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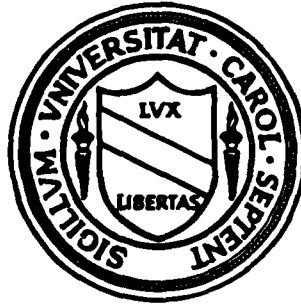
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EXTREME VALUE THEORY FOR DEPENDENT SEQUENCES VIA

THE STEIN-CHEN METHOD OF POISSON APPROXIMATION

by

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EXTREME VALUE THEORY FOR DEPENDENT SEQUENCES VIA
THE STEIN-CHEN METHOD OF POISSON APPROXIMATION

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Summary

In 1970 Stein introduced a new method for bounding the approximation error in central limit theory for dependent variables. This was subsequently developed by Chen for Poisson approximation and has proved very successful in the areas to which it has been applied. Here we show how the method can be applied to extreme value theory for dependent sequences, focussing particularly on the nonstationary case. The method gives new and shorter proofs of some known results, with explicit bounds for the approximation error.

1. POISSON APPROXIMATION

Much of extreme value theory is concerned with convergence to a Poisson distribution or a Poisson process of random variables or processes generated by exceedances over high thresholds. It might therefore be expected that general theorems of convergence to Poisson limits will be relevant to this theory. In fact, such general theorems are relevant, but the main application to date has been to a rather specialised class of problems connected with symmetric statistics. The purpose of the present note is to introduce the possibility of using such results in the extreme value theory of discrete-time stochastic processes, as developed by Leadbetter, Lindgren and Rootzén (1983).

There is a long history of work on Poisson approximation, summarised in the article of Serfling (1978). In recent years, a new approach has been developed. This started with a paper of Stein (1970) on the normal approximation for dependent sequences, and was developed in the Poisson context by Chen (1975). The method has proved highly successful in the extreme value properties of random variables of the form $g(X_{i_1}, \ldots, X_{i_m})$ where g is a symmetric function of m arguments, $X_1, \ldots, X_n$ are independent or exchangeable random variables and the multi-index $(i_1, \ldots, i_m)$ ranges over a class of m-subsets of $\{1, \ldots, n\}$ (Barbour and Eagleson, 1983, 1984). A particular advantage of the method is that it does not require that the individual X_i have common distribution. This means that it has the potential to be applied to extreme value theory for non-stationary sequences, a subject developed by Hüsler (1983, 1986). We shall show how the Stein-Chen method leads to an alternative version of Hüsler's main results, with the additional advantage in giving an explicit upper bound for the approximation error.

We begin by outlining the Stein-Chen method. Suppose Y_i, $i \in I$, is a set of 0-1 random variables with $p_i = P\{Y_i = 1\} = 1 - P\{Y_i = 0\}$. Define

$$(1.1) \qquad W = \sum_{i \in I} Y_i, \quad \lambda = \sum_{i \in I} p_i .$$

Under suitable circumstances W will be approximately Poisson with mean λ. The purpose of the method is to provide an upper bound for the distance in total variation between the distribution of W and the Poisson distribution. The method is first to calculate an upper bound for

$$(1.2) \qquad |E\{Wf(W) - \lambda f(W+1)\}|$$

where f is a real-valued function on Z_+, and then to choose a particular f which allows (1.2) to be related directly to the desired distance. We shall throughout be working with a function f for which

$$(1.3) \qquad \|f\| = \max\{|f(w)|, w \in Z_+\}, \quad \Delta f = \max\{|f(w+1) - f(w)|, w \in Z_+\}$$

are finite (with known upper bounds).

Suppose, for each $i \in I$, J_i is a class of "near neighbors" of i. The idea is that J_i should be small compared with i, but, for $j \notin J_i$, Y_i and Y_j are nearly independent - in some cases exactly so, in others governed by a mixing-type condition. Assume $i \in J_i$. Let

$$(1.4) \qquad W^{(i)} = W - Y_i, \quad V^{(i)} = \sum_{j \in I-J_i} Y_j .$$

Then we may write

$$(1.5) \qquad |E\{Wf(W) - \lambda f(W+1)\}| = |\sum_{i \in I} E\{Y_i f(W^{(i)}+1)\} - \lambda E\{f(W+1)\}|$$

$$= |\sum_{i \in I} E[Y_i\{f(W^{(i)}+1) - f(V^{(i)}+1)\}] + \sum_{i \in I} E\{(Y_i-p_i) f(V^{(i)}+1)\}|$$

$$+ \sum_{i \in I} E[p_i \{ f(V^{(i)}+1) - f(W+1) \}] |$$

$$\leq \Delta f \sum_{i \in I} \sum_{\substack{j \in J_i \\ j \neq i}} E\{Y_i Y_j\}$$

$$+ | \sum_{i \in I} E\{(Y_i - p_i) f(V^{(i)}+1)\} | + \Delta f \sum_{i \in I} \sum_{j \in J_i} p_i p_j .$$

The method then proceeds by bounding each of the three terms in (1.5).

