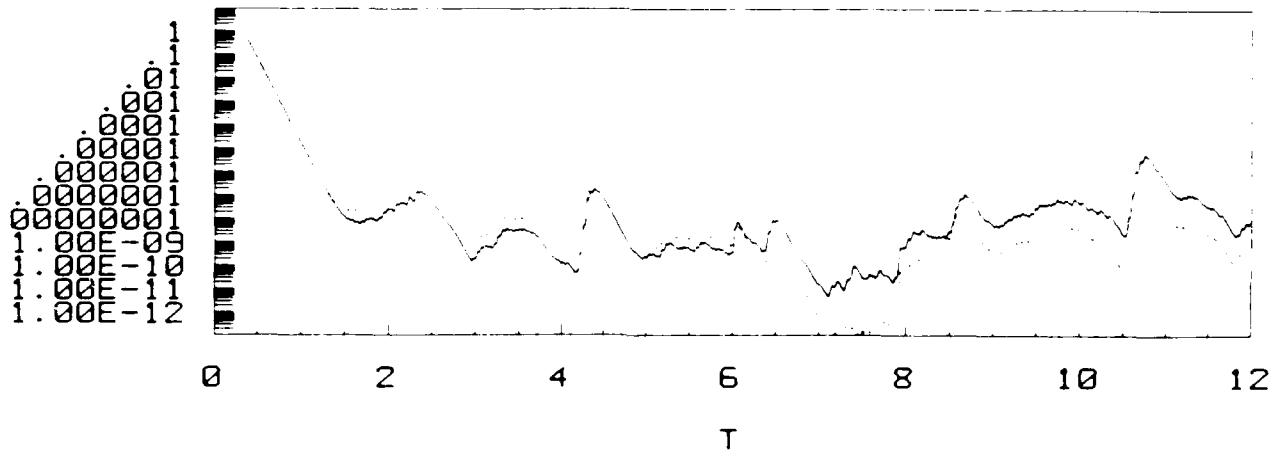
U1(1,1), U2(1,1), SIM 3



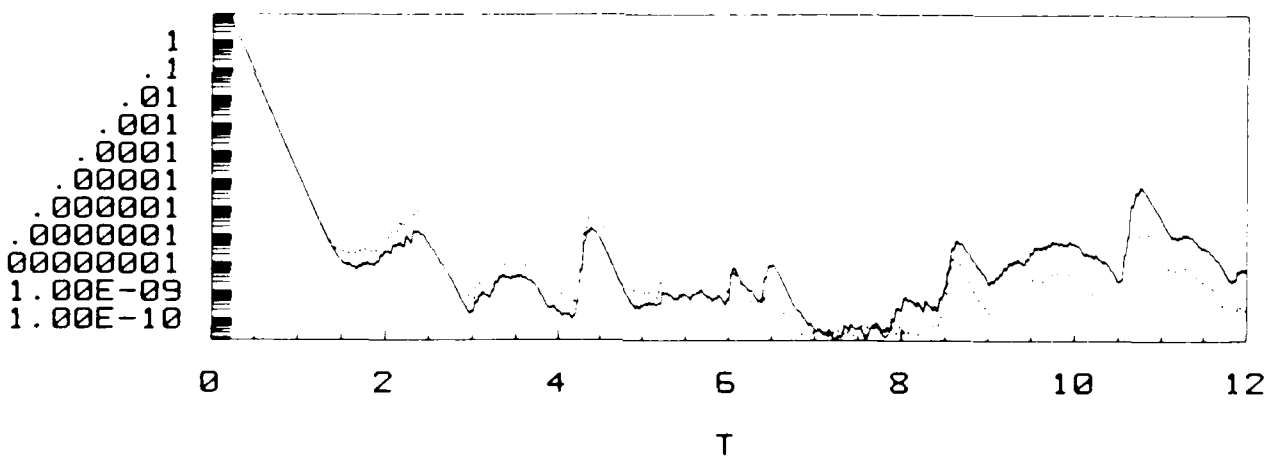
U1(1,1), U2(1,1), SIM 4

— Estimator 1
- - - Estimator 2

P-10



