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**FEEDBACK CONTROL OF DISTRIBUTED PARAMETER SYSTEMS
WITH APPLICATIONS TO LARGE SPACE STRUCTURES**

12/28/83 - 9/1/87

Principal Investigator:

Mark J. Balas

Formerly with

**Electrical, Computer and Systems Engineering Department
Rensselaer Polytechnic Institute
Troy, New York 12180-3590
(518) 266-6683**

On sabbatical leave for 1985/86 at

**Laboratory for Electromagnetic and Electronic Systems
Massachusetts Institute of Technology
Cambridge, MA 02139
(617) 253-2068**

Presently with

**Department of Aerospace Engineering Sciences and
Center for Space Structures and Controls
University of Colorado
Campus Box 429
Boulder, Colorado 80309-0429
(303) 492-3177**

1.0 Finite-Dimensional Controllers for Infinite-Dimensional Systems

Many engineering systems, such as large aerospace structures, exhibit distributed parameter system (DPS) behavior and their dynamics must be described on infinite-dimensional state-spaces. The most fundamental constraint for feedback control of DPS is that the controller algorithm be finite-dimensional in order to be realized with an on-line computer and a finite number of control actuators and sensors. This is especially serious since there is no guarantee that a finite-dimensional controller can stabilize a DPS; in fact, there are examples to the contrary.

The most obvious, and most often used, approach to design of finite-dimensional controllers for DPS is to make a finite-dimensional approximation, i.e. reduced-order model (ROM), of the open-loop DPS and then design the controller directly from the ROM. In particular, when the exact modes of the DPS are known, such a controller can be designed from a low-order modal approximation; however, this does not necessarily lead to a stable closed-loop design.

We have taken a very different approach: make finite-dimensional approximations of infinite-dimensional controllers which stabilize the DPS. We concentrate on exponential stability, rather than weak or strong, stability of the DPS because of its robustness to bounded perturbations. We have shown conditions under which Galerkin approximation schemes of a quite general nature can yield finite-dimensional stabilizing controllers for linear DPS. Furthermore, in the special case where the modes of the DPS are known, this leads to a

special form of the controller: it is the old modal controller augmented by a residual mode filter which counteracts the observation spillover terms in the sensor outputs to produce stable closed-loop operation. Even when exact modal knowledge is not available, good approximate modal data can be used effectively for such designs.

2.0 Direct Adaptive Control of Linear DPS

To our knowledge there has never been a proof of stable adaptation of a finite-dimensional adaptive algorithm in closed-loop with a linear DPS. In analytical terms this is a nonlinear, time-varying, infinite-dimensional stability problem. We have taken a model reference adaptive control approach using command generator tracker concepts to produce a finite-dimensional direct adaptive controller for linear DPS defined on Hilbert spaces. We have shown that such schemes are Lagrange stable, i.e. all trajectories enter a small neighborhood of the origin and never leave; the size of this neighborhood can be estimated; see Ph.D. Thesis of J. Wen.

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Publications

The following papers have been published or accepted for publication.

- (a) "Linear Distributed Parameter Systems: Closed-Loop Exponential Stability with a Finite-Dimensional Controller," *Automatica*, Vol. 20, 1984, pp. 371-377.
- (b) "A Connection Between State-Space and Doubly Coprime Fractional Representations," (with C. Nett and C. Jacobson), *IEEE Transactions on Automatic Control*, Vol. AC-29, 1984, pp. 831-832.
- (c) "Optimal Quasi-Static Shape Control for Large Aerospace Antennae," *J. Optimization Theory and Application*, Vol. 46, No. 2, June 1985.
- (d) "Finite-Dimensional Control of Distributed Parameter Systems by Galerkin Approximation of Infinite-Dimensional Controllers," *J. Math. Analysis and Application* (to appear).
- (e) "New Results in Finite-Dimensional Control of Stabilization of Linear Distributed Parameter System," *IEEE Control and Decision Conference*, Las Vegas, Nevada, December 1984.
- (f) "Galerkin Modeling and Feedback Control of Linear Distributed Parameter Systems" (with J.G. Hsieh), 27th Midwest Symposium on Circuits and Systems, West Virginia University, Morgantown, West Virginia, June 1984.
- (g) "Some Results on Closed-Loop Stability with Reduced-Order Controllers Using Aggregation Methods" (with A. Jain), *ibid.*
- (h) "Reduced-Order Modeling and Finite-Dimensional Stabilization of Linear Distributed Parameter Systems Via Galerkin's Technique" (with J.-G Hsieh), 22nd Allerton Conference on Communication, Control and Computing, Monticello, Illinois, October 1984.
- (i) "New Directions in Asymptotically Stable Finite-Dimensional Adaptive Control of Linear Distributed Parameter Systems," Workshop on Identification and Control of Flexible Structures, Jet Propulsion Laboratory, Pasadena, California, June 1984.
- (j) "Stable Direct Adaptive Control of Linear Infinite-Dimensional Systems Using a Command Generator Tracker Approach" (with H. Kaufman and J. Wen), *ibid.*
- (k) "Reduced-Order Adaptive Control of Distributed Parameter Systems" (with Z. Islam), 18th Annual Conference on Information Sciences and Systems, Princeton University, Princeton, New Jersey, March 1984.