In remains to choose f. Let A denote an arbitrary subset of Z_+ and ~~t~~ $f(0) = 0$,

$$(1.6) \qquad f(w) = -\lambda^{-w} e^\lambda (w-1)! \ \{ P_\lambda (A \cap U_w) - P_\lambda(A) P_\lambda(U_w) \}, \ w \geq 1$$

where P_λ denotes the Poisson probability distribution with mean λ ($P_\lambda(B)$ $= \sum_{j \in B} \lambda^j e^{-\lambda}/j!$) and $U_w = \{0,1,\ldots,w-1\}$. Then

$$(1.7) \qquad I(w \in A) - P_\lambda(A) = wf(w) - \lambda f(w+1)$$

so (1.2) provides an upper bound on $|P\{W \in A\} - P_\lambda(A)|$. Moreover, for such f we have

$$(1.8) \qquad ||f|| \leq \min(1, 1.4\lambda^{-\frac{1}{2}}), \ \Delta f \leq \min(1, \lambda^{-1})$$

(Barbour and Eagleson, 1983) so the upper bound derived from (1.5) is in fact a universal bound, valid for all A. This is the total variation distance between the distribution of W and P_λ.

In Barbour and Eagleson (1984), this method was applied to a situation in which I is a class of two-member subsets of $\{1,\ldots,n\}$, and Y_i and Y_j are independent whenever $i \cap j = \phi$. Their proof corresponds to defining J_i to be the class of all j such that $i \cap j \neq \phi$. With this definition the middle term of (1.5) is 0, so the result depends solely on the first and third terms.

For extreme value theory in dependent sequences, it is natural to try to apply the method when $I = \{1,\ldots,n\}$ and there is some form of mixing condition on the Y_i. Such a theorem was in fact given by Chen (1975), but it requires ϕ-mixing, which is much too strong an assumption, and the final result is hard to interpret - there is nothing corresponding explicitly to Condition D' (see Section 2), but we know from the long history of extreme value theory that some condition of this form is necessary. We therefore outline an alternative approach which leads directly to some results in extreme value theory for nonstationary sequences (Hüsler, 1983, 1986).

2. EXTREMES IN NONSTATIONARY SEQUENCES

Suppose, for some $n \geq 1$, $X_1, \ldots, X_n$ are random variables with marginal distribution functions $F_1, \ldots, F_n$ and let $\{u_{ni}, 1 \leq i \leq n\}$ denote a sequence of boundary values. Define Y_i to be 1 if $X_i > u_{ni}$, 0 otherwise, and let $p_i = P\{Y_i = 1\}$, $p_n^* = \max(p_i, 1 \leq i \leq n)$, $T_n = \sum_1^n Y_i$, $\tau_n = E(T_n) = \sum_1^n p_i$. Assume:

Condition D   Let $\mathcal{B}(i,j)$ denote the σ-algebra generated by $(Y_i, Y_{i+1}, \ldots, Y_j)$ for $1 \leq i \leq j \leq n$. Then, for each $\ell \geq 1$,

$$(2.1) \quad \sup\{|P(AB) - P(A)P(B)| : A \in \mathcal{B}(1,k), B \in \mathcal{B}(k+\ell,n), 1 \leq k \leq n-\ell\} \leq \alpha(n,\ell).$$

Condition D'   For each $r \geq 1$, there exist intervals of the form

$$I_1 = \{1, \ldots, i_1\}, \ I_2 = \{i_1+1, \ldots, i_2\}, \ldots, I_r = \{i_{r-1}+1, \ldots, n\}$$

and subsets $I_k^* \subseteq I_k$ ($1 \leq k \leq r$) for which

$$(2.2) \qquad \sum_{i \in I_k} p_i \leq C_n/r, \quad 1 \leq k \leq r,$$

$$(2.3) \quad \sum_{\substack{i \in I_k^* \\ \ }} \sum_{\substack{j \in I_k^* \\ i \neq j}} E(Y_i Y_j) \leq \alpha^*(n,r),$$

$$(2.4) \qquad \sum_{i \in I_k - I_k^*} p_i \leq g(r)/r.$$

Then we have:

Theorem 1   Let ℓ and r be such that conditions D and D' hold. Then for arbitrary $A \subseteq Z_+$,

$$(2.5) \quad |P\{T_n \in A\} - P_{\tau_n}(A)| \leq r\alpha(n,\ell) + 3r\alpha^*(n,r) + \frac{C_n^2}{r} + 2r\ell p_n^* + 2g(r).$$

This is in the form of an upper bound for finite n, rather than a limit theorem as $n \to \infty$. However, if we make some assumptions about the asymptotic

behaviour of the various constants involved in Theorem 1, we can derive a limit theorem as a corollary:

<u>Corollary 2</u>   Suppose the conditions of Theorem 1 hold for each $n \geq 1$ and that

(i)   the sequence $\{\tau_n, n \geq 1\}$ satisfies

(2.6) $\tau_n \to \tau$ as $n \to \infty$, where $0 < \tau < \infty$,

(ii)   the sequence $\{C_n, n \geq 1\}$ satisfies

(2.7) $C_n \leq C < \infty$,

(iii) There exists a sequence ℓ_n, $n \geq 1$, such that

(2.8) $\alpha(n, \ell_n) \to 0$, $\ell_n p_n^* \to 0$,

(iv)   As $r \to \infty$,

(2.9) $\lim\sup\limits_{n \to \infty} r\, a^*(n, r) \to 0$, $g(r) \to 0$.

The $P\{T_n \in A\} \to P_\tau(A)$ for each $A \subseteq Z_+$.

An immediate consequence of Corollary 2 is:

(2.10) $P\{X_i \leq u_{ni} \text{ for } i = 1, \ldots, n\} \to e^{-\tau}$ as $n \to \infty$.

The result (2.10) was obtained by Hüsler (1986) under minor variations in the assumptions. In Condition D', Hüsler made slightly different assumptions about the construction of the I_k's, but his assumptions are effectively equivalent to (2.2) - (2.4) and (2.9). Hüsler's condition D is different from ours in that the $B(i,j)$ of (2.1) consist only of events of the form

$$\{Y_{i_1} = Y_{i_2} = \ldots = Y_{i_p} = 0\} \text{ for some } i \leq i_1 < \ldots < i_p \leq j.$$

With this modification, (2.1) together with (2.8) is exactly Hüsler condition D. It is not clear whether the modification is of any practical significance; a similar modification to D has been adopted in a different context by Hsing, Hüsler and Leadbetter (1986).

Before giving the proof of Theorem 1, we note the following:

Lemma 3   If P_λ and P_μ are Poisson distributions with means λ and μ, then for all $A \in Z_+$,

$$|P_\lambda(A) - P_\mu(A)| \le |\lambda - \mu|.$$

Proof   Suppose $\lambda > \mu$. Let X and Y - X be independent Poisson with means μ and $\lambda - \mu$. Then

$$|P\{X \in A\} - P\{Y \in A\}| \le P\{X \ne Y\} \le \lambda - \mu.$$

Lemma 4   Suppose B and C are two σ-fields with the property that, for any events $B \in B$, $C \in C$,

$$|P(BC) - P(B)P(C)| \le \varepsilon.$$

Let X and Y be two bounded random variables, ranges [a,b] and [c,d], measurable with respect to B and C respectively. Then

$$|EXY - (EX)(EY)| \le \varepsilon(b-a)(d-c).$$

Proof   If $a = c = 0$, $b = d = 1$ then

$$|EXY - (EX)(EY)|$$

$$= \left| \int_0^1 \int_0^1 [P\{X > x, \ Y > y\} - P\{X > x\}P\{Y > y\}]dxdy \right| \le \varepsilon.$$

The general case follows by linear transformation.