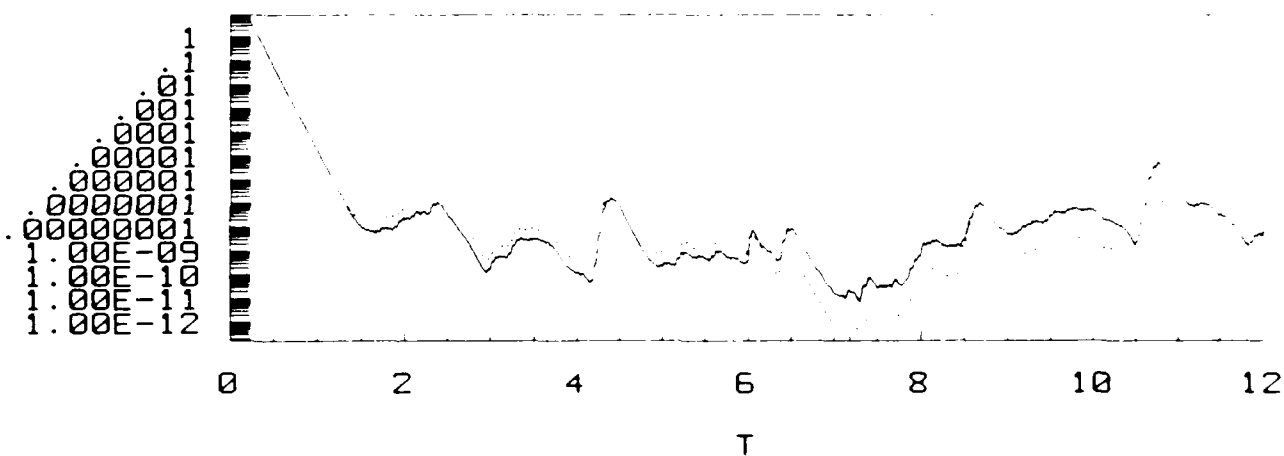
U1(2,2), U2(2,2), SIM 1



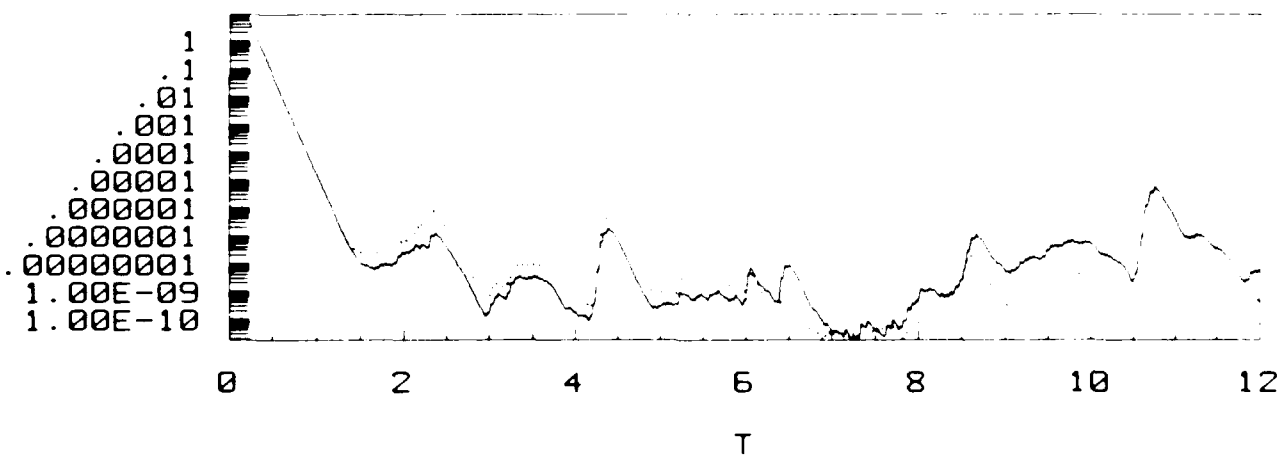
U1(2,2), U2(2,2), SIM 2

— Estimator 1
- - - Estimator 2

P-11