- (l) "The Structure of Discrete-Time Finite-Dimensional Control of Distributed Parameter Systems," J. Math. Analysis and Application, Vol. 102, 1984, pp. 519-538.
- (m) "Exponentially Stabilizing Finite-Dimensional Controllers for Linear Distributed Parameter Systems Described on a Banach Space," IEEE Control 85 Conference, University of Cambridge, England, July 1985.
- (n) "Finite-Dimensional Stabilization of Linear Distributed Parameter Systems Via Galerkin's Technique" (with J.-G. Hsieh), Fifth VPI SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, Virginia, June 1985.
- (o) "Direct Adaptive Control in Hilbert Space" (with J. Wen), *ibid*; also presented at the Fourth Yale Workshop on Applications of Adaptive Systems Theory, Yale University, New Haven, CT, MA 1985.
- (p) "Finite Dimensional Controllers for Linear Distributed Parameter Systems: Exponential Stability using Residual Mode Filters," IFAC Workshop on Estimation and Control of Uncertain Systems, Boston, MA, June 1985.
- (q) Invited participant in Workshop on Control Systems Governed by Partial Differential Equations with Applications to Large Flexible Structures, Clearwater, Florida, March 1985.
- (r) "Exponentially Stabilizing Finite-Dimensional Controllers for Linear DPS: Galerkin Approximation of Infinite-Dimensional Controllers," J. Math. Analysis of Appl. (to appear).
- (s) "Boundary and Interior Control of Parabolic DPS Using Finite-Dimensional Compensators" (with Z. Gu), 19th Annual Conference on Information Sciences and Systems, The Johns Hopkins University, Baltimore, MD, March 1985.

ADAPTIVE RESIDUAL MODE FILTER CONTROL OF
DISTRIBUTED PARAMETER SYSTEMS FOR LARGE SPACE STRUCTURE APPLICATIONS

by

Jang J. Ouyang

An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

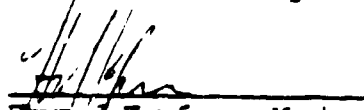
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The original of the complete thesis is on
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Institute Library

Approved by the
Examining Committee:


Mark J. Balas, Thesis Adviser


Robert E. O'Malley, Jr., Member


Howard Kaufman, Member


Alan A. Desrochers, Member

Rensselaer Polytechnic Institute
Troy, New York

May 1987

Abstract

Modal control has often been proposed as a way to design stabilizing low order controllers for Distributed Parameter Systems (DPS). However, it is well known that such controllers, designed from a reduced order modal model, do not necessarily stabilize the actual DPS. In this paper, we prove that exponential closed-loop stability can always be achieved by the addition of a very low order Residual Mode Filter (RMF). Due to the uncertainty of modal data for the Large Space Structure (LSS), a real-time adaptive controller has been designed and tested successfully via computer simulation. The controller, implemented in a digital minicomputer, consists of a modal Reduced-Order Model (ROM) controller, a bank of RMFs and a bank of Frequency Locked Loops (FLL) with associated bandpass filters for real-time parameter identification. Three DPS examples: A simply supported Euler-Bernoulli beam, a telegraph equation and a Space based Laser (SBL) beam expander are presented to illustrate the application of this concept.