Proof of Theorem 1   For each k let $I_k^{**} \subseteq I_k^*$ be formed from I_k^* by deleting the ℓ rightmost points of I_k. This ensures that $|i - j| > \ell$ whenever $i \in I_k^{**}$, $j \in I_m^{**}$, $k \ne m$. Let

$$I = \bigcup_{k=1}^{r} I_k^{**}, \quad W = \sum_{i \in I} Y_i, \quad \lambda = \sum_{i \in I} p_i.$$

The proof proceeds by obtaining a Poisson approximation for W using the method of Section 1. For each $i \in I_k^{**}$ let $J_i = I_k^{**}$. Since $V^{(i)}$ in (1.4) depends only on k, we write $V^{(i)} = U^{(k)}$ whenever $i \in I_k^{**}$. Consider (1.5). The third term may be written

$$(2.11) \qquad \Delta f \sum_{k=1}^{r} (\sum_{i \in I_k^{**}} \sum_{j \in I_k^{**}} p_i p_j) \leq \frac{\Delta f C_n^2}{r}$$

using (2.2). The first term may be written

$$(2.12) \qquad \Delta f \sum_{k=1}^{r} [\sum_{\substack{i,j \in I_k^{**} \\ i \neq j}} E\{Y_i Y_j\}] \leq \Delta f \, r\alpha^*(n,r)$$

using (2.3). Therefore we concentrate on the second term in (1.5), which may be written in the form

$$(2.13) \qquad | \sum_{k=1}^{r} E\{(S_k - \mu_k) f(U^{(k)} + 1)\} |$$

where $S_k = \sum_{i \in I_k^{**}} Y_i$ and $\mu_k = E(S_k)$. However, we may write

$$(2.14) \qquad E\{(S_k - \mu_k) f(U^{(k)} + 1)\} = E\{(S_k - \mu_k) I(S_k \leq 1) f(U^{(k)} + 1)\}$$

$$+ E\{(S_k - \mu_k) I(S_k > 1) f(U^{(k)} + 1)\}.$$

The second term is bounded by

$$\|f\| \, E\{S_k I(S_k > 1)\} \leq \|f\| \, E\{S_k (S_k - 1)\}$$

$$= \|f\| \, E\{ \sum_{i \neq j \in I_k^{**}} Y_i Y_j \} \leq \|f\| \, \alpha^*(n,r)$$

while by Lemma 3 the first term in (2.14) is bounded by

$$\|f\| \, \alpha(n,\ell) + |E\{(S_k - \mu_k) I(S_k \leq 1)\} E\{f(U^{(k)} + 1)\}|$$

$$\leq \|f\| \, [\alpha(n,\ell) + E\{S_k - \mu_k\} I(S_k > 1)\}] \leq \|f\| \, [\alpha(n,\ell) + \alpha^*(n,r)].$$

Thus an upper bound on (2.13) is

$$(2.15) \qquad r\|f\| \; [\alpha(n,\ell) + 2\alpha^*(n,r)] \; .$$

The sum of (2.11), (2.12) and (2.15) now gives a bound on the total variation distance between the distribution of W and P_λ. The total variation distance between W and T_n is at most

$$(2.16) \qquad P\{ \sum_{k=1}^{r} \sum_{i \in I_k - I_k^{**}} Y_i \neq 0 \} \leq \sum_{k=1}^{r} \sum_{i \in I_k - I_k^{**}} p_i \leq r\ell p_n^* + g(r) \; .$$

The total variation distance between P_λ and P_{τ_n} is, by Lemma 4, at most

$$\tau_n - \lambda = \sum_{k=1}^{r} \sum_{i \in I_k - I_k^{**}} p_i$$

which is also bounded by (2.16). Finally, adding (2.11), (2.12), (2.15) and twice (2.16), using also (1.8), gives the result.

<u>Proof of Corollary 2</u> It suffices to show that $r = r_n$ and $\ell = \ell_n$ can be chosen so that the right hand side of (2.5) tends to 0. Let ℓ_n be the sequence that satisfies (2.8) and write

$$\varepsilon_n = \alpha(n, \ell_n) + 2\ell_n p_n^* \; ,$$

$$g_1(r) = 2g(r) + 3 \limsup_{n \to \infty} r\alpha^*(n, r) \; .$$

Then the bound in (2.5) becomes

$$r\varepsilon_n + g_1(r) + \frac{C^2}{r} \; .$$

Choosing $r = r_n$ so that $r_n \sim \varepsilon_n^{-\frac{1}{2}}$, for example, guarantees that this tends to 0.

3. PROCESSES WITH LOCAL DEPENDENCE

Condition D' is of course crucial to the results of Section 2. For many familiar kinds of stochastic processes it is not satisfied, and much attention has been given to such cases in recent years. The general picture is that exceedances of the boundary occur in clusters, and the limiting distribution of the number of exceedances is compound Poisson (Hsing, Hüsler and Leadbetter, 1986, Alpuim, 1987). Although the Stein-Chen method does not yield explicit rates of convergence in this case, it nevertheless suggests an alternative method of proof of limit theorems.