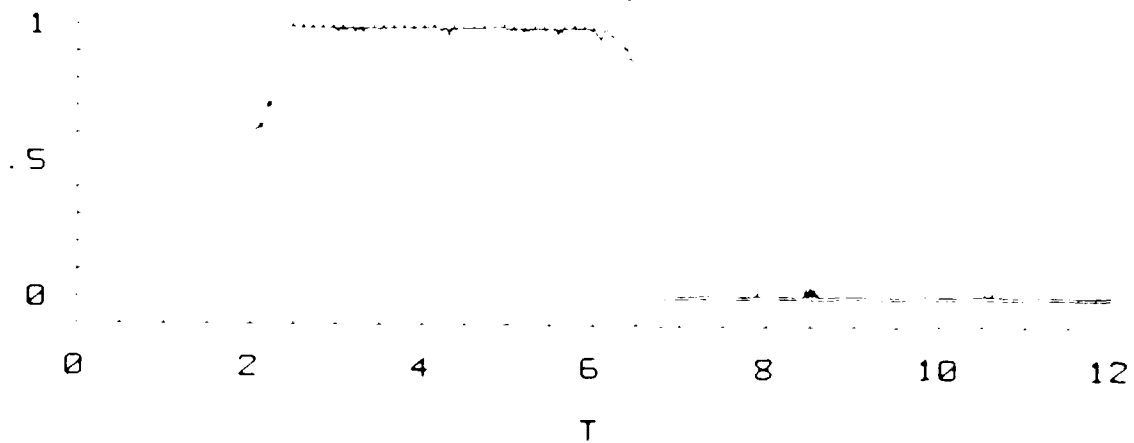
U1(2,2), U2(2,2), SIM 3



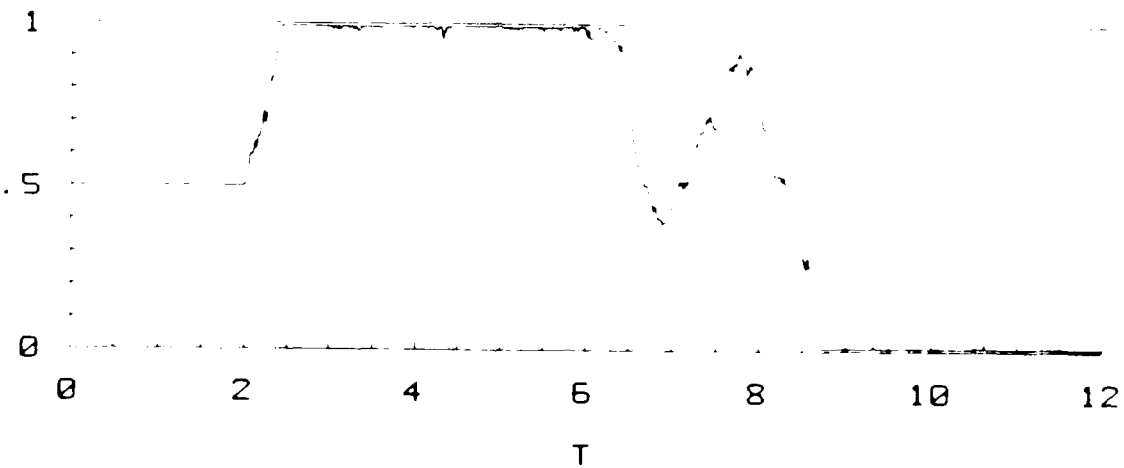
U1(2,2), U2(2,2), SIM 4

Estimated Weighting Coefficient
