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Statistical Inference for Stochastic Processes

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Alan F. Karr
Department of Mathematical Sciences
G.W.C. Whiting School of Engineering
The Johns Hopkins University
Baltimore, Maryland 21218

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CONTENTS

Summary

This is the final technical report for Air Force Office of Scientific Research grant number 82-0029, 'Statistical Inference for Stochastic Processes,' for the period January 1, 1982 - December 31, 1986. During this time, a research program of high international visibility and impact was conducted, which resulted not only in numerous publications — including the highly regarded book *Point Processes and their Statistical Inference* — but also in many scientific visits, conference addresses and seminar presentations.

The principal research accomplishments may be grouped and summarized as follows:

- Inference for Point Processes
  - State estimation for Cox processes with unknown law
    - Mixed Poisson processes: publication [3]
    - General Cox processes, nonparametric models: publication [5]
    - General Cox processes, parametric models: publication [12]
  - Inference for stationary point processes
    - Estimation of Palm distributions: publications [1] and [9]
    - Estimation of distributions: publication [9]
    - State estimation: publication [9]
  - Inference for multiplicative intensity models
    - Maximum likelihood estimation using the method of sieves: publication [10]
  - Inference for thinned point processes
    - Nonparametric estimation: publication [4]
    - State estimation: publication [1]
- Inference for 0-1 Markov processes
  - Parameter estimation: publications [1] and [2]

  - State estimation: publication [2]
  - Combined inference and state estimation: publication [2]

- Inference for stationary random fields
  - Nonparametric estimation of covariances and spectra from Poisson samples: publication [6]

- Additional topics
  - Poisson approximation: publications [11] and [13]
  - Properties of randomized stopping times for Markov processes: publication [8]
  - Nonparametric survival analysis: publication [12]
  - Applications of the Cox regression model: publication [7]

Principal Research Accomplishments

State estimation for Cox processes. A simple point process N and diffuse random measure M (on the same space and defined over the same probability space) comprise a *Cox pair* if conditional on M, N is a Poisson process with mean measure M. We say also that N is a *Cox process* (or doubly stochastic Poisson process) with *directing measure* M.

In applications ranging from signal detection to image analysis to modeling of precipitation, the directing measure is

- Of primary physical importance
- Not observable.

Rather, only the Cox process N is observable, and one seeks to reconstruct — for each realization ω and with minimal error — the value $M(\omega)$ from $N(\omega)$.

The optimal state estimators are conditional expectations $E[M(\cdot)|N]$, which are themselves random measures. More generally, the entire conditional distribution $P\{M \in (\cdot)|N\}$ should be calculated. In [5] a complete solution to this problem is derived.

Theorem. Let N be a Cox process with directing measure M. For each set A, let

$$\mathcal{F}^N(A) = \sigma(N(B) : B \subseteq A)$$

be the σ-algebra corresponding to (complete, uncorrupted) observation of N over A. Then provided that $E[M(A)] < \infty$,

$$P\{M \in \Gamma | \mathcal{F}^N(A)\} = \left. \frac{E[e^{-M_\mu(A)} 1(M_\mu \in \Gamma)]}{E[e^{-M_\mu(A)}]} \right|_{\mu = N_A}, \tag{1}$$

for each set Γ, where the M_μ are unreduced Palm processes of M (cf. [1,5]), and N_A is the restriction of N to A (i.e., the observations).

It follows, for example, that for each set B,

$$E[M(B)|\mathcal{F}^N(A)] = \left. \frac{E[e^{M_\mu(A)} M_\mu(B)]}{E[e^{-M_\mu(A)}]} \right|_{\mu = N_A}.$$

However, while this is a completely general solution to the state estimation problem, implementation of (1) requires knowledge of the probability law of M (or of N — the two determine each other uniquely), which is often unavailable in practice. In [1,3,5,12] we address various facets of the problem of *combined inference and state estimation* for Cox processes, formulated in the following manner.

1. Suppose that E is compact and for simplicity that $A = E$. Let (N_i, M_i), $i = 1, 2, \ldots$ be i. i. d. (independent, identically distributed) copies of a Cox pair (N, M), such that the N_i are observable whereas the M_i are not. Assume that the law of M is unknown. Suppose that $N_1, \ldots, N_n$ have been observed and that we desire an approximation to the 'true' state estimator $E[e^{-M_{n+1}(f)}|N_{n+1}]$.

2. The first key observation is that for each function f,

$$E[e^{-M_{n+1}}(f)|N_{n+1}] = \left. \frac{L_M(\mu, 1+f)}{L_M(\mu, 1)} \right|_{\mu=N_A}, \tag{2}$$

   where $L_M(\mu, f) = E[e^{-M_\mu(f)}]$ is the Laplace functional of the Palm process M_μ.

3. Second, a key lemma in [5] establishes that M_μ ar 'he reduced Palm process N_μ, which satisfies

$$P\{N_\mu \in (\cdot)\} = P\{N - \mu \in (\cdot)|N - \mu \geq 0\}, \tag{3}$$

   form a Cox pair, so that for functions g with $0 \leq g < 1$,

$$L_M(\mu, g) = L_N(\mu, -\log(1-g)), \tag{4}$$

   where $L_N(\mu, \cdot)$ is the Laplace functional of N_μ.

4. We now invoke the *principle of separation* long used in electrical engineering. Since by (1) and (4),

$$E[e^{M_{n+1}(f)}|N_{n+1}] = \frac{L_N(N_{n+1}, -\log f)}{L_N(N_{n+1}, \infty)},$$

if we were able to estimate L_N we could use the *pseudo-state estimators*

$$\hat{E}[e^{M_{n+1}(f)}|N_{n+1}] = \frac{\hat{L}_N(N_{n+1}, -\log f)}{\hat{L}_N(N_{n+1}, \infty)}, \tag{5}$$

where $\hat{L}_N$ is an estimator based on $N_1, \ldots, N_{n+1}$.

5. On the basis of (3) we construct estimators

$$\hat{L}_N(\mu, g) = \frac{e^{\mu(g)} \sum_{i=1}^{n} e^{-N_i(g)} \prod_{j=1}^{\ell_n} 1(N_i(A_{nj} \geq \mu(A_{nj}))}{\sum_{i=1}^{n} \prod_{j=1}^{\ell_n} 1(N_i(A_{nj} \geq \mu(A_{nj}))}, \tag{6}$$

where (A_{nj}) is a null array of partitions of E.

These estimators have been shown [5] under appropriate hypotheses to be

- Strongly uniformly consistent
- Pointwise asymptotically normal;

however, principal result is the following.