__Theorem 5__   Suppose all the conditions of Theorem 1 hold except (2.3). Define

$$(3.1) \qquad S_k = \sum_{i \in I_k} Y_i, \quad \tau_n(j) = \sum_{k=1}^{r} P\{S_k = j\}, \quad j = 1, 2, \ldots$$

Fix $B \subseteq Z_+ - \{0\}$ and define

$$(3.2) \qquad Z_k = I(S_k \in B), \quad W = \sum_{k=1}^{r} Z_k, \quad \lambda = \sum_{j \in B} \tau_n(j).$$

Then for any $A \subseteq Z_+$,

$$(3.3) \qquad |P\{W \in A\} - P_\lambda(A)| \le r\alpha(n, \ell) + \frac{c_n^2}{r} + 2r\ell p_n^* + 2g(r).$$

The relevance of this to the compound Poisson limit is seen from:

__Lemma 6__   Suppose $\{Y_{nk}, n \ge 1, 1 \le k \le r_n\}$ is an integer valued array such that, for each $B \subseteq Z_+ - \{0\}$, the random variable

$$\sum_{k=1}^{r_n} I(Y_{nk} \in B)$$

converges to a Poisson limit with mean $\sum_{j \in B} \tau(j)$. Assume the uniform integrability condition

$$(3.4) \qquad \lim_{m \to \infty} \lim_{n \to \infty} \sup \sum_k P\{Y_{nk} > m\} = 0.$$

Let $\tau = \sum_1^\infty \tau(j)$ (assumed finite), $\pi(j) = \tau(j)/\tau$

and define the generating function

$$(3.5) \qquad \phi(z) = \sum_{j=1}^\infty z^j \, \pi(j)$$

Then $\sum_{k=1}^{r_n} Y_{nk}$ converges as $n \to \infty$ to a compound Poisson distribution with generating function

$$(3.6) \qquad \exp\{\tau\phi(z) - \tau\}.$$

Thus, provided (3.3) tends to 0 as $n \to \infty$ for each B, Lemma 6 implies that the limiting distribution of T_n is compound Poisson. The full result is:

<u>Corollary 7</u>   Suppose the conditions of Theorem 5 hold, together with (2.7), (2.8) and the second half of (2.9). Suppose, for each $j \geq 1$,

$$(3.7) \qquad \tau_n(j) \to \tau \pi(j)$$

where $0 < \tau < \infty$ and $\{\pi(j), j \geq 1\}$ is a proper probability distribution with generating function given by (3.5). Then the distribution of

$$T_n = \sum_{i=1}^n I(X_i > u_{ni})$$

converges to a compound Poisson distribution with generating function (3.5).

Clearly, these results are not as easily applied as those of Section 2, since (3.1) and (3.7) require detailed calculations of the local fluctuations of the process. However, a number of specific examples have now been worked out for which it is possible to make such calculations explicitly, e.g. Davis and Resnick (1985), Hsing (1986). The general question, of how broad is the

condition (3.7), is harder to answer. In the stationary case the $\pi(j)$'s, if they exist at all, are properties of the process and do not depend on the precise sequence of boundaries (Hsing, Hüsler and Leadbetter, 1986). In the non-stationary case, the discussion of extremal index in Hüsler (1986) makes it clear that no such universal result can hold, though we might still expect (3.7) to be valid if the process is nearly stationary in some sense, e.g. a stationary process modified by some slowly moving trend which does not affect the local fluctuations.

<u>Proof of Theorem 5</u>   Define I_k^{**} as in the proof of Theorem 1, and set

$$(3.8) \qquad S_k^* = \sum_{i \in I_k^{**}} Y_i, \quad \tau_n^*(j) = \sum_{k=1}^{r} P\{S_k^* = j\}, \quad j=1,2,\ldots,$$

$$(3.9) \qquad Z_k^* = I(S_k^* \in B), \quad W^* = \sum_{k=1}^{r} Z_k^*, \quad \lambda^* = \sum_{j \in B} \tau_n^*(j).$$

As in the proof of Theorem 1, both

$$|P\{W \in A\} - P\{W^* \in A\}|$$

and

$$|P_\lambda(A) - P_{\lambda *}(A)|$$

are bounded by

$$(3.10) \qquad r \ell p_n^* + g(r)$$

so we concentrate on the approximation of W^* by P_λ^*. With $J_k = \{k\}$, $W^{(k)} = W^* - Z_k^*$, (1.5) - (1.8) lead to the inequality

$$(3.11) \quad |P\{W^* \in A\} - P_{\lambda *}(A)| \le \left| \sum_{k=1}^{r} E\{(Z_k^* - EZ_k^*)f(W^{(k)} + 1)\} \right| + \sum_{k=1}^{r} (EZ_k^*)^2.$$