- - - Actual Weighting Coefficient

P-12



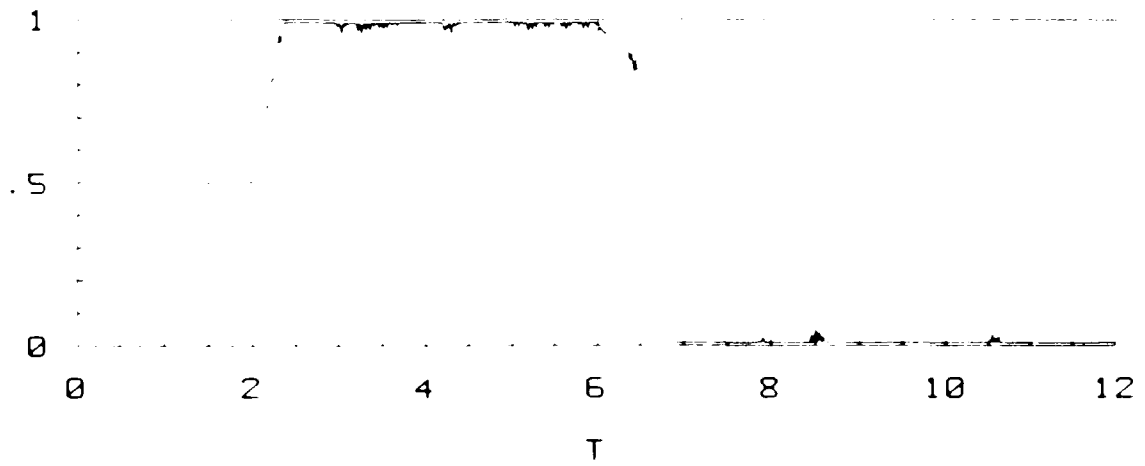
A1, SIM 1



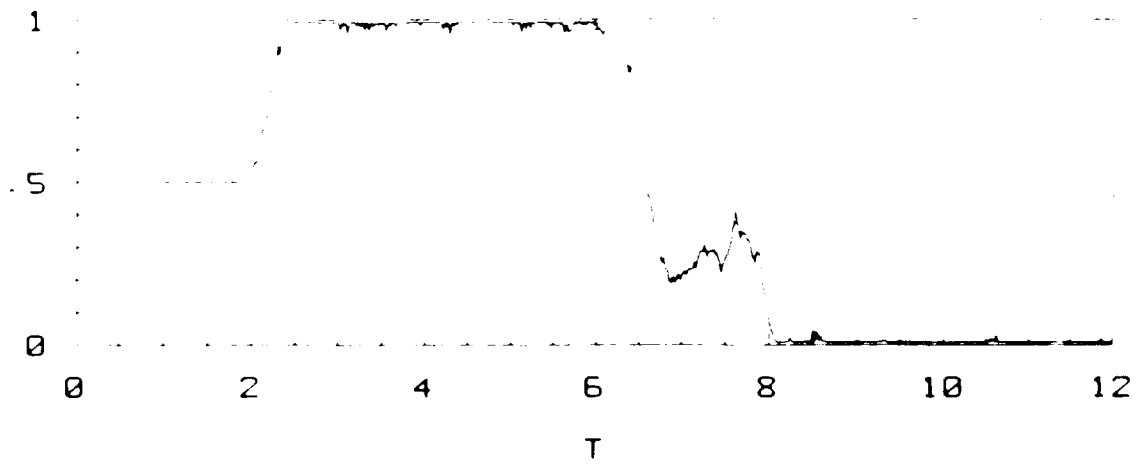
A1, SIM 2

--- Estimated Weighting Coefficient