Theorem. For f a function satisfying $0 < f \leq 1$ and $\delta > 0$,

$$\lim_{n \to \infty} n^{-1/2+\delta} E\left[\left(\hat{E}[e^{M_{n+1}(f)}|N_{n+1}] - E[e^{M_{n+1}(f)}|N_{n+1}]\right)^2\right] = 0. \tag{7}$$

While very general, this theorem is disappointing in the sense that the rate of (L^2-)convergence, $n^{-1/4}$, is distinctly less than one would wish (based on central limit theory, the hoped-for rate of convergence is $n^{-1/2}$). In [3] more precise results we obtained for *mixed Poisson processes*, which are Cox processes in which the directing measure has the particular form $M = Y\nu_0$, where Y is a positive random variable with unknown distribution F and ν_0 is a fixed (but unknown) measure on E. In this case,

$$E[e^{-M(f)}|N_A] = \frac{\int F(du) e^{-u\nu_0(A)} u^{N(A)} e^{-u\nu_0(f)}}{\int F(du) e^{-u\nu_0(A)} u^{N(A)}},$$

and special structure may be used to estimate ν_0 and the integrals

$$K(k) = \int F(du) e^{-u\nu_0(A)} u^k e^{-u\nu_0(f)}$$

separately.

Similarly, in [5] we stipulate a parametric model $\{P_\theta : \theta \in \Theta\}$ and are able to obtain optimal rates of convergence in (7).

Inference for Stationary Point Processes. Let N be a point process on (for simplicity) $\mathbf{R}^d$, assumed stationary with respect to translation operators $\theta_x \omega = \omega \tau_x^{-1}$, where $\tau_x y = y - x$, (cf. [1]) and suppose that the underlying probability measure P is unknown. The data comprise *single realizations* of N observed over compact, convex sets K; the problems of interest are

- Estimation of moment measures, cumulant measures and the spectral measure of N

- Estimation of the Palm measure P^*, which satisfies

$$E\left[\int G(N\tau_x^{-1}, x)N(dx)\right] = E^*\left[\int G(N, x)dx\right]$$

  for appropriate functionals G, and has the heuristic interpretation that

$$P^*\{N \in \Gamma\}/P^*(\Omega) = P\{N \in \Gamma | N(\{0\}) = 1\}$$

- Estimation of P

- 'Large sample' behavior of estimators as $K \uparrow \mathbf{R}^d$.

To estimate the integral

$$P^*(H) = E^*[N(H)] = \int_\Omega H dP^* \tag{8}$$

we use the unbiased estimators

$$\hat{P}^*(H) = \frac{1}{\lambda(K)} \int_K H(N\tau_x^{-1})N(dx). \tag{9}$$

These can be written as

$$\hat{P}^*(H) = \frac{1}{\lambda(K)} \sum_{j: X_j \in K} H(\varepsilon_0 + \sum_{i \neq j} \varepsilon_{X_i - X_j}),$$

so that $\hat{P}^{\cdot}(H)$ is simply a sum of evaluations of H at translations of N placing each point in turn at the origin.

Particular choices of H lead, for example, to estimators of reduced moment measures. Taking $H \equiv 1$ yields the estimators $\hat{\nu} = \hat{P}^{\cdot}(1) = N(K)/\lambda(K)$ of the intensity $\nu = E^{\cdot}[1]$. Similarly, taking $H(N) = N(f)$, where f is a function with compact support, gives the estimators

$$\hat{\mu}^2_{\cdot}(f) = \frac{1}{\lambda(K)} \int_K N(dx) \int f(y-x) N(dy) \tag{10}$$

of the reduced second moment measure $\mu^2_{\cdot}$.

Given a convex set K, we define $\delta(K)$ to be the supremum of the radii of Euclidean balls that are subsets of K. Assuming that $\delta(K) \to \infty$ allows K to grow in a general yet nevertheless restrained manner; in particular, K must expand in all directions, although not necessarily at the same rate.

The main result on strong uniform consistency, proved in [9], improves previous consistency results by the addition of uniformity.

Theorem. If

a) P is ergodic;

b) $0 < \nu < \infty$;

c) $\mathcal{H}$ is a uniformly bounded set of functions on Ω that is compact in the topology of uniform convergence on compact subsets of Ω;

then almost surely

$$\lim_{\delta(K)\to\infty} \sup_{H\in\mathcal{H}} |\hat{P}^{\cdot}(H) - P^{\cdot}(H)| = 0. \tag{11}$$

Consequences include strong uniform consistency of the estimators $\hat{\mu}^2_{\cdot}$: for $\mathcal{K}$ a compact, uniformly bounded subset of $C_+(E)$, each element of which has support in the same compact subset of $\mathbf{R}^d$,

$$\lim_{\delta(K)\to\infty} \sup_{f\in\mathcal{K}} |\hat{\mu}^2_{\cdot}(f) - \mu^2_{\cdot}(f)| = 0 \tag{12}$$

almost surely, as well as for corresponding estimators of the spectral measure and spectral density function.

Available results on asymptotic normality are rather less satisfactory, inasmuch as they are proved only under very strong and virtually uncheckable assumptions, using classical techniques of showing that cumulants of orders three and greater converge to zero.

Theorem. If

a) P is ergodic;

b) Under P, moments of N of all orders exist;

c) All reduced cumulant measures of N have finite total variation;

then for each H,

$$\lim_{\delta(K)\to\infty} \sqrt{\lambda(K)}\left[\hat{P}^{*}(H) - P^{*}(H)\right] \xrightarrow{d} N(0, \sigma^2(H)), \tag{13}$$

where the variance $\sigma^2(H)$ depends on P.

The full force of the strong uniform consistency established in (11) is used in [9] to prove pointwise consistency for estimators of the probability P itself. Assumimg that the intensity ν is known we use as estimators of $P(H) = E[H(N)]$,

$$\hat{P}(H) = \frac{1}{\nu\lambda(K)^2}\int_K N(dx)\int_K H(N\tau_{x-y}^{-1})dy \tag{14}$$

Theorem. Assume that P is ergodic, that $0 < \nu < \infty$ and that H is bounded and continuous. Then almost surely

$$\lim_{\delta(K)\to\infty} |\hat{P}(H) - P(H)| = 0. \tag{15}$$

Additional issues addressed in [9] include

- Poisson approximations complementing the central limit theorem
- Linear state estimation when the probability P is unknown.

Inference for multiplicative intensity models. Let $N^{(1)}$, $N^{(2)}$, ... be independent copies of a point process N on $[0,1]$ whose stochastic intensity, under the probability measure P_α, is $\lambda(\alpha)_t = \alpha_t\lambda_t$, where λ is an predictable process and α is an unknown element of $L^1_+[0,1]$. Both the $N^{(i)}$ and the baseline stochastic intensities $\lambda^{(i)}$ are observable, and goal is to estimate the deterministic factor α.

