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TYPE OF REPORT 13b. TIME Final FROM 1	COVERED Jul 86 <sup>TO</sup> 30 Jun 8	14 DATE OF REP 7 June 19	ORT (Year, Mon 187	th, Day) 15 PA 3	GE COUNT
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PROGRESS & FORECAST REPORT

Grant No. AFOSR-86-0179 January 1987

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Troy, New York



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1) Solidification Fronts: Asymptotic equations are derived to model the evolution of the solid-liquid interface which occurs in the directional solidification of binary alloys. In the limit where  $GD/VC_o(g + f_c)$  is small, the Kuramoto-Sivashinsky equation is obtained. Here G is the imposed temperature gradient, D is the diffusion coefficient of the impurity, V is the imposed transport velocity, g is equal to minus the slope of the liquidus line. Co is the impurity concentration at the liquid side of a planar interface, and  $f_c$  is a coefficient reflecting deviations from local thermal and intermittancy contained The cusping in the equilibrium. Kuramoto-Sivashinsky equation may help to explain the creation of defect points and the onset of loss of coherency. In the limit where the segregation coefficient is small, an alternative to the Sivashinsky equation is obtained. In the new equation a quadratic nonlinearity is replaced by an exponential term. The new equation is believed to more accurately describe the solidification profile.

2) <u>Viscous Phase Transition</u>: Two equations are derived and analyzed which model the dynamics of viscous first order phase transitions. Inclusion of both viscous and gradient energy terms yields the viscous Cahn-Hilliard equation,

$$c_{+} = \Delta(f(c) + \nu c_{+} - K\Delta c) . \qquad (1)$$

Here c(x,t) is a concentration, f(c) is an intrinsic chemical potential which should be nonmonotone in systems exhibiting phase separation and  $\nu$  and K are respectively coefficients of viscosity and gradient energy. In the limit  $K \rightarrow 0$ , equation (1) reduces to a viscous diffusion equation

$$c_{+} = \Delta(f(c) + \nu c_{+})$$
 (2)

and in the limit  $\nu \rightarrow 0$ , (1) reduces to the well-known Cahn-Hilliard equation.

$$c_{+} = \nu(f(c) - K\Delta c)$$

Equations (1) - (3) can be viewed as viscous and/or gradient energy regularizations of the backwards-forwards heat equation. A comparison theorem shows that early stages in the evolution of (1) follow closely the behavior of (2) for small K. The asymptotic behavior of (1) and (2) are contrasted. Equation (2) admits the possibility of discontinuous equilibria. Equation (1) initially develops transients which probably evolve and decay in much the same way as is believed to occur in the Cahn-Hilliard equation.

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Report

Appendix I

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Appendix III

#### (ii) Report:

### Solidification Fronts:

If deviations from local thermal equilibrium are taken into account, then it is possible to obtain a Kuramoto-Sivashinsky equation as a model equation for the solid-liquid interface present in the directional solidification of a dilute binary alloy. The derivation is given in a revised version of a paper which has been accepted for publication in Physica D (Appendix I). The AFOSR acknowledgement will be added in proof. In the small distribution coefficient limit, a similar derivation gives rise to a Sivashinsky equation. (Here local thermal equilibrium is assumed.) In a forthcoming paper, an alternative derivation gives rise to a similar equation in which the quadratic nonlinearity is replaced by an exponential term. This new equation is now being studied. A rough abstract of this forthcoming paper is included here (Appendix II).

## Viscous Phase Transitions:

Appendix III contains a contribution to the Heriot-Watt <u>Symposium on</u> <u>Non-classical Continuum Mechanics</u>. A more detailed account of our results will appear in

- [1] Novick-Cohen, A. and Pego, R.L., Viscosity and the dynamics of phase decomposition, in preparation.
- [2] Novick-Cohen, A. and Pego, R.L., On the viscous Cahn-Hilliard equation, in preparation.
- [3] Novick-Cohen, A. and Pego, R.L., Analysis of a viscous diffusion equation, in preparation.