- - - Actual Weighting Coefficient

P-13



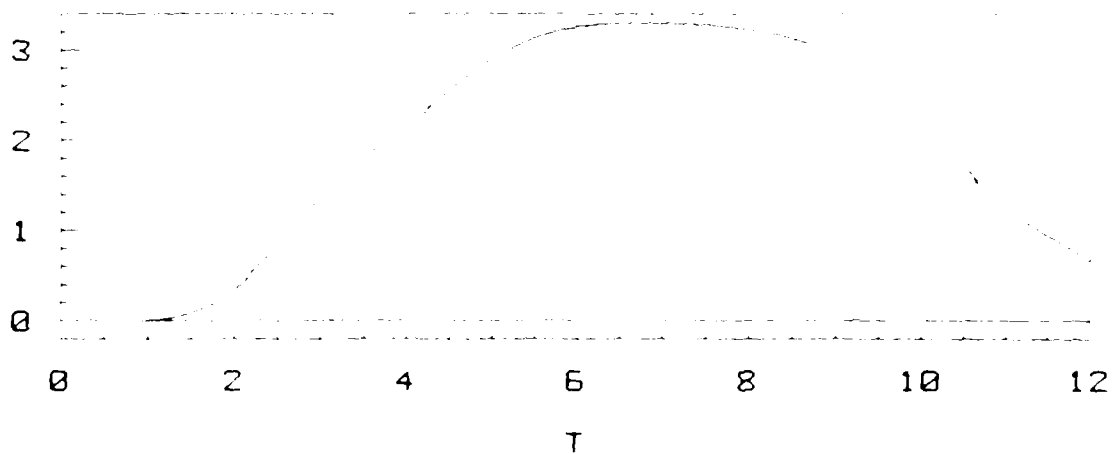
A1, SIM 3



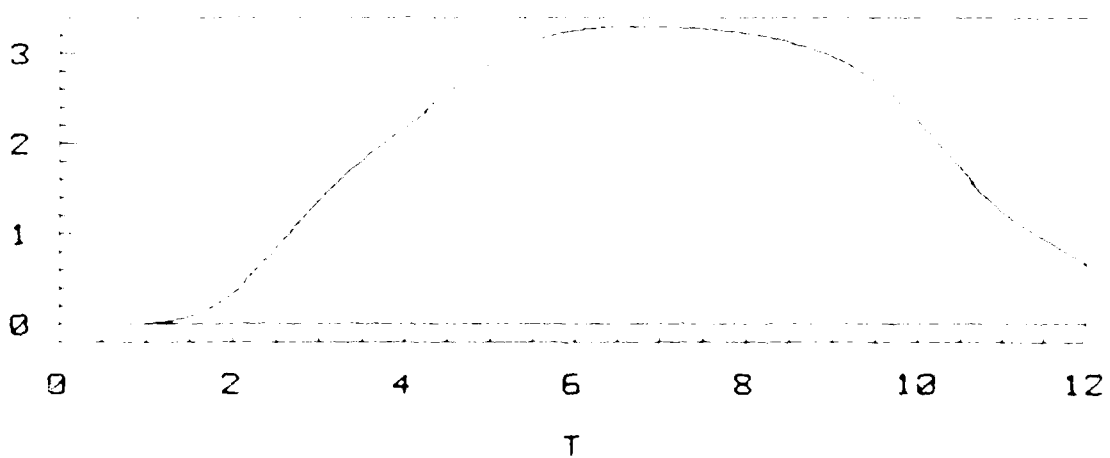
A1, SIM 4

— Commanded Output
- - - Actual Output