Given the data $(N^{(1)}, \lambda^{(1)}), \ldots, (N^{(n)}, \lambda^{(n)})$, the log-likelihood function

$$L_n(\alpha) = \int_0^1 \lambda_s^n(1 - \alpha_s)ds + \int_0^1 (\log\alpha_s)dN_s^n, \tag{16}$$

where $N^n = \sum_{i=1} N^{(i)}$ and $\lambda^n = \sum_{i=1}^n \lambda^{(i)}$, is unbounded above, rendering direct estimation of α by maximum likelihood techniques impossible.

Martingale estimators, one attractive alternative, estimate processes

$$B_t(\alpha) = \int_0^t \alpha_s 1(\lambda_s > 0) ds, \tag{17}$$

which are surrogates for indefinite integrals $\int_0^t \alpha_s ds$, via

$$\hat{B}_t = \int_0^t 1(\lambda_s > 0)\lambda_s^{-1} dN_s.$$

Martingale estimators are easy to calculate, as are their variances, which can likewise be estimated. Moreover, potent martingale central theorems may be applied to establish asymptotic normality of the estimators. Despite all this, martingale estimators admit shortcomings nevertheless, arguably the most severe of which is that they do not estimate α itself but rather the indefinite integrals in (17).

We employ the *method of sieves*, developed by Grenander and others. In our setting, it operates in the following manner: let $I = L_+^1[0,1]$ be the index set of the statistical model, and let the log-likelihood functions L_n be given by (16). For sample size n, we

- Replace I by a compact subset I_n, over which .iere does exist a maximizer $\hat{\alpha} = \hat{\alpha}_n$ of L_n;

- Let the restrictions become successively weaker as more data is obtained.

Given proper balancing of the rate at which the I_n increase with n, these estimators $\hat{\alpha}$ are consistent.

More precisely, for each $a > 0$, let $I(a)$ be the family of absolutely continuous $\alpha \in I$ such that $a \le \alpha \le a^{-1}$ and $|\alpha'|/\alpha \le a_{-1}$. These are suitable restrictions of I; the *sieve mesh* a measures the roughness of elements of $I(a)$. Then the following theorem [10] indicates how a should depend on n.

Theorem. Assume that

a) The function $m_s(\alpha) = E_\alpha[\lambda_s]$ is bounded and bounded away from zero on $[0,1]$;

b) The 'entropy'

$$H(\alpha) = -\int_0^1 [1 - \alpha_s + \alpha_s \log(\alpha_s)] m_s(\alpha) ds$$

is finite;

c) $\int_0^1 \mathrm{Var}_\alpha(\lambda_s) ds < \infty$;

d) $E_\alpha[N_1^2] < \infty$.

Then for each n and a there exists a maximizer $\hat{\alpha}(n,a)$ of L_n within $I(a)$, and for $a_n = n^{-1/4+\eta}$ with $0 < \eta < 1/4$, the estimators $\hat{\alpha} = \hat{\alpha}(n, a_n)$ satisfy

$$\lim_{n\to\infty} \| \hat{\alpha} - \alpha \|_1 = 0 \tag{18}$$

almost surely with respect to P_α.

Local asymptotic normality of log-likelihood processes

$$L_n(\alpha, t) = \int_0^t \lambda_s^n (1 - \alpha_s) ds + \int_0^t (\log \alpha_s) dN_s^n$$

can be established as well; it is of interest in its own right and also leads to a central limit theorem for the integrated estimation error.

Theorem. Let α and $\alpha*$ be elements of I such that

$$\int_0^1 \left[\frac{(\alpha*_s)^2}{\alpha_s}\right] m_s(\alpha) ds < \infty$$

and

$$\int_0^1 \left[\frac{(\alpha*_s)^3}{a^2 a_s}\right] m_s(\alpha) ds < \infty.$$

Then under P_α the processes

$$\left(L_n(\alpha + n^{-1/2}\alpha*, t) - L_n(\alpha, t) + \frac{1}{2}\int_0^t \left[\frac{(\alpha*_s)^2}{\alpha_s}\right] m_s(\alpha) ds\right)$$

converge in distribution to a Gaussian martingale with (independent increments and) variance function

$$V_t(\alpha, \alpha*) = \int_0^t \left[\frac{(\alpha*_s)^2}{\alpha_s}\right] m_s(\alpha) ds.$$

One can deduce from this theorem that the integrated sieve estimators — interestingly — satisfy the same central limit theorem as do martingale estimators.

Inference for thinned point processes. Given a point process $N = \sum_i \varepsilon_{X_i}$ and a function p with $0 < p < 1$, let U_i be random variables such that

- the U_i are conditionally independent given N
- for each i, $P\{U_i = 1|N\} = 1 - P\{U_i = 0|N\} = p(X_i)$;

then the point process

$$N' = \sum_i U_i \varepsilon_{X_i}$$

is called a *p-thinning* of N. Heuristically, points of N are, randomly and independently, either retained in their original location in N' or else deleted entirely; a point of N at x is retained with probability $p(x)$.

Among the computational relationships between N and N' are complementary expressions for the Laplace functionals:

$$\begin{aligned} L_{N'}(f) &= L_N(-\log[1 - p + pe^{-f}]) \\ L_N(g) &= L_{N'}\left(-\log\frac{p - 1 + e^{-g}}{p}\right) \end{aligned}$$

If p is known, then the laws of N and N' determine each other uniquely.

When the underlying process N cannot be observed, but only the p-thinning N' is observable, the state estimation problem for N is to reconstruct, for each realization ω, the unobserved value $N(\omega)$ from the observations $N'(\omega)$. The probability law of N and the function p (and hence the law of N') are stipulated to be known.

Of course, since N' is observed, we need actually only reconstruct the point process $N - N'$ of deleted points. In the following result, the entire conditional distribution of N' given $N - N'$ is expressed in terms of the reduced Palm distributions of N.

Theorem. Let N' be the p-thinning of N, and let $Q(\mu, d\nu)$ denote the reduced Palm distributions of N. Then

$$P\{N - N' \in d\nu|N'\} = \frac{\exp[-\nu(-\log(1-p))]Q(N', d\nu)}{\int \exp[-\eta(-\log(1-p))]Q(N', d\eta)}. \tag{19}$$

In other settings, estimation of p and the law of N from observation of i. i. d. copies N_i' of the p-thinning N' may be the principal objective. We have done this in a completely nonparametric manner as follows. Let $\{A_{nj} : n \geq 1, 1 \leq j \leq \ell_n\}$ be a null array of partitions of E; then under the assumption that the mean measure μ of N is known and given the observations $N_1', \ldots, N_n'$ we use the estimators

$$\hat{p}(x) = \frac{1}{n\mu(A_{nj})} \sum_{i=1}^{n} N_i'(A_{nj}) \tag{20}$$

for p and

$$\hat{L}(g) = \hat{L}'\left(\log \frac{\hat{p} - 1 + e^{-g}}{\hat{p}}\right), \tag{21}$$

for the Laplace functional L of N, where

$$\hat{L}'(f) = \frac{1}{n} \sum_{i=1}^{n} \exp[-N_i'(f)]$$

is the sequence of *empirical Laplace functionals* associated with (N_i').