By (2.2), $EZ_n^* \le C_n/r$ and hence the second term in (3.11) is at most C_n^2/r. By Lemma 4, (2.1) and (1.8), the first term in (3.11) is at most $r\alpha(n,\ell)$. Hence

(3.12) $\left| P\{W^* \in A\} - P_{\lambda^*}(A) \right| \leq r\alpha(n,\ell) + C_n^2/r$

and the result follows by combining (3.10) and (3.12).

Proof of Lemma 6   The assumptions of Lemma 6 imply that

$$U_{nj} = \sum_{k=1}^{r_n} I(Y_{nk} = j), \quad j = 1, 2, \ldots$$

converge in distribution as $n \to \infty$ to independent Poisson variables $U_j (j \geq 1)$ with $E(U_j) = \tau \pi(j)$. Hence

(3.13) $$\sum_{k=1}^{r_n} Y_{nk} = \sum_{j=1}^{\infty} j U_{nj}$$

converges in distribution to $\sum j U_j$. Condition (3.4) allows this operation to be rigorously justified, by showing that (3.13) can be approximated arbitrarily closely, uniformly in n, by the sum from $j = 1$ to m. Finally, it is easily checked that $\sum j U_j$ has the generating function (3.6).

Proof of Corollary 7   Write S_{nk} in place of S_k in Theorem 5, r_n in place of r, ℓ_n in place of ℓ. This is to emphasise the dependence of these quantities on n. Choosing ℓ_n so that (2.8) is satisfied, we have by the same argument as in Corollary 2 that the right hand side of (3.3) tends to 0 as $n \to \infty$, provided r_n is chosen appropriately. This is true for each B. Now apply Lemma 6, identifying Y_{nk} with S_{nk}. To verify (3.4) we have

$$\sum_k P\{S_{nk} > m\} \leq m^{-1} \sum_k E(S_{nk}) \leq m^{-1} C$$

which is independent of n, so (3.4) is satisfied. The result then follows from the conclusion of Lemma 6.

4. SOME FURTHER REMARKS

1. Hüsler (1986) considered only the case when the $\{X_i\}$ are drawn from a single sequence of random variables. Obviously our method applies equally well to triangular arrays. Perhaps that is true of Hüsler's method as well, but Hüsler did not mention the point.

2. An obvious application of Theorem 1 is to rates of convergence in dependent extreme value theory. This topic has been extensively developed for extreme values of i.i.d. random variables, but the only substantial contribution to the dependent case is the work of Rootzén (1983) on stationary Gaussian sequences.

3. The method is also applicable to the more general problem of Poisson convergences of point processes generated by high-level exceedances. For example, in the stationary case Leadbetter, Lindgren and Rootzén (1983, Section 5.7) give conditions under which the two-dimensional point process N_n with points at $(j/n, u_n^{-1}(X_j))$ $(j=1,\ldots,n)$, with some suitably defined functions u_n^{-1}, converges to a limiting process N which is homogeneous Poisson on $(0,1) \times (0,\infty)$. The same method as in Theorem 1 can be used to bound the total variation distance between the distributions of $N_n(B)$ and $N(B)$, for arbitrary measurable B.

4. A final, intriguing possibility is the use of this method to obtain improved approximations. In the independent case, the calculations leading to (1.5) show that

$$(4.1) \qquad E\{Wf(W) - \lambda f(W+1)\} = \sum_i p_i E\{f(W^{(i)}+1) - f(W+1)\}$$

$$= -\sum_i p_i E\{Y_i \Delta f(W^{(i)}+1)\}$$

$$= -\sum_i p_i^2 E\{\Delta f(W^{(i)}+1)\}$$

where $\Delta f(w) = f(w+1) - f(w)$. The right hand side of (4.1) can be approximated by

$$(4.2) \qquad \cdot \qquad - \sum_i p_i^2 E\{\Delta f(T+1)\}$$

where T has a Poisson distribution with mean λ. Chen (1975) shows that this procedure leads to an improved approximation for the distribution of W, with error of order $\sum p_i^3$ instead of $\sum p_i^2$. This idea has never been applied in the dependent case but the possibility exists, if one of the three terms in (1.5) dominates the other two, of doing further analysis along these lines.

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