P-14



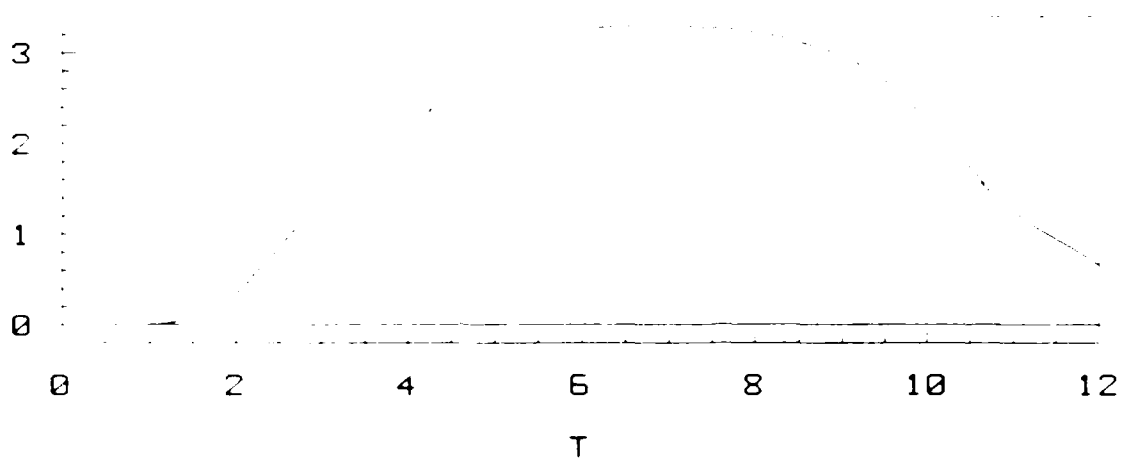
FLIGHT PATH ANGLE, HOST AND MODEL, SIM 1



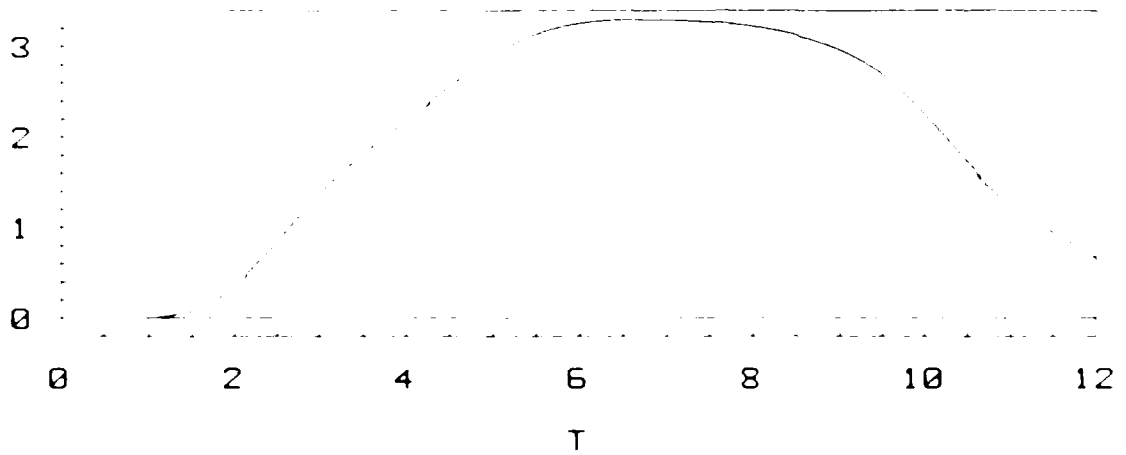
FLIGHT PATH ANGLE, HOST AND MODEL, SIM 2