In [4], the following properties of these estimators are established.

Theorem. Assume that

a) $\mu\{x : p(x) = 0\} = 0$;

b) μ is diffuse;

c) p is continuous and $p(x) > 0$ for all x;

d) $\max_j$ diam $A_{nj} \to 0$;

e) $E[N(E)^4] < \infty$;

f) There is $\delta < 1$ such that as $n \to \infty$,

$$\ell_n \max_j \left\{ \frac{E[N(A_{nj})^3]}{\mu(A_{nj})^3} \right\} = O(n^\delta).$$

Then almost surely

$$\lim_{n\to\infty} \sup_{x \in E} |\hat{p}(x) - p(x)| = 0; \tag{22}$$

and for each compact set K of functions

$$\lim_{n\to\infty} \sup_{g \in K} |\hat{L}(g) - L(g)| = 0. \tag{23}$$

A variety of other results appears in [4] as well, including central limit theorems for these estimators.

Inference for binary Markov processes. Let $X = (X_t)$ be a Markov process with state space $S = \{0, 1\}$ and infinitesimal generator

$$A = \begin{bmatrix} -a & a \\ b & -b \end{bmatrix},$$

where a and b are positive numbers, possibly unknown. In [2] we addressed issues of

- Statistical inference, i.e., estimation of a and b
- State estimation
- Combined statistical inference and state estimation.

for X under a wide variety of forms of *partial observation*. These include

- Regularly spaced discrete observations $X_{n\Delta}$, where Δ is the sampling interval
- 'Jittered' regular samples $X_{n\Delta+\epsilon_n}$, where the ϵ_n are i. i. d. random variables, independent of X, satisfying $|\epsilon_n| \leq \Delta/2$
- Poisson samples X_{T_n}, where the T_n are the arrival times in a Poisson process N independent of X, whose rate may be unknown
- Poisson samples observable only when $X_t = 1$
- Observability determined by an alternating renewal process independent of X
- Observation of only the 'level crossing' times $U_1, U_2, \ldots$ at which X enters state 1
- Observation of the time-averaged data $Y_n = \int_{n\Delta-\gamma}^{n\Delta} X_t dt$
- Observation of a random time change $Z_t = X_{\tau_t}$.

In each case, *ad hoc* estimators $\hat{a}$ and $\hat{b}$, which exploit the special structure of the problem, are devised, and shown to be strongly consistent and asymptotically normal, and optimal state estimators $\hat{X}_t$ for unobserved values of X are calculated, either explicitly or as solutions of stochastic differential equations. Rather than present an exhaustive list of results, we illustrate with the case of Poisson samples and state 0 unobservable.

Let N be a Poisson process with arrival times T_n such that X and N are independent, and suppose that a, b and the rate λ of N are all unknown. Neither X nor N is completely observable; instead the observations are the point process

$$N_t^* = \int_0^t X_s dN_s,$$

whose arrival times are those T_n for which $X_{T_n} = 1$. It is known that this process is a Cox process, and also a renewal process, whose interarrival distribution F satisfies

$$\int_0^\infty e^{-\alpha u}[1 - F(u)]du = \frac{a + b + \alpha}{\alpha^2 + \alpha(a + b + \lambda) + \lambda a}.$$

In particular, (a, b, λ) is uniquely determined by the three values

$$\begin{aligned} A &= \int_0^\infty [1 - F(u)]du \\ B &= \int_0^\infty e^{-u}[1 - F(u)]du \\ C &= \int_0^\infty e^{-2u}[1 - F(u)]du. \end{aligned}$$

With W_k denoting the interarrival times of the observed renewal process N^*, appropriate estimators for A, B and C, given observation of N^* over $[0, t]$, are corresponding functionals of the empirical distribution function $\hat{F}$ given by

$$\hat{F}(u) = \frac{1}{N^*(t)} \sum_{k=1}^{N^*(t)} 1(W_k \leq u). \tag{24}$$

That is,

$$\hat{A} = \int_0^\infty [1 - \hat{F}(u)]du = \frac{1}{N^*(t)} \sum_{k=1}^{N^*(t)} W_k$$

$$\hat{B} = \int_0^\infty e^{-u}[1 - \hat{F}(u)]du = \frac{1}{N^*(t)} \sum_{k=1}^{N^*(t)} (1 - e^{-W_k})$$

$$\hat{C} = \int_0^\infty e^{-2u}[1 - \hat{F}(u)]du = \frac{1}{2N^*(t)} \sum_{k=1}^{N^*(t)} (1 - e^{-2W_k}).$$

Then, since there is a function H (computed in [2]) such that

$$(a, b, \lambda) = H(A, B, C),$$

we arrive at estimators

$$(\hat{a}, \hat{b}, \hat{\lambda}) = H(\hat{A}, \hat{B}, \hat{C}). \tag{25}$$

Although the situation is rendered more complicated by the presence of the 'random sample size' $N^*(t)$ in (24), limit theory for empirical distribution functions can nevertheless be applied to yield the following large sample properties for the estimators of (25).

Theorem. As $n \to \infty$,

$$(\hat{a}, \hat{b}, \hat{\lambda}) \to (a, b, \lambda) \tag{26}$$

almost surely.

Theorem. As $n \to \infty$,

$$\sqrt{n}\left[(\hat{A}, \hat{B}, \hat{C}) - (A, B, C)\right] \xrightarrow{d} N(0, \Sigma),$$

where the covariance matrix Σ is computed in [2], and hence

$$\sqrt{n}\left[(\hat{a}, \hat{b}, \hat{\lambda}) - (a, b, \lambda)\right] \xrightarrow{d} N(J_H \Sigma J_H^t), \tag{27}$$

where J_H is the Jacobian of the transformation H.

State estimation for X given observations $\mathcal{G}_t = \sigma(N_u^* : u \leq t)$ of N^* entails principally solving the *filtering problem* of calculating the optimal state estimators

$$\hat{X}_t = E[X_t | \mathcal{G}_t]. \tag{28}$$

No closed form solution is known; however, we have shown that the process $\hat{X}_t$ can be calculated recursively, as the solution of a stochastic differential equation.

Theorem. The state estimators $\hat{X}$ of (28) are the unique solution of the stochastic differential equation

$$d\hat{X}_t = \left[-b\hat{X}_t + a(1 - \hat{X}_t)\right] dt + (1 - \hat{X}_{t-}) \left[dN_t^* - \lambda \hat{X}_{t-} dt\right]. \tag{29}$$

For extension to general Markov processes with finite state space, see [1, Chapter 10].