REDUCED ORDER STABILITY AND CONTROL OF LINEAR
DISTRIBUTED PARAMETER SYSTEMS

by

Abhinandan Jain

A Thesis Submitted to the Graduate
Faculty of Rensselaer Polytechnic Institute
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Major Subject: Electrical Engineering

Approved by the
Examining Committee:

M. J. Balas, Thesis Advisor

H. Kaufman, Member

A. Z. Manitius, Member

G. Habetler, Member

Rensselaer Polytechnic Institute
Troy, New York

December 1986

ABSTRACT

For distributed parameter systems (DPS), which are inherently infinite dimensional, a full order compensator is not feasible and reduced order compensators must be designed. Often reduced order compensator designs are based on a reduced order model (ROM) of the system. Issues related to such compensators are the subject of this thesis, and algebraic techniques are the main tool used to study them. Using a special kind of fractional representation, we obtain several properties of a parametrization of stabilizing compensators including a bound on the order of a stabilizing compensator. Using this fractional representation for the ROM, a new method for the design of reduced order stabilizing compensators is developed. The method is simple to use and is illustrated by an example in which we design a first order stabilizing compensator for an unstable DPS governed by the telegraph equation. The method can be extended to problems where features such as sensitivity minimization and disturbance rejection are additional design goals. We show that every exponentially stabilizing compensator is in fact an observer-based compensator for a ROM of a stabilizable and detectable realization of the plant. Moreover, this ROM is associated with a pair of stabilizing subspaces for the system. We also studied properties of a fixed order optimal LQG compensator, which are known to be observer-based compensators for a ROM of the system. We show that a fixed order optimal LQG compensator is also optimal for another realization of the plant, and here again the ROM is associated with stabilizing subspaces. Under some conditions, the compensator is related via modal projections to the full order optimal LQG compensator.

DIRECT ADAPTIVE CONTROL IN HILBERT SPACE

by

John Ting-Yung Wen

An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

The original of the complete thesis is on
file in the Rensselaer Polytechnic
Institute library

Approved by the
Examining Committee:

Mark J. Balas, Thesis Advisor

Andrejz Manitius, Member

Marshall Slemrod, Member

Howard Kaufman, Member

Rensselaer Polytechnic Institute
Troy, New York

June 1985
(For Graduation August 1985)

ABSTRACT

Though great advances have been reported in adaptive control for single-input/single-output (SISO) systems and some multi-input/multi-output (MIMO) systems, a large amount of a priori structural information of the plant is needed for most of the methods proposed. This is unsatisfying because all physical systems have some unmodelled dynamics and structure and operate in noisy environment. In fact, in many high performance control system design, the distributed nature of the plant must be taken into account. These distributed parameter systems may be modelled by delayed differential equations, partial differential equations or integral equations. They must be analysed in the appropriate infinite-dimensional state space.

A particular approach, model reference adaptive control (MRAC) with command generator tracker (CGT) concept, adopts a set of assumptions that are not system dimension dependent. The method has been applied successfully to some finite-dimensional systems and show promise for the infinite-dimensional state space generalization. In this thesis, the scheme is modified in order to make the transition of this theory from finite dimensions to the infinite-dimensional Hilbert Space mathematically rigorous. As a bonus, the modified scheme also improves performance in the finite-dimensional case in terms of rate of convergence, robustness with respect to unmodelled dynamics, input and output disturbances and parameter variation. We have also relaxed the model matching condition to an asymptotic form. The proposed modification can also be applied to other adaptive control schemes yielding improvement in robustness. Three DPS examples, heat

equation and beam equation with marginally stable rigidbody modes and first order delay equation, and many finite-dimensional examples are given to illustrate the application of this scheme.

FINITE-DIMENSIONAL FEEDBACK CONTROL OF A CLASS OF
LINEAR INFINITE-DIMENSIONAL SYSTEMS VIA GALERKIN APPROXIMATION

by

Jer-Guang Hsieh

An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

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Major Subject: Electrical Engineering

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Institute Library

Approved by the
Examining Committee:

Mark J. Balas, Thesis Adviser

George N. Saridis, Member

George J. Habetler, Member

Marshall Slemrod, Member

Rensselaer Polytechnic Institute
Troy, New York

December 1984

ABSTRACT

Control of the distributed parameter systems (DPS) usually involves dynamical descriptions on infinite-dimensional Hilbert or Banach spaces of functions. For a linear stabilizable and detectable DPS, an infinite-dimensional feedback controller exists so that the closed-loop system is stable. However, this infinite-dimensional controller generally cannot be implemented with practical computers and devices. Consequently, the reduced order modeling of DPS becomes extremely crucial for finite-dimensional controller design. In this thesis, Galerkin (or finite element) methods are presented as a way to develop finite-dimensional Galerkin controllers by approximating the infinite-dimensional stabilizing controller for a class of linear DPS. This investigation will give a very simple sufficient condition, i.e. the test of nonsingularity of a matrix for a finite-dimensional Galerkin feedback controller to perform as well as the infinite-dimensional stabilizing controller.

INVESTIGATION OF APPLICATION OF AN EXTENDED KALMAN FILTER FOR
PARAMETER ESTIMATION IN DISTRIBUTED PARAMETER SYSTEMS

by

Alan B. Jenkin

A Thesis Submitted to the Graduate
Faculty of Rensselaer Polytechnic Institute
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE

Approved:


Professor Mark J. Balas
Thesis Advisor

Rensselaer Polytechnic Institute
Troy, New York

December 1984

ABSTRACT

A candidate method of on-line parameter estimation for distributed parameter systems (DPS) has been tested in the case of an elastic vibrating string via computer simulation. This method incorporates the use of a constant-gain extended Kalman filter, the design of which is based on a finite dimensional approximation or reduced order model (ROM) of the DPS. Because the unknown parameters are treated as additional ROM states to be estimated by the filter, the resulting filtering problem becomes non-linear and is therefore linearized about a desired DPS state to facilitate calculation of the filter gains. Time varying error covariance update equations are not included for on-line implementation because of the computational burden imposed by relatively high filter order, and subsequently the time invariant filter-gains are calculated by solving the steady state Riccati error covariance equations. For successful filter operation the DPS must be kept away from its unforced equilibrium so that the parameters are identifiable.

Results from simulations of a variety of vibrating string cases are promising and indicate that the designer has a fairly high degree of control over filter performance.

BOUNDARY AND INTERIOR CONTROL OF PARABOLIC DPS
USING FINITE DIMENSIONAL COMPENSATORS

by.

Zhougmei Gu

A Thesis Submitted to the Graduate
Faculty of Rensselaer Polytechnic Institute
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE

Approved:

Mark J. Balas
Thesis Advisor

Rensselaer Polytechnic Institute
Troy, New York

August 1984

ABSTRACT

Many high performance systems must be modelled by partial differential equations. Generally speaking, it is not possible to implement an infinite dimensional feedback controller for these distributed parameter systems. It is proved that, for a class of parabolic distributed parameter systems with bounded or unbounded feedback control and observation which satisfy certain conditions, there exist finite dimensional compensators.

MODELING AND CONTROL OF WEB GUIDING PROCESSES

by

Lisa Sievers

An Abstract of a Thesis Submitted to the graduate

Faculty of Rensselaer Polytechnic Institute

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Major Subject: **Computer and Systems Engineering**

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Institute Library

Approved by the
Examining Committee:

Mark J. Balas, Thesis Adviser

Alan A. Desrochers, Member

Donald A. Drew, Member

Victor Roytburd, Member

Rensselaer Polytechnic Institute
Troy, New York

August 1987

Abstract

Three distributed parameter models for the lateral dynamic behavior of a moving web in an n-roller system were investigated. A span of web moving longitudinally and under tension, was modeled as either a string, an Euler-Bernoulli beam, or a Timoshenko beam. A quasi-static simplification based upon spectral separation was made to simplify the models. Boundary conditions at the web-roller interface were determined from the kinematics of web roller interaction. The kinematics of web contact on a roller was also investigated. Actuator dynamics were introduced through the formulation of the boundary conditions. Closed-loop simulations, using a state space version of the simplified model, were in agreement with experimental results only when the web was modeled as a Timoshenko beam. The model was manipulated into a form suitable for modern multivariable control design.

Disturbance rejection of a forced input was the objective of the control design. It was shown that disturbance rejection can be achieved at a specified location in the web path if one actuator is used in the control loop. More importantly, it was shown that complete disturbance rejection everywhere downstream ^{from} of the control loop is possible if two actuators are used in the control system.

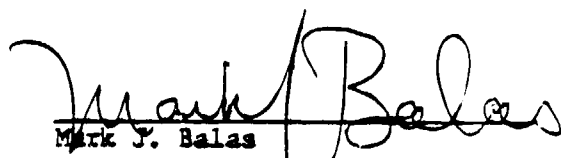
OPTIMAL SHAPE CONTROL OF QUASI-STATIC LINEAR DISTRIBUTED
PARAMETER SYSTEMS FOR APPLICATION
TO LARGE AEROSPACE ANTENNAE

by

Lisa Anne Sievers

A Thesis Submitted to the Graduate
Faculty of Rensselaer Polytechnic Institute
in Partial Fulfillment of the
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MASTER OF SCIENCE

Approved by:


Mark J. Balas
Thesis Advisor

Rensselaer Polytechnic Institute
Troy, New York

(For Graduation May 1985)

ABSTRACT

We propose an on-line control approach which will adjust the steady-state shape of a large antenna arbitrarily close to any achievable desired profile. The method makes use of distributed parameter system theory and allows refocusing using a limited number of control actuators and sensors.

The controller gains are calculated by approximating the solution to an infinite-dimensional optimal quasi-static control problem. The Galerkin (finite element) approximation method is used for model reduction. We prove that both gain and state convergence can be achieved by using the proposed approximation scheme. We also prove that the performance index of the approximate controller converges monotonically to the performance index of the infinite-dimensional optimal quasi-static control problem.

SOME ASPECTS OF THE STRUCTURE AND STABILITY OF A
CLASS OF LINEAR DISTRIBUTED SYSTEMS

by

Clas A. Jacobson

Department of Electrical, Computer, and Systems Engineering
Rensselaer Polytechnic Institute
Troy, New York 12181

ABSTRACT

In this paper the structure with respect to dynamic output feedback stabilizability of a class of linear distributed parameter systems is examined. The class of linear distributed parameter systems considered are time-invariant state space systems on infinite dimensional Banach spaces with bounded sensing and control.

Connections are made with the algebra of transfer functions developed by Callier and Desoer and the state space systems under consideration. These connections show the necessity of an assumption used in the literature known as the spectrum decomposition assumption for state space systems which are exponentially stabilizable and detectable. Connections between input-output stability and state space exponential stability are made. These connections coupled with recent results of Nett on the parameterization of all finite dimensional input-output stabilizing compensators allow the extension to be made to parameterize all finite dimensional exponentially stabilizing compensators for the class of state space distributed parameter systems under study.

THE FRACTIONAL REPRESENTATION APPROACH TO ROBUST
LINEAR FEEDBACK DESIGN: A SELF-CONTAINED EXPOSITION

by

Carl N. Nett

Dept. of ECSE
RPI
Troy, NY 12181

ABSTRACT

The problem of robust linear feedback design is addressed via fractional representation theory. As such, the results presented here apply to many classes of MIMO linear systems of engineering interest, including multidimensional, distributed, time-varying, and discrete-time systems.

The exposition is essentially self-contained and begins with a consolidation and slight extension of previous work in the area of fractional representation theory. The theory is then extended to encompass the problem of robust regulation, which consists of tracking and disturbance rejection in the face of plant uncertainty. In particular, a necessary and sufficient condition for robust regulation is derived in the form of an internal model principle, from which it follows that robust tracking is equivalent to robust disturbance rejection. The set of all robust regulators of a given plant is then characterized in terms of the set of all stabilizing controllers of a related plant. This allows development of an explicit affine parameterization of the set of all robust regulators of a given plant. This result appears to be particularly useful in the design of optimal sensitivity robust regulators, as it is shown that such a design can be formulated in terms of an optimal sensitivity stabilizing controller of a related plant.

In conclusion, the widespread applicability of the theory and its utility in design are demonstrated by giving essentially complete results concerning the problem of finite-dimensional robust regulation of LTI distributed systems. Necessary and sufficient conditions are given for the existence of a proper finite-dimensional robust regulator of a given strictly proper distributed plant. An explicit affine parameterization of a set of robust regulators is given, and it is shown that this set contains the set of all proper finite-dimensional robust regulators as a dense subset. This allows an "optimal" design to be selected from the first set, and the optimal performance to be approximated arbitrarily closely with a finite-dimensional design. All results are constructive in nature, and hence form the basis for a practical design methodology.