— Commanded Output
- - - Actual output

P-15



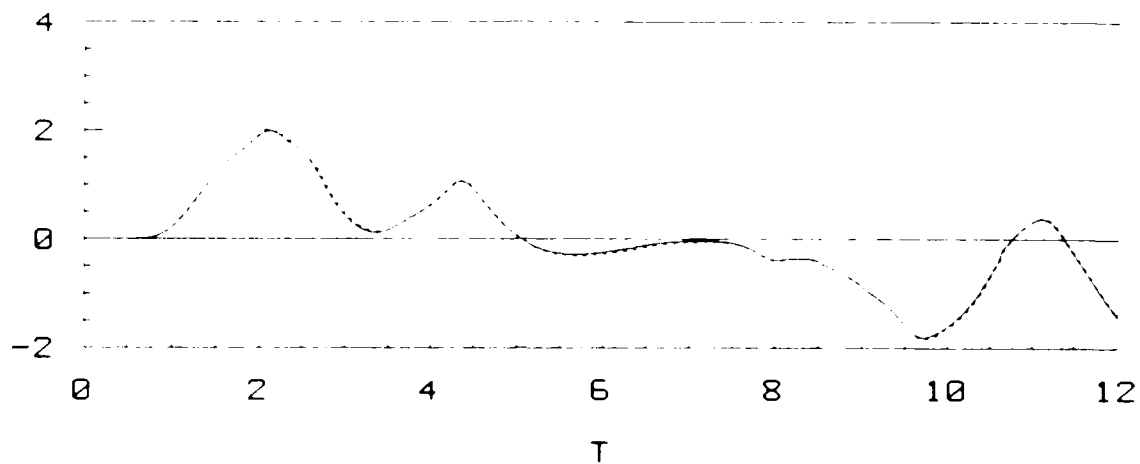
FLIGHT PATH ANGLE, HOST AND MODEL, SIM 3



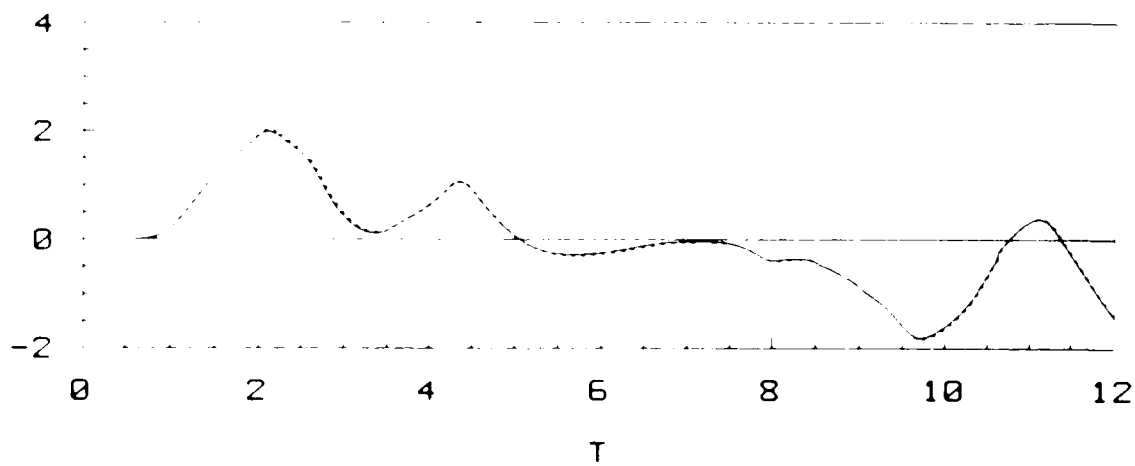
FLIGHT PATH ANGLE, HOST AND MODEL, SIM 4

— — — Commanded Output
- - - Actual Output

P-16



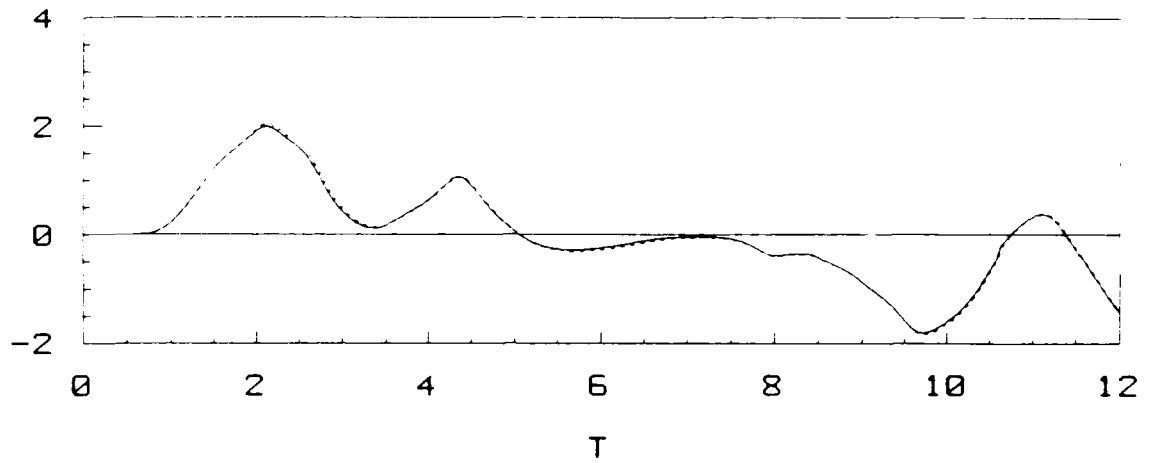
PITCH RATE, HOST AND MODEL, SIM 1



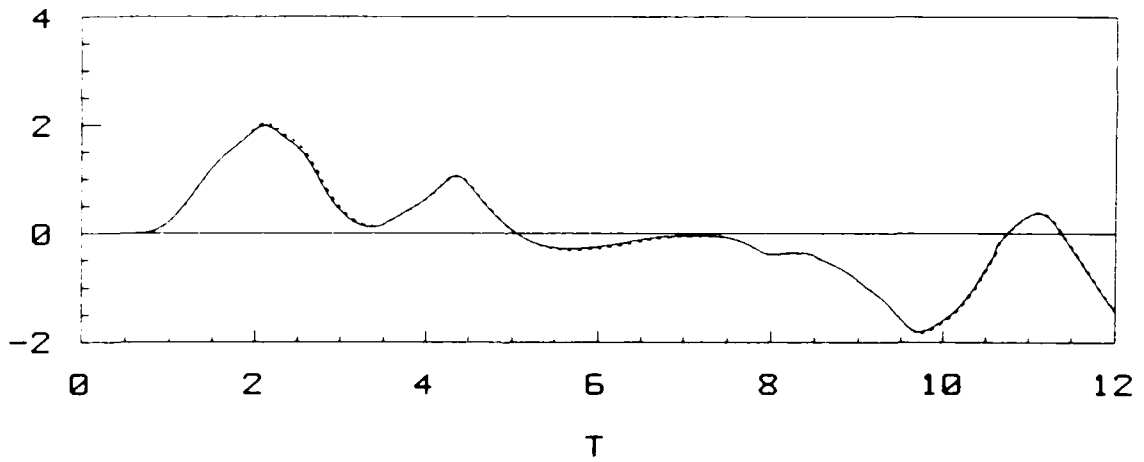
PITCH RATE, HOST AND MODEL, SIM 2

— Commanded Output
- - - Actual Output

P-17



PITCH RATE, HOST AND MODEL, SIM 3



PITCH RATE, HOST AND MODEL, SIM 4

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Abstract

Adaptive control of aircraft model-following systems has shown promising results for in-flight simulation, but the computational expense and slow convergence of conventional parameter estimation techniques for higher order models inhibits their direct use for in-flight simulation. Computer simulations of adaptive systems usually assume some knowledge of model parameters in order to maintain tracking fidelity at a reasonable computational cost as parameters change. This thesis incorporates a-priori information into a multiple-model estimation algorithm which assigns a probability weighting of each estimator within a "bank" of estimators. Final parameter estimates used in adaptive control are formed as a probabilistic weighted sum of individual estimates. Simulations of the system show excellent tracking performance throughout the flight envelope. A moving bank scheme for use over a wide range of flight conditions is recommended as a further area of study.

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