Inference for random fields. Let $Y = \{Y(x); x \in \mathbf{R}^d\}$ be a stationary random field on $\mathbf{R}^d$ with unknown mean

$$m = E[Y(x)]$$

and unknown covariance function

$$R(y) = \mathrm{Cov}(Y(x), Y(x + y)).$$

Let $N = \sum_i \varepsilon_{X_i}$ be a stationary Poisson process on $\mathbf{R}^d$ with (possibly unknown) intensity ν, and assume that Y and N are independent. We stipulate that

- N is observable
- Y is observable only at the points of N.

Thus the observations are the *marked point process*

$$\bar{N} = \sum_i \varepsilon_{(X_i, Y(X_i))} \tag{30}$$

over sets of the form $K \times \mathbf{R}$, where K is compact and convex, and the principal statistical issues are

- Estimation of m and R
- Large sample properties of estimators as $\delta(K) \to \infty$
- State estimation for unobserved values of Y.

This model is applicable in a wide variety of situations; its principal properties are established in [6].

A fundamental question is whether inference is even possible at all; that is, does the law of $\bar{N}$ determine that of Y?

Theorem. If Y is continuous in probability, then the probability law of $\bar{N}$ determines uniquely that of Y.

Estimation of the mean value m is straightforward, even if ν is unknown: the appropriate estimator given observation of $\bar{N}$ over $K \times \mathbf{R}$ is

$$\hat{m} = \frac{\int_K Y dN}{N(K)} = \frac{\sum_{X_i \in K} Y(X_i)}{N(K)}. \tag{31}$$

Provided that the covariance function R is integrable, i.e.,

$$\int_{\mathbf{R}^d} |R(y)| dy < \infty \tag{32}$$

these estimators have the following properties. (Here and below, Lebesgue measure is denoted by λ or simply by dy.)

Theorem. If (32) holds, then the estimators $\hat{m}$ of (31) are consistent in quadratic mean: as $\delta(K) \to \infty$,

$$E[(\hat{m} - m)^2] \to 0$$

and asymptotically normal:

$$\sqrt{\lambda(K)}(\hat{m} - m) \xrightarrow{d} N(0, \sigma^2),$$

where

$$\sigma^2 = \int R(y) dy + \frac{[R(0) + m^2]}{\nu}.$$

In order to simplify the discussion of estimation for R we assume that ν is known; however, it can be replaced throughout by the estimator $\hat{\nu} = N(K)/\lambda(K)$.

As estimator of R we employ

$$\hat{R}(x) = \frac{1}{\nu^2 \lambda(K)} \int_K \int_K w_K(x - x_1 + x_2) Y(x_1) Y(x_2) N^{(2)}(dx_1, dx_2), \tag{33}$$

where $N^{(2)}(dx_1, dx_2) = N(dx_1)(N - \varepsilon_{x_1})(dx_2)$ and $w_K(x) = \alpha_K^d w(x/\alpha_K)$, with w a positive, bounded, isotropic density function on $\mathbf{R}^d$ and the α_K positive numbers satisfying $\alpha_K \to 0$ and $\alpha_K^d \lambda(K) \to \infty$ as $\delta(K) \to \infty$.

Weak consistency of the estimators $\hat{R}$ is implied by the following result.

Theorem. Assume that R is continuous and fulfills (32), and that the fourth-order cumulant Q of Y exists and satisfies

$$\sup_{x_1, x_2} \int |Q(x + x_1, x, x_2)| dx < \infty.$$

Then as $\delta(K) \to \infty$,

$$E[\hat{R}(x)] \to R(x) \tag{34}$$

for each x and

$$\lambda(K)\alpha_K^d \mathrm{Cov}(\hat{R}(x_1), \hat{R}(x_2)) \to 1(x_1 = \pm x_2) S(x_1), \tag{35}$$

where

$$S(x) = \frac{\nu^2[Q(0, x, x) + 2R(x)^2 + R(0)^2]}{\int w(y)^2 dy}. \tag{36}$$

Asymptotic normality is more intricate.

Theorem. If Y has finite moments of all orders, if for each k the kth order cumulant function $Q^{(k)}$ satisfies

$$\int \left| Q^{(k)}(z_1, \ldots, z_{k-1}) \right| dz_1 \ldots dz_{k-1} < \infty,$$

if R is twice continuously differentiable and if $\int |y|^2 w(y) dy < \infty$, then as $\delta(K) \to \infty$,

$$\sqrt{\lambda(K)\alpha_K^d}[\hat{R}(x) - R(x)] \xrightarrow{d} N(0, S(x)), \tag{37}$$

for each x, where S is given by (36).

Additional aspects treated in [6] include

- Construction of strongly consistent estimators for the set-indexed process $Q(A) = \int_A R(x) dx$

- Optimal state estimation for unobserved values of Y, using linear state estimators

$$\hat{Y}(x) = \int_K h(x,z)Y(z)N(dz).$$

The optimal function h is characterized as the solution to a particular integral equation.

Work in Progess as of December 31, 1986

For positron emission tomography — an increasingly important form of medical imaging — the basic model is that observed data constitute a Poisson process N with intensity function

$$\mu(u) = \int p(u|x)\lambda(x)dx, \tag{38}$$

where

- p is a known point-spread function
- λ is the unknown intensity function of the Poisson process of positron-electron annihilations.

The goal is to estimate λ, the key quantity of physical interest.

To date we have addressed several crucial mathematical issues, albeit mainly for the simplified problem of estimating the unknown intensity function λ of a Poisson process N (i.e., it is not assumed that (38) holds) on d-dimensional Euclidean space R^d. These include the following.

1. Given as data i. i. d. copies $N_1, \ldots, N_n$ of N, define the superposition $N^n = \sum_{i=1}^n N_i$; N^n, a Poisson process with intensity function $n\lambda$, and also a sufficient statistic for λ. The log-likelihood function (omitting a term not dependent on λ)

   $$L_n(\lambda) = -n \int \lambda(x)dx + \int [\log \lambda(x)] N^n(dx), \tag{39}$$

   is, however, not bounded above, and therefore does not admit a maximizer.

2. The method of sieves provides an effective means of approaching the problem: the parameter space of possible intensity functions is restricted in a manner that becomes successively weaker as more data is obtained. We have shown that maximizers of (39) within the restricted sets do exist, and satisfy desirable statistical properties. Specifically, for each $h > 0$ we introduce the sieve S_h of functions λ of the form

   $$\lambda(x) = \int k_h(x, y) F(dy), \tag{40}$$

where h is a smoothing parameter, k_h is a kernel and F is a probability measure on R^d, but is otherwise unrestricted. (Note the resemblance between (40) and (38)!) We have taken k_h to be a circularly symmetric Gaussian kernel with variance h:

$$k_h(x,y) = (2\pi h)^{-d/2}\exp[-\|x-y\|^2/2h]. \tag{41}$$

The smaller h, the rougher λ can be. We have established that under minor restrictions on the kernels, for each n and h there exists a maximizing element $\hat{F} = \hat{F}(n,h)$ of L_n, which under (40) becomes a function of F.

3. Computation of the restricted maximum likelihood estimators is far from straightforward. Based on the similarity between (38) and (40) and on the 'incomplete data' aspect of the model, we have investigated use of the EM algorithm of Dempster, Laird and Rubin. The basic iteration for the algorithm is

$$F^1(dy) = F^0(dy)\int \frac{k_h(x,y)}{\int k_h(x,z)F^0(dz)}\frac{N^n(dx)}{N^n(R^d)}, \tag{42}$$

where F^0 is an initial estimate and F^1 the new estimate. Although we have made progress regarding convergence of this infinite-dimensional iterative algorithm, our results remain incomplete.

4. In order to obtain the statistically desirable property of consistency for estimators

$$\hat{\lambda}(x) = \hat{\lambda}_n = \int k_h(x,y)\hat{F}_n(dy)$$

the sieve mesh h must be allowed to depend on the sample size n and in particular to converge to zero as n converges to infinity. We have proved the following theorem.

Theorem. If
a) λ is continuous and has compact support;
b) $|\int \lambda(x)\log(\lambda(x))dx| < \infty$;
then provided that $h_n = n^{-1/8+\epsilon}$ for some $\epsilon > 0$,

$$\|\hat{\lambda}(n,h_n) - \lambda\|_1 \to 0 \tag{43}$$

as $n \to \infty$, in the sense of almost sure convergence.

Under weaker conditions and with the sieve mesh permitted to decrease more rapidly, (43) holds in the sense of convergence in probability.

List of Publications

1. *Point Processes and their Statistical Inference* (Marcel Dekker, New York, 1986).

2. Estimation and reconstruction for zero-one Markov processes. *Stochastic Process. Appl.* **16** (1984) 219–255.

3. Combined nonparametric inference and state estimation for mixed Poisson processes. *Z. Wahrscheinlichkeitstheorie und verw. Geb.* **66** (1984) 81–96.

4. Inference for thinned point processes, with application to Cox processes. *J. Multivariate Anal.* **16** (1985) 368–392.

5. State estimation for Cox processes with unknown probability law. *Stochastic Process. Appl.* **20** (1985) 115–131.

6. Inference for stationary random fields given Poisson samples. *Adv. Appl. Prob.* **18** (1986) 406–422.

7. Flood frequency analysis using the Cox regression model (with J. A. Smith). *Water Resources Res.* **22** (1986) 890–896.

8. Structural properties of random times (with A. O. Pittenger). *Probab. Th. Rel. Fields* **72** (1986) 395–416.

9. Estimation of Palm measures of stationary point processes. *Probab. Th. Rel. Fields* **74** (1987) 55-69.

10. Maximum likelihood estimation in the multiplicative intensity model, via sieves. *Ann. Statist.* **15** (1987) 473–490.

11. Poisson approximation of Bernoulli point processes and their superpositions, via coupling (with R. J. Serfling). *Stochastic Process. Appl.* (to appear).

12. State estimation for Cox processes with unknown law: parametric models. *Stochastic Process. Appl.* (to appear).

13. Poisson approximation in selected metrics by coupling and semigroup methods with applications (with P. Deheuvels, D. Pfeifer and R.J. Serfling). *J. Statist. Planning Inf.* (to appear).

14. Nonparametric survival analysis with time-dependent covariate effects: a penalized partial likelihood approach (with D. M. Zucker). (Submitted to *Ann. Statist.*, 10-86).

Doctoral Dissertations

1. David M. Zucker, 'Survival Data Regression Analysis with Time-Dependent Covariate Effects,' 1986

2. Edward L. Chornoboy, 'Maximum Likelihood Techniques for the Identification of Neural Point Processes,' 1986 (with L. P. Schramm)

CURRICULUM VITA — Alan F. Karr

Personal Data

Name: Alan F. Karr

PII Redacted [redacted]

Address: G. W. C. Whiting School of Engineering
The Johns Hopkins University, Baltimore, MD 21218

Telephone: 301-338-7395

Education

B. S., Industrial Engineering, Northwestern University, 1969
(with highest distinction)

M. S., Industrial Engineering, Northwestern University, 1970

Ph. D., Applied Mathematics, Northwestern University, 1973

Academic Employment

Assistant Professor, Mathematical Sciences, The Johns Hopkins University, 1973–1979

Associate Professor, Mathematical Sciences, The Johns Hopkins University, 1979–1983

Professor, Mathematical Sciences, The Johns Hopkins University, 1983–present

Visiting Research Professor, Statistics, University of North Carolina at Chapel Hill, 1983

Chair, Mathematical Sciences, The Johns Hopkins University, 1985–1986

Associate Dean for Academic Affairs, G. W. C. Whiting School of Engineering, The Johns Hopkins University, 1986–present

Acting Chair, Computer Science, The Johns Hopkins University, 1986

Other Employment

Research Staff Member, Institute for Defense Analyses, 1972–1973

Consultant, Institute for Defense Analyses, 1973–1984

Consultant, Interstate Commission for the Potomac River Basin, 1981

Consultant, U.S. Army Night Vision Laboratory, 1987–present

Honors

Member, Tau Beta Pi

1976 Prize, Military Applications Section, Operations Research Society of America (with L. B. Anderson, J. W. Blankenship)

Fellow, Institute of Mathematical Statistics

Fellowships and Grants

1. National Science Foundation Traineeship, Northwestern University, 1969–1972

2. Royal E. Cabell Fellowship, Northwestern University, 1971–1972

3. Principal Investigator, National Science Foundation Grant MCS-80-03560, 'Studies in Stochastic Processes,' The Johns Hopkins University, July 1, 1980–June 30, 1981

4. Principal Investigator, Air Force Office of Scientific Research Grant 82-0029, 'Estimation and Reconstruction for Stochastic Processes and Deterministic Functions,' The Johns Hopkins University, January 1–December 31, 1982

5. Principal Investigator, Air Force Office of Scientific Research Grant 82-0029A, 'Inference and Reconstruction for Stochastic Processes and Deterministic Functions,' The Johns Hopkins University, January 1–December 31, 1983

6. Principal Investigator, Air Force Office of Scientific Research Grant 82-0029B, 'Inference and State Estimation for Stochastic Point Processes,' The Johns Hopkins University, January 1–December 31, 1984

7. Principal Investigator, Air Force Office of Scientific Research Grant 82-0029C, 'Statistical Inference for Stochastic Point Processes,' The Johns Hopkins University, January 1–December 31, 1985

8. Principal Investigator, Air Force Office of Scientific Research Grant 82-0029D, 'Statistical Inference for Stochastic Processes.' The Johns Hopkins University, January 1–December 31, 1986

9. Co-investigator, Office of Water Research and Technology, Department of the Interior, Grant 14-34-0001-0407, 'Policy Analysis of Reservoir Operation in the Potomac River Basin,' The Johns Hopkins University, January 1–December 31, 1979 (with J. L. Cohon, C. S. ReVelle)

10. Co-administrator, Office of Naval Research Grant NR-042-489, 'Research Conference on Queueing Networks and Applications,' The Johns Hopkins University, April 1, 1982–March 31, 1983 (with R. J. Serfling)

11. Co-administrator, Office of Naval Research Grant N00014-85-G-0117, 'Research Conference on Combinatorial Aspects of Matrix Analysis,' The Johns Hopkins University, February 1, 1985–January 31, 1986 (with R. J. Serfling)

Professional Activities

Member, American Mathematical Society, American Society for Engineering Education, American Statistical Association, Bernoulli Society for Mathematical Statistics and Probability, Institute for Mathematical Statistics, Operations Research Society of America, Society for Industrial and Applied Mathematics

Associate Editor, *Operations Research Letters*, 1982–present

Associate Editor, *Mathematics of Operations Research*, 1985–present

Editor, *SIAM Journal on Applied Mathematics*, 1985–present

Referee, *Advances in Applied Probability, Annals of Probability, Annals of Statistics, IEEE Transactions on Information Theory, Journal of Geophysical Research, Journal of Multivariate Analysis, Management Science, Mathematics of Operations Research, Operations Research, Scandinavian Journal of Statistics, Stochastic Processes and their Applications* and numerous other journals

Proposal reviewer, Air Force Office of Scientific Research, Army Research Office, National Science Foundation, Natural Sciences and Engineering Research Council of Canada

Editor, Series in the Mathematical Sciences, The Johns Hopkins University Press

Editor-at-Large, Marcel Dekker, Inc.

Manuscript reviewer, Birkhäuser-Boston, Springer-Verlag

Founder, Chesapeake Bay-Delaware Bay Regional Probability-Statistics Days

Selected Conference Addresses

Invited address, 'A partially observed Poisson process,' Ninth Conference on Stochastic Processes and their Applications, Evanston, IL, August, 1979

Contributed address, 'State estimation for Cox processes on general spaces,' Tenth Conference on Stochastic Processes and their Applications, Montreal, PQ, August, 1981

Contributed address, 'Reconstruction of partially observed binary Markov processes,' Eleventh Conference on Stochastic Processes and their Applications, Clermont-Ferrand, France, June, 1982

Invited address, 'Nonparametric inference and state estimation for Cox processes,' Joint Statistics Meetings, Cincinnati, OH, August, 1982

Invited address, 'Statistical inference and state estimation for binary Markov processes,' Joint ORSA/TIMS National Meeting, Chicago, IL, April, 1983

Invited lectures, 'Inference for point processes and Markov processes,' Humboldt-Universität, Berlin, DDR, May, 1983

Invited lectures, 'Inference for point processes,' Summer School in Probability, Turku, Finland, June, 1983

Invited lectures, 'Inference for point processes,' University of North Carolina at Chapel Hill, October–December, 1983

Invited address, 'Inference for stationary point processes,' Mini-Conference on Inference for Stochastic Processes, Lexington, KY, May, 1984

Invited lecture, 'Inference for stationary point processes,' University of Copenhagen, Denmark, June, 1984

Contributed address, 'Maximum likelihood estimation in the multiplicative intensity model' Fourteenth Conference on Stochastic Processes and their Applications, Göteborg, Sweden, June, 1984

Invited address, 'Estimation of Palm measures of stationary point processes,' École Polytechnique, Paris, June, 1984

Invited address, 'Inference for Stochastic Processes: A Survey,' (with R. J. Serfling) Joint Statistics Meetings, Las Vegas, NV, August, 1985

Contributed address, 'Statistical inference for stationary random fields based on Poisson samples,' 45th Session of the International Statistical Institute, Amsterdam, Netherlands, August, 1985

Contributed address, 'Combined inference and state estimation for Cox processes: the parametric case,' Satellite Meeting on Mathematical Statistics and Probability to the 45th Session of the ISI, Maastricht, Netherlands, August, 1985

Invited address, 'State estimation for Cox processes,' Research Workshop on Asymptotic Statistical Inference, Edinburgh, Scotland, June, 1986

Invited address, 'Estimation of intensity functions of Poisson processes via the method of sieves, with application to positron emission tomography,' (with M. I. Miller, D. L. Snyder), Joint Statistics Meetings, Chicago, August, 1986

Invited address, 'Estimation of intensity functions of Poisson processes via the method of sieves, with application to positron emission tomography,' Mathematisches Forschungsinstitut Oberwolfach, FRG, 1986

Invited lectures, 'Statistical problems arising in image analysis,' Summer School in Probability, Lahti, Finland, June, 1987

Invited address, 'Palm distributions of point processes and their applications to statistical inference,' AMS/IMS/SIAM Joint Research Conference in the Mathematical Sciences, Ithaca, NY, August, 1987

Invited address, 'Maximum likelihood estimation in the multiplicative intensity model, via sieves,' Satellite Meeting on Mathematical Statistics and Probability to the 46th Session of the ISI, Kyoto, Japan, September, 1987

Doctoral Students

James A. Smith, 'Point Process Models of Rainfall,' 1980 (with J. L. Cohon) (currently at Interstate Commission for the Potomac River Basin, Rockville, MD)

David M. Zucker, 'Survival Data Regression Analysis with Time-Dependent Covariate Effects,' 1986 (currently at National Heart, Lung and Blood Institute, Bethesda, MD)

Edward L. Chornoboy, 'Maximum Likelihood Techniques for the Identification of Neural Point Processes,' 1986 (with L. P. Schramm) (currently at Washington University, St. Louis)

Selected Teaching

Introduction to Engineering (2 times)
Introduction to Probability
Introduction to Statistics (2 times)
Elementary Stochastic Processes (2 times)
Modern Algebra for Applications
Probability Theory (7 times)
Stochastic Processes I (6 times)
Statistical Theory
Topics in Probability: Image Analysis
Analysis and Probability (6 times)
Topics in Applied Mathematics: Approximation Theory
Applied Probability Models
Stochastic Processes II (2 times)
Diffusion Processes (2 times)
Brownian Motion and Potential Theory
Random Measures and Point Processes (2 times)
Inference for Point Processes (2 times)
Stochastic Processes for Inference
Inference for Stochastic Processes (2 times)
Inference for Diffusion Processes

Book

1. *Point Processes and their Statistical Inference* (Marcel Dekker, New York, 1986).

Research Papers

1. Weak convergence of a sequence of Markov chains. *Z. Wahrscheinlichkeitstheorie und verw. Geb.* **33** (1975) 41–48.

2. Two models for optimal allocation of aircraft sorties (with J. Bracken, J. E. Falk). *Opns. Res.* **23** (1975) 979–995.

3. Stability of one-dimensional systems of colliding particles. *J. Appl. Prob.* **13** (1976) 155–158.

4. Two extreme value processes arising in hydrology. *J. Appl. Prob.* **13** (1976) 190–194.

5. The role of Maxwell-Boltzmann and Bose-Einstein statistics in point pattern analysis (with A. M. Liebetrau). *Geographical Anal.* **9** (1977) 418–422.

6. Lévy random measures. *Ann. Probability* **6** (1978) 57–71.

7. The inverse balayage problem for Markov chains (with A. O. Pittenger). *Stochastic Process. Appl.* **7** (1978) 165–178.

8. Markov chains and processes with a prescribed invariant measure. *Stochastic Process. Appl.* **7** (1978) 277–290.

9. Derived random measures. *Stochastic Process. Appl.* **8** (1978) 159–169.

10. An inverse balayage problem for Brownian motion (with A. O. Pittenger). *Ann. Probability* **7** (1979) 186–191.

11. Classical limit theorems for measure-valued Markov processes. *J. Multivariate Anal.* **9** (1979) 234–247.

12. The inverse balayage problem for Markov chains, II (with A. O. Pittenger). *Stochastic Process. Appl.* **9** (1979) 35–53.

13. Some inverse problems involving conditional expectations. *J. Multivariate Anal.* **11** (1981) 17–39.

14. Natural clades differ from 'random' clades: simulations and analysis (with S. M. Stanley, P. W. Signor, III, and S. Lidgard). *Paleobiology* **7** (1981) 115–127.

15. A partially observed Poisson process. *Stochastic Process. Appl.* **12** (1982) 249–269.

16. State estimation for Cox processes on general spaces. *Stochastic Process. Appl.* **14** (1983) 209–232.

17. Extreme points of certain sets of probability measures, with applications. *Math. Opns. Res.* **8** (1983) 74–85.

18. Error bounds for reconstruction of a function f from a finite sequence $\langle \mathrm{sgn}(f(t_i) + x_i)) \rangle$ (with R. J. Serfling). *SIAM J. Appl. Math.* **43** (1983) 476–490.

19. A point process model of summer season rainfall occurrences (with J. A. Smith). *Water Resources Res.* **19** (1983) 95–103.

20. Estimation and reconstruction for zero-one Markov processes. *Stochastic Process. Appl.* **16** (1984) 219–255.

21. Combined nonparametric inference and state es ation for mixed Poisson processes. *Z. Wahrscheinlichkeitstheorie und verw. Geb.* **66** (1984) 81–96.

22. The martingale method: introductory sketch and access to the literature. *Opns. Res. Lett.* **3** (1984) 59–63.

23. Statistical inference for point process models of rainfall (with J. A. Smith). *Water Resources Res.* **21** (1985) 73–79.

24. Inference for thinned point processes, with application to Cox processes. *J. Multivariate Anal.* **16** (1985) 368–392.

25. Integer Prim-Read solutions to a class of target defense problems (with S. A. Burr, J. E. Falk). *Opns. Res.* **33** (1985) 726–745.

26. Nonlinear response to sustained load processes (with K. C. Chou, R. B. Corotis). *J. Structural Engg., ASCE* **111** (1985) 142–157.

27. State estimation for Cox processes with unknown probability law. *Stochastic Process. Appl.* **20** (1985) 115–131.

28. Parameter estimation for a model of space-time rainfall (with J. A. Smith). *Water Resources Res.* **21** (1985) 1251–1257.

29. Inference for stationary random fields given Poisson samples. *Adv. Appl. Prob.* **18** (1986) 406–422.

30. Flood frequency analysis using the Cox regression model (with J. A. Smith). *Water Resources Res.* **22** (1986) 890–896.

31. Structural properties of random times (with A. O. Pittenger). *Probab. Th. Rel. Fields* **72** (1986) 395–416.

32. Estimation of Palm measures of stationary point processes. *Probab. Th. Rel. Fields* **74** (1987) 55–69.

33. Maximum likelihood estimation in the multiplicative intensity model, via sieves. *Ann. Statist.* **15** (1987) 473–490.

34. Poisson approximation of Bernouili point processes and their superpositions, via coupling (with R. J. Serfling). *Stochastic Process. Appl.* (to appear).

35. State estimation for Cox processes with unknown law: parametric models. *Stochastic Process. Appl.* (to appear).

36. Poisson approximation in selected metrics by coupling and semigroup methods with applications (with P. Deheuvels, D. Pfeifer and R.J. Serfling). *J. Statist. Planning Inf.* (to appear).

Review and Survey Articles

1. Lanchester attrition processes and theater-level combat models. In *Mathematics of Conflict*, M. Shubik, ed., 89–126 (North-Holland, Amsterdam, 1983).

2. Point process, stationary. In *Encyclopedia of Statistical Sciences*, **VII**, N. L. Johnson and S. Kotz, eds., 15–19 (Wiley, New York, 1986).

3. Poisson processes. In *Encyclopedia of Statistical Sciences*, **VII**, N. L. Johnson and S. Kotz, eds., 29–32 (Wiley, New York, 1986).

4. Stochastic point process. In *Encyclopedia of Statistical Sciences*, **VIII**, N. L. Johnson and S. Kotz, eds. (Wiley, New York)(to appear).

5. Markov processes. In *Handbook of Operations Research and Management Science*, D. P. Heyman and M. J. Sobel, eds. (North-Holland, Amsterdam) (to appear).

6. Martingales. In *The New Palgrave: A Dictionary of Economics*, J. Eatwell, M. Milgate and P. Newman, eds. (London, Macmillan) (to appear).

<u>Book Reviews</u>

1. Review of *Markov Chains: Theory and Applications* by D. Isaacson and R. Madsen. *SIAM Rev.* **20** (1978) 606–607.

2. Review of *Infinitely Divisible Point Processes* by K. Matthes, J. Kerstan and J. Mecke. *J. Amer. Statist. Assoc.* **75** (1980) 750–751.

3. Review of *Stochastic Processes and Integration* by M. M. Rao. *SIAM Rev.* **24** (1982) 238–239.

4. Review of *A Second Course in Stochastic Processes* by S. Karlin and H. M. Taylor. *SIAM Rev.* **25** (1983) 280–281.

5. Review of *Stochastic Processes* by S. M. Ross. *SIAM Rev.* **26** (1984) 4[illegible]–449.

6. Review of *Stochastic Models in Operations Research*, **I** and **II**, by D. P. Heyman and M. J. Sobel. *American Scientist* **72** (1984) 406–407.

7. Review of *Random Measures*, 3rd ed., by O. Kallenberg. *J. Amer. Statist. Assoc.* **82** (1987) 346–347.

Manuscripts Submitted

1. Nonparametric survival analysis with time-dependent covariate effects: a penalized partial likelihood approach (with D. M. Zucker). (To *Ann. Statist.*, 10